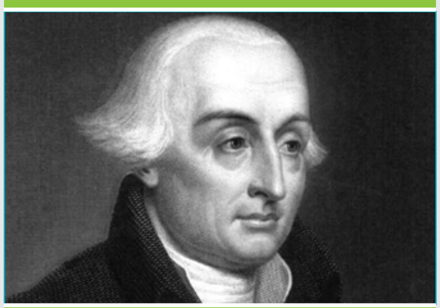
5 Numerical Methods



Joseph-Louis Lagrange
(25.01.1736 - 10.04.1813)

Introduction

Numerical Analysis is a branch of Mathematics which leads to approximate solution by repeated applications of four basic operations of Algebra. The knowledge of finite differences is essential for the study of Numerical Analysis.

Joseph-Louis Lagrange was an Italian mathematician and astronomer. he made significant contributions to the fields of analysis, number theory, and both classical and celestial mechanics.

Learning Objectives

After studying this chapter, students will be able to understand

- the finite differences
- how to find the polynomial using finite differences
- how to find the relations between the operators
- how to find the missing terms
- how to interpolate the values of a given series using Newton's interpolation formulae
- how to apply the Lagrange's interpolation formula

5.1 Finite Differences

Consider the arguments $x_0, x_1, x_2, \cdots, x_n$ and the entries $y_0, y_1, y_2, \cdots, y_n$. $y = f(x)$ be a function of x. Let us assume that the values of x are in increasing order and equally spaced with a space length h. Then the values of x may be taken to be $x_0, x_0+h, x_0+2h, \cdots, x_0+nh$ and the function assumes the values $f(x_0), f(x_0+h), f(x_0+2h), \cdots, f(x_0+nh)$. Here we study some of the finite differences of the function $y = f\left(x\right)$.

5.1.1 Forward Difference Operator, Backward Difference Operator and Shifting Operator

Forward Difference Operator (Δ):

Let $y = f(x)$ be a given function of x. Let $y_0, y_1, y_2, \cdots, y_n$ be the values of y at $x = x_0, x_1, x_2, \ldots, x_n$ respectively. Then $y_1 - y_0,\ y_2 - y_1,\ y_3 - y_2, \ldots, y_n - y_{n-1}$ are called the first (forward) differences of the function y. They are denoted by $\Delta y_0, \Delta y_1, \Delta y_2, \ldots, \Delta y_{n-1}$ respectively.

(i.e) $\Delta y_0 = y_1 - y_0, \Delta y_1 = y_2 - y_1, \Delta y_2 = y_3 - y_2, ..., \Delta y_{n-1} = y_n - y_{n-1}$

In general, $\Delta y_n = y_{n+1} - y_n,\ n = 0,1,2,3,...$

The symbol Δ is called the forward difference operator and pronounced as delta.

The forward difference operator Δ can also be defined as $\Delta f(x) = f(x+h) - f(x)$, h is the equal interval of spacing.

Proof of these properties are not included in our syllabus:

Properties of the operator Δ :

Property 1: If c is a constant then $\Delta c = 0$

Proof: Let $f(x) = c$

$\therefore f(x+h) = c$ (where 'h' is the interval of difference)

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta c = c - c = 0$$

Property 2: Δ is distributive i.e. $\Delta(f(x) + g(x)) = \Delta f(x) + \Delta g(x)$

Proof: $\Delta\left[f(x) + g(x)\right]$

$$= \left[f(x+h) + g(x+h)\right] - \left[f(x) + g(x)\right]$$

$$= f(x+h) + g(x+h) - f(x) - g(x)$$

$$= f(x+h) - f(x) + g(x+h) - g(x)$$

$$= \Delta f(x) + \Delta g(x)$$

Similarly we can show that $\Delta\left[f(x) - g(x)\right] = \Delta f(x) - \Delta g(x)$

In general, $\Delta\left[f_1(x) + f_2(x) ...... + f_n(x)\right] = \Delta f_1(x) + \Delta f_2(x) + ... + \Delta f_n(x)$

Property 3: If c is a constant then $\Delta c\, f(x) = c\,\Delta f(x)$

Proof: $\Delta[c\, f(x)] = c\, f(x+h) - c\, f(x)$

$$= c[f(x+h) - f(x)]$$

$$= c\,\Delta f(x)$$

Results without proof

1. If m and n are positive integers then $\Delta^m . \Delta^n\ f(x) = \Delta^{m+n} f(x)$
2. $\Delta\left[f(x)\ g(x)\right] = f(x)\Delta g(x) + g(x)\Delta f(x)$
3. $\Delta\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x).\ g(x+h).}$

The differences of the first differences denoted by $\Delta^2 y_0, \Delta^2 y_1, ..., \Delta^2 y_n$ are called second differences, where

$$\Delta^2 y_n = \Delta(\Delta y_n) = \Delta(y_{n+1} - y_n)$$

$$= \Delta y_{n+1} - \Delta y_n$$

$$\Delta^2 y_n = \Delta y_{n+1} - \Delta y_n,\ n = 0,1,2,...$$

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$$

$$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$$

Similarly the differences of second differences are called third differences.

$$\Delta^3 y_n = \Delta^2 y_{n+1} - \Delta^2 y_n,\ \text{n} = 0,1,2,...$$

In particular,

$$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$$

$$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$$

In general kth differences of yn is

$$\Delta^k y_n = \Delta^{k-1} y_{n+1} - \Delta^{k-1} y_n,\ \text{n} = 0,1,2,...$$

Note

$$\Delta^k f(x) = \Delta^{k-1} f(x+h) - \Delta^{k-1} f(x)$$

It is convenient to represent the above differences in a table as shown below.

Forward Difference Table for y :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0	y_0					
		Δy_0				
x_1	y_1		$\Delta^2 y_0$			
		Δy_1	ASS	$\Delta^3 y_0$		
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$	
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$	
		Δy_3		$\Delta^3 y_2$		
x_4	y_4		$\Delta^2 y_3$			
		Δy_4				
x_5	y_5					

The forward difference table for $f(x)$ is given below.

x	$f(x)$				
		$\Delta\ f(x)$			
$x+h$	$f(x+h)$		$\Delta^2\ f(x)$		
		$\Delta\ f(x+h)$		$\Delta^3 f(x)$	
$x+2h$	$f(x+2h)$		$\Delta^2\ f(x+h)$		$\Delta^4\ f(x)$
		$\Delta\ f(x+2h)$		$\Delta^3\ f(x+h)$	
$x+3h$	$f(x+3h)$		$\Delta^2\ f(x+2h)$		
		$\Delta\ f(x+3h)$			
$x+4h$	$f(x+4h)$				

Backward Difference operator (∇):

Let $y = f(x)$ be a given function of x. Let $y_0, y_1, ..., y_n$ be the values of y at $x = x_0, x_1, x_2, ..., x_n$ respectively. Then

$$y_1 - y_0 = \nabla y_1$$

$$y_2 - y_1 = \nabla y_2$$

$$y_n - y_{n-1} = \nabla y_n$$

are called the first(backward) differences.

The operator ∇ is called backward difference operator and pronounced as nepla.

Second (backward) differences:

$$\nabla^2 y_n = \nabla y_n - \nabla y_{n+1},\quad n = 1,2,3,\ldots$$

Third (backward) differences:

$$\nabla^3 y_n = \nabla^2 y_n - \nabla^2 y_{n-1} \quad n = 1,2,3,\ldots$$

In general, k^{th} (backward) differences:

$$\nabla^k y_n = \nabla^{k-1} y_n - \nabla^{k-1} y_{n-1} \quad n = 1,2,3,\ldots$$

Backward difference table:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
x_0	y_0				
		∇y_1			
x_1	y_1		$\nabla^2 y_2$		
		∇y_2		$\nabla^3 y_3$	
x_2	y_2		$\nabla^2 y_3$		$\nabla^4 y_4$
		∇y_3		$\nabla^3 y_4$	
x_3	y_3		$\nabla^2 y_4$		
		∇y_4			
x_4	y_4				

Backward differences can also be defined as follows.

$$\nabla f(x) = f(x) - f(x-h)$$

First differences:

$$\nabla f(x+h) = f(x+h) - f(x)$$

$$\nabla f(x+2h) = f(x+2h) - f(x+h),$$

h is the interval of spacing.

Second differences:

$$\nabla^2 f(x+h) = \nabla(\nabla f(x+h) = \nabla(f(x+h) - f(x))$$
$$= \nabla f(x+h) - \nabla f(x)$$

$$\nabla^2 f(x+2h) = \nabla f(x+2h) - \nabla f(x+h)$$

Third differences:

$$\nabla^3 f(x+h) = \nabla^2 f(x+h) - \nabla^2 f(x)$$

$$\nabla^3 f(x+2h) = \nabla^2 f(x+2h) - \nabla^2 f(x+h)$$

Here we note that,

$$\nabla f(x+h) = f(x+h) - f(x) = \Delta f(x)$$

$$\nabla f(x+2h) = f(x+2h) - f(x+h)$$
$$= \Delta f(x+h)$$

$$\nabla^2 f(x+2h) = \nabla f(x+2h) - \nabla f(x+h)$$
$$= \Delta f(x+h) - \Delta f(x)$$
$$= \Delta^2 f(x)$$

In general, $\nabla^n f(x+nh) = \Delta^n f(x)$

Shifting operator (E):

Let $y = f(x)$ be a given function of x and $x_0, x_0+h, x_0+2h, x_0+3h, \ldots, x_0+nh$ be the consecutive values of x. Then the operator E is defined as

$$E\left[f(x_0)\right] = f(x_0+h)$$

E is called the shifting operator . It is also called the displacement operator.

$$E\left[f\left(x_0+h\right)\right] = f(x_0+2h),$$
$$E[f(x_0+2h)] = f(x_0+3h),\ldots,$$
$$E[f(x_0+(n-1)h)] = f(x_0+nh)$$

> $E\left[f(x)\right] = f(x+h)$, h is the (equal) interval of spacing

$E^2 f(x)$ means that the operator E is applied twice on $f(x)$

(i.e) $E^2\ f(x) = E\left[E\, f(x)\right]$

$= E[f(x+h) = f(x+2h)$

In general ,

> $E^n f(x) = f(x+nh)$ and
> $E^{-n} f(x) = f(x-nh)$

Properties of the operator E:

1. $E\left[f_1(x) + f_2(x) + \ldots.. + f_n(x)\right] = Ef_1(x) +$ $Ef_2(x) \ldots. + \ldots. + E\left[f_n(x)\right]$
2. $E\left[c\, f(x)\right] = cE\left[f(x)\right]$ c is a constant

3. $E^m\left[E^n f(x)\right]=E^n\left(E^m f(x)\right)=E^{m+n} f(x)$

4. If 'n' is a positive integer, then $E^n\left[E^{-n}\left(f(x)\right)\right]=f(x)$.

Note

Let $y = f(x)$ be given function of x. Let $y_0, y_1, y_2, \cdots, y_n$ be the values of y at $x = x_0, x_1, x_2, ..., x_n$. Then E can also be defined as

$Ey_0 = y_1, \quad Ey_1 = y_2, ..., Ey_{n-1} = y_n$

$E\left[Ey_0\right] = E(y_1) = y_2$ and

In general $E^n y_0 = y_n$

Relations between the operators Δ, ∇ and E:

1. $\Delta \equiv E - 1$

Proof: From the definition of Δ we know that

$\Delta f(x) = f(x+h) - f(x)$

and $E[f(x)] = f(x+h)$

where h is the interval of difference.

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta f(x) = Ef(x) - f(x)$$

$$\Rightarrow \Delta f(x) = (E-1) f(x)$$

$$\Delta \equiv E - 1$$

$$\therefore \quad E \equiv 1 + \Delta$$

2. $E\Delta \equiv \Delta E$

Proof:

$$\begin{aligned} E(\Delta f(x)) &= E\left[f(x+h) - f(x)\right] \\ &= E f(x+h) - E f(x) \\ &= f(x+2h) - f(x+h) \\ &= \Delta f(x+h) \\ &= \Delta E f(x) \end{aligned}$$

$$\therefore E\Delta \equiv \Delta E$$

3. $\nabla \equiv \dfrac{E-1}{E}$

Proof:

$$\begin{aligned} \nabla f(x) &= f(x) - f(x-h) \\ &= f(x) - E^{-1} f(x) \\ &= (1 - E^{-1}) f(x) \\ \Rightarrow \nabla &\equiv 1 - E^{-1} \\ \text{i.e., } \nabla &\equiv 1 - \frac{1}{E} \\ \text{Hence, } \nabla &\equiv \frac{E-1}{E} \end{aligned}$$

DO YOU KNOW?

(i) $(1+\Delta)(1-\nabla) = 1$

(ii) $\Delta\nabla \equiv \Delta - \nabla$

(iii) $\nabla \equiv E^{-1}\Delta$

Example 5.1

Construct a forward difference table for the following data

x	0	10	20	30
y	0	0.174	0.347	0.518

Solution:

The Forward difference table is given below:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	0			
		0.174		
10	0.174		–0.001	
		0.173		–0.001
20	0.347		–0.002	
		0.171		
30	0.518			

Example 5.2

Construct a forward difference table for $y = f(x) = x^3 + 2x + 1$ for $x = 1,2,3,4,5$.

Solution:

$y = f(x) = x^3 + 2x + 1$ for $x = 1,2,3,4,5$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	4				
		9			
2	13		12		
		21		6	
3	34		18		0
		39		6	
4	73		24		
		63			
5	136				

Example 5.3

By constructing a difference table and using the second order differences as constant, find the sixth term of the series 8,12,19,29,42…

Solution:

Let k be the sixth term of the series in the difference table.

First we find the forward differences.

x	y	Δy	$\Delta^2 y$
1	8		
		4	
2	12		3
		7	
3	19		3
		10	
4	29		3
		13	
5	42		k–55
		k–42	
6	k		

Given that the second differences are constant

$\therefore \quad k - 55 = 3$

$k = 58$

$\therefore$ the sixth term of the series is 58

Example 5.4

Find (i) Δe^{ax} (ii) $\Delta^2 e^x$ (iii) $\Delta \log x$

Solution:

(i) $\Delta e^{ax} = e^{a(x+h)} - e^{ax}$

$= e^{ax}.e^{ah} - e^{ax} \quad \left[\because a^{m+n} = a^m.a^n\right]$

$= e^{ax}\left[e^{ah} - 1\right]$

(ii) $\Delta^2 e^x = \Delta.\left[\Delta e^x\right]$

$= \Delta\left[e^{x+h} - e^x\right]$

$= \Delta\left[e^x e^h - e^x\right]$

$= \Delta e^x\left[e^h - 1\right]$

$= \left(e^h - 1\right)\Delta e^x$

$= \left(e^h - 1\right).\left(e^h - 1\right).e^x$

$= \left(e^h - 1\right)^2.e^x$

(iii) $\Delta \log x = \log(x+h) - \log x$

$= \log\dfrac{x+h}{x}$

$= \log\left(\dfrac{x}{x} + \dfrac{h}{x}\right)$

$= \log\left(1 + \dfrac{h}{x}\right)$

Example 5.5

Evaluate $\Delta\left[\dfrac{5x+12}{x^2+5x+6}\right]$ by taking '1' as the interval of differencing.

Solution:

$\Delta\left[\dfrac{5x+12}{x^2+5x+6}\right]$

By Partial fraction method

$\dfrac{5x+12}{x^2+5x+6} = \dfrac{A}{x+3} + \dfrac{B}{x+2}$

$A = \dfrac{5x+12}{x+2}\left[x = -3\right]$

$= \dfrac{-15+12}{-1} = \dfrac{-3}{-1} = 3$

$$B = \frac{5x+12}{x+3}\left[x=-2\right] = \frac{2}{1} = 2$$

$$\frac{5x+12}{x^2+5x+6} = \left[\frac{3}{x+3}+\frac{2}{x+2}\right]$$

$$\Delta\left[\frac{5x+12}{x^2+5x+6}\right] = \Delta\left[\frac{3}{x+3}+\frac{2}{x+2}\right]$$

$$=\left[\frac{3}{x+1+3}-\frac{3}{x+3}\right]+\left\{\frac{2}{x+1+2}-\frac{2}{x+2}\right\}$$

$$=3\left[\frac{1}{x+4}-\frac{1}{x+3}\right]+2\left[\frac{1}{x+3}-\frac{1}{x+2}\right]$$

$$=\left[\frac{-3}{(x+4)(x+3)}-\frac{2}{(x+3)(x+2)}\right]$$

$$=\frac{-5x-14}{(x+2)(x+3)(x+4)}$$

Example 5.6

Evaluate $\Delta^2\left(\frac{1}{x}\right)$ by taking '1' as the interval of differencing.

Solution:

$$\Delta^2\left(\frac{1}{x}\right) = \Delta\left(\Delta\left(\frac{1}{x}\right)\right)$$

Now $\Delta\left[\frac{1}{x}\right] = \frac{1}{x+1}-\frac{1}{x}$

$$\Delta^2\left(\frac{1}{x}\right) = \Delta\left(\frac{1}{1+x}-\frac{1}{x}\right)$$

$$= \Delta\left(\frac{1}{1+x}\right)-\Delta\left(\frac{1}{x}\right)$$

Similarly $\Delta^2\left(\frac{1}{x}\right) = \frac{2}{x(x+1)(x+2)}$

DO YOU KNOW?

$$\Delta^n\left(\frac{1}{x}\right) = \frac{(-1)^n n!}{x(x+1)(x+2)......(x+n)}$$

Example 5.7

Prove that $f(4)=f(3)+\Delta f(2)+\Delta^2 f(1)+\Delta^3 f(1)$ taking '1' as the interval of differencing.

Solution:

We know that $f(4)-f(3)=\Delta f(3)$

$$f(4)-f(3) = \Delta f(3)$$

$$=\Delta\left[f(2)+\Delta f(2)\right] \quad \because \left[f(3)-f(2)=\Delta f(2)\right]$$

$$=\Delta f(2)+\Delta^2 f(2)$$

$$=\Delta f(2)+\Delta^2\left[f(1)+\Delta f(1)\right]$$

$$\therefore f(4)=f(3)+\Delta f(2)+\Delta^2 f(1)+\Delta^3 f(1).$$

Example 5.8

Given $U_0=1, U_1=11, U_2=21, U_3=28$ and $U_4=29$ find $\Delta^4 U_0$

Solution :

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$\Delta^4 U_0 = (E-1)^4 U_0$$

$$=\left(E^4-4E^3+6E^2-4E+1\right)U_0$$

$$=E^4U_0-4E^3U_0+6E^2U_0-4EU_0+U_0$$

$$=U_4-4U_3+6U_2-4U_1+U_0$$

$$=29-4(28)+6(21)-4(11)+1.$$

$$=156-156=0$$

Example 5.9

Given $y_3=2, y_4=-6, y_5=8, y_6=9$ and $y_7=17$ Calculate $\Delta^4 y_3$

Solution :

Given $y_3=2, y_4=-6, y_5=8, y_6=9$ and $y_7=17$

$$\Delta^4 y_3 = (E-1)^4 y_3$$
$$= (E^4 - 4E^3 + 6E^2 - 4E + 1) y_3$$
$$= E^4 y_3 - 4E^3 y_3 + 6E^2 y_3 - 4E y_3 + y_3$$
$$= y_7 - 4y_6 + 6y_5 - 4y_4 + y_3$$
$$= 17 - 4(9) + 6(8) - 4(-6) + 2$$
$$= 17 - 36 + 48 + 24 + 2 = 55$$

5.1.2 Finding the missing terms

Using the difference operators and shifting operator we can able to find the missing terms.

Example 5.10

From the following table find the missing value

x	2	3	4	5	6
$f(x)$	45.0	49.2	54.1	-	67.4

Solution:

Since only four values of $f(x)$ are given, the polynomial which fits the data is of degree three. Hence fourth differences are zeros.

(ie) $\Delta^4 y_0 = 0, \therefore (E-1)^4 y_0 = 0$

$$(E^4 - 4E^3 + 6E^2 - 4E + 1) y_0 = 0$$
$$E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0 = 0$$
$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$
$$67.4 - 4y_3 + 6(54.1) - 4(49.2) + 45 = 0$$
$$240.2 = 4y_3 \qquad \therefore y_3 = 60.05$$

Example 5.11

Estimate the production for 1964 and 1966 from the following data

Year	1961	1962	1963	1964	1965	1966	1967
Production	200	220	260	-	350	-	430

Solution:

Since five values are given, the polynomial which fits the data is of degree four.

Hence $\Delta^5 y_k = 0$ (ie) $(E-1)^5 y_k = 0$

i.e., $(E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1) y_k = 0$

$$E^5 y_k - 5E^4 y_k + 10E^3 y_k - 10E^2 y_k + 5E y_k - y_k = 0 \quad (1)$$

Put $k = 0$ in (1)

$$E^5 y_0 - 5E^4 y_0 + 10E^3 y_0 - 10E^2 y_0 + 5E y_0 - y_0 = 0$$
$$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$$
$$y_5 - 5(350) + 10y_3 - 10(260) + 5(220) - 200 = 0$$
$$y_5 + 10y_3 = 3450 \quad (2)$$

Put $k = 1$ in (1)

$$E^5 y_1 - 5E^4 y_1 + 10E^3 y_1 - 10E^3 y_1 - y_1 = 0$$
$$y_6 - 5y_5 + 10y_4 - 10y_3 - y_1 = 0$$
$$430 - 5y_5 + 10(350) - 10y_3 + 5(260) - 220 = 0$$
$$5y_5 + 10y_3 = 5010 \quad (3)$$

(3) – (2) ⇒ $4y_5 = 1560$

$$y_5 = 390$$

From (1) $390 + 10y_3 = 3450$

$$10y_3 = 3450 - 390$$
$$y_3 \cong 306$$

					1					
				1		2		1		
			1		3		3		1	
		1		4		6		4		1
	1		5		10		10		5		1

Exercise 5.1

1. Evaluate $\Delta(\log ax)$.
2. If $y = x^3 - x^2 + x - 1$ calculate the values of y for $x = 0,1,2,3,4,5$ and form the forward differences table.
3. If $h = 1$ then prove that $(E^{-1}\Delta)x^3 = 3x^2 - 3x + 1$.

4. If $f(x)=x^2+3x$ than show that $\Delta f(x)=2x+4$.

5. Evaluate $\Delta\left[\frac{1}{(x+1)(x+2)}\right]$ by taking '1' as the interval of differencing

6. Find the missing entry in the following table

x	0	1	2	3	4
y_x	1	3	9	-	81

7. Following are the population of a district

Year (x)	1881	1891	1901	1911	1921	1931
Population (y) Thousands	363	391	421	-	467	501

Find the population of the year 1911

8. Find the missing entries from the following.

x	0	1	2	3	4	5
$y=f(x)$	0	-	8	15	-	35

5.2 Interpolation

Consider the profit of a manufacturing company in various years as given below:

Year (x)	1986	1987	1988	1990	1991	1992
Profit (Rs. in lakhs)	25	29	24	30	32	31

The profit for the year 1989 is not available .To estimate the profit for 1989 we use the technique called interpolation. Let x denote the year and y denote the profit. The independent variable x is called the argument and the dependent variable y is called the entry. If y is to be estimated for the value of x between two extreme points in a set of values, it is called interpolation.

DO YOU KNOW?

If y is to be estimated for the values of x which lies outside the given set of the values of it, is called extrapolation.

5.2.1 Methods of interpolation

There are two methods for interpolation. One is Graphical method and the other one is algebraic method.

5.2.2 Graphical method

We are given the 'n' values of x and the corresponding values of y for given function $y=f(x)$. we plot these n observed points $(x_i, y_i), i=1,2,3....$ and draw a free hand curve passing through these plotted points. From the graph so obtained, we can find out the value of y for any intermediate value of x. There is one drawback in the graphic method which states that the value of y obtained is the estimated value of y. The estimated value of y differs from the actual value of y.

Example 5.12

Using graphic method, find the value of y when x = 38 from the following data:

x	10	20	30	40	50	60
y	63	55	44	34	29	22

Solution:

Steps in Graphic method:

Take a suitable scale for the values of x and y, and plot the various points on the graph paper for given values of x and y.

Draw a suitable curve passing through the plotted points.

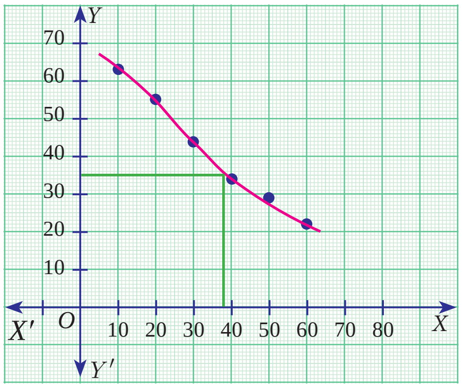


Fig. 5.1

Find the point corresponding to the value x = 38 on the curve and then read the corresponding value of y on the y- axis, which will be the required interpolated value.

From the graph in Fig. 5.1 we find that for $x = 38$, the value of y is equal to 35.

5.2.3 Algebraic method

Newton's Gregory forward interpolation formula (or) Newton's forward interpolation formula (for equal intervals).

DO YOU KNOW?

The first two terms will give the linear interpolation and the first three terms will give a parabolic interpolation and so on.

Let $y = f(x)$ denote a polynomial of degree n which takes $(n+1)$ values. Let them be $y_0, y_1, y_2, \dots y_n$ corresponding to the values $x_0, x_1, \dots x_n$ respectively.

The values of x $(x_0, x_1, x_2, \dots x_n)$ are at equidistant.

(i.e.) $x_1 = x_0 + h,\ x_2 = x_0 + 2h,\ x_3 = x_0 + 3h, \dots, x_n = x_0 + nh$

Then the value of $f(x)$ at $x = x_0+nh$ is given by

$$f(x_0 + nh) = f(x_0) + \frac{n}{1!}\Delta f(x_0) + \frac{n(n-1)}{2!}\Delta^2 f(x_0) + \frac{n(n-1)(n-2)}{3!}\Delta^3 f(x_0) + \dots$$

$$\text{(or)}\quad y_{(x=x_0+nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \dots \text{ where } n = \frac{x-x_0}{h}$$

Newton's Gregory backward interpolation Formula.

Note

Newton's forward interpolation formula is used when the value of y is required near the beginning of the table.

In general Newton's forward interpolation formula not to be used when the value of y is required near the end of the table. For this we use another formula, called Newton's Gregory backward interpolation formula.

Then the value of $f(x)$ at $x = x_n + nh$ is given by

$$f(x_n + nh) = f(x_n) + \frac{n}{1!}\nabla f(x_n) + \frac{n(n+1)}{2!}\nabla^2 f(x_n) + \frac{n(n+1)(n+2)}{3!}\nabla^3 f(x_n) + \dots$$

$$\text{(or)}\quad y_{(x=x_n+nh)} = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \dots \text{ when } n = \frac{x-x_n}{h}$$

Note

Newton's backward interpolation formula is used when the value of y is required near the end of the table.

Example 5.13

Using Newton's formula for interpolation estimate the population for the year 1905 from the table:

Year	1891	1901	1911	1921	1931
Population	98,752	1,32,285	1,68,076	1,95,670	2,46,050

Solution

To find the population for the year 1905 (i.e) the value of y at $x = 1905$.

Since the value of y is required near the beginning of the table, we use the Newton's forward interpolation formula.

$$y_{(x=x_0+nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \dots$$

To find y at $x = 1905$ $\therefore x_0 + nh = 1905$, $x_0 = 1891, h = 10$.

$1891 + n(10) = 1905 \quad \Rightarrow n = 1.4$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	98,752				
		33,533			
1901	1,32,285		2,258		
		35,791		−10,435	
1911	1,68,076		-8,177		41,358
		27,614		30,293	
1921	1,95,690		22,746		
		50,360			
1931	2,46,050				

$$y_{(x=1905)} = 98,752 + (1.4)(33533) + \frac{(1.4)(0.4)}{2}(2258) + \frac{(1.4)(0.4)(-0.6)}{6}(-10435) + \frac{(1.4)(0.6)(-0.6)(-1.6)}{24}(41358)$$

= 98752 + 46946.2 + 632.24 + 584.36 + 1389.63

= 148304.43

≅ 1,48,304

Example 5.14

The values of $y = f(x)$ for $x = 0, 1, 2, \ldots, 6$ are given by

x	0	1	2	3	4	5	6
y	2	4	10	16	20	24	38

Estimate the value of y (3.2) using forward interpolation formula by choosing the four values that will give the best approximation.

Solution:

Since we apply the forward interpolation formula,last four values of $f(x)$ are taken into consideration (Take the values from x = 3).

The forward interpolation formula is

$$y_{(x=x_0+nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \ldots$$

$x_0 + nh = 3.2, x_0 = 3, h = 1$

$\therefore \quad n = \frac{1}{5}$

The difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
3	16			
		4		
4	20		0	
		4		10
5	24		10	
		14		
6	38			

$$y_{(x=3.2)} = 16 + \frac{1}{5}(4) + \frac{\frac{1}{5}\left(\frac{-4}{5}\right)}{2}(0) + \frac{\frac{1}{5}\left(\frac{-4}{5}\right)\left(\frac{-9}{5}\right)}{6} \times 10$$

$= 16 + 0.8 + 0 + 0.48 = 17.28$

Example 5.15

From the following table find the number of students who obtained marks less than 45.

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	31	42	51	35	31

Solution:

Let x be the marks and y be the number of students.

By converting the given series into cumulative frequency distribution, the difference table is as follows.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Less than 40	31				
		42			
50	73		9		
		51		−25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

$$y_{(x=x_0+nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \ldots$$

To find y at $x = 45$ $\therefore x_0 + nh = 45$, $x_0 = 40, h = 10$ $\Rightarrow n = \frac{1}{2}$

$$y_{(x=45)} = 31 + \frac{1}{2} \times 42 + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)}{2}(9) + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{6} \times (-25) + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right)}{24} \times (37)$$

$$= 31 + 21 - \frac{9}{8} - \frac{25}{16} - \frac{37 \times 15}{384}$$

$= 47.867 \cong 48$

Example 5.16

Using appropriate interpolation formula find the number of students whose weight is between 60 and 70 from the data given below:

Weight in lbs	0-40	40-60	60-80	80-100	100-120
No. of Students	250	120	100	70	50

Solution:

Let x be the weight and y be the number of students.

Difference table of cumulative frequencies are given below.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40	250				
		120			
60	370		–20		
		100		–10	
80	470		–30		20
		70		10	
100	540		–20		
		50			
120	590				

Let us calculate the number of students whose weight is below 70. For this we use forward difference formula.

$$y_{(x=x_0+nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \ldots$$

To find y at $x = 70$ $\therefore$ $x_0 + nh = 70$, $x_0 = 40, h = 20$

$40 + n(20) = 70$ $\Rightarrow n = 1.5$

$$\therefore\ y_{(x=70)} = 250 + 1.5(120) + \frac{(1.5)(0.5)}{2!}(-20) + \frac{(1.5)(0.5)(-0.5)}{3!}(-10) + \frac{(1.5)(0.5)(-0.5)(-1.5)}{4!}(20)$$

$= 250 + 180 - 7.5 + 0.625 + 0.46875$

$= 423.59$

$\cong 424$

Number of students whose weight is between 60 and 70 $= y(70) - y(60) = 424 - 370 = 54$

Example 5.17

The population of a certain town is as follows:

Year : x	1941	1951	1961	1971	1981	1991
Population in lakhs : y	20	24	29	36	46	51

Using appropriate interpolation formula, estimate the population during the period 1946.

Solution:

x	1941	1951	1961	1971	1981	1991
y	20	24	29	36	46	51

Here we find the population for year1946. (i.e) the value of y at x=1946. Since the value of y is required near the beginning of the table, we use the Newton's forward interpolation formula.

$$y_{(x=x_0+nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + ...$$

To find y at $x = 1946$

$\therefore\ x_0 + nh = 1946,\ x_0 = 1941,\ h = 10$

$1941 + n(10) = 1946 \Rightarrow n = 0.5$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1941	20					
		4				
1951	24		1			
		5		1		
1961	29		2		0	
		7		1		−9
1971	36		3		−9	
		10		−8		
1981	46		−5			
		5				
1991	51					

$$y_{(x=1946)} = 20 + \frac{0.5}{1!}(4) + \frac{0.5(0.5-1)}{2!}(1) + \frac{0.5(0.5-1)(0.5-2)}{3!}(1) + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4!}(0) + \frac{0.5(0.5-1)(0.5-2)(0.5-3)(0.5-4)}{5!}(-9)$$

$$= 20 + 2 - 0.125 + 0.0625 - 0.24609$$

$= 21.69$ lakhs

Example 5.18

The following data are taken from the steam table.

Tempreture C^0	140	150	160	170	180
Pressure kg f/cm^2	3.685	4.854	6.302	8.076	10.225

Find the pressure at temperature $t = 175^0$

Solution:

Since the pressure required is at the end of the table, we apply Backward interpolation formula. Let temperature be x and the pressure be y.

$$y_{(x=x_n+nh)} = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + ...$$

To find y at $x = 175$

$\therefore x_n + nh = 175,\ x_n = 180, h = 10 \Rightarrow n = -0.5$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
140	3.685				
		1.169			
150	4.854		0.279		
		1.448		0.047	
160	6.032		0.326		0.002
		1.774		0.049	
170	8.076		0.375		
		2.149			
180	10.225				

$$y_{(x=175)} = 10.225 + (-0.5)(2.149) + \frac{(-0.5)(0-5)}{2!}(0.375) + \frac{(-0.5)(0-5)(1.5)}{3!}(0.049) + \frac{(-0-5)(0.5)(1.5)(2.5)}{4!}(0.002)$$

$$= 10.225 - 1.0745 - 0.046875 - 0.0030625 - 0.000078125$$

$= 9.10048438 = 9.1$

Example 5.19

Calculate the value of y when $x = 7.5$ from the table given below:

x	1	2	3	4	5	6	7	8
y	1	8	27	64	125	216	343	512

Solution:

Since the required value is at the end of the table, apply backward interpolation formula.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1	1				
		7			
2	8		12		
		19		6	
3	27		18		0
		37		6	
4	64		24		0
		61		6	
5	125		30		0
		91		6	
6	216		36		0
		127		6	
7	343		42		
		169			
8	512				

$$y_{(x=x_n+nh)} = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \dots$$

To find y at $x = 7.5 \therefore x_n + nh = 7.5$, $x_n = 8, h = 1 \Rightarrow n = -0.5$

$$y_{(x=7.5)} = 512 + \frac{-0.5}{1!}169 + \frac{-0.5(-0.5+1)}{2!}42 + \frac{-0.5(-0.5+1)(-0.5+2)}{3!}6$$

$= 421.88$

Example 5.20

From the following table of half- yearly premium for policies maturing at different ages. Estimate the premium for policies maturing at the age of 63.

Age	45	50	55	60	65
Premium	114.84	96.16	83.32	74.48	68.48

Solution:

Let age $= x$ and premium $= y$

To find y at $x = 63$. So apply Newton's backward interpolation formula.

$$y_{(x=x_n+nh)} = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \dots$$

To find y at $x = 63$ $\therefore x_n + nh = 63$, $x_n = 65, h = 5 \Rightarrow n = -\frac{2}{5}$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
45	114.84				
		-18.68			
50	96.16		5.84		
		-12.84		-1.84	
55	83.32		4		0.68
		-8.84		-1.16	
60	74.48		2.84		
		-6			
65	68.48				

$$y_{(x=63)} = 68.48 + \frac{\frac{-2}{5}}{1!}(-6) + \frac{\frac{-2}{5}\left(\frac{-2}{5}+1\right)}{2!}2.84 + \frac{\frac{-2}{5}\left(\frac{-2}{5}+1\right)\left(\frac{-2}{5}+2\right)}{3!}(-1.16) + \frac{\frac{-2}{5}\left(\frac{-2}{5}+1\right)\left(\frac{-2}{5}+2\right)\left(\frac{-2}{5}+3\right)}{3!}(0.68)$$

$= 68.48 + 2.4 - 0.3408 + 0.07424 - 0 - 0.028288$

$y(63) = 70.437.$

Example 5.21

Find a polynomial of degree two which takes the values.

x	0	1	2	3	4	5	6	7
y	1	2	4	7	11	16	22	29

Solution:

We will use Newton's backward interpolation formula to find the polynomial.

$$y_{(x=x_n+nh)} = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \dots$$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
0	1			
		1		
1	2		1	
		2		0
2	4		1	
		3		0
3	7		1	
		4		0
4	11		1	
		5		0
5	16		1	
		6		0
6	22		1	
		7		
7	29			

To find y in terms of x $\therefore x_n + nh = x$, $x_n = 7, h = 1 \Rightarrow n = x - 7$.

$$y_{(x)} = 29 + (x-7)(7) + \frac{(x-7)(x-6)}{2}(1)$$

$$= 29 + 7x - 49 + \frac{1}{2}\left(x^2 - 13x + 42\right)$$

$$= \frac{1}{2}\left[58 + 14x - 98 + x^2 - 13x + 42\right]$$

$$= \frac{1}{2}\left[x^2 + x + 2\right]$$

5.2.4 Lagrange's interpolation formula

The Newton's forward and backward interpolation formulae can be used only when the values of x are at equidistant. If the values of x are at equidistant or not at equidistant, we use Lagrange's interpolation formula.

Let $y = f(x)$ be a function such that $f(x)$ takes the values $y_0, y_1, y_2, \ldots\ldots, y_n$ corresponding to $x = x_0, x_1, x_2 \ldots, x_n$ That is $y_i = f(x_i), i = 0, 1, 2, \ldots, n$. Now, there are $(n+1)$ paired values $(x_i, y_i), i = 0, 1, 2, \ldots, n$ and hence $f(x)$ can be represented by a polynomial function of degree n in x.

Then the Lagrange's formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}y_1 + \ldots + \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})}y_n$$

Example 5.22

Using Lagrange's interpolation formula find y(10) from the following table:

x	5	6	9	11
y	12	13	14	16

Solution:

Here the intervals are unequal. By Lagrange's interpolation formula we have

$$x_0 = 5, x_1 = 6, x_2 = 9, x_3 = 11$$

$$y_0 = 12, y_1 = 13, y_2 = 14, y_3 = 16$$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$$

$$= \frac{(x-6)(x-9)(x-11)}{(5-6)(5-6)(5-11)}(12) + \frac{(x-5)(x-9)(x-11)}{(6-5)(6-9)(6-9)}(13) + \frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)(9-11)}(14) + \frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)}(16)$$

Put $x = 10$

$$y(10) = f(10) = \frac{4(1)(-1)}{(-1)(-4)(-6)}(12) + \frac{(5)(1)(-1)}{(1)(-3)(-5)}$$

$$(13) + \frac{5\,(4)(-1)}{4(3)(-2)}(14) + \frac{(5)(4)(1)}{6(5)(2)}(16)$$

$$= \frac{1}{6}(12) - \frac{13}{3} + \frac{5(14)}{3\times 2} + \frac{4\times 16}{12}$$

$= 14.6663$

Exercise 5.2

1. Using graphic method, find the value of y when x = 48 from the following data:

x	40	50	60	70
y	6.2	7.2	9.1	12

2. The following data relates to indirect labour expenses and the level of output

Months	Jan	Feb	Mar
Units of output	200	300	400
Indirect labour expenses (Rs)	2500	2800	3100

Months	Apr	May	June
Units of output	640	540	580
Indirect labour expenses (Rs)	3820	3220	3640

Estimate the expenses at a level of output of 350 units, by using graphic method.

3. Using Newton's forward interpolation formula find the cubic polynomial.

x	0	1	2	3
$f(x)$	1	2	1	10

4. The population of a city in a censes taken once in 10 years is given below. Estimate the population in the year 1955.

Year	1951	1961	1971	1981
Population in lakhs	35	42	58	84

5. In an examination the number of candidates who secured marks between certain interval were as follows:

Marks	0-19	20-39	40-59	60-79	80-99
No. of candidates	41	62	65	50	17

Estimate the number of candidates whose marks are lessthan 70.

6. Find the value of $f(x)$ when $x = 32$ from the following table:

x	30	35	40	45	50
$f(x)$	15.9	14.9	14.1	13.3	12.5

7. The following data gives the melting point of a alloy of lead and zinc where 't' is the temperature in degree c and P is the percentage of lead in the alloy.

P	40	50	60	70	80	90
T	180	204	226	250	276	304

Find the melting point of the alloy containing 84 percent lead.

8. Find $f(2.8)$ from the following table:

x	0	1	2	3
$f(x)$	1	2	11	34

9. Using interpolation estimate the output of a factory in 1986 from the following data.

Year	1974	1978	1982	1990
Output in 1000 tones	25	60	80	170

10. Use Lagrange's formula and estimate from the following data the number of workers getting income not exceeding Rs. 26 per month.

Income not exceeding (₹)	15	25	30	35
No. of workers	36	40	45	48

11. Using interpolation estimate the business done in 1985 from the following data

Year	1982	1983	1984	1986
Business done (in lakhs)	150	235	365	525

12. Using interpolation, find the value of $f(x)$ when $x = 15$

x	3	7	11	19
$f(x)$	42	43	47	60

Exercise 5.3

Choose the correct Answer

1. $\Delta^2 y_0 =$

   (a) $y_2 - 2y_1 + y_0$ (b) $y_2 + 2y_1 - y_0$

   (c) $y_2 + 2y_1 + y_0$ (d) $y_2 + y_1 + 2y_0$

2. $\Delta f(x) =$

   (a) $f(x+h)$ (b) $f(x) - f(x+h)$

   (c) $f(x+h) - f(x)$ (d) $f(x) - f(x-h)$

3. $E \equiv$

   (a) $1+\Delta$

   (b) $1-\Delta$

   (c) $1+\nabla$

   (d) $1-\nabla$

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4. If h=1, then $\Delta(x^2) =$

   (a) $2x$ (b) $2x-1$

   (c) $2x+1$ (d) 1

5. If c is a constant then $\Delta c =$

   (a) c (b) Δ

   (c) Δ^2 (d) 0

6. If m and n are positive integers then $\Delta^m \Delta^n f(x) =$

   (a) $\Delta^{m+n} f(x)$ (b) $\Delta^m f(x)$

   (c) $\Delta^n f(x)$ (d) $\Delta^{m-n} f(x)$

7. If 'n' is a positive integer $\Delta^n\left[\Delta^{-n} f(x)\right]$

   (a) $f(2x)$ (b) $f(x+h)$

   (c) $f(x)$ (d) $\Delta f(x)$

8. $E f(x) =$

   (a) $f(x-h)$ (b) $f(x)$

   (c) $f(x+h)$ (d) $f(x+2h)$

9. $\nabla \equiv$

   (a) 1+E (b) 1-E

   (c) $1-E^{-1}$ (d) $1+E^{-1}$

10. $\nabla f(a) =$

    (a) $f(a) + f(a-h)$

    (b) $f(a) - f(a+h)$

    (c) $f(a) - f(a-h)$

    (d) $f(a)$

11. For the given points (x_0, y_0) and (x_1, y_1) the Lagrange's formula is

    (a) $y(x) = \dfrac{x - x_1}{x_0 - x_1} y_0 + \dfrac{x - x_0}{x_1 - x_0} y_1$

    (b) $y(x) = \dfrac{x_1 - x}{x_0 - x_1} y_0 + \dfrac{x - x_0}{x_1 - x_0} y_1$

    (c) $y(x) = \dfrac{x - x_1}{x_0 - x_1} y_1 + \dfrac{x - x_0}{x_1 - x_0} y_0$

    (d) $y(x) = \dfrac{x_1 - x}{x_0 - x_1} y_1 + \dfrac{x - x_0}{x_1 - x_0} y_0$

12. Lagrange's interpolation formula can be used for

    (a) equal intervals only

    (b) unequal intervals only

    (c) both equal and unequal intervals

    (d) none of these.

13. If $f(x) = x^2 + 2x + 2$ and the interval of differencing is unity then $\Delta f(x)$

    (a) $2x-3$ (b) $2x+3$

    (c) $x+3$ (d) $x-3$

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14. For the given data find the value of $\Delta^3 y_0$ is

x	5	6	9	11
y	12	13	15	18

(a) 1 (b) 0 (c) 2 (d) –1

Miscellaneous Problems

1. If $f(x)=e^{ax}$ then show that $f(0)$, $\Delta f(0), \Delta^2 f(0)$ are in G.P.

2. Prove that $i)(1+\Delta)(1-\nabla)=1$
   $ii)\Delta\nabla=\Delta-\nabla$ (iii) $E\nabla=\Delta=\nabla E$

3. A second degree polynomial passes though the point (1,-1) (2,-1) (3,1) (4,5). Find the polynomial.

4. Find the missing figures in the following table:

x	0	5	10	15	20	25
y	7	11	-	18	-	32

5. Find $f(0.5)$ if $f(-1)=202$, $f(0)=175$, $f(1)=82$ and $f(2)=55$.

6. From the following data find y at x = 43 and x = 84.

x	40	50	60	70	80	90
y	184	204	226	250	276	304

7. The area A of circle of diameter 'd' is given for the following values

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the approximate values for the areas of circles of diameter 82 and 91 respectively.

8. If $u_0=560$, $u_1=556$, $u_2=520$, $u_4=385$, show that $u_3=465$.

9. From the following table obtain a polynomial of degree y in x.

x	1	2	3	4	5
y	1	–1	1	–1	1

10. Using Lagrange's interpolation formula find a polynomial which passes through the points (0, –12), (1, 0), (3, 6) and (4,12).

Summary

In this chapter we have acquired the knowledge of

- $\Delta f(x)=f(x+h)-f(x)$
- $\nabla f(x)=f(x)-f(x-h)$
- $\nabla f(x+h)=\Delta f(x)$
- $Ef(x)=f(x+h)$
- $E^n f(x)=f(x+nh)$
- Newton's forward interpolation formula:

$$y_{(x=x_0+nh)}=y_0+\frac{n}{1!}\Delta y_0+\frac{n(n-1)}{2!}\Delta^2 y_0+\frac{n(n-1)(n-2)}{3!}\Delta^3 y_0+\ldots$$

- Newton's backward interpolation formula:

$$y_{(x=x_n+nh)}=y_n+\frac{n}{1!}\nabla y_n+\frac{n(n+1)}{2!}\nabla^2 y_n+\frac{n(n+1)(n+2)}{3!}\nabla^3 y_n+\ldots$$

- Lagrange's interpolation formula:

$$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}y_1 + \ldots + \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})}y_n$$

GLOSSARY (கலைச்சொற்கள்)	
Algebraic methods	இயற்கணித முறைகள்
Backward difference operator	பின்நோக்கு வேறுபாட்டுச் செயலி
Extrapolation	புறச்செருகல்
Finite differences	திட்டமான வேறுபாடுகள்
Forward difference operator	முன்நோக்கு வேறுபாட்டுச் செயலி
Graphic method	வரைபட முறை
Gregory- Newton's formulae	கிரிகோரி–நியுட்டனின் சூத்திரங்கள்
Interpolation	இடைச்செருகல்
Lagrange's formula	இலக்ராஞ்சியின் சூத்திரம்
Numerical	எண்ணியியல்
Policy	காப்பீடு
Shifting operator	இடப்பெயர்வுச் செயலி