13 Chapter

Functions

Quantities of various characters such as length, area, mass, temperature and volume either have constant values or they vary based on the values of other quantities. Such quantities are called constant and variable respectively.

Function is a concept of mathematics that studies the dependence between variable quantities in the process of their change. For instance, with a change in the side of a square, the area of the square also varies. The question of how the change in the side of the square affects the area is answered by a mathematical relationship between the area of the square and the side of the square.

Let the variable x take on numerical values from the set D.

A function is a rule that attributes to every number x from D one definite number y where y belongs to the set of real numbers.

Here, x is called the independent variable and y is called the dependent variable.

The set D is referred to as the *domain of definition* of the function and the set of all values attained by the variable y is called the *range of the function*.

In other words, a variable y is said to be the value of function of a variable x in the domain of definition D if to each value of x belonging to this domain there corresponds a definite value of the variable y.

This is symbolised as $y = f(x)$ where f denotes the rule by which y varies with x.

BASIC METHODS OF REPRESENTING FUNCTIONS

Analytical Representation

This is essentially representation through a formula.

This representation could be a uniform formula in the entire domain, for example, $y = 3x^2$

or

by several formulae which are different for different parts of the domain.

Example: $y = 3x^2$ if $x < 0$

and $y = x^2$ if $x > 0$

In analytical representations, the domain of the function is generally understood as the set of values for which the equation makes sense.

For instance, if $y = x^2$ represents the area of a square then we get that the domain of the function is $x > 0$.

Problems based on the analytical representation of functions have been a favourite for the XLRI exam and have also become very common in the CAT over the past few years. Other exams are also moving towards asking questions based on this representation of functions.

Tabular Representation of Functions

For representing functions through a table, we simply write down a sequence of values of the independent variable x and then write down the corresponding values of the dependent variable y. Thus, we have tables of logarithms, trigonometric values and so forth, which are essentially tabular representations of functions.

The types of problems that appear based on tabular representation have been restricted to questions that give a table and then ask the student to trace the appropriate analytical representation or graph of the function based on the table.

Graphical Representation of Functions

This is a very important way to represent functions. The process is: on the coordinate xy plane for every value of x from the domain D of the function, a point $P(x, y)$ is constructed whose abscissa is x and whose ordinate y is got by putting the particular value of x in the formula representing the function.

For example, for plotting the function $y = x^2$, we first decide on the values of x for which we need to plot the graph.

Thus we can take $x = 0$ and get $y = 0$ (means the point (0, 0) is on the graph).

Then for $x = 1$, $y = 1$; for $x = 2$, $y = 4$; for $x = 3$, $y = 9$ and for $x = –1$ $y = 1$; for $x = –2$, $y = 4$, and so on.

EVEN AND ODD FUNCTIONS

Even Functions

Let a function $y = f(x)$ be given in a certain interval. The function is said to be even if for any value of x

Æ $f(x) = f(–x)$

Properties of even functions:

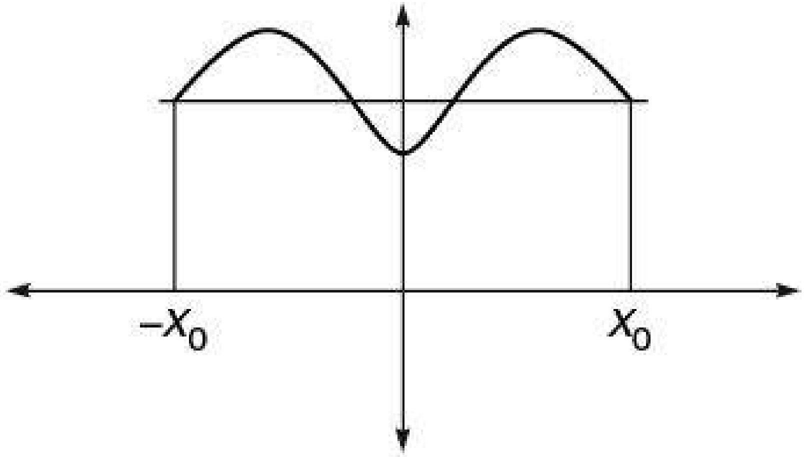
(a) The sum, difference, product and quotient of an even function is also an even function.

(b) The graph of an even function is symmetrical about the y-axis.

However, when y is the independent variable, it is symmetrical about the x-axis. In other words, if $x = f(y)$ is an even function, then the graph of this function will be symmetrical about the x-axis. Example: $x = y^2$.

Examples of even functions: $y = x^2$, $y = x^4$, $y = –3x^8$, $y = x^2 + 3$, $y = x^4/5$, $y = | x |$ are all even functions.

The symmetry about the y-axis of an even function is illustrated below.



Odd Functions

Let a function $y = f(x)$ be given in a certain interval. The function is said to be odd if for any value of x

Æ $f(x) = -f(-x)$

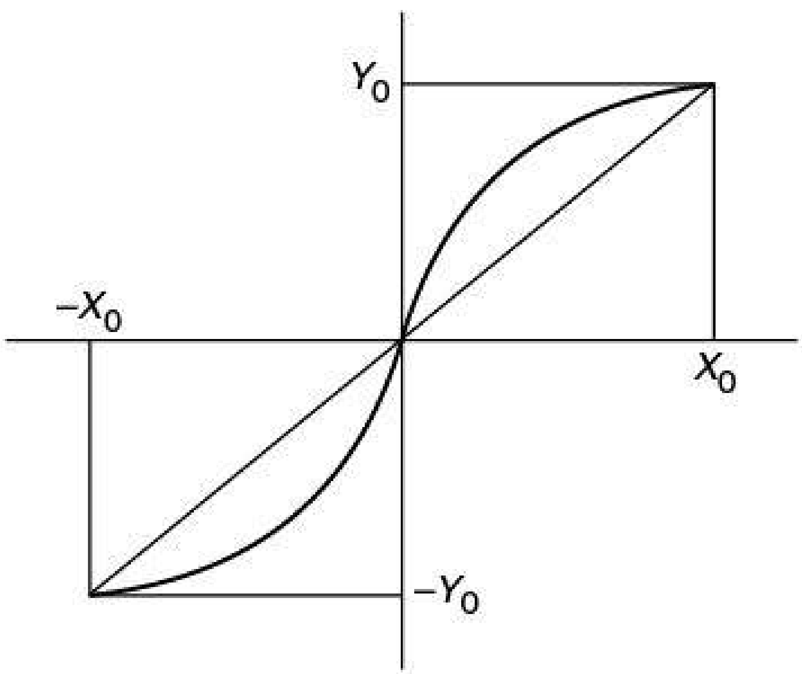
Properties of odd functions.

(a) The sum and difference of an odd function is an odd function.

(b) The product and quotient of an odd function is an even function.

(c) The graph of an odd function is symmetrical about the origin.

The symmetry about the origin of an odd function is illustrated below.



Examples of odd functions $y = x^3$, $y = x^5$, $y = x^3 + x$, $y = x/(x^2 + 1)$.

Not all functions need be even or odd. However, every function can be represented as the sum of an even function and an odd function.

Inverse of a Function

Let there be a function $y = f(x)$, which is defined for the domain D and has a range R.

Then, by definition, for every value of the independent variable x in the domain D, there exists a certain value of the dependent variable y. In certain cases the same value of the dependent variable y can be got for different values of x. For example, if $y = x^2$, then for $x = 2$ and $x = -2$ give the value of y as 4.

In such a case, the inverse function of the function $y = f(x)$ *does not exist*.

However, if a function $y = f(x)$ is such that for every value of y (from the range of the function R) there corresponds one and only one value of x from the domain D, then the inverse function of $y = f(x)$ exists and is given by $x = g(y)$. Here it can be noticed that x becomes the dependent variable and y becomes the independent variable. Hence, this function has a domain R and a range D.

Under the above situation, the graph of $y = f(x)$ and $x = g(y)$ are one and the same.

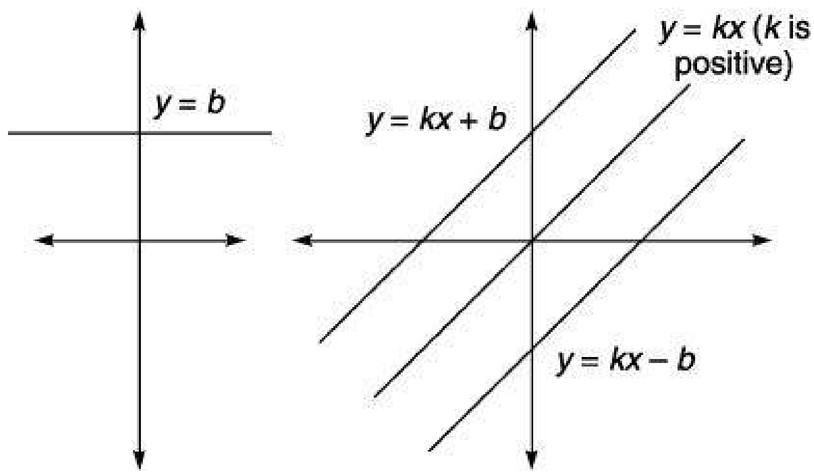
However, when denoting the inverse of the function, we normally denote the independent variable by y and, hence, the inverse function of $y = f(x)$ is denoted by $y = g(x)$ and not by $x = g(y)$.

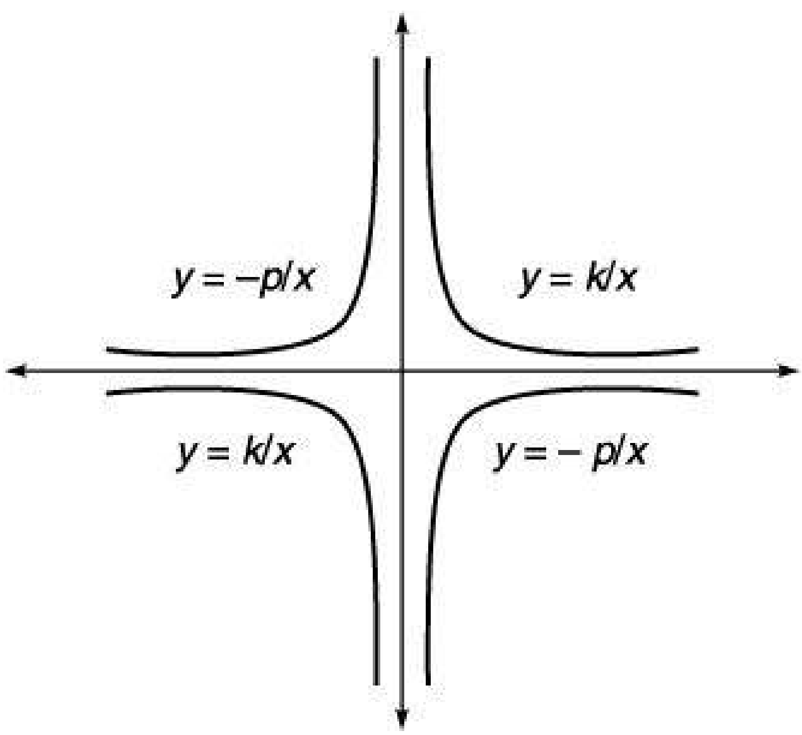
The graphs of two inverse functions when this change is used are symmetrical about the line $y = x$ (which is the bisector of the first and the third quadrants).

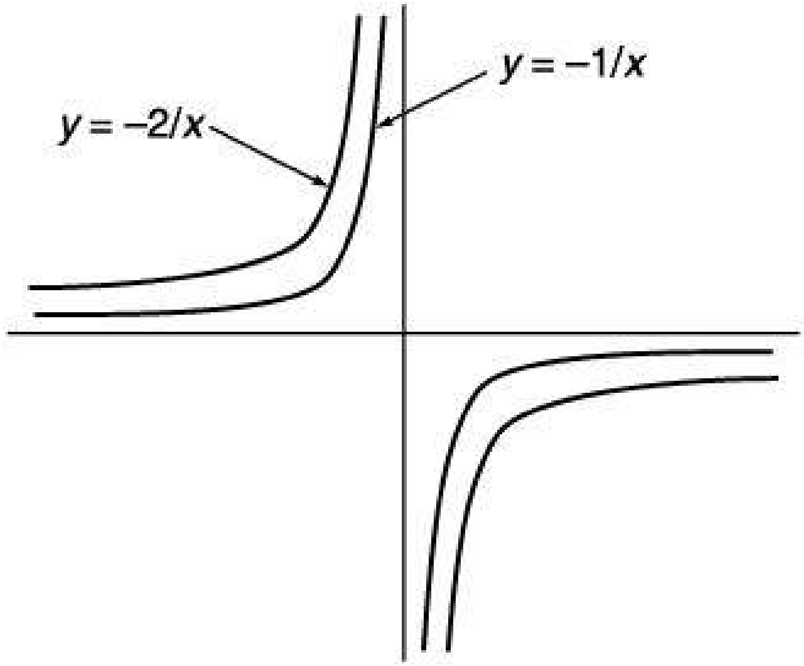
Graphs of Some Simple Functions The student is advised to familiarise himself/herself with the following figures.

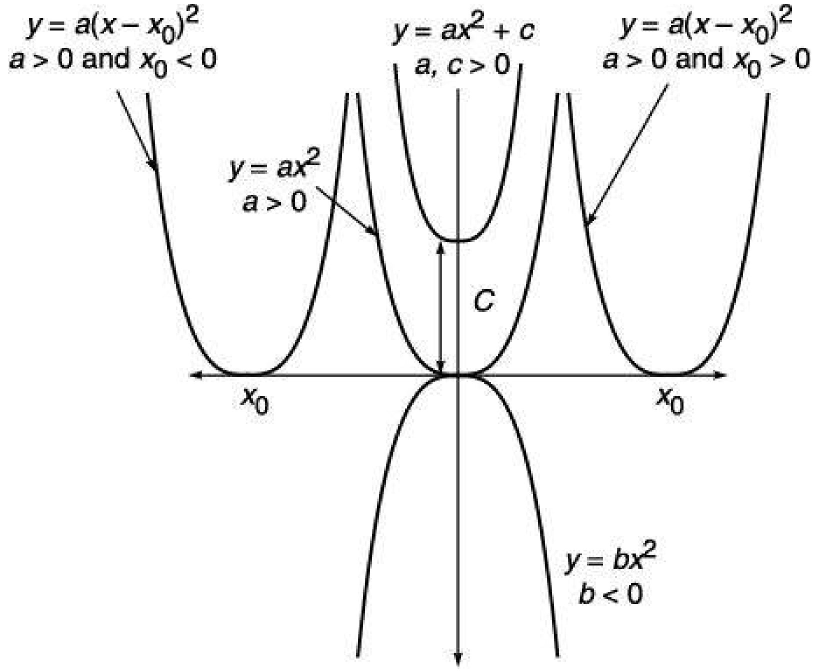
Graphs of $y = b$, $y = kx$, $y = kx + b$, $y = kx - b$.

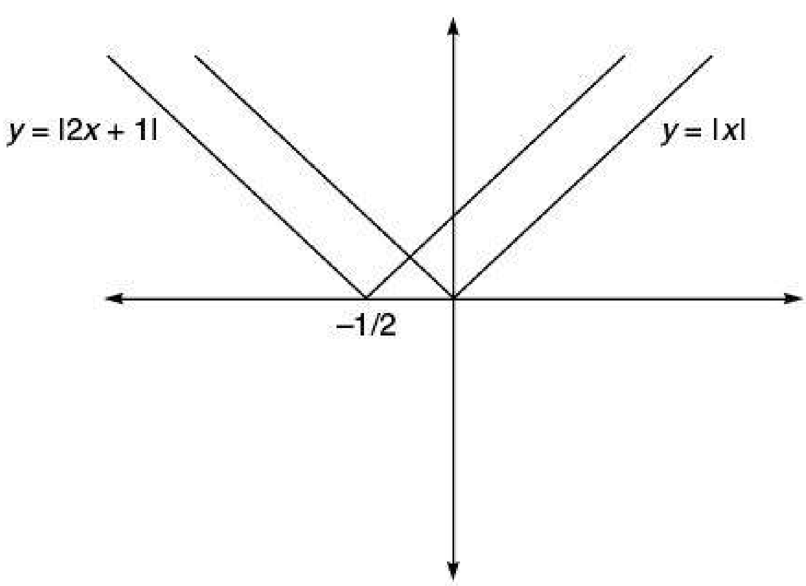
Note the shifting of the line when a positive number b is added and subtracted to the function's equation.











SHIFTING OF GRAPHS

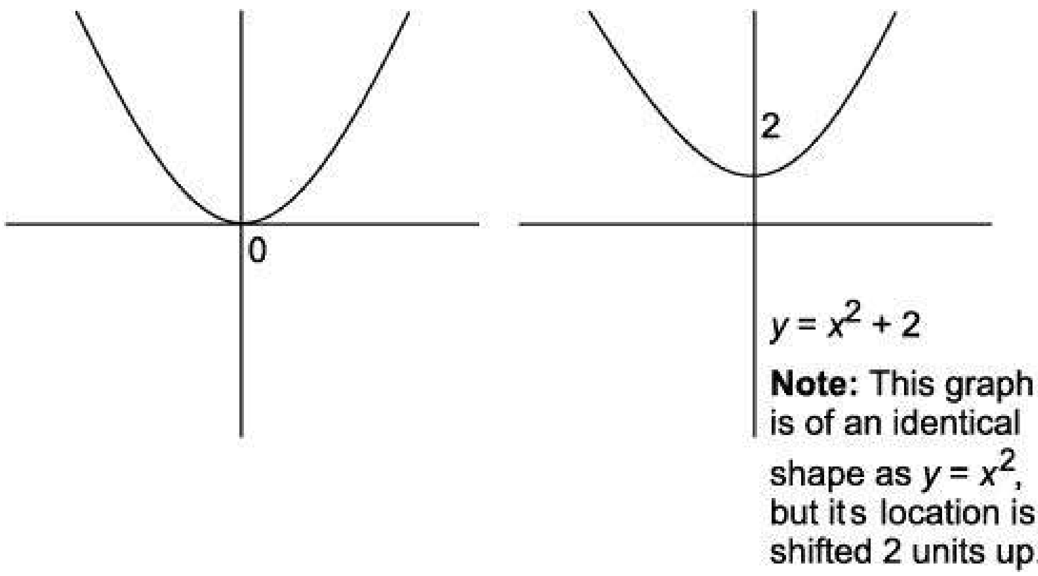
The ability to visualize how graphs shift when the basic analytical expression is changed is a very important skill. For instance if you knew how to visualize the graph of $(x + 2)^2 - 5$, it will definitely add a lot of value to your ability to solve questions of functions and all related chapters of block 5 graphically.

In order to be able to do so, you first need to understand the following points clearly:

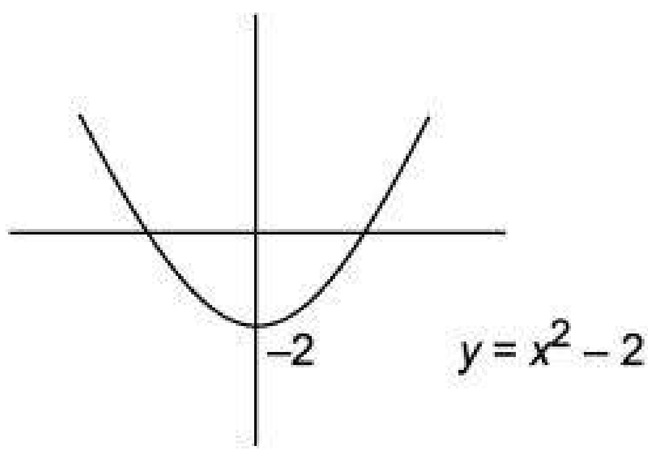
(1) The relationship between the graph of $y = f(x)$ and $y = f(x) + c$ (where c represents a positive constant.): The shape of the graph of $y = f(x) + c$ will be the same as that of the $y = f(x)$ graph. The only difference would be in terms of the fact that $f(x) + c$ is shifted c units up on the $x - y$ plot.

The following figure will make it clear for you:

Example: Relationship between $y = x^2$ and $y = x^2 + 2$.

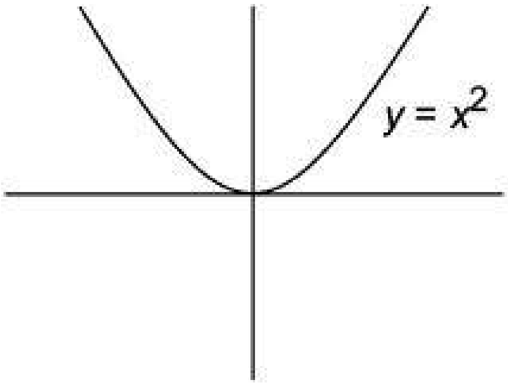


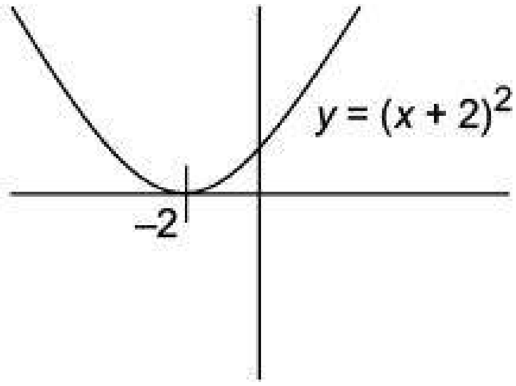
(2) The relationship between $y = f(x)$ and $y = f(x) - c$: In this case while the shape remains the same, the position of the graph gets shifted c units down.



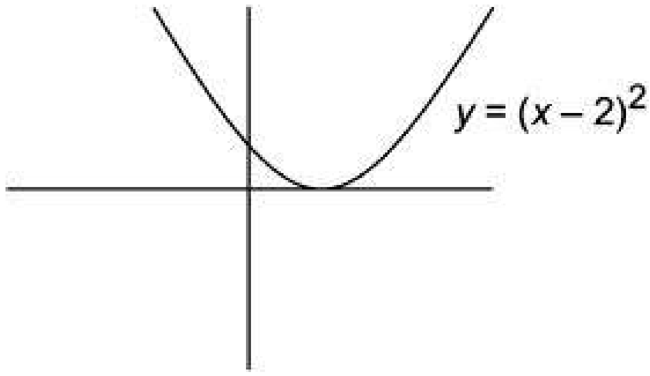
(3) The relationship between $y = f(x)$ and $y = f(x + c)$: In this case the graph will get shifted c units to the left. (Remember, c was a positive constant)

Example:





(4) **The relationship between $y = f(x)$ and $y = f(x - c)$:** In this case the graph will get shifted c units to the right on the $x - y$ plane.



COMBINING MOVEMENTS

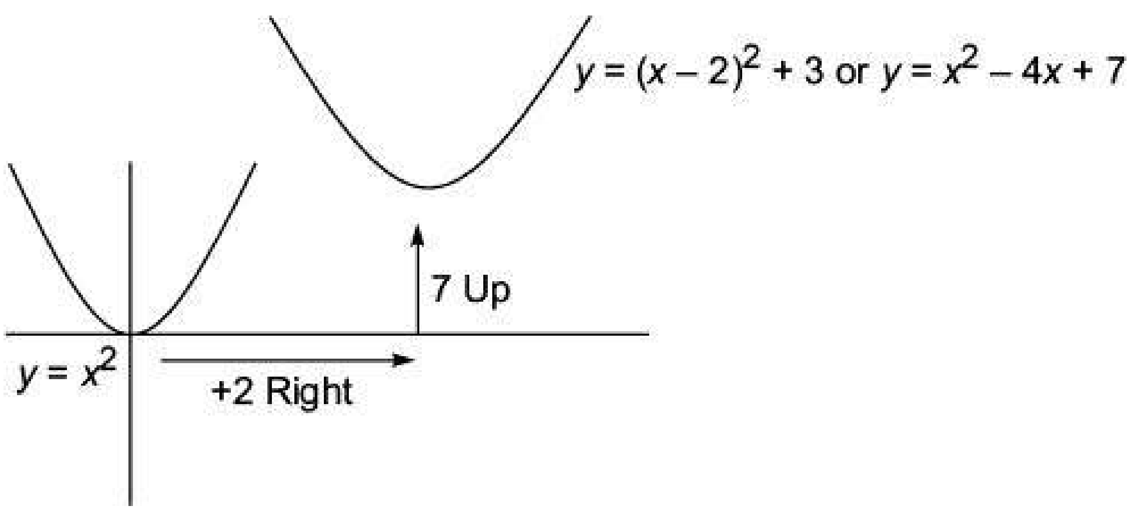
It is best understood through an example:

Visualizing a graph for a function like $x^2 - 4x + 7$.

First convert $x^2 - 4x + 7$ into $(x - 2)^2 + 3$

[**Note:** In order to do this conversion, the key point of your thinking should be on the $-4x$. Your first focus has to be to put down a bracket $(x - a)^2$ which on expansion gives $-4x$ as the middle term. When you think this way, you will get $(x - 2)^2$. On expansion $(x - 2)^2 = x^2 - 4x + 4$. But you wanted $x^2 - 4x + 7$. Hence add +3 to $(x - 2)^2$. Hence the expression $x^2 - 4x + 7$ is equivalent to $(x - 2)^2 + 3$.]

To visualize $(x - 2)^2 + 3$ shift the x^2 graph two units right [to account for $(x - 2)^2$] and 3 units up [to account for the +3] on the $x - y$ plot. This will give you the required plot.



Task for the student: I would now like to challenge and encourage you to think of how to add and multiply functions graphically.

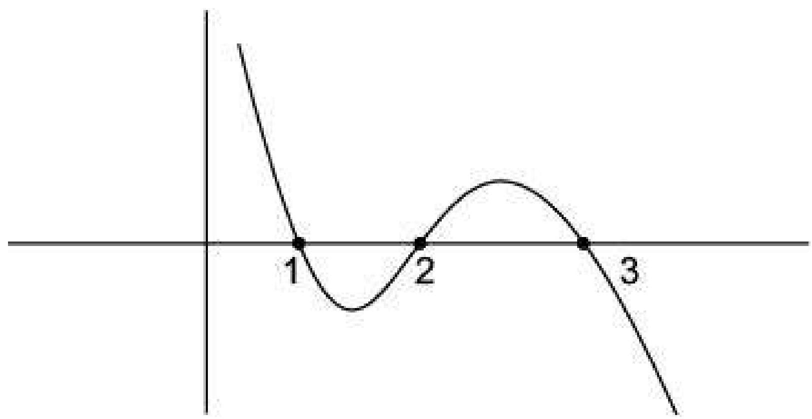
INEQUALITIES

The Logical Graphical Process for Solving Inequalities

Your knowledge of the standard graphs of functions and how these shift can help you immensely while solving inequalities.

Thus, for instance if you are given an inequality question based on a quadratic function like $ax^2 + bx + c < 0$ (and a is positive) you should realize that the curve will be *U* shaped. And the inequality would be satisfied between the roots of the quadratic equation $ax^2 + bx + c = 0$. [Remember, we have already seen and understood that the solution of an equation $f(x) = 0$ is seen at the points where the graph of $y = f(x)$ cuts the x axis.]

Similarly, for a cubic curve like the one shown below, you should realize that it is greater than 0 to the left of the point 1 shown in the figure. This is also true between points 2 and 3. At the same time the function is less than zero between points 1 and 2 and to the right of point 3. (on the x-axis.)

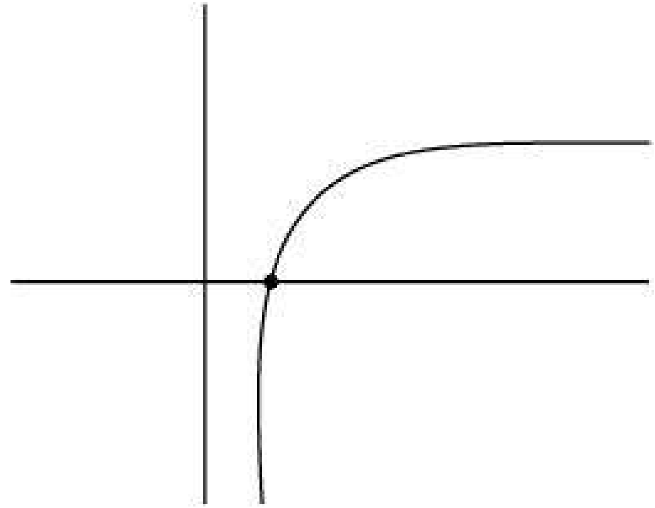


Another important point to note is that in the case of strict inequalities (i.e. inequalities with the '<' or '>' sign) the answer will also consist of strict inequalities only. On the other hand in the case of slack inequalities (inequalities having £ or ≥ sign) the solution of the inequality will also have a slack inequality sign.

LOGARITHMS

Graphical View of the Logarithmic Function

The typical logarithmic function is shown in the graph below:



Note the following points about the logarithmic function $y = \log x$.

(1) It is only defined for positive values of x.

(2) For values of x below 1, the logarithmic function is negative. At the same time for $x = 1$, the logarithmic function has a value of 0. (Irrespective of the base)

(3) The value of $\log x$ becomes 2, when the value of x becomes equal to the square of the base.

(4) As we go further right on the x axis, the graph keeps increasing. However, this increase becomes more and more gradual and hence the shape of the graph becomes increasingly flatter as we move further on the x axis.

WORKED-OUT PROBLEMS

Problem 13.1 Find the domain of the definition of the function $y = 1 /(x^2 - 2x)^{1/2}$

(a) (–•, –2)　　(b) (–•, + •) except [0, 2]

(c) (2, + •)　　(d) (–•, 0)

Solution For the function to be defined, the expression under the square root should be non-negative and the denominator should not be equal to zero.

So, $x^2 - 2x > 0$ and $(x^2 - 2x)$ π 0

or, $x (x - 2) > 0$ or $(x^2 - 2x)$ π 0

So, x won't lie in between 0 and 2 and x π 0, x π 2.

So, x will be x (–• , + •) excluding the range 0 £ x is £ 2.

In exam situations, to solve the above problem, you should check the options as below.

In fact, for solving all questions on functions, the student should explore the option-based approach.

Often you will find that going through the option-based approach will help you save a significant amount of time. The student should try to improve his/her selection of values through practice so that he/she is able to eliminate the maximum number of options on the basis of every check. The student should develop a knack for disproving three options so that the fourth automatically becomes the answer. It should also be noted that if an option cannot be disproved, it means that it is the correct option.

What I am trying to say will be clear from the following solution process.

For this question, if we check at $x = 3$, the function is defined. However, $x = 3$ is outside the ambit of option *a* and *d*. Hence, *a* and *d* are rejected on the basis of just one value check, and *b* or *c* has to be the answer.

Alternately, you can try to disprove each and every option one by one.

Problem 13.2 Which of the following is an even function?

(a) $| x^2 | - 5x$　　(b) $x^4 + x^5$

(c) $e^{2x} + e^{-2x}$　　(d) $| x |^2/x$

Solution Use options for solving.

If a function is even it should satisfy the equation $f(x) = f(-x)$.

We now check the four options to see which of them represents an even function.

Checking option (a) $f(x) = | x^2 | - 5x$

Putting $-x$ in the place of x.

$f (x) = | (-x)^2 | - 5(-x)$

$= | x^2 | + 5(x)$ π $f(x)$

Checking option (b) $f(x) = x^4 + x^5$.

Putting $(-x)$ at the place of x,

$f(-x) = (-x)^4 + (-x)^5 = x^4 - x^5$ π $f(x)$

Checking option (c), $f(x) = e^{2x} + e^{-2x}$

Putting $(-x)$ at the place of x.

$f(-x) = e^{-2x} + e^{-(-2x)} = e^{-2x} + e^{2x} = f(x)$

So (c) is the answer.

You do not need to go further to check for *d*. However, if you had checked, you would have been able to disprove it as follows:

Checking option (d), $f(x) = |x|^2/x$

Putting $f(x)$ at the place of x,

$f(-x) = |-x|^2/-x = |x^2|/-x–\pi f(-x)$

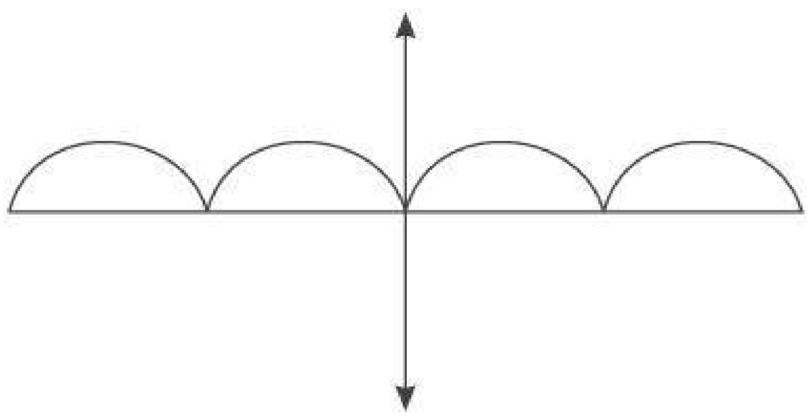
Directions for Questions 13.3–13.6:

Mark (a) if $f(-x) = f(x)$

Mark (b) if $f(-x) = -f(x)$

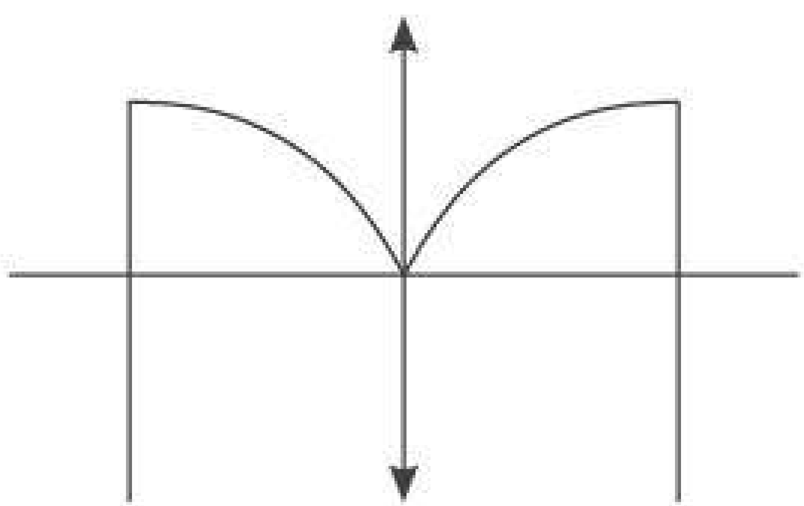
Mark (c) if neither (a) nor (b)

Problem 13.3



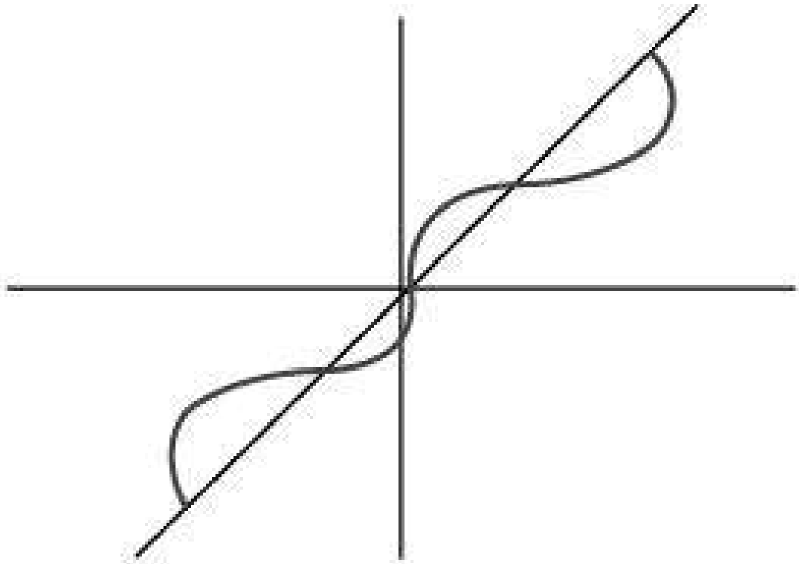
Solution The graph is symmetrical about the *y*-axis. This is the definition of an even function. So (a).

Problem 13.4



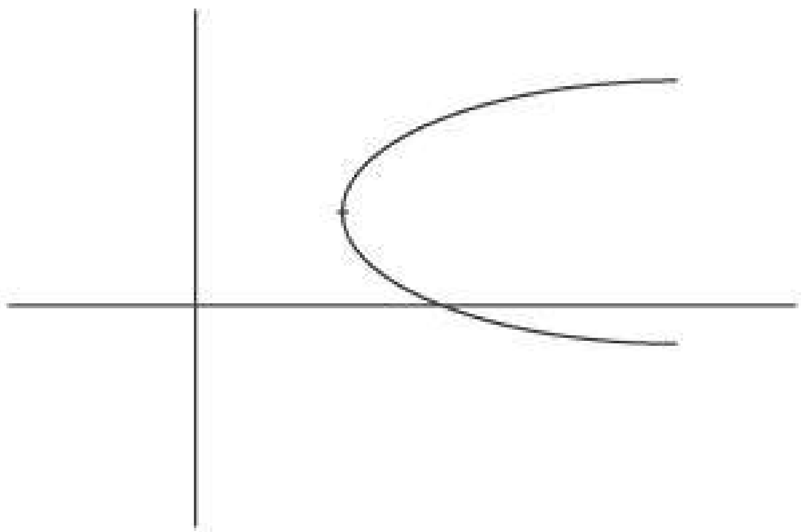
Solution The graph is symmetrical about the *y*-axis. This is the definition of an even function. So (a).

Problem 13.5



Solution The graph is symmetrical about origin. This is the definition of an odd function. So (b).

Problem 13.6



Solution The graph is neither symmetrical about the y-axis nor about origin. So (c).

Problem 13.7 Which of the following two functions are identical?

(a) $f(x) = x^2/x$ (b) $g(x) = (\sqrt{x})^2$

(c) $h(x) = x$

(i) (a) and (b) (ii) (b) and (c)

(iii) (a) and (c) (iv) None of these

Solution For two functions to be identical, their domains should be equal.

Checking the domains of $f(x)$, $g(x)$ and $h(x)$,

$f(x) = x^2/x$, x should not be equal to zero.

So, domain will be all real numbers except at $x = 0$.

$g(x) = (\sqrt{x})^2$, x should be non-negative.

So, domain will be all positive real numbers.

$h(x) = x$, x is defined every where.

So, we can see that none of them have the same domain.

Hence, (*d*) is the correct option.

Problem 13.8 If $f(x) = 1/x$, $g(x) = 1/(1 - x)$ and $h(x) = x^2$, then find *fogoh* (2).

(a) –1 (b) 1

(c) 1/2 (d) None of these

Solution *fogoh* (2) is the same as $f(g(h(2)))$

To solve this, open the innermost bracket first. This means that we first resolve the function $h(2)$. Since $h(2) = 4$ we will get

$f(g(h(2))) = f(g(4)) = f(-1/3) = -3$. Hence, the option (d) is the correct answer.

Read the instructions below and solve Problems 11.9 and 11.10.

$A * B = A^3 - B^3$

$A + B = A - B$

$A - B = A/B$

Problem 13.9 Find the value of (3 * 4) – (8 + 12).

(a) 9 (b) 9.25

(c) –9.25 (d) None of these

Solution Such problems should be solved by the BODMAS rule for sequencing of operations.

Solving, thus, we get: (3 * 4) – (8 + 12)

= –37 – (–4). [Note here that the '–' sign between –37 and –4 is the operation defined above.]

= 37/4 = 9.25

Problem 13.10 Which of the following operation will give the sum of the reciprocals of *x* and *y* and unity?

(a) $(x + y) * (x - y)$

(b) $[(x * y) - x] - y$

(c) $[\{(x * y) - (x + y)\} - x] - y$

(d) $(x + y) - (x - y)$

For solving questions containing a function in the question as well as a function in the options (where values are absent), the safest process for students weak at math is to assume certain convenient values of the variables in the expression and checking for the correct option that gives us equality with the expression in the question. The advantages of this process of solution is that there is very little scope for making mistakes. Besides, if the expression is not simple and directly visible, this process takes far less time as compared to simplifying the expression from one form to another.

This process will be clear after perusing the following solution to the above problem.

Solution The problem statement above defines the expression: $(1/x) + (1/y) + 1$ and asks us to find out which of the four options is equal to this expression. If we try to simplify, we can start from the problem expression and rewrite it to get the correct option. However, in the above case this will become extremely complicated since the symbols are indirect. Hence, if we have to solve through simplifying, we should start from the options one by one and try to get the problem expression. However, this is easier said than done and for this particular problem, going through this approach will take you at least two minutes plus.

Hence, consider the following approach:

Take the values of x and y as 1 each. Then,

$(1/x) + (1/y) + 1 = 3$

Put the value of x and y as 1 each in each of the four options that we have to consider.

Option (a) will give a value of –1 π 3. Hence, option (a) is incorrect.

Opton (b) will give a value of 0. 0 π 3. Hence, option (b) is incorrect.

Option (c) gives an answer of 3. Hence, option (c) could be the answer.

Option (d) gives an answer of 0. 0 π 3. Hence, option (d) is incorrect.

Now since options (a), (b) and (d) are incorrect and option (c) is the only possibility left, it has to be the answer.

[Note that in case there is a fourth option of ‘none of these’ then we have to be careful before marking the answer.]

LEVEL OF DIFFICULTY (I)

1. Find the domain of the definition of the function $y = |x|$.

   (a) 0 £ x (b) –• < x < + •

   (c) x < + • (d) 0 £ x < + •

2. Find the domain of the definition of the function $y = \sqrt{x}$.

   (a) –• < x < + • (b) x £ 0

   (c) $x > 0$ (d) $x \geq 0$

3. Find the domain of the definition of the function $y = \left|\sqrt{x}\right|$.

   (a) $x \geq 0$ (b) –• < x < + •

   (c) $x > 0$ (d) x < + •

4. Find the domain of the definition of the function $y = (x – 2)^{1/2} + (8 – x)^{1/2}$.

   (a) All the real values except 2 £ x £ 8

   (b) 2 £ x

   (c) 2 £ x £ 8

   (d) x £ 8

5. Find the domain of the definition of the function $y = (9 – x^2)^{1/2}$.

   (a) –3 £ x £ 3 (b) (–•, –3] » [3, •)

   (c) –3 £ x (d) x £ 3

6. Find the domain of the definition of the function $y = 1/(x^2 – 4x + 3)$.

   (a) 1 £ x £ 3

   (b) (–•, –3) » (3, •)

   (c) $x = (1, 3)$

   (d) –• < x < •, excluding 1, 3

7. The values of x for which the functions $f(x) = x$ and $g(x) = (\sqrt{x})^2$ are identical is

   (a) – • < x < + • (b) $x \geq 0$

   (c) $x > 0$ (d) x £ 0

8. The values of x for which the functions $f(x) = x$ and $g(x) = x^2/x$ are identical is

   (a) Set of real numbers excluding 0

   (b) Set of real numbers

   (c) $x \geq 0$

   (d) $x > 0$

9.

If $f(x) = \sqrt{x^3}$, then $f(3x)$ will be equal to

(a) $\sqrt{3x^3}$

(b) $3\sqrt{x^3}$

(c) $3\sqrt{(3x^3)}$

(d) $3\sqrt{x^5}$

10. If $f(x) = e^x$, then the value of 7 $f(x)$ will be equal to

(a) e^{7x}

(b) $7e^x$

(c) 7 e^{7x}

(d) e^x

11. If $f(x) = \log x^2$ and $g(x) = 2 \log x$, then $f(x)$ and $g(x)$ are identical for

(a) –• < x < + •

(b) 0 £ x < •

(c) –• x £ 0

(d) 0 < x < •

12. If $f(x)$ is an even function, then the graph $y = f(x)$ will be symmetrical about

(a) x-axis

(b) y-axis

(c) Both the axes

(d) None of these

13. If $f(x)$ is an odd function, then the graph $y = f(x)$ will be symmetrical about

(a) x-axis

(b) y-axis

(c) Both the axes

(d) origin

14. Which of the following is an even function?

(a) x^{-8}

(b) x^3

(c) x^{-33}

(d) x^{73}

15. Which of the following is not an odd function?

(a) $(x + 1)^3$

(b) x^{23}

(c) x^{53}

(d) x^{77}

16. For what value of x, $x^2 + 10x + 11$ will give the minimum value?

(a) 5

(b) +10

(c) –5

(d) –10

17. In the above question, what will be the minimum value of the function?

(a) –14

(b) 11

(c) 86

(d) 0

18. Find the maximum value of the function $1/(x^2 - 3x + 2)$.

(a) 11/4

(b) 1/4

(c) 0

(d) None of these

19. Find the minimum value of the function $f(x) = \log_2 (x^2 - 2x + 5)$.

(a) –4

(b) 2

(c) 4 (d) –2

20. $f(x)$ is any function and $f^{-1}(x)$ is known as inverse of $f(x)$, then $f^{-1}(x)$ of $f(x) = \frac{1}{x} + 1$ is

(a) $\frac{1}{x} - 1$ (b) $x - 1$

(c) $\frac{1}{(x-1)}$ (d) $\frac{1}{x+1}$

Directions for Questions 21 to 23: Read the instructions below and solve.

$f(x) = f(x - 2) - f(x - 1)$, x is a natural number

$f(1) = 0$, $f(2) = 1$

21. The value of $f(8)$ is

(a) 0 (b) 13

(c) –5 (d) –9

22. The value of $f(7) + f(4)$ is

(a) 11 (b) –6

(c) –12 (d) 12

23. What will be the value of $\sum_{n=1}^{9} f(n)$?

(a) –12 (b) –15

(c) –14 (d) –13

24. What will be the domain of the definition of the function $f(x) = {}^{8-x}C_{5-x}$ for positive values of x?

(a) {1, 2, 3} (b) {1, 2, 3, 4}

(c) {1, 2, 3, 4, 5} (d) {1, 2, 3, 4, 5, 6, 7, 8}

Directions for Questions 25 to 38:

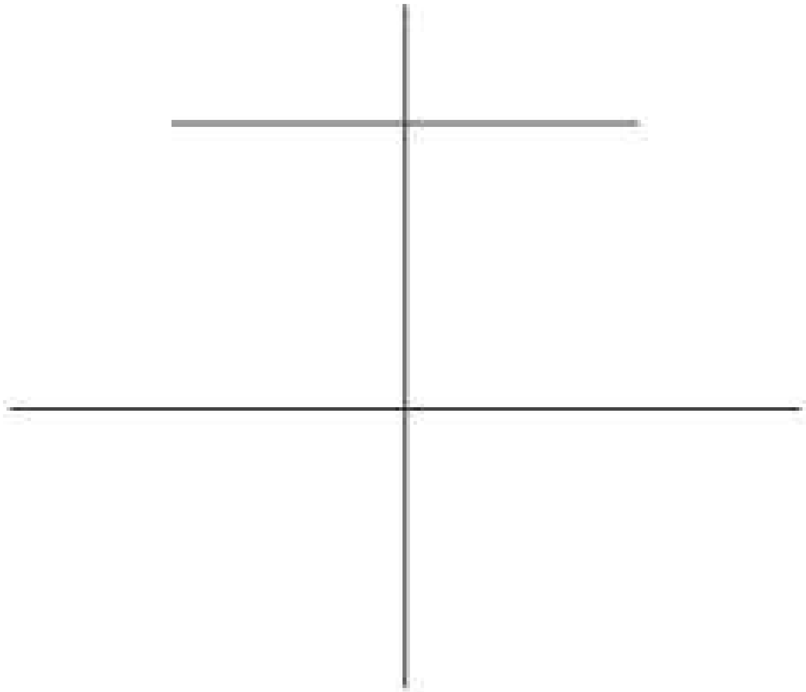
Mark *a* if $f(-x) = f(x)$

Mark *b* if $f(-x) = -f(x)$

Mark *c* if neither *a* nor *b* is true

Mark *d* if $f(x)$ does not exist at at least one point of the domain.

25.



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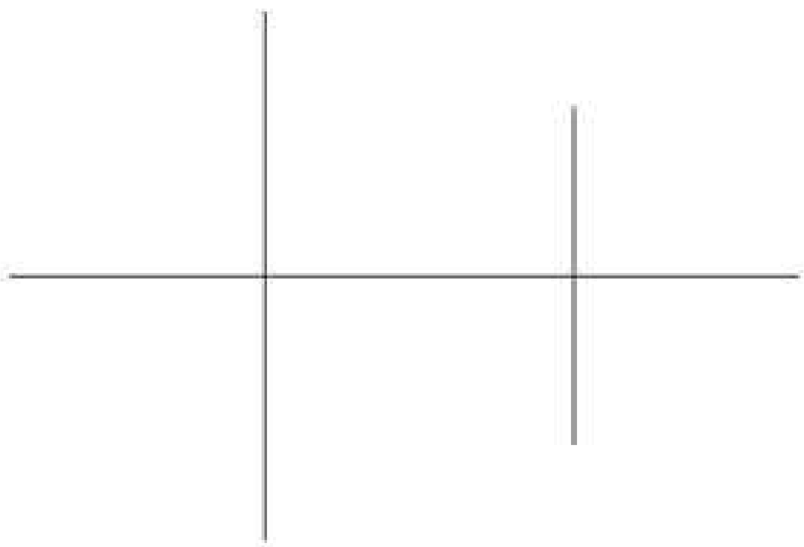

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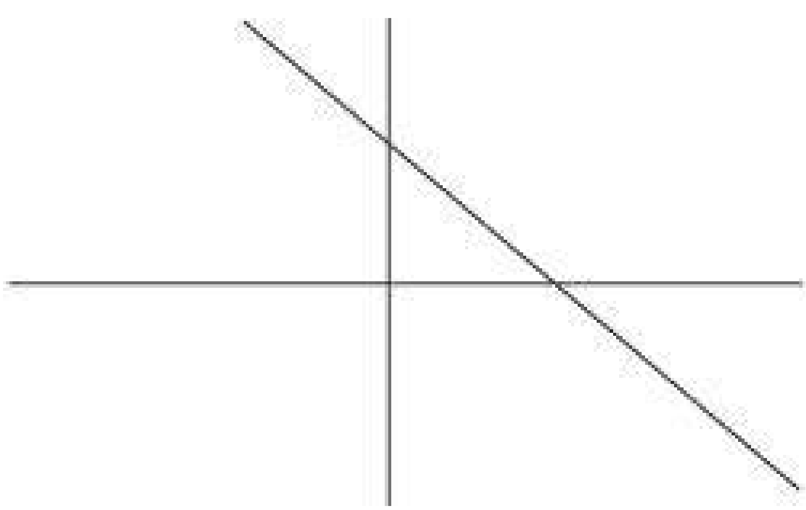

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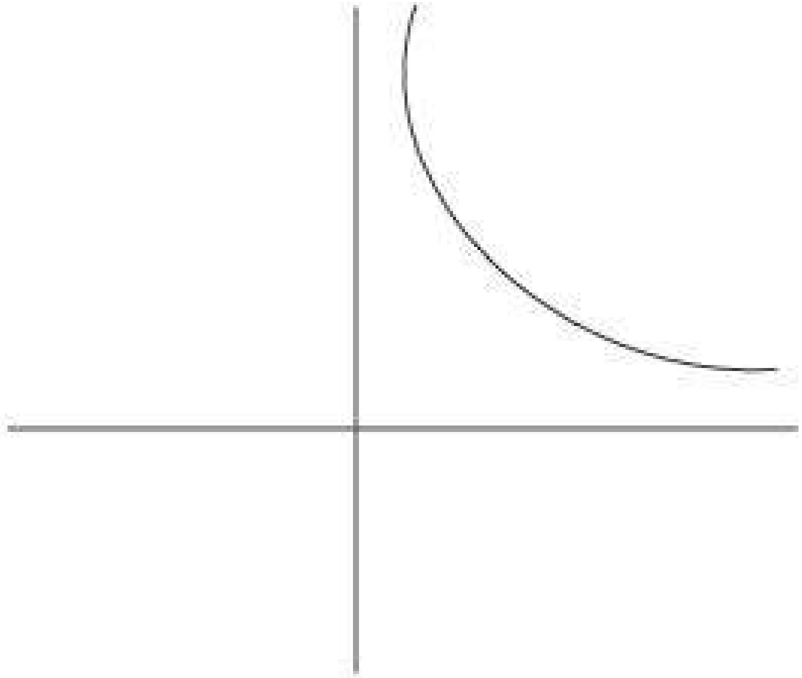

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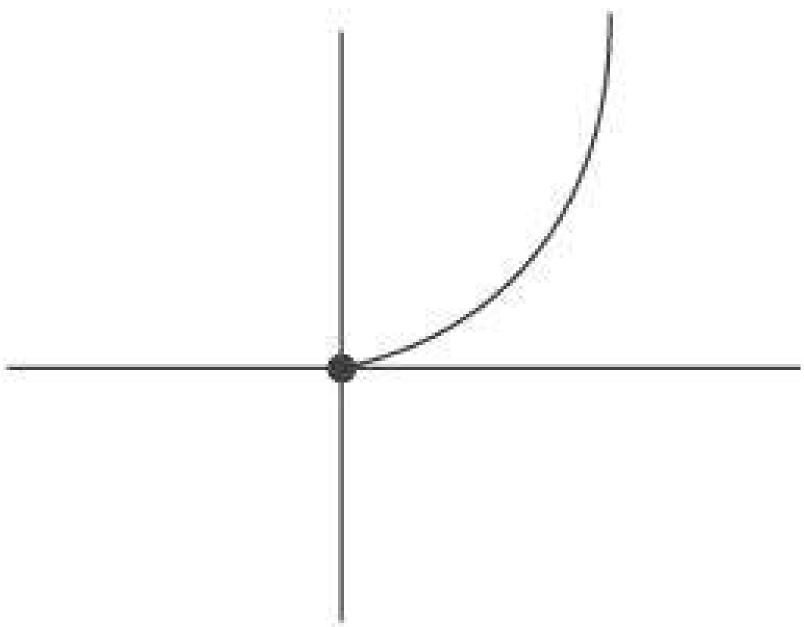

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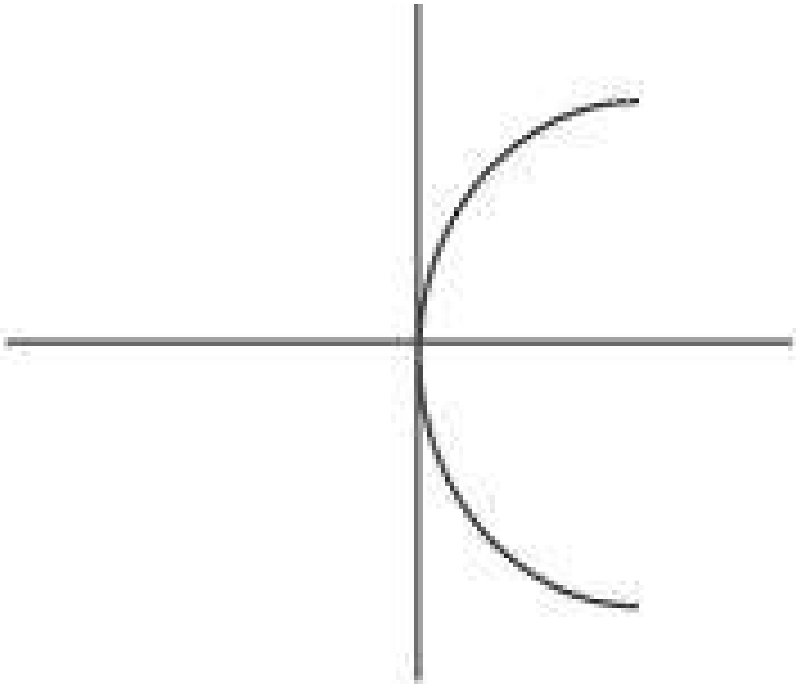

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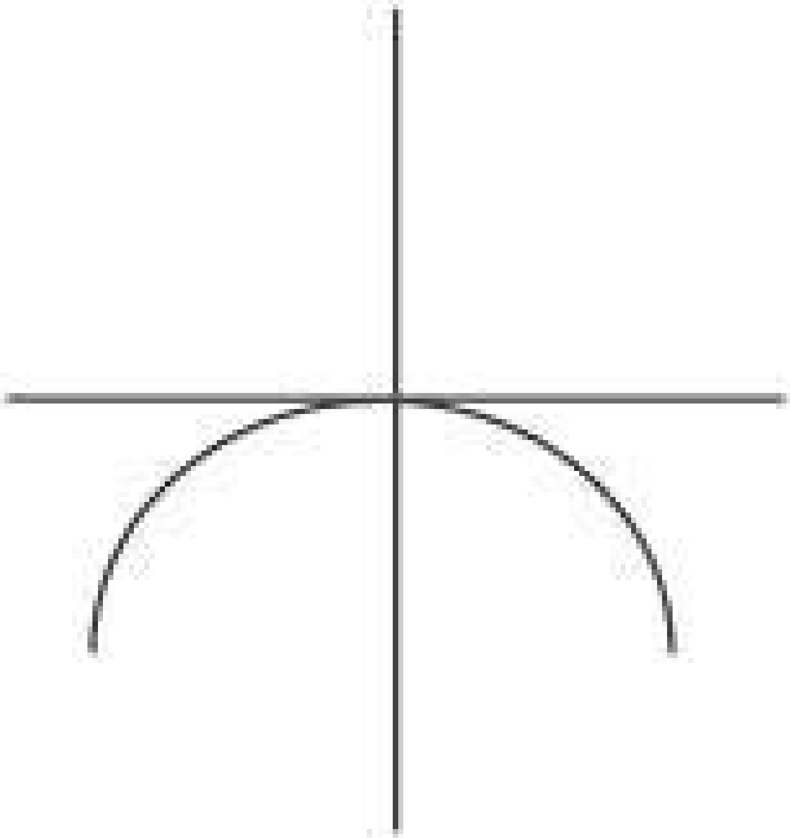

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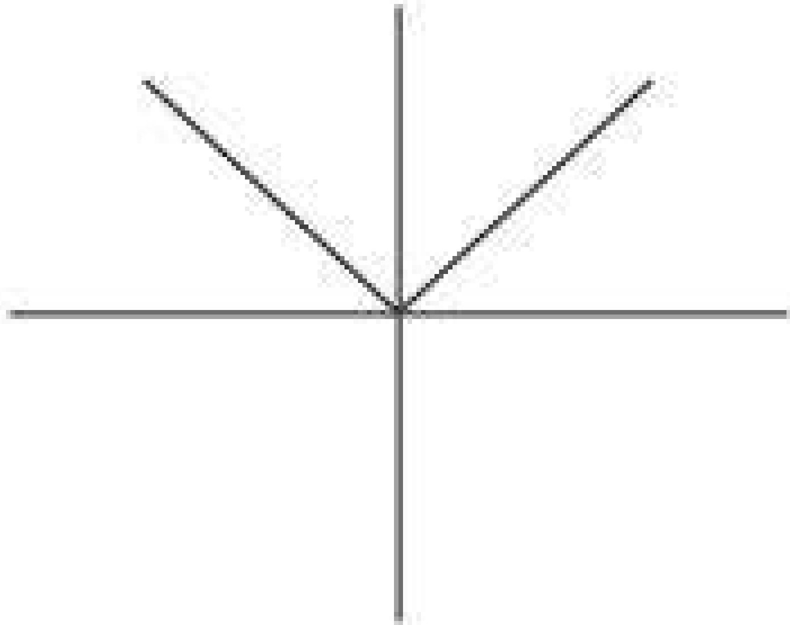

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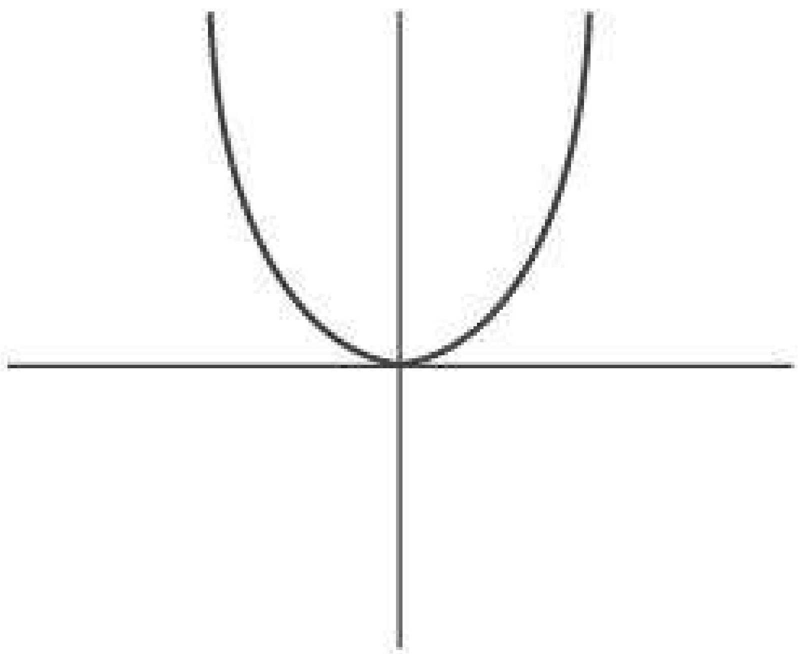

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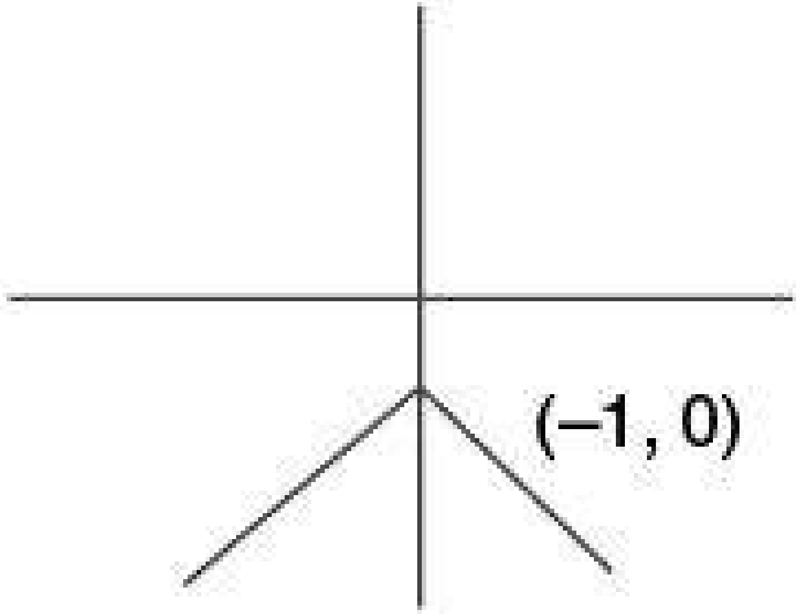

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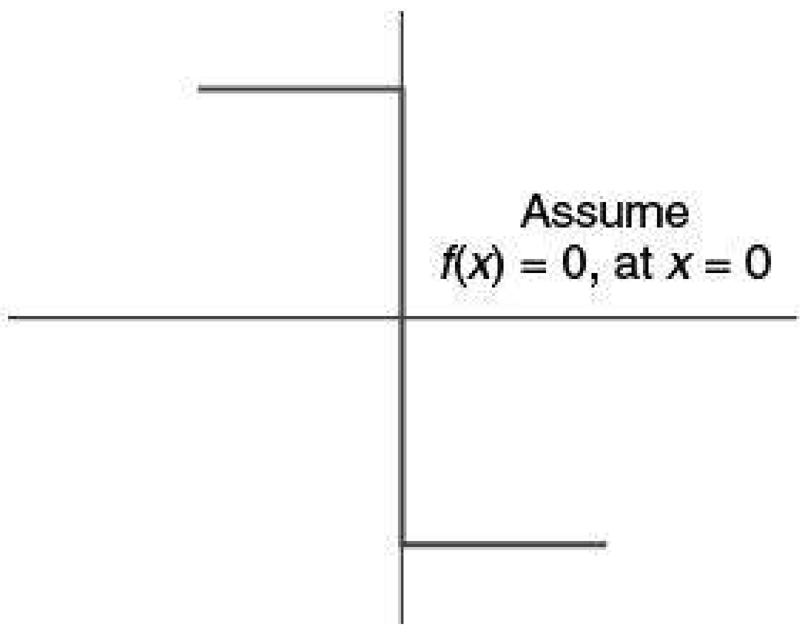

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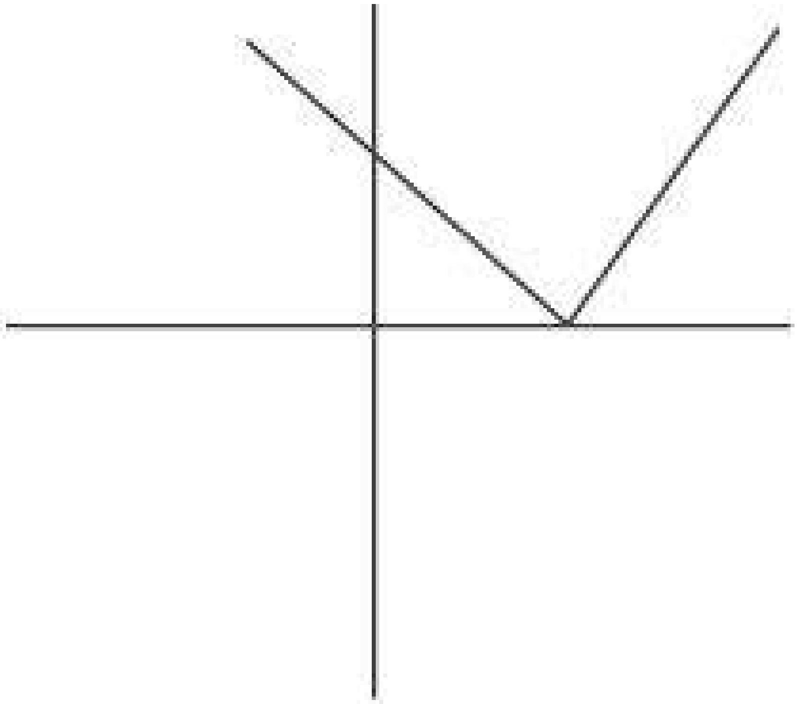

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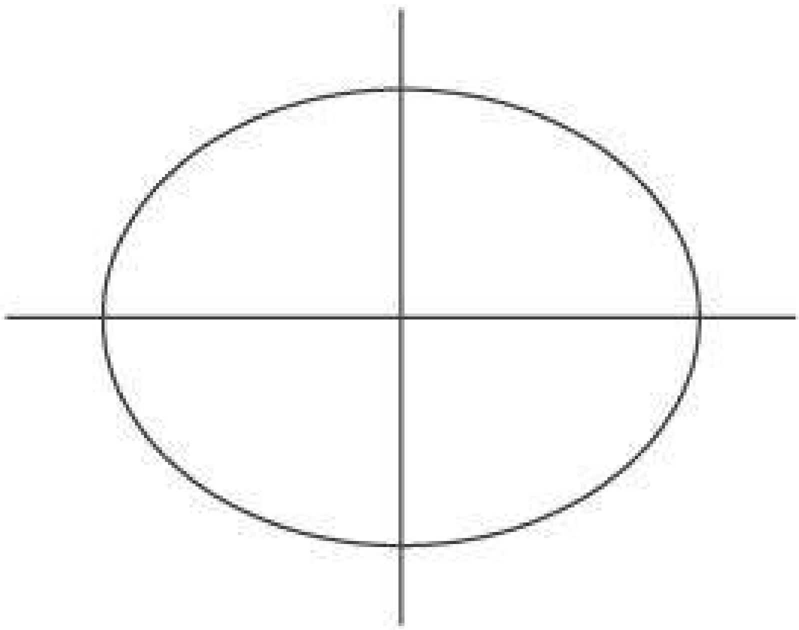

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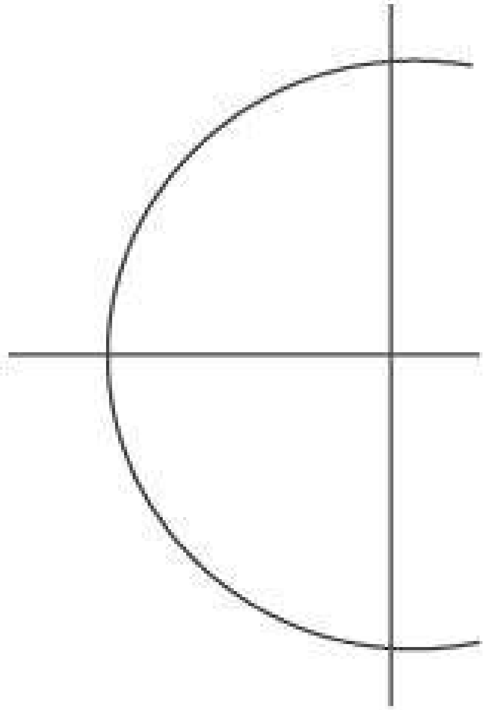


Directions for Questions 39 to 43: Define the following functions:

(i) $a @ b = \frac{a+b}{2}$

(ii) $a \# b = a^2 - b^2$

(iii) $(a ! \mathrm{b}) = \frac{a-b}{2}$

39. Find the value of {[(3@4)!(3#2)] @ [(4!3)@(2#3)]}.

(a) –0.75 (b) –1

(c) –1.5 (d) –2.25

40. Find the value of (4#3)@(2!3).

(a) 3.25 (b) 3.5

(c) 6.5 (d) 7

41. Which of the following has a value of 0.25 for a = 0 and b = 0.5?

(a) $a @ b$ (b) $a \# b$

(c) Either a or b (d) Cannot be determined

42. Which of the following expressions has a value of 4 for a = 5 and b = 3?

(a) $\frac{(a!b)}{(a\#b)}$ (b) (a!b)(a@b)

(c) $\frac{(a\#b)}{(a!b)(a@b)}$ (d) Both (b) and (c)

43. If we define $a\$b$ as $a^3 - b^3$, then for integers a, $b > 2$ and $a > b$ which of the following will always be true?

(a) $(a@b) > (a!b)$ (b) $(a@b) \geq (a!b)$

(c) $(a\#b) < (a\$b)$ (d) Both *a* and *c*

Directions for Questions 44 to 48: Define the following functions:

(a) $(a\ M\ b) = a - b$ (b) $(a\ D\ b) = a + b$

(c) $(a\ H\ b) = (ab)$ (d) $(a\ P\ b) = a/b$

44. Which of the following functions will represent $a^2 - b^2$?

(a) $(a\ M\ b)\ H\ (a\ D\ b)$ (b) $(a\ H\ b)\ M\ (a\ P\ b)$

(c) $(a\ D\ b)/(a\ M\ b)$ (d) None of these

45. Which of the following represents a^2?

(a) $(a\ M\ b)\ H\ (a\ D\ b) + b^2$

(b) $(a\ H\ b)\ M\ (a\ P\ b) + b^2$

(c) $\frac{(a\ M\ b)}{(a\ P\ b)}$

(d) Both (a) and (c)

46. What is the value of $(3M4H2D4P8M2)$?

(a) 6.5 (b) 6

(c) –6.5 (d) None of these

47. Which of the four functions defined has the maximum value?

(a) $(a\ M\ b)$ (b) $(a\ D\ b)$

(c) $(a\ P\ b)$ (d) Cannot be determined

48. Which of the four functions defined has the minimum value?

(a) $(a\ M\ b)$ (b) $(a\ D\ b)$

(c) $(a\ H\ b)$ (d) Cannot be determined

49. If $0 < a < 1$ and $0 < b < 1$ and $a > b$, which of the 4 expressions will take the highest value?

(a) $(a\ M\ b)$ (b) $(a\ D\ b)$

(c) $(a\ P\ b)$ (d) Cannot be determined

50. If $0 < a < 1$ and $0 < b < 1$ and if $a < b$, which of the following expressions will have the highest value?

(a) $(a\ M\ b)$ (b) $(a\ D\ b)$

(c) $(a\ P\ b)$ (d) Cannot be determined

51. A function $F(n)$ is defined as $F(n-1) = \frac{1}{(2 - F(n))}$ for all natural numbers '*n*'. If $F(1) = 3$, then what is the value of $[F(1)] + [F(2)] + \ldots + [F(1000)]$?

(Here, $[x]$ is equal to the greatest integer less than or equal to 'x')

(a) 1001 (b) 1002

(c) 3003 (d) None of these

52. For the above question find the value of the expression: $F(1) \times F(2) \times F(3) \times F(4) \times \ldots F(1000)$

(a) 2001 (b) 1999

(c) 2004 (d) 1997

53. A function $f(x)$ is defined for all real values of x as $f(x) = ax^2 + bx + c$. If $f(3) = f(-3) = 18$, $f(0) = 15$, then what is the value of $f(12)$?

(a) 63 (b) 159

(c) 102 (d) None of these

54. Two operations, for real numbers x and y, are defined as given below.

(i) $M (x \text{ q } y) = (x + y)^2$

(ii) $f(x \text{ y } y) = (x - y)^2$

If $M(x^2 \text{ q } y^2) = 361$ and $M(x^2 \text{ y } y^2) = 49$, then what is the value of the square root of $((x^2 y^2) + 3)$?

(a) ±81 (b) ±9

(c) ±7 (d) ±11

55. The function $\text{Y}(m) = [m]$, where $[m]$ represents the greatest integer less than or equal to m. Two real numbers x and y are such that $\text{Y}(4x + 5) = 5y + 3$ and $\text{Y}(3y + 7) = x + 4$, then find the value of $x^2 \times y^2$.

(a) 1 (b) 2

(c) 4 (d) None of these

56. A certain function always obeys the rule: If $f(x.y) = f(x).\, f(y)$ where x and y are positive real numbers. A certain Mr. Mogambo found that the value of $f(128) = 4$, then find the value of the variable $M = f(0.5).\, f(1).\, f(2).\, f(4).\, f(8).\, f(16).\, f(32).\, f(64).\, f(128).\, f(256)$

(a) 128 (b) 256

(c) 512 (d) 1024

57. x and y are non negative integers such that $4x + 6y = 20$, and x^2 £ $M/y^{2/3}$ for all values of x, y. What is the minimum value of M ?

(a) $2^{2/3}$ (b) $2^{1/3}$

(c) $2^{4/3}$ (d) $4^{2/3}$

58. Let $\text{Y}(x) = \dfrac{x+3}{2}$ and $\text{q}(x) = 3x^2 + 2$. Find the value of q (Y(–7)).

(a) 12 (b) 14

(c) 50 (d) 42

59. If $F(a + b) = F(a).F(b) \div 2$, where $F(b)$ π 0 and $F(a)$π 0, then what is the value of $F(12b)$?

(a) $(F(b))^{12}$

(b) $F(b))^{12} \div 2$

(c) $(F(b))^{12} \div 2^{12}$

(d) $(F(b))^{12} \div 2^{11}$

60. A function a = q(b) is said to be reflexive if b = q(a). Which of the following is/are reflexive functions?

(i) $\frac{3b+5}{4b-3}$

(ii) $\frac{3b+5}{5b-2}$

(iii) $\frac{2b+12}{12b-2}$

(a) All of these are reflexive

(b) Only (i) and (ii) are reflexive

(c) Only (i) and (iii) are reflexive

(d) None of these are reflexive.

LEVEL OF DIFFICULTY (II)

1. Find the domain of the definition of the function $y = 1/ (4 - x^2)^{1/2}$.

   (a) (–2, 2)

   (b) [–2, 2]

   (c) (–•, –2) » (2, •) excluding –2 and 2

   (d) (2, •)

2. The domain of definition of the function

   $y = \dfrac{1}{\log_{10}(1-x)} + (x + 2)^{1/2}$ is

   (a) (–3, –2) (b) [0, 1)

   (c) [–2, 1] (d) [–2, 1) excluding 0

3. The domain of definition of $y = [\log_{10}\left(\dfrac{5x - x^2}{4}\right)]^{1/2}$ is

   (a) [1, 4] (b) [–4, –1]

   (c) [0, 5] (d) [–1, 5]

4. Which of the following functions is an odd function?

   (a) $2^{-x.x}$ (b) $2^{x- x.x.x.x}$

   (c) Both (a) and (b) (d) Neither (a) nor (b)

5. The domain of definition of $y = [1- |x|]^{1/2}$ is

   (a) [–1, 0] (b) [0, 1]

   (c) (–1, 1) (d) [–1, 1]

6. The domain of definition of $y = [3/(4 - x^2)] + \log_{10} (x^3 - x)$ is

   (a) (–1, 0) » (1, •) (b) Not 2 or –2

   (c) (a) and (b) together (d) None of these

7. If $f(t) = 2^t$, then $f(0)$, $f(1)$, $f(2)$ are in

   (a) AP (b) HP

   (c) GP (d) Cannot be said

8. Centre of a circle $x^2 + y^2 = 16$ is at (0, 0). What will be the new centre of the circle if it gets shifted 3 units down and 2 units left?

   (a) (2, 3) (b) (–2, –3)

   (c) (–2, 3) (d) (2, –3)

9. If $u(t) = 4t - 5$, $v(t) = t^2$ and $f(t) = 1/t$, then the formula for $u(f(v(t)))$ is

(a) $\frac{1}{(4t-5)^2}$

(b) $\frac{4}{(t-5)}$

(c) $\frac{4}{t^2} - 5$

(d) None of these

10. If $f(t) = \sqrt{t}$, $g(t) = t/4$ and $h(t) = 4t - 8$, then the formula for $g(f(h(t)))$ will be

(a) $\frac{\sqrt{t-2}}{4}$

(b) $2\sqrt{t} - 8$

(c) $\frac{\sqrt{(4t-8)}}{4}$

(d) $\frac{\sqrt{(t-8)}}{4}$

11. In the above question, find the value of $h(g(f(t)))$.

(a) $\sqrt{t} - 8$

(b) $2\sqrt{t-8}$

(c) $\frac{\sqrt{t}+8}{4}$

(d) None of these

12. In question number 10, find the formula of $f(h(g(t)))$.

(a) $\sqrt{t} - 8$

(b) $\sqrt{(t-8)}$

(c) $2\sqrt{t} - 8$

(d) None of these

13. The values of x, for which the functions $f(x) = x$, $g(x) = (\sqrt{x})^2$ and $h(x) = x^2/x$ are identical, is

(a) 0 £ x

(b) $0 < x$

(c) All real values

(d) All real values except 0

14. Which of the following is an even function?

(a) e^x

(b) e^{-x}

(c) $e^x + e^{-x}$

(d) $\frac{e^x + e^{-x}}{e^x - e^{-x}}$

15. The graph of $y = (x + 3)^3 + 1$ is the graph of $y = x^3$ shifted

(a) 3 units to the right and 1 unit down

(b) 3 units to the left and 1 unit down

(c) 3 units to the left and 1 unit up

(d) 3 units to the right and 1 unit up

16. If $f(x) = 5x^3$ and $g(x) = 3x^5$, then $f(x).g(x)$ will be

(a) Even function

(b) Odd function

(c) Both

(d) None of these

17. If $f(x) = x^2$ and $g(x) = \log_e x$, then $f(x) + g(x)$ will be

(a) Even function (b) Odd function

(c) Both (d) Neither (a) nor (b)

18. If $f(x) = x^3$ and $g(x) = x^2/5$, then $f(x) - g(x)$ will be

(a) Odd function (b) Even function

(c) Neither (a) nor (b) (d) Both

19. $f(x)$ is any function and $f^{-1}(x)$ is known as inverse of $f(x)$, then $f^{-1}(x)$ of $f(x) = 1/(x - 2)$ is

(a) $\frac{1}{x} + 2$ (b) $\frac{1}{(x+2)}$

(c) $\frac{1}{x} + 0.5$ (d) None of these

20. $f(x)$ is any function and $f^{-1}(x)$ is known as inverse of $f(x)$, then $f^{-1}(x)$ of $f(x) = e^x$ is

(a) $-e^x$ (b) e^{-x}

(c) $\log_e x$ (d) None of these

21. $f(x)$ is any function and $f^{-1}(x)$ is known as inverse of $f(x)$, then $f^{-1}(x)$ of $f(x) = x/(x - 1)$, x π 1 is

(a) $x/(1 + x)$ (b) $\frac{x}{x^2 - 1}$

(c) $x/(x - 1)$ (d) $-x/(x + 1)$

22. Which of the following functions will have a minimum value at $x = -3$?

(a) $f(x) = 2x^3 - 4x + 3$ (b) $f(x) = 4x^4 - 3x + 5$

(c) $f(x) = x^6 - 2x - 6$ (d) None of these

Directions for Questions 23 to 32:

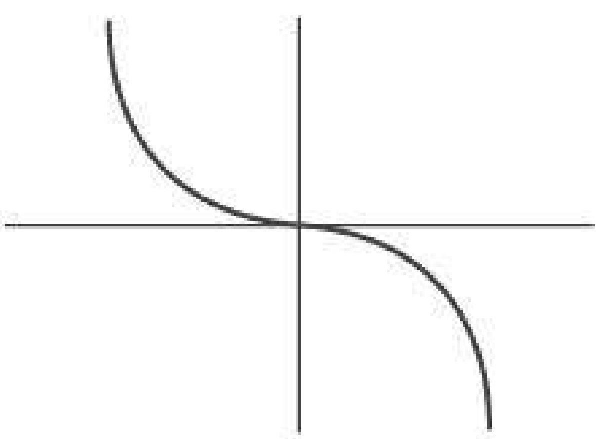
Mark (a) if $f(-x) = f(x)$

Mark (b) if $f(-x) = -f(x)$

Mark (c) if neither (a) nor (b) is true

Mark (d) if $f(x)$ does not exist at at least one point of the domain.

23.



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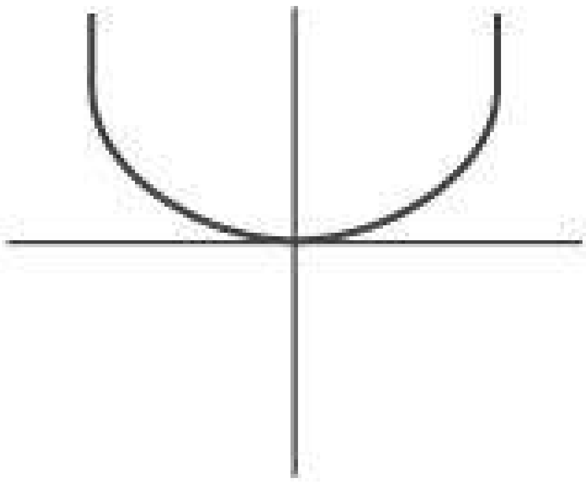

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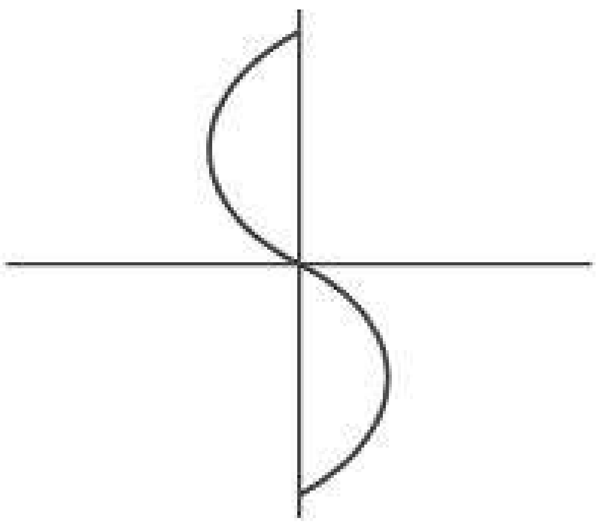

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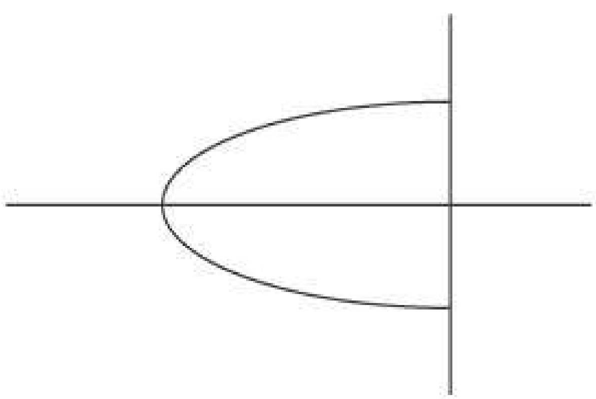

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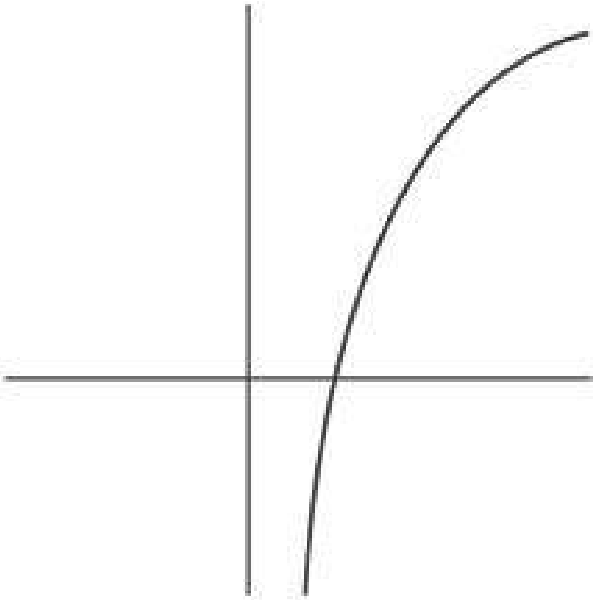

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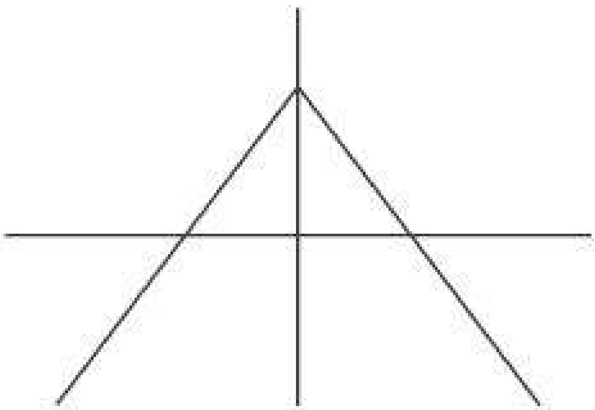

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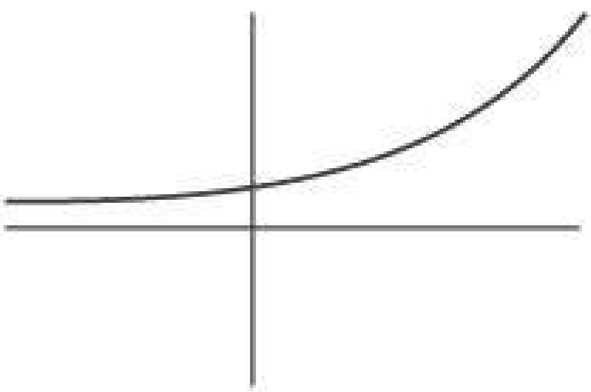

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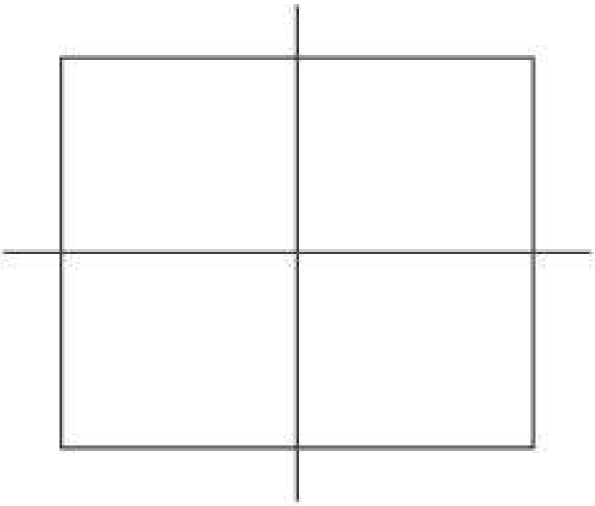

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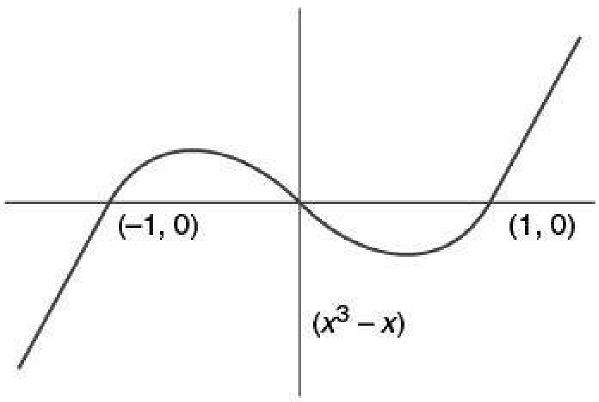

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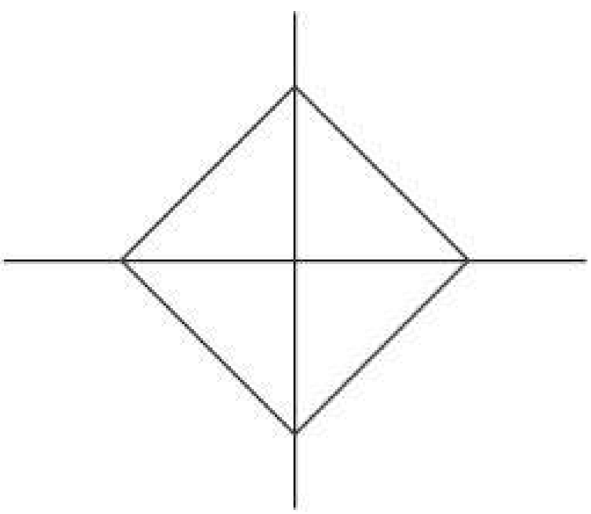
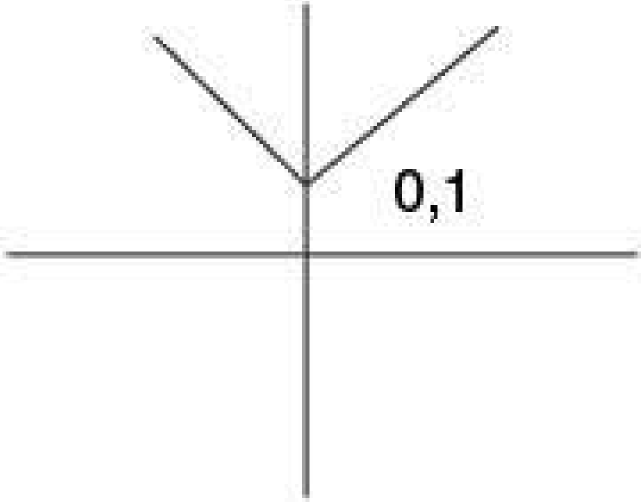
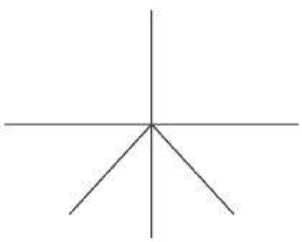


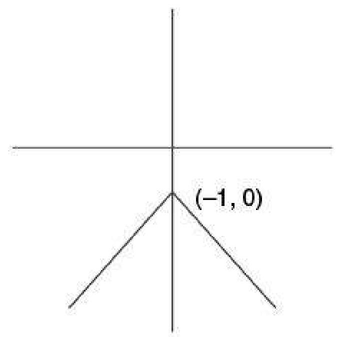
Directions for Questions 33 to 36: If $f(x)$ is represented by the graph below.



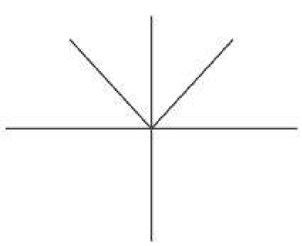
33. Which of the following will represent the function $-f(x)$?
34. Which of the following will represent the function $-f(x) + 1$?
35. Which of the following will represent the function $f(x) - 1$?
36. Which of the following will represent the function $f(x) + 1$?



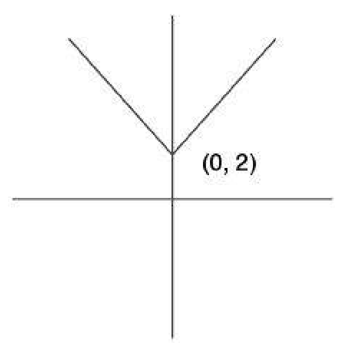
(a)



(b)



(c)



(d)

Directions for Questions 37 to 40: Define the following functions:

$f(x, y, z) = xy + yz + zx$

$g(x, y, z) = x^2y + y^2z + z^2x$ and

$h(x, y, z) = 3\,xyz$

Find the value of the following expressions:

37. $h[f(2, 3, 1), g(3, 4, 2), h(1/3, 1/3, 3)]$

    (a) 0 (b) 23760

(c) 2640 (d) None of these

38. $g[f(1, 0, 0), g(0, 1, 0), h(1, 1, 1)]$

(a) 0 (b) 9

(c)12 (d) None of these

39. $f[f(1, 1, 1), g(1, 1, 1), h(1, 1, 1)]$

(a) 9 (b) 18

(c)27 (d) None of these

40. $f(1, 2, 3) - g(1, 2, 3) + h(1, 2, 3)$

(a) –6 (b) 6

(c)12 (d) 8

41. If $f(x) = 1/g(x)$, then which of the following is correct?

(a) $f(f(g(f(x)))) = f(g(g(f(f(x)))))$

(b) $f(g(g(f(f(x))))) = f(f(g(g(g(x)))))$

(c) $g(g(f(f(g(f(x)))))) = f(f(g(g(f(g(x))))))$

(d) $f(g(g(g(f(x))))) = g(g(f(f(f(x)))))$

42. If $f(x) = 1/g(x)$, then the minimum value of $f(x) + g(x)$, $f(x) > 0$ and $g(x) > 0$, will be

(a) 0

(b) 2

(c) Depends upon the value of $f(x)$ and $g(x)$

(d) None of these

Directions for Questions 43 to 45:

If $R(a/b)$ = Remainder when a is divided by b;

$Q(a/b)$ = Quotient obtained when a is divided by b;

$SQ(a)$ = Smallest integer just bigger than square root of a.

43. If $a = 12$, $b = 5$, then find the value of $SQ[R\{(a + b)/b\}]$.

(a) 0 (b) 1

(c) 2 (d) 3

44. If $a = 9$, $b = 7$, then the value of $Q[[SQ(ab)+b]/a]$ will be

(a) 0 (b) 1

(c) 2 (d) None of these

45. If $a = 18$, $b = 2$ and $c = 7$, then find the value of $Q[\{SQ(ab) + R(a/c)\}/b]$.

(a) 3 (b) 4

(c) 5 (d) 6

Directions for Questions 46 to 48: Read the following passage and try to answer questions based on them.

$[x]$ = Greatest integer less than or equal to x

$\{x\}$ = Smallest integer greater than or equal to x.

46. If x is not an integer, what is the value of $([x] – \{x\})$?

(a) 0 (b) 1

(c) –1 (d) 2

47. If x is not an integer, then $(\{x\} + [x])$ is

(a) An even number

(b) An odd integer

(c) $> 3x$

(d) $< x$

48. What is the value of x if $5 < x < 6$ and $\{x\} + [x] = 2x$?

(a) 5.2 (b) 5.8

(c) 5.5 (d) 5.76

49. If $f(t) = t^2 + 2$ and $g(t) = (1/t) + 2$, then for $t = 2$, $f[g(t)] – g[f(t)]$ = ?

(a) 1.2 (b) 2.6

(c) 4.34 (d) None of these

50. Given $f(t) = kt + 1$ and $g(t) = 3t + 2$. If $fog = gof$, find k.

(a) 2 (b) 3

(c) 5 (d) 4

51. Let $F(x)$ be a function such that $F(x)\ F(x + 1) = – F(x – 1)F(x–2)F(x–3)F(x–4)$ for all $x \geq 0$. Given the values of If $F(83) = 81$ and $F(77) = 9$, then the value of $F(102)$ equals to

(a)27 (b) 54

(c) 729 (d) Data Insufficient

52. Let $f(x) = 121– x^2$, $g(x) = |x – 8| + |x + 8|$ and $h(x) = \min(f(x), g(x)\}$. What is the number of integer values of x for which $h(x)$ is equal to a positive integral value?

(a) 17 (b) 19

(c) 21 (d) 23

53. If the function $R(x) = \max(x^2 – 8, 3x, 8)$, then what is the max value of $R(x)$?

(a) 4 (b) $\frac{1+\sqrt{5}}{2}$

(c) • (d) 0

54. If the function $R(x) = \min(x^2 – 8, 3x, 8)$, what is the max value of $R(x)$?

(a)4 (b) 8

(c) • (d) None of these

55. The minimum value of $ax^2 + bx + c$ is 7/8 at $x = 5/4$. Find the value of the expression at $x = 5$, if the value of the expression at $x = 1$ is 1.

(a) 75 (b) 79

(c) 121 (d) 129

56. Find the range of the function $f(x) = (x + 4)(5 - x)(x + 1)$.

(a) [–2, 3] (b) (–•, 20]

(c) (– •,+ •) (d) [–20, •)

57. The function $f(x)$ is defined for positive integers and is defined as:

$f(x) = 6^x - 3$, if x is a number in the form $2n$.

$= 6^x + 4$, if x is a number in the form $2n + 1$.

What is the remainder when $f(1) + f(2) + f(3) + ... + f(1001)$ is divided by 2?

(a) 1 (b) 0

(c) –1 (d) None of the above

58. p, q and r are three non-negative integers such that $p + q + r = 10$. The maximum value of $pq + qr + pr + pqr$ is

(a) ≥ 40 and < 50 (b) ≥ 50 and < 60

(c) ≥ 60 and < 70 (d) ≥ 70 and < 80

59. A function a(x) is defined for x as $3a(x) + 2a(2 - x) = (x + 3)^2$. What is the value of $[G(-5)]$ where $[x]$ represents the greatest integer less than or equal to x?

(a) 37 (b) –38

(c) –37 (d) Cannot be determined

60. For a positive integer x, $f(x + 2) = 3 + f(x)$, when x is even and $f(x + 2) = x + f(x)$, when x is odd. If $f(1) = 6$ and $f(2) = 4$, then find $f(f(f(f(1)))) + f(f(f(f(2))))$.

(a) 1375 (b) 1425

(c) 1275 (d) None of these

LEVEL OF DIFFICULTY (III)

1. Find the domain of the definition of the function
$y = 1/(x - |x|)^{1/2}$.

(a) –• < x < •
(b) –• < x < 0
(c) 0 < x < •
(d) No where

2. Find the domain of the definition of the function
$y = (x - 1)^{1/2} + 2(1 - x)^{1/2} + (x^2 + 3)^{1/2}$.

(a) $x = 0$
(b) [1, •)
(c) [–1, 1]
(d) 1

3. Find the domain of the definition of the function
$y = \log_{10} [(x - 5)/(x^2 - 10x + 24)] - (x + 4)^{1/2}$.

(a) $x > 6$
(b) $4 < x < 5$
(c) Both *a* and *b*
(d) None of these

4. Find the domain of the definition of the function
$y = [(x - 3)/(x + 3)]^{1/2} + [(1 - x)/(1 + x)]^{1/2}$.

(a) $x > 3$
(b) $x < -3$
(c) -3 £ x £ 3
(d) Nowhere

5. Find the domain of the definition of the function
$y = (2x^2 + x + 1)^{-3/4}$.

(a) $x \geq 0$
(b) All x except $x = 0$
(c) –3 £ x £ 3
(d) Everywhere

6. Find the domain of the definition of the function
$y = (x^2 - 2x - 3)^{1/2} - 1/(-2 + 3x - x^2)^{1/2}$.

(a) $x > 0$
(b) $-1 < x < 0$
(c) x^2
(d) None of these

7. Find the domain of the definition of the function
$y = \log_{10} [1 - \log_{10}(x^2 - 5x + 16)]$.

(a) (2, 3]
(b) [2, 3)
(c) [2, 3]
(d) None of these

8. If $f(t) = (t - 1)/(t + 1)$, then $f(f(t))$ will be equal to

(a) $1/t$
(b) $-1/t$
(c) t
(d) $-t$

9. If $f(x) = e^x$ and $g(x) = \log_e x$ then value of *fog* will be

(a) x
(b) 0

(c) 1

(d) e

10. In the above question, find the value of *gof*.

(a) x

(b) 0

(c) 1

(d) e

11. The function $y = 1/x$ shifted 1 unit down and 1 unit right is given by

(a) $y - 1 = 1/(x + 1)$

(b) $y - 1 = 1/(x - 1)$

(c) $y + 1 = 1/(x - 1)$

(d) $y + 1 = 1/(x + 1)$

12. Which of the following functions is an even function?

(a) $f(t) = (a^t + a^{-t})/(a^t - a^{-t})$

(b) $f(t) = (a^t + 1)/(a^t - 1)$

(c) $f(t) = t.\ (a^t - 1)/(a^t + 1)$

(d) None of these

13. Which of the following functions is not an odd function?

(a) $f(t) = \log_2(t + \sqrt{t^2} + 1)$

(b) $f(t) = (a^t + a^{-t})/(a^t - a^{-t})$

(c) $f(t) = (a^t + 1)/(a^t - 1)$

(d) All of these

14. Find *fof* if $f(t) = t/(1 + t^2)^{1/2}$.

(a) $1/(1+ 2t^2)^{1/2}$

(b) $t/(1 + 2t^2)^{1/2}$

(c) $(1 + 2t^2)$

(d) None of these

15. At what integral value of x will the function $\frac{(x^2 + 3x + 1)}{(x^2 - 3x + 1)}$ attain its maximum value?

(a) 3

(b) 4

(c) –3

(d) None of these

16. Inverse of $f(t) = (10^t - 10^{-t})/(10^t + 10^{-t})$ is

(a) $1/2 \log \{(1 - t)/(1 + t)\}$

(b) $0.5 \log \{(t - 1)/(t + 1)\}$

(c) $1/2 \log_{10} (2^t - 1)$

(d) None of these

17. If $f(x) = |x - 2|$, then which of the following is always true?

(a) $f(x) = (f(x))^2$

(b) $f(x) = f(-x)$

(c) $f(x) = x - 2$

(d) None of these

Directions for Questions 18 to 20: Read the instructions below and solve:

$f(x) = f(x - 2) - f(x - 1)$, x is a natural number

$f(1) = 0$, $f(2) = 1$

18. The value of $f(x)$ is negative for

    (a) All $x > 2$

    (b) All odd $x (x > 2)$

    (c) For all even $x (x > 0)$

    (d) $f(x)$ is always positive

19. The value of $f[f(6)]$ is

    (a) 5 (b) – 1

    (c) – 3 (d) – 2

20. The value of $f(6) - f(8)$ is

    (a) $f(4) + f(5)$ (b) $f(7)$

    (c) $-\{f(7) + f(5)\}$ (d) $-f(5)$

21. Which of the following is not an even function ?

    (a) $f(x) = e^x + e^{-x}$ (b) $f(x) = e^x - e^{-x}$

    (c) $f(x) = e^{2x} + e^{-2x}$ (d) None of these

22. If $f(x)$ is a function satisfying $f(x)$. $f(1/x) = f(x) + f(1/x)$ and $f(4) = 65$, what will be the value of $f(6)$?

    (a) 37 (b) 217

    (c) 64 (d) None of these

Directions for Questions 23 to 34:

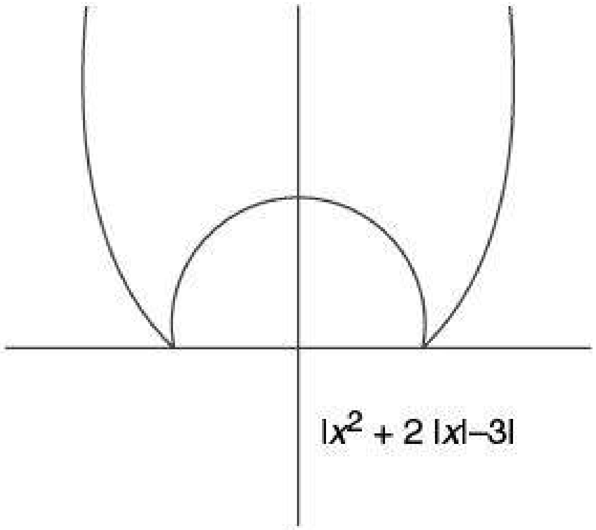
Mark (a) if $f(-x) = f(x)$,

Mark (b) if $f(-x) = -f(x)$

Mark (c) if neither (a) nor (b) is true

Mark (d) if $f(x)$ does not exist at at least one point of the domain.

23.



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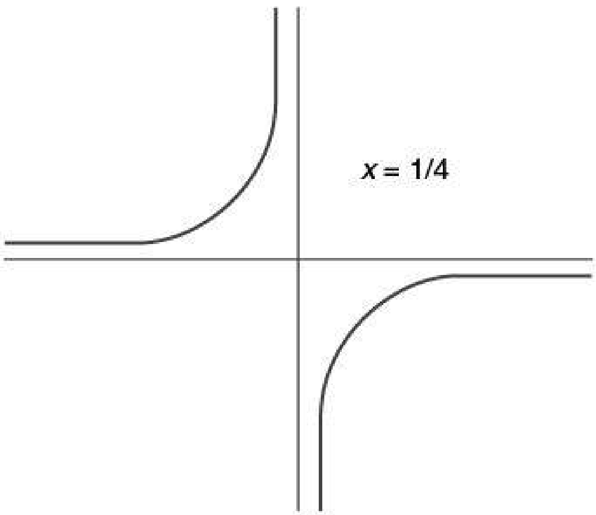

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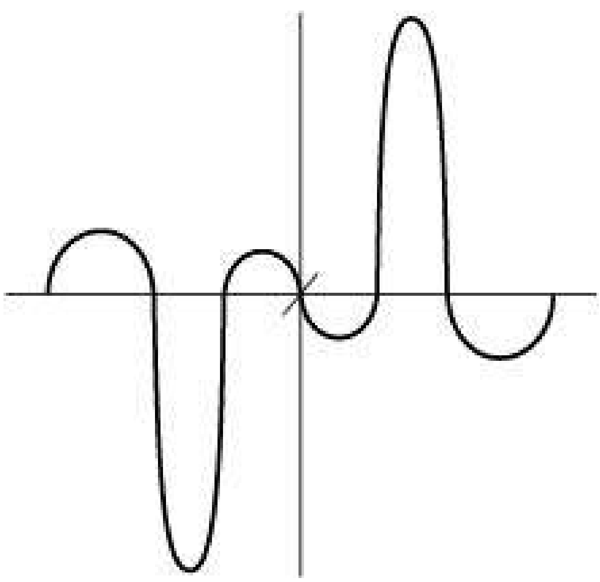

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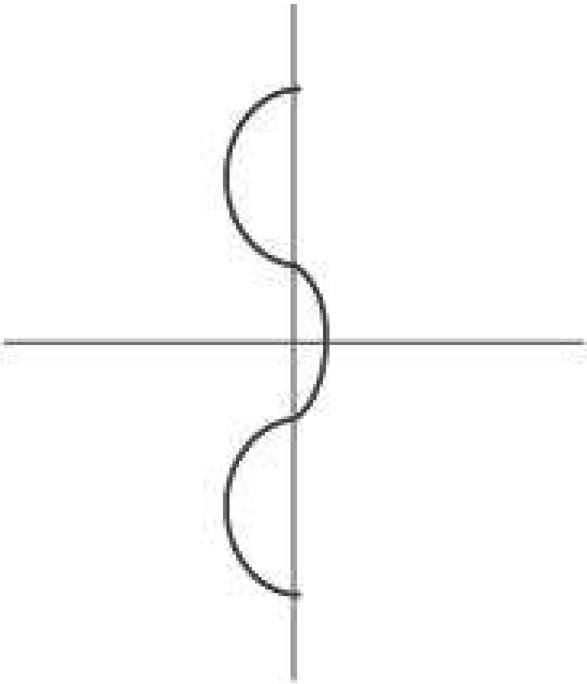

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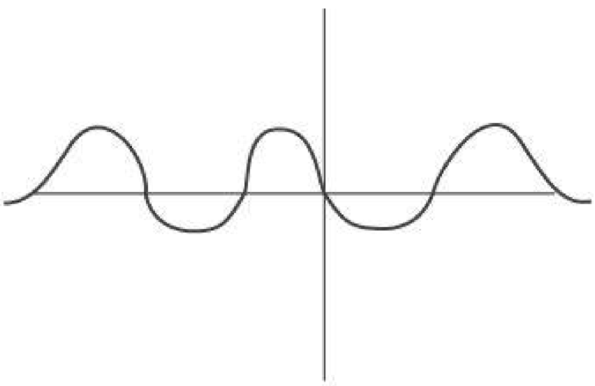

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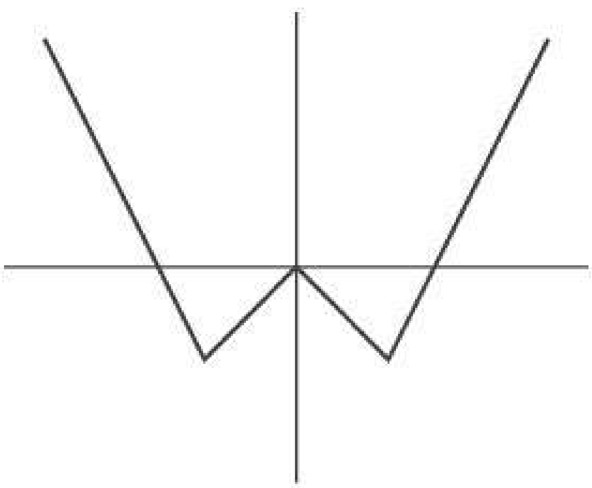
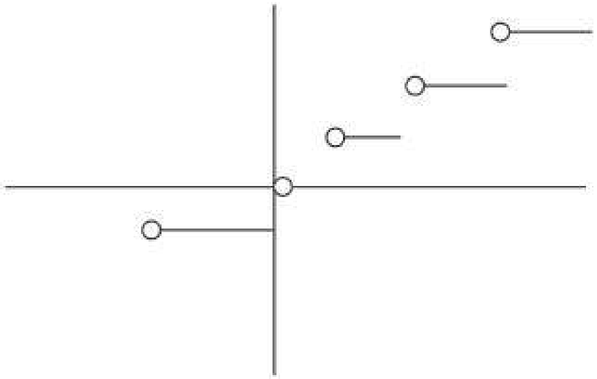


29.



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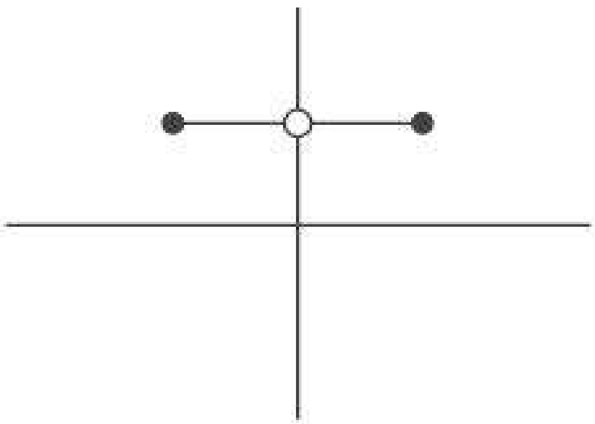

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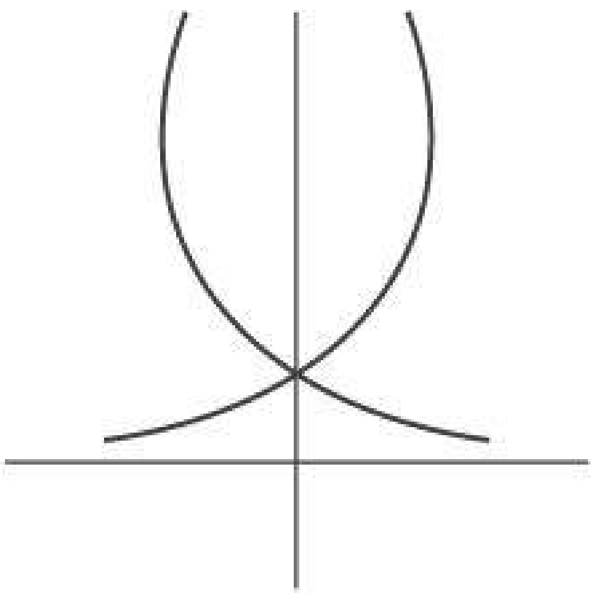

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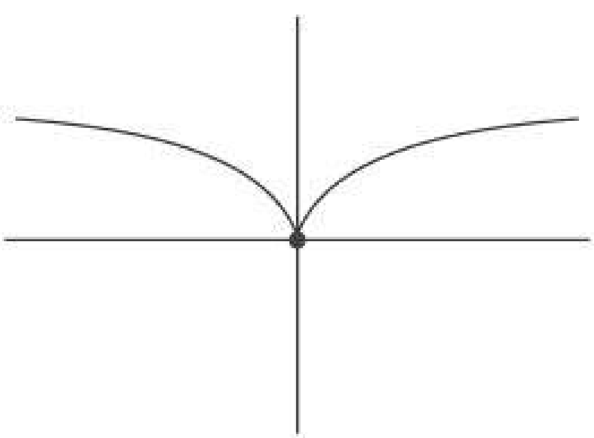

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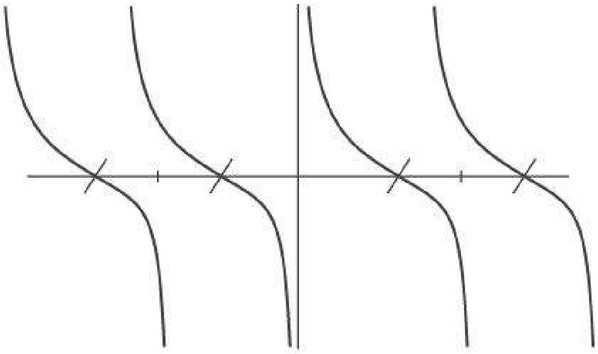

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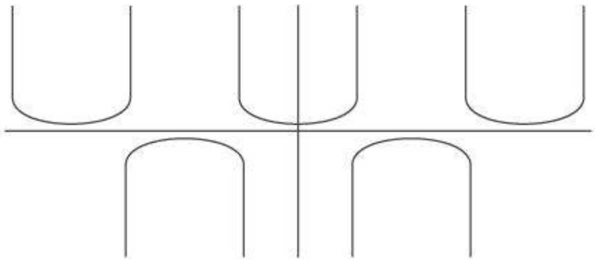


Directions for Questions 35 to 40: Define the functions:

A(*x*, *y*, *z*) = Max (max (*x*, *y*), min (*y*, *z*) min (*x*, *z*))

B(*x*, *y*, *z*) = Max (max (*x*, *y*), min (*y*, *z*) max (*x*, *z*))

C(*x*, *y*, *z*) = Max (min (*x*, *y*), min (*y*, *z*) min (*x*, *z*))

D(*x*, *y*, *z*) = Min (max (*x*, *y*), max (*y*, *z*) max (*x*, *z*))

Max (*x*, *y*, *z*) = Maximum of *x*, *y* and *z*.

Min (*x*, *y*, *z*) = Minimum of *x*, *y* and *z*.

Assume that *x*, *y* and *z* are distinct integers.

35. For what condition will *A*(*x*, *y*, *z*) be equal to Max (*x*, *y*, *z*)?

    (a) When *x* is maximum (b) When *y* is maximum

    (c) When *z* is maximum (d) Either (a) or (b)

36. For what condition will *B*(*x*, *y*, *z*) be equal to Min (*x*, *y*, *z*)?

    (a) When *y* is minimum (b) When *z* is minimum

    (c) Either (a) or (b) (d) Never

37. For what condition will *A*(*x*, *y*, *z*) not be equal to *B* (*x*, *y*, *z*)?

    (a) $x > y > z$ (b) $y > z > x$

    (c) $z > y > x$ (d) None of these

38. Under what condition will *C*(*x*, *y*, *z*) be equal to *B*(*x*, *y*, *z*)?

    (a) $x > y > z$ (b) $z > y > x$

    (c) Both *a* and *b* (d) Never

39. Which of the following will always be true?

    (I) *A*(*x*, *y*, *z*) will always be greater than Min (*x*, *y*, *z*)

    (II) *B* (*x*, *y*, *z*) will always be lower than Max (*x*, *y*, *z*)

(III) $A(x, y, z)$ will never be greater than $B(x, y, z)$

(a) I only (b) III only

(c) Both *a* and *b* (d) All the three

40. The highest value amongst the following will be

(a) Max/Min (b) A/B

(c) C/D (d) Cannot be determined

Directions for Questions 41 to 49: Suppose x and y are real numbers. Let $f(x, y) = |x + y|$ $F(f(x, y)) = -f(x, y)$ and $G(f(x, y)) = -F(f(x, y))$

41. Which one of the following is true?

(a) $F(f(x, y)).G(f(x, y)) = -F(f(x, y)).\ G.(f(x, y))$

(b) $F(f(x, y)).G(f(x, y)) < -F(f(x, y)).\ G.(f(x, y))$

(c) $G(f(x, y)).f(x, y) = F(f(x, y)).\ (f(x, y)$

(d) $G(f(x, y)).F(f(x, y)) = f(x, y).\ f(x, y)$

42. Which of the following has a^2 as the result?

(a) $F(f(a, -a)).G(f(a, -a))$

(b) $-F(f(a, a).G(f(a, a))/4$

(c) $F(f(a, a)).G(f(a, a))/2^2$

(d) $f(a, a).f(a, a)$

43. Find the value of the expression.

$$\frac{G(f(3,2))+F(f(-1,2))}{f(2,-3)+G(f(1,2)}\ \ldots$$

(a) 3/2 (b) 2/3

(c) 1 (d) 2

44. Which of the following is equal to

$$\frac{G(f(32,13))+F(f(15,-5))}{f(2,3)+G(f(1.5,0.5))}\ ?$$

(a) $\dfrac{2G(f(1,2))+(f(-3,1))}{G(f(2,6)+F(f(-8,2))}$

(b) $\dfrac{3.G(f(3,4))+F(f(1,0))}{f(1,1)+G(f(2,0))}$

(c) $\dfrac{(f(3,4))+F(f(1,2))}{G(f(1,1))}$

(d) None of these

Now if

$A(f(x, y) = f(x, y)$

$B(f(x, y) = -f(x, y)$

$C(f(x, y) = f(x, y)$

$D(f(x, y) = -f(x, y)$ and similarly

$Z\ (f(x, y) = -f(x, y)$

Now, solve the following:

45. Find the value of $A(f(0, 1)) + B(f(1, 2)) + C(f(2, 3)) +\ldots+ Z(f(25, 26))$.

(a) –50 (b) –52

(c) –26 (d) None of these

46. Which of the following is true?

(i) $A(f(0, 1)) < B(f(1, 2)) < C(f(2, 3)\ldots$

(ii) $A(f(0, 1)).\ B(f(1, 2)) > B(f(1, 2)).C(f(2, 3)) > C(f(2, 3)).\ D(f(3, 4))$

(iii) $A(f(0, 0)) = B(f(0, 0)) = C(f(0, 0)) = \ldots = Z(f(0, 0))$

(a) only (i) and (ii) (b) only (ii) and (iii)

(c) only (ii) (d) only (i)

47. If max (x, y, z) = maximum of x, y and z

Min (x, y, z) = minimum of x, y and z

$f(x, y) = |x + y|$

$F(f(x, y)) = -f(x, y)$

$G(f(x, y) = -F(f(x, y))$

Then find the value of the following expression:

Min (max $[f(2, 3), F(f(3, 4)), G(f(4, 5))]$, min $[f(1, 2), F(f(-1, 2)), G(f(1, -2))]$, max $[f(-3\ -4),$ $f(-5\ -1), G(f(-4, -6))]$)

(a) –1 (b) –7

(c) –6 (d) –10

48. Which of the following is the value of

Max. $[f(a, b), F(f(b, c), G(f(c, d))]$

For all $a > b > c > d$?

(a) Anything but positive

(b) Anything but negative

(c) Negative or positive

(d) Any real value

49. If another function is defined as $P(x, y) = \frac{F(f(x, y))}{(x, y)}$ which of the following is second lowest in value?

(a) Value of $P(x, y)$ for $x = 2$ and $y = 1$

(b) Value of $P(x, y)$ for $x = 3$ and $y = 4$

(c) Value of $P(x, y)$ for $x = 3$ and $y = 5$

(d) Value of $P(x, y)$ for $x = 3$ and $y = 2$

50. If $f(s) = (b^s + b^{-s})/2$, where $b > 0$. Find $f(s + t) + f(s - t)$.

(a) $f(s) - f(t)$

(b) $2\, f(s).f(t)$

(c) $4\, f(s).f(t)$

(d) $f(s) + f(t)$

Questions 51 to 60 are all actual questions from the XAT exam.

51. A0, A1, A2,...... is a sequence of numbers with

A0 = 1, A1 = 3, and At = (t +1) At–1 – t At–2 = 2, 3, 4,....

Conclusion I. A8 = 77

Conclusion II. A10 = 121

Conclusion III. A12 = 145

(a) Using the given statement, only Conclusion I can be derived.

(b) Using the given statement, only Conclusion II can be derived.

(c) Using the given statement, only Conclusion III can be derived.

(d) Using the given statement, Conclusion I, II and III can be derived.

(e) Using the given statement, none of the three Conclusions I, II and III can be derived.

52. A, B, C be real numbers satisfying A < B < C, A + B + C = 6 and AB + BC + CA = 9

Conclusion I. 1 < B < 3

Conclusion II. 2 < A < 3

Conclusion III. 0 < C < 1

(a) Using the given statement, only Conclusion I can be derived.

(b) Using the given statement, only Conclusion II can be derived.

(c) Using the given statement, only Conclusion III can be derived.

(d) Using the given statement, Conclusion I, II and III can be derived.

(e) Using the given statement, none of the three Conclusions I, II and III can be derived.

53. If $F(x, n)$ be the number of ways of distributing "x" toys to "n" children so that each child receives at the most 2 toys, then $F(4, 3)$ = ____?

(a) 2

(b) 6

(c) 3

(d) 4

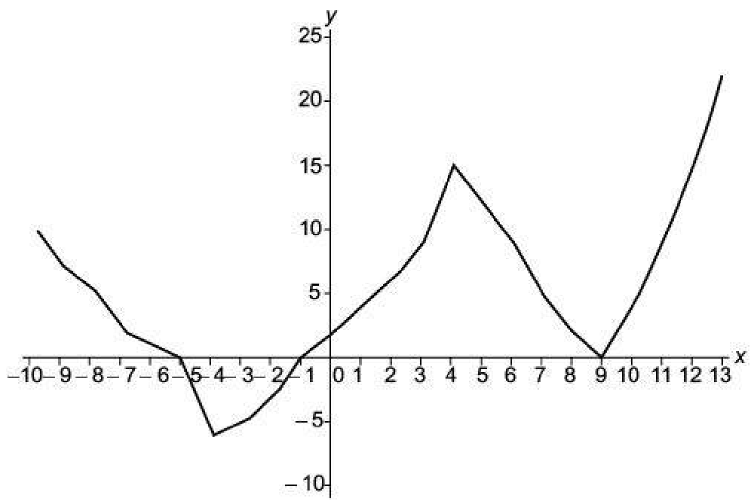
(e) 5

54. The figure below shows the graph of a function $f(x)$. How many solutions does the equation $f(f(x)) = 15$ have?

(a) 5 (b) 6

(c) 7 (d) 8

(e) Cannot be determined from the given graph



55. A sequence of positive integers is defined as $A_{n+1} = A^2_{n+1}$ for each $n \geq 0$. What is the value of Greatest Common Divisor of A900 and A_{1000}?

I. A0 = 1 II. $A_1 = 2$

56. A manufacturer produces two types of products— A and B, which are subjected to two types of operations, viz., grinding and polishing. Each unit of product A takes 2 hours of grinding and 3 hours of polishing whereas product B takes 3 hours of grinding and 2 hours of polishing. The manufacturer has 10 grinders and 15 polishers. Each grinder operates for 12 hours/day and each polisher 10 hours/day. The profit margin per unit of A and B are ` 5/- and ` 7/- respectively. If the manufacturer utilises all his resources for producing these two types of items, what is the maximum profit that the manufacturer can earn?

(a) ` 280/- (b) ` 294/-

(c) ` 515/- (d) ` 550/-

(e) None of the above

57. Consider a function $f(x) = x^4 + x^3 + x^2 + x + 1$, where x is a positive integer greater than 1. What will be the remainder if $f(x^5)$ is divided by $f(x)$?

(a) 1 (b) 4

(c) 5 (d) A monomial in x

(e) A polynomial in x

58. For all real numbers x, except $x = 0$ and $x = 1$, the function F is defined by $F\left(\frac{x}{x-1}\right) = \frac{1}{x}$

If $0 < a < 90°$ then $F((\text{cosec a})^2) =$

(a) $(\sin a)^2$

(b) $(\cos a)^2$

(c) $(\tan a)^2$

(d) $(\cot a)^2$

(e) $(\sec a)^2$

59. $F(x)$ is a fourth order polynomial with integer coefficients and with no common factor. The roots of $F(x)$ are –2, –1, 1, 2. If p is a prime number greater than 97, then the largest integer that divides $F(p)$ for all values of p is:

(a) 72

(b) 120

(c) 240

(d) 360

(e) None of the above.

60. If $x = (9 + 4\sqrt{5})^{48} = [x] + f$, where $[x]$ is defined as integral part of x and f is a fraction, then $x(1 – f)$ equals–

(a) 1

(b) Less than 1

(c) More than 1

(d) Between 1 and 2

(e) None of the above

ANSWER KEY

Level of Difficulty (I)

1. (b)	2. (d)	3. (a)	4. (c)
5. (a)	6. (d)	7. (b)	8. (a)
9. (c)	10. (b)	11. (d)	12. (b)
13. (d)	14. (a)	15. (a)	16. (c)
17. (a)	18. (d)	19. (b)	20. (c)
21. (b)	22. (b)	23. (a)	24. (c)
25. (a)	26. (d)	27. (c)	28. (c)
29. (c)	30. (d)	31. (a)	32. (a)
33. (a)	34. (a)	35. (b)	36. (c)
37. (d)	38. (d)	39. (c)	40. (a)
41. (a)	42. (d)	43. (d)	44. (a)
45. (a)	46. (c)	47. (d)	48. (d)
49. (d)	50. (d)	51. (b)	52. (a)
53. (a)	54. (b)	55. (d)	56. (d)
57. (c)	58. (b)	59. (d)	60. (c)

Level of Difficulty (II)

1. (a)	2. (d)	3. (a)	4. (d)

5. (d)	6. (c)	7. (c)	8. (b)
9. (c)	10. (c)	11. (a)	12. (b)
13. (b)	14. (c)	15. (c)	16. (a)
17. (d)	18. (c)	19. (a)	20. (c)
21. (c)	22. (d)	23. (b)	24. (a)
25. (d)	26. (d)	27. (c)	28. (a)
29. (c)	30. (d)	31. (b)	32. (d)
33. (b)	34. (a)	35. (c)	36. (d)
37. (c)	38. (a)	39. (c)	40. (b)
41. (c)	42. (b)	43. (c)	44. (b)
45. (c)	46. (c)	47. (b)	48. (c)
49. (d)	50. (a)	51. (a)	52. (c)
53. (c)	54. (b)	55. (b)	56. (c)
57. (a)	58. (c)	59. (c)	60. (a)

Level of Difficulty (III)

1. (d)	2. (d)	3. (c)	4. (d)
5. (d)	6. (d)	7. (d)	8. (b)
9. (a)	10. (a)	11. (c)	12. (c)
13. (a)	14. (b)	15. (a)	16. (b)
17. (d)	18. (b)	19. (c)	20. (b)
21. (b)	22. (b)	23. (a)	24. (b)
25. (b)	26. (d)	27. (b)	28. (a)
29. (c)	30. (a)	31. (d)	32. (a)
33. (d)	34. (a)	35. (d)	36. (d)
37. (c)	38. (d)	39. (c)	40. (d)
41. (b)	42. (b)	43. (c)	44. (b)
45. (c)	46. (b)	47. (a)	48. (b)
49. (b)	50. (b)	51. (e)	52. (a)
53. (b)	54. (e)	55. (d)	56. (b)
57. (c)	58. (b)	59. (d)	60. (a)

Solutions and Shortcuts

Level of Difficulty (I)

1. *y* = È*x*È will be defined for all values of *x*. From = – • to + •
   Hence, option (b).
2. For $y = \sqrt{x}$ to be defined, *x* should be non-negative. i.e. $x \geq 0$.
3. Since the function contains *a* $\sqrt{x}$ in it, $x \geq 0$ would be the domain.
4. For $(x - 2)^{1/2}$ to be defined $x \geq 2$.
   For $(8 - x)^{1/2}$ to be defined *x* £ 8.
   Thus, 2 £ *x* £ 8 would be the required domain.
5. $(9 - x^2) \geq 0$ fi –3 £ *x* £ 3.

6. The function would be defined for all values of x except where the denominator viz: $x^2 - 4x + 3$ becomes equal to zero.

   The roots of $x^2 - 4x + 3x = 0$ being 1, 3, it follows that the domain of definition of the function would be all values of x except $x = 1$ and $x = 3$.

7. $f(x) = x$ and $g(x) = (\sqrt{x})^2$ would be identical if $\sqrt{x}$ is defined.

   Hence, $x \geq 0$ would be the answer.

8. $f(x) = x$ is defined for all values of x.

   $g(x) = x^2/x$ also returns the same values as $f(x)$ except at $x = 0$ where it is not defined.

   Hence. option (a).

9. $f(x) = \sqrt{x^3}$ fi $f(3x) = \sqrt{(3x)^3} = 3\sqrt{3x^3}$.

   Option (c) is correct.

10. $7\,f(x) = 7\,e^x$.

11. While $\log x^2$ is defined for – • < x < • , $2 \log x$ is only defined for 0 < x < •. Thus, the two functions are identical for 0 < x < •.

12. y – axis by definition.

13. Origin by definition.

14. x^{-8} is even since $f(x) = f(-x)$ in this case.

15. $(x + 1)^3$ is not odd as $f(x)$ π $-f(-x)$.

16. $dy/dx = 2x + 10 = 0$ fi $x = -5$.

17. Required value $= (-5)^2 + 10(-5) + 11$

    $= 25 - 50 + 11 = -14$.

18. Since the denominator $x^2 - 3x + 2$ has real roots, the maximum value would be infinity.

19. The minimum value of the function would occur at the minimum value of $(x^2 - 2x + 5)$ as this quadratic function has imaginary roots.

    For $\quad y = x^2 - 2x + 5$

    $dy/dx = 2x - 2 = 0$ fi $x = 1$

    fi $\quad x^2 - 2x + 5 = 4$.

    Thus, minimum value of the argument of the log is 4.

    So minimum value of the function is $\log_2 4 = 2$.

20. $y = 1/x + 1$

    Hence, $y - 1 = 1/x$

    fi $\quad x = 1/(y - 1)$

    Thus $\quad f^{-1}(x) = 1/(x - 1)$.

21–23.

$f(1) = 0, f(2) = 1,$

$f(3) = f(1) - f(2) = -1$

$f(4) = f(2) - f(3) = 2$

$f(5) = f(3) – f(4) = –3$

$f(6) = f(4) – f(5) = 5$

$f(7) = f(5) – f(6) = –8$

$f(8) = f(6) – f(7) = 13$

$f(9) = f(7) – f(8) = –21$

21. 13
22. –8 + 2 = –6
23. 0 + 1 – 1 + 2 – 3 + 5 – 8 + 13 – 21 = –12.
24. For any ${}^nC_r$, n should be positive and $r \geq 0$.

    Thus, for positive x, $5 – x \geq 0$

    fi x = 1, 2, 3, 4, 5.

Directions for Questions 25 to 38: You essentially have to mark (a) if it is an even function, mark (b) if it is an odd function, mark (c) if the function is neither even nor odd.

Also, option (d) would occur if the function does not exist atleast one point of the domain. This means one of two things.

Either the function is returning two values for one value of x. (as in questions 26, 30, 37 and 38) or the function has a break in between (not seen in any of these questions).

We see even functions in: 25, 31, 32, 33 and 34, [Symmetry about the y axis].

We see odd functions in question 35.

While the figures in Questions 27, 28, 29 and 36 are neither odd nor even.

39. {[(3@4)! (3 #2)] @ [(4!3) @ (2 # 3)]}

    {[(3.5) ! (5)] @ [(0.5) @ (–5)]}

    {[–0.75] @ [–2.25]} = –1.5.
40. (7) @ (–0.5) = 3.25.
41. 0 @ 0.5 = 0.25. Thus, *a*
42. b = (1) (4) = 4.

    $C = \frac{(16)}{(1)(4)}$

    16/4 = 4

    Hence, both (b) and (c).
43. (a) will always be true because $(a + b)/2$ would always be greater then $(a – b)/2$ for the given value range.

    Further, $a^2 – b^2$ would always be less than $a^3 – b^3$. Thus, option (d) is correct.

44–48.

44. Option $a = (a – b)(a + b) = a^2 – b^2$
45. Option $a = (a^2 – b^2) + b^2 = a^2$.
46. 3 – 4 × 2 + 4/8 – 2 = 3 – 8 + 0.5 – 2 = – 6.5

    (using BODMAS rule)

47. The maximum would depend on the values of a and b. Thus, cannot be determined.

48. The minimum would depend on the values of a and b. Thus, cannot be determined.

49. Any of $(a + b)$ or a/b could be greater and thus we cannot determine this.

50. Again $(a + b)$ or a/b can both be greater than each other depending on the values we take for a and b.

E.g. for $a = 0.9$ and $b = 0.91$, $a + b > a/b$.

For $a = 0.1$ and $b = 0.11$, $a + b < a/b$

51. Given that $F(n-1) = \frac{1}{(2-F(n))}$, we can rewrite the expression as $F(n) = (2F(n-1) - 1)/((F(n-1))$.

For $n = 2$: $F(2) = \frac{6-1}{3} \Rightarrow F(2)\frac{5}{3}$.

The value of $F(3)$ would come out as 7/5 and $F(4)$ comes out as 9/7 and so on. What we realise is that for each value of n, after and including $n = 2$, the value of $F(n) = \frac{2n+1}{2n-1}$.

This means that the greatest integral value of $F(n)$ would always be 1 for $n = 2$ to $n = 1000$.

Thus, the value of the given expression would turn out to be:

$3 + 1 \times 999 = 1002$. Option (b) is the correct answer.

52. From the solution to the previous question, we already know how the value of the given functions at $n = 1, 2, 3$ and so on would behave.

Thus, we can try to see what happens when we write down the first few terms of the expression:

$F(1) \times F(2) \times F(3) \times F(4) \times \ldots. F(1000)$

$= 3\times\frac{5}{3}\times\frac{7}{5}\times\frac{9}{7}\times\cdots\times\frac{2001}{1999}$.

53. Since $f(0) = 15$, we get $c = 15$.

Next, we have $f(3) = f(-3) = 18$. Using this information, we get:

$9a + 3b + c = 9a - 3b + c$ Æ $3b = -3b$

\ $6b = 0$ Æ $b = 0$.

Also, since

$f(3) = 9a + 2b + c = 18$ Æ we get: $9a + 15 = 18$ Æ $a = 1/3$

The quadratic function becomes $f(x) = x^2/3 + 15$. $f(12) = 144/3 + 15 = 63$.

54. What you need to understand about $M(x^2$ q $y^2)$ is that it is the square of the sum of two squares. Since $M(x^2$ q $y^2) = 361$, we get $(x^2 + y^2)^2 = 361$, which means that the sum of the sqaures of x and y viz. $x^2 + y^2 = 19$. (Note it cannot be –19 as we are talking about the sum of two squares, which cannot be negative under any circumstance).

Also, from $M(x^2$ y $y^2) = 49$,we get $(x^2 - y^2)^2 = 49$, Æ $(x^2 - y^2) = \pm 7$

Based on these two values, we can solve for two distinct situations:

(a) When $x^2 + y^2 = 19$ and $x^2 - y^2 = 7$, we get $x^2 = 13$ and $y^2 = 6$

(b) When $x^2 + y^2 = 19$ and $x^2 - y^2 = -7$, we get $x^2 = 6$ and $y^2 = 13$

In both cases, we can see that the value of: $((x^2 y^2) + 3)$ would come out as $13 \times 6 + 3 = 81$ and the square root of its value would turn out to ± 9. Option (b) is correct.

55. The first thing you need to understand while solving this question is that, since $[m]$ will always be integral, hence $Y(4x + 5)$ will also be integral. Since $Y(4x + 5) = 5y + 3$, naturally, the value of $5y + 3$ will also be integral. This means that y is an integer. By a similar logic, the value of x will also be an integer considering the second equation: $Y(3y + 7) = x + 4$.

Using, this logic we know that $Y(4x + 5) = 4x + 5$ (because, whenever m is an integer the value of $[m] = m$).

This leads us to two linear equations as follows:

$4x + 5 = 5y +$ ...(i)

$3y + 7 = x + 4$ ...(ii)

Solving simultaneously, we will get: $x = -2$ and $y = -3$. Thus, $x^2 \times y^2 = 4 \times 9 = 36$.

56. Since $f(128) = 4$, we can see that the product of $f(256).\ f(0.5) = f(256 \times 0.5) = f(128) = 4$.

Similarly, the products $f(1).\ f(128) = f(2).\ f(64)$

$= f(4).\ f(32) = f(8).\ f(16) = 4$.

Thus, $M = 4 \times 4 \times 4 \times 4 \times 4 = 1024$.

Option (d) is the correct answer.

57. The only values of x and y that satisfy the equation $4x + 6y = 20$ are $x = 2$ and $y = 2$ (since, x, y are non negative integers). This gives us: $4 \le M/2^{2/3}$. M has to be greater than $2^{4/3}$ for this expression to be satisfied. Option (c) is correct.

58. $q(Y(-7)) = q(-2) = 14$. Option (b) is correct.

59. $F(2b) = F(b + b) = F(b).F(b) \div 2 = (F(b))^2 \div 2$

Similarly, $F(3b) = F(b + b + b) = F(b + b).F(b) \div 2 = \{F(b)^2 \div 2\}.\{F(b)\} \div 2 = (F(b))^3 \div 2^2$

Similarly, $F(4b) = (F(b))^4 \div 2^3$.

Hence, $F(12b) = (F(b))^{12} \div 2^{11}$. Option (d) is correct.

60. To test for a reflexive function as defined in the problem use the following steps:

Step 1: To start with, assume a value of '*b*' and derive a value for 'a' using the given function.

Step 2: Then, insert the value you got for '*a*' in the first step into the value of '*b*' and get a new value of '*a*'. This value of '*a*' should be equal to the first value of '*b*' that you used in the first step. If this occurs the function would be reflexive. Else it is not reflexive.

Checking for the expression in (i) if we take $b = 1$, we get:

$a = 8/1 = 8$. Inserting, $b = 8$ in the function gives us $a = 29/29 = 1$. Hence, the function given in (i) is reflexive.

Similarly checking the other two functions, we get that the function in (ii) is not reflexive while the function in (iii) is reflexive.

Thus, Option (c) is the correct answer.

Level of Difficulty (II)

1. For the function to be defined $4 - x^2 > 0$

This happens when $-2 < x < 2$.

Option (a) is correct.

2. For the function to be defined two things should happen

(a) $(1 - x) > 0$ fi $x < 1$ and

(b) $(x + 2) \geq 0$ fi $x \geq -2$. Also x π 0

Thus, option (d) is correct.

3. $\frac{5x - x^2}{4} \geq 1$ fi 1 £ x £ 4.

4. Neither $2^{-x◊x}$ nor $2^{x-x◊x◊x◊x}$ is an odd function as for neither of them is $f(x) = -f(-x)$.

5. $1 - |x|$ should be non negative.

[–1, 1] would satisfy this.

6. $4 - x^2$ π 0 and $(x^3 - x) > 0$ fi (–1, 0) » (1, •) but not 2 or –2.

7. $f(0) = 1$, $f(1) = 2$ and $f(2) = 4$

Hence, they are in G.P.

8. x would become – 2 and $y = -3$.

9. $u(f(v(t))) = u(f(t^2)) = u(1/t^2) = \left(\frac{4}{t^2}\right) - 5$.

10. $g(f(h(t))) = g(f(4t - 8)) = g\left(\sqrt{4t - 8}\right)$

$= \frac{\sqrt{4t - 8}}{4}$

11. $h(g(f(t)) = h(g(\sqrt{t})) = h(\sqrt{t}/4)$

$= \sqrt{t} - 8$

12. $f(h(g(t))) = f(h(t/4)) = f(t - 8) = \sqrt{t - 8}$.

13. All three functions would give the same values for $x > 0$. As $g(x)$ is not defined for negative x, and $h(x)$ is not defined for $x = 0$.

14. $e^x + e^{-x} = e^{-x} + e^x$

Hence, this is an even function.

15. $(x + 3)^3$ would be shifted 3 units to the left and hence $(x + 3)^3 + 1$ would shift 3 units to the left and 1 unit up. Option (c) is correct.

16. $f(x)$ ◊ $g(x) = 15x^8$ which is an even function. Thus, option (a) is correct.

17. $(x^2 + \log_e x)$ would be neither odd nor even since it obeys neither of the rules for even function ($f(x) = f(-x)$) nor for odd functions ($f(x) = -f(-x)$).

18. $(x^3 - x^2/5) = f(x) - g(x)$ is neither even nor odd.

19. $y = 1/(x - 2)$ fi $(x - 2) = 1/y$ fi $x = 1/y + 2$.

Hence, $f^{-1}(x) = 1/x + 2$.

20. $y = e^x$

fi $\log_e y = x$.

fi $f^{-1}(x) = \log_e x$.

21. $y = x/(x - 1)$

fi $(x - 1)/x = 1/y$

fi $1 - (1/x) = 1/y$

fi $1/x = 1 - 1/y$ fi $1/x = (y - 1)/y$

fi $x = y/(y - 1)$

Hence, $f^{-1}(x) = x/(x - 1)$.

22. If you differentiate each function with respect to x, and equate it to 0 you would see that for none of the three options will get you a value of $x = -3$ as its solution. Thus, option (d) viz. None of these is correct.

Directions for Questions 23 to 32: You essentially have to mark (a) if it is an even function, mark (b) if it is an odd function, mark (c) if the function is neither even nor odd.

Also, option (d) would occur if the function does not exist atleast one point of the domain. This means one of two things.

Either the function is returning two values for one value of x or the function has a break in between. This is seen in Questions 25, 26, 30 and 32.

We see even functions in Questions 24 and 28. [Symmetry about the y axis]. We see odd functions in Questions 23 and 31.

While the figures in Questions 27 and 29 are neither odd nor even.

Even fi 24, 28,

Odd 23, 31.

Neither 27, 29,

doesn't exist: 25, 26, 30 and 32.

33. $-f(x)$ would be the mirror image of the function, about the 'x' axis which is seen in option (b).
34. $-f(x) + 1$ would be mirror image about the x axis and then shifted up by 1. Option (a) satisfies this.
35. $f(x) - 1$ would shift down by 1 unit. Thus option (c) is correct.
36. $f(x) + 1$would shift by 1 unit. Thus, option (d) is correct.
37. The given function would become $h[11, 80, 1] = 2640$.
38. The given function would become $g[0, 0, 3] = 0$.
39. The given function would become $f[3, 3, 3] = 27$.
40. $f(1, 2, 3) - g(1, 2, 3) + h(1, 2, 3) = 11 - 23 + 18 = 6$.
41. The number of g's and f's should be equal on the LHS and RHS since both these functions are essentially inverse of each other.

    Option (c) is correct.
42. The required minimum value would occur at $f(x) = g(x) = 1$.
43. $SQ\,[R[(a + b)/b]] = SQ\,[R[17/5]]$ fi $SQ\,[2] = 2$.
44. $Q\,[[SQ(63) + 7\,]/9] = Q\,[[\,8 + 7]/9] = Q\,[15/9] = 1$.
45. $Q\,[[SA\,(36) + R\,(18/7)]/2] = Q\,[(7 + 4)/2] = Q\,[11/2] = 5$.

46. $[x] - \{x\} = -1$

47. $[x] + \{x\}$ will always be odd as the values are consecutive integers.

48. At $x = 5.5$, the given equation can be seen to be satisfied as: $6 + 5 = 2 \times 5.5 = 11$.

49. $f(g(t)) - g(f(t)) = f(2.5) - g(6) = 8.25 - 2.166 = 6.0833$.

50. $fog = f(3t + 2) = K(3t + 2) + 1$

$gof = g(kt + 1) = 3(kt + 1) + 2$

$K(3t + 2) + 1 = 3(kt + 1) + 2$

fi $2k + 1 = 5$

fi $k = 2$.

51. When the value of $x = 81$ and 82 is substituted in the given expression, we get,

$F(81)\, F(82) = -F(80)\, F(79)\, F(78)\, F(77)$ ...(i)

$F(82)\, F(83) = -F(81)\, F(80)\, F(79)\, F(78)$ ...(ii)

On dividing (i) by (a), we get

$\frac{F(81)}{F(83)} = \frac{F(77)}{F(81)}$ fi $F(81) \times F(81) = 81 \times 9$

fi $F(81) = 27$

Option (a) is the correct answer.

52. In order to understand this question, you first need to develop your thought process about what the value of $h(x)$ is in various cases. A little bit of trial and error would show you that the value of $h(x)$ since it depends on the minimum of $f(x)$ and $g(x)$, would definitely be dependant on the value of $f(x)$ once x becomes greater than 11 or less than –11. Also, the value of $g(x)$ is fixed as an integer at 16. It can be observed that when $f(x) > g(x)$ i.e. $f(x) > 16$, the values of $h(x)$ would also be 16 and hence would be a positive integer.

With this thought when you look at the expression of $f(x) = 121 - x^2$, you realise that the value of x can be –10, –9, –8, –7, ...0, 1, 2, 3,....8, 9, 10, i.e., 21 values of x when $h(x) = g(x) = 16$. When we use $x = 11$ or $x = -11$, the vlaue of $f(x) = \min(0, 16) = 0$ and is not a positive integral value.

Hence, the correct answer is Option (c).

53. Since, $R(x)$ is the maximum amongst the three given functions, its value would always be equal to the highest amongst the three. It is easy to imagine that $x^2 - 8$ and $3x$ are increasing functions, therefore the value of the function is continuously increasing as you increase the value of x. Similarly $x^2 - 8$ would be increasing continuously as you go farther and farther down on the negative side of the x-axis. Hence, the maximum value of $R(x)$ would be infinity. Option (c) is the correct answer.

54. In this case, the value of the function, is the minimum of the three values. If you visualise the graphs of the three functions (viz: $y = x^2 - 8$, $y = 3x$ and $y = 8$) you realise that the function $y = 3x$ (being a straight line) will keep going to negative infinity as you move to the left of zero on the negative side of the x-axis.

Hence, the minimum value of the function $R(x)$ after a certain point (when x is negative) would get dictated by the value of $3x$. This point will be the intersection of the line $y = 3x$ and the function $y = x^2 - 8$ when x is negative.

The two intersection points of the line ($3x$) and the quadratic curve ($x^2 – 8$) would be got by equating $3x = x^2 – 8$. Solving this equation tells us that the intersection points are:

$\frac{3-\sqrt{41}}{2}$ and $\frac{3+\sqrt{41}}{2}$.

$R(x)$ would depend on the following structures based on the value of x:

(i) When x is smaller than $\frac{3-\sqrt{41}}{2}$, the value of the function $R(x)$ would be given by the value of $3x$.

(ii) When x is between $\frac{3-\sqrt{41}}{2}$ and 4 the value of the function $R(x)$ would be given by the value of $x^2 – 8$, since that would be the least amongst the three functions.

(iii) After $x = 4$, on the positive side of the x-axis, the value of the function would be defined by the third function viz: $y = 8$.

A close look at these three ranges would give you that amongst these three ranges, the third range would yield the highest value of $R(x)$. Hence, the maximum possible value of $R(x) = 8$. Option (b) is correct.

55. The expression is $2x^2 – 5x + 4$, and its value at $x = 5$ would be equal to $100 – 25 + 4 = 79$. Option (b) is correct.

56. At $x = 0$, the value of the function is 20 and this value rejects the first option. Taking some higher values of x, we realise that on the positive side, the value of the function will become negative when we take x greater than 5 since the value of $(5 – x)$ would be negative. Also, the value of $f(x)$ would start tending to –•, as we take bigger values of x.

Similarly, on the negative side, when we take the value of x lower than –4, $f(x)$ becomes positive and when we take it farther away from 0 on the negative side, the value of $f(x)$ would continue tending to +•. Hence, Option (c) is the correct answer.

57. The remainder when $6^x + 4$ is divided by 2 would be 0 in every case (when x is odd)

Also, when x is even, we would get $6^x – 3$ as an odd number. In every case the remainder would be 1 (when it is divided by 2.)

Between $f(2)$, $f(4)$, $f(6)$,….$f(1000)$ there are 500 instances when x is even. In each of these instances the remainder would be 0 and hence the remainder would be 0 (in total). Option (b) is correct.

58. The product of p, q and r will be maximum if p, q and r are as symmetrical as possible. Therefore, the possible combination is (4, 3, 3).

Hence, maximum value of $pq + qr + pr + pqr = 4 \times 3 + 4 \times 3 + 3 \times 3 + 4 \times 3 \times 3 = 69$.

Hence, Option (c) is correct.

59. The equation given in the question is: $3a(x) + 2a(2–x) = (x + 3)^2$ ….(i)

Replacing x by $(2–x)$ in the above equation, we get

$3a(2–x) + 2a(x) = (5–x)^2$

Solving the above pairs of equation, we get

$a(x) = (x^2 + 38 – 23)/5$

Thus, $G(-5) = -188/5 = -37.6$. The value of $[-37.6] = -37$. Hence, Option (c) is the correct answer.

60. The first thing you do in this question is to create the chain of values of $f(x)$ for $x = 1, 2, 3$ and so on.

The chain of values would look something like this:

	When x is odd			*When x is even*	
$f(1)$	Value is given	6	$f(2)$	Value is given	4
$f(3)$	$= 1 + f(1)$	7	$f(4)$	$= 3 + f(2)$	7
$f(5)$	$= 3 + f(3)$	10	$f(6)$	$= 3 + f(4)$	10
$f(7)$	$= 5 + f(5)$	15	$f(8)$	$= 3 + f(6)$	13
$f(9)$	$= 7 + f(7)$	22	$f(10)$	$= 3 + f(8)$	16
$f(11)$	$= 9 + f(9)$	31			

In order to evaluate the value of the embedded function represented by $f(f(f(f(f(f(1))))))$, we can use the above values and think as follows:

$f(f(f(f(1)))) = f(f(f(6))) = f(f(10)) = f(16) = 25$

Also, $f(f(f(f(2) = f(f(f(4))) = f(f(7)) = f(15) = 55$

Hence, the product of the two values is $25 \times 55 = 1375$.

Option (a) is correct.

Level of Difficulty (III)

1. $x - |x|$ is either negative for $x < 0$ or 0 for $x \geq 0$. Thus, option (d) is correct.
2. The domain should simultaneously satisfy:

   $x - 1 \geq 0$, $(1 - x) \geq 0$ and $(x^2 + 3) \geq 0$.

   Gives us: $x \geq 1$ and x £ 1

   The only value that satisfies these two simultaneously is $x = 1$.
3. For the function to exist, the argument of the logarithmic function should be positive. Also, $(x + 4) \geq 0$ should be obeyed simultaneously.

   For $\frac{(x-5)}{(x^2 - 10x + 24)}$ to be positive both numerator and denominator should have the same sign.

   Considering all this, we get:

   $4 < x < 5$ and $x > 6$.

   Option (c) is correct.
4. Both the brackets should be non-negative and neither $(x + 3)$ nor $(1 + x)$ should be 0.

   For $(x - 3)/(x + 3)$ to be non negative we have $x > 3$ or $x < -3$.

   Also for $(1 - x)/(1 + x)$ to be non-negative $-1 < x < 1$. Since there is no interference in the two ranges, Option (d) would be correct.
8. $f(f(t)) = f((t - 1)/(t + 1))$

$= \left[\left(\frac{t-1}{t+1}\right)-1\right] \Big/ \left[\left(\frac{t-1}{t+1}\right)+1\right] = \frac{t-1-t-1}{t-1+t+1}$

$= -2/2t = -1/t$.

9. fog = $f(\log_e x) = e^{\log ex} = x$.

10. gof = $g(e^x) = \log_e e^x = x$.

11. Looking at the options, one unit right means x is replaced by $(x - 1)$. Also, 1 unit down means –1 on the RHS.

    Thus, $(y + 1) = 1/(x - 1)$

12. For none of these is $f(t) = f(-t)$

    Thus, Option (d) is correct.

13. Option (b) is odd because:

    $\frac{a^{-t}+a^{t}}{a^{t}-a^{-t}} = -1 \times \left(\frac{a^{-t}+a^{t}}{a^{-t}-a^{t}}\right)$

    Similarly option (c) is also representing an odd function. The function in option (a) is not odd.

14. $f(f(t)) = f[t/(1+ t^2)^{1/2}] = t/(1 + 2t^2)^{1/2}$.

15. By trial and error it is clear that at $x = 3$, the value of the function is 19. At other values of 'x' the value of the function is less than 19.

17. Take different values of n to check each option. Each of Options (a), (b) and (c) can be ruled out. Hence, Option (d) is correct.

    Solutions to 18 to 20:

    $f(1) = 0, f(2) = 1,$

    $f(3) = f(1) - f(2) = -1$

    $f(4) = f(2) - f(3) = 2$

    $f(5) = f(3) - f(4) = -3$

    $f(6) = f(4) - f(5) = 5$

    $f(7) = f(5) - f(6) = -8$

    $f(8) = f(6) - f(7) = 13$

18. It can be seen that $f(x)$ is positive wherever x is even and negative whenever x is odd.

19. $f(f(6) = f(5) = -3$.

20. $f(6) - f(8) = 5 - 13 = -8 = f(7)$.

21. Option (b) is not even since $e^x - e^{-x} \pi e^{-x} - e^x$.

22. We have $f(x) \Diamond f(1/x) = f(x) + f(1/x)$

    fi $f(1/x) [f(x) - 1] = f(x)$

    For $x = 4$, we have $f(1/4) [f(4) - 1] = f(4)$

    fi $f(1/4) [64] = 65$

    fi $f(1/4) = 65/64 = 1/64 + 1$

    This means $f(x) = x^3 + 1$

    For $f(6)$ we have $f(6) = 216 + 1 = 217$.

Directions for Questions 23 to 34: You essentially have to mark (a) if it is an even function, mark (b) if it is an odd function, mark (c) if the function is neither even nor odd.

Also, Option (d) would occur if the function does not exist at, atleast one point of the domain. This means one of two things.

Either the function is returning two values for one value of x or the function has a break in between (as in questions 26, 31 and 33).

We see even functions in Questions 23, 28, 30, 32 and 34 [Symmetry about the y axis]. We see odd functions in Questions 24, 25 and 27.

While the figure in Question 29 is neither odd nor even.

Solutions to 35–40:

In order to solve this set of questions first analyse each of the functions:

$A(x, y, z)$ = will always return the value of the highest between x and y.

$B(x, y, z)$ will return the value of the maximum amongst x, y and z.

$C(x, y, z)$ and $D(x, y, z)$ would return the second highest values in all cases while max (x, y, z) and min(x, y, z) would return the maximum and minimum values amongst x, y, and z respectively.

35. When either x or y is maximum.
36. This would never happen.
37. When z is maximum, A and B would give different values. Thus, option (c) is correct.
38. Never.
39. I and III are always true.
40. We cannot determine this because it would depend on whether the integers x, y, and z are positive or negative.

Solutions to 41 to 49:

$F(x, y)$ is always positive

$F(f(x, y)$ is always negative

$G(f(x, y)$ is always positive

41. $F \times G$ would always be negative while $- F \times G$ would always be positive.

    Hence, Option (b) $F \times G < - F \times G$ is correct.
42. Option (b) can be seen to give us $4a^2/4 = a^2$.
43. $(5 - 1)/(1+ 3) = 4/4 = 1$.
44. The given expression = $(45 - 10)/(5 + 2) = 35/7 = 5$.

    Option (b) = $20/4 = 5$.

Directions for Questions 45 to 49: Do the following analysis:

$A(f(x, y))$ is positive

$B(f(x, y))$ is negative

$C(f(x, y))$ is positive

$D(f(x, y))$ is positive

$E(f(x, y))$ is positive and so on.

45. $1 - 3 + 5 - 7 + 9 - 11 + \ldots - 51$

= (1 + 5 + 9 + 13 + … 49) – (3 + 7 + 11 … + 51)

= –26

46. Verify each statement to see that (ii) and (iii) are true.

47. The given expression becomes:

Min (max [5, – 7, 9], min [3, – 1 , 1], max [7, 6, 10])

= Min [9, – 1, 10]

= – 1.

48. The given expression becomes:

Max [È*a* + *b*È, – È*b* + *c*È, È*c* + *d*È]

This would never be negative.

49. The respective values are:

–3/2, –7/12, –8/15, and –5/6.

Option (b) is second lowest.

50. Let $s = 1$, $t = 2$ and $b = 3$

Then, $f(s + t) + f(s – t)$

$= f(3) + f(–1) = (3^3 + 3^{–3})/2 + (3^{–1}+ 3^1)/2$

= [(27 + (1/27)]/2 + [3 + (1/3)]/2

= 730/54 + 10/6

= 820/54 = 410/27

Option (b) 2 $f(s) \times f(t)$ gives the same value.

51. This question is based on the logic of a chain function. Given the relationship

A*t* = (*t* + 1) A*t* – 1 – *t* A*t* – 2

We can clearly see that the value of A2 would depend on the values of A_0 and A_1. Putting $t = 2$ in the expression, we get:

A_2 = 3A1 – 2A0 = 7; A_3 = 19; A_4 = 67 and A_5 = 307. Clearly, A6 onwards will be larger than 307 and hence none of the three conclusions are true. Option (e) is the correct answer.

52. In order to solve this question, we would need to check each of the value ranges given in the conclusions: Checking whether Conclusion I is possible

For B = 2, we get A + C = 4 (since A + B + C = 6). This transforms the second equation AB + BC + CA = 9 to:

2(A + C) + CA = 9 Æ CA = 1.

Solving CA = 1 and A + C = 4 we get: (4 – A)A = 1 Æ A^2 –4A + 1 = 0 Æ A = 2 + $3^{1/2}$ and C = 2– $3^{1/2}$. Both these numbers are real and it satisfies A < B < C and hence, Conclusion I is true.

Checking Conclusion II: If we chose A = 2.5, the condition is not satisfied since we get the other two variables as (3.5 + $11.25^{1/2}$) ÷ 2 ª 3.4 and (3.5 – $11.25^{1/2}$) ÷ 2 ª 0.1. In this case, A is no longer the least value and hence Conclusion II is rejected.

Checking Conclusion III we can see that 0 < C < 1 cannot be possible since C being the largest of the three values has to be greater than 3 (the largest amongst A, B, and C would be greater than the average of A, B, C).

Option (a) is correct.

53. The number of ways of distributing n identical things to r people such that any person can get any number of things including 0 is always given by ${}^{n+r-1}C_{r-1}$. In the case of $F(4,3)$, the value of $n = 4$ and $r = 3$ and hence the total number of ways without any constraints would be given by ${}^{4+3-1}C_{3-1}$ $= {}^{6}C_2 = 15$. However, out of these 15 ways of distributing the toys, we cannot count any way in which more than 2 toys are given to any one child. Hence, we need to reduce as follows:

The distribution of 4 toys as (3, 1, 0) amongst three children A, B and C can be done in 3! = 6 ways.

Also, the distribution of 4 toys as (4, 0, 0) amongst three children A, B and C can be done in 3 ways.

Hence, the value of $F(4, 3) = 15 – 6 – 3 = 6$.

Option (b) is correct.

54. $f (f (x) = 15$ when $f (x) = 4$ or $f (x) = 12$ in the given function. The graph given in the figure becomes equal to 4 at 4 points and it becomes equal to 12 at 2 points in the figure. This gives us 6 points in the given figure when $f (f (x) =15$. However, the given function is continuous beyond the part of it which is shown between –10 and +13 in the figure. Hence, we do not know how many more solutions to $f (f (x) = 15$ would be there. Hence, Option (e) is the correct answer.

55. The given function is a chain function where the value of A_{n+1} depends on the value A_n.

Thus for $n = 0$, $A1 = A_0^2 + 1$.

For n = 1, $A_2 = A_1^2 + 1$ and so on.

In such functions, if you know the value of the function at any one point, the value of the function can be calculated for any value till infinity.

Hence, Statement I is sufficient by itself to find the value of the GCD of A_{900} and A_{1000}.

So also, the Statement II is sufficient by itself to find the value of the GCD of A_{900} and A_{1000}.

Hence, Option (d) is correct.

56. This question can be solved by first putting up the information in the form of a table as follows:

	Product A	*Product B*	*No of machines available*	*No of Hours/day per Machine.*	*Total Hrs. per day available for each activity*
Grinding	2 hr	3 hr	10	12	120
Polishing	3 hr	2 hr	15	10	150
Profit	` 5	` 7			

On the surface, the profit of Product B being higher, we can think about maximising the number of units of Product B. Grinding would be the constraint when we maximise Product B production and we can produce a maximum of 120 ÷ 3 = 40 units of Product B to get a profit of ` 280. The clue that this is not the correct answer comes from the fact that there is a lot of 'polishing' time left in this situation. In order to try to increase the profit we can check that if we reduce production of Product B and try to increase the production of Product A, does the profit go up?

When we reduce the production of Product B by 2 units, the production of Product A goes up by 3 units and the profit goes up by +1 (–2 × 7 + 3 × 5 gives a net effect of +1). In this case, the grinding time remains the same (as there is a reduction of 2 units × 3 hours/unit = 6 hours in grinding time due to the reduction in Product B's production, but there is also a simultaneous increase of 6 hours in the use of the grinders in producing 3 units of Product A). Given that a reduction in the production of Product B, with a simultaneous maximum possible increase in the production of Product A, results in an increase in the profit, we would like to do this as much as possible. To think about it from this point this situation can be tabulated as under for better understanding:

	Product A Production (A)	*Product B Production (B)*	*Grinding Machine Usage = 3A + 2B*	*Polishing Machine Usage = 2A + 3B*	*Time Left on Grinding Machine*	*Time Left on Polishing Machine*	*Profit = 7A+5B*
Case 1	40	0	120	80	0	70	280
Case 2	38	3	120	85	0	65	281
Case 3	36	6	120	90	0	60	281
The limiting case would occur when we reduce the time left on the polishing machine to 0. That would happen in the following case:							
Optimal case	12	42	120	150	0	0	294

Hence, the answer would be 294.

57. The value of $f(x)$ as given is: $f(x) = x^4 + x^3 + x^2 + x + 1 = 1 + x + x^2 + x^3 + x^4 + x^5$. This can be visualised as a geometric progression with 5 terms with the first term 1 and common ratio x. The sum of the GP = $f(x) = \frac{x^5-1}{x-1}$

The value of $f(x^5) = x^{20} + x^{15} + x^{10} + x^5 + 1$ and this can be rewritten as:

$F(x^5) = (x^{20}–1) + (x^{15}–1) + (x^{10}–1) + (x^5–1) + 5$. When this expression is divided by $f(x) = \frac{x^5-1}{x-1}$ we get each of the first four terms of the expression would be divisible by it, i.e. $(x^{20}–1)$ would be divisible by $f(x) = \frac{x^5-1}{x-1}$ and would leave no remainder (because $x^{20}–1$ can be rewritten in the form $(x^5–1) \times (x^{15} + x^{10} + x^5 + 1)$ and when you divide this expression by $\left(\frac{x^5-1}{x-1}\right)$ we get the remainder as 0.)

A similar logic would also hold for the terms $(x^{15}–1)$, $(x^{10}–1)$ and $(x^5–1)$. The only term that would leave a remainder would be 5 when it is divided by $\left(\frac{x^5-1}{x-1}\right)$

Also, for $x \geq 2$ we can see the value of $\left(\frac{x^5-1}{x-1}\right)$ would be more than 5. Hence, the remainder would always be 5 and Option (c) is the correct answer.

58. Start by putting $\frac{x}{x-1}$ = (cosec a)^2 in the given expression

$$F\left(\frac{x}{x-1}\right)=\frac{1}{x}$$

Now for 0 < a < 90°

$\frac{x}{x-1}$ = (cosec a)^2 fi $x = \frac{1}{1-\sin^2\alpha} \Rightarrow \frac{1}{x}$ = cos^2 a

Hence, Option (b) is correct.

59. Given that the roots of the equation $F(x)$= 0 are –2, –1, 1 and 2 respectively and the $F(x)$ is a polynomial with the highest power of x as x^4, we can create the value of

$F(x) = (x + 2)(x + 1)(x –1)(x − 2)$

Hence, $F(p) = (P + 2)(P + 1)(P –1)(P − 2)$

It is given to us that P is a prime number greater than 97. Hence, p would always be of the form 6n ± 1 where n is a natural number greater than or equal to 17.

Thus, we get two cases for $F(p)$.

Case 1: If $p = 6n + 1$.

F(6n + 1) = (6n + 3)(6n + 2)(6n)(6n –1)

= 3(2n + 1)· 2(3n + 1)(6n)(6n –1)

= (36)(2n + 1)(3n + 1)(n)(6n –1) …(i)

If you try to look for divisibility of this expression by numbers given in the options for various values of $n \geq 17$, we see that the for n = 17 and 18 both 360 divides the value of $F(p)$. However at n = 19, none of the values in the four options divides 36 × 39 × 58 × 19 × 113. In this case however, at n = 19, 6n + 1 is not a prime number hence, this case is not to be considered. Whenever we put a value of n as a value greater than 17, such that 6n+1 becomes a prime number, we also see that the value of $F(p)$ is divisible by 360. This divisibility by 360 happens since the expression (2n + 1)(3n + 1)(n)(6n –1) … is always divisible by 10 in all such cases. A similar logic can be worked out when we take $p = gn$–1. Hence, the Option (d) is the correct answer.

60. In order to solve this question, we start from the value of $x = (9 + 4\sqrt{5})^{48}$.

Let the value of $x(1 – f) = xy$. (We are assuming $(1 – f) = y$, which means that y is between 0 to 1).

The value of $x = (9 + 4\sqrt{5})^{48}$ can be rewritten as $[{}^{48}C_0\, 9^{48} + {}^{48}C_1\, 9^{47}(4\sqrt{5}) + {}^{48}C_2\, 9^{46}(4\sqrt{5})^2 + \ldots + {}^{48}C_{47}(9)(4\sqrt{5})^{47} + {}^{48}C_{48}\,(4\sqrt{5})^{48}]$ using the binary theorem.

In this value, it is going to be all the odd powers of the $(4\sqrt{5})$ which would account for the value of 'f' in the value of x. Thus, for instance it can be seen that the terms ${}^{48}C_0\, 9^{48}$, ${}^{48}C_2\, 9^{46}(4\sqrt{5})^2$, …. ${}^{48}C_{48}\,(4\sqrt{5})^{48}$ would all be integers. It is only the terms: ${}^{48}C_1 9^{47}\,(4\sqrt{5})$, ${}^{48}C_3\, 9^{45}(4\sqrt{5})^3$,… ${}^{48}C_{47}\,(9)(4\sqrt{5})^{47}$ which would give us the value of 'f' in the value of x.

Hence, $x(1–f) = x\,[1 – {}^{48}C_1\, 9^{47}\,(4\sqrt{5}) – {}^{48}C_3\, 9^{45}\,(4\sqrt{5})^3 –\ldots – {}^{48}C_{47}\,(9)(4\sqrt{5})^{47}]$

In order to think further from this point, you would need the following thought. Let $y = (9 - 4\sqrt{5})^{48}$.

Also, $x + y = \{^{48}C_0\, 9^{48} + {}^{48}C_1\, 9^{47}(4\sqrt{5}) + {}^{48}C_2\, 9^{46}(4\sqrt{5})^2 + \ldots + {}^{48}C_{47}\, (9)(4\sqrt{5})^{47} + {}^{48}C_{48}\, (4\sqrt{5})^{48}\} + \{^{48}C_0\, 9^{48} - {}^{48}C_1\, 9^{47}\, (4\sqrt{5}) + {}^{48}C_2\, 9^{46}\, (4\sqrt{5})^2 + \ldots - {}^{48}C_{47}\, (9)\, (4\sqrt{5})^{47} + {}^{48}C_{48}\, (4\sqrt{5})^{48}\} = 2\{^{48}C_0\, 9^{48} + {}^{48}C_2\, 9^{46}\, (4\sqrt{5})^2 + \ldots {}^{48}C_{48}\, (4\sqrt{5})^{48}\}$ – the bracket in this expression has only retained the even terms which are integral. Hence, the value of $x+y$ is an integer.

Further, $x+y = [x] + f$ and hence, if $x+y$ is an integer, $[x] + f + y$ would also be an integer. This automatically means that $f+y$ must be an integer (as $[x]$ is an integer).

Now, the value of y is between 0 to 1 and hence when we add the fractional part of x i.e. 'f' to y, and we need to make it an integer, the only possible integer that $f + y$ can be equal to is 1.

Thus, if $f + y = 1$ Æ $y = (1 - f)$.

In order to find the value of $x(1 - f)$ we can find the value of $x \times y$.

Then, $x(1 - f) = x \times y = (9 + 4\sqrt{5})^{48} \times (9 - 4\sqrt{5})^{48} = (81 - 80)^{48} = 1$

$x(1 - f) = 1$