CLASS : XII EXERCISE 6.2 CHAPTER – 6 7

INCREASING & DECREASING FUNCTIONS

Q No. 1 Show that the function given by $f(x) = 3x+17$ is strictly increasing function.

Sol:

Here $f(x) = 3x+17$, $D_f = R$

$f'(x) = \frac{d}{dx}(3x+17) = 3 > 0$

$\therefore f'(x) > 0 \ \forall x \in R.$

$\therefore f(x) = 3x+17$ is strictly increasing function.

Q No. 2 Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R.

Sol:

Here $f(x) = e^{2x}$, $D_f = R$

Also $f'(x) = \frac{d}{dx}(e^{2x}) = e^{2x}.2 = 2e^{2x} > 0 \ \forall x \in R$

$\therefore f'(x) > 0 \ \forall x \in D_f.$

$\therefore f(x)$ is strictly increasing function.

Q No 3. Show that function given by $f(x) = \sin x$ is

(a) Strictly increasing in $(0, \frac{\pi}{2})$ (b) Strictly decreasing in $(\frac{\pi}{2}, \pi)$ (c) Neither ↑ nor ↓ in $(0, \pi)$

Sol:

Here $f(x) = \sin x$, $D_f = R$

and $f'(x) = \frac{d}{dx}(\sin x) = \cos x.$

(a) Now $\forall x \in (0, \pi/2)$, $\cos x > 0$ ie $f'(x) = \cos x > 0$

$\Rightarrow \forall x \in (0, \pi/2)$; $f'(x) > 0$

$\therefore f(x)$ is strictly increasing(↑) in $(0, \pi/2)$

(b) Since $f'(x) = \cos x < 0 \ \forall x \in (\pi/2, \pi)$

$\therefore f(x)$ is strictly decreasing (↓) in $(\pi/2, \pi)$

(c) From (a) and (b) we can say that $f(x) = \sin$ is neither ↑ nor ↓ in $(0, \pi)$

Q No. 4. Find the interval in which the function f given by $f(x) = 2x^2 - 3x$ is
(a) strictly increasing (b) strictly decreasing.

Sol

Given $f(x) = 2x^2 - 3x$.

$$f'(x) = \frac{d}{dx}(2x^2 - 3x) = 4x - 3.$$

(a) Now f will be strictly increasing when $f'(x) > 0$

i.e. $4x - 3 > 0$ i.e. $x > \frac{3}{4}$ i.e. $x \in \left(\frac{3}{4}, \infty\right)$

(b) f will be strictly decreasing when $f'(x) < 0$

i.e. $4x - 3 < 0$ i.e. $x < \frac{3}{4}$ i.e. $x \in \left(-\infty, \frac{3}{4}\right)$

Q No. 5 Find intervals in which function given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is (a) strictly ↑ (b) strictly ↓

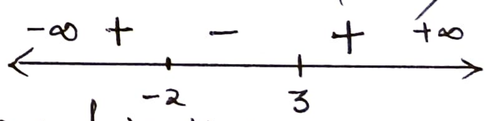
Sol.

Here $f(x) = 2x^3 - 3x^2 - 36x + 7$

$$f'(x) = 6x^2 - 6x - 36$$
$$= 6[x^2 - x - 6] = 6(x-3)(x+2)$$

$$f'(x) = 0 \Rightarrow 6(x-3)(x+2) = 0$$
$$\Rightarrow x = 3 \text{ or } x = -2 \quad (\text{Critical points})$$

Using Wavy Curve Method or Method of Intervals we find that



$f'(x) > 0$ when $x \in (-\infty, -2) \cup (3, \infty)$

and $f'(x) < 0$ when $x \in (-2, 3)$

∴ (a) $f(x)$ is strictly increasing when $x \in (-\infty, -2) \cup (3, \infty)$

(b) $f(x)$ is strictly decreasing when $x \in (-2, 3)$

Q No 6: Find the intervals for which the following functions are strictly increasing or decreasing:

(a) $x^2 + 2x - 5$ (b) $10 - 6x - 2x^2$ (c) $-2x^3 - 9x^2 - 12x + 1$

(d) $6 - 9x - x^2$ (e) $(x+1)^3 (x-3)^3$

Sol (a) $f(x) = x^2+2x-5$, $D_f = R$

and $f'(x) = 2x+2 = 2(x+1)$

Now $f'(x) > 0$ if $2(x+1) > 0$ ie if $x+1 > 0$

ie. if $x > -1$ ie. if $x \in (-1, \infty)$

$\therefore$ f is strictly increasing when $x > -1$ or $x \in (-1, \infty)$

Again. $f'(x) < 0$ if $2(x+1) < 0$ ie if $x+1 < 0$

i.e. if $x < -1$ ie if $x \in (-\infty, -1)$

$\therefore$ f is strictly decreasing when $x < -1$ or $x \in (-\infty, -1)$

(b) $f(x) = 10-6x-2x^2$, $D_f = R$

and $f'(x) = -6-4x = -2(3+2x)$

Now $f'(x) > 0$ if $-2(3+2x) > 0$

ie. if $3+2x < 0$ (Dividing by -2 inequality is reversed)

ie if $2x < -3$

ie if $x < -3/2$

ie $x \in (-\infty, -3/2)$

$\therefore$ f is strictly increasing in $(-\infty, -3/2)$

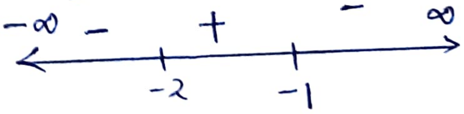
and f will be strictly decreasing in $(-\frac{3}{2}, \infty)$

(c) $f(x) = -2x^3 - 9x^2 - 12x + 1$, $D_f = R$

$f'(x) = -6x^2 - 18x - 12 = -6(x^2+3x+2) = -6(x+1)(x+2)$

$f'(x) = 0 \Rightarrow -6(x+1)(x+2) = 0 \Rightarrow x = -1, -2$

Using Wavy Curve method.



$f'(x) > 0$ when $x \in (-2, -1)$

$f'(x) < 0$ when $x \in (-\infty, -2) \cup (-1, \infty)$

(Keep in mind that constant multiplied with factors is $-$ve)

$\therefore$ f is strictly increasing in $(-2, -1)$

and f is strictly decreasing in $(-\infty, -2) \cup (-1, \infty)$

(d) $f(x) = 6-9x-x^2$, $D_f = R$

and $\therefore$ $f'(x) = -9-2x$

Now $f'(x) > 0$ if $-9-2x > 0$ ie $-9 > 2x$

ie $2x < -9$ ie $x < -\frac{9}{2}$ ie $x \in (-\infty, -9/2)$

$\therefore$ f is strictly increasing in $(-\infty, -9/2)$

and f will be strictly decreasing in $(-\frac{9}{2}, \infty)$

(e) $f(x) = (x+1)^3 (x-3)^3$, $D_f = R$

$f'(x) = (x+1)^3 \frac{d}{dx}(x-3)^3 + (x-3)^3 \cdot \frac{d}{dx}(x+1)^3$

$= (x+1)^3 \cdot 3(x-3)^2 \cdot 1 + (x-3)^2 3(x+1)^2 \cdot 1$

$= 3(x+1)^2 (x-3)^2 [x+1+x-3] = 3(x+1)^2(x-3)^2(2x-2)$

$= 6(x+1)^2(x-3)^2(x-1)$ and $\therefore$ $x = -1, 1, 3$ are critic-al points

Now We know that $6(x+1)^2(x-3)^2 \geq 0 \ \forall x \in R$

$\therefore$ $f'(x) > 0$ if $6(x+1)^2(x-3)^2(x-1) > 0$

ie if $x-1 > 0$ ie if $x > 1$ ie ~~if~~ $x \in (1, \infty) - \{3\}$

$\therefore$ f is strictly increasing in $(1, 3) \cup (3, \infty)$

Illy f will be strictly decreasing in

$(-\infty, -1) \cup (-1, 1)$ as $x = -1$ is critical point where $f'(x) = 0$

Q No 7: Show that $y = \log(1+x) - \frac{2x}{2+x}$; $x > -1$ is an increasing function of x throughout its domain.

Sol:

$y = \log(1+x) - \frac{2x}{2+x}$; $x > -1$.

$\frac{dy}{dx} = \frac{1}{1+x} - 2\frac{d}{dx}\left(\frac{x}{2+x}\right)$

$= \frac{1}{1+x} - 2\left\{\frac{(2+x)(1) - x(0+1)}{(2+x)^2}\right\}$

$$\frac{dy}{dx} = \frac{x^2}{(1+x)(2+x)^2}$$

Now as $x > -1$

$\Rightarrow x+1 > 0$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2}{(1+x)(2+x)^2} \geq 0 \quad \forall\ x > -1$$

Hence y is an increasing function of x throughout its domain.

Q No. 8: Find the values of x for which $y = [x(x-2)]^2$ is an increasing function.

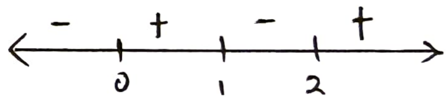
Sol:

Here $y = [x(x-2)]^2 = (x^2 - 2x)^2$

$$\frac{dy}{dx} = 2(x^2 - 2x)(2x-2) = 4x(x-2)(x-1)$$

Now $f'(x) = 0 \Rightarrow x = 0, 1, 2.$

Using Wavy curve method.



$f'(x) > 0$ if $x \in (0,1) \cup (2, \infty)$

and $f'(x) < 0$ if $x \in (-\infty, 0) \cup (1,2)$

$\therefore$ y is increasing function if $x \in (0,1) \cup (2,\infty)$

Q No 9: Prove that $y = \dfrac{4\sin\theta}{(2+\cos\theta)} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$

Sol:

Given $y = \dfrac{4\sin\theta}{(2+\cos\theta)} - \theta, \quad \theta \in \left[0, \frac{\pi}{2}\right]$

$$\Rightarrow \frac{dy}{d\theta} = \frac{(2+\cos\theta)(4\cos\theta) - 4\sin\theta(0-\sin\theta)}{(2+\cos\theta)^2} - 1$$

$$\frac{dy}{dx} = \frac{8\cos\theta + 4\cos^2\theta + 4\sin^2\theta - 4 - \cos^2\theta - 4\cos\theta}{(2+\cos\theta)^2}$$

$$= \frac{4\cos\theta - \cos^2\theta}{(2+\cos\theta)^2} = \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$$

Now for $\theta \in (0, \frac{\pi}{2})$ $0 < \cos\theta < 1$.

$\Rightarrow \frac{dy}{dx} = \frac{(4-\cos\theta)\cos\theta}{(2+\cos\theta)^2} > 0 \quad \forall \theta \in (0, \pi/2)$

$\Rightarrow$ y is an increasing function in $[0, \frac{\pi}{2}]$

Q.No 10 Prove that the logarithmic function is strictly increasing on $(0, \infty)$.

Sol:

Let $f(x) = \log x$; $D_f = (0, \infty)$

Now $f'(x) = \frac{1}{x}$.

Now $\forall x \in D_f$ i.e $x \in (0, \infty)$

$f'(x) = \frac{1}{x} > 0 \Rightarrow f'(x) > 0 \ \forall x \in (0, \infty)$

$\Rightarrow$ f is strictly increasing function on $(0, \infty)$

Q.No 11. Prove that the function f given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on $(-1, 1)$

Sol.

Given $f(x) = x^2 - x + 1$

$\Rightarrow f'(x) = 2x - 1$

Now $f'(x) > 0$ if $2x - 1 > 0$ ie if $x > \frac{1}{2}$ i.e $x \in (\frac{1}{2}, \infty)$

and $f'(x) < 0$ if $2x - 1 < 0$ if $x < \frac{1}{2}$ i.e $x \in (-\infty, \frac{1}{2})$

$\Rightarrow f'(x) > 0$ for $x \in (\frac{1}{2}, 1)$

and $f'(x) < 0$ for $(-1, \frac{1}{2})$

$\Rightarrow$ f is increasing function on $\left(\frac{1}{2}, 1\right)$ and decreasing on $\left(-1, \frac{1}{2}\right)$

Hence f is neither increasing nor decreasing on $(-1, 1)$

Q No 12: Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$?

(A) $\cos x$ (B) $\cos 2x$ (C) $\cos 3x$ (D) $\tan x$.

Sol. (A) Let $f(x) = \cos x$, then

$f'(x) = -\sin < 0 \ \forall x \in (0, \pi/2)$

$\Rightarrow f(x)$ is strictly decreasing in $(0, \pi/2)$

(B) $f(x) = \cos 2x \Rightarrow f'(x) = -2\sin 2x$.

Now $0 < x < \pi/2$

$\Rightarrow 0 < 2x < \pi$.

$\Rightarrow -2\sin 2x < 0 \ \forall x \in (0, \pi/2)$ $\left[\because \sin\theta \text{ is } +\text{ve in } (0, \pi)\right]$

$\Rightarrow f(x) = \cos 2x$ is strictly decreasing in $(0, \pi/2)$

(C) $f(x) = \cos 3x \Rightarrow f'(x) = -3\sin 3x$

Now $0 < x < \pi/2 \Rightarrow 0 < 3x < \frac{3\pi}{2}$.

Now $\sin 3x$ is +ve in $(0, \pi)$ and −ve in $(\pi, 3\pi/2)$

$\Rightarrow f'(x) = -3\sin 3x$ assumes +ve as well as −ve values in $(0, \pi/2)$

$\Rightarrow f$ is neither increasing nor decreasing on $(0, \pi/2)$

(D) $f(x) = \tan x$, then $f'(x) = \sec^2 x$

and $f'(x) = \sec^2 x = (\sec x)^2 > 0 \ \forall x \in (0, \pi/2)$

$\Rightarrow f(x)$ is strictly increasing on $(0, \pi/2)$

Thus, functions given A and B are strictly decreasing on $(0, \pi/2)$

Q No. 13 On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ strictly decreasing?

(A) $(0,1)$ (B) $\left(\frac{\pi}{2}, \pi\right)$ (C) $(0, \pi/2)$ (D) None of these.

Sol. Given $f(x) = x^{100} + \sin x - 1$, $D_f = R$

$f'(x) = 100x^{99} + \cos x \neq 0.$

(A) $f'(x) > 0 \quad \forall\, x \in (0,1)$

$\therefore$ $f(x)$ is strictly increasing on $(0,1)$

(B) $f'(x) > 0 \;\forall\; x \in \left(\frac{\pi}{2}, \pi\right)$.

$\therefore$ $f(x)$ is strictly increasing on $(\pi/2, \pi)$

(C) $f'(x) > 0 \;\forall\; x \in (0, \pi/2) \Rightarrow f(x)$ is strictly increasing

$\therefore$ $f(x)$ is not decreasing is any of given interval.

$\therefore$ D is correct option.

Q No. 14: Find the least value of a such that the functions f given by $f(x) = x^2 + ax + 1$ is strictly increasing on $(1,2)$?

Sol: Given $f(x) = x^2 + ax + 1$; $1 < x < 2$.

$\therefore$ $f'(x) = 2x + a$.

Now for $1 < x < 2$

$2 < 2x < 4$

$\Rightarrow 2 + a < 2x + a < 4 + a. \Rightarrow 2 + a < f'(x) < 4 + a.$

Now for $f(x)$ to be increasing strictly on $(1,2)$

$f'(x) > 0$

ie if $2 + a \geq 0$ ie if $a \geq -2$

$\therefore$ Required least value of a for which $f(x) = x^2 + ax + 1$ is strictly increasing on $(1,2)$ is -2

Q No 15 Let I be any interval disjoint from $[-1, 1]$. Prove that function given by $f(x) = x + \frac{1}{x}$ is strictly increasing on I.

Sol. Given $f(x) = x + \frac{1}{x}$; $x \in I$

$$\Rightarrow \quad f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}.$$

Now $f'(x) = \frac{x^2-1}{x^2} > 0 \quad \forall\ x$ for which $x^2 - 1 > 0$ $\left[\because x^2 > 0 \ \forall\ x > 0\right]$

ie $\quad (x-1)(x+1) > 0$

ie $\quad x \in (-\infty, -1) \cup (+1, \infty)$

ie $\quad x \in R - [-1, 1]$

$\therefore$ $f(x)$ is strictly in-creasing on I.

Q No 16 Prove that the function f given by $f(x) = \log(\sin x)$ is st. increasing on $(0, \pi/2)$ and st. decreasing on $(\pi/2, \pi)$

Sol: Given $f(x) = \log(\sin x)$

$$\Rightarrow \quad f'(x) = \frac{1}{\sin x} \cdot \cos x = \cot x.$$

Now As $\cot x > 0 \ \forall\ x \in (0, \pi/2)$ and $\cot x < 0 \ \forall x \in (\pi/2, \pi)$

$\therefore$ $f(x)$ is strictly increasing on $(0, \pi/2)$ and strictly decreasing on $(\frac{\pi}{2}, \pi)$

Q No 17. Prove that the function given by $f(x) = \log \cos x$ is strictly decreasing on $(0, \pi/2)$ and strictly decreasing on $(\pi/2, \pi)$

Sol: ★ Statement of the question is not correct as $\log(\cos x)$ is not defined for $x \in (\pi/2, \pi)$ as $\cos x < 0 \ \forall\ x \in (\pi/2, \pi)$

Correct statement should be $f(x) = \log|\cos x|$ for $x \in (\pi/2, \pi)$

Now Given $f(x) = \log \cos x$, $x \in (0, \pi/2) \cup (\pi/2, \pi)$

$$\Rightarrow \quad f'(x) = \frac{1}{\cos x}(-\sin x) = -\tan x$$

As $\tan x > 0 \ \forall \ x \in (0, \pi/2)$ and $\tan x < 0 \ \forall \ x \in (\pi/2, \pi)$

$\Rightarrow$ $f'(x) < 0 \ \forall \ x \in (0, \pi/2)$ and $f'(x) > 0 \ \forall \ x \in (\pi/2, \pi)$

$\Rightarrow$ f is strictly decreasing on $(0, \pi/2)$ and.
f is strictly increasing on $(\pi/2, \pi)$

Q No. 18 : Prove that $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R.

Sol. Given $f(x) = x^3 - 3x^2 + 3x - 100$

$\Rightarrow$ $f'(x) = 3x^2 - 6x + 3 = 3(x^2 - 2x + 1) = 3(x-1)^2$

Now as $3(x-1)^2 \geq 0 \ \forall \ x \in R$

$\Rightarrow$ $f'(x) \geq 0 \ \forall \ x \in R$

$\Rightarrow$ $f(x)$ is increasing on R.

Q No 19 : The interval in which $y = x^2 e^{-x}$ is increasing is
(A) $(-\infty, \infty)$ (B) $(-2, 0)$ (C) $(2, \infty)$ (D) $(0, 2)$

Sol

$$y = x^2 e^{-x}$$

$$\therefore \frac{dy}{dx} = x^2 . e^{-x}(-1) + e^{-x} . 2x = xe^{-x}[-x+2]$$

$$= xe^{-x}(2-x)$$

Now y is strictly increasing if $\frac{dy}{dx} > 0$

ie if $x . e^{-x}(2-x) > 0$

ie if $x(2-x) > 0$ $\quad [\because e^{-x} = \frac{1}{e^x} > 0 \ \forall \ x \in R]$

ie if $x(x-2) < 0$

ie if $0 < x < 2$

ie if $x \in (0, 2)$

$\therefore$ Option D is correct interval.

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