

MOTION

1. Introduction

When a body does not change its position with time, we can say that the body is at **rest**. While if a body changes its position with time, it is said to be in **motion**.

- An object is said to be a **point object** if it changes its position by distances which are much greater than its size.
- A point or some stationary object with respect to which a body continuously changes its position in the state of motion is known as **origin** or **reference point**.

2. Type of motions :

(a) According to Directions

- **One dimensional motion** is the motion of a particle moving along a straight line.
- **Two dimensional motion** A particle moving along a curved path in a plane has 2-dimensional motion.
- **Three dimensional motion** Particle moving in space has 3-dimensional motion.

(b) According to state of motion

Uniform Motion

- A body is said to be in a state of uniform motion if it travels equal distances in equal intervals of time.
- If the time distance graph is a straight line the motion is said to be uniform motion.

Non-uniform motion

- A body has a non-uniform motion if it travels unequal distances in equal intervals of time. e.g. a freely falling body.
- Time - distance graph for a body with non-uniform motion is a curved line.

3. Distance & Displacement

- The path length between the initial and final positions of the particle gives the **distance** covered by the particle.
- The minimum distance between the initial and final positions of a body during that time interval is called **displacement**.

Difference between distance and displacement.

- Distance travelled is a scalar quantity while displacement is a vector quantity.
eg. if a body moves along the circumference of a circle of radius r , then the distance travelled is given by $2\pi r$, while the displacement is given by zero.
- When a body continuously moves in the same straight line and in the same direction then displacement will be equal to the distance travelled. But if the body changes its direction while moving, then the displacement is smaller than the distance travelled.

$$\text{Displacement} \leq \text{Distance}$$

- Displacement in any interval of time may be zero, positive or negative.

Ex.1 A person travels a distance of 5 m towards east, then 4 m towards north and then 2 m towards west.

- Calculate the total distance travelled.
- Calculate the resultant displacement.

Sol. (i) Total distance travelled by the person

$$= 5 \text{ m} + 4 \text{ m} + 2 \text{ m} = 11 \text{ m}$$

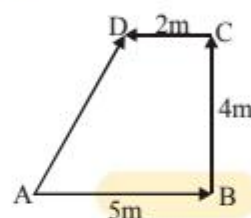
- To calculate the resultant displacement, we choose a convenient scale, where 1 cm represents 1 m. We draw a 5 cm long line AB towards east and then 4 cm long line BC towards north. Finally, a 2 cm long line CD towards west. The resultant displacement is calculated by joining the initial position A to the final position D. We measure AD = 5 cm.

Since 1 cm = 1 m

$$\therefore 5 \text{ cm} = 5 \text{ m}$$

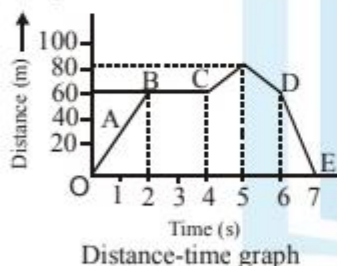
Hence, the displacement of the person

= 5m towards AD.



Ex.2 A body is moving in a straight line. Its distances from origin are shown with time in Fig. A, B, C, D and E represent different parts of its motion. Find the following :

- Displacement of the body in first 2 seconds.
- Total distance travelled in 7 seconds.
- Displacement in 7 seconds



Sol. (i) Displacement of the body in first 2s = 60m

- From $t = 0$ to $t = 7$ s, the body has moved a distance of 80 m from origin and it has again come back to origin. Therefore, the total distance covered = $80 \times 2 = 160$ m
- Since the body has come back to its initial position, the displacement is zero.

4. Speed and Velocity

- The 'distance' travelled by a body in unit time interval is called its **speed**. When the position of a body changes in particular direction, then speed is denoted by 'velocity'. i.e. the rate of change of displacement of a body is called its **velocity**.
- Speed is a scalar quantity while velocity is a vector quantity.

$$\text{Speed} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$$

Where Δs = displacement in time interval Δt .

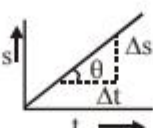
➤ Velocity = $\frac{\Delta s}{\Delta t}$

Where Δs = distance travelled in time interval Δt .

➤ Unit : In M.K.S. system = ms^{-1}

In C.G.S. system = ms^{-1}

- If time distance graph is a straight line, then speed can be given by the slope of the line, i.e.

$$v = \frac{\Delta s}{\Delta t} = \text{slope} = \frac{s_2 - s_1}{t_2 - t_1}$$


- The area of velocity time graph gives distance travelled.

Types of speed

(a) Average and Instantaneous speed

Average speed :

It is obtained by dividing the total distance travelled by the total time interval. i.e.

$$\text{average speed} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$$

- Average speed is a scalar, while average velocity is a vector.
- For a given time interval average velocity is single valued, while average speed can have many values depending on path following.
- If after motion body comes back to its initial position $\vec{v}_{av} = 0$ [as $\Delta \vec{r} = 0$], but $v_{av} > 0$ and finite (as $\Delta s > 0$)
- For a moving body average speed can never be -ve or zero (unless $t \rightarrow \infty$), while average velocity can be i.e. $v_{av} > 0$ while $\vec{v}_{av} > =$ or < 0
- In general average speed is not equal to magnitude of average velocity (as $\Delta s \neq |\Delta \vec{r}|$). However it can be so if the motion is along a straight line without change in direction (as $\Delta s = |\Delta \vec{r}|$)
- If a particle travels distances L_1, L_2, L_3 at speeds v_1, v_2, v_3 etc respectively, then

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{L_1 + L_2 + \dots + L_n}{\frac{L_1}{v_1} + \frac{L_2}{v_2} + \dots + \frac{L_n}{v_n}} = \frac{\sum L_i}{\sum \frac{L_i}{v_i}}$$

- If a particle travels at speeds v_1, v_2 etc for intervals t_1, t_2 etc respectively, then

$$v_{av} = \frac{v_1 t_1 + v_2 t_2 + \dots}{t_1 + t_2 + \dots} = \frac{\sum v_i t_i}{\sum t_i}$$

- If a particle moves a distance at speed v_1 and comes back with speed v_2 , then

$$\text{average speed } v_{av} = \frac{2v_1 v_2}{v_1 + v_2} \quad (\vec{v}_{av} = 0)$$

- If a particle moves for two equal time intervals

$$v_{av} = \frac{v_1 + v_2}{2}$$

Instantaneous speed :

The speed of a body at a particular instant of time is called its instantaneous speed. i.e. instantaneous speed (v)

$$= \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

(b) Uniform and Non uniform speed**Uniform speed :**

If the time speed graph of an object is a straight line parallel to time axis then the body is moving with a uniform speed.

Non-uniform speed :

If the speed of a body is changing with respect to time it is moving with a non-uniform speed.

Ex.3 The distance between two points A and B is 100 m. A person moves from A to B with a speed of 20 m/s and from B to A with a speed of 25 m/s. Calculate average speed and average velocity.

Sol. (i) Distance from A to B = 100 m

Distance from B to A = 100 m

Thus, total distance = 200 m

Time taken to move from A to B, is given by

$$t_1 = \frac{\text{distance}}{\text{velocity}} = \frac{100}{20} = 5 \text{ seconds}$$

Time taken from B to A, is given by

$$t_2 = \frac{\text{distance}}{\text{velocity}} = \frac{100}{25} = 4 \text{ seconds}$$

Total time taken = $t_1 + t_2 = 5 + 4 = 9 \text{ sec.}$

\therefore Average speed of the person

$$= \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{200}{9} \text{ m/s} = 22.2 \text{ m/s}$$

(ii) Since person comes back to initial position A, displacement will be zero, resulting zero average velocity.

Ex.4 A car moves with a speed of 40 km/hr for first hour, then with a speed of 60 km/hr for next half hour and finally with a speed of 30 km/hr for next $1\frac{1}{2}$ hours. Calculate the average speed of the car.

Sol. Distance travelled in first hour, is given by

$$s_1 = \text{speed} \times \text{time} = 40 \text{ km/hr} \times 1 \text{ hr} = 40 \text{ km}$$

Distance travelled in next half an hour, is given by

$$s_2 = \text{speed} \times \text{time} = 60 \text{ km/hr} \times \frac{1}{2} \text{ hr} = 30 \text{ km}$$

Distance travelled in last $1\frac{1}{2}$ hours, is given by

$$s_3 = \text{speed} \times \text{time} = 30 \text{ km/hr} \times \frac{3}{2} \text{ hr} = 45 \text{ km}$$

Thus, total distance travelled = $s_1 + s_2 + s_3$

$$= 40 + 30 + 45 = 115 \text{ km}$$

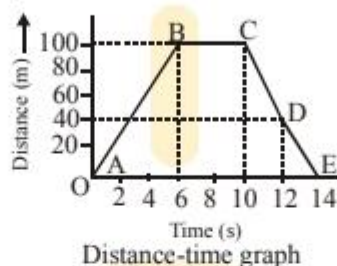
Total time taken = $1 + \frac{1}{2} + 1\frac{1}{2} = 3 \text{ hours}$

$$\therefore \text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{115 \text{ km}}{3 \text{ hrs}} = 38.33 \text{ km/hr}$$

Ex.5 Figure shows time distance graph of an object. Calculate the following :

- Which part of the graph shows that the body is at rest ?
- Average speed in first 10 s.
- Speeds in different parts of motion.



Sol. (i) The part BC shows that the body is at rest.

- (ii) In first 10 seconds, distance travelled = 100m

$$\therefore \text{Thus, average speed} = \frac{\text{Distance covered}}{\text{Time taken}} = \frac{100}{10} = 10 \text{ m/s}$$

- (iii) Speed of the object in part AB is given by slope = $\frac{100}{6} = 50/3 \text{ m/s}$

Speed of object in part BC = 0 m/s

Speed of object in part CD

$$= \frac{100 - 40}{12 - 10} = \frac{60}{2} = 30 \text{ m/s}$$

Speed of object in part DE

$$= \frac{40 - 0}{14 - 12} = \frac{40}{2} = 20 \text{ m/s}$$

Ex.6 Figure shows the time-displacement graph of an object. Calculate the following:

- Average velocity in the first 4s.
- Displacement at the end of 10s.
- After how much time, the object reaches its original position ?
- Velocity at $t = 2 \text{ s}$, and at $t = 6 \text{ s}$

Sol. (i) Average velocity in first 4 s, is given by

$$v = \frac{\text{Total displacement (in 4 seconds)}}{4 \text{ s}} = \frac{20}{4} = 5 \text{ m/s}$$

- (ii) Displacement at the end of 10 s = 20 m

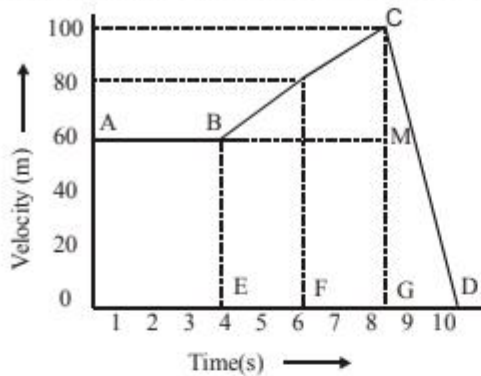
- (iii) It is clear from the graph that the object comes to its original position.
i.e. displacement = 0, after 7 seconds and again after 13 seconds.

- (iv) At $t = 2 \text{ s}$, $v = \text{slope} = \frac{0 - 20}{3 - 1} = \frac{-20}{2} = -10 \text{ m/s}$

$$\text{At } t = 6 \text{ s}, v = \frac{0 - 20}{7 - 5} = \frac{-20}{2} = -10 \text{ m/s}$$

Ex.7 Time-velocity graph of a particle is shown in Fig.

- (i) Calculate the distance travelled in first 8 seconds.



Sol. Distance travelled in first 8s is given by area OABCG
 = area of rectangle OAMG
 + area of triangle BMC
 $= 8 \times 60 + \frac{1}{2} \times 4 \times 40$
 $= 480 + 80 = 560 \text{ m.}$

5. Acceleration

- Rate of change of velocity is called acceleration
- The change in velocity may be in magnitude or in direction or both.

$$\text{i.e. } a = \frac{v - u}{t}$$

- Unit of acceleration = m/s^2 or ms^{-2}

Types of acceleration

- **Uniform & Non uniform acceleration**

Uniform acceleration

If a body travels in a straight line and its velocity increases by equal amounts in equal intervals of time then it is said to be in state of uniform acceleration e.g. motion of a freely falling body.

Non uniform acceleration

A body has a non-uniform acceleration if its velocity increases by unequal amounts in equal intervals of time.

- **Average & Instantaneous acceleration**

Average acceleration :

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

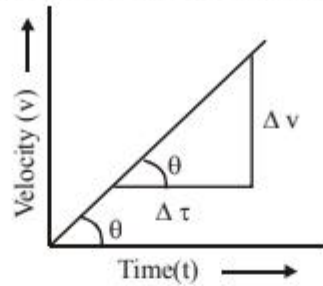
[here it is assumed that acceleration remains the same during the time interval Δt .]

If a body travels with a uniform acceleration a_1 for a time interval t_1 and with uniform acceleration a_2 for a time interval t_2 then

$$a_{av} = \frac{(a_1 t_1 + a_2 t_2)}{(t_1 + t_2)}$$

Instantaneous acceleration :

The acceleration of a body at any instant is called its instantaneous acceleration.



e.g. $a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

- If the velocity of a body decreases, then it will experience a negative acceleration which is called deceleration or retardation.
- **Acceleration is determined by the slope of time-velocity graph.**

$$\tan \theta = \frac{dv}{dt}$$

- (i) If the time velocity graph is a straight line, acceleration remains constants.
- (ii) If the slope of the straight line is positive, positive acceleration occurs.
- (iii) If the slope of the straight line is negative, negative acceleration or retardation occurs.
- (iv) Larger the slope ($\tan \theta$) longer will be the straight line.
- (v) If the time velocity graph is a curve, then the acceleration changes continuously.

Ex.8 Time-velocity graph of a body is shown in the fig. Find its acceleration in m/s^2 .

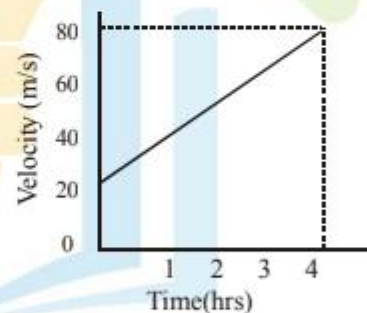
Sol. As it is clear from the figure,

At $t = 0$ s, $v = 20$ m/s

At $t = 4$ s, $v = 80$ m/s

\therefore Acceleration, $a = \frac{\text{Change in velocity}}{\text{Time interval}}$

$$= \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{(80 - 20) \text{ m/s}}{(4 - 0)} = 15 \text{ m/s}^2$$



Ex.9 Time-velocity graph of a particle is shown in fig. Find its instantaneous acceleration at following intervals :

- (i) at $t = 3$ s
- (ii) at $t = 6$ s
- (iii) at $t = 9$ s

Sol. (i) Instantaneous acceleration at $t = 3\text{ s}$, is given by

$$a = \text{slope of line AB} = \text{zero}$$

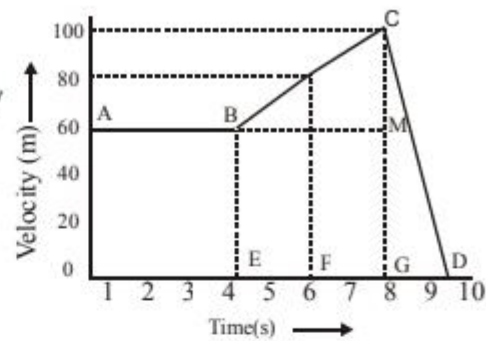
(ii) Instantaneous acceleration at $t = 6\text{ s}$, is given by

$$a = \text{slope of line}$$

$$BC = \frac{CM}{BM} = \frac{100 - 60}{8 - 4} = -10 \text{ m/s}^2$$

(iii) Instantaneous acceleration at $t = 9\text{ s}$, is given by

$$a = \text{slope of line CD} = \frac{0 - 100}{10 - 8} = -50 \text{ m/s}^2$$



Ex.10 Starting from rest, Deepak paddles his bicycle to attain a velocity of 6 m/s in 30 seconds then he applies brakes so that the velocity of the bicycle comes down to 4 m/s in the next 5 seconds . Calculate the acceleration of the bicycle in both the cases.

Sol. (i) Initial velocity, $u = 0$, final velocity, $v = 6\text{ m/s}$, time, $t = 30\text{ s}$

Using the equation $v = u + at$, we have

$$a = \frac{v - u}{t}$$

substituting the given values of u , v and t in the above equation, we get

$$a = \frac{6 - 0}{30} = 0.2 \text{ m/s}^2 ;$$

which is positive acceleration.

(ii) Initial velocity, $u = 6\text{ m/s}$, final velocity, $v = 4\text{ m/s}$, time, $t = 5\text{ s}$, then

$$a = \frac{v - u}{t} = \frac{4 - 6}{5} = -0.4 \text{ m/s}^2 ;$$

which is retardation.

Note : The acceleration of the case (i) is positive and is negative in the case (ii).

6. Equations of Motion

➤ Motion under uniform acceleration

Suppose a body starts with initial velocity u , moving with an acceleration attains a velocity v after time t travels a distance s , then motion can be described by following equations.

(a) $v = u + at$

(b) $s = ut + \frac{1}{2}at^2$

(c) $v^2 = u^2 + 2as$

➤ The equations of motion under gravity can be obtained by replacing acceleration by acceleration due to gravity (g) and can be written as follows :

- When the body is coming towards the centre of earth
 - (a) $v = u + gt$;
 - (b) $h = ut + \frac{1}{2}gt^2$;
 - (c) $v^2 = u^2 + 2gh$
- When a body is thrown upwards with some initial velocity, then a retardation produced due to attraction of the earth. In equations of motion, a is replaced by $(-g)$ and thus equations become.
 - (a) $v = u - gt$;
 - (b) $h = ut - \frac{1}{2}gt^2$;
 - (c) $v^2 = u^2 - 2gh$
- Distance covered by a body in n^{th} sec. i.e.

$$s_n = u + \frac{1}{2}a(2n - 1)$$

7. Body Falling Freely Under Gravity

Assuming $u = 0$ for a freely falling body :

t is given

$$v = gt$$

$$h = \frac{1}{2}gt^2$$

h is given

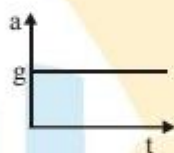
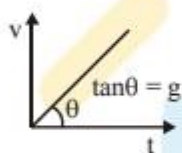
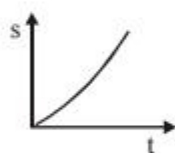
$$t = \sqrt{\frac{2h}{g}}$$

$$v = \sqrt{2gh}$$

v is given

$$t = \frac{v}{g}$$

$$h = \frac{v^2}{2g}$$



- As $h = \left(\frac{1}{2}\right)gt^2$ i.e. $h \propto t^2$
Distance fallen in time $t, 2t, 3t$ etc will be in the ratio of $1^2 : 2^2 : 3^2 \dots$ i.e. square of integers.
- The distance fallen in n^{th} sec $= \frac{1}{2}g(2n-1)$
so distance fallen in $1^{\text{st}}, 2^{\text{nd}}, 3^{\text{rd}}$ sec will be in the ratio $1 : 3 : 5$ i.e. odd integers only.
- Body is projected vertically up :
Taking initial position as origin and direction of motion (i.e. vertically up) as positive.
 - (a) At the highest point $v = 0$
 - (b) $a = -g$

t is given

$$u = gt$$

$$h = \frac{1}{2}gt^2$$

h is given

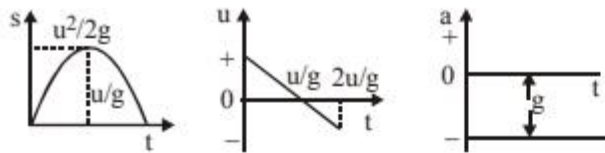
$$t = \sqrt{\frac{2h}{g}}$$

$$u = \sqrt{2gh}$$

u is given

$$t = \frac{u}{g}$$

$$h = \frac{u^2}{2g}$$



➤ It is clear that in case of motion under gravity

- Time taken to go up is equal to the time taken to fall down through the same distance.
- The speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.
- The body returns to the starting point with the same speed with which it was thrown.

Ex.11 A body starts moving with an initial velocity 50 m/s and acceleration 20 m/s^2 . How much distance it will cover in 4s ? Also, calculate its average speed during this time interval.

Sol. Given : $u = 50 \text{ m/s}$, $a = 20 \text{ m/s}^2$,
 $t = 4\text{s}$, $s = ?$

$$s = ut + \frac{1}{2}at^2 = 50 \times 4 + \frac{1}{2} \times 20 \times (4)^2$$

$$= 200 + 160 = 360 \text{ m}$$

Average speed during this interval,

$$\bar{v} = \frac{\text{distance travelled}}{\text{time interval}} = \frac{360}{4} = 90 \text{ m/s}$$

Ex.12 A body is moving with a speed of 20 m/s. When certain force is applied, an acceleration of 4 m/s^2 is produced. After how much time its velocity will be 80 m/s ?

Sol. Given : $u = 20 \text{ m/s}$, $a = 4 \text{ m/s}^2$,
 $v = 80 \text{ m/s}$, $t = ?$

Using equation, $v = u + at$, we get

$$80 = 20 + 4 \times t$$

$$\text{or } 4t = 80 - 20 = 60$$

$$\text{or } t = 15 \text{ s}$$

Therefore, after 15 seconds, the velocity of the body will be 80 m/s.

Ex.13 A body starts from rest and moves with a constant acceleration. It travels a distance s_1 in first 10 s, and a distance s_2 in next 10 s. Find the relation between s_2 and s_1 .

Sol. Given : $u = 0$, $t_1 = 10 \text{ s}$

∴ Distance travelled in first 10 seconds, is given by

$$s_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times a \times (10)^2$$

$$= 50a \quad \dots(1)$$

To calculate the distance travelled in next 10s, we first calculate distance travelled in 20 s and then subtract distance travelled in first 10 s.

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times a \times (20)^2$$

$$= 200a \quad \dots(2)$$

∴ Distance travelled in 10th second interval,

$$s_2 = s - s_1 = 200a - 50a \quad \dots(3)$$

$$\text{or } s_2 = 150a$$

$$\text{Now, } \frac{s_2}{s_1} = \frac{150a}{50a} = \frac{3}{1}$$

$$\text{or } s_2 = 3s_1$$

Ex.14 A train is moving with a velocity 400 m/s. With the application of brakes a retardation of 10 m/s² is produced. Calculate the following :

- After how much time it will stop ?
- How much distance will it travel before it stops ?

Sol. (i) Given: $u = 400 \text{ m/s}$, $a = -10 \text{ m/s}^2$, $v = 0$, $t = ?$

Using equation, $v = u + at$, we get

$$0 = 400 + (-10) \times t$$

$$\text{or } t = 40 \text{ s}$$

- For calculating the distance travelled, we use equation,

$$v^2 = u^2 + 2as, \text{ we get}$$

$$(0)^2 = (400)^2 + 2 \times (-10) \times s$$

$$\text{or } 20s = 400 \times 400$$

$$\text{or } s = 8000 \text{ m} = 8 \text{ km}$$

Ex.15 A body is thrown vertically upwards with an initial velocity of 19.6 m/s. If $g = -9.8 \text{ m/s}^2$. Calculate the following :

- The maximum height attained by the body.
- After how much time will it come back to the ground ?

Sol.(i) Given: $u = 19.6 \text{ m/s}$, $g = -9.8 \text{ m/s}^2$, $v = 0$, $h = ?$

Using equation $v^2 = u^2 + 2gh$, we get

$$(0)^2 = (19.6)^2 + 2(-9.8) \times h$$

$$\text{or } h = \frac{19.6 \times 19.6}{2 \times 9.8} = 19.6 \text{ m}$$

- Time taken to reach the maximum height can be calculated by the equation,

$$v = u + gt$$

$$\text{or } 0 = 19.6 + (-9.8) \times t$$

$$\text{or } t = 2\text{s}$$

In the same time, it will come back to its original position.

$$\therefore \text{Total time} = 2 \times 2 = 4\text{s}$$

Ex.16 From the top of a tower of height 490 m, a shell is fired horizontally with a velocity 100 m/s. At what distance from the bottom of the tower, the shell will hit the ground ?

Sol. We know that the horizontal motion and the vertical motion are independent of each other. Now for vertical motion, we have $u = 0$, $h = 490 \text{ m}$, $g = 9.8 \text{ m/s}^2$, $t = ?$

Using equation, $h = ut + \frac{1}{2}gt^2$, we get

$$490 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$$\text{or } t^2 = \frac{490}{4.9} = 100$$

$$\text{or } t = 10 \text{ s}$$

∴ It takes 10 seconds to reach the ground.

Now, horizontal distance

$$= \text{horizontal velocity} \times \text{time}$$

$$= 100 \text{ m/s} \times 10 \text{ s} = 1000 \text{ m}$$

∴ The shell will strike the ground at a distance of 1000 m from the bottom of the tower.

Ex.17 The brakes applied to a car produce a negative acceleration of 6 m/s^2 . If the car takes 2 seconds to stop after applying the brakes, calculate the distance it travels during this time.

Sol. Given: $a = -6 \text{ m/s}^2$, $t = 2 \text{ s}$ and $v = 0 \text{ m/s}$

Using the equation,

$$v = u + at, \text{ we get}$$

$$0 = u + (-6) \times 2$$

$$[\because \text{Since final velocity, } v = 0]$$

$$\text{or } u = 12 \text{ m/s}$$

Now by using the equation

$$s = ut + \frac{1}{2}at^2,$$

we get

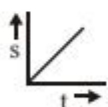
$$\text{or } s = 12 \times 2 + \frac{1}{2} \times (-6) \times (2)^2 = 12 \text{ m}$$

Thus, the car will move 12 m before it stops after applying the brakes.

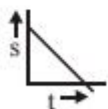
9. Various Graphs Related to Motion

Displacement- time graph :

- The straight line inclined to time axis in s-t graph represents constant velocity.



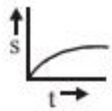
- In s-t graph the straight line inclined to time axis at angle greater than 90° shows negative velocity



- Body with accelerated motion

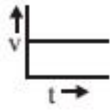


- Body with decelerated motion



Velocity -time graph :

- For the body having constant velocity or zero acceleration.



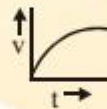
- The body is moving with constant retardation and its initial velocity is not zero.



- The body is accelerated and the initial velocity is zero.

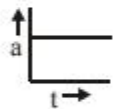


- The body is decelerated

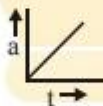


Acceleration time graph :

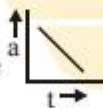
- Acceleration is constant



- Acceleration is increasing and is +ve



- Acceleration is decreasing and is -ve



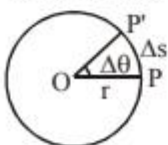
10. Circular Motion

When a body moves in such a way that its distance from a fixed point always remains constant, then its motion is said to be the circular motion.

Types of circular motion

(A) Uniform circular motion :

- If the radius vector sweeps out equal angles in equal times, then its motion is said to be uniform circular motion.



- In uniform circular motion speed remains const.
- Linear velocity, being a vector quantity, its direction changes continuously.

- The direction of velocity is along the tangent at every point.

Angular velocity :

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- A vector quantity
- Direction is perpendicular to plane of rotation
- Note :** If the particle is revolving in the clockwise direction then the direction of angular velocity is perpendicular to the plane downwards. Whereas in case of anticlockwise direction the direction will be upwards.
- Unit is Radian/sec.
- In uniform circular motion the direction of angular velocity is along the axis of rotation which is constant throughout.
- Angular velocity remains constant in magnitude as well as in direction.
- $v = r\omega$ where r = radius of the circle.

Centripetal acceleration

- In uniform circular motion the particle experiences an acceleration called the centripetal acceleration.
- $a_c = \frac{v^2}{r}$
- The direction of centripetal acceleration is along the radius towards the centre.

Centripetal force :

- Always acts towards centre.
- Centripetal force is required to move a particle in a circle.
- Because F_c is always perpendicular to velocity or displacement, hence the work done by this force will always be zero.

Note :

- Circular motion in horizontal plane is usually uniform circular motion.
- Remember that equations of motion are not applicable for circular motion.

(B) Non-Uniform circular motion :

- In non-uniform circular motion speed is not constant. i.e. $|\vec{v}| \neq \text{constant}$

Time period :

- It is the time taken to complete one complete revolution.
- In one revolution, angle subtended is 2π and if T is time period, then the angular velocity is given by

$$\omega = \frac{2\pi}{T} \quad \text{or} \quad T = \frac{2\pi}{\omega}$$

Frequency :

- Frequency is defined as the no. of revolutions per second.

$$\text{i.e. } n = \frac{1}{T} = \frac{\omega}{2\pi}$$

Ex.18 A particle moves in a circle of radius 2 m and completes 5 revolutions in 10 seconds. Calculate the following :

- (i) Angular velocity and (ii) Linear velocity.

Sol. Since, it completes 5 revolutions in 10 seconds.

$$\therefore \text{Time period} = \frac{10}{5} = 2\text{s}$$

(i) Now angular velocity, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \text{ rad/s}$

(ii) Linear velocity is given by

$$v = r\omega = 2\pi$$

$$\therefore v = 2\pi \text{ m/s}$$

Ex.19 The length of second's needle in a watch is 1.2 cm. Calculate the following :

(i) Angular velocity and

(ii) Linear velocity of the tip of the needle.

Sol. (i) We know that the second's needle in a watch completes one revolution in 60 seconds.

$$\therefore \text{Time period, } T = 60 \text{ s}$$

Angular velocity,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30} \text{ rad/s}$$

(ii) Length of the needle = 1.2 cm = Radius of the circle

Linear velocity of the tip of the needle is given by

$$v = r\omega = 1.2 \times \frac{\pi}{30} = \frac{\pi}{25}$$

$$\text{or } v = \frac{\pi}{25} = 1.266 \times 10^{-1} \text{ cm/sec.}$$

Ex.20 Earth revolves around the sun in 365 days. Calculate its angular velocity.

Sol. Time period,

$$T = 365 \text{ days}$$

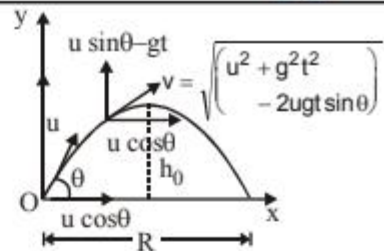
$$= 365 \times 24 \times 60 \times 60 \text{ seconds}$$

$$\therefore \text{Angular velocity, } \omega = \frac{2\pi}{T} = \frac{2\pi}{365 \times 24 \times 60 \times 60} \text{ rad/s} = 1.99 \times 10^{-7} \text{ rad/s.}$$

11. Projectile Motion

- Any particle, which once thrown, moves freely in gravitational field of the earth, is defined as a projectile.
- It is an example of two dimensional motion with constant acceleration.
- Projectile motion can be considered as two simultaneous motions in mutually perpendicular directions viz. (a) horizontal and (b) vertical
- The particle is thrown from the ground level at an angle θ from the horizontal velocity u .
- Initial velocity can be resolved into two components :
 $u \cos\theta$ = horizontal component,

$u \sin \theta = \text{vertical component}$



- The horizontal component of velocity ($u \cos \theta$) remains constant whereas the vertical component changes constantly due to acceleration due to gravity 'g'.]

In horizontal direction

In vertical direction

(a) initial velocity

(a) initial velocity

$$u_x = u \cos \theta$$

$$u_y = u \sin \theta \text{ (upward)}$$

(b) acceleration = 0
(downward)

(b) acceleration $a = g$

(c) velocity after time t

(c) velocity after time t ,

$$v_x = u \cos \theta$$

$$v_y = u_y - gt \\ = u \sin \theta - gt$$

(d) horizontal displacement

(d) vertical displacement

after time t

$$x = u_x t$$

$$y = u_y t - \frac{1}{2} g t^2$$

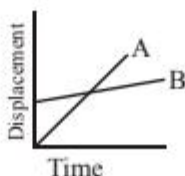
$$= u \cos \theta t$$

$$= u \sin \theta t - \left(\frac{1}{2} \right) g t^2$$

EXERCISE - 1

(A) VERY SHORT ANSWER-TYPE QUESTIONS

- Q.1** Can the speed of a body moving with a constant velocity change ?
- Q.2** Can the velocity of a body moving with a uniform speed change ?
- Q.3** Can average velocity of a moving body be zero ?
- Q.4** Can average speed of a moving body be zero ?
- Q.5** Time-displacement graph is a straight line parallel to the time axis. What is its velocity and the acceleration ?
- Q.6** What does the slope of time distance graph represent ?
- Q.7** What does the slope of time displacement graph represent ?
- Q.8** What does the slope of time-velocity graph represent ?
- Q.9** What is the acceleration of a body moving with constant velocity ?
- Q.10** What is the unit of acceleration in MKS system ?
- Q.11** A stone is thrown upwards, reaches a height h and comes back. What are the distance moved and displacement ?
- Q.12** A particle moves along the circumference of a circle in half cycle. Calculate the distance travelled and displacement.
- Q.13** What does area under time-velocity graph represent ?
- Q.14** What does area under time-acceleration graph represent ?
- Q.15** Define circular motion.
- Q.16** Define uniform circular motion.
- Q.17** What is the relation between linear velocity and angular velocity ?
- Q.18** Does uniform circular motion has accelerated motion or no acceleration at all ?
- Q.19** What is the direction of angular velocity ?
- Q.20** In uniform circular motion, does the angular velocity remain constant or if changes with time.
- Q.21** A car starts moving with 20 m/s and its velocity becomes 80 m/s after 6 sec. Calculate its acceleration.
- Q.22** A body is thrown vertically up with a velocity 98 m/s. How much high it will rise ? ($g = 9.8 \text{ m/s}^2$).
- Q.23** A body falls from a height of 500 m. In how much time, will it strike the ground ?
- Q.24** Time-displacement graphs of two bodies A and B are shown in the Fig. Which one has larger velocity ?



Q.25 The velocity of a body is 72 km/hr. Calculate its value in m/s.

(B) SHORT ANSWER-TYPE QUESTIONS

Q.26 Define state of motion.

Q.27 Define velocity.

Q.28 Differentiate between the following :

- (i) speed and velocity,
- (ii) distance and displacement

Q.29 Displacement of a body can be zero even when the distance travelled is not zero. Explain.

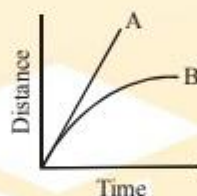
Q.30 What do you mean by negative and positive acceleration ? Explain.

Q.31 A train is moving with a constant speed of 40 km/hr. Draw time-speed graph. From this, draw time-distance graph upto 5 hours from the start.

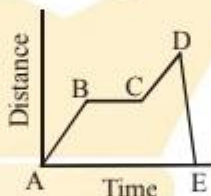
Q.32 Draw the graph for uniform motion.

- (i) Displacement - Time (ii) Velocity - Time

Q.33 In the given fig., A and B represent uniform motion or accelerated motion.

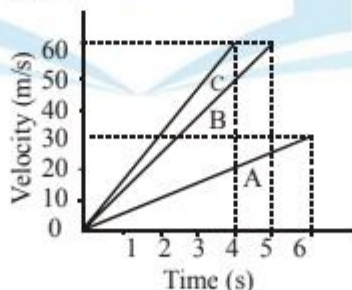


Q.34 In the given Fig. what type of motion are represented by the parts AB, BC, CD and DE.



Q.35 For a moving body distance travelled is directly proportional to the time. What do you conclude about its speed ?

Q.36 Figure shows the time velocity graphs for three bodies A, B and C.



- (i) Which body has minimum acceleration ?
- (ii) Which body has maximum acceleration ?

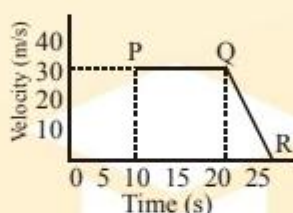
Q.37 A body starting with initial velocity u moves with a constant acceleration a . Find the expression for distance travelled in n th seconds.

Q.38 A body starting from rest moves with a constant acceleration. It moves a distance s_1 in first 5 seconds and a distance s_2 in next 5 seconds. Prove that $\Delta s_2 = 3s_1$.

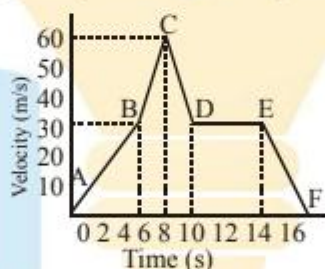
- Q.39** An engine is moving with a velocity 44 m/s. After applying the brakes, it stops after covering a distance of 121 m. Calculate retardation and time taken by the engine to stop.
- Q.40** A body is thrown vertically up with an initial velocity of 60 m/s. If $g = 10 \text{ m/s}^2$, at what time, it will be at a height of 100 m.

(C) LONG ANSWER-TYPE QUESTIONS

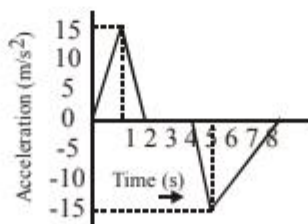
- Q.41** What do you mean by average speed ? How will you find average speed from time-distance graph ?
- Q.42** What is the difference between time-speed and time-velocity graph ? In what condition, they are similar ?
- Q.43** What do you mean by acceleration ? How do you find acceleration from time-velocity graph?
- Q.44** How do you find change in velocity from time-velocity graph ?
- Q.45** Time-velocity graph of a body is shown in fig. Calculate the following :
- Distance travelled in first 10 s
 - Acceleration at $t = 15 \text{ s}$
 - Acceleration between $t = 20 \text{ s}$ to $t = 25 \text{ s}$.



- Q.46** Time velocity graph of a moving body is shown in fig. Calculate the following :



- Change in velocity during $t = 6 \text{ s}$ to $t = 8 \text{ s}$
 - Average acceleration during $t = 10 \text{ s}$ to $t = 12 \text{ s}$.
 - In which time interval acceleration will be zero.
 - Acceleration during $t = 14 \text{ s}$ to $t = 16 \text{ s}$.
- Q.47** Time-acceleration graph of a moving body is shown in fig. Calculate the following :



- Time interval in which acceleration will be zero.
- Acceleration at $t = 5 \text{ s}$.
- Change in velocity during time interval $t = 4 \text{ s}$ to $t = 8 \text{ s}$.

- Q.48** A body moves with a velocity of 2 m/s for 5 s, then its velocity uniformly increases to 10 m/s in 5s. Thereafter the velocity begins to decrease at a uniform rate until it comes to rest after 10 s.
- Plot a velocity-time graph and distance-time graph for the motion of the body.
 - Mark the portions of the graph to show when it is non-uniform.
 - From the graph, find the total distance moved by the body after 2s and 12 s and in the last 10 s.
- Q.49** A car in a 100 m race, leaves 4 m in first second, 30 m in next 4s, 52 m in another 4s and finishes the race in 10s. Calculate the following :
- Average velocity of the car.
 - Time interval in which the average velocity attained by the car maximum.
 - Plot the distance-time graph for the motion of the car.
 - Distance moved by the car at the end of 6 s with the help of the graph.

(D) FILL IN THE BLANKS

- Q.50** SI unit of displacement is
- Q.51** Displacement is a quantity while distance is a quantity.
- Q.52** A scooter acquires a speed of 36 km/hr in 5s after the start. The acceleration of the scooter is
- Q.53** A racing car has a uniform acceleration of 4 m/s^2 . If it starts from rest the distance travelled by the car is
- Q.54** The unit of angular velocity is
- Q.55** The SI unit of velocity is
- Q.56** A cyclist is running at a speed of 10 m/s. If the radius of each wheel of the bicycle is 45 cm, the angular velocity of the wheel is
- Q.57** A person travels a distance πR along the circumference of a circle. Displacement of the particle is
- Q.58** The slope of velocity-time graph gives
- Q.59** A body moves from A to B with a speed of 40 km/hr and comes back to A with a speed of 60 km/hr. The average speed during the complete journey is
- Q.60** Rate of change of displacement represents
- Q.61** Rate of change of velocity represents
- Q.62** $10 \text{ m/s} = \dots\dots\dots \text{ km/hr}$.
- Q.63** If acceleration is positive then velocity
- Q.64** If acceleration is negative then velocity

(E) TRUE/FALSE-TYPE QUESTIONS

- Q.65** Distance moved and displacement are always equal.
- Q.66** Distance moved may be greater or equal to the displacement.

- Q.67** Speed and velocity both are vector quantities.
- Q.68** A body is moving with a uniform speed. Its velocity is necessarily constant.
- Q.69** A body is moving with a constant speed. Still it may possess acceleration.
- Q.70** A body is moving with a constant velocity. Its acceleration is zero.
- Q.71** Slope of displacement time graph gives the velocity of the particle.
- Q.72** Slope of velocity-time graph gives the acceleration of the particle.
- Q.73** Area under velocity-time graph of a particle gives the displacement.
- Q.74** A body is moving a uniform speed along the circumference of a circle. Its acceleration is zero.
- Q.75** Displacement of a particle is proportional to the time elapsed. It has a non-uniform velocity.
- Q.76** Displacement of a particle is proportional to time. Its acceleration is zero.
- Q.77** A body is projected vertically up. At the highest point of its path its velocity is zero.
- Q.78** A body is projected vertically up. At the highest point of its path the acceleration is zero.

(F) SINGLE CORRECT ANSWER TYPE QUESTIONS

- Q.79** A car increases its speed from 36 km/hr to 72 km/hr in 10 s. Its acceleration is
(A) 1 m/s^2 (B) 2 m/s^2 (C) 3.6 m/s^2 (D) 5 m/s^2
- Q.80** The rate of change of displacement with time is called
(A) speed (B) velocity
(C) angular velocity (D) acceleration
- Q.81** The velocity of an object is directly proportional to the time elapsed. The object has
(A) uniform speed
(B) uniform velocity
(C) uniform acceleration
(D) variable acceleration
- Q.82** A body whose speed is constant
(A) has a constant velocity
(B) must be accelerated
(C) might be accelerated
(D) can not be accelerated
- Q.83** An object is moving with a constant velocity of 10 m/s, then its acceleration is
(A) zero (B) 10 m/s^2
(C) 5 m/s^2 (D) 20 m/s^2
- Q.84** A particle moves with a uniform positive acceleration. Its velocity-time graph is a
(A) straight line parallel to time axis
(B) straight line parallel to velocity axis
(C) straight line inclined to time axis
(D) parabola

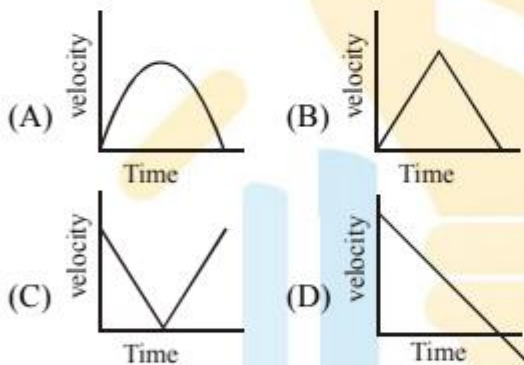
- Q.85** A ball is thrown vertically upwards, It rises to a height of 20 m and comes back to the same place, then the
- (A) total distance covered by the ball is zero
 - (B) net displacement of the ball is zero
 - (C) displacement is 40 m
 - (D) both distance moved and displacement are 40 m each

- Q.86** A body moves with a constant acceleration upto 10s. It travels a distance s_1 in first 5s and a distance s_2 in next 5s, then
- (A) $s_2 = s_1$
 - (B) $s_2 = 2s_1$
 - (C) $s_2 = 3s_1$
 - (D) $s_2 = 4s_1$

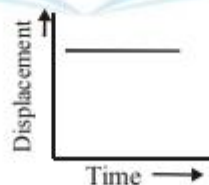
- Q.87** The slope of speed-time graph gives
- (A) displacement
 - (B) velocity
 - (C) acceleration
 - (D) momentum

- Q.88** A particle moves in a straight line. It travels for first half time with a velocity v_1 and for next half time with a velocity v_2 . The average velocity over the complete path is
- (A) $\frac{v_1 + v_2}{2}$
 - (B) $\frac{2v_1v_2}{v_1 + v_2}$
 - (C) $\frac{v_1 - v_2}{2}$
 - (D) $v_1 + v_2$

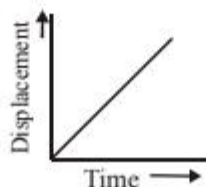
- Q.89** A particle is thrown vertically up with some initial velocity and comes back to the ground. Its velocity-time graph is represented by



- Q.90** The following time-displacement graph (fig.) represents
- (A) zero velocity
 - (B) constant velocity
 - (C) increasing velocity
 - (D) decreasing velocity

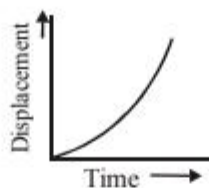


- Q.91** The following time-displacement graph (fig.) represents
- (A) zero velocity
 - (B) constant velocity
 - (C) increasing velocity
 - (D) decreasing velocity



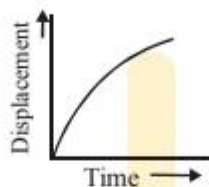
Q.92 The following time-displacement graph (fig.) represents

- (A) zero velocity
- (B) constant velocity
- (C) increasing velocity
- (D) decreasing velocity



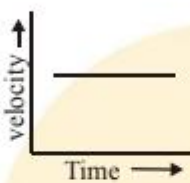
Q.93 The following time-displacement graph (fig.) represents

- (A) zero velocity
- (B) constant velocity
- (C) increasing velocity
- (D) decreasing velocity



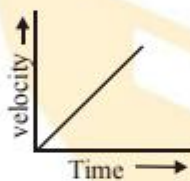
Q.94 The following time-velocity graph (fig.) represents

- (A) zero acceleration
- (B) constant acceleration
- (C) increasing acceleration
- (D) decreasing acceleration



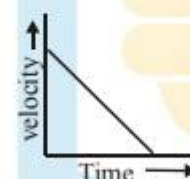
Q.95 The following time-velocity graph (fig.) represents

- (A) zero acceleration
- (B) positive acceleration
- (C) negative acceleration
- (D) variable acceleration



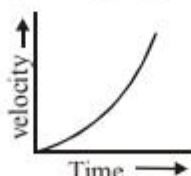
Q.96 The following time-velocity graph (fig.) represents

- (A) zero acceleration
- (B) positive acceleration
- (C) negative acceleration
- (D) variable acceleration



Q.97 The following time-velocity graph (fig.) represents

- (A) zero acceleration
- (B) constant acceleration
- (C) increasing acceleration
- (D) decreasing acceleration



(G) ONE OR MORE THAN ONE CORRECT ANSWER TYPE QUESTIONS

Q.98 Which of the following quantities is/are a scalar quantity ?

- (A) displacement
- (B) distance
- (C) speed
- (D) velocity

- Q.99** Which of the following quantities is/are a vector quantity ?
 (A) Mass (B) Length
 (C) Force (D) Momentum
- Q.100** Which of the following is/are incorrect statement ?
 (A) speed is a scalar quantity while velocity is a vector quantity
 (B) speed may be positive or negative but velocity is always positive.
 (C) The change in the position of a body in a particular direction is called distance.
 (D) displacement can be positive as well as negative.
- Q.101** In case of uniform circular motion—
 (A) speed remains constant
 (B) velocity remains constant
 (C) direction of velocity is along the tangent at every point
 (D) direction of velocity is radial
- Q.102** Choose the correct statement :
 (A) The average velocity of a moving body can be zero
 (B) The average velocity of a moving body cannot be zero
 (C) The average speed of a moving body can be zero.
 (D) The average speed of a moving body can not be zero.
- Q.103** A particle moves with a uniform positive acceleration then—
 (A) its velocity time graph will be a straight line inclined to time axis
 (B) its acceleration time graph will be a straight line parallel to time axis
 (C) its velocity time graph will be a straight line parallel to time axis.
 (D) its acceleration time graph will be a straight line inclined to time axis.
- Q.104** A ball is thrown vertically upwards. It rises to a height of 10m and comes back to ground, then
 (A) the net displacement of the ball is zero.
 (B) the total distance covered by the ball is zero.
 (C) both distance moved and displacement are 20 m each.
 (D) total distance covered by the ball is 20m
- Q.105** Motion under gravity is an example of:
 (A) motion with uniform acceleration
 (B) non uniform motion
 (C) uniform motion
 (D) motion with non-uniform acceleration

(H) MATCH THE COLUMN

Q.106

	Column-I	Column-II
(A)	area of v-t curve	(P) acceleration
(B)	area of a-t curve	(Q) displacement
(C)	slope of v-t curve	(c) velocity

Q.107

	Column-I	Column-II
(A)	Uniform circular motion	(P) velocity
(B)	Freely falling body	(Q) centripetal force
(C)	motion with constant velocity	(R) uniform acceleration
(D)	slope of x-t curve	(S) 0 acceleration

(I) ASSERTION-REASON-TYPE QUESTIONS

The following questions consist of two statements each printed as Assertion & Reason. While answering these you are to choose any one of the following four responses.

- (A) If both Assertion & Reason are true & the Reason is a correct explanation of the Assertion.
 (B) If both Assertion & Reason are true but the Reason is not a correct explanation of the Assertion.
 (C) If Assertion is true but Reason is false.
 (D) If Reason is true but Assertion is false.

Q.108 Assertion : Uniform circular motion is an example of accelerated motion.

Reason : The direction of motion of a body moving along a circular path changes at every point.

Q.109 Assertion : A freely falling body is an example of uniform motion.

Reason : The distance travelled in equal intervals of time is not equal during motion.

Q.110 Assertion : A freely falling body is an example of uniform accelerated motion.

Reason : The freely falling body moves under the influence of acceleration due to gravity. Which remains constant.

Q.111 Assertion : A body having zero velocity may have acceleration.

Reason : A freely falling body has zero velocity as well as zero acceleration.

Q.112 Assertion : a ball thrown vertically up takes the same time to go up and come down.

Reason : The acceleration due to gravity is same while in going up and coming down.

Q.113 Assertion : Average speed does not mean the magnitude of the average velocity vector.

Reason : The magnitude of average velocity is always less than or equal to the average speed.

Q.114 Assertion : For the motion of planets in elliptical orbits around the sun, average velocity is zero.

Reason : The displacement is zero.

Q.115 Assertion : Newton's equation of motion can not be applied for uniform circular motion.

Reason : In uniform circular motion acceleration is not uniform.

Q.116 Assertion : The speed of a particle can not be negative.

Reason : speed is a scalar quantity and it gives the magnitude of the velocity.