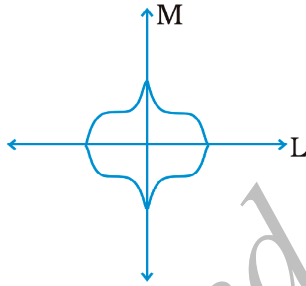


4. To complete the figure we need it to be symmetrical about line M also. Draw the remaining part of figure as shown.

This figure has two lines of symmetry i.e. line L and line M.

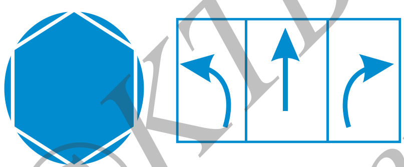
Try taking similar pieces and adding to them so that the figure has two lines of symmetry.

Some shapes have only one line of symmetry; some have two lines of symmetry; and some have three or more.

Can you think of a figure that has six lines of symmetry?

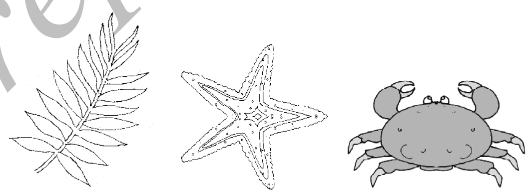
Symmetry, symmetry everywhere!

- Many road signs you see everyday have lines of symmetry. Here, are a few.

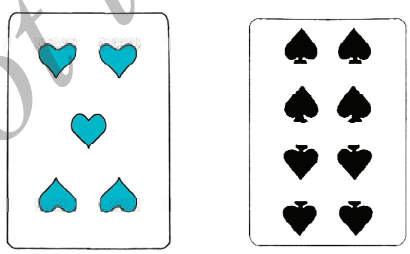


Identify a few more symmetric road signs and draw them. Do not forget to mark the lines of symmetry.

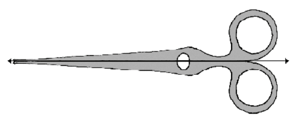
- The nature has plenty of things having symmetry in their shapes; look at these:



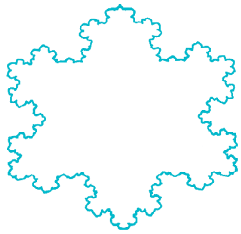
- The designs on some playing cards have line symmetry. Identify them for the following cards.



- Here is a pair of scissors!
  How many lines of symmetry does it have?

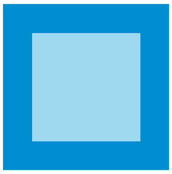


- Observe this beautiful figure. It is a symmetric pattern known as Koch's Snowflake. (If you have access to a computer, browse through the topic "Fractals" and find more such beauties!). Find the lines of symmetry in this figure.

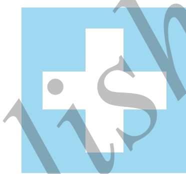


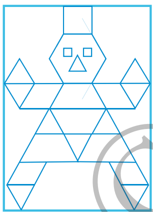
EXERCISE 13.2

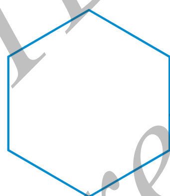
1. Find the number of lines of symmety for each of the following shapes :

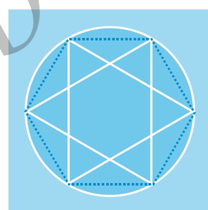
(a) 

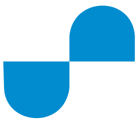
(b) 

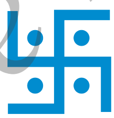
(c) 

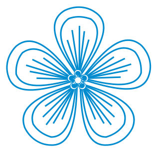
(d) 

(e) 

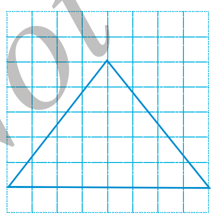
(f) 

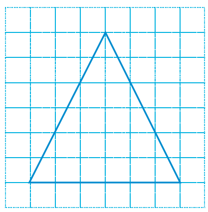
(g) 

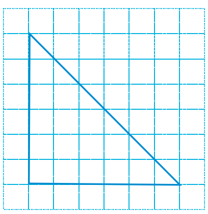
(h) 

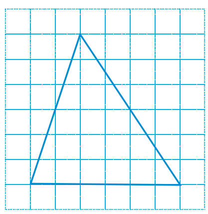
(i) 

2. Copy the triangle in each of the following figures on squared paper. In each case, draw the line(s) of symmetry, if any and identify the type of triangle. (Some of you may like to trace the figures and try paper-folding first!)

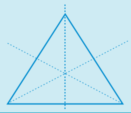
(a) 

(b) 

(c) 

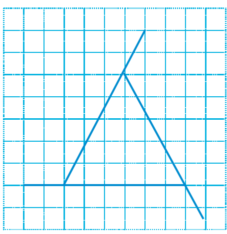
(d) 

3. Complete the following table.

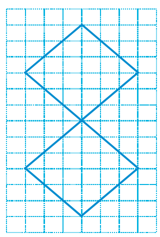
Shape	Rough figure	Number of lines of symmetry
Equilateral triangle		3
Square		
Rectangle		
Isosceles triangle		
Rhombus		
Circle		

4. Can you draw a triangle which has
   (a) exactly one line of symmetry?
   (b) exactly two lines of symmetry?
   (c) exactly three lines of symmetry?
   (d) no lines of symmetry?
   Sketch a rough figure in each case.
5. On a squared paper, sketch the following:
   (a) A triangle with a horizontal line of symmetry but no vertical line of symmetry.
   (b) A quadrilateral with both horizontal and vertical lines of symmetry.
   (c) A quadrilateral with a horizontal line of symmetry but no vertical line of symmetry.
   (d) A hexagon with exactly two lines of symmetry.
   (e) A hexagon with six lines of symmetry.
   (**Hint :** It will be helpful if you first draw the lines of symmetry and then complete the figures.)
6. Trace each figure and draw the lines of symmetry, if any:

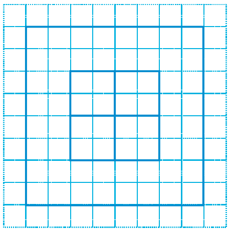
(a)



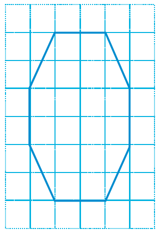
(b)



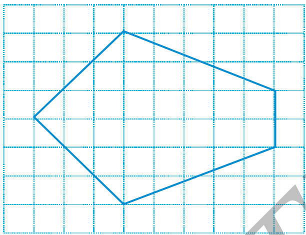
(c)



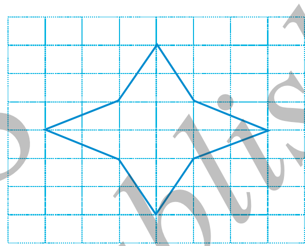
(d)



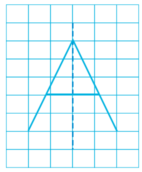
(e)



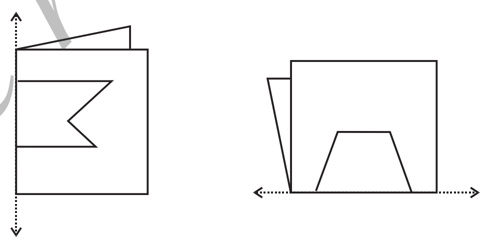
(f)



7. Consider the letters of English alphabets, A to Z. List among them the letters which have
   (a) vertical lines of symmetry (like A)
   (b) horizontal lines of symmetry (like B)
   (c) no lines of symmetry (like Q)



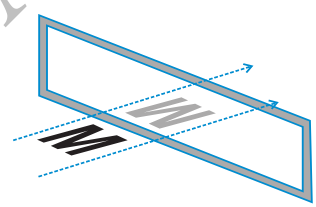
8. Given here are figures of a few folded sheets and designs drawn about the fold. In each case, draw a rough diagram of the complete figure that would be seen when the design is cut off.

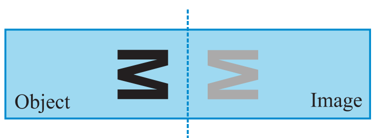


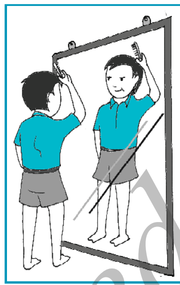
13.5 Reflection and Symmetry

Line symmetry and mirror reflection are naturally related and linked to each other.

Here is a picture showing the reflection of the English letter M. You can imagine that the mirror is invisible and can just see the letter M and its image.





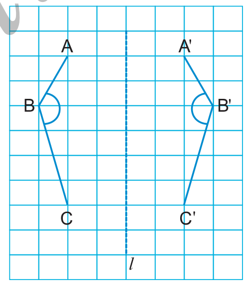


The object and its image are symmetrical with reference to the mirror line. If the paper is folded, the mirror line becomes the line of symmetry. We then say that the image is the reflection of the object in the mirror line. You can also see that when an object is reflected, there is no change in the lengths and angles; i.e. the lengths and angles of the object and the corresponding lengths and angles of the image are the same. However, in one aspect there is a change, i.e. there is a difference between the object and the image. Can you guess what the difference is?

(**Hint :** Look yourself into a mirror).

Do This

On a squared sheet, draw the figure ABC and find its mirror image A'B'C' with *l* as the mirror line.



Compare the lengths of

AB and A'B'; BC and B'C'; AC and A'C'.

Are they different?

Does reflection change length of a line segment?

Compare the measures of the angles (use protractor to measure) ABC and A'B'C'.

Does reflection change the size of an angle?

Join AA', BB' and CC'. Use your protractor to measure the angles between the lines *l* and *AA'*, *l* and *BB'*, *l* and *CC'*.

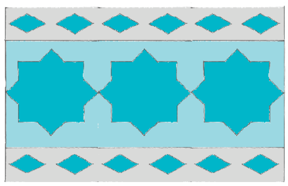
What do you conclude about the angle between the mirror line *l* and the line segment joining a point and its reflected image?

Try These

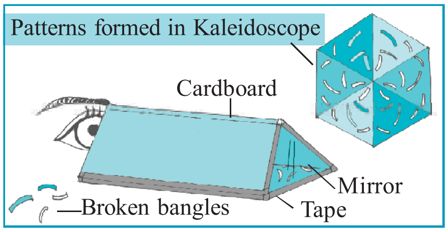
If you are 100 cm in front of a mirror, where does your image appear to be? If you move towards the mirror, how does your image move?

Paper decoration

Use thin rectangular coloured paper. Fold it several times and create some intricate patterns by cutting the paper, like the one shown here. Identify the line symmetries in the repeating design. Use such decorative paper cut-outs for festive occasions.



Kaleidoscope

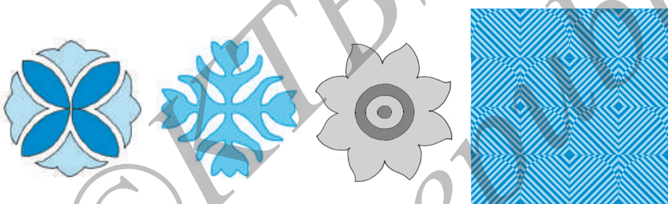


A kaleidoscope uses mirrors to produce images that have several lines of symmetry (as shown here for example). Usually, two mirrors strips forming a V-shape are used. The angle between the mirrors determines the number of lines of symmetry.

Make a kaleidoscope and try to learn more about the symmetric images produced.

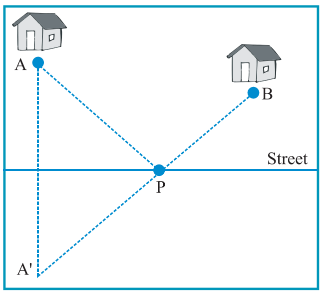
Album

Collect symmetrical designs you come across and prepare an album. Here are a few samples.



An application of reflectional symmetry

A paper-delivery boy wants to park his cycle at some point P on the street and delivers the newspapers to houses A and B. Where should he park the cycle so that his walking distance AP + BP will be least?

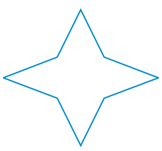


You can use reflectional symmetry here. Let A' be the image of A in the mirror line which is the street here. Then the point P is the ideal place to park the cycle (where the mirror line and A'B meet). Can you say why?

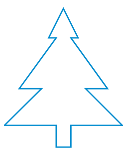
EXERCISE 13.3

1. Find the number of lines of symmetry in each of the following shapes. How will you check your answers?

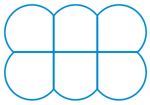
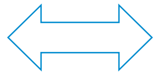
(a)



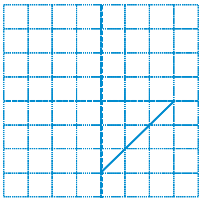
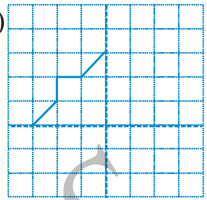
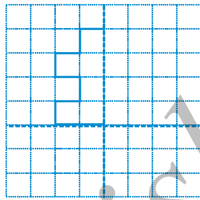
(b)

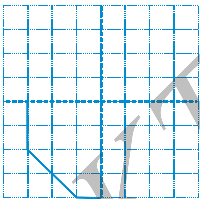
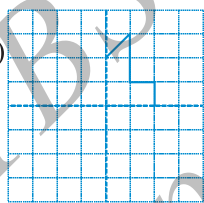
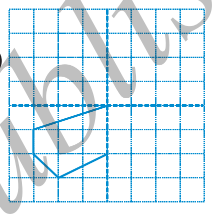


(c)

(d)  (e)  (f) 

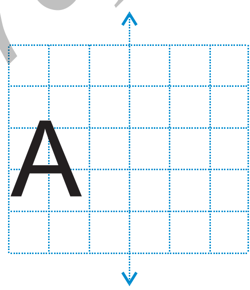
2. Copy the following drawing on squared paper. Complete each one of them such that the resulting figure has two dotted lines as two lines of symmetry.

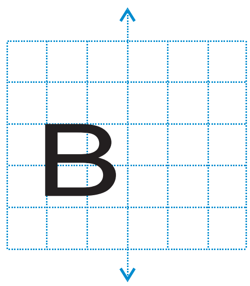
(a)  (b)  (c) 

(d)  (e)  (f) 

How did you go about completing the picture?

3. In each figure alongside, a letter of the alphabet is shown along with a vertical line. Take the mirror image of the letter in the given line. Find which letters look the same after reflection (i.e. which letters look the same in the image) and which do not. Can you guess why?



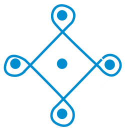


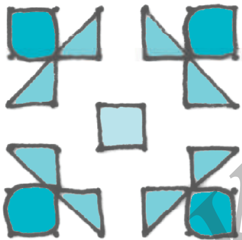
Try for O E M N P H L T S V X

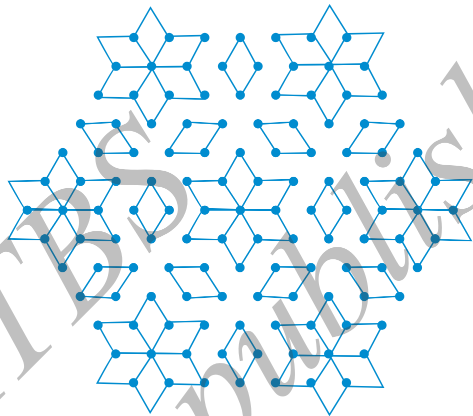
Rangoli patterns

Kolams and Rangoli are popular in our country. A few samples are given here. Note the use of symmetry in them. Collect as many patterns as possible of these and prepare an album.

Try and locate symmetric portions of these patterns alongwith the lines of symmetry.







What have we discussed?

1. A figure has *line symmetry* if a line can be drawn dividing the figure into two identical parts. The line is called a *line of symmetry*.
2. A figure may have no line of symmetry, only one line of symmetry, two lines of symmetry or multiple lines of symmetry. Here are some examples.

Number of lines of symmetry	*Example*
No line of symmetry	A scalene triangle
Only one line of symmetry	An isosceles triangle
Two lines of symmetry	A rectangle
Three lines of symmetry	An equilateral triangle

3. The line symmetry is closely related to mirror reflection. When dealing with mirror reflection, we have to take into account the left ↔ right changes in orientation.

Symmetry has plenty of applications in everyday life as in art, architecture, textile technology, design creations, geometrical reasoning, Kolams, Rangoli etc.

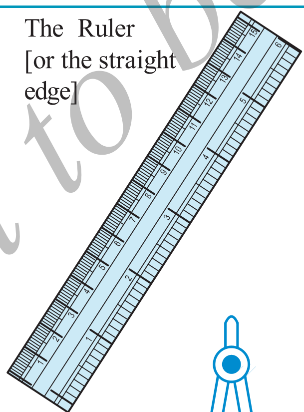
Practical Geometry

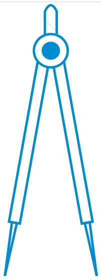
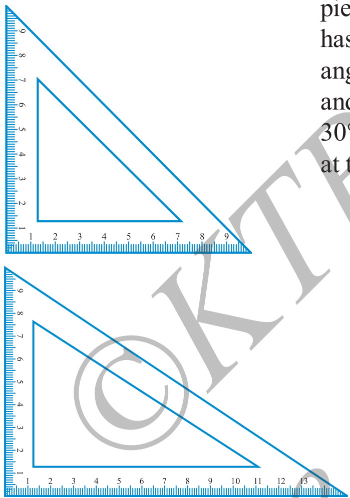
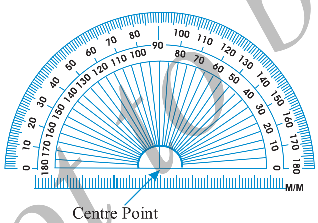
Chapter 14

14.1 Introduction

We see a number of shapes with which we are familiar. We also make a lot of pictures. These pictures include different shapes. We have learnt about some of these shapes in earlier chapters as well. Why don't you list those shapes that you know about alongwith how they appear?

In this chapter we shall learn to make these shapes. In making these shapes we need to use some tools. We shall begin with listing these tools, describing them and looking at how they are used.

S.No.	Name and figure	Description	Use
1.	The Ruler [or the straight edge] 	A ruler ideally has no markings on it. However, the ruler in your instruments box is graduated into centimetres along one edge (and sometimes into inches along the other edge).	To draw line segments and to measure their lengths.
2.	The Compasses 	A pair – a pointer on one end and a pencil on the other.	To mark off equal lengths but not to measure them. To draw arcs and circles.

3.	The Divider		A pair of pointers	To compare lengths.
4.	Set-Squares 		Two triangular pieces – one of them has 45°, 45°, 90° angles at the vertices and the other has 30°, 60°, 90° angles at the vertices.	To draw perpendicular and parallel lines.
5.	The Protractor 		A semi-circular device graduated into 180 degree-parts. The measure starts from 0° on the right hand side and ends with 180° on the left hand side and vice-versa.	To draw and measure angles.

We are going to consider **"Ruler and compasses constructions"**, using ruler, only to draw lines, and compasses, only to draw arcs.

Be careful while doing these constructions.

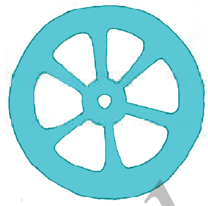
Here are some tips to help you.

(a) Draw thin lines and mark points lightly.

(b) Maintain instruments with sharp tips and fine edges.

(c) Have two pencils in the box, one for insertion into the compasses and the other to draw lines or curves and mark points.

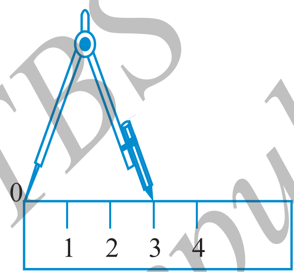
14.2 The Circle

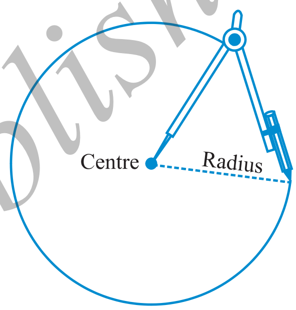


Look at the wheel shown here. Every point on its boundary is at an equal distance from its centre. Can you mention a few such objects and draw them? Think about five such objects which have this shape.

14.2.1 Construction of a circle when its radius is known

Suppose we want to draw a circle of radius 3 cm. We need to use our compasses. Here are the steps to follow.





Step 1 Open the compasses for the required radius of 3cm.

Step 2 Mark a point with a sharp pencil where we want the centre of the circle to be. Name it as O.

Step 3 Place the pointer of the compasses on O.

Step 4 Turn the compasses slowly to draw the circle. Be careful to complete the movement around in one instant.

Think, discuss and write

How many circles can you draw with a given centre O and a point, say P?

EXERCISE 14.1

1. Draw a circle of radius 3.2 cm.
2. With the same centre O, draw two circles of radii 4 cm and 2.5 cm.
3. Draw a circle and any two of its diameters. If you join the ends of these diameters, what is the figure obtained? What figure is obtained if the diameters are perpendicular to each other? How do you check your answer?
4. Draw any circle and mark points A, B and C such that
   (a) A is on the circle. (b) B is in the interior of the circle.
   (c) C is in the exterior of the circle.
5. Let A, B be the centres of two circles of equal radii; draw them so that each one of them passes through the centre of the other. Let them intersect at C and D. Examine whether $\overline{AB}$ and $\overline{CD}$ are at right angles.

14.3 A Line Segment

Remember that a line segment has two end points. This makes it possible to measure its length with a ruler.

If we know the length of a line segment, it becomes possible to represent it by a diagram. Let us see how we do this.

14.3.1 Construction of a line segment of a given length

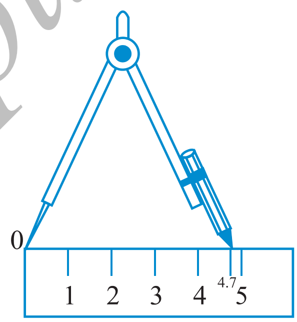
Suppose we want to draw a line segment of length 4.7 cm. We can use our ruler and mark two points A and B which are 4.7 cm apart. Join A and B and get $\overline{AB}$. While marking the points A and B, we should look straight down at the measuring device. Otherwise we will get an incorrect value.

Use of ruler and compasses

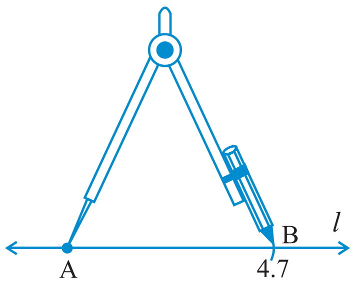
A better method would be to use compasses to construct a line segment of a given length.

Step 1 Draw a line *l*. Mark a point A on a line *l*.

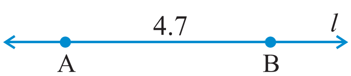
Step 2 Place the compasses pointer on the zero mark of the ruler. Open it to place the pencil point upto the 4.7cm mark.



Step 3 Taking caution that the opening of the compasses has not changed, place the pointer on A and swing an arc to cut *l* at B.



Step 4 $\overline{AB}$ is a line segment of required length.



EXERCISE 14.2

1. Draw a line segment of length 7.3 cm using a ruler.
2. Construct a line segment of length 5.6 cm using ruler and compasses.
3. Construct $\overline{AB}$ of length 7.8 cm. From this, cut off $\overline{AC}$ of length 4.7 cm. Measure $\overline{BC}$.
4. Given $\overline{AB}$ of length 3.9 cm, construct $\overline{PQ}$ such that the length of $\overline{PQ}$ is twice that of $\overline{AB}$. Verify by measurement.

(**Hint :** Construct $\overline{PX}$ such that length of $\overline{PX}$ = length of $\overline{AB}$; then cut off $\overline{XQ}$ such that $\overline{XQ}$ also has the length of $\overline{AB}$.)

5. Given $\overline{AB}$ of length 7.3 cm and $\overline{CD}$ of length 3.4 cm, construct a line segment $\overline{XY}$ such that the length of $\overline{XY}$ is equal to the difference between the lengths of $\overline{AB}$ and $\overline{CD}$. Verify by measurement.

14.3.2 Constructing a copy of a given line segment

Suppose you want to draw a line segment whose length is equal to that of a given line segment $\overline{AB}$.

A quick and natural approach is to use your ruler (which is marked with centimetres and millimetres) to measure the length of $\overline{AB}$ and then use the same length to draw another line segment $\overline{CD}$.

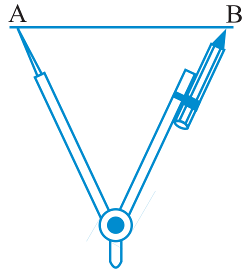
A second approach would be to use a transparent sheet and trace $\overline{AB}$ onto another portion of the paper. But these methods may not always give accurate results.

A better approach would be to use ruler and compasses for making this construction.

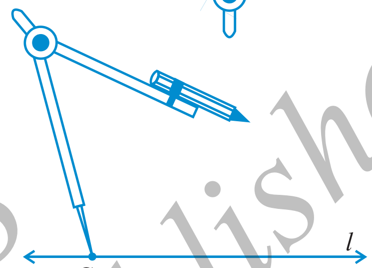
To make a copy of $\overline{AB}$.

Step 1 Given $\overline{AB}$ whose length is not known.

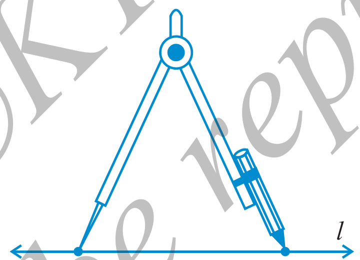
Step 2 Fix the compasses pointer on A and the pencil end on B. The opening of the instrument now gives the length of $\overline{AB}$.



Step 3 Draw any line *l*. Choose a point C on *l*. Without changing the compasses setting, place the pointer on C.



Step 4 Swing an arc that cuts *l* at a point, say, D. Now $\overline{CD}$ is a copy of $\overline{AB}$.



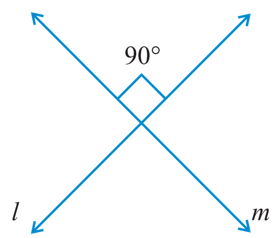
EXERCISE 14.3

1. Draw any line segment $\overline{PQ}$. Without measuring $\overline{PQ}$, construct a copy of $\overline{PQ}$.
2. Given some line segment $\overline{AB}$, whose length you do not know, construct $\overline{PQ}$ such that the length of $\overline{PQ}$ is twice that of $\overline{AB}$.

14.4 Perpendiculars

You know that two lines (or rays or segments) are said to be perpendicular if they intersect such that the angles formed between them are right angles.

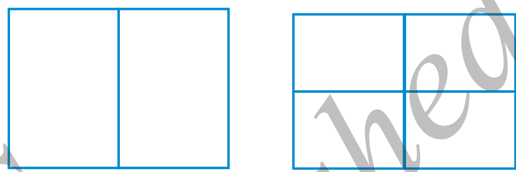
In the figure, the lines *l* and *m* are perpendicular.



The corners of a foolscap paper or your notebook indicate lines meeting at right angles.

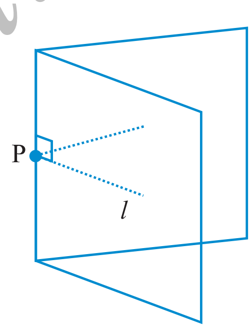
Do This

Where else do you see perpendicular lines around you?



Take a piece of paper. Fold it down the middle and make the crease. Fold the paper once again down the middle in the other direction. Make the crease and open out the page. The two creases are perpendicular to each other.

14.4.1 Perpendicular to a line through a point on it



Given a line *l* drawn on a paper sheet and a point P lying on the line. It is easy to have a perpendicular to *l* through P.

We can simply fold the paper such that the lines on both sides of the fold overlap each other.

Tracing paper or any transparent paper could be better for this activity. Let us take such a paper and draw any line *l* on it. Let us mark a point P anywhere on *l*.

Fold the sheet such that *l* is reflected on itself; adjust the fold so that the crease passes through the marked point P. Open out; the crease is perpendicular to *l*.

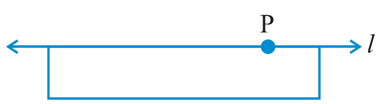
Think, discuss and write

How would you check if it is perpendicular? Note that it passes through P as required.

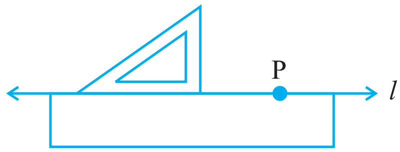
A challenge : Drawing perpendicular using ruler and a set-square (An optional activity).

Step 1 A line *l* and a point P are given. Note that P is on the line *l*.

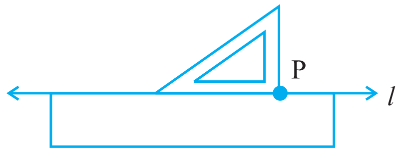
Step 2 Place a ruler with one of its edges along *l*. Hold this firmly.



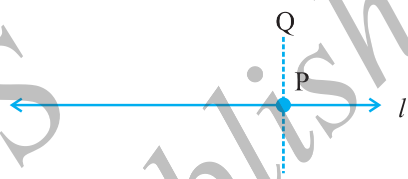
Step 3 Place a set-square with one of its edges along the already aligned edge of the ruler such that the right angled corner is in contact with the ruler.



Step 4 Slide the set-square along the edge of ruler until its right angled corner coincides with P.



Step 5 Hold the set-square firmly in this position. Draw $\overline{PQ}$ along the edge of the set-square.



$\overline{PQ}$ is perpendicular to l. (How do you use the $\perp$ symbol to say this?).

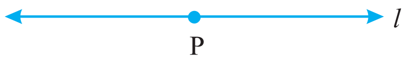
Verify this by measuring the angle at P.

Can we use another set-square in the place of the 'ruler'? Think about it.

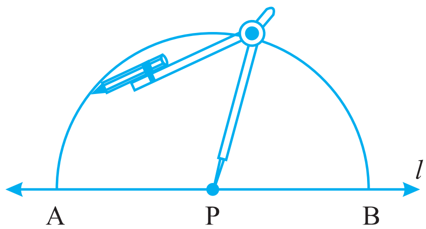
Method of ruler and compasses

As is the preferred practice in Geometry, the dropping of a perpendicular can be achieved through the "ruler-compasses" construction as follows :

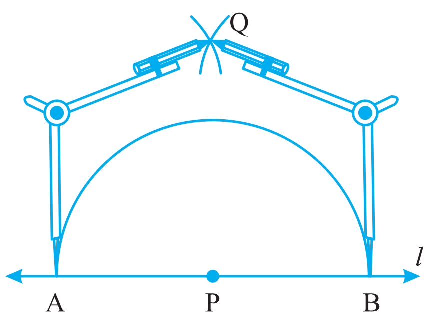
Step 1 Given a point P on a line l.



Step 2 With P as centre and a convenient radius, construct an arc intersecting the line l at two points A and B.

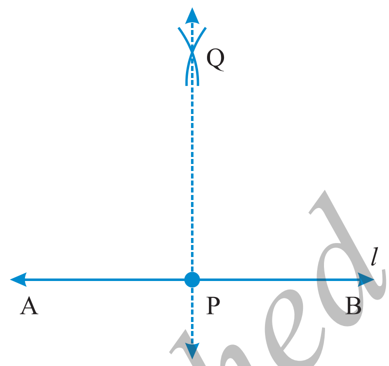


Step 3 With A and B as centres and a radius greater than AP construct two arcs, which cut each other at Q.



Step 4 Join PQ. Then $\overleftrightarrow{PQ}$ is perpendicular to l.

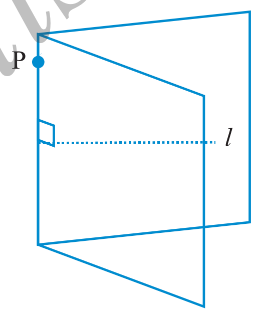
We write $\overleftrightarrow{PQ} \perp l$.



14.4.2 Perpendicular to a line through a point not on it

Do This

(Paper folding)



If we are given a line l and a point P not lying on it and we want to draw a perpendicular to l through P, we can again do it by a simple paper folding as before.

Take a sheet of paper (preferably transparent).

Draw any line l on it.

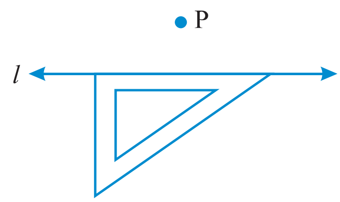
Mark a point P away from l.

Fold the sheet such that the crease passes through P. The parts of the line l on both sides of the fold should overlap each other.

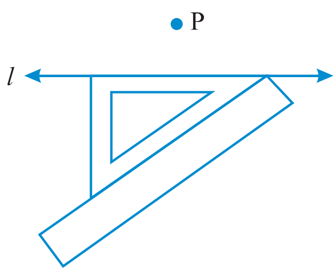
Open out. The crease is perpendicular to l and passes through P.

Method using ruler and a set-square (An optional activity)

Step 1 Let l be the given line and P be a point outside l.

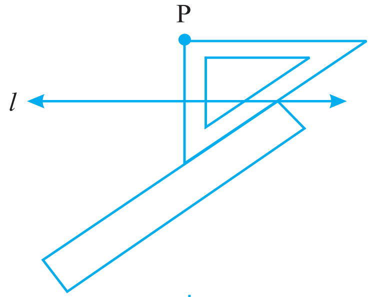


Step 2 Place a set-square on l such that one arm of its right angle aligns along l.



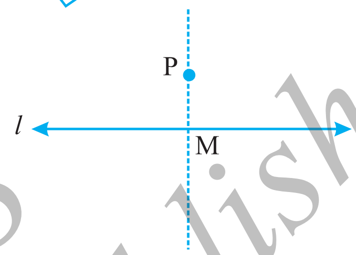
Step 3 Place a ruler along the edge opposite to the right angle of the set-square.

Step 4 Hold the ruler fixed. Slide the set-square along the ruler till the point P touches the other arm of the set-square.



Step 5 Join PM along the edge through P, meeting l at M.

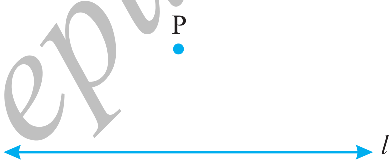
Now $\overleftrightarrow{PM} \perp l$.



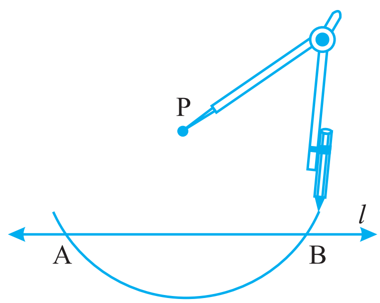
Method using ruler and compasses

A more convenient and accurate method, of course, is the ruler-compasses method.

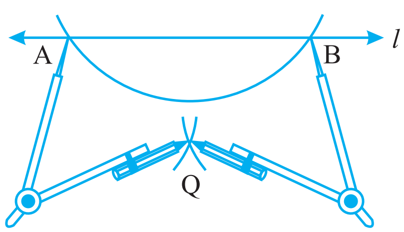
Step 1 Given a line l and a point P not on it.



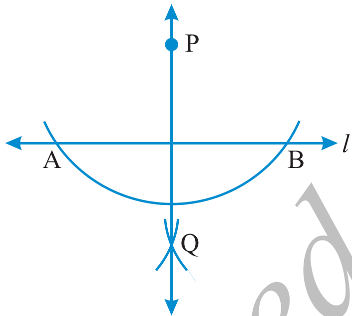
Step 2 With P as centre, draw an arc which intersects line l at two points A and B.



Step 3 Using the same radius and with A and B as centres, construct two arcs that intersect at a point, say Q, on the other side.



Step 4 Join PQ. Thus, $\overline{PQ}$ is perpendicular to *l*.



EXERCISE 14.4

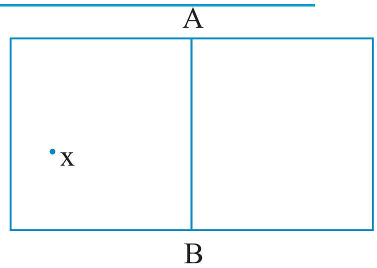
1. Draw any line segment $\overline{AB}$. Mark any point M on it. Through M, draw a perpendicular to $\overline{AB}$. (use ruler and compasses)
2. Draw any line segment $\overline{PQ}$. Take any point R not on it. Through R, draw a perpendicular to $\overline{PQ}$. (use ruler and set-square)
3. Draw a line *l* and a point X on it. Through X, draw a line segment $\overline{XY}$ perpendicular to *l*.

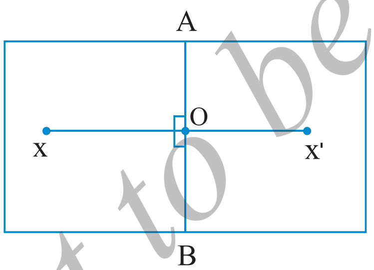
   Now draw a perpendicular to $\overline{XY}$ at Y. (use ruler and compasses)

14.4.3 The perpendicular bisector of a line segment

Do This

Fold a sheet of paper. Let $\overline{AB}$ be the fold. Place an ink-dot X, as shown, anywhere. Find the image X' of X, with $\overline{AB}$ as the mirror line.





Let $\overline{AB}$ and $\overline{XX'}$ intersect at O.

Is OX = OX'? Why?

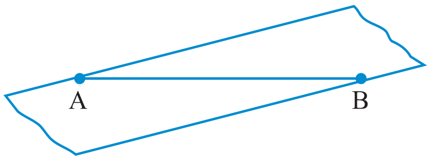
This means that $\overline{AB}$ divides $\overline{XX'}$ into two parts of equal length. $\overline{AB}$ bisects $\overline{XX'}$ or $\overline{AB}$ is a bisector of $\overline{XX'}$. Note also that AOX and BOX are right angles. (Why?).

Hence, $\overline{AB}$ is the perpendicular bisector of $\overline{XX'}$. We see only a part of $\overline{AB}$ in the figure. Is the perpendicular bisector of a line joining two points the same as the axis of symmetry?

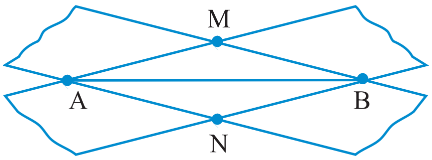
Do This

(Transparent tapes)

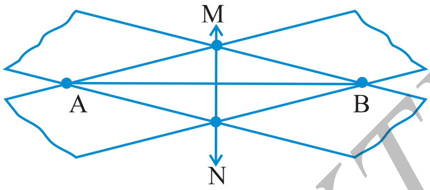
Step 1 Draw a line segment $\overline{AB}$.



Step 2 Place a strip of a transparent rectangular tape diagonally across $\overline{AB}$ with the edges of the tape on the end points A and B, as shown in the figure.



Step 3 Repeat the process by placing another tape over A and B just diagonally across the previous one. The two strips cross at M and N.

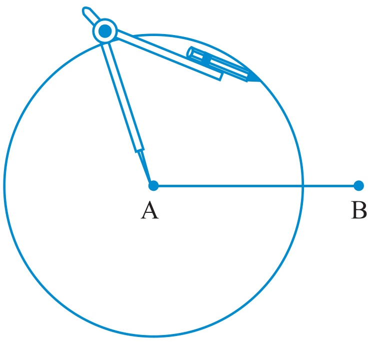


Step 4 Join M and N. Is $\overline{MN}$ a bisector of $\overline{AB}$? Measure and verify. Is it also the perpendicular bisector of $\overline{AB}$? Where is the mid point of $\overline{AB}$?

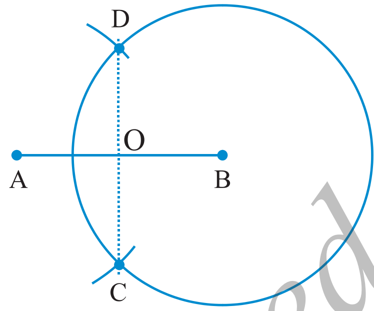
Construction using ruler and compasses

Step 1 Draw a line segment $\overline{AB}$ of any length.

Step 2 With A as centre, using compasses, draw a circle. The radius of your circle should be more than half the length of $\overline{AB}$.



Step 3 With the same radius and with B as centre, draw another circle using compasses. Let it cut the previous circle at C and D.



Step 4 Join $\overline{CD}$. It cuts $\overline{AB}$ at O. Use your divider to verify that O is the midpoint of $\overline{AB}$. Also verify that COA and COB are right angles. Therefore, $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.

In the above construction, we needed the two points C and D to determine $\overline{CD}$. Is it necessary to draw the whole circle to find them? Is it not enough if we draw merely small arcs to locate them? In fact, that is what we do in practice!

Try These

In Step 2 of the construction using ruler and compasses, what would happen if we take the length of radius to be smaller than half the length of $\overline{AB}$?

EXERCISE 14.5

1. Draw $\overline{AB}$ of length 7.3 cm and find its axis of symmetry.
2. Draw a line segment of length 9.5 cm and construct its perpendicular bisector.
3. Draw the perpendicular bisector of $\overline{XY}$ whose length is 10.3 cm.
   (a) Take any point P on the bisector drawn. Examine whether PX = PY.
   (b) If M is the mid point of $\overline{XY}$, what can you say about the lengths MX and XY?
4. Draw a line segment of length 12.8 cm. Using compasses, divide it into four equal parts. Verify by actual measurement.
5. With $\overline{PQ}$ of length 6.1 cm as diameter, draw a circle.
6. Draw a circle with centre C and radius 3.4 cm. Draw any chord $\overline{AB}$. Construct the perpendicular bisector of $\overline{AB}$ and examine if it passes through C.
7. Repeat Question 6, if $\overline{AB}$ happens to be a diameter.
8. Draw a circle of radius 4 cm. Draw any two of its chords. Construct the perpendicular bisectors of these chords. Where do they meet?
9. Draw any angle with vertex O. Take a point A on one of its arms and B on another such that OA = OB. Draw the perpendicular bisectors of $\overline{OA}$ and $\overline{OB}$. Let them meet at P. Is PA = PB ?

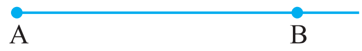
14.5 Angles

14.5.1 Constructing an angle of a given measure

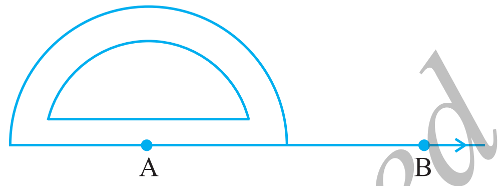
Suppose we want an angle of measure 40°.

Here are the steps to follow :

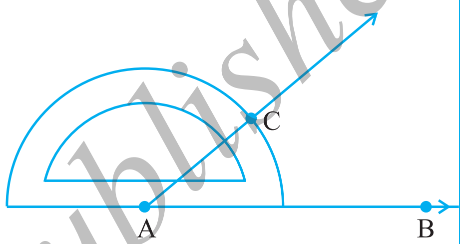
Step 1 Draw $\overline{AB}$ of any length.



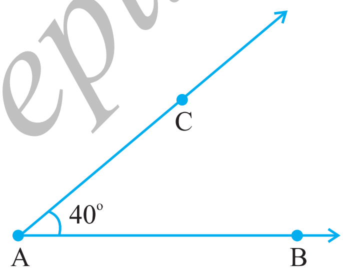
Step 2 Place the centre of the protractor at A and the zero edge along $\overline{AB}$.



Step 3 Start with zero near B. Mark point C at 40°.

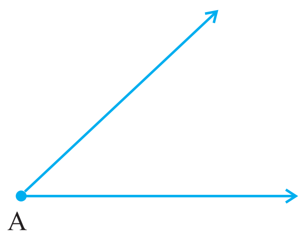


Step 4 Join AC. ∠BAC is the required angle.



14.5.2 Constructing a copy of an angle of unknown measure

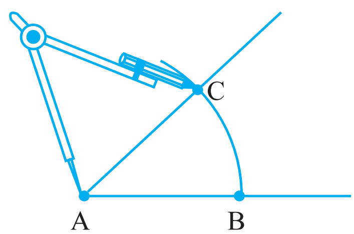
Suppose an angle (whose measure we do not know) is given and we want to make a copy of this angle. As usual, we will have to use only a straight edge and the compasses.



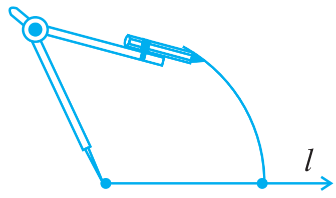
Given ∠A, whose measure is not known.

Step 1 Draw a line *l* and choose a point P on it.

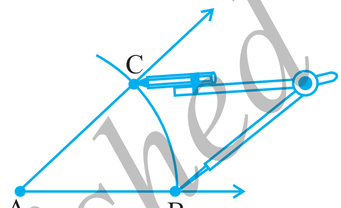
Step 2 Place the compasses at A and draw an arc to cut the rays of ∠A at B and C.



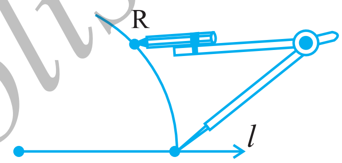
Step 3 Use the same compasses setting to draw an arc with P as centre, cutting *l* in Q.



Step 4 Set your compasses to the length BC with the same radius.

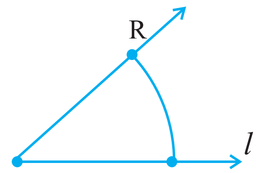


Step 5 Place the compasses pointer at Q and draw the arc to cut the arc drawn earlier in R.



Step 6 Join PR. This gives us ∠P. It has the same measure as ∠A.

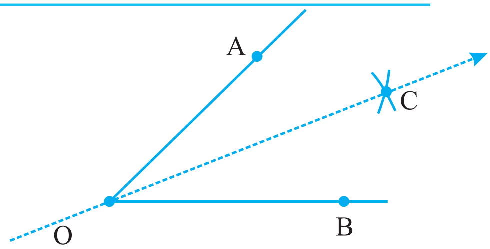
This means ∠QPR has same measure as ∠BAC.



14.5.3 Bisector of an angle

Do This

Take a sheet of paper. Mark a point O on it. With O as initial point, draw two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$. You get ∠AOB. Fold the sheet through O such that the rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ coincide. Let OC be the crease of paper which is obtained after unfolding the paper.

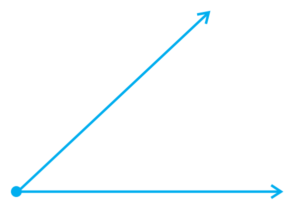


OC is clearly a line of symmetry for ∠AOB.

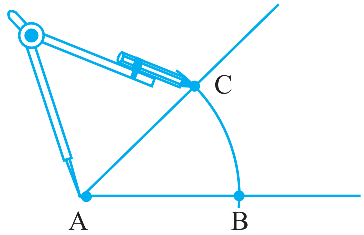
Measure ∠AOC and ∠COB. Are they equal? OC the line of symmetry, is therefore known as the angle bisector of ∠AOB.

Construction with ruler and compasses

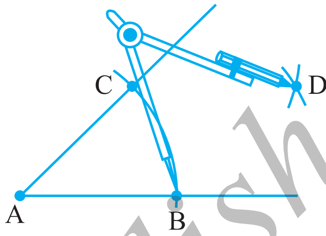
Let an angle, say, ∠A be given.



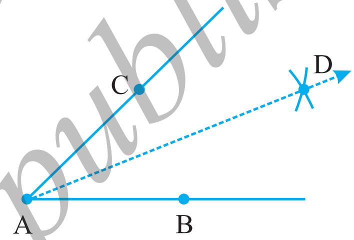
Step 1 With A as centre and using compasses, draw an arc that cuts both rays of ∠A. Label the points of intersection as B and C.



Step 2 With B as centre, draw (in the interior of ∠A) an arc whose radius is more than half the length BC.



Step 3 With the same radius and with C as centre, draw another arc in the interior of ∠A. Let the two arcs intersect at D. Then $\overline{AD}$ is the required bisector of ∠A.



Try These

In Step 2 above, what would happen if we take radius to be smaller than half the length BC?

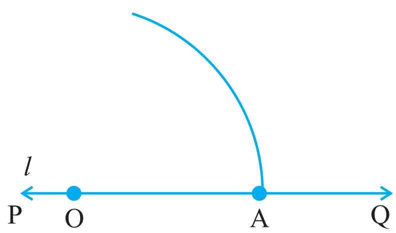
14.5.4 Angles of special measures

There are some elegant and accurate methods to construct some angles of special sizes which do not require the use of the protractor. We discuss a few here.

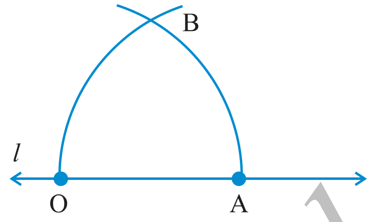
Constructing a 60° angle

Step 1 Draw a line *l* and mark a point O on it.

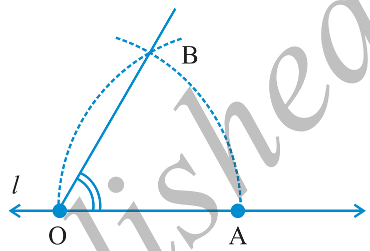
Step 2 Place the pointer of the compasses at O and draw an arc of convenient radius which cuts the line $\overleftrightarrow{PQ}$ at a point say, A.



Step 3 With the pointer at A (as centre), now draw an arc that passes through O.



Step 4 Let the two arcs intersect at B. Join OB. We get $\angle BOA$ whose measure is 60°.



Constructing a 30° angle

Construct an angle of 60° as shown earlier. Now, bisect this angle. Each angle is 30°, verify by using a protractor.

Try These

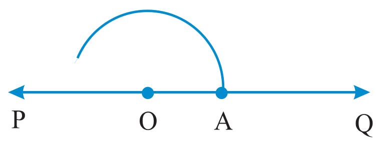
How will you construct a 15° angle?

Constructing a 120° angle

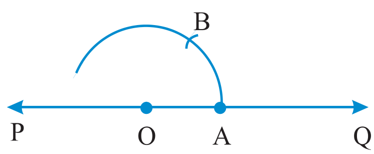
An angle of 120° is nothing but twice of an angle of 60°. Therefore, it can be constructed as follows :

Step 1 Draw any line PQ and take a point O on it.

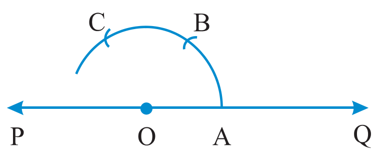
Step 2 Place the pointer of the compasses at O and draw an arc of convenient radius which cuts the line at A.



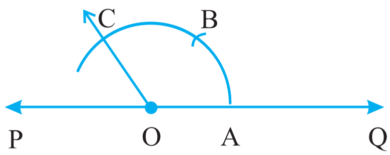
Step 3 Without disturbing the radius on the compasses, draw an arc with A as centre which cuts the first arc at B.



Step 4 Again without disturbing the radius on the compasses and with B as centre, draw an arc which cuts the first arc at C.



Step 5 Join OC, ∠COA is the required angle whose measure is 120°.



Try These

How will you construct a 150° angle?

Try These

How will you construct a 45° angle?

Constructing a 90° angle

Construct a perpendicular to a line from a point lying on it, as discussed earlier. This is the required 90° angle.

EXERCISE 14.6

1. Draw ∠POQ of measure 75° and find its line of symmetry.
2. Draw an angle of measure 147° and construct its bisector.
3. Draw a right angle and construct its bisector.
4. Draw an angle of measure 153° and divide it into four equal parts.
5. Construct with ruler and compasses, angles of following measures:
   (a) 60° (b) 30° (c) 90° (d) 120° (e) 45° (f) 135°
6. Draw an angle of measure 45° and bisect it.
7. Draw an angle of measure 135° and bisect it.
8. Draw an angle of 70º. Make a copy of it using only a straight edge and compasses.
9. Draw an angle of 40º. Copy its supplementary angle.

What have we discussed ?

This chapter deals with methods of drawing geometrical shapes.

1. We use the following mathematical instruments to construct shapes:
   (i) A graduated ruler (ii) The compasses
   (iii) The divider (iv) Set-squares (v) The protractor
2. Using the ruler and compasses, the following constructions can be made:
   (i) A circle, when the length of its radius is known.
   (ii) A line segment, if its length is given.
   (iii) A copy of a line segment.
   (iv) A perpendicular to a line through a point
   (a) on the line (b) not on the line.

(v) The perpendicular bisector of a line segment of given length.

(vi) An angle of a given measure.

(vii) A copy of an angle.

(viii) The bisector of a given angle.

(ix) Some angles of special measures such as

(a) 90° (b) 45° (c) 60° (d) 30° (e) 120° (f) 135°

EXERCISE 7.1

1. (i) $\frac{2}{4}$ (ii) $\frac{8}{9}$ (iii) $\frac{4}{8}$ (iv) $\frac{1}{4}$ (v) $\frac{3}{7}$ (vi) $\frac{3}{12}$
(vii) $\frac{10}{10}$ (viii) $\frac{4}{9}$ (ix) $\frac{4}{8}$ (x) $\frac{1}{2}$

3. Shaded portions do not represent the given fractions.

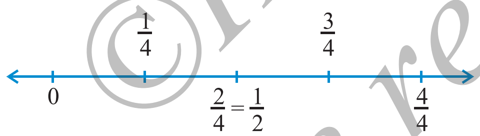
4. $\frac{8}{24}$ 5. $\frac{40}{60}$

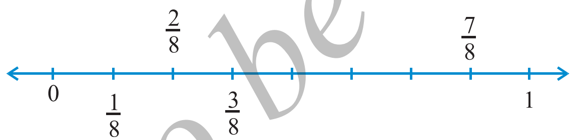
6. (a) Arya will divide each sandwich into three equal parts, and give one part of each sandwich to each one of them.
(b) $\frac{1}{3}$ 7. $\frac{2}{3}$ 8. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; $\frac{5}{11}$

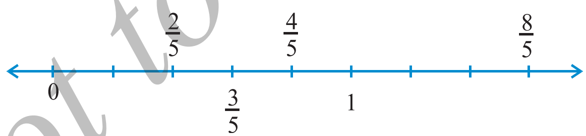
9. 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113; $\frac{4}{12}$

10. $\frac{4}{8}$ 11. $\frac{3}{8}, \frac{5}{8}$

EXERCISE 7.2

1. (a)


(b)


(c)


2. (a) $6\frac{2}{3}$ (b) $2\frac{1}{5}$ (c) $2\frac{3}{7}$ (d) $5\frac{3}{5}$ (e) $3\frac{1}{6}$ (f) $3\frac{8}{9}$

3. (a) $\frac{31}{4}$ (b) $\frac{41}{7}$ (c) $\frac{17}{6}$ (d) $\frac{53}{5}$ (e) $\frac{66}{7}$ (f) $\frac{76}{9}$

EXERCISE 7.3

1. (a) $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$; Yes (b) $\frac{4}{12}, \frac{3}{9}, \frac{2}{6}, \frac{1}{3}, \frac{6}{15}$; No

2. (a) $\frac{1}{2}$ (b) $\frac{4}{6}$ (c) $\frac{3}{9}$ (d) $\frac{2}{8}$ (e) $\frac{3}{4}$ (i) $\frac{6}{18}$

(ii) $\frac{4}{8}$ (iii) $\frac{12}{16}$ (iv) $\frac{8}{12}$ (v) $\frac{4}{16}$

(a), (ii); (b), (iv); (c), (i); (d), (v); (e), (iii)

3. (a) 28 (b) 16 (c) 12 (d) 20 (e) 3

4. (a) $\frac{12}{20}$ (b) $\frac{9}{15}$ (c) $\frac{18}{30}$ (d) $\frac{27}{45}$

5. (a) $\frac{9}{12}$ (b) $\frac{3}{4}$

6. (a) equivalent (b) not equivalent (c) not equivalent

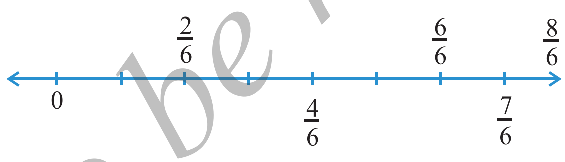
7. (a) $\frac{4}{5}$ (b) $\frac{5}{2}$ (c) $\frac{6}{7}$ (d) $\frac{3}{13}$ (e) $\frac{1}{4}$

8. Ramesh $\rightarrow \frac{10}{20}=\frac{1}{2}$, Sheelu $\rightarrow \frac{25}{50}=\frac{1}{2}$, Jamaal $\rightarrow \frac{40}{80}=\frac{1}{2}$. Yes

9. (i) $\rightarrow$ (d) (ii) $\rightarrow$ (e) (iii) $\rightarrow$ (a) (iv) $\rightarrow$ (c) (v) $\rightarrow$ (b)

EXERCISE 7.4

1. (a) $\frac{1}{8}<\frac{3}{8}<\frac{4}{8}<\frac{6}{8}$ (b) $\frac{3}{9}<\frac{4}{9}<\frac{6}{9}<\frac{8}{9}$

(c) 

$\frac{5}{6}>\frac{2}{6}$, $\frac{3}{6}>\frac{0}{6}$, $\frac{1}{6}<\frac{6}{6}$, $\frac{8}{6}>\frac{5}{6}$

2. (a) $\frac{3}{6}<\frac{5}{6}$ (b) $\frac{1}{7}<\frac{1}{4}$ (c) $\frac{4}{5}<\frac{5}{5}$ (d) $\frac{3}{5}>\frac{3}{7}$

4. (a) $\frac{1}{6}<\frac{1}{3}$ (b) $\frac{3}{4}>\frac{2}{6}$ (c) $\frac{2}{3}>\frac{2}{4}$ (d) $\frac{6}{6}=\frac{3}{3}$ (e) $\frac{5}{6}<\frac{5}{5}$

5. (a) $\frac{1}{2}>\frac{1}{5}$ (b) $\frac{2}{4}=\frac{3}{6}$ (c) $\frac{3}{5}<\frac{2}{3}$ (d) $\frac{3}{4}>\frac{2}{8}$

(e) $\frac{3}{5}<\frac{6}{5}$ (f) $\frac{7}{9}>\frac{3}{9}$ (g) $\frac{1}{4}=\frac{2}{8}$ (h) $\frac{6}{10}<\frac{4}{5}$

(i) $\frac{3}{4}<\frac{7}{8}$ (j) $\frac{6}{10}=\frac{3}{5}$ (k) $\frac{5}{7}=\frac{15}{21}$

6. (a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{4}{25}$ (d) $\frac{4}{25}$ (e) $\frac{1}{6}$ (f) $\frac{1}{5}$

(g) $\frac{1}{5}$ (h) $\frac{1}{6}$ (i) $\frac{4}{25}$ (j) $\frac{1}{6}$ (k) $\frac{1}{6}$ (l) $\frac{4}{25}$

(a), (e), (h), (j), (k) ; (b), (f), (g) ; (c), (d), (i), (l)

7. (a) No ; $\frac{5}{9}=\frac{25}{45}, \frac{4}{5}=\frac{36}{45}$ and $\frac{25}{45}\neq\frac{36}{45}$

(b) No ; $\frac{9}{16}=\frac{81}{144}, \frac{5}{9}=\frac{80}{144}$ and $\frac{81}{144}\neq\frac{80}{144}$ (c) Yes ; $\frac{4}{5}=\frac{16}{20}$

(d) No ; $\frac{1}{15}=\frac{2}{30}$ and $\frac{2}{30}\neq\frac{4}{30}$

8. Ila has read less 9. Rohit

10. Same fraction ($\frac{4}{5}$) of students got first class in both the classes.

EXERCISE 7.5

1. (a) + (b) – (c) +

2. (a) $\frac{1}{9}$ (b) $\frac{11}{15}$ (c) $\frac{2}{7}$ (d) 1 (e) $\frac{1}{3}$

(f) 1 (g) $\frac{1}{3}$ (h) $\frac{1}{4}$ (i) $\frac{3}{5}$

3. The complete wall.

4. (a) $\frac{4}{10}$ $(=\frac{2}{5})$ (b) $\frac{8}{21}$ (c) $\frac{6}{6}$ $(=1)$ (d) $\frac{7}{27}$ 5. $\frac{2}{7}$

EXERCISE 7.6

1. (a) $\frac{17}{21}$ (b) $\frac{23}{30}$ (c) $\frac{46}{63}$ (d) $\frac{22}{21}$ (e) $\frac{17}{30}$

(f) $\frac{22}{15}$ (g) $\frac{5}{12}$ (h) $\frac{3}{6}$ $(=\frac{1}{2})$ (i) $\frac{23}{12}$ (j) $\frac{6}{6}$ $(=1)$ (k) 5

(l) $\frac{95}{12}$ (m) $\frac{9}{5}$ (n) $\frac{5}{6}$

2. $\frac{23}{20}$ metre 3. $2\frac{5}{6}$

4. (a) $\frac{7}{8}$ (b) $\frac{7}{10}$ (c) $\frac{1}{3}$

5. (a)

$\frac{2}{3}$	$\frac{4}{3}$	2
$\frac{1}{3}$	$\frac{2}{3}$	1
$\frac{1}{3}$	$\frac{2}{3}$	1

(b)

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{7}{12}$
$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{4}$

6. Length of the other piece = $\frac{5}{8}$ metre

7. The distance walked by Nandini = $\frac{4}{10}$ ($\frac{2}{5}$) km

8. Asha's bookshelf is more full; by $\frac{13}{30}$

9. Rahul takes less time; by $\frac{9}{20}$ minutes

EXERCISE 8.1

1.

	Hundreds (100)	Tens (10)	Ones (1)	Tenths $(\frac{1}{10})$
(a)	0	3	1	2
(b)	1	1	0	4

2.

	Hundreds (100)	Tens (10)	Ones (1)	Tenths $(\frac{1}{10})$
(a)	0	1	9	4
(b)	0	0	0	3
(c)	0	1	0	6
(d)	2	0	5	9

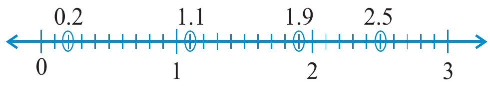
3. (a) 0.7 (b) 20.9 (c) 14.6 (d) 102.0 (e) 600.8

4. (a) 0.5 (b) 3.7 (c) 265.1 (d) 70.8 (e) 8.8
(f) 4.2 (g) 1.5 (h) 0.4 (i) 2.4 (j) 3.6
(k) 4.5

5. (a) $\frac{6}{10}, \frac{3}{5}$ (b) $\frac{25}{10}, \frac{5}{2}$ (c) 1, 1 (d) $\frac{38}{10}, \frac{19}{5}$ (e) $\frac{137}{10}, \frac{137}{10}$
(f) $\frac{212}{10}, \frac{106}{5}$ (g) $\frac{64}{10}, \frac{32}{5}$

6. (a) 0.2cm (b) 3.0 cm (c) 11.6 cm (d) 4.2 cm
(e) 16.2 cm (f) 8.3 cm

7. (a) 0 and 1; 1 (b) 5 and 6; 5 (c) 2 and 3; 3 (d) 6 and 7; 6
(e) 9 and 10; 9 (f) 4 and 5; 5

8. 

9. A, 0.8 cm; B, 1.3 cm; C, 2.2 cm; D, 2.9 cm

10. (a) 9.5 cm (b) 6.5 cm

EXERCISE 8.2

1.

	Ones	Tenths	Hundredths	Number
(a)	0	2	6	0.26
(b)	1	3	8	1.38
(c)	1	2	8	1.28

2. (a) 3.25 (b) 102.63 (c) 30.025 (d) 211.902 (e) 12.241

3.

	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
(a)	0	0	0	2	9	0
(b)	0	0	2	0	8	0
(c)	0	1	9	6	0	0
(d)	1	4	8	3	2	0
(e)	2	0	0	8	1	2

4. (a) 29.41 (b) 137.05 (c) 0.764 (d) 23.206 (e) 725.09

5. (a) Zero point zero three (b) One point two zero
(c) One hundred eight point five six (d) Ten point zero seven
(e) Zero point zero three two (f) Five point zero zero eight

6. (a) 0 and 0.1 (b) 0.4 and 0.5 (c) 0.1 and 0.2
(d) 0.6 and 0.7 (e) 0.9 and 1.0 (f) 0.5 and 0.6

7. (a) $\frac{3}{5}$ (b) $\frac{1}{20}$ (c) $\frac{3}{4}$ (d) $\frac{9}{50}$ (e) $\frac{1}{4}$
(f) $\frac{1}{8}$ (g) $\frac{33}{500}$

EXERCISE 8.3

1. (a) 0.4 (b) 0.07 (c) 3 (d) 0.5 (e) 1.23
(f) 0.19 (g) both are same (h) 1.490 (i) both are same (j) 5.64

EXERCISE 8.4

1. (a) ₹ 0.05 (b) ₹ 0.75 (c) ₹ 0.20 (d) ₹ 50.90 (e) ₹ 7.25
2. (a) 0.15 m (b) 0.06 m (c) 2.45 m (d) 9.07 m (e) 4.19 m
3. (a) 0.5 cm (b) 6.0 cm (c) 16.4 cm (d) 9.8 cm (e) 9.3 cm

4. (a) 0.008 km (b) 0.088 km (c) 8.888 km (d) 70.005 km

5. (a) 0.002 kg (b) 0.1 kg (c) 3.750 kg (d) 5.008 kg (e) 26.05 kg

EXERCISE 8.5

1. (a) 38.587 (b) 29.432 (c) 27.63 (d) 38.355 (e) 13.175 (f) 343.89

2. ₹ 68.35 **3.** ₹ 26.30 **4.** 5.25 m

5. 3.042 km **6.** 22.775 km **7.** 18.270 kg

EXERCISE 8.6

1. (a) ₹ 2.50 (b) 47.46 m (c) ₹ 3.04 (d) 3.155 km (e) 1.793 kg

2. (a) 3.476 (b) 5.78 (c) 11.71 (d) 1.753

3. ₹ 14.35 **4.** ₹ 6.75 **5.** 15.55 m

6. 9.850 km **7.** 4.425 kg

EXERCISE 9.1

1.

Marks	Tally marks	Number of students
1	||	2
2	|||	3
3	|||	3
4	|||| ||	7
5	|||| |	6
6	|||| ||	7
7	||||	5
8	||||	4
9	|||	3

(a) 12 (b) 8

2.

Sweets	Tally marks	Number of students
Ladoo	|||| |||| |	11
Barfi	|||	3
Jalebi	|||| ||	7
Rasgulla	|||| ||||	9
		30

(b) Ladoo

3.

Numbers	Tally marks	How many times?
1	||||/ ||	7
2	||||/ |	6
3	||||/	5
4	||||/	4
5	|||| |||| |	11
6	|||| ||	7

(a) 4 (b) 5 (c) 1 and 6

4. (i) Village D (ii) Village C (iii) 3 (iv) 28

5. (a) VIII (b) No (c) 12

6. (a) Number of bulbs sold on Friday are 14. Similarly, number of bulbs sold on other days can be found.

(b) Maximum number of bulbs were sold on Sunday.

(c) Same number of bulbs were sold on Wednesday and Saturday.

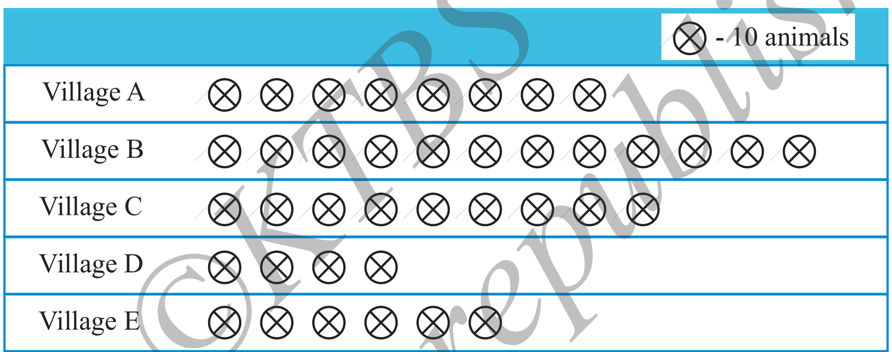
(d) Minimum number of bulbs were sold on Wednesday and Saturday.

(e) 10 Cartons

7. (a) Martin (b) 700 (c) Anwar, Martin, Ranjit Singh

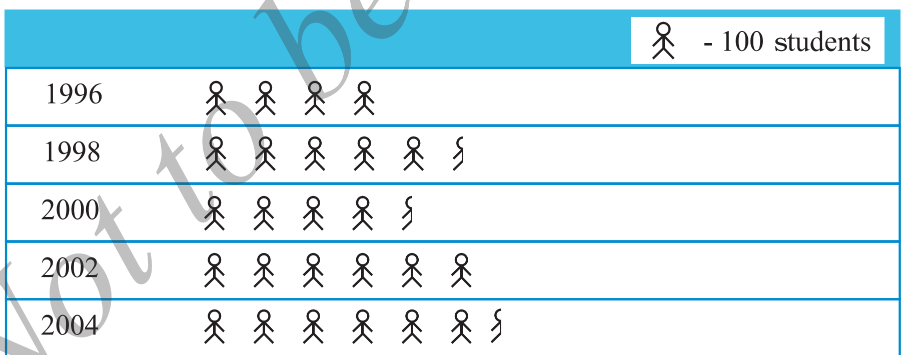
EXERCISE 9.2

1.



(a) 6 (b) Village B (c) Village C

2.



A (a) 6 (b) 5 complete and 1 incomplete B Second

EXERCISE 9.3

1. (a) 2002 (b) 1998

2. (a) This bar graph shows the number of shirts sold from Monday to Saturday

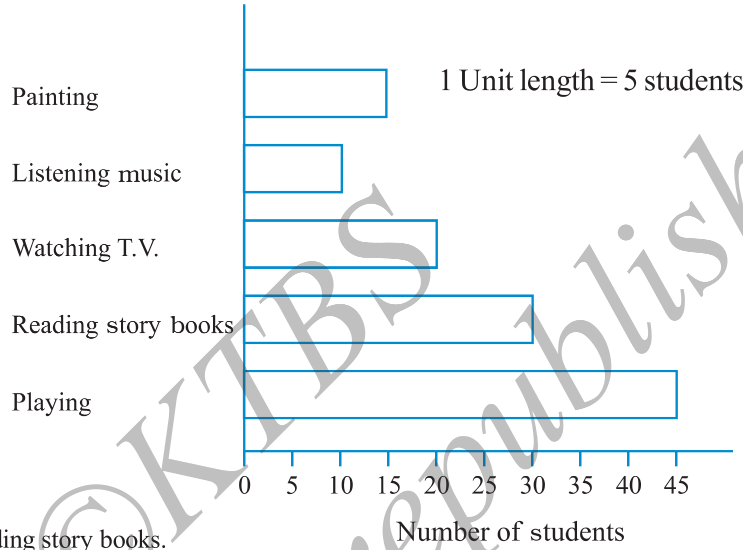
(b) 1 unit = 5 shirts (c) Saturday, 60

(d) Tuesday (e) 35

3. (a) This bar graph shows the marks obtained by Aziz in different subjects.
   (b) Hindi (c) Social Studies
   (d) Hindi – 80, English – 60, Mathematics – 70, Science – 50 and Social Studies – 40.

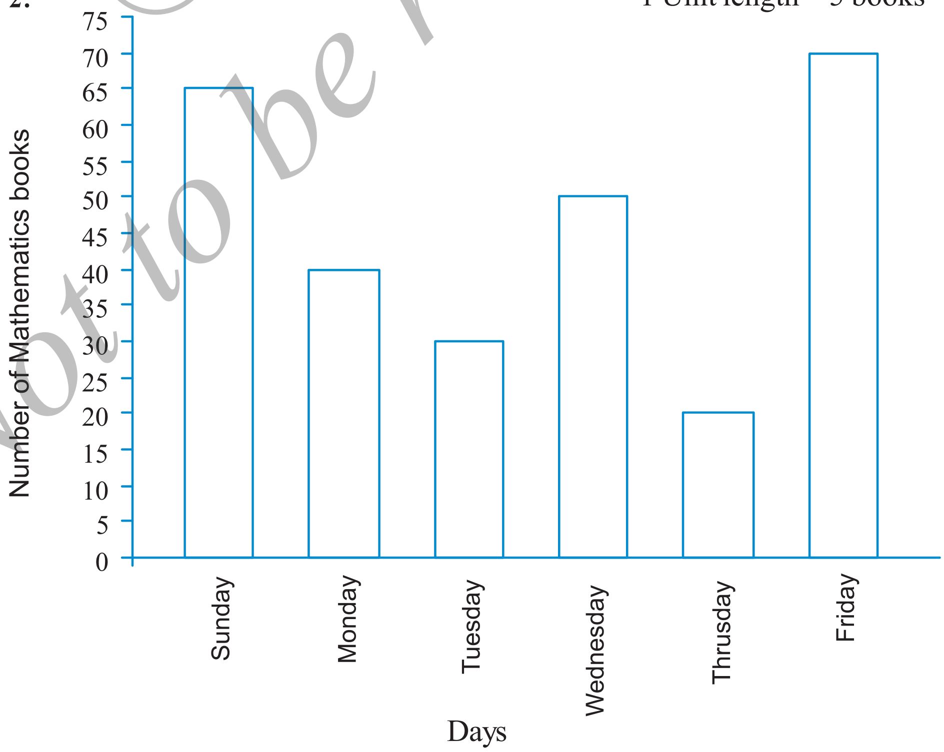
EXERCISE 9.4

1.

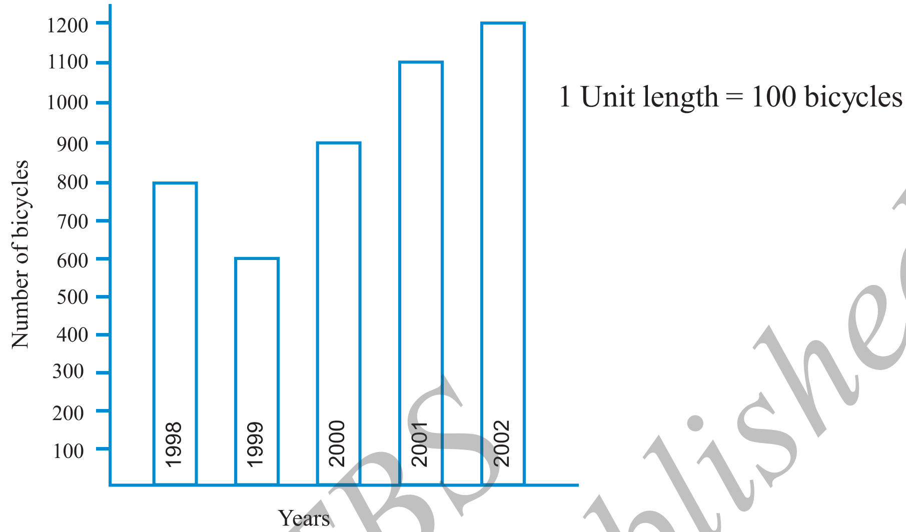


Reading story books.

2.

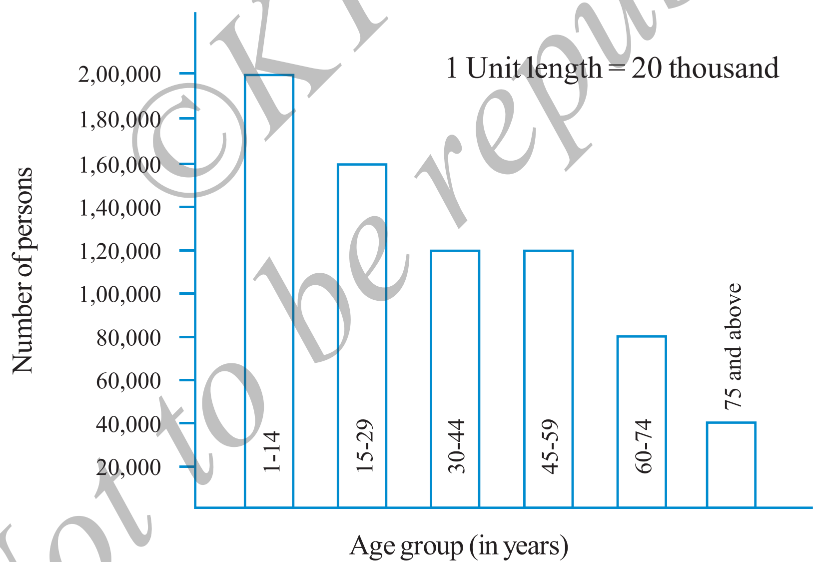


3.



(a) 2002 (b) 1999

4.



(a) 30 – 44, 45 – 59 (b) 1 lakh 20 thousand

EXERCISE 10.1

1. (a) 12 cm (b) 133 cm (c) 60 cm (d) 20 cm (e) 15 cm (f) 52 cm
2. 100 cm or 1 m
3. 7.5 m
4. 106 cm
5. 9.6 km
6. (a) 12 cm (b) 27 cm (c) 22 cm
7. 39 cm
8. 48 m
9. 5 m
10. 20 cm
11. (a) 7.5 cm (b) 10 cm (c) 5 cm
12. 10 cm
13. ₹ 20,000
14. ₹ 7200
15. Bulbul

16. (a) 100 cm (b) 100 cm (c) 100 cm (d) 100 cm
All the figures have same perimeter.

17. (a) 6 m (b) 10 m (c) Cross has greater perimeter

EXERCISE 10.2

1. (a) 9 sq units (b) 5 sq units (c) 4 sq units (d) 8 sq units (e) 10 sq units
(f) 4 sq units (g) 6 sq units (h) 5 sq units (i) 9 sq units (j) 4 sq units
(k) 5 sq units (l) 8 sq units (m) 14 sq units (n) 18 sq units

EXERCISE 10.3

1. (a) 12 sq cm (b) 252 sq cm (c) 6 sq km (d) 1.40 sq m
2. (a) 100 sq cm (b) 196 sq cm (c) 25 sq m
3. (c) largest area (b) smallest area
4. 6 m **5.** ₹ 8000 **6.** 3 sq m **7.** 14 sq m
8. 11 sq m **9.** 15 sq m
10. (a) 28 sq cm (b) 9 sq cm
11. (a) 40 sq cm (b) 245 sq cm (c) 9 sq cm
12. (a) 240 tiles (b) 42 tiles

EXERCISE 11.1

1. (a) $2n$ (b) $3n$ (c) $3n$ (d) $2n$ (e) $5n$
(f) $5n$ (g) $6n$
2. (a) and (d); The number of matchsticks required in each of them is 2
3. $5n$ **4.** $50b$ **5.** $5s$
6. t km **7.** $8r$, 64, 80 **8.** $(x - 4)$ years **9.** $l + 5$
10. $2x + 10$
11. (a) $3x + 1$, x = number of squares
(b) $2x + 1$, x = number of triangles

EXERCISE 11.2

1. $3l$ **2.** $6l$ **3.** $12l$ **4.** $d = 2r$
5. (a + b) + c = a + (b + c)

EXERCISE 11.3

2. (c), (d)
3. (a) Addition, subtraction, addition, subtraction
(b) Multiplication, division, multiplication
(c) Multiplication and addition, multiplication and subtraction
(d) Multiplication, multiplication and addition, multiplication and subtraction

4. (a) $p + 7$ (b) $p - 7$ (c) $7p$ (d) $\frac{p}{7}$
(e) $-m - 7$ (f) $-5p$ (g) $\frac{-p}{5}$ (h) $-5p$

5. (a) $2m + 11$ (b) $2m - 11$ (c) $5y + 3$ (d) $5y - 3$
(e) $-8y$ (f) $-8y + 5$ (g) $16 - 5y$ (h) $-5y + 16$

6. (a) $t + 4,\ t - 4,\ 4t,\ \frac{t}{4},\ \frac{4}{t},\ 4 - t,\ 4 + t$ (b) $2y + 7,\ 2y - 7,\ 7y + 2$, ..., ...,

EXERCISE 11.4

1. (a) (i) $y + 5$ (ii) $y - 3$ (iii) $6y$ (iv) $6y - 2$ (v) $3y + 5$
(b) $(3b - 4)$ metres (c) length = $5h$ cm, breadth = $5h - 10$ cm
(d) $s + 8,\ s - 7,\ 4s - 10$ (e) $(5v + 20)$ km

2. (a) A book costs three times the cost of a notebook.
(b) Tony's box contains 8 times the marbles on the table.
(c) Total number of students in the school is 20 times that of our class.
(d) Jaggu's uncle is 4 times older than Jaggu and Jaggu's aunt is 3 years younger than his uncle.
(e) The total number of dots is 5 times the number of rows.

EXERCISE 11.5

1. (a) an equation with variable x (e) an equation with variable x
(f) an equation with variable x (h) an equation with variable n
(j) an equation with variable p (k) an equation with variable y
(o) an equation with variable x

2. (a) No (b) Yes (c) No (d) No
(e) No (f) Yes (g) No (h) No
(i) Yes (j) Yes (k) No (l) No
(m) No (n) No (o) No (p) No (q) Yes

3. (a) 12 (b) 8 (c) 10 (d) 14
(e) 4 (f) – 2

4. (a) 6 (b) 7 (c) 12 (d) 10

5. (i) 22 (ii) 16 (iii) 17 (iv) 11

EXERCISE 12.1

1. (a) 4 : 3 (b) 4 : 7
2. (a) 1 : 2 (b) 2 : 5
3. (a) 3 : 2 (b) 2 : 7 (c) 2 : 7
4. 3 : 4 **5.** 5, 12, 25, Yes
6. (a) 3 : 4 (b) 14 : 9 (c) 3 : 11 (d) 2 : 3
7. (a) 1 : 3 (b) 4 : 15 (c) 11 : 20 (d) 1 : 4
8. (a) 3 : 1 (b) 1 : 2
9. 17 : 550
10. (a) 115 : 216 (b) 101 : 115 (c) 101 : 216
11. (a) 3 : 1 (b) 16 : 15 (c) 5 : 12

12. 15 : 7 **13.** 20 ; 100 **14.** 12 and 8 **15.** ₹ 20 and ₹ 16

16. (a) 3 : 1 (b) 10 : 3 (c) 13 : 6 (d) 15 : 1

EXERCISE 12.2

1. (a) Yes (b) No (c) No (d) No
(e) Yes (f) Yes

2. (a) T (b) T (c) F (d) T
(e) F (f) T

3. (a) T (b) T (c) T (d) T (e) F

4. (a) Yes, Middle Terms – 1 m, ₹ 40; Extreme Terms – 25 cm, ₹ 160
(b) Yes, Middle Terms – 65 litres, 6 bottles; Extreme Terms – 39 litres, 10 bottles
(c) No.
(d) Yes, Middle Terms – 2.5 litres, ₹ 4 ; Extreme Terms – 200 ml, ₹ 50

EXERCISE 12.3

1. ₹ 1,050 **2.** ₹ 9,000 **3.** 644 mm

4. (a) ₹ 146.40 (b) 10 kg

5. 5 degrees **6.** ₹ 60, 000 **7.** 24 bananas **8.** 5 kg

9. 300 litres **10.** Manish **11.** Anup

EXERCISE 13.1

1. Four examples are the blackboard, the table top, a pair of scissors, the computer disc etc.

2. The line l_2

3. Except (c) all others are symmetric.

EXERCISE 13.2

1. (a) 4 (b) 4 (c) 4 (d) 1
(e) 6 (f) 6 (g) 0 (h) 0 (i) 5

3. Number of lines of symmetry are :
Equilateral triangle – 3; Square – 4; Rectangle – 2; Isosceles triangle – 1;
Rhombus – 2; Circle – countless.

4. (a) Yes; an isosceles triangle. (b) No.
(c) Yes; an equilateral triangle. (d) Yes; a scalene triangle.

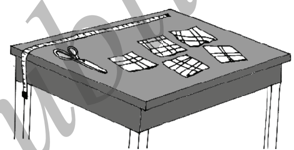
7. (a) A,H,I,M,O,T,U,V,W,X,Y (b) B, C, D, E, H, I, K, O, X
(c) F, G, J, L, N, P, Q, R, S, Z

EXERCISE 13.3

1. Number of lines of symmetry to be marked :
(a) 4 (b) 1 (c) 2 (d) 2
(e) 1 (f) 2

BRAIN-TEASERS

1. From a basket of mangoes when counted in twos there was one extra, counted in threes there were two extra, counted in fours there were three extra, counted in fives there were four extra, counted in sixes there were five extra. But counted in sevens there were no extra. Atleast how many mangoes were there in the basket?
2. A boy was asked to find the LCM of 3, 5, 12 and another number. But while calculating, he wrote 21 instead of 12 and yet came with the correct answer. What could be the fourth number?
3. There were five pieces of cloth of lengths 15 m, 21 m, 36 m, 42 m, 48 m. But all of them could be measured in whole units of a measuring rod. What could be the largest length of the rod?



4. There are three cans. One of them holds exactly 10 litres of milk and is full. The other two cans can hold 7 litres and 3 litres respectively. There is no graduation mark on the cans. A customer asks for 5 litres of milk. How would you give him the amount he ask? He would not be satisfied by eye estimates.
5. Which two digit numbers when added to 27 get reversed?
6. Cement mortar was being prepared by mixing cement to sand in the ratio of 1:6 by volume. In a cement mortar of 42 units of volume, how much more cement needs to be added to enrich the mortar to the ratio 2:9?
7. In a solution of common salt in water, the ratio of salt to water was 30:70 as per weight. If we evaporate 100 grams of water from one kilogram of this solution, what will be the ratio of the salt to water by weight?
8. Half a swarm of bees went to collect honey from a mustard field. Three fourth of the rest went to a rose garden. The rest ten were still undecided. How many bees were there in all?

9. Fifteen children are sitting in a circle. They are asked to pass a handkerchief to the child next to the child immediately after them.

   The game stops once the handkerchief returns to the child it started from. This can be written as follows : $1\rightarrow3\rightarrow5\rightarrow7\rightarrow9\rightarrow11\rightarrow13\rightarrow15\rightarrow2\rightarrow4\rightarrow6\rightarrow8\rightarrow10\rightarrow12\rightarrow14\rightarrow1$. Here, we see that every child gets the handkerchief.

(i) What would happen if the handkerchief were passed to the left leaving two children in between? Would every child get the handkerchief?

(ii) What if we leave three children in between? What do you see?

In which cases every child gets the handkerchief and in which cases not? Try the same game with 16, 17, 18, 19, 20 children. What do you see?

10. Take two numbers 9 and 16. Divide 9 by 16 to get the remainder. What is the remainder when 2×9 is divided by 16, 3×9 divided by 16, 4×9 divided by 16, 5×9 divided by 16... 15×9 divided by 16. List the remainders. Take the numbers 12 and 14. List the remainders of 12, 12×2, 12×3, 12×4, 12×5, 12×6, 12×7, 12×8, 12×9, 12×10, 12×11, 12×12, 12×13 when divided by 14. Do you see any difference between above two cases?

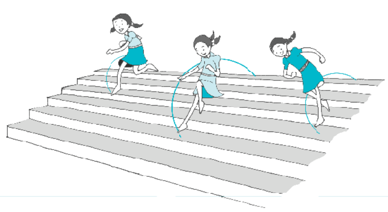
11. You have been given two cans with capacities 9 and 5 litres respectively. There is no graduation marks on the cans nor is eye estimation possible. How can you collect 3 litres of water from a tap? (You are allowed to pour out water from the can). If the cans had capacities 8 and 6 litres respectively, could you collect 5 litres?

12. The area of the east wall of an auditorium is 108 sq m, the area of the north wall is 135 sq m and the area of the floor is 180 sq m. Find the height of the auditorium.

13. If we subtract 4 from the digit at the units place of a two digit number and add 4 to the digit at the tens place then the resulting number is doubled. Find the number.

14. Two boatmen start simultaneously from the opposite shores of a river and they cross each other after 45 minutes of their starting from the respective shores. They rowed till they reached the opposite shore and returned immediately after reaching the shores. When will they cross each other again?

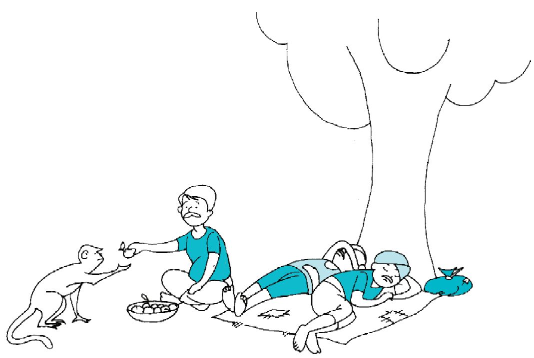
15. Three girls are climbing down a staircase. One girl climbs down two steps at one go. The second girl three steps at one go and the third climbs down four steps. They started together from the beginning of the staircase



leaving their foot marks. They all came down in complete steps and had their foot marks together at the bottom of the staircase. In how many steps would there be only one pair of foot mark?

Are there any steps on which there would be no foot marks.

16. A group of soldiers was asked to fall in line making rows of three. It was found that there was one soldier extra. Then they were asked to stand in rows of five. It was found there were left 2 soldiers. They were asked to stand in rows of seven. Then there were three soldiers who could not be adjusted. At least how many soldiers were there in the group?
17. Get 100 using four 9's and some of the symbols like +, –, ×, ¸ etc.
18. How many digits would be in the product 2 × 2 × 2 ..... × 2 (30 times)?
19. A man would be 5 minutes late to reach his destination if he rides his bike at 30 km. per hour. But he would be 10 minutes early if he rides at the speed of 40 km per hour. What is the distance of his destination from where he starts?
20. The ratio of speeds of two vehicles is 2:3. If the first vehicle covers 50 km in 3 hours, what distance would the second vehicle covers in 2 hours?
21. The ratio of income to expenditure of Mr. Natarajan is 7:5. If he saves ₹ 2000 a month, what could be his income?
22. The ratio of the length to breadth of a lawn is 3:5. It costs ₹ 3200 to fence it at the rate of ₹ 2 a metre. What would be the cost of developing the lawn at the rate of ₹10 per square metre.
23. If one counts one for the thumb, two for the index finger, three for the middle finger, four for the ring finger, five for the little finger and continues counting backwards, six for the ring finger, seven for the middle finger, eight for the index finger, 9 for the thumb, ten for the index finger, eleven for the middle finger, twelve for the ring finger, thirteen for the little finger, fourteen for the ring finger and so on. Which finger will be counted as one thousand?
24. Three friends plucked some mangoes from a mango grove and collected them together in a pile and took nap after that. After some time, one of the friends woke up and divided the mangoes into three equal numbers. There was one



mango extra. He gave it to a monkey nearby, took one part for himself and slept again. Next the second friend got up unaware of what has happened, divided the rest of the mangoes into three equal shares. There was an extra mango. He gave it to the monkey, took one share for himself and slept again. Next the third friend got up not knowing what happened and divided the mangoes into three equal shares. There was an extra mango. He gave it to the monkey, took one share for himself and went to sleep again. After some time, all of them got up together to find 30 mangoes. How many mangoes did the friends pluck initially?

25. **The peculiar number**

    There is a number which is very peculiar. This number is three times the sum of its digits. Can you find the number?

26. Ten saplings are to be planted in straight lines in such way that each line has exactly four of them.

27. What will be the next number in the sequence?
    (a) 1, 5, 9, 13, 17, 21, ...
    (b) 2, 7, 12, 17, 22, ...
    (c) 2, 6, 12, 20, 30, ...
    (d) 1, 2, 3, 5, 8, 13, ...
    (e) 1, 3, 6, 10, 15, ...

28. Observe the pattern in the following statement:

    $31 \times 39 = 13 \times 93$

    The two numbers on each side are co-prime and are obtained by **reversing the digits** of respective numbers. Try to write some more pairs of such numbers.

ANSWERS

1. 119
2. 28
3. 3 m
4. The man takes an empty vessel other than these.

   With the help of 3 litres can he takes out 9 litres of milk from the 10 litres can and pours it in the extra can. So, 1 litre milk remains in the 10 litres can. With the help of 7 litres can he takes out 7 litres of milk from the extra can and pours it in the 10 litres can. The 10 litres can now has $1 + 7 = 8$ litres of milk.

With the help of 3 litres can he takes out 3 litres milk from the 10 litres can. The 10 litres can now has 8 – 3 = 5 litres of milk, which he gives to the customer.

5. 14, 25, 36, 47, 58, 69
6. 2 units
7. 1 : 2
8. 80
9. (i) No, all children would not get it.

   (ii) All would get it.
10. 9, 2, 11, 4, 13, 6, 15, 8, 1, 10, 3, 12, 5, 14, 7.

    12, 10, 8, 6, 4, 2, 0, 12, 10, 8, 6, 4.
11. Fill the 9 litres can. Remove 5 litres from it using the 5 litres can. Empty the 5 litres can. Pour 4 litres remaining in the 9 litres can to the 5 litres can.

    Fill the 9 litres can again. Fill the remaining 5 litres can from the water in it. This leaves 8 litres in the 9 litres can. Empty the 5 litres can. Fill it from the 9 litres can. You now have 3 litres left in the 9 litres can.
12. Height = 9m
13. 36
14. 90 minutes
15. Steps with one pair of foot marks – 2, 3, 9, 10

    Steps with no foot marks – 1, 5, 7, 11
16. 52
17. $99 + \frac{9}{9}$
18. 10
19. 30 km
20. 50 km
21. ₹ 7000 per month

22. ₹ 15,00,000

23. Index finger

24. 106 mangoes

25. 27

26. One arrangement could be 

27. (a) 25 (b) 27 (c) 42 (d) 21 (e) 21

28. One such pair is $13 \times 62 = 31 \times 26$.