A TEXT BOOK OF
BASIC MATHEMATICS
PRESCRIBED FOR
FIRST YEAR PRE-UNIVERSITY

AS PER NEW SYLLABUS FROM 2013-14 ONWARDS

(Based on NCERT Guidelines and CBSE Pattern)

DIRECTORATE OF PRE UNIVERSITY EDUCATION

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A Text book of Basic Mathematics for I PUC

First Edition : 2013

Note: All possible care has been taken in the editing, proof reading and printing of this book. But in case of any omission/mistake/ misprint which might have crept in the book , we thankfully acknowledge them and correct them in the next edition. Any suggestions for the improvement of this book are most welcome.

Ashta Graphics

Director's Message

Dear Students,

We at the Department of Pre-university Education, Karnataka strive to empower each student to dream big and equip them with the tools that enable them to reach new heights and successfully deal with the challenges of life. As Swami Vivekananda said, "**Real education is that which enables one to stand on one's own legs**".

The course contents in this book are designed with the objective of equipping you well for the next level of study.

We wish you well on your journey and look forward to you becoming a responsible citizen of the nation and give back to the betterment of the society.

With best wishes,

Sd/-
C. Shikha, IAS
Director
Department of Pre University Education
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ACKNOWLEDGEMENTS

The Directorate of Pre-University Education and the Pre-University Board gratefully acknowledge the valuable contribution of the revised syllabus Text Committee Members.

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PREFACE

This book of **BASIC MATHEMATICS** is written in accordance with the New Syllabus prescribed by the pre-university Board of Karnataka for the **Academic year 2013-14** according to the CBSE pattern with the guidance of NCERT for the first year pre-university students of Commerce and Arts. Good Effort has been put forward to prepare this book in a very lucid and unambiguous manner.

A large number of worked examples and exercise problems are included so that the students understand the basic concept of the subject easily. Our endeavour is to provide a good student-friendly text book. We hope that this book would serve the purpose of making the study of mathematics interesting as well as stimulating.

This Book is designed as a self contained, comprehensive class room text book for the first year Pre-university students. Great Care has been taken while framing the syllabus keeping in mind that the students should be able to develop good skills by problem solving technique in various branch of mathematics namely algebra, commercial arithmetic, trigonometry and analytical geometry. We, the committee chairman, co-ordinator and the members are extremely grateful to the Directorate of Pre-University Department for having given an opportunity along with guidance, encouragement and support in this endeavour.

Our sincere thanks to the textbook review committee for their valuable guidance and suggestion in sculpting / framing this text book.

Our thanks are due to the publishers, M/s Excellent DTP & Enterprises for their efforts in bringing out this work in an elegant manner.

Inspite of our best efforts if some misprint and mistake have escaped our notice we thankfully acknowledge them and sincerely incorporate them in the subsequent edition. Any suggestions for improvement of this book are most welcome.

From
The Syllabus Committee
Chairman, Co-ordinator and the
Members of I PUC Basic Mathematics

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ALGEBRA

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CHAPTER 1

NUMBER THEORY

1.0 Introduction:

In this chapter we shall study about the numbers and their properties and applications.

1.1 Natural numbers:

Counting numbers 1, 2, 3, 4, 5, … are called natural numbers. They are also called positive integers N = { 1, 2, 3, 4, ….. }

1.2 Whole numbers:

All the natural numbers together with zero (0) form the set of whole numbers. If 0 is added to any natural number 'n' then it gives the same number n

$0 + n = n + 0 = n \quad \forall\ n \varepsilon N$

W = { 0, 1, 2, 3 4, 5 ……. }

1.3 Integers:

The set of all positive integers, negative integers together with 0 is called the set of integers and is denoted by Z

Z = {…. -3, -2, -1, 0, 1, 2, 3 …..}

1.4 Odd and even integers:

The integers 1, 3, 5, 7 …….. are called odd integers.

The integers 0, 2, 4, 6 ….. -2, -4, -6 ….. are called even integers

Note:
- i) 1, 2, 3, 4 …. are called positive integers
- ii) -1, -2, -3, -4 …….. are called negative integers
- iii) Zero (0) is neither positive nor negative. We also assume that it is neither even nor odd.

1.5 Prime numbers:

An integer p > 1 is said to be a prime number if it has no other divisors except one and itself. Eg : 2, 3, 5, 7, 11, 13 ………….

Note:
- i) 2 is the only even prime number
- ii) Prime numbers are infinite

1.6 Composite numbers:

An integer which is not a prime number is called a composite number

Eg : 4, 6, 8, 9, 10, 12, 14 ……….

A composite number n has a divisor other than $\pm$ 1 and $\pm$ **n**

Note: 0 and 1 are neither prime nor composite

1.7 Fundamental theorem of Arithmetic.

Every composite number can be expressed as a product of primes and this decomposition is unique. Apart from the order in which the prime factors occurs.

For **Eg.** i) The factors of 35 are 5 and 7

$\therefore 35 = 5^1 \times 7^1$

ii) The prime factors of 24 are $24 = 2^3 \times 3^1$

iii) The prime factors of 28 are $28 = 2^2 \times 7^1$

This is called **PRIME FACTORIZATION** or **CANONICAL REPRESENTATION.**

The above theorem can be applied to find out

i) Number of positive divisors of a number

ii) The sum of all the positive divisors of a number.

Let n be a composite number. We know that any composite number can be expressed as a product of Primes

Let **n** = $P_1^{\alpha_1}, P_2^{\alpha_2}, P_3^{\alpha_3} \ldots\ldots P_n^{\alpha_n}$ where $P_1, P_2, P_3, P_{4}, \ldots P_n$ are distinct primes.

Let T(n) denote the number of positive divisors of n and S(n) denote the sum of all positive divisors of n. Then we have

$$T_{(n)} = (1+\alpha_1)(1+\alpha_2)\ldots\ldots\ldots(1+\alpha_n)$$

$$S(n) = \frac{P_1^{\alpha_1+1}-1}{P_1-1} \times \frac{P_2^{\alpha_2+1}-1}{P_2-1} \times \ldots\ldots\ldots \times \frac{P_n^{\alpha_n+1}-1}{P_n-1}$$

WORKED EXAMPLES :

Example 1 Find the numbers of positive divisors and the sum of all positive divisors of the following numbers.

(i) 60 (ii) 360 (iii) 825 (iv) 1024 (v) 960

Solution:

i) 60

$$\begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \\ \hline \end{array}$$

$60 = 2^2 \times 3^1 \times 5^1$

$= P_1^{\alpha_1} \times P_2^{\alpha_2} \times P_3^{\alpha_3}$

$P_1 = 2 \quad \alpha_1 = 2, \qquad P_2 = 3, \qquad \alpha_2 = 1, \qquad P_3 = 5, \qquad \alpha_3 = 1$

$T(n) = (1+\alpha_1)(1+\alpha_2)(1+\alpha_3)$

$T(60) = (1+2)(1+1)(1+1)$

$= (3)\ (2)\ (2)$

$= (3)\ (4)$

$= 12$

$$S(n) = \frac{P_1^{\alpha_1+1}-1}{P_1-1} \times \frac{P_2^{\alpha_2+1}-1}{P_2-1} \times \dots\dots\dots \frac{P_n^{\alpha_n+1}-1}{P_n-1}$$

$$S(60) = \frac{2^3-1}{2-1} \times \frac{3^2-1}{3-1} \times \frac{5^2-1}{5-1}$$

$$= \frac{7}{1} \times \frac{8}{2} \times \frac{24}{4}$$

$= 7 \times 4 \times 6$

$= 7 \times 24$

$= 168$

ii) 360

$$\begin{array}{r|l}
2 & 360 \\ \hline
2 & 180 \\ \hline
3 & 90 \\ \hline
3 & 30 \\ \hline
2 & 10 \\ \hline
 & 5 \\ \hline
\end{array}$$

$360 = 2^3 \times 3^2 \times 5^1$

$n = P_1^{\alpha_1} \times P_2^{\alpha_2} \times P_3^{\alpha_3}$

$P_1 = 2,\ \alpha_1 = 3,\ P_2 = 3,\ \alpha_2 = 2,\ P_3 = 5,\ \alpha_3 = 1$

$T(n) = (1+\alpha_1)(1+\alpha_2)(1+\alpha_3)$

$T(360) = (3+1)(1+2) \quad (1+1)$

$= (4) \quad (3) \quad (2)$

$= 24$

$$S(n) = \frac{P_1^{\alpha_1+1}-1}{P_1-1} \times \frac{P_2^{\alpha_2+1}-1}{P_2-1} \ldots\ldots\ldots \times \frac{P_n^{\alpha_n+1}-1}{P_n-1}$$

$$= \frac{2^4-1}{2-1} \times \frac{3^3-1}{3-1} \times \frac{5^2-1}{5-1}$$

$$= \frac{15}{1} \times \frac{26}{2} \times \frac{24}{1}$$

$= 1170$

iii) 825

$$\begin{array}{r|l} 3 & 825 \\ \hline 5 & 275 \\ \hline 5 & 55 \\ \hline & 11 \\ \hline \end{array}$$

$825 = 3^1 \times 5^2 \times 11^1$

$P_1 = 3, \alpha_1 = 1,$ $P_2 = 5,$ $\alpha_2 = 2,$ $P_3 = 11,$ $\alpha_3 = 1$

$$\begin{aligned} T(n) &= (1+\alpha_1)(1+\alpha_2)(1+\alpha_3) \\ &= (1+1)(1+2)(1+1) \\ &= (2)(3)(2) \\ &= 12 \end{aligned}$$

$$S(n) = \frac{P_1^{\alpha_1+1}-1}{P_1-1} \times \frac{P_2^{\alpha_2+1}-1}{P_2-1} \cdots\cdots\cdots \frac{P_n^{\alpha_n+1}-1}{P_n-1}$$

$$= \frac{3^2-1}{3-1} \times \frac{5^3-1}{5-1} \times \frac{11^2-1}{11-1}$$

$$= \frac{8}{2} \times \frac{124}{4} \times \frac{120}{10}$$

$$= 4 \times 31 \times 12 = 1488$$

iv) 1024

$$\begin{array}{r|l} 2 & 1024 \\ \hline 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \\ \hline \end{array}$$

$1024 = 2^{10}$

$P_1 = 2 \qquad \alpha_1 = 10$

$T(1024) = (1 + 10) = 11$

$$S(1024) = \frac{2^{11} - 1}{2 - 1}$$

$$= 2047$$

v) 960

$$\begin{array}{r|l} 2 & 960 \\ \hline 2 & 480 \\ \hline 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \\ \hline \end{array}$$

$960 = 2^6 \times 3^1 \times 5^1$

$P_1 = 2, \quad \alpha_1 = 6, \quad P_2 = 3, \ \alpha_2 = 1, \quad P_3 = 5, \ \alpha_3 = 1$

$$\begin{aligned} T(n) &= (1+\alpha_1)(1+\alpha_2)(1+\alpha_3) \\ &= (1+6)(1+1)(1+1) \\ &= (7)\quad(2)\quad(2) \\ &= 28 \end{aligned}$$

$$S(n) = \frac{P_1^{\alpha_1+1}-1}{P_1-1} \times \frac{P_2^{\alpha_2+1}-1}{P_2-1} \cdots\cdots\cdots \frac{P_n^{\alpha_n+1}-1}{P_n-1}$$

$$\begin{aligned} &= \frac{2^7-1}{2-1} \times \frac{3^2-1}{3-1} \times \frac{5^2-1}{5-1} \\ &= 127 \times \frac{8}{2} \times \frac{24}{2} \\ &= 127 \times 4 \times 6 \\ &= 3048 \end{aligned}$$

EXERCISE : 1.1

1 MARK QUESTIONS:

1. Find the numbers of divisors of the following numbers.
   i) $3^4 \times 5^3 \times 7^2$ ii) 4896 iii) 1644 iv) 672 iv) 768

2. Find the sum of all positive divisors of the following
   i) 672 ii) 768 iii) 6498 iv) 39744 v) 1026

3. Give the canonical representation of the following
   i) 96 ii) 140 iii) 156 iv) 306 v) 5005

ANSWERS OF : 1.1

1. i) 60 ii) 36 iii) 12 iv) 24 v) 18

2. (i) 2016 ii) 2044 iii) 81489 iv) 121920 v) 2400

3. i) $96 = 2^5 \times 3^1$
   ii) $140 = 2^2 \times 5^1 \times 7^1$
   iii) $156 = 2^1 \times 3^2 \times 13^1$
   iv) $306 = 2^2 \times 3^1 \times 17^1$
   v) $5005 = 5^1 \times 7^1 \times 11^1 \times 13^1$

1.8 Least Common Multiple

The least number which is exactly divisible by each one of the given numbers is called their LCM.

Rules to find LCM

Rule I : To find the LCM by **Prime factorization method.**

Express each number as the product of primes then LCM = Product of highest powers of all the factors.

Example 1 Find the LCM of 12, 21 and 24

Solution :
$12 = 2^2 \times 3^1$
$21 = 3^1 \times 7^1$
$24 = 2^3 \times 3^1$

LCM $= 2^3\ 3^1 \times 7^1$
$= 8 \times 21$
$= 168$

Example 2 Find the LCM of 36, 40 and 48

Solution Factors of 36:

$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \\ \hline \end{array}$$

$\therefore$ **$36 = 2^2 \times 3^2$**

Factors of 40:

$$\begin{array}{r|l} 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 1 \\ \hline & 5 \\ \hline \end{array}$$

$$\therefore \mathbf{40 = 2^3 \times 5^1}$$

Factors of 48:

$$\begin{array}{r|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$$\therefore \mathbf{48 = 2^4 \times 3^1}$$

LCM of 36, 40 and 48

$= 2^4 \times 3^2 \times 5^1$

$= 720$

Rule II LCM by division method

1. Write the given numbers in a horizontal line, separating them by commas.
2. Divide by any one of the prime numbers 2, 3, 5, 7 etc. which will exactly divide atleast any two of the numbers.
3. Write the quotient and undivided numbers in a line below the first line.
4. Repeat the process until we get a line of numbers which are prime to one another.
5. The product of all the divisiors and the numbers on the last line will be the required LCM

Example 3 Find the LCM of 12, 18, 24

Solution :

$$\begin{array}{r|l} 2 & 12, 18, 24 \\ \hline 2 & 6, 9, 12 \\ \hline 2 & 2, 3, 4 \\ \hline & 1, 3, 2 \\ \hline \end{array}$$

LCM : $2 \times 2 \times 3 \times 1 \times 3 \times 2 = 72$

Example 4 Find the LCM of 30, 60, 90

Solution :

$$\begin{array}{r|l} 2 & 30, 60, 90 \\ \hline 2 & 10, 20, 30 \\ \hline 5 & 5, 10, 15 \\ \hline & 1, 2, 3 \\ \hline \end{array}$$

LCM : $3 \times 2 \times 5 \times 2 \times 3 = 180$

Example 5 Find the LCM of 12, 15 and 18

Solution :

$$\begin{array}{r|l} 3 & 12, 15, 18 \\ \hline 2 & 4, \ 5, \ 6 \\ \hline & 2, \ 5, \ 3 \\ \hline \end{array}$$

LCM $= 3 \times 2 \times 2 \times 5 \times 3 = 180$

Example 6 Find the LCM of 48, 96 and 74

Solution :

$$\begin{array}{r|l} 2 & 48, 96, 74 \\ \hline 2 & 24, 48, 37 \\ \hline 2 & 12, 24, 37 \\ \hline 2 & 6, 12, 37 \\ \hline 3 & 3, \ 6, \ 37 \\ \hline & 1, \ 2, \ 37 \\ \hline \end{array}$$

LCM $= 48 \times 2 \times 37 = 3552$

1.9 To find out the LCM of fractions :

If $\frac{a}{b}$, $\frac{c}{d}$, $\frac{e}{f}$ are the proper fractions, then LCM is given by

$$\text{LCM} = \frac{LCM\ of\ numerators}{HCF\ of\ denominators}$$

Example 7 Find the LCM of $\frac{1}{3}$, $\frac{5}{6}$, $\frac{5}{7}$

Solution : First find LCM of numerators

$$\begin{array}{r|l} 5 & 1,\ 5,\ 5 \\ \hline & 1,\ 1,\ 1 \\ \hline \end{array} \qquad \text{LCM} = 5$$

HCF of denominators 3, 6, 7 = 1

$$\text{LCM} = \frac{LCM\ of\ numerators}{HCF\ of\ denominators} = \frac{5}{1} = 5$$

Example 8 Three vessels can hold 9, 15, 24 litres of water. Find the least quantity of water which can be filled by these vessels an exact numbers of times.

Solution :

Let us find the LCM

$$\begin{array}{r|l} 3 & 9, 15, 24 \\ \hline & 3, 5, 8 \\ \hline \end{array}$$

LCM = $15 \times 8 \times 3 = 120 \times 3 = 360$

Example 9 Three bells call at intervals 30 sec., 40 sec., 50 sec., respectively. They start together. After how many minutes will next bell fall together.

Solution : Find the LCM of 30, 40 and 50

$$\begin{array}{r|l} 2 & 30, 40, 50 \\ \hline 5 & 15, 20, 25 \\ \hline 2 & 3,\ 4,\ 5 \\ \hline & 3,\ 2,\ 5 \\ \hline \end{array}$$

LCM = $30 \times 20 = 600$

Hence the bells will fall together after 600 sec.

Or $\frac{600}{60}$ = 10 minutes

Example 10 Three measuring rods are 60 cm, 80 cm, and 100 cm in length. What is the length of cloth that can be measured exact number of times using any one of these rods.

Solution : Required length = LCM of (60, 80 and 100) cm.

$$\begin{array}{r|l} 2 & 60, 80, 100 \\ \hline 2 & 30, 40, 50 \\ \hline 5 & 15, 20, 25 \\ \hline & 3,\ 4,\ 5 \end{array}$$

LCM = $2 \times 2 \times 5 \times 60$
= 1200 cm
= 12 m

Example 11 Find the numbers which when divided by 36, 40 and 48 leaves the same remainder 5.

Solution : Let us find out the LCM of 36, 40, 48

$36 = 2^2 \times 3^2$
$40 = 2^3 \times 5$
$48 = 2^4 \times 3$

LCM = $2^4 \times 3^2 \times 5^1$
= 16×45
= 720

Since 5 has to be the remainder, we have to add 5 to 720
i.e. 720 + 5 = 725

EXERCISE : 1.2

3 MARK QUESTIONS:

1. Find the L.C.M. of the following:
   numbers by factorization method
   1) 6, 9, 12   ii) 8, 10, 12,   iii) 12, 14, 16
   iv) 5, 10, 15   v) 16, 20, 24

2. Find the LCM of the following numbers by division method.
   i) 72, 64   ii) 48, 96, 72   iii) 12, 15, 18,
   iv) 25, 75, 150   v) 4, 12, 24

3. Find the least integer divisible by 18 and 24

4 Find the LCM of
   i) $\frac{1}{3}, \quad \frac{5}{6}, \quad \frac{2}{9}$ *ii)* $\frac{6}{7}, \quad \frac{5}{14}, \quad \frac{8}{21}$

5. Three scales are 65 cm, 85 cm, and 95 cm in length. What is the length of the cloth that can be measured exact number of times using any one of these three scales.

6. Find the number which when divided by 16, 20 and 40 leaves the same remainder 4.

ANSWERS : 1.2

1. i) 36   ii) 120   iii) 336   iv) 30   v) 240
2. i) 576   ii) 576   iii) 180   iv) 150   v) 24
3. 72
4. i) $\frac{10}{3}$ ii) $\frac{120}{7}$
5. 20995
6. 124

1.9 Highest Common Factor : (HCF) or (Greatest Common Divisor GCD) or (Greatest common measure)

Definition: An integer d is called as the GCD or HCF of 2 integers ‘a’ and ‘b’ (both of them are not zero) if

i) $d|a$ and $d|b$

ii) Every common divisor of ‘a’ and ‘b’ divides ‘d’
i.e. $x|a$, $x|b$ $\Rightarrow x|d$
usually GCD of ‘a’ and ‘b’ is denoted by d = (a, b)

For e.g. HCF of 8 and 16 is 8 and is denoted by (8, 16) = 8
Note: i) HCF of 2 numbers is a unique positive integer.
ii) If (a, b) = d then d = (-a, b) = (a, -b) = (-a, -b) = a positive integer.

Rules to find HCF (or GCD)

RULE 1: BY PRIME FACTORIZATION METHOD

1) Express each number as the product of primes, then HCF = Product of least powers of common factors.

Example 12 Find the HCF of 12, 15, 18

Solution :

$12 = 2^2\ 3^1$
$15 = 3^1 \times 5^1$
$18 = 3^2 \times 2^1$
HCF = Product of least powers of common factors = 3

Example 13 Find the H.C.F. of 16, 24, 48

Solution :

$$\begin{array}{r|l} 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \\ \hline \end{array}$$

$16 = 2^4$

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \\ \hline \end{array}$$

$$24 = 2^3 \times 3^1$$

$$\begin{array}{r|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline \multicolumn{1}{r}{} & 3 \end{array}$$

$$48 = 2^4 \times 3^1$$

HCF = Product of least powers of common factors

$= 2^3$

$= 8$

Rule 2 By Division method

Let ‘a’ and ‘b’ be any 2 integers, without loss of generality we can assume $b > a > 0$.

on dividing ‘b’ by ‘a’ let q be the quotient and r_1 be the remainder if $r_1 = 0$, then ‘a’ by divides ‘b’ thus HCF = a

If $r_1 \neq 0$, divide a by r_1, and let q_2 be the quotient and r_2 the remainder. If $r_2 = 0$ then r_1 is the HCF of ‘a’ and ‘b’.

If $r_2 \neq 0$ divide r_1 by r_2 to get the quotient $q_{.3}$ and remainder r_3. If $r_3 = 0$ then HCF $= r_2$.

If $r_3 \neq 0$, continue this process of dividing each divisor by the remainder till the remainder becomes zero. The last non-zero remainder is the HCF of ‘a’ and ‘b’ – This is known as **EUCLID’S ALGORITHM**.

Example 14 Find the HCF of 55 and 210

Solution :

$$
\begin{array}{l}
55\ )\ 210\ (\ 3 \\
\quad\ \ \underline{165} \qquad\qquad\qquad\qquad\qquad r_1 = 45\ ;\text{ Divide 55 by 45} \\
\quad\ \ 45\ )\ 55\ (\ 1 \\
\qquad\quad\ \ \underline{45} \\
\qquad\quad\ \ 10\ )\ 45\ (\ 4 \qquad\qquad\quad r_2 = 10\text{ Divide 45 by 10} \\
\qquad\qquad\quad \underline{40} \\
\qquad\qquad\quad\ 5\ )\ 10\ (\ 2 \qquad\qquad r_3 = 5\text{ Divide by 10 by 5} \\
\qquad\qquad\qquad\ \ \underline{10} \\
\qquad\qquad\qquad\ \ 00 \qquad\qquad\qquad r_4 = 0
\end{array}
$$

The last non zero remainder is 5

Therefore 5 is the HCF of 55 and 210

In symbols we write this as

(55, 210) = 5

Example 15 Find the HCF 18 and 24

Solution :

$$
\begin{array}{l}
18\ )\ 24\ (\ 1 \\
\quad\ \ \underline{18} \\
\qquad 6\ )\ 18\ (\ 3 \\
\qquad\quad\ \ \underline{18} \\
\qquad\quad\ \ 00
\end{array}
$$

$\therefore$ HCF = 6

i.e., (18, 24) = 6

Example 16 Find the H.C.F. of 165, 225 and 435

Solution :

Let us first find the HCF of 165 and 225

165) 225 (1
$\underline{165}$
60) 165 (2
$\underline{120}$
45) 60 (1
$\underline{45}$
15) 45 (3
$\underline{45}$
00

HCF of 165 and 225 is 15

Let us find out the H.C.F. of 15 and 435

15) 435 (29
435
00

$\therefore$ HCF of 165, 225 and 435 is 15

1.10 Relation between LCM and HCF

If A and B are two numbers then the product of their LCM and HCF is equal to the product of two numbers.

LCM $\times$ HCF = A $\times$ B

$$\Rightarrow \text{HCF} = \frac{A \times B}{LCM}$$

$$\Rightarrow \text{LCM} = \frac{A \times B}{HCF}$$

1.11 To find the HCF of fractions

If $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$ are the proper fractions, the HCF is given by

$$\text{HCF} = \frac{HCF\ of\ numerators\ a, c, e}{LCM\ of\ denominators\ b, d, f}$$

Example 17 The cost of a chair is ₹600 and the cost of a table is ₹900. Find the least sum of money that a person must possess in order to purchase a whole number of chairs or tables.

Solution :

Let us find the HCF of 600 and 900

600) 900 (1
<u>600</u>
300) 600 (2
<u>600</u>
00

HCF = 300

$$\text{LCM} = \frac{600 \times 900}{300}$$

$= 1800$

The person must possess the least sum of ₹1800

Example 18 Find the greatest number which divides 39, 48 and 90 leaving remainders 6, 4 and 2 respectively.

Solution:

Let the number be **h**

Given that h divides 39 and leaves the remainder 6

$\therefore$ h divides $39 - 6 = 33$

Likewise h divides 48 and leaves the remainder 4

$48 - 4 = 44$

And when h divides 90, it leaves the remainder 2

$\therefore$ 90 – 2 = 88

$\therefore$ h is the HCF of 33, 44, 88

$33 = 11^1 \times 3^1$

$44 = 11^1 \ 2^2$

$88 = 11^1 \times 2^3$

$\therefore$ HCF of 33, 44, 88 is 11

$\therefore$ h = 11 $\quad\therefore$ the required number is 11

Example 19 If the product of two numbers is 216 and their LCM is 36. Find their HCF

Solution :

ab = 216 LCM = 36 HCF = ?

HCF $\times$ LCM = ab = 216

$$\text{HCF} = \frac{216}{LCM}$$

$$\text{HCF} = \frac{216}{36} = 6$$

1.12 Rational numbers:

A number in the simplest form $\frac{p}{q}$ where p and q are integers and q $\neq$ 0 is called a **reational number.** The set of all rational numbers is denoted by Q.

Eg $\frac{4}{7}, \frac{3}{5}, \frac{-2}{3}$ *etc*

Note:

1. Every integer is a rational number

2. Zero (0) is a rational number since $0 = \frac{0}{1}$ is of the form $\frac{p}{q}$, where p and q are integer and q $\neq$ 0.

3. Square root of a positive integer, which is a perfect square is rational number:

Eg: $\sqrt{16} = 4$, $\sqrt{25} = 5$

Rational number in lowest terms

A rational number of the form p/q where $q \neq 0$ is said to be in the lowest form if the integers p and q have no other common factors other than 1 i.e., if p and q are co-prime or HCF of p and q is 1

Eg: $\frac{7}{14}$ is a rational number but not in the lowest form. But $\frac{7}{14} = \frac{1}{2}$ is a rational number in lowest form.

All rational numbers when expressed in decimal form are either terminating decimals or recurring decimals.

Example 20

1) $\frac{1}{4} = 0.25$ is a terminating decimal.

2) $\frac{2}{5} = 0.4$ is a terminating decimal

3) $\frac{1}{3} = 0.\bar{3}\bar{3}\bar{3}\bar{3}$........ is a non terminating recurring decimal

4) $\frac{1}{7} = 0.\overline{142857}\,\overline{142857}\,\overline{142857}$
.... is a non terminating recurring decimal.

Example 21 Express each of the following rational numbers as decimals.

Solution :

i) $\frac{3}{4} = 0.75$

ii) $\frac{15}{8} = 1.875$

iii) $\frac{8}{125} = \frac{8 \times 2^3}{125 \times 2^3} = \frac{64}{1000} = 0.064$

Recurring decimal: A decimal representation in which all the digits after a certain stage are repeated is called a recurring decimal.

For Eg : $\frac{1}{3} = 0.33333 \ldots\ldots = 0.\bar{3}$

Mixed Recurring decimal : A decimal in which some digits after the decimal point is not repeated and then some digit or digits are repeated is called a 'Mixed recurring decimal'

For example : $6.12555 = 6.12\bar{5}$

Note:

1. Let $x = \frac{p}{q}$ be a rational number such that prime factorization of q is of the form $2^n \times 5^m$. Then decimal expansion of $\frac{p}{q}$ terminates.

2. If $\frac{p}{q}$ is a rational number such that prime factorization of q is not of the form $2^n \times 5^m$. Then decimal expansion of $\frac{p}{q}$ is non-terminating.

Example 23 Without actually performing the long division state whether the following rational numbers will have a terminating decimal expansion or non- terminating repeating decimal expansion.

Solution : **i)** $\frac{17}{3125}$

Prime factorization of $3125 = 5^5 = 2^0\ x\ 5^5 = (2^n\ .\ 5^m$ form)

$\therefore \frac{17}{3125}$ is a rational number having decimal expansion.

ii) $\frac{13}{8}$

Prime factorization of $8 = 2^3 = (2^m .5^n$ form)

$\therefore$ $\frac{13}{8}$ is a rational having decimal expansion.

iii) $\frac{31}{243}$

Prime factorization of $243 = 3^5$. It is not of the form $2^m\ 5^n$.

$\therefore$ $\frac{31}{243}$ is a non terminating decimal expansion

iv) $\frac{64}{343}$

Prime factorization of $343 = 7^3$. It is not of the form $2^m.\ 5^n$.

$\therefore$ the rational number $\frac{64}{343}$ is a non terminating decimal expansion.

1.13 Irrational numbers: A number which cannot be put in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$ is called as irrational number.

i.e. a number which is not rational is irrational.

Eg: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$ etc. are irrational.

Note:

1. Negative of an irrational number is an irrational number eg.: $-\sqrt{2}$
2. Sum of rational and irrational number is always an irrational number.

   Eg. $2+\sqrt{3}$
3. Product of a non-zero rational number and an irrational number is irrational.

   Eg. $\frac{2}{3}\times(\sqrt{3}+1)$

Example 24 Prove that $\sqrt{2}$ is an irrational number.

Solution : We shall prove it by contradiction.

If possible Let $\sqrt{2}$ be a rational number

Let $\sqrt{2} = \frac{p}{q}$ where p and q are integers and $q \neq 0$.

Further let p and q are coprime i.e. H.C.F. of p and q = 1.

$\sqrt{2} = \frac{p}{q} \Rightarrow \sqrt{2}q = p$

$\Rightarrow 2q^2 = p^2$

$\Rightarrow$ 2 divides p^2 $\Rightarrow$ 2 divides p

$\Rightarrow$ p is even

Let p = 2k where k is an integer $p^2 = 4k^2$

$2q^2 = 4k^2$

$q^2 = 2k^2 \Rightarrow q^2$ is even

$\Rightarrow$ q is even.

Now p is even and q is even which implies p and q have a common factor 2. which is a contradiction of the fact that p and q are co-prime.

$\therefore$ our assumption that $\sqrt{2}$ is rational is wrong and hence $\sqrt{2}$ is irrational.

Example 25 Prove that $3 + \sqrt{5}$ is an irrational number;

Solution : If possible, let 3 + “5 be a rational number

$\Rightarrow \quad 3 + \sqrt{5} = \frac{a}{b}$ where a and b are integers and $b \neq 0$

$\Rightarrow \quad \sqrt{5} = \frac{a}{b} - 3$

= rational – rational

= rational

Irrational $\sqrt{5}$ = rational

A rational number cannot be equal to an irrational number

$\therefore$ our assumption is wrong. Hence $3+\sqrt{5}$ is irrational.

EXERCISE : 1.3

3 MARK QUESTIONS

1. Find the HCF of the following by faction sation method.
   i) 144, 720 ii) 60, 72, 84 iii) 108, 216
   iv) 34, 85, 153 v) 165, 225

2. Find the HCF of the following mumbers by division method.
   i) 72, 96 ii) 104, 130 iii) 45, 90, 180
   iv) 8, 16, 24 v) 12, 15, 24

3. Find the largest integer which divides 105 and 315

4. Find the greatest interger which divides 42, 52, 86 leaving remainder 6, 4 and 2 respectively.

5. Find the HCF of $\frac{8}{9}, \frac{32}{81}, \frac{16}{27}$

6. If the HCF of two numbers is 42 and their product is 52920. Find their L.CM

7. The HCF of two numbers is 16 and their LCM is 160. If one of the numbers is 64. Find the other numbers.

ANSWERS : 1.3

1.	i) 144	ii) 12	iii) 108	iv) 17	v) 15
2.	i) 24	ii) 26	iii) 45	iv) 8	v)3
3.	105	4. 12	5. $\frac{8}{81}$	6. 1260	7. 40

DEFINITION OF A SURD: An irrational root of a rational number is called a surd.

For eg.: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}+3, 3+\sqrt{7}$

TRANSCENDENTAL NUMBER = An irrational number which is not a surd is called a transcendental number. π is a transcendental number.

1.14 Real Numbers: Rational numbers and irrational numbers taken together are said to form the set of Real numbers. The set of real numbers is denoted by R.

i.e. A real number is either rational or irrational $R = Q \cup Q'$ where Q is set of rational and Q' is the set of irrationals.

Every real number (whether rational or irrational) can be represented by a unique point on the number line called real number line and conversely.

Properties of Real numbers:

1. For any real number a, only one of the following is true.
   i) $a > 0$ or ii) $a < 0$ or iii) $a = 0$

2. For any two real numbers a and b only one of the following is true.
   i) $a < b$ or ii) $a > b$ or iii) $a = b$

3. If a, b, c are real numbers such that $a > b, b > c$ then $a > c$

4. If a and b are any two real numbers such that $ab = 0$ then $a = 0$ or $b = 0$

5. If a, b, c, are any three real numbers then

   1) $\left.\begin{array}{l} a + b = b + a \\ a.\,b = b.\,a \end{array}\right\}$ Commutative laws

   2) $\left.\begin{array}{l} a + (b + c) = (a + b) + c \\ a.\,(b.c) = (a.\,b).\,c \end{array}\right\}$ Associative laws

   3) $a.(b + c) = a.b + a.c$ (Distributive law)

6. Square of a real number is always positive.
   For eg. $(-9)^2 = 81$
   $(5)^2 = 25$

1.15 Complex numbers:

A number of the form a + ib where a, b εR, the set of real numbers and $i = \sqrt{-1}$ is called as a complex number. Where 'a' is called as real part and 'b' is called the imaginary part.

Example: $1 + 6i$, $3 - 4i$, $2 + \sqrt{3}i$ etc. are complex numbers.

Algebra of complex numbers:

Addition and subtraction of two complex numbers:

Definition: If $z_1 = a + ib$ and $z_2 = c + id$ are two complex numbers, then

$z_1 + z_2 = (a + ib) + (c + id)$ is the complex number $= (a + c) + i\,(b + d)$

$z_1 - z_2 = (a + ib) - (c + id)$

$= (a - c) + i\,(b - d)$

Eg. $z_1 = 4 + 5i$, $z_2 = 1 + 7i$

i) $z_1 + z_2 = 5 + 12i$ ii) $z_1 - z_2 = 3 - 2i$

MULTIPLICATION OF TWO COMPLEX NUMBERS.

If $z_1 = a + ib$ and $z_2 = c + id$ are two complex numbers then

$z_1 z_2 = (a + ib)(c + id)$

$= (ac - bd) + i\,(ad + bc)$

Example: $(2 + 3i)(4 + 3i) = (8 - 9) + i\,(18) = -1 + 18i$.

CONJUGATE OF A COMPLEX NUMBER

If $z = a + ib$ is a complex number then $a - ib$ is called as its conjugate and is denoted by $\bar{z}$.

Example if $z = 2 + 3i$ then $\bar{z} = 2 - 3i$

If $z = 5 - 4i$ then $\bar{z} = 5 + 4i$

DIVISION OF A COMPLEX NUMBER BY A NON-ZERO COMPLEX NUMBER.

Let $z_1 = a + ib$ and $z_2 = c + id$ be two complex numbers

$\frac{z_1}{z_2} = \frac{a+ib}{c+id}$ is a complex number

$$\frac{a+ib}{c+id} = \frac{c-id}{c-id}$$

$$= \left[\frac{ac+bd}{c^2+d^2}\right] + i\left[\frac{bc-ac}{c^2+d^2}\right]$$

Example: $\frac{1+i}{1-i} = \frac{(1+i)}{(1-i)} \times \frac{(1+i)}{(1+i)}$

$= \qquad \frac{1+i^2+2i}{1-i^2}$

$= \qquad \frac{1-1+2i}{2} = \frac{2i}{2} = 0 + i1$

a + ib = 0 + i1

a = 0

b = 1

IDENTITY PROPERTY

z + 0 = 0 + z = z

$\forall z \neq$ 0 which is a complex number, 1 + 0i is the multiplicative identity.

1 × z = z × 1 = z

INVERSE PROPERTY : For every complex number z there exists some - z belonging to the set of complex numbers such that z + - z = - z + z

- z is the additive inverse of z

For every complex number z except zero there exists some $\frac{1}{Z}$ belonging to the set of complex numbers such that

$z \times \frac{1}{z} = \frac{1}{z} \times z = 1$

$\frac{1}{z}$

is the multiplicative inverse of z

5. COMMUTATIVE PROPERTY:

For every complex numbers z_1 , z_2

1) $z_1 + z_2 = z_2 + z_1$

2) $z_1 \times z_2 = z_2 \times z_1$

6. a) **LEFT DISTRIBUTIVE PROPERTY**

For every complex numbers z_1 , $z_{2,}$ z_3

$z_1 (z_2 + z_3) = z_1\ z_2 + z_1\ z_3$

b) **RIGHT DISTRIBUTIVE PROPERTY:**

$(z_1 + z_2)\ z_3 = z_1 z_3 + z_2 z_3$

MODULUS OF A COMPLEX NUMBER:

If z = a + ib

$|z| = \sqrt{a^2 + b^2}$

EQUALITY OF COMPLEX NUMBERS

DEFINITION: If a + ib and c + id are two complex umbers then a + ib = c + id if and only if a = c and b = d

Example:

1. a + ib = 4 + 3i
   $\Rightarrow$ a = 4, b = 3

2. a + ib = 0 + 0i
   $\Rightarrow$ a = 0, b = 0

PROPERTIES OF COMPLEX NUMBERS:

Let C be the set of complex numbers then

1. Closure property : For every complex numbers z_1 and z_2 ,
   $z_1 + z_2$ belongs to the set of complex numbers.Also
   $z_1 . z_2$ belongs to the set of complex numbers

2. Associative property: For every complex numbers z_1 , z_2 and z_3
   1) $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$
   2) $z_1 \times (z_2 \times z_3) = (z_1 \times z_2) \times z_3$

3. Identities : For every complex number z
   0 + 0i is the additive identity

Examples:

Evaluate the following:

1. $(1 + i) + (2 + 3i)$
   $= 3 + 4i$
   Real part = 3, imaginary part = 4

2. $(1 + i)(1 - i)$
   $= \quad 1 - i^2$
   $= \quad 1 + 1 = 2$
   Real part = 2, imaginary part = 0

3. $(1 + i)(-2 + 3i)$
   $= (-5) + 1i$
   Real Part = -5 imaginary part = 1

4. $\dfrac{(1+2i)}{(3-4i)}$

$$= \frac{1+2i}{(3-4i)} \times \frac{(3+4i)}{(3+41)}$$

$$= \frac{(1+2i)(3+4i)}{9+16}$$

$$= \frac{(3-8)+i(10)}{25}$$

$$= \frac{-5+10}{25} = -\frac{1}{5} + \frac{2i}{5}$$

Real part = $-\dfrac{1}{5}$ Imaginary part $= \dfrac{2}{5}$

5. $\dfrac{1}{4+3i}$

$= \dfrac{1}{4+3i} \times \dfrac{4-3i}{4-3i}$

$= \dfrac{4-3i}{16+9}$

$= \dfrac{4}{25} - \dfrac{3i}{25}$

Real part $= \dfrac{4}{25}$

Imaginary part $= \dfrac{-3}{25}$

EXERCISE : 1.4

I. 5 MARKS QUESTIONS:

1. Prove that $\sqrt{2}$ is an irrational number
2. Prove that $\sqrt{5}$ is an irrational number
3. Prove that $2 + \sqrt{3}$ is an irrational number
4. Prove that $2 + 3\sqrt{5}$ is an irrational number

II. 2 MARKS QUESTIONS:

Without actually performing the long division state whether the following rational numbers will have terminating decimal expansion. Or nonterminating decimal expansion also write down the decimal expansion of the rational numbers which have the terminating decimal expansions.

$i)\ \dfrac{29}{1600}$ $\quad ii)\ \dfrac{139}{2^3 \times 5^2}$ $\quad iii)\ \dfrac{129}{7^2}$ $\quad iv)\ \dfrac{77}{210}$

III. 1 MARK QUESTIONS:

Write the real and imaginary parts of the following complex numbers.

i) $3 + 5i$ ii) $4 - i$ iii) $3i$ iv) 6 v) $\dfrac{2}{3} - \dfrac{4i}{5}$

ANSWERS : 1.4

II. i) 0.018125 (Terminating decimal)

ii) .695 (Terminating decimal)

iii) 2.632653061 ……….. (Non – Terminating Decimal)

iv) 0.3666666 ………… (Non – Terminating Decimal)

III. i) Real part = 3 Imaginary Part = 5

ii) Real part = 4 Imaginary Part = - 1

iii) Real part = 0 Imaginary Part = 3

iv) Real Part = 6 Imaginary Part = 0

v) Real Part $= \frac{2}{3}$ Imaginary Part $= \frac{-4}{5}$

EXERCISE : 1.5

I. Express the following in the form of a + ib

1. If $z_1 = 2 + 3i$ $z_2 = 1 - i$
   Find i) $z_1 + z_2$ ii) $z_1 - z_2$

2. If $z = 2 + 3i$ Find 1) $z + \bar{z}$
   2) $z \cdot \bar{z}$

3. Evaluate 1) $\frac{-1+5i}{2}$ 2) $\frac{7+3i}{52}$

4. Evaluate : $(1 + i)(4 - 3i)(1 - i)$

ANSWERS : 1.5

1. i) $3 + 2i$ ii) $1 + 4i$

2. i) 4 ii) 13

3. i) $\frac{-1+5i}{2}$ ii) $\frac{7+3i}{52}$

4. $8 - 6i$

CHAPTER 2

SETS, RELATIONS AND FUNCTIONS

2.0 Introduction: The theory of sets was developed by German Mathematician George Cantor. He is regarded as the father of set theory. It is proved to be of great importance in the foundation of relations and functions, sequences, Geometry, Probability theory etc. Also it has wide application in logic and philosophy.

2.1 Sets: A set is a well defined collection of distinct objects. Each member is called the element of the set.

Note:
1. A set is always represented by capital letters
2. If a is an element of set A then we write a ε A.
3. If b is not an element of set A then we write b $\notin$ A

Examples:
1. The set of boys in class V^{th}A.
2. The set of even natural numbers
3. The set of days of a week
4. The set of vowels in the English alphabet.

2.2 Methods of describing a set:

A set can be represented in two forms
1. Roster form or Tabular form
2. Set builder form or rule form

Roster Form: In the roster Form, all the elements are listed and separated by commas and are enclosed within brackets.

A = The set of all even numbers between 0 and 10
Roster Method given by A = {2, 4, 6, 8}

Set builder Form: In this method all the elements of a set possess a single common property, which is not possessed by any element outside the set.

If A = {1, 2, 3, 4, 5} then the set builder form is represented by
A = $\{ x : x \ \varepsilon \ N \text{ and } x < 6\}$

2.3 Null set or Empty set: A set containing no elements is called an empty set. It is denoted by ϕ or { }

For **eg.:** A = {The set of all even prime numbers other than 2 }

A = ϕ or { }

2. A = set of all natural numbers < 0

A = ϕ or{ }

2.4 Singleton set: A set containing only one element is called a singleton set.

Eg. 1. A = $\{ x : x - 1 = 0 , x \varepsilon N\}$

A = { 1 }

2. B = $\{x : x$ is an even prime number}

B = { 2 }

2.5 Finite set and infinite set: A set is called a finite set if it contains finite numbers of elements.

Example 1 A = {1, 2, 3}

n(A) = 3

2 B = {set of prime numbers < 9}

B = {2, 3, 5,7}

n(B) = 4

A set which is not finite is called an infinite set.

Examples 1. The set of natural numbers

2. The set of real numbers

2.6 Equal and Equivalent sets:

Equal Sets: Two sets A and B are said to be equal if they have exactly the same elements.

Ex. 1. A = {1, 3, 8}

B = {8, 3, 1}

Then A = B as A and B have the same elements.

2. A = { $x : x$ is a letter in the word 'flow}

B = { $x : x$ is a letter in the word 'wolf')

Then A = B as A and B have the same elements

Equivalent sets: Two finite sets A and B are said to be equivalent if they have the same cardinal number i.e. if the same number of elements. i.e. if $n(A) = n(B)$.

Let $A = \{a, e, i, o, u\}$

$B = \{1, 2, 3, 4, 5\}$

Then $n(A) = 5$ and $n(B) = 5$

$\Rightarrow$ the sets A and B are equivalent.

2.7 Subset:

If each and every element of A is an element of B, then A is called a subset of B or A is contained in B. We write $A \subset B$.

Example 1 $A = \{1, 2\}$

$B = \{1, 2, 3, 4\}$

$A \subset B$

Note:

1. If atleast one element of A does not belong to set B then A is not a subset of B. It is symbolically represented by $A \subseteq B$
2. Every set A is a subset of itself i.e. $A \subseteq A$.
3. ϕ is a subset of every set.
4. If $A \subseteq B$ and $B \subseteq A$ then $A = B$

Example 2 Set of Natural Numbers $\subseteq$ set of whole numbers

Super Set: Set A and B are two non empty sets such that A is contained in B and $A \neq B$ then B is called the super set of A

It is symbolically represented by $B \supset A$

EXAMPLE : Set of complex numbers is a super set of set of real numbers

PROPER SUBSET: A is called a proper subset of B if each and every element of A is contained in B and $A \neq B$. It is symbolically represented by $A \subset B$ and is read as 'A' is a proper subset of B

$A = \{a, b, c\}$

$B = \{a, b, c, d\}$

$A \subset B$

Power Set : A set formed by all the subsets of a set A as its elements is called the Power set of A and is denoted by p(A)

Examples:

1. Set A = {a, b}
   The subsets of A are {a}, {b} {a, b} and {a, b} then
   P(A) = {Ø, {a}, {b}, {a, b}}
   n (p (A)) = 2^2 = 4

Note: n (p (A)) = 2^n where n is the number of elements of the set A.

2. B = {1, 2, 3}
   The subsets of B are Ø, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}
   ∴ The power set of the given set B is
   P(B) = { Ø, {1}, {2}, {3}, {1, 2}, {1, 3} {2, 3}, {1, 2, 3}}
   n (P (B)) = 2^3 = 8

2.8 Universal set: If all the sets under consideration are subsets of a set U. Then U is called the Universal set.

Example: For the set of integers Z, the universal set can be set of real numbers R or the set of complex numbers C.

Cardinal number of a finite set: The number of elements of a finite set A is called the cardinal number and is represented by n(A).

A = {1, 2, 3, 4, 5, 6}

Cardinal number of set A = n (A) = 6

2.9 Operation on Sets:

a) **Union of Sets:** Let A and B be any two sets. Then the union of A and B denoted by $A \cup B$ is defined to be the set of all those elements. which are in A or in B or in both.

Examples:

1. Let A = {a, b, c} B = {c, d, e, f}
   ∴ $A \cup B$ = {a, b, c, d, e, f}

2. A = {1, 2, 3, 4, 5}
   B = { 1, 2, ……………… 9}
   $A \cup B$ = {1, 2, 3 ……….. ..9}

Note: 1) $A \cup A = A$

2) $A \cup \varnothing = A$

3) $\varnothing \cup \varnothing = \varnothing$

4) If $A \subseteq B$ then $A \cup B = B$

b) **Intersection of sets:** Let A and B be any two sets. Then the intersection of A and B denoted by $A \cap B$ is defined to be the set of all common elements between A and B

Example:

1. Let $A = \{a, b, c, d\}$
   $B = \{c, d, e, f, g, h\}$
   $A \cap B = \{c,d\}$

2. Let $A = \{1, 2, 3, 4, 5\}$
   $B = \{1, 2, 3, \ldots\ldots.. 9\}$
   $A \cap B = \{1, 2, 3, 4, 5\} = A$ itself

Note: 1. $A \cap A = A$

2. $A \cap \varnothing = \varnothing$

3 $\varnothing \cap \varnothing = \varnothing$

4. It $A \subseteq B$ then $A \cap B =$ A itself.

c) **Difference between any two sets** : Let A and B be any two sets. Then the difference A-B is defined to be the set of all those elements of A which are not in B.

It is also called the complement of B w.r.t. A. Similarly B – A is defined to be the set of all those elements of B which are not in A. It is also called the complement of A w.r.t. B.

Example:

1. Let $A = \{a, b, c, d\}$
   $B = \{d, e, f, g, h, i\}$
   $A - B = \{a, b, c\}$
   $B - A = \{e, f, g, h, i\}$

d) **Symmetric difference:** Let A and B be any two sets. Then the symmetric difference between A and B is defined to be $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$

Example: $A = \{a, b, c, d\}$

$B = \{d, e, f, g, h, i\}$

$A - B = \{a, b, c\}$

$B - A = \{e, f, g, h, i\}$

$A \Delta B = (A - B) \cup (B - A)$

$A \Delta B = \{a,b,c,e,f,g,h,i\}$

2.10 Complement of a set w.r.t. Universal set

Let A be any set. Then the complement of A w.r.t. U is the set of all those elements of U which are not in A and is denoted by $U - A$ or A' or A^C

2.11 Algebra of Sets

1. **Commutative laws**: If A and B are any two sets then

1) $A \cup B = B \cup A$ and 2) $A \cap B = B \cap A$

Example: Let

$A = \{1, 2, 3\}$

$B = \{2, 3, 4\}$

$A \cup B = \{1, 2, 3, 4\}$

$B \cup A = \{1, 2, 3, 4\}$

$\therefore A \cup B = B \cup A$

$A \cap B = \{2, 3\}$

$B \cap A = \{2, 3\}$

$\therefore A \cap B = B \cap A$

2. **Associative laws:** If A, B, C are three sets then

i) $A \cup (B \cup C) = (A \cup B) \cup C$ and

ii) $A \cap (B \cap C) = (A \cap B) \cap C$

Example: Let $A = \{1, 2, 3, 4\}$
$B = \{3, 4, 5, 6\}$
$C = \{5, 6, 7, 8\}$

Verify the associative laws

Solution:

i) $B \cup C = \{3, 4, 5, 6, 7, 8\}$

$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 7, 8\}$ (1)

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$ (2)

From (1) and (2) we have

$\therefore A \cup (B \cup C) = (A \cup B) \cup C$

ii) $A \cap B = \{3, 4\}$ (3)

$(A \cap B) \cap C = \{3, 4\}, \cap \{5, 6, 7, 8\} = \phi$

$B \cap C = \{5, 6\}$

$A \cap (B \cap C) = \{1, 2, 3, 4\} \cap \{5, 6\} = \phi$ (4)

From (3) and (4) we have

$A \cap (B \cap C) = (A \cap B) \cap C$

3. **Distributive Laws.** If A, B, C are any three sets then

i) $A \cup B \cap C) = A \cup B \cap (A \cup C)$ and

ii) $A \cap (B \cup C) = (A \cap B) \ (A \cap C)$

Example: Let $A = \{1, 2, 3, 4\}$
$B = \{3, 4, 5, 6\}$
$C = \{4, 5, 6, 7, 8\}$

$B \cap C = \{4, 5, 6\}$

$A \cup (B \cap C) = \{1, 2, 3, 4,) \cup \{4, 5, 6\}$
$= \{1, 2, 3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$(A \cup B) \cap (A \cup C)$

$= \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 5, 6, 7, 8\}$

$= \{1, 2, 3, 4, 5, 6\}$

$\therefore \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ii) $B \cup C = \{3, 4, 5, 6, 7, 8\}$

$A \cap (B \cup C) = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8\}$

$= \{3, 4\}$

$A \cap B = \{3, 4\}$

$A \cap C = \{4\}$

$(A \cap B) \cup (A \cap C) = \{3, 4\}$

Thus we have

$\therefore \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. **De Morgan's Laws:** If A and B are any two sets then

i) $(A \cup B)^1 = A^1 \cap B^1$ and ii) $(A \cap B)^1 = A^1 \cup B^1$

Example: U = (1, 2, 3, 4, 5, 6, 8, 9)

$A = \{1, 2, 3, 4, 5\}$

$B = \{3, 4, 5, 6, 7\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$ (1)

$(A \cup B)^1 = \{8, 9\}$ (1)

$A^1 = \{6, 7, 8, 9\}$

$B^1 = \{1, 2, 8, 9\}$

$A^1 \cap B^1 = \{8, 9\}$ (2)

From (1) and (2) we have

$(A \cup B)^1 = A^1 \cap B^1 = \{8, 9\}$

ii) $A \cap B = \{3, 4, 5\}$ (3)

$(A \cap B)^1 = \{1, 2, 6, 7, 8, 9\}$

$A^1 = \{6, 7, 8, 9\}$

$B^1 = \{1, 2, 8, 9\}$

$A^1 \cup B^1 = \{1, 2, 6, 7, 8, 9\}$ (4)

From (3) & (4) we have

$(A \cap B)^1 = A^1 \cup B^1 = \{1, 2, 6, 7, 8, 9\}$

1. Let $A = \{a, b, c, d\}$

   $B = \{c, d, e., f, g, h\}$

   $A \cap B = \{c,d\}$

2. Let $A = \{1, 2, 3, 4, 5\}$

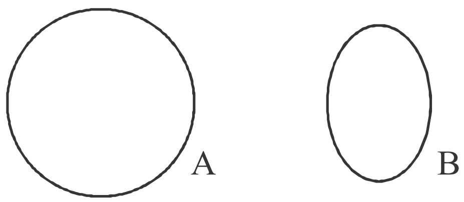
   $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

   $A \cap B = \{1, 2, 3, 4, 5\} = A$ itself

Note:
1. $A \cap A = A$
2. $A \cap \emptyset = \emptyset$
3 $\emptyset \cap \emptyset = \emptyset$
4. It $A \subseteq B$ then $A \cap B =$ A itself.

2.12 Venn Diagrams: Diagramatic representation of sets and properties of sets is called Venn diagram in the name of the famous mathematician Venn who devised it.

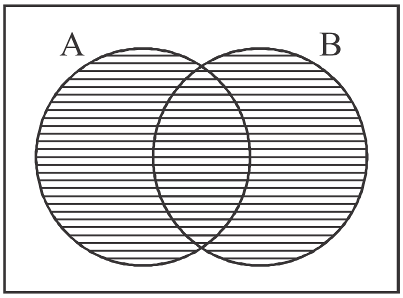
1. A set is represented by a circle or a closed figure.



2. Universal set:

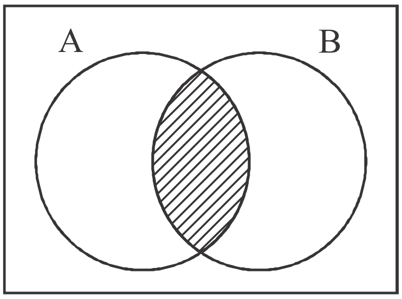
U= 

3. A$\cup$B

U = 

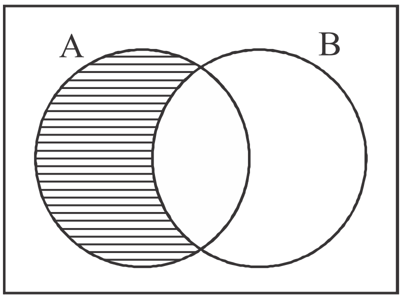
The shaded portion is A$\cup$B

4. A$\cap$B

U = 

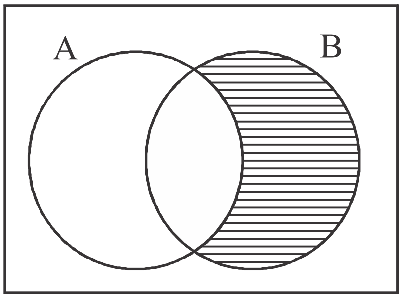
The shaded portion is A$\cap$B

5. A - B

U = 

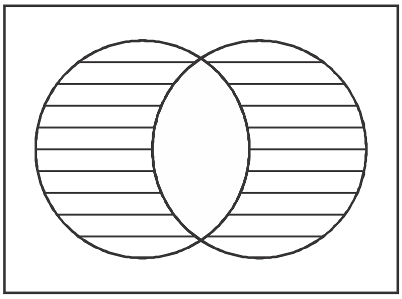
The shaded portion is A – B

6. B – A



The shaded portion is B – A

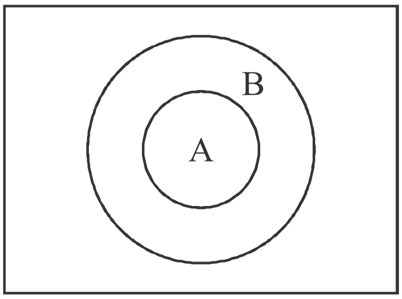
7. A ΔB



The shaded portion is
AΔB = (A-B) ∪ (B-A)

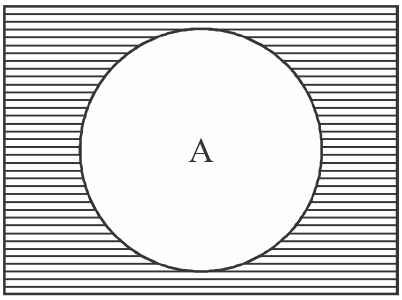
8. Subset A⊆B

U =



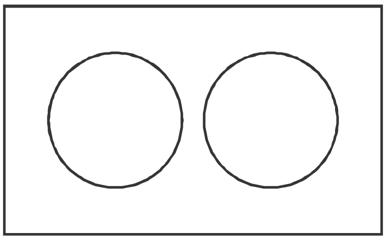
9. A' or U-A

U =



The shaded portion is U-A or A' in U

10. Disjoint sets

U = 

Note that here $A \cap B = \phi$ here

LIST OF FORMULAE:

1. If A and B are two finite sets, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

   Note: If $A \cap B = \emptyset$ then $n(A \cap B) = 0$ $\therefore$ we get

   $n(A \cup B) = n(A) + n(B)$

2. If A, B and C are three finite sets then

   $n(A \cup B \cup C) = n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cap B)$

   $- n(B \cap C) - n(A \cap C)$

3. If A and A^1 both are finite sets, then $n(A^1) = n(U) - n(A)$ where U is the universal set.

4. If A and B are two finite sets, then

   a) $n(A - B) = n(A) - n(A \cap B)$

   b) $n(B - A) = n(B) - n(A \cap B)$

WORKED EXAMPLES:

Example 1 Which of the following are sets?

a)	{2, 4, 6, 8……..}	Ans.: is a set
b)	{10, 12, 15} ……..)	Ans. is a set
c)	All interesting books	Ans.: Not a set
d)	All orange flowers	Ans.: is a set
e}	All rivers of India	Ans.: is a set

Example 2 Represent the following sets in both Roster form and Rule method.

a) Set of factors of 20

Roster form $\{\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20\}$

Rule Form : $\{x : x$ is a factor of $20\}$

b) Set of all prime Numbers less than 10

Roster form = $\{2, 3, 5, 7\}$

Rule form = $\{x : x$ is a prime number $< 10\}$

Example 3 Convert the following sets from roster form to rule form.

a) $A = \{8, 16, 24 \ldots\ldots\ldots\}$

Solution: $A = \{x : x$ is a positive multiple of $8\}$

b) $B = \{5, 10, 15, 20, 25 \ldots\ldots\ldots 50\}$

Solution: $B = \{x : x$ is a positive multiple of $5 < 50\}$

c) $C = \{a, e, i, o, u\}$

Solution: $C = \{x{:}x$ is a vowel in English alphabet$\}$

d) $D = \{2\}$

Solution: $D = \{x : x$ is an even prime number$\}$

e) $E = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

Solution : $E = \{x : x$ is a prime number $< 25\}$

Example 4 Convert the following sets from rule to roster form.

Solution : $A = \{P, R, O, E\}$

a) The set of all letters of the word ROPE

b) $B = \{x : x^2 - 5x + 6 = 0\}$

Solution : $B = \{2, 3\}$

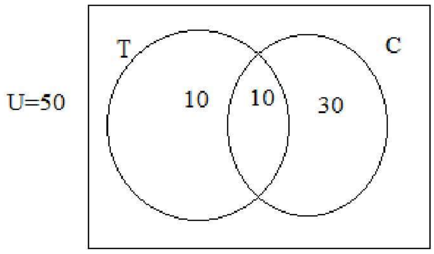
c) $A = \{n : n$ is a prime number $< 20\}$

Solution: $A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

d) $A = \{n:$ n is a positive factor of $25\}$

Solution: $A = \{25, 5, 1\}$

Example 5 Out of 50 people, 20 people drink tea, 10 take both tea, and coffee. How many take atleast one of the two drinks.



n (T∪C) = n (T) + n (C) – n (T∩C)

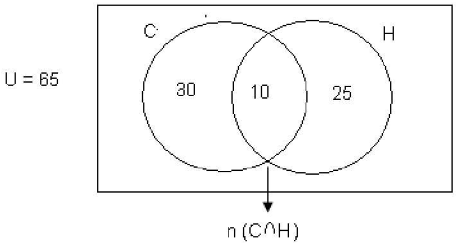
50 = 20 + n (C) - 10

40 = n (C)

Number of people taking atleast one of the two drinks

10 + 10 + 30 = 40 + 10 = 50

Example 6 In a group of 65 people, 40 like Cricket, 10 like hockey and cricket both. How many like Cricket only and not hockey? How many like hockey?



n (C∪H) = n(C) + n (H) – n (C∩H)

65 =40 + n(H) – 10

65 =30 + n(H)

n(H) = 35

Since n (C∩H) = 10 number of people like hockey 35.

Example 7 In a survey of 100 persons it was found that 28 read magazine A,30 read magazine B,42 read magazine C,8 read magazine A and B,10 read magazine A and C,5 read magazines B and C while 3 read all the three magazines.Find:

i) How many read none of the three magazines?

ii) How many read only magazine C?

Solution :

Given (U)=100, n(A)=28, n(B)=30, n(C)=42, $n(A\cap B)=8$, $n(A\cap C)=10$,

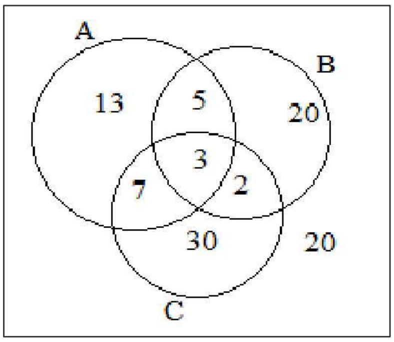
$n(B\cap C)=5$, $n(A\cap B\cap C)=3$

i) Number of people who read none of the magazines

$= 100-(13+5+20+7+3+2+30)$

$= 100-80=20$

ii) Number of people who read only magazine C only =30

U = 

Exampale 8 Write all the possible subsets of A = {a, b, c}

Solution: Ø , {a} {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}

EXERCISE : 2.1

1 MARK QUESTIONS:

1. Which of the following are sets?
   a) {4, 8, 12…..}
   b) {10, 6, 8, a, m, ……}
   c) All good films
   d) All interesting subjects
   e) Students of a particular college.

2. Represent the following sets in both roster form and rule form
   a) Set of even natural numbers less than 30
   b) Set of all multiples of 3
   c) Set of all integers between – 3 and + 3
   d) Set of Prime numbers < 20
   e) Set of all positive factors of 25

3. Convert the following sets from roster form to rule form
   a) A = { 4, 8, 12….}
   b) B = { 6, 12, 18…..}
   c) C = {a, b, c, d}
   d) D = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}
   e) E = { June, July}

4. Convert the following sets from rule to roster form
   a) A = { x : x is a letter in the word 'ENGINEERING'}
   b) B = {n : $n^2 - 7n + 12 = 0$}

5. Write all the possible subsets of the following
   a) A = {1, 2, 3}
   b) B = { a, b}

6. If A = {5, 6, 7}, Find P (A)

7. If A has 4 elements, how many elements will P (A) have?

8. If A = {2, 3, 4} write all the proper subsets of A

ANSWERS : 2.1

1. a) Is a set
   b) Is not a set
   c) Is not a set
   d) Is not a set
   e) Is a set

2. a) Roster form A = {2, 4, 6, 8, 10, 12………… 28}
   Rule method = { x : x = 2n, $\forall$ n < 14} n $\in$ N

b) Roster form A = {3, 6, 9, ……}

Rule method A = $\{x : x = 3n,\ \forall\ n\ \varepsilon\ N\}$

c) Roster form A = { , -2, -1, 0, 1, 2, 0}

Rule method A = $\{x : x, -3 < n < +3\,\forall\ , n\ \varepsilon\ Z\}$

3. .

a) A = $\{x : x = 4n\ \ \forall\ n\ \varepsilon\ N\}$

b) B = $\{x : x = 6n\ \ \forall\ n\ \varepsilon\ N\}$

c) C = {x : x is the first four letters of English alphabet}

d) D = {x : x is a day of the week}

e) E = {x : x is a month of the year starting with 'J}

4. a) A = { E, N, G, I, N, R }

b) B = {3, 4}

5. a) The subsets of A are

{1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3} ε ϕ

b) The subsets of B are

{a}, {b}, {a, b} ε ϕ

6. P (A) = { {5},{6}, {7}, {5, 6}, {6, 7}, {5, 7}, {5, 6, 7}, ϕ}

7. n $\left(P(A)\right) = 2^4 = 16$

8. Proper subsets of A are {2}, {3}, {4}, {2, 3}, {2, 4},{3,4},ϕ

EXERCISE : 2.2

5 MARK QUESTIONS:

1. In a class of 50 students, 15 do not participate in any games, 25 play Cricket and 20 play football. Find the number of students who play both.

2. In a class of 100 students, 35 play football, 45 play basket ball, 35 play indoor games 10 play football and basket ball, 15 play basket ball and indoor games, 5 play football, basketball and indoor games.If 15 do not play any games then find How many play football and indoor games?

3. Out of 250 people, 160 drink coffee, 90 drink tea, 85 drink milk, 45 drink coffee and tea, 35 drink tea and milk, 20 drink all the three. How many will drink Coffee and Milk?

4. In a class of 150 students, It was found that 95 like burgers and 79 like pizzas. Assuming every student like at least one of the above, find the number of students who like both burgers and pizzas. Show the result through venn diagram.

5. Out of 85 students of class I P.U. A who took up a combined test in English and Hindi. If 63 students passed in both, 12 failed in English and 4 failed only in English, use venn diagram to find how many (a) failed in Hindi (b) passed in English, (c) passed in Hindi.

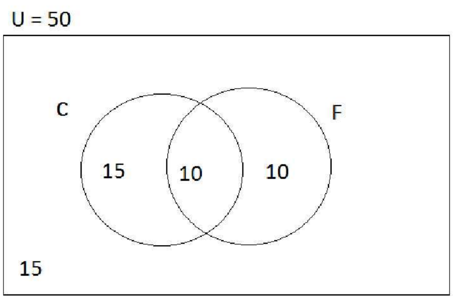
6. In a college $\left(\frac{2}{5}\right)^{th}$ of the students play Basket ball and $\left(\frac{3}{4}\right)^{th}$ play volley ball. If 50 students play none of these two games and 125 play both, use venn diagram to find the number of students in the college.

7. In a certain college with 500 students, 300 take milk and 250 take tea. Find how many take (a) milk only, (b) tea only, (c) both milk and tea.

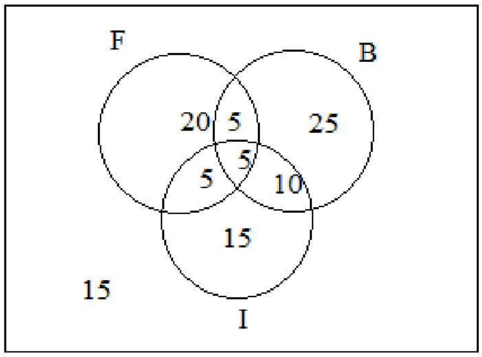
8. In a class of 150 students, each student is required to take at least one of the two subject namely Biology or Economics. If 75 students have taken Biology and 25 have taken both Biology and Economics, how many have taken Economics? Show the result with the help of venn diagram.

ANSWERS : 2.2

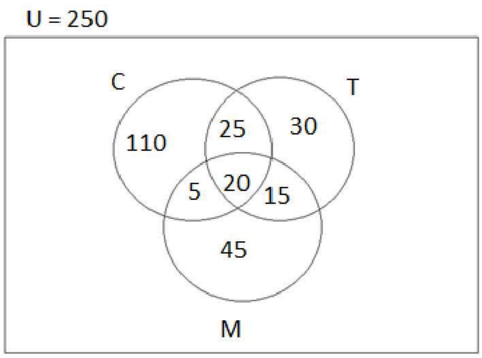
1. The number of students who play both are = 10.



2. 10 play football and indoor games.

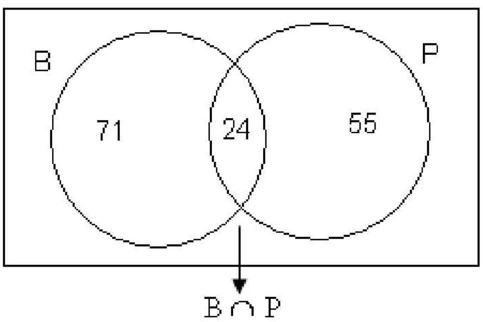


3. 25 Students will drink both coffee and milk.

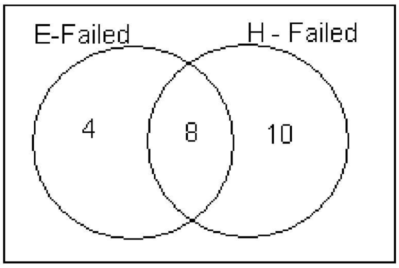


4. 24 like both burgers and pizzas.

U=150



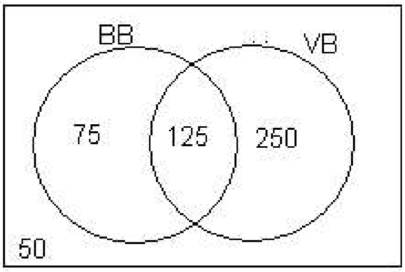
5. U=85



a) 18 failed in Hindi
b) 73 passed in English
c) 67 passed in Hindi

6. Total number of students in the college is 500.

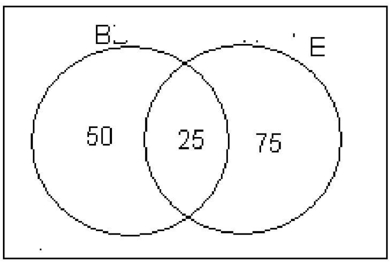
U=500



7 a) 250 take milk only
b) 200 take tea only
c) 50 take both milk and tea.

8.

U=150



100 Students have failed economics.

RELATIONS AND FUNCTIONS

2.13 Ordered Pairs: Let a ε A, b ε B, then the ordered pair of elements a and b is denoted by (a, b)

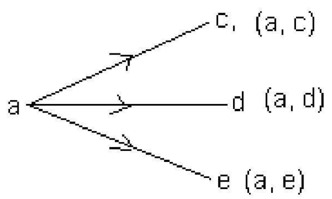
a is the 1^{st} element
b is the 2^{nd} element

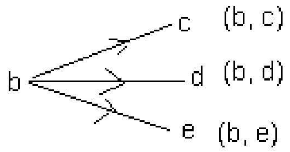
2.14 Equality of ordered pairs:

Two ordered pairs (a, b) and (c, d) are said to be equal of a = c and b = d

2.15 Cartesian Product: Cartesian product of 2 sets A and B is denoted by A B is the set of all the ordered pairs (a, b) $\forall a$ ε A and b ε B.

A × B ={ (a, b), ∀ a ε A and b ε B}
If A = {a, b}, B = {c, d, e}
A × B = { (a, c), (a, d), (a, e), (b, c), (b, d), (b, e)}





Note:

1. If A = Ø or B = Ø then A × B = Ø
2. If A ≠ Ø or B ≠ Ø then A × B ≠ Ø
3. If A × B = Ø iff A = Ø or B = Ø
4. If A = B then A × B = A^2
5. If a ε A, b ε B and c ε C
   Then
   (A × B × C) = { (a, b, c) ∀ a ε A, b ε B, c ε C}

2.16 Worked Examples:

Example 1

$A = \{a, b, c\}$ $\quad$ $B = \{c, d\}$

Find (i) $A \times B$ $\quad$ ii) $B \times A$ $\quad$ iii) $A \times A$ $\quad$ iv) $B \times B$

Solution :

i. $A \times B = \{ (a, c), (a, d), (b, c), (b, d), (c, c), (c, d) \}$

ii. $B \times A = \{ (c, a), (c, b), (c, c), (d, a), (d, b), (d, c) \}$

iii. $A \times A = \{ (a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c) \}$

iv. $B \times B = \{ (c, c), (c, d), (d, c), (d, d) \}$

Example 2

If $A = \{1, 2\}$, $B = \{2, 3\}$, $C = \{3, 4\}$

Find 1) $A \times (B \cup C)$

2) $(A \times B)\ (A \times C)$

3) $A \times (B \cap C)$

4) $(A \times B) \cap (A \cup C)$

Solution :

1) $\{1, 2\} \times (2, 3, 4)$
$\{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4)\}$

2) $\{ (1, 2), (1, 3), (2, 2), (2, 3)$
$U \{ (1, 3), (1, 4), (2, 3), (2, 4) \}$
$= \{ (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4) \}$

3) $\{ 1, 2 \} \times \{ 3\}$
$\{ (1, 3), (2, 3) \}$

4) $\{ (1, 2), (1, 3), (2, 2), (2, 3) \} \cap \{ (1, 3), (1, 4), (2, 3), (2, 4)\}$
$\{ (1, 3), (2, 3) \}$

Example 3 If A = {a, b, c, d} B = {d, e, f, g}

Find 1) $(A - B) \times A$

2) $A \cap B \times B$

Solution :

$\{a, b, c\} \times \{a, b, c, d\}$

$\{ (a, a), (a, b), (a, c), (a, d), (b, a), (b, b), (b, c) (b, d),$
$(c, a), (c, b), (c, c), (c, d) \}$

ii) $\{ d\} \quad \{d, e, f, g\}$

$\{ (d, d), (d, e), (d, f), (d, g) \}$

Example 4 **If**

A = {a, b, c} B = {d} C = { e }

Verify : $A \times (B - C) = (A \times B) - (A \times C)$

Solution:

L H S

$A \times (B - C)$

$= \{ a, b, c\} \times \{ d \}$

$\{ (a, d), (b, d), (c. d) \}]$

RHS

$(A \times B) - (A \times C)$

$\{ (a, d), (b, d), (c, d) \} - \{ (a, e), (b, e), (c, e) \}$

$\{ (a, d), (b, d), (c, d) \}$

LHS = **RHS =** { (a, d), (b, d), (c, d) }

Example 5 **If**

$A = \{ x : x^2 - 7x + 12 = 0\}$

$B = \{2, 4\} \quad C = \{4, 5 \}$

Find $(A - B) \times (B - C)$

Solution :

$A - B = \{3, 4\} - \{2, 4\} = \{ 3 \}$

$B - A = \{2, 4\} - \{4, 5\} = \{ 2 \}$

$(A - B) \times (B - A) = \{ 3 \} \times \{ 2 \}$

$= \{ (3, 2) \}$

Example 6 If A = {c, e, f}, B = {f, g, h}

C = {g, h, i }

Find $(A \cap B) \times (B \cap C)$

Solution :

$A \cap B = \{ f \}$

$B \cap C = \{g, h\}$

$\{ f \} \times \{g, h\}$

$\{(f, g), (f, h)\}$

Example 7

If $A \times B = \{ (a, 1), (a, 2), (a, 3) (b, 1), (b, 2), (b, e) \}$

Find A and B

Solution :

A = {a, b} B = { 1, 2, 3 }

Example 8 If $(2x, x + y) = (8, 4)$

Find x and y

$2x = 8$

$x = 4$

$4 + y = 4$

$y = 0$

Example 9 $A = \{ x : x^2 - 9 = 0, x < 0 \}$

$B = \{x: x \ \varepsilon N < 3\}$

Find 1) $A \times B$

2) $B \times A$

Solution :

$A=\{-3\}$, $B=\{1,2\}$

1) $A \times B$

$= \{ (-3, 1), (-3, 2) \}$

2) $B \times A$

$\{ (1, -3), (2, -3) \}$

Example 10 If $U = \{a, b, c, d \,\ldots\ldots\ldots\, g\}$

$A = \{a, b, c, d\}$

$B = \{b, d, f, g\}$

$C = \{c, d, e, f\}$

Find 1) $(A \cap C)^1$ in U

2) $(B - C)^1$ in U

Solution :

1. $(A \cap C)^1$ in $U = \{c, d\}^1 = \{a, b, e, f, g\}$
2. $(B - C)^1$ in $U = \{b, g\}^1$

$= \{a, c, d, e, f\}$

EXERCISE : 2.3

2 MARKS QUESTIONS:

1. If $A = \{a, b, c, d\}$, $B = \{b, d, f, g\}$

$C = \{c, d, e, f\}$ $U = \{a, b, c, d, e, f, g, h\}$

Find i) $A \times B^1$

ii) $(A \cap B) \times A$

iii) $(A - B) \times B^1$

iv) $(A \cap B)^1 \times B$

2. $A = \{x : x^2 - 5x + 6 = 0, \; x \in N\}$

$B = \{x - 7 + 12 = 0, \; x \in N\}$

$C = \{x : x^2 - 9 = 0, \; x \in Z\}$

Find i) $(A \cap B \cap C) \times A$

ii) $(A - B) \times B$

iii) $B \cap C \times C$

iv) $(A - B) \times (B - A)$

v) $(A \times B) - (A \times C)$

3. If $A=\{3, 5, 7\}$
$B=\{5, 7, 9\}$
$C=\{7, 9, 11\}$

Find i) $(A \cap B \cap C) \times C$
ii) $(A \cap B) \times (B \cap C)$
iii) $(A - B) \times (B - C)$
iv) $(A \cap B) \times (B - A)$

4. If $A \times B = \{ (-1, a), (-1, b), (-2, a), (-2, b), (3, a), (3, b) \}$
Find A and B

5. If $(2x + 4, 3x + y) = (8, 0)$
Find x and y

6. If $A = \{1, 3, 5\}$ $B = \{ 5\}$ $C = \{ 7 \}$
Verify $A \times (B - C) = (A \times B) - (A \times C)$

7. If $A = \{ 1, 4, 7\}$ $B = (c, d\}$ S.T. $A \times B \neq B \times A$

8. If $A = (2, 4, 6)$ $B = \{3, 5\}$
Find i) $A \times B$ ii) $A \times A$ iii) $B \times B$ iv) $B \times A$

9. If $(x+ y, x - y) = (5, 1)$ Find x and y

10. If $A = \{4, 6\}$ $B = \{6, 8, 10\}$ $C = (8, 10, 12\}$
Then Verify: $A \times (B - C) = (A \times B) - (A \times C)$

ANSWERS OF : 2.3

Solution 1:

i) $\{a, b, c, d\} \times \{a, c, e, h\}$
$\{ (a, a), (a, c), (a, e), (a, h), (b, a), (b, c), (b, e), (b, h),$
$(c, a), (c, c), (c. e), (c, h), (d, a), (d, c), (d, e), (d, h) \}$

ii) $\{b, d \} \times \{a, b, c, d\}$
$\{ (b, a), (b,b), (b, c), (b, d), (d, a), (d, b), (d, c), (d, d) \}$

iii) $\{a, c\} \times \{a, c, e, h\}$
$\{(a, a,), (a, c), (a, e), (a, h), (c, a), (c, c), (c, e), (c, h) \}$

iv) $\{e, h\} \times \{b, d, f, g\}$
$\{ (e, b), (e, d), (e, f), (e, g), (h, b), (h, d), (h, f), (h, g) \}$

Solution 2 :

i) $\{ (3, 2), (3, 3) \}$

ii) $\{ (2, 3), (2, 4) \}$

iii) $\{ (3, 3), (3, -3) \}$

iv) $\{ (2, 4) \}$

v) $\{ (2, 3), (2, 4), (3, 3), (3, 4) \}$
$- \{ (2, 3), (3, 3), (2, -3), (3, -3)\}$
$= \{(2, 4), (3, 4) \}$

Solution 3 :

i) $\{ (7, 7), (7, 9), (7, 11)$
ii) $\{ (5, 7), (5, 9), (7, 7), (7, 9) \}$
iii) $\{ (3, 5) \}$
iv) $\{ (5, 9), (7, 9) \}$

Solution 4 : $A = \{ -1, -2, 3\}$ $B = \{a, b\}$

Solution 5 : $x = 2$, $y = -6$

Solution 6 : LHS = RHS = $\{ (1, 5), (3, 5), (5, 5) \}$

Solution 7 :

$A \times B = \{ (1, c), (1, d), (4, c), (4, d), (7, c), (7, d)$
$B \times A = \{ (c, 1), (c, 4), (c, 7), (d, 1), (d, 4), (d, 7) \}$
$A \times B \neq B \times A$

Solution 8 :

i) $A \times B = \{ (2, 3), (2, 5), (4, 3), (4, 5), (6, 3), (6, 5)$
ii) $A \times A = \{ (2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}$
iii) $B \times B = \{ (3, 3), (3, 5), (5, 3), (5, 5) \}$
iv) $B \times A = \{ (3, 2), (3, 4), (3, 6), (5, 2), (5, 4), (5, 6) \}$

Solution 9 : $x = 3$

$y = 2$

Solution 10 :

LHS = RHS = { (4, 6), (6, 6) }

2.17 Relation : Let A and B be two empty sets, A relation R from A to B is a subset of $A \times B$ i.e. R is a relation from A to B

if $R \subseteq A \times B$

1. A relation 'R' from A to A is a subset of $A \times A$
2. If set A has m elements, set B has n elements then A B has mn elements.
3. $A \times B$ has 2^{mn} subsets
4. If there exists 2^{mn} relations from A to B
   Then there are 2^{mn} relations from B to A
5. If R is a relation from A to B and (x, y) εR then this is denoted by x Ry

2.18 Domain and Range of a relation:

Let A and B be 2 non-empty sets and R be a relation from A to B

i.e. $R \subseteq A \times B$. The domain of R is defined as the collection of all the first elements of the ordered pairs (a, b) εR.

i.e. Domain of R = {a ε A :(a, b) ε R}

The Range of R is defined as the set of all the second elements of the ordered pairs (a, b) ε R

i.e. Range of R = {b ε R : (a, b) ε R }

Example: Let A = {b, c, d} B = {c, d, e}

If R is a relation from A to B defined by is 'next letter in Eng Alphabet'

Then R^{-1} = { (b, c), (c, d), (d, e) }

2.19 Inverse relation: Let R be a relation from A to B. The inverse relation of R is denoted by R^{-1} and is a relation from B to A

i.e. sub set of $(B \times A)$ defined as follows

$R^{-1} = \{ (y, x) \ \forall \ (x, \ y) \ \varepsilon \ R \}$
Let $A = \{1, 2\}$ $\quad B = \{a, b\}$
If R is a relation from A to B defined by
$R = \{ (1, a), \ (1, b), (2, a)\}$
Then $R^{-1} = \{ (a, 1), (b, 1), (a, 2) \}$

2.20 Types of relations:

1) **Identity relation**.

Let A be a non empty set the relation I_A defined by $I_A = \{ (a, a) : a \ \varepsilon \ A\}$ is called the identity relation on A.

2) **Null relation (void relation)**

Let A be a non-empty set. We know that $\emptyset \subset A \times A$ and hence $\emptyset$ is a relation on A. This relation is called as the null relation on A.

3) **Universal Relation:** Let A be a non empty set . we know that

$(A \times A) \subseteq (A \times A)$ and hence AA is a relation on A. This relation is called as Universal Relation on A.

4) **Reflexive Relation:** A relation R on a non empty set A is called a reflexive relation if $(a, a) \ \varepsilon \ R \ \forall \ a \ \varepsilon \ R$

If $A = \{ a, b, c \}$

$R_1 = \{(a, a), \ (b, b)\}$

$R_2 = \{ (a, a), (b, b), \ (c. c) \}$ are reflexive relations on A

Note : The identity relation and universal relation on a non empty set are reflexive relations.

5) **Symmetric Relation:** A relation R on a non empty set A is called a symmetric relation, if $(a, b) \ \varepsilon \ R \Rightarrow (b, a) \ \varepsilon \ R$

Example: $A = \{1, 2, 3, 4, 5 \}$
$R_1 = \{ (2, 3), (3, 2), \ (3, 4), (4, 3) \}$
And $R_2 = \{(1, 5), (5, 1), (2, 5), (5, 2) \}$
are symmetric relations on A.

Note : The universal relation on a non empty set is a symmetric relation.

6) **Transitive Relation:** A relation R on a non empty set A is called as a Transitive Relation if (a, b), (b, c) ε R ⇒ (a, c) ε R

Example: Let R be a relation on the set of naturals defined by 'is a factor of'

xRy ⇒ x is a factor of y

yRz ⇒ y is a factor of z

⇒ xRz ⇒ x is a factor of z

7) **Equivalence Relation:** A relation R on a non empty set A is called an equivalence relation if it is reflexive, symmetric and transitive.

On set 'L' of straight lines in a plane

i) **Reflexive Relation:**

line 'l_1' is parallel to itself.

∴ R is reflexive.

ii) Symmetric relation:

l_1 is parallel to line l_2

then l_2 is parallel to line l_1

(l_1, l_2) ε R ⇒ (l_2, l_1) ε R

∴ R is symmetric relation.

iii) **Transitive Relation:**

If a line l_1 is parallel to l_2 and a line l_2 is parallel to l_3, then we know l_1 is parallel to l_3 . If (l_1, l_2), (l_2, l_3) ε R ⇒ (l_1, l_3) ε R

∴ R is transitive.

Since 'R' is reflexive, symmetric and transitive,

∴ it is an equivalence relation.

8) **Anti Symmetric Relation:** A relation R on a non empty set A is called as anti symmetric relation if (a, b), (b, a) ε R

⇒ a = b

Example: Consider the relation R defined by 'is less than or equal to on the set of integers,

If x, y ε R such that $x \leq$ y and y $\leq x$ then x = y.

∴ R is an anti symmetric relation.

2.21 Worked Examples:

Example 1

If A = { 1, 2, 3, 4, 5} B = {1, 2, 3, 4} and
R is a relation from A to B defined by
R = { (x, y) : $y = 2x + 1$ }, Find R .

Solution 1 : R = { (1, 3) }

Example 2 R is a relation on the set of natural numbers N standing for x is related to y

if $x = 3y$, Find R

Solution 2 :

R = { (3, 1), (6, 2) (9, 3) .... }

Example 3

Given A = {1, 2, 4} B = {2, 3, 5} and R_1 and R_2 are relations from A to B by 'is less than'.Find R

Solution 3:

R = { (1, 2), (1, 3), (1, 5), (2, 3), (2, 5), (4, 5) }

Example 4

Give an example of a relation which is

a) reflexive, but not symmetric and not transitive
b) symmetric but not reflexive and not transitive
c) Transitive but not reflexive and not symmetric

Solution 4:

Let A = {1, 2, 3}

a) R = { (1, 1), (2, 2), (3, 3), (1, 2), (3, 1) }
R is reflexive but not symmetric and not transitive

b) R_1 = { (1, 1), (3, 3), (2, 1), (1, 2) }
R_1 is symmetric but not reflexive and not transitive.

c) R_2 = { (1, 1), (2, 2), (1, 2) }
R_2 is transitive but not reflexive and not symmetric

Example 5

A relation R is defined on the set of integers by

R = { (x, y) : x - y is a multiple of a non zero integer 5 } show that R is an equivalence relation on Z.

Solution 5 :

1) **Reflexive Relation**

x R $x \Rightarrow$ x - x is a multiple if 5. $\forall$ x ε Z

$\therefore$ R is reflexive.

ii) **Symmetric relation:**

x R y $\Rightarrow x$ - y is a multiple of 5

$\Rightarrow$ 5 | x - y [5 divides x-y]

$\Rightarrow$ 5 | - (y - x)

$\Rightarrow$ 5 | y - x

$\Rightarrow$ (y, x) εR

$\therefore$ R is Symmetric

iii) **Transitive relation:**

(x, y) εR, (y, z) εR

5 | x - y and 5 | y – x

$\Rightarrow$ 5 | (x - y) + (y – z)

$\Rightarrow$ 5 | x - z

$\Rightarrow$ (x, z) εR

$\therefore$ R is transitive.

Since R is reflexive, symmetric and transitive then R is an equivalence relation.

Example 6

Show that the relation 'is congruent to' is an equivalence relation on a set T of triangles.

Solution 6:

Reflexive relation:

x R x $\Rightarrow$ A triangle T_1 is congruent to itself

$\therefore$ R is reflextive

Symmetric relation: If a triangle T_1 is congruent to triangle T_2 then T_2 is congruent to T_1

$\therefore$ R is symmetric

Transitive relation: If a triangle T_1 is congruent to T_2 and T_2 is congruent to T_3 then T_1 is congruent to T_3.

$\therefore$ R is transitive.

Since R is reflexive, symmetric and transitive

$\therefore$ it is an equivalence relation.

EXERCISE : 2.4

1 MARK QUESTIONS:

1. Given S = {1, 2, 3, 4} state which of the following is a relation on S.
   a) $R_1 = \{(1, 2), (2, 0), (3, 1)\}$
   b) $R_2 = \{(1, 3), (4, 2), (2, 4), (1, 4)\}$
   c) $R_3 = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$
   d) $R_4 = \{(1, 2), (0, 2), (5, 1), (3, 2)\}$

2. If A = { 1, 2, 3, 4, 5} Find the following relations from A to B
   a) $R_1 = \{(x, y) : x > y\}$
   b) $R_2 = \{(x, y) : x$ divides y)
   c) $R_3 = \{(x, y) : y = x + 3\}$
   d) $R_4 = \{(x, y) : x = y\}$

3. A relation R on a collection of set of integers defined by R = { $(x, y) : x - y$ is a multiple of 3}. Show that R is an equivalence relation on Z.

2 MARKS QUESTIONS:

Q.4 If A = {2, 3}, B = {3, 4} Find the number of relations that can be defined from A to B.

Q.5 Find the domain and range of the following relations.
1) R = { (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7) }
2) R = { $(x, y) : y = x^3$, x is a positive prime number less than 10}

Q.6 If R^{-1} = { (2, 4), (1, 2), (3, 1), (3, 2)} Find R

3 MARKS QUESTIONS:

Q.7 List all the relations on the set
A = {a, b }

Q.8 Define equivalence relation. Give one example.

Q.9 If A = { 4, 5, 6, 7}
R_1 = { (4, 4), (5, 6), (6, 7), (7, 7) }
Represent R_1 by diagram.

ANSWERS OF : 2.4

1. a) R_1 is not a relation on S
   b) R_2 is a relation on S
   c) R_3 is a relation on S
   d) R_4 is not a relation on S

2. a) R_1 = { (2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4) }
   b) R_2 = { (1, 1), (2, 2) (3, 3) (4, 4), (5, 5) (1, 2), (1, 3), (1, 4), (1, 5), (2, 4) }
   c) R_3 = { (1, 4), (2, 5) }
   d) R_4 = { (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) }

3. i) $x - x$ is a multiple of 3 $\Rightarrow x$ R x $\therefore$ R is reflexive.

   ii) x - y is a multiple of 3 $\Rightarrow x$ R y
   $\Rightarrow$ y - x is a multiple of 3 $\Rightarrow$ yR x
   $\therefore$ R is symmetric

   iii) 3 | x - y and 3| y – z $\Rightarrow x$ R y and y R z.
   $\Rightarrow$ 3 | (x - y) + (y – z)
   $\Rightarrow$ 3 | (x - z)
   $\Rightarrow x$ R Z
   $\therefore$ R is transitive

Since R is reflexive, Symmetric and transitive $\therefore$ it is an equavalence relation.

4. 2^4 = 16 Relations

5. 1) Domain = { 1 } and Range = {2, 3, 4, 5, 6, 7}
   2) Relation R = { (2, 8) (3, 27), (5, 125), (7, 343) }
   Domain = {2, 3, 5, 7} and Range = {8, 27, 125, 343}

6. R = { (4, 2), (2, 1), (1, 3) (2, 3) }

7. A $\times$ A= { (a, a), (a, b), (b, a), (b, b) }

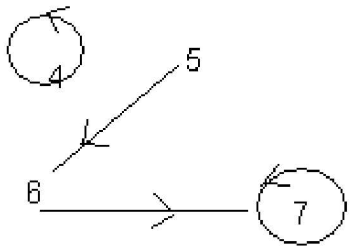
   Hence all relation on set A are

   { (a, a)}, {(a, b) }, {(b, a)}, {(b, b)}
   {(a, a), (a, b)}, {(a, a), (b, b)}, {(a, b), (b, a) }
   {(a, a), (a, b), (b, a)}, { (a, a), (a, b), (b, b) }
   { (a, b), (b, a), (b, b)}, {(a, a), (b, b), (b, a) }
   { (a, a), (a, b), (b, a), (b, b)}, {(a, a), (b, a) }
   { (a, b), (b, b) } {(b, a), (b, b) }

8. If a relation is reflexive, symmetric and transitive then it is an equivalence relation.

   For example in set of triangles 'is similar to' is an equivalence relation.

9.



2.22 Functions:

Set X and Y be two non-empty sets

A subset f of X×Y is called as a function iff the following conditions hold good.

i) For each x ε X , there exists a unique y εY such that (x, y) εf
ii) Elements of x should not be repeated.

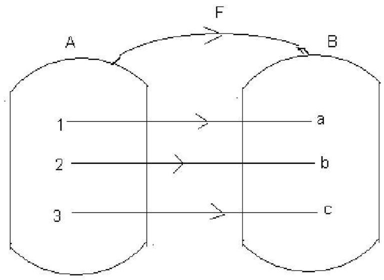
Note:

1) If (x_1, y_1) εf and (x, y_2) εf then $y_1 = y_2$
2) y εY is called as the image of the element x ε X
3) x is called as the pre-image of y ε Y.
4) The set X is called as the Domain of the function f.
5) The set Y is called as the Range of the function f.

Example 1: Consider A = {1, 2, 3} B = {a, b, c}

Let F = { (1, a), (2, b), (3, c)}

This can be shown in the diagram.



Here

F (1) = a

F (2) = b

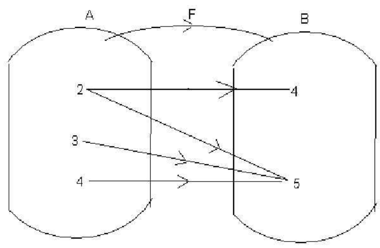
F (3) = c

In all the representations, every element of A is associated with a unique element of B and there is no element of set B which is not mapped on to any element of A.

Example 2: A = {2, 3, 4} B = {4, 5}

Solution : Let F ={ (2, 4), (2, 5), (3, 5), (4, 5) }

Since two ordered pairs in F have the same first component. ∴ is not a functions.



2.23 Domain, Co-domain and Range of a function:

Let F : A → B i.e. F is a function or mapping from A to B. The set A is called as domain of F and the set B is called as Co-domain of F. The set consisting of all images is called as the range of F which in symbols is written on F(A)

F = { (1, a), (2, b), (3, c), (4, d)}
Domain of F = A= {1, 2, 3, 4}
Range of F =F(A)= {a, b, c, d}
Co-domain of F = B= {a, b, c, d)

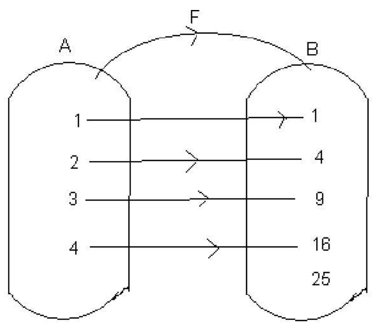
Note: 1) F (A) $\subseteq$ B

2) Every function is a relation but the converse is not true.

2.24 Different types of Functions:

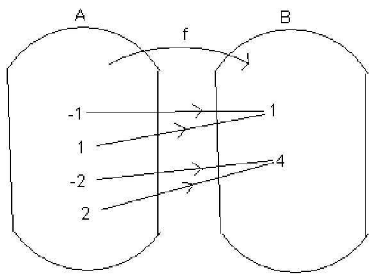
1. Into functions

The mapping f : A $\rightarrow$ B is called as 'Into' if there is at least one element of set B which is not an image. (or which has no pre-image in the set A) symbolically we write f : A $\rightarrow$ B.



2. Many – one function:

The function f : A $\rightarrow$ B is called as many one function if different elements of set A have the same image in B.

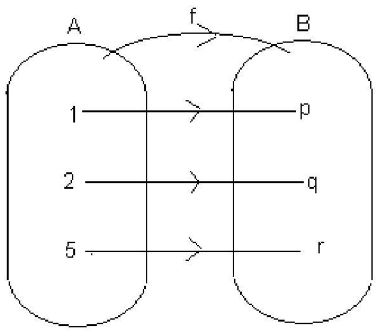


3. **One – One function (injective mapping)**

A function f : A → B is said to be a one-one function if different elements of sets A have different images in set B.

Symbolically we can write if

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
$\forall\ x_1, x_2\ \varepsilon\ A$

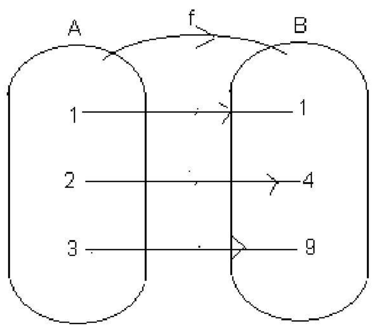


f (1) = p
f (3) = q
f (5) = r

4. **Onto function (surjective mapping)**

The mapping f : A → B is called as on to if every element of set B is the image of some element of set A.

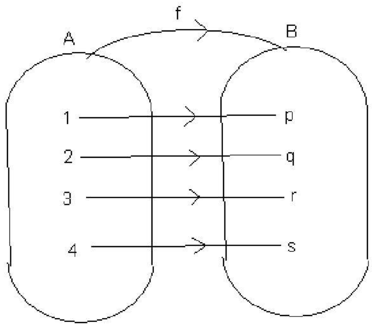
Example : $f(x) = x^2 \quad \forall\ x\ \varepsilon N$ and $x \leq 3$



f (1) = 1
f (2) = 4
f (3) = 9

5. **Bijection Or one – one and onto function**

A function f : A → B is said to be bijective of f is both one – one and onto function.



One – one and onto

f (1) = p   f (3) = r

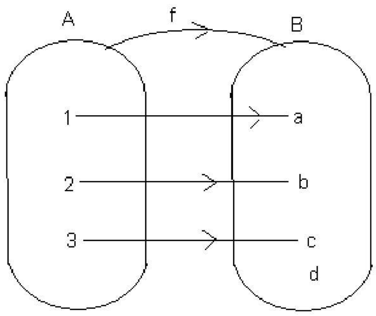
f (2) = q   f (4) = s

6. **one-one into mapping:**

A function f = A → B is said to be one – one into function if the Range of function is a subset of B

(i) $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

i.e.there must be atleast one element of set B which is not the image of some element of set A.



f (1) = a

f (2) = b

f (3) = c

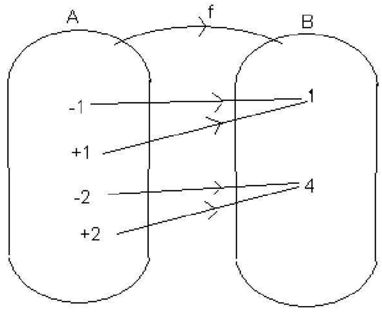
Element dεB is not the image of any element of set A.

7. **Many – one onto mapping:**

A mapping f : A $\rightarrow$ B is said to be many-one onto iff i) Range of function is equal to B

(ii) $f(x_1) = f(x_2) \Rightarrow x_1 \neq x_2$.

i.e. 2 or more elements of set A have the same image in set B.



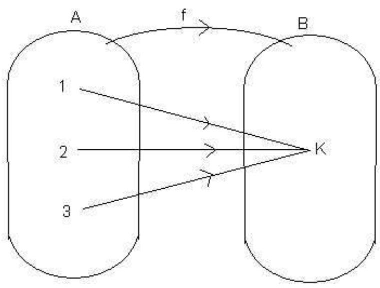
f (-1) = 1
f (1) = 1
f (-2) = 4
f (2) = 4

2 distinct elements -1 and 1 of set A are mapped on to same element 1 of set B

2 distinct element -2 and +2 of set A are mapped onto the same element 4 of set B.

8. **Constant function:**

If f : A $\rightarrow$ B and $f(x) = k \ \forall \ x \ \varepsilon$ A and k εB i.e. if each and every element of a εA is mapped on to a single element k of the co-domain B then f is called as a constant function.

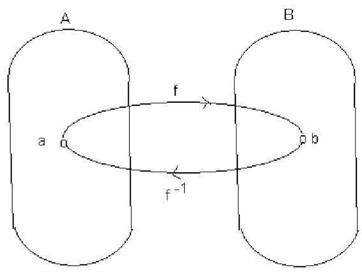


$f(1) = k$
$f(2) = k$
$f(3) = k$

9. **Inverse function :**

Let $f : A \rightarrow B$ be a one-one, onto function from A to B. Then for each $b \in B$.

$f^{-1}(b) \in A$ exists and is unique so that $f^{-1} : B \rightarrow A$ is a function defined by $f^{-1}(b) = a$ iff $f(a) = b$

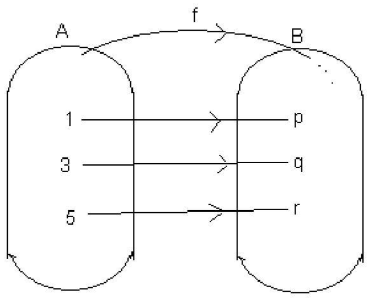


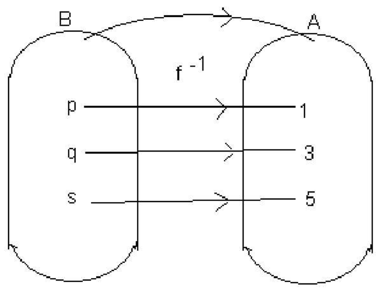
NOTE:

If f is bijective function
Then f^{-1} is also a bijective function

Example 3

Solution :





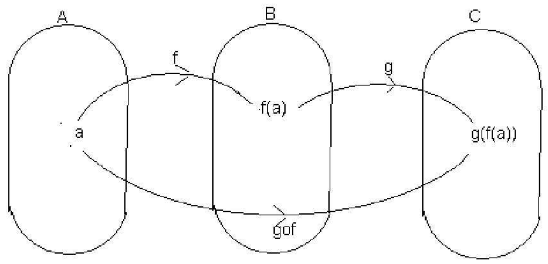
$f : A \rightarrow B$
$f(1) = p$ $\qquad f(3) = q$ $\qquad f(5) = r$

$f^{-1} : B \rightarrow A$
$f^{-1}(p) = 1$ $\qquad f^{-1}(q) = 3$ $\qquad f^{1}(r) = 5$

Both f and f^{-1} are bijective functions.

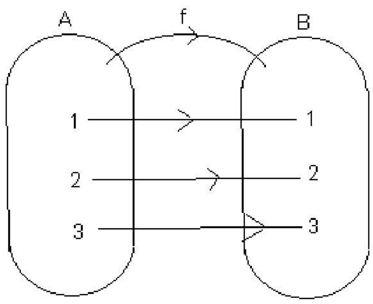
10. **Composite functions:**

Let A, B and C be any three non-empty lets let f : A → B and g : B → C be any two functions. Define a function gof : A → C as (gof) a = g(f(a)) ∀ a εA since f(a) ε B g(f(a)) εC. Thus (gof) so obtained is called as composition of f and g. Similarly we can define the composite function. (fog) x = f (g(x)



11. **Identity function:**

A mapping f : A → A is said to be an identity function if f(x) = x ∀ x εA. It is symbolically represented by I_A i.e I_A : A → A



2.25 WORKED EXAMPLES:

Example 1 A function f(x) is defined as f(x) = 3 + 5. Find the values of

i) f (- 1) ii) f (2) iii) f (3) iv) f (5) v) f(-2)

Solution 1:

i) f (-1) = 3 (- 1) + 5 = 2
ii) f (2) = 3 (2) + 5 = 11
iii) f (3) = 3 (3) + 5 = 14
iv) f (5) = 3 (5) + 5 = 20
v) f(- 2) = 3 (-2) + 5 = -1

Example 2 Examine which of the following relations are functions?

i) R = { (2, 1),) (1, 3), (3, 4) }

ii) R ={(1, 2), (1,3), (1, 4) }

iii) R = {(1, 2), (2, 3), (3, 4), (5, 6)

Solution 2:

i) is a function

ii) is not a function

iii) is a function

Example 3 Let N be the set of natural numbers such that

R = {(x, y) : y = 3x+ 4, x, y ε N} Write the domain and range of the function.

Solution 3:

Relation R = {(1, 7), (2, 10), (3, 13) ....}

Domain = {1, 2, 3 ....}

R = {7, 10, 13 ....}

Example 4

$$T(F) = \frac{5}{9}\ (F-32)$$

The function to which maps temperature in Fahrenheit into temperature in degree Celsius is defined as above.

Find i) T(32), ii) T(-40) iii) T (-49)

Solution 4 :

i) T(32) $= \dfrac{5(0)}{9} = 0$

ii) T (-40) $= \dfrac{5(-40-32)}{9}$

$= \dfrac{5}{9}\times -72$

$=$ - 40

iii) T (-49) $= \dfrac{5(-49-32)}{9}$

$= \dfrac{5(-81)}{9}$

$= -9 \times 5$

$= -45.$

Example 5 Find the range of the following functions:

i) $f(x) = x^2 + 5$, $x > 0$

ii) $f(x) = 2x + 3$, $x > 0$

Solution 5 :

i) f (1) = 6
f (2) = 9
f (3) = 14
f (4) = 21

Range of f = {6, 9, 14, 21, ……..}

ii) f (1) = 5
f (2) = 7
f (3) = 9
f (4) = 11

Range of f = {5, 7, 9, 11 ……}

Example 6 If $f(x) = x^2$

Find the value of $\dfrac{f(2)-f(1)}{(2-1)}$

Solution 6 :

f (2) = 4
f (1) = 1

$f(2) - f(1) = \dfrac{4-1}{1} = 3$

Example 7 Find the domain and Range of the function.

$$f(x) = \frac{x^2+2x+1}{x^2-8x-12}, \; x \in R.$$

Solution 7 :

$$f(1) = \frac{4}{-19} = -\frac{4}{19}$$

$$f(3) = \frac{9+6+1}{9-24+12}$$

$$= \frac{16}{-3}$$

$$f(5) = \frac{5^2+10+1}{5^2-8(5)+12}$$

$$= \frac{36}{-3} = -12$$

Domain = {1, 3, 5 ……….. }

Range = {-4/19 , -16/3, -12 ……… }

Example 8 Let f = {(1, 2), (2, 3), (3, 4) } be a function from Z to Z where Z is the set of integers defined by f(x) = ax + b $\forall$ some integers a and b. Determine a & b.

Solution 8 :

2 = a + b (i)
3 = 2a + b (ii)
-1 = - a from (i) and (ii)
$\therefore$ a = 1
$\therefore$ b = 1

Example 9: f(x) = x + 1 and g(x) = ^{2} + 1

Find i) fog (1)ii) fog(2) iii) gof (1), iv) gof(2) v) fog(3)

Solution 9 :

i) fog (1) $= f(g(1))$

$= f(2)$

$= 2 + 1 = 3$

ii) fog(2) $= f(g(2))$

$= f(5)$

$= 5 + 1$

$= 6$

iii) gof (1)

$= g(f(1))$

$= g(2)$

$= 4 + 1 = 5$

iv) gof (2)

$= g(f(2))$

$= g(3)$

$= 9 + 1$

$= 10$

v) fog (3)

$f(g(3))$

$f(10) = 10 + 1$

$= 11$

Example 10 If $f(x) = x - 1$ and $g(x) = 2x^2 - 3$

Find i) fog(1) ii) gof(2)

i) fog (1)

$= f(g(1))$

$= f(-1)$

$= -2$

ii) gof (2)

$= g(f(2))$

$= g(1)$

$= 2 - 3$

$= -1$

EXERCISE : 2.5

1 MARK QUESTIONS.

Q.1 A function f(x) is defined as f(x) = 2 + 1. Find the value of

i) f(1) ii) f(2) iii) f(-1) iv) f(-2) v) f(-3)

Q.2 Examine which of the following relations are functions?

i) R = {(1, 3), (1, 5), (1, 8), (2, 5) }

ii) R = { (1, a), (2, b) (3, c) }

iii) R = { (2, 1), (3, 2), (4, 1) }

Q.3 Let N be the set of natural numbers such that

R = {(x, y) : y = 2, , y ε N} Find the domain and range of the function?

Q.4 Find the range of the function:

i) $f(x) = x^2 + 2x + 1$, $x > 0$

ii) $f(x) = 3x + 5$, $x > 0$

2 MARK QUESTIONS.

Q.5 If $(x) = x^3$ Find the value of $\frac{f(3)-f(2)}{3-2}$

Q.6 Find the domain and range of the function

$F(x) = \frac{x^2-2x+1}{x^2-9x+13}$ where x ε N

Q.7 Let f = {(1, 1), (2, 3), (0, -1) } be a function from Zto Z defined by

$f(x) = ax + b$ ∀ some integers a and b. Determine a & b.

Q.8 If f(x) = x and g(x) = $x^3 + 1$

Find i) fog (1) ii) fog (2) iii) gof (1) v) gof (-1)

Q.9 If f(x) = $2x + 1$ g(x)= $x^2 + 2x + 1$

Find i) fog (2) ii) gof (3)

Q.10 If f(x) = x^2 and g(x) = $x + 1$

Find i) fog (x) ii) gof (x)

ANSWERS : 2.5

1. i) $f(1) = 3$ ii) $f(2) = 5$, iii) $f(-1) = -1$
   iv) $f(-2) = -4 + 1 = -3$ v) $f(-3) = -5$

2. i) is not a function
   ii) is a function
   iii) is a function

3. D = $\{1, 2, 3 \ldots\ldots\ldots\}$
   F = $\{2, 4, 6, 8 \ldots\ldots\}$

4. i) R = $\{4, 9, 16, \ldots\ldots\}$
   ii) R = $\{8, 11, 14 \ldots\ldots\}$

5. 19

6. D = $\{1, 2, 3, \ldots\ldots\ldots\}$

   R = $\{0, -1, \frac{-4}{5} \ldots\ldots\}$

7. $a = 2$, $b = -1$

8. i) 2 ii) 9 iii) 2 iv) 0

9. i) 19 ii) 64

10. i) $(x+1)^2$ ii) $x^2 + 1$

CHAPTER 3

THEORY OF INDICES

3.1 Introduction:

The basics of indices is important for many calculations. In this chapter we have discussed the laws of indices and problems based on those laws. This is further used to derive logarithms.

3.2 Meaning of a^n :

If a is any real number and n is a non-zero positive integer, then

$a^n = a \times a \times a$ - - - n factors

Here 'a' is called the base

n is called the index, power or exponent.

3.3. Laws of Indices:

FIRST LAW: If 'a' is a any non zero real number, m and n are two positive integers, then

$a^m . a^n = a^{m+n}$

Proof: $a^m . a^n = (a \ a \times a \times \ldots\ldots \text{ m times })(a\times a\times a\times$ - - - - n times)

$= (a \times a \times a \times \ldots\ldots (m + n) \text{ times.}$

$= a^{m+n}$

Similarly , $a^m. a^n. a^p. a^k \ldots = a^{m+n+p+k\ldots}$

SECOND LAW: If a is any non zero real number m and n are any two positive integers, then

$$\frac{a^m}{a^n} = \begin{cases} a^{m-n} \ if \ m > n \\ a^{n-m} \ if \ m < n \end{cases}$$

Proof:

Case 1) If $m > n$, $\frac{a^m}{a^n} = a^{m-n}$

$$\frac{a^m}{a^n} = \frac{a \times a \times a \times ....m \text{ times}}{a \times a \times a \times .....n \text{ times}}$$

$= a \times a \times a \times .....$ (m –n) times

$= a^{m-n}$

Case 2) If $m < n$ $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$

$$\frac{a^m}{a^n} = \frac{a \times a \times ....m \text{ times}}{a \times a \times .....n \text{ times}}$$

$$= \frac{1}{a^{n-m}}$$

THIRD LAW: If a is any non zero real number, m and n are two positive integers, then

$(a^m)^n = a^{mn}$

Proof : $(a^m)^n = a^m. a^m. a^m.$ ——— n times

= (a. a. a …. m times) (a. a. a. ….. n times) ——— n times

= a. a. a ——— mn times

$= a^{mn}$

Note :

1. If a is any non zero integer, then $a^0 = 1$

   Ex. $2^0 = 1$, $\left(\frac{1}{3}\right)^0 = 1$

2. If a is any non zero integer and m is a positive integer, then $a^{-m} = \frac{1}{a^m}$

   Ex. $3^{-2} = \frac{1}{3^2}$, $a^{-1} = \frac{1}{a}$

3. If a is any non zero integer, p & q are positive integer (q ≠ o), then

$$\sqrt[q]{a^p} = a^{p/q}$$

Ex. $\sqrt[5]{3^2} = 3^{2/5}$, $\sqrt[3]{\left(\frac{2}{3}\right)^2} = \left(\frac{2}{3}\right)^{2/3}$

WORKED EXAMPLES

Example 1 Simplify : $\left(\frac{9}{4}\right)^{-3/2}$

Solution: $\left(\frac{9}{4}\right)^{-3/2} = \left(\frac{4}{9}\right)^{3/2} = \left(\frac{2^2}{3^2}\right)^{3/2} = \frac{2^{2\times3/2}}{3^{2\times3/2}} = \frac{2^3}{3^3} = \frac{8}{27}$

Example 2 Simplify : $\frac{a^{m+n}.a^{2m-n}}{a^{m-n}}$

Solution: $= a^{m+n+2m-n-m+n} = a^{2m+n}$

Example 3 Simplify : $\frac{3^{n+1}+3^n}{3^n-3^{n-1}}$

Solution: $\frac{3^{n+1}+3^n}{3^n-3^{n-1}} = \frac{3^n.3^1+3^n}{3^n-3^n3^{-1}} = \frac{3^n(3+1)}{3^n(1-\frac{1}{3})} = 4 \times \frac{3}{2} = 6$

Example 4 Simplify: $\frac{2^{(2^0)}+(2^0)^3}{(2^3)^2+(2^2)^0}$

Solution : $\frac{2^{(2^0)}+(2^0)^3}{(2^3)^2+(2^2)^0} = \frac{2^1+1^3}{2^6+1} = \frac{2+1}{64+1} = \frac{3}{65}$

Example 5 Prove that : $\dfrac{1}{1+x^{p-q}}+\dfrac{1}{1+x^{q-p}}=1$

Solution : LHS $= \dfrac{1}{1+\dfrac{x^p}{x^q}} + \dfrac{1}{1+\dfrac{x^q}{x^p}}$

$= \dfrac{x^q}{x^q+x^p}+\dfrac{x^p}{x^p+x^q}$

$= \dfrac{x^p+x^q}{x^p+x^q}$

$=$ 1 = RHS

Example 6 Prove that

Solution : LHS $= (x^{b-c})^a \,.\, (x^{c-a})^b .\, (x^{a-b})^c = 1$

$= x^{ab-ac} .\, x^{bc-ab} \,.\, x^{ac-bc}$

$= x^{ab-ac+bc-ab+ac-bc} \quad = x^0 = 1 =$ RHS

Example 7 Prove that : $\left(\dfrac{x^{a+b}}{x^{b-c}}\right)^{a-c} . \left(\dfrac{x^{b+c}}{x^{c-a}}\right)^{b-a} . \left(\dfrac{x^{c+a}}{x^{a-b}}\right)^{c-b} = 1$

Solution : LHS $= (x^{a+b-b+c})^{a-c} \; (x^{b+c-c+a})^{b-a} \; (x^{c+a-a+b})^{c-b}$

$= (x^{a+c})^{a-c} \,.\, (x^{b+a})^{b-a} \; (x^{c+b})^{c-b}$

$= x^{a^2-c^2} .\; x^{b^2-a^2} .\, x^{c^2-b^2}$

$= x^{a^2-c^2+b^2-a^2+c^2-b^2}$

$= x^0 = 1 =$ RHS

Example 8 If $a^x = b$, $b^y = c$, $c^z = a$. Show that $xyz = 1$

Solution : Given $a^x = b$

$(a^x)^{yz} = b^{yz}$

$\Rightarrow$ $a^{xyz} = b^{yz}$

$\Rightarrow$ $a^{xyz} = c^z$ $(\because b^y = c)$

$\Rightarrow$ $a^{xyz} = a^1$ $(\because c^z = a)$

$\Rightarrow$ $xyz = 1$ (Since the bases are same the powers are equal)

Example 9 If $a = 3^x$, $b = 3^y$, $c = 3^z$ and $ab = c^2$ prove that $x + y = 2z$

Solution : Given : $a = 3^x$, $b = 3^y$, $c = 3^z$

Consider $ab = c^2$

$3^x . 3^y = (3^z)^2$

$3^{x+y} = 3^{2z}$

$x+y = 2z$

Example 10 If $45^{1/x} = 3^{1/y} = 5^{1/z}$, Prove that $x = 2y + z$

Solution : Let $45^{1/x} = 3^{1/y} = 5^{1/z} = k$

$(45)^{1/x} = k \Rightarrow 45 = k^x$

$3^{1/y} = k \Rightarrow 3 = k^y$

$5^{1/z} = k \Rightarrow 5 = k^z$

Consider $k^x = 45$

$= 9 \times 5$

$= 3^2 \times 5$

$= (k^y)^2 \quad k^z$

$= k^{2y} k^z$

$k^x = k^{2y+z}$

$x = 2y + z$ (Since the bases are same the powers are equal)

Example 11 If $a^{1/3} + b^{2/3} + c = 0$ then show that $(a + b^2 + c^3)^3 = 27ab^2c^3$

Solution : Given $a^{1/3} + b^{2/3} + c = 0$

$a^{1/3} + b^{2/3} = - c$

Cubing both sides, we get

$(a^{1/3} + b^{2/3})^3 = (- c)^3$

$(a^{1/3})^3 + (b^{2/3})^3 + 3a^{1/3}\, b^{2/3}\, (a^{1/3} + b^{2/3}) = - c^3$

$a + b^2 + 3a^{1/3}\, b^{2/3}\, (- c) = - c^3 \quad (\because a^{1/3} + b^{2/3} = - c)$

$a + b^2 + c^3 = 3a^{1/3}\, b^{2/3}\, c$

cubing both sides.

$(a + b^2 + c^3)^3 = 27\, ab^2c^3$

Example 12 Solve: $8.4^x - 9.2^x + 1 = 0$

Solution : $8.4^x - 9.2^x + 1 = 0$

$8.(2^x)^2 - 9.2^x + 1 = 0$ .... (1)

Put $2^x = y$

1) $\Rightarrow$ $8y^2 - 9y + 1 = 0$

$8y^2 - 8y - 1y + 1 = 0$

$8y (y - 1) - 1 (y - 1) = 0$

$(y - 1) (8y - 1) = 0$

$y = 1 \qquad 8y = 1$

$y = 1$ or $\qquad y = 1/8$

$\Rightarrow \; 2^x = 1$ or $2^x = 2^{-3}$

$\Rightarrow \; 2^x = 2^0$ or $2^x = 2^{-3}$

$\Rightarrow \; x = 0$ or $x = - 3$

Example 13 If abc=1, then Prove that : $\frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}+\frac{1}{1+c+a^{-1}}=1$

Solution : L.H.S $= \frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}+\frac{1}{1+c+a^{-1}}$

$$= \frac{b}{b+ab+1}+\frac{1}{1+b+ab}+\frac{a}{a+ac+1} \quad (\because abc=1)$$

$$= \frac{b}{b+ab+1}+\frac{1}{1+b+ab}+\frac{a}{a+\frac{1}{b}+1} \quad \text{(ac=1/b)}$$

$$= \frac{b}{b+ab+1}+\frac{1}{1+b+ab}+\frac{ab}{ab+1+b}$$

$$= \frac{b+1+ab}{b+ab+1}$$

1 = RHS

Example 14 Simplify:

$$\sqrt[bc]{\frac{x^b}{x^c}} \times \sqrt[ac]{\frac{x^c}{x^a}} \times \sqrt[ab]{\frac{x^a}{x^b}}$$

Solution:

$$\sqrt[bc]{\frac{x^b}{x^c}} \times \sqrt[ac]{\frac{x^c}{x^a}} \times \sqrt[ab]{\frac{x^a}{x^b}}$$

$$= \quad (x^{b-c})^{1/bc} \quad (x^{c-a})^{1/ac} \quad (x^{a-b})^{1/ab}$$

$$= \quad x^{\frac{b-c}{bc}} \quad x^{\frac{c-a}{ac}} \quad x^{\frac{a-b}{ab}}$$

$$= \quad x^{\frac{b-c}{bc}+\frac{c-a}{ac}+\frac{a-b}{ab}}$$

$$= \quad x^{\frac{ab-ac+bc-ab+ac=bc}{abc}}$$

$$= \quad x^0$$

$$= \quad 1$$

Example 15 Show that

$$\sum \frac{1}{1+x^{a-b}+x^{a-c}} = 1$$

Solution : Consider

$$\sum \frac{1}{1+x^{a-b}+x^{a-c}}$$

$$= \frac{1}{1+x^{a-b}+x^{a-c}} + \frac{1}{1+x^{b-c}+x^{b-a}} + \frac{1}{1+x^{c-a}+x^{c-b}}$$

Again consider

$$\frac{1}{1+x^{a-b}+x^{a-c}} = \frac{x^{-a}}{x^{-a}+x^{-b}+x^{-c}}$$

(multiply both numerator and denominator by x^{-a})

Similarly multiply both numerator and denominator of 2nd term and 3rd term of LHS by x^{-b} and x^{-c} respectively.

$$\because \frac{1}{1+x^{a-b}+x^{a-c}} = \frac{x^{-a}}{x^{-a}+x^{-b}+x^{-c}} + \frac{x^{-b}}{x^{-b}+x^{-c}+x^{-a}} + \frac{x^{-c}}{x^{-c}+x^{-a}+x^{-b}}$$

$$= \frac{x^{-a}+x^{-b}+x^{-c}}{x^{-a}+x^{-b}+x^{-c}}$$

$$= 1$$

Example 16 Show that : $\dfrac{y^{-1}}{x^{-1}+y^{-1}} + \dfrac{y^{-1}}{x^{-1}-y^{-1}} = \dfrac{2xy}{y^2-x^2}$

Solution : LHS $= \dfrac{1/y}{\dfrac{1}{x}+\dfrac{1}{y}} + \dfrac{1/y}{\dfrac{1}{x}-\dfrac{1}{y}}$

$$\frac{1}{y}\times\frac{xy}{y+x}+\frac{1}{y}\times\frac{xy}{y-x}$$

$$\frac{x}{y+x}+\frac{x}{y-x}$$

$$= \quad \frac{x(y-x)+x(y+x)}{y^2-x^2}$$

$$= \quad \frac{xy-x^2+xy+x^2}{y^2-x^2}$$

$$= \quad \frac{2xy}{y^2-x^2} \quad = \quad \text{RHS}$$

Example 17 **Simplify:** $2(3^{-2})+\left(\frac{1}{3}\right)^{-3}+3^2$

Solution : $\frac{2}{3^2}+3^3+3^2$

$$= \quad \frac{2+243+81}{9}$$

$$= \quad \frac{326}{9}$$

Example 18 **Simplify:** $\frac{(2^2)^0-2^{(3^0)}}{(2^0)^6-2^{(2^2)}}$

Solution : $\frac{(2^2)^0-2^{(3^0)}}{(2^0)^6-2^{(2^2)}}$

$$= \quad \frac{(4)^0-2^1}{(1)^6-2^4}$$

$$= \quad \frac{1-2}{1-16}=\frac{-1}{-15}=\frac{1}{15}$$

Example 19 Simplify: $\frac{2^b(2^{b-1})^3}{2^{b+1}4^b}.\frac{16^{b/2}}{8}$

Solution : $\frac{2^b(2^{b-1})^3}{2^{b+1}4^b}.\frac{16^{b/2}}{8}$

$= \frac{2^b 2^{3b-3}.(2^4)^{b/2}}{2^{b+1}2^{2b}2^3}$

$= 2^{b+3b-3+2b-b-1-2b-3}$

$= 2^{3b-7}$

Example 20 Solve: $2^{2x} - 6.2^x + 8 = 0$

Solution : Consider $(2^x)^2 - 6.2^x + 8 = 0$

Put $2^x = y$

$\Rightarrow$ $y^2 - 6y + 8 = 0$

$y^2 - 2y - 4y + 8 = 0$

$y(y - 2) - 4(y - 2) = 0$

$(y - 2)(y - 4) = 0$

$y = 2$ or $y = 4$

But $y = 2^x$

$2^x=2$ or $2^x=4=2^2$

$\therefore$ x=1 or x=2

EXERCISE : 3.1

I One mark questions

Simplify the following:

1) $(5)^{5^0} + (5^2)^0$ 2) $\left(\frac{5x^3}{y}\right)^2$ 3) $a^{x+y}\, a^{2x-y}$

4) $\left[\left\{\sqrt[3]{x^2}\right\}^3\right]^{1/2}$ 5) $(x^{1/2} + y^{1/2})\ (x^{1/2} - y^{1/2})$

II. Two marks questions.

Simplify the following:

1) $\dfrac{a^{2m+n}.a^{3m+n}}{a^{4m+2n}}$ 2) $\dfrac{2^{n+1}+2^{n-1}}{2^{n}+2^{n+2}}$ 3) $\dfrac{(3^0)^3+(3^2)^0}{(3^2)^2+3^{(3^0)}}$

4) $\left(\dfrac{x^a}{x^b}\right)^{a+b}\left(\dfrac{x^b}{x^c}\right)^{b+c}\left(\dfrac{x^c}{x^a}\right)^{c+a}$ 5) $\dfrac{2^{7b-2a}.8^{2a-b}}{16^{a+b}}$

III. Three marks questions:

1) Prove that $\left(\dfrac{x^a}{x^b}\right)^{a^2+ab+b^2}\left(\dfrac{x^b}{x^c}\right)^{b^2+bc+c^2}\left(\dfrac{x^c}{x^a}\right)^{c^2+ca+a^2} = 1$

2) If $a^x = b^y = c^z$ and $b^2 = ac$. Show that $\dfrac{1}{x}+\dfrac{1}{z}=\dfrac{2}{y}$

3) If $p^x = q^y = r^z = s^w$ and $pq = rs$. Prove that $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{z}+\dfrac{1}{w}$

4) If $3^x = 5^y = 15^z$, show that $z(x + y) = xy$

5) If $2^{1/a} = 3^{1/b} = (54)^{1/c}$ show that $a + 3b = c$.

IV. Five marks questions:

1) If $a^x = bc$, $b^y = ca$ and $c^z = ab$, show that $xyz = x+y+z+2$

2) If $a^{1/3} + b^{1/3} + c^{1/3} = 0$, then show that $(a + b + c)^3 = 27\,abc$

3) If $x^{5/3} - y^{1/3} - z = 0$, then show that $(x^5 - y - z^3)^3 = 27x^5yz^3$

4) If $x = 2^{1/3} + 2^{-1/3}$ prove that $2x^3 - 6x - 4 = 0$

5) Solve : a) $3^{2x} + 10.3^x + 9 = 0$

b) $5.5^{2y} - 26.5^y + 5 = 0$

c) $5^{2x+1} + (25)^{x+1} = 150$

d) $\dfrac{7^{3-2x}}{7^{7-5x}} = 49^{3x-14}$

ANSWERS : 3.1

I)	1) 6	2) $25\dfrac{x^6}{y^2}$	3) a^{3x}	4) x	5) $x - y$
II	1) a^m	2) $\dfrac{1}{2}$	3) $\dfrac{1}{42}$	4) 1	5) 1
IV	5) a) 0, 2	b) 1, -1	c) 1/2	d) 8	

Chapter 4

LOGARITHMS

4.1 Introduction: John Napier (1550 – 1617), a Scottish Mathematician, invented the logarithms in 1614 and further modified by Henry Briggs (1556-1630) who introduced common logarithms. The word Logarithms was derived from two Greek words, Logos which means a ratio and arithmos, meaning number. Logarithms is used to simplify calculations. Logarithm to base e is known as Natural Logarithm and Logarithm to base 10 is known as common logarithm.

4.2 Definition of Logarithm:

If a is a positive real number other than 1 and $y = a^x$, then x is called the logarithm of y to the base a and is denoted by $x = \log_a y$.

i.e. $y = a^x$ (exponential form)
$\Leftrightarrow x = \log_a y$ (logarithmic form)

Note:

1) If a is any non zero real number, then $a^0 = 1 \Leftrightarrow \log_a 1 = 0$. i.e. the logarithm of 1 to any base is zero.
2) If a is any non zero real number, then $a^1 = a \Leftrightarrow \log_a a = 1$. i.e. the logarithm of any number to the base of the same number is equal to 1.

4.3. Laws of Logarithms:

First Law : $\log_a mn = \log_a m + \log_a n$

Proof : Let $\log_a m = x \Leftrightarrow a^x = m$
$\log_a n = y \Leftrightarrow a^y = n$
$\log_a mn = z \Leftrightarrow a^z = mn$

Consider $a^z = mn$
$a^z = a^x . a^y$
$a^z = a^{x+y}$
$z = x + y$ (when bases are same, powers are equal)
$\Rightarrow \log_a mn = \log_a m + \log_a n$

Note: Similarly $\log_a (mnl - - -) = \log_a m + \log_a n + \log_a l + - - -$

Second Law: $\log_a \frac{m}{n} = \log_a m - \log_a n$

Proof: Let $\log_a m = x \Leftrightarrow a^x = m$

$\log_a n = y \Leftrightarrow a^y = n$

$$\log_a \left(\frac{m}{n}\right) = z \Leftrightarrow a^z = \frac{m}{n}$$

Consider $a^z = \frac{m}{n}$

$$a^z = \frac{a^x}{a^y}$$

$$a^z = a^{x-y}$$

$$z = x - y$$

$$\Rightarrow \quad \log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$$

Third Law: $\log_a m^n = n \log_a m$

Proof: Let $\log_a m = x \Leftrightarrow a^x = m$

Consider $m = a^x$

$m^n = (a^x)^n$

$\therefore \log_a m^n = nx$ (by definition of Logarithm)

$\log_a m^n = n \log_a m$

Fourth Law: (Change of base)

$$\log_n m = \frac{\log_a m}{\log_a n}$$

Proof : Let $\log_n m = x \Leftrightarrow m = n^x$

Now consider $m = n^x$

Taking log on both sides to the base a,

$\log_a m = \log_a n^x$

$= x\log_a n$ (By using third law)

$$\therefore \quad x = \frac{\log_a m}{\log_a n}$$

$$\therefore \quad \log_n m = \frac{\log_a m}{\log_a n}$$

Corollary:

1. $\log_b a = \dfrac{\log_a a}{\log_a b} = \dfrac{1}{\log_a b}$
2. $\log_b a \,.\, \log_c b = \log_c a$

WORKED EXAMPLES:

Example 1

Express each of the following in exponential form

a) $\log_5 125 = 3$ b) $\log_2 ½ = -1$

Solution:

a) $\log_5 125 = 3$

$\Rightarrow \quad 5^3 = 125$

b) $\log_2 ½ = - 1$

$\Rightarrow \quad 2^{-1} = \dfrac{1}{2}$

Example 2

Express the following in the logarithmic form.

a) $3^3 = 27$ b) $5^{-1} = 0.2$

Solution:

a) $3^3 = 27$

$\Rightarrow \quad \log_3 \ 27 = 3$

b) $5^{-1} = 0.2$

$\Rightarrow \quad \log_5 0.2 = - 1$

Example 3

Find the value of $\log_{\sqrt{3}} 27$

Solution: Let $\log_{\sqrt{3}} 27 = x$

$\Rightarrow \quad (\sqrt{3})^x = 27$

$\Rightarrow \quad (3^{1/2})^x = 27$

$\Rightarrow \quad 3^{x/2} = 3^3$

$\Rightarrow \quad \frac{x}{2} = 3$ (when bases are same, powers are equal)

$\Rightarrow \quad x = 6$

Example 4 Solve for x :

a) $\log_7 x = 2$ b) $\log_{\sqrt{x}} 4 = 2$

Solution: a) $\log_7 x = 2$

$\Rightarrow \quad x = 7^2$

$\Rightarrow \quad x = 49$

b) $\log_{\sqrt{x}} 4 = 2$

$\Rightarrow \quad (\sqrt{x})^2 = 4$

$\Rightarrow \quad x = 4$

EXERCISE : 4.1

I. One mark questions.

1. Express the following in the logarithmic form.

(a) $2^5 = 32$ b) $3^{-2} = \frac{1}{9}$ C) $5^{-2} = 0.04$

2. Express the following in the exponential form:

a) $\log_9 81 = 2$ b) $\log_{10} 0.01 = -2$ c) $\log_2 \frac{1}{4} = -2$

3. Solve for x:

a) $\log_2 \sqrt{32} = x$ b) $\log_{0.1} 10 = x$

c) $\log_{\sqrt[3]{5}} x = 6$ d) $\log_x 625 = 4$

ANSWERS : 4.1

I. 1. a) $\log_2 32 = 5$ b) $\log_3 1/9 = -2$ c) $\log_5 0.04 = -2$

2. a) $9^2 = 81$ b) $10^{-2} = 0.01$ c) $2^{-2} = \frac{1}{4}$

3. a) 5/2 b) -1 c) 25 d) 5

WORKED EXAMPLE:

Example 1 Prove that:

$$\log\frac{9}{5} + \log\frac{15}{9} - \log\frac{3}{2} = \log 2$$

Solution :

$$\begin{aligned}
\text{LHS} &= \log\frac{9}{5} + \log\frac{15}{9} - \log\frac{3}{2} \\
&= \log\left(\frac{9}{5} \times \frac{15}{9}\right) - \log\frac{3}{2} \\
&= \log 3 - \log\frac{3}{2} \\
&= \log\left(3.\frac{2}{3}\right) \\
&= \log 2 = \text{R.H.S}
\end{aligned}$$

Example 2 Prove that:

$$2\log\frac{3}{7}+\log\frac{49}{9}=0$$

Solution:

$$
\begin{aligned}
\text{LHS} &= 2\ \log\ \frac{3}{7}+\log\frac{49}{9}\\
&= \log\ \left(\frac{3}{7}\right)^2+\log\frac{49}{9}\\
&= \log\ \frac{9}{49}+\log\frac{49}{9}\\
&= \log\ \left(\frac{9}{49}\times\frac{49}{9}\right)\\
&= \log\ 1\\
&= 0 = \text{R.H.S.}
\end{aligned}
$$

Example 3 Prove that

$$\log_{b^2} a^2.\log_{c^2} b^2.\log_{a^2} c^2=1$$

Solution : By using the change of base formula, we get

$$
\begin{aligned}
\text{LHS} &= \frac{\log_K a^2}{\log_K b^2}.\frac{\log_K b^2}{\log_K c^2}.\frac{\log_K c^2}{\log_K a^2}\\
&= 1 = \text{R.H.S.}
\end{aligned}
$$

Example 4 If $\log_k x + \log_k y + \log_k z = 0$, show that $xyz = 1$

Solution : Given $\log_k x + \log_k y + \log_k z = 0$

$\Rightarrow \quad \log_k xyz = 0$

$\Rightarrow \quad xyz = 1$ ($\because$ $\log 1 = 0$)

Example 5 Show that : $\log_{bc} a = \dfrac{\log_b a}{1+\log_b c}$

Solution:

$$\text{LHS} = \log_{bc} a$$

$$= \frac{\log_b a}{\log_b bc} \quad \text{(by using change of base formula)}$$

$$= \frac{\log_b a}{\log_b b + \log_b c}$$

$$= \frac{\log_b a}{1+\log_b c}$$

$$= \text{R.H.S.}$$

Example 6 If $x = \log_{2a} a$, $y = \log_{3a} 2a$, $z = \log_{4a} 3a$ show that $xyz + 1 = 2yz$

Solution: Consider

$$xyz + 1 = \log_{2a} a . \ \log_{3a} 2a \ . \ \log_{4a} 3a + 1$$

$$= \log_{3a} a \ . \ \log_{4a} 3a \ . + 1$$

$$(\because \ \log_b a \ . \ \log_c b = \log_c a)$$

$$= \log_{4a} a + 1$$

$$= \log_{4a} a + \log_{4a} 4a \qquad (\because \ \log_a a = 1)$$

$$= \log_{4a} 4a^2$$

$$= \log_{4a}(2a)^2$$

$$= 2 \log_{4a} 2a \quad (\because \ \log a^n = m \log a)$$

$$= 2 \log_{3a} 2a \ . \ \log_{4a} 3a$$

$$= 2\, yz$$

$$= \text{RHS}$$

Example 7 Prove that

$$\frac{1}{\log_{a^2b^2}(abc)}+\frac{1}{\log_{b^2c^2}(abc)}+\frac{1}{\log_{c^2a^2}(abc)}=4$$

$$\text{LHS} = \frac{1}{\log_{a^2b^2}(abc)}+\frac{1}{\log_{b^2c^2}(abc)}+\frac{1}{\log_{c^2a^2}(abc)}$$

$= \log_{abc} a^2b^2 + \log_{abc} b^2c^2 + \log_{abc} c^2a^2 \quad \left[\because \frac{1}{\log_a b}=\log_b a\right]$

$= \log_{abc} (a^2\, b^2 .\, b^2c^2 \,.\, c^2a^2)$

$= \log_{abc} a^4\, b^4\, c^4$

$= \log_{abc} (abc)^4$

$= 4 \log_{abc} abc \; [\because \log x^m = m \log x]$

$= 4\,(1)$

$= 4$

= R.H.S.

Example 8 Show that

$x^{\log y - \log z} \,.\, y^{\log z - \log x} .\; z^{\log x - \log y} = 1$

Solution: Consider the common base as k

Let $\log_K x = a \Rightarrow k^a = x$

$\text{Log}_K y = b \Rightarrow k^b = y$

$\text{Log}_K z = c \Rightarrow k^c = z$

Now, LHS $= x^{\log y - \log z} .\; y^{\log z - \log x} \,.\, z^{\log x - \log y}$

$= (k^a)^{b-c} \quad (k^b)^{c-a} \quad (K^c)^{a-b}$

$= k^{ab-ac} \,.\, k^{bc-ba} \,.\, k^{ca-cb}$

$= k^{ab-ac+bc-ba+ca-cb}$

$= k^0$

$= 1$ = R.H.S.

Example 9 Solve : $\log x + \log (x-4) - \log (x-6) = 0$

Solution :

$\log x + \log (x-4) - \log (x-6) = 0$

$\log x(x-4) - \log (x-6) = 0$

$$\log \frac{x^2-4x}{x-6} = 0$$

$$\frac{x^2-4x}{x-6} = 1 \qquad (\because \log 1 = 0)$$

$x^2 - 4x = x - 6$

$x^2 - 4x - x + 6 = 0$

$x^2 - 5x + 6 = 0$

$x^2 - 2x - 3x + 6 = 0$

$x(x-2) - 3(x-2) = 0$

$(x-2)(x-3) = 0$

$x = 2$ or $x = 3$

Example 10 If $\log \left(\frac{a-b}{4}\right) = \log\sqrt{a} + \log\sqrt{b}$, show that $(a+b)^2 = 20ab$

Solution :

$$\log \left(\frac{a-b}{4}\right) = \log\sqrt{a} + \log\sqrt{b}$$

$$\log \left(\frac{a-b}{4}\right) = \log\sqrt{ab}$$

$$\frac{a-b}{4} = \sqrt{ab}$$

$a - b = 4\sqrt{ab}$

Squaring both sides, we get

$(a-b)^2 = 16ab$

$a^2 + b^2 - 2ab = 16ab$

$a^2 + b^2 = 16ab + 2ab$

$a^2 + b^2 = 18ab$

$a^2 + b^2 + 2ab = 18ab + 2ab$

$(a+b)^2 = 20ab$

Example 11 If $x^2 + y^2 = 12xy$,
show that $2 \log (x - y) = \log 2 + \log 5 + \log x + \log y$

Solution: Consider $2 \log (x-y) = \log (x - y)^2$

$= \log (x^2 + y^2 - 2xy)$

$= \log (12xy - 2xy)$ $[\because\ x^2 + y^2 = 12xy]$

$= \log 10xy$

$= \log (2.5.\ x.\ y)$

$= \log 2 + \log 5 + \log x + \log y.$

Example 12 Solve $\log_9 x + 2\log_{27} x + 3\log_3 x = 25$

Solution : $\log_9 x + 2\log_{27} x + 3\log_3 x = 25$

$$\Rightarrow \frac{\log_3 x}{\log_3 9} + \frac{2\log_3 x}{\log_3 27} + \frac{3\log_3 x}{\log_3 3} = 25 \quad \text{[change the base to 3]}$$

$$\Rightarrow \frac{\log_3 x}{\log_3 3^2} + \frac{2\log_3 x}{\log_3 3^3} + 3\log_3 x = 25 \quad (\log_a a = 1)$$

$$\Rightarrow \frac{\log_3 x}{2\ \log_3 3} + \frac{2\log_3 x}{3\log_3 3} + 3\log\ \ x = 25$$

$$\Rightarrow \log_3 x \left[\frac{1}{2} + \frac{2}{3} + 3\right] = 25$$

$$\Rightarrow \log_3 x \left[\frac{3 + 4 + 18}{6}\right] = 25$$

$$\Rightarrow \log_3 x \left(\frac{25}{6}\right) = 25$$

$\Rightarrow \log_3 x = 6$

$\Rightarrow x = 3^6$

$x = 729$

EXERCISE : 4.2

I. Two marks questions:

1. Find the value of $\log\sqrt{\frac{9}{4}}-\log\frac{3}{2}$

2. Prove that $\log\frac{7}{8}+\log\frac{32}{49}-\log\frac{4}{14}=\log 2$

3. Prove that $2\log\frac{3}{5}+3\log\frac{5}{2}-\log\frac{45}{8}=0$

4. Prove that $\log\frac{12}{15}+2\log\frac{6}{8}+\frac{1}{3}\log\frac{8}{27}=\log\frac{3}{10}$

5. Prove that $\log\sqrt{\frac{a}{b}}+\log\sqrt{\frac{b}{c}}+\log\sqrt{\frac{c}{a}}=0$

6. Prove that $\left(\frac{1}{\log_b a}\right)\left(\frac{1}{\log_a b}\right)=1$

II. Three marks questions:

1. Prove that $\log_y x^2 \cdot \log_z y^3 \cdot \log_x z^5 = 30$

2. Prove that $\frac{1}{\log_2 4}+\frac{1}{\log_8 4}+\frac{1}{\log_{16} 4}=4$

3. Prove that $\log_4 8 \cdot \log_2 32 \cdot \log_{16} 4 = 15/4$.

4. Solve : $\log x + \log (x + 1) = 2$

5. Solve : $\log_2 x + \log_4 x = 3$

6. Solve : $\log_x 9 + \log_x 4 = 2$

7. If $x = \log_2 9$ $y = \log_9 7$ $z = \log_7 4$, show that $xyz = 2$

8. Prove that

$$\log x + \log x^2 + \log x^3 + \ldots + \log x^n = \frac{1}{2}n(n+1)\log x$$

III. **Five marks questions** :

1. If $\log\left(\frac{a+b}{4}\right)=\frac{1}{2}$ [log a + log b], show that $(a-b)^2 = 12ab$

2. If log (a – b) = $\frac{1}{2}$ log (5ab), show that $a^2 + b^2 = 7ab$

3. If $m^2 + n^2 = 15$ mn, show that
   2 log (m – n) = log 13 + log m + log n.

4. If $m^2 + n^2 = 20$ mn, show that
   2 log (m + n) = log 2 + log 11 + log m + log n

5. If $\log_a (bc) = x$, $\log_b (ca) = y$, $\log_c(ab) = z$,
   Show that $\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}=1$

6. Solve : $\log x + \log (2x - 1) - \log (x + 1) = 2$

7. Solve : $2 \log_2 x + 3\log_4 x + 5\log_8 x = 62$

ANSWERS : 4.2

I. 1) 0

II. 4) -2 5) 4 6) 6

III. $2, \frac{-1}{2}$ 7) 4096

4.4 COMMON LOGARITHMS:

Logarithms to the base 10 are known as common logarithms

By definition of logarithms, consider the following examples.

$10^0 = 1 \quad \Leftrightarrow \quad \log_{10} 1 = 0$

$10^1 = 10 \quad \Leftrightarrow \quad \log_{10} 10 = 1$

$10^2 = 100 \quad \Leftrightarrow \quad \log_{10} 100 = 2$

$10^3 = 1000 \quad \Leftrightarrow \quad \log_{10} 1000 = 3$

$10^{-1} = 0.1 \quad \Leftrightarrow \quad \log_{10} 0.1 = -1$

$10^{-2} = 0.01 \quad \Leftrightarrow \quad \log_{10} 0.01 = -2$

$10^{-3} = 0.001 \quad \Leftrightarrow \quad \log_{10} 0.001 = -3$

From the above examples we observe that the logarithm of a number between1 and 10 lies between 0 and 1, logarithm of a number between 10 and 100 lies between 1 and 2, logarithm of number between 0.01 and 0.1 lies between – 2 and -1 and so on.

CHARACTERISTIC AND MANTISSA:

Logarithm of a number to the base 10 has two parts. One is the integral part called the characteristic and the other is the fractional part called Mantissa.

Ex. Log 25.63 = 1.4085

Here, 1 is the characteristic and 0.4085 is the mantissa.

Logarithmic tables:

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	5	7	8	10	11	12
32	5061	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5196	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11

Analyse the above table. Let us find the value of log (33.76) by using the table. Characteristic can be found out by inspection and Mantissa is found out by using the table. In log (33.76), before decimal there are 2 digits. Therefore, characteristic is 2 -1 = 1. While finding Mantissa, neglect the decimal and consider the four numbers from the beginning . Check the first 2 numbers. i.e. 33 in the first column. Move along the horizontal row beginning with 33 and check the four digit number where the column headed by 7 will meet the number will be 5276. Further move towards the mean differences column and check the number where the column headed by 6 is reached. Here we find the number as 8. Add this number to 5276 i.e. 5276 + 8 = 5284

$$\therefore \log (33.76) = 1.5284$$

Similarly $\log (337.6) = 2.5284$

$\log (3376) = 3.5284$

$\log (0.3376) = \overline{1}.5284$

$\log (0.03376) = \overline{2}.5284$

We observe from the above example that though the characteristic differs with the change of decimal place, mantissa remains the same.

Note : Mantissa is always positive.

Antilogarithmic table.

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1658	0	1	1	2	2	2	3	3	3
22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4

Analyse the above table, let us consider an example, say $\log x = 1.2456 \Rightarrow x = \text{antilog } 1.2456$. While finding the antilog 1.2456, ignore the characteristic and consider the four digits after decimal point. Check the first column in the table for 0.24. Beginning with 0.24 move along the horizontal row to meet the four digit number where the column headed by 5 will meet. The number is 1758. Further move to the mean differences column. Read the number where the column headed by 6 will reach add the number to 1758. i.e. 1758 + 2 = 1760. Add 1 to characteristic i.e. 1+ 1 = 2 Place decimal after the first 2 digits.

i.e. Antilog (1. 2456) = 17.60 Antilog ($\bar{2}$ .2456) = 0.02456

Similarly Antilog (0.2456) = 1.760

Antilog ($\bar{1}$ .2456) = 0.2456

WORKED EXAMPLES:

Example 1 Find the value of each of the following using logarithmic tables:

a) 12.56×10.73 b) $0.632 \times 5.673 \times 0.3213$

c) $\dfrac{213.781 \times 7.434}{6.321}$ d) $\dfrac{0.5634 \times 0.0635}{2.563 \times 12.5}$

e) $\dfrac{\sqrt{14.5} \times \sqrt[3]{8.571}}{(16.751)^{2/3}}$

a) 12.56×10.73

Solution: Let $x = 12.56 \times 10.73$

Taking log both sides, we get

$\log x = \log (12.56 \times 10.73)$

$= \log 12.56 + \log 10.73$ ($\because$ log mn = log m + log n)

$= 1.0990 + 1.0306$

$= 2.1296$

$x = \text{Antilog } (2.1296)$

$= 134.8$

b) $0.632 \times 5.673 \times 0.3213$

Solution: Let $x = 0.632 \times 5.673 \times 0.3213$

$\log x = \log [0.632 \times 5.673 \times 0.3213]$

$= \log 0.632 + \log 5.673 + \log 0.3213$

$= \bar{1}.8007 + 0.7568 + \bar{1}.5069$

$= 0.0644$

$x = \text{Antilog } (0.0644)$

$= 1.160$

c) $\dfrac{213.781 \times 7.434}{6.321}$

Solution :

Let $x = \dfrac{213.781 \times 7.434}{6.321}$

$$\log x = \log\left[\frac{213.781 \times 7.434}{6.321}\right]$$

$= \log 213.781 + \log 7.434 - \log 6.321$

$= 2.3298 + 0.8712 - 0.8008.$

$= 2.4002$

$x = \text{Antilog } 2.4002$

$x = 251.3$

d) $\dfrac{0.5634 \times 0.0635}{2.563 \times 12.5}$

Solution :

Let $x = \dfrac{0.5634 \times 0.0635}{2.563 \times 12.5}$

$$\log x = \log\left[\frac{0.5634 \times 0.0635}{2.563 \times 12.5}\right]$$

$= \log 0.5634 + \log 0.0635 - \log 2.563 - \log 12.5$

$= \overline{1}.7508 + \overline{2}.8028 - 0.4087 - 1.0969$

$= 2.5536 - 1.5056$

$= \overline{2} + 0.5536 - 1.5056$

$= - 2 - 0.952$

$= - 2 .952$

$= - 3 + 3\text{-}2.952$

$= - 3 + 0.048$

$= \overline{3}.048$

x = Antilog ($\bar{3}$ .048)

= 0.001117

= 1.117×10^{-3}

3) $\dfrac{\sqrt{14.5} \times \sqrt[3]{8.571}}{(16.751)^{2/3}}$

Solution : Let $x = \dfrac{\sqrt{14.5} \times \sqrt[3]{8.571}}{(16.751)^{2/3}}$

$$\log x = \log \frac{\sqrt{14.5} + \sqrt[3]{8.571}}{(16.751)^{2/3}}$$

$$= \log \sqrt{14.5} + \log \sqrt[3]{8.571} - \log(16.751)^{2/3}$$

$$= \frac{1}{2}\log 14.5 + \frac{1}{3}\log 8.571 - \frac{2}{3}\log(16.751)$$

$$= \frac{1}{2}(1.1614) + \frac{1}{3}(0.9331) - \frac{2}{3}(1.2240)$$

= 0.5807 + 0.3110 + 0.816

= 0.0757

x = antilog 0.0757

= 1.191

Example 2 If log 5 = 0.6990, find the number of digits in the integral part of 5^{23}

Solution : Consider log (5^{23}) = 23 log 5

= 23 (0.6990)

= 16.077

Since the characterstic of log (5^{23}) is 16, there are 17 digits in the integral part of 5^{23}

Example 3 If log 6 = 0.7782, find the number of zeros between the decimal point and the first significant figure is $(0.6)^{30}$

Solution : Consider $\log (0.6)^{30} = 30 \log (0.6)$

$= 30 (\bar{1}.7782)$

$= 30 (- 1 + 0.7782)$

$= - 30 + 23.346$

$= - 30 + 23 + 0.346$

$= - 7 + 0.346$

$= \bar{7}.346$

Here the characteristic is $\bar{7}$. Hence the required number of zeros is $7 - 1 = 6$

EXERCISE : 4.3

I. One Mark questions:

1. Using logarithmic tables, find the logarithm of each of the following numbers.

   a) 563.5 b) 12 c) 0.0057 d) 0.00063

2. Using Logarithmic tables, find the antilogarithm of each of the following:

   a) 1.563 b) 0.643 c) $\bar{3}.673$ d) 0.078

II. Two marks questions:

1. Find the number of digits in the integral part of

   a) 3^{20} b) $(1.456)^{15}$ c) $(3.546)^{20}$

2. Find the number of zeros between the decimal point and the first significant figure in a) $(0.7)^{55}$ b) $(5.63)^{-8}$

III. Three marks questions:

1. Using tables, find the value of

   a) 0.7321×0.563 b) 2.345×12.72 c) $\sqrt{3.56} \times \sqrt[3]{8.634}$

IV. Five Marks questions:

1. Using tables, find the value of

a) $\dfrac{5.6348 \times 25.645}{12.72}$ b) $\dfrac{0.5679 \times 0.0789}{0.0073 \times 0.123}$

c) $\dfrac{\sqrt{6.43} \times 0.5789}{(13.46)^{3/2}}$ d) $\dfrac{12.567 \times 15.674}{0.5968 \times 19.78}$

ANSWERS : 4.3

I.	1.	a) 2.7509	b) 1.0792	c) 3.7559	d) 4.7993
	2.	a) 36.56	b) 4.395	c) 0.004710	d) 1.197
II.	1.	a) 10	b) 3	c) 11	
	2.	a) 8	b) 6		
III.	1.	a) 0.4122	b) 29.83	c) 3.871	
IV.	1.	a) 11.36	b) 49.90	c) 0.0297	d) 16.69

Chapter 5

PROGRESSIONS

5.1 Introduction: In this chapter we shall study particular types of sequences such as Arithmetic Progression, Geometric Progression Harmonic Progression and their corresponding series.

5.2 Sequence: A sequence is an orderly arrangement of numbers according to some rule.

Ex. 2, 4, 6, 8 - - - form a sequence.

Finite and Infinite sequence:

A sequence having a finite number of terms is a finite sequence whereas the sequence having an infinite number of terms is an infinite sequence.

Ex. i) 3, 7, 9, 10, 11 is a finite sequence

ii) 1, 2, 3, 4, 5, - - - - is an infinite sequence.

5.3 Series:

The sum of the terms of a sequence is called the series of the corresponding sequence.

Ex. : 1 + 2 + 3 + 4 + 5 . - - - form a series.

Finite and Infinite sequence:

A series having a finite number of terms is a finite series where as the series having an infinite number of terms is an infinite series.

Ex. 1) $\frac{1}{2}+1+\frac{3}{2}+2$ is a finite series

2) 4 + 5 + 6 + 7 - - - is an infinite series.

5.4 ARITHMETIC PROGRESSION:

Definition: An arithmetic progression is a sequence in which the difference between a term and its proceding term is a constant. The constant is known as **common difference** and it is denoted by 'd'.

Ex. 1, 3, 5, 7 - - form an A.P. where 1 is the first term and 2 is the common difference.

5.5 n^{th} term of an A.P.

Let, the first term $T_1 = a = a + (1 - 1)\,d$

Similarly second term $T_2 = a + d = a + (2 - 1)\,d$

third term $T_3 = a + 2d = a + (3 - 1)\,d$

Last term $\mathbf{T_n = a + (n - 1)\,d}$

Where n = number of terms

a = first term

d = common difference

The general form of an A.P. is given by **a, a + d, a + 2d, - - -, a + (n – 1) d.**

5.6 Sum to 'n' terms of an Arithmetic progression:

Let S_n be the sum to 'n' terms of an A.P.

i.e. $S_n = a + (a + d) + (a + 2d) + - - - + l$ .... (1)

Where 1 is the last term

Also $S_n = l + (l - d) + (l - 2d) + - - - + a$ ...... (2)

(1) + (2) $\Rightarrow$

$$2S_n = (a + l) + (a + l) + (a + l) + \text{——} + (a + l)$$

$$2S_n = n\,(a + l)$$

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{n}{2}\,[a + a + (n - 1)\,d] \qquad [\because l = T_n = a + (n - 1)\,d]$$

$$S_n = \frac{n}{2}\,[2a + (n - 1)\,d]$$

WORKED EXAMPLES:

Example 1 Find the 11^{th} term of the A.P. 3, 5, 7, 9 - - - -

Solution : Here, $a = 3$, $d = 5 - 3 = 2$

$n = 11$

We know that

$$T_n = a + (n - 1)\, d$$
$$T_{11} = 3 + (11 - 1)\, 2$$
$$= 3 + (10)\, 2$$
$$= 3 + 20$$
$$= 23$$

Example 2 Find the 8^{th} term of an A.P. -2, -4, -6 - -

Solution : Here, $a = -2$, $d = -4 - (-2) = -4 + 2 = -2$, $n=8$

$T_8 = -2 + (8 - 1)(-2)$

$= -2 + 7(-2)$

$= -2 - 14 = -16$

Example 3 Find the common difference of an A.P. whose first term is 6 and 12^{th} term is 72.

Solution : Given $a = 6$, $T_{12} = 72$, $d=?$

$T_{12} = a + 11d$

$\Rightarrow$ $72 = 6 + 11d$

$72 - 6 = 11d$

$11d = 66$

$$d = \frac{66}{11} = 6$$

$\therefore$ The common difference is 6.

Example 4 If the 3^{rd} term of an A.P. is 11 and 10^{th} term is 32. Find the A.P.

Solution : Given : $T_3 = 11 \Rightarrow a + 2d = 11$ ———— (1)

$T_{10} = 32 \Rightarrow a + 9d = 32$ ———— (2)

$$\begin{array}{r} a+2d=11 \\ \overset{(-)}{a}\overset{(+)}{-9d}\overset{(-)}{=32} \\ \hline -7d=21 \end{array}$$

$\Rightarrow$

$$d=\frac{-21}{-7}=3$$

(1) $\Rightarrow$ a = 11 – 2d

= 11 – 2 (3)

= 11 – 6

= 5

$\therefore$ A.P is 5, 8, 11, - - -

Example 5 Which term of an AP $\frac{1}{2},1,\frac{3}{2}$ - - - is 5?

Solution : Here, $a=\frac{1}{2}$, $d=1-\frac{1}{2}=\frac{1}{2}$, $T_n=5$

$\therefore$ $T_n = a + (n-1)d$

$$\Rightarrow 5=\frac{1}{2}+(n-1)\frac{1}{2}$$

$$\frac{5}{1}-\frac{1}{2}=(n-1)\frac{1}{2}$$

$$\frac{9}{2}=(n-1)\frac{1}{2}$$

9 = n – 1

n = 9 + 1

= 10

$\therefore$ T_{10} is 5

Example 6 Is 8 is the term of an A.P. $\frac{1}{3}, \frac{4}{3}, \frac{7}{3}$ —— ?

Solution : Here, $a = \frac{1}{3}$, $d = \frac{4}{3} - \frac{1}{3} = \frac{3}{3} = 1$

$T_n = 8$

Consider, $T_n = a + (n - 1) d$

$8 = \frac{1}{3} + (n - 1)\, 1$

$8 - \frac{1}{3} = n - 1$

$\frac{23}{3} = n - 1$

$\therefore\ n = \frac{23}{3} + 1 = \frac{26}{3}$

n is always a positive integer. Here $n = \frac{26}{3}$ is a fraction. Therefore 8 is not the term of the given A.P.

Example 7 Find the three numbers which are in A.P. whose sum is 18 and their product is 210.

Solution : Let the three numbers all

$a - d, a, a + d$

By data,

$a - d + a + a + d = 18$

$3a = 18$

$a = \frac{18}{3} = 6$

$(a - d)(a)(a + d) = 210$

$a(a^2 - d^2) = 210$

$6(36 - d^2) = 210$

$216 - 6d^2 = 210$

$6d^2 = 216 - 210$

$6d^2 = 6$

$$d^2 = \frac{6}{6} = 1$$

$$d = \pm 1$$

when $a = 6, d = 1$ the required numbers are 5, 6, 7, when $a = 6, = -1$ the required numbers are 7, 6, 5

Examples 8 The sum of four numbers which are in AP is 28 and 10 times the least number is 4 times the greatest number. Find the numbers.

Solution : Let the four numbers are

$a - 3d, a - d, a + d, a + 3d$

Given $a - 3d + a - d + a + d + a + 3d = 28$

$4a = 28$

$$\therefore \quad a = \frac{28}{4} = 7$$

Also $10(a - 3d) = 4(a + 3d)$

$10a - 39d = 4a + 12d$

$10a = 4a = 30d + 12d$

$42d = 6a$

$42d = 6(7)$

$42d = 42$

$$d = \frac{42}{42} = 1$$

$\therefore$ The numbers are 4, 6, 8, 10

Example 9 Find the sum of 15 elements of an A.P. 4, 5, 6, 7, 8 - - -

Solution : Here, $a = 4$, $d = 5 - 4 = 1$, $n = 15$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2(4) + (15-1)1]$$

$$\frac{15}{2}[8 + 14]$$

$= \quad 22$

$= \quad 15 \quad 11$

$= \quad 165$

Example 10 Evaluate : $3 + 5 + 7 \ldots\ldots + 61$

Solution : Here, $a = 3$, $d = 5 - 3 = 2$, $l = T_n = 61$

Consider, $T_n = a + (n-1)d$

$61 = 3 + (n-1)2$

$61 - 3 = 2n - 2$

$58 = 2n - 2$

$2n = 58 + 2$

$2n = 60$

$$\therefore \quad n = \frac{60}{2} = 30$$

Now, $S_n = \frac{n}{2}(a + l)$

$$S_{30} = \frac{30}{2}(3 + 61)$$

$= 15\,(64)$

$= 960$

Example 11 How many terms of an A.P. -5, -7, -9, - - - will make the sum – 140?

Solution : Here, $a = -5, \quad d = -7 - (-5) = -7 + 5 = -2$

$S_n = -140$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$-140 = \frac{n}{2}[2(-5) + (n-1)(-2)]$

$-280 = n[-10 - 2n + 2]$

$-280 = n[-2n - 8]$

$-280 = -2n^2 - 8n$

$2n^2 + 8n - 280 = 0$

$\Rightarrow \quad n^2 + 4n - 140 = 0$

$n^2 + 14n - 10n - 140 = 0$

$n(n + 14) - 10(n + 14) = 0$

$(n + 14)(n - 10) = 0$

$n = -14 \quad n = 10$

$n = -14$ is discarded.

$\therefore$ 10 terms are required to make the sum – 140

Example 12 Find the sum of all even numbers from 20 to 120.

Solution : Here, $a = 20$

$d = 2$

$l = T_n = 120$

Now $T_n = a + (n-1)d$

$120 = 20 + (n-1)2$

$120 - 20 = 2n - 2$

$100 + 2 = 2n$

$2n = 102$

$n = \frac{102}{2} = 51$

Now consider,

$$S_n = \frac{n}{2}(a+ l)$$

$$S_{51} = \frac{51}{2}(20 + 120)$$

$$= \frac{51}{2} \times 140$$

$$= 3,570$$

Example 13 Find the sum of all numbers between 50 and 200 which are divisible by 11.

Solution : AP is given by

55, 66, 77, - - - 198

Here $a = 55$, $l = T_n = 198$, $d = 11$

Now, $T_n = a + (n - 1) d$

$198 = 55 + (n - 1) 11$

$198 - 55 = (n - 1) 11$

$$\frac{143}{11} = n - 1$$

$n - 1 = 13$

$n = 13 + 1$

$n = 14$

Now consider:

$$S_n = \frac{n}{2}(a + l)$$

$$S_{14} = \frac{14}{2}(55 + 198)$$

+

$$= 7 (253)$$

$$= 1,771$$

Example 14 Ankur Choudhary agrees to pay the rent ₹30,000 for the first year, ₹32,000 for the second year and so on Each year the rent is increased by ₹2,000/- Find the total amount he paid for 10 years.

Solution : a=30,000

d=2,000

n=10

$$\text{now } S_n = \frac{n}{2}[2a + (n -1)\, d]$$

$$S_{10} = \frac{10}{2}(2\,(30{,}000) + (10 - 1)\, 2{,}000]$$

$$= 5\,[60{,}000 + 9\,(2{,}000)]$$

$$= 5\,[60{,}000 + 18{,}000]$$

$$= 5\,[78{,}000]$$

$$= 3{,}90{,}000$$

∴ He paid ₹3,90,000 for 10 years.

Example 15 Sambhav buys a used Bike for ₹18,000. He pays ₹12,000 cash and agrees to pay the balance in annual instalments of ₹500 plus 10% interest on the unpaid amount. How much will the Bike cost him?

Solution : Total cost of the bike = ₹18,000

Initial payment = ₹12,000

Balance to be paid = ₹6,000

$$1^{st} \text{ instalment} = 500 + \frac{6{,}000 \times 10}{100} = ₹1{,}000$$

$$2^{nd} \text{ Instalment} = 500 + \frac{5{,}500 \times 10}{100} = ₹1{,}050$$

$$3^{rd} \text{ Instalment} = 500 + \frac{5000 \times 10}{100} = ₹1{,}000$$

$$\text{Last instalment} = 500 + \frac{500 \times 10}{100} = ₹550$$

∴ Cost of the bike = 12,000 + [1,100 + 1,050 + 1,000 a ....+ 550]

The terms with in the bracket forms an A.P. with

$a = 1{,}100 \quad$ and $\quad d = -50$

Now, $T_n = a + (n-1)\,d$

$550 = 1{,}100 + (n-1)(-50)$

$550 - 1{,}100 = -50n + 50$

$50n = 600$

$\therefore \quad n = \dfrac{600}{50} = 12$

$\therefore$ the number of instalments = 12

Now, $\quad S_{12} = \dfrac{12}{2}\,[1{,}100 + 550]$

$= 6\,[1{,}650]$

$= 9{,}900$

$\therefore$ Cost of the bike $= 12{,}000 + 9{,}900 =$ ₹21,900

EXERCISE : 5.1

I. One mark questions:

1. Find the
   a) 12^{th} term of the A.P. 1, 4, 7 - - - - -
   b) 13^{th} term of the A.P. $\frac{1}{3}, \frac{2}{3}, 1, - - -$
   c) 30^{th} term of the A.P. -2, -5, -8, - - -
   d) 10^{th} term of the A.P. 0.5, 0.7, 0.9 - - -

2. Find the sum of the following Arithmetic Progression.
   a) 2, 6, 10, - - - - to 10 terms
   b) -7, -8, -9 - - - to 15 terms
   c) $\frac{3}{2}, 2, \frac{5}{2}$, - - - to 18 terms
   d) 1.1, 1.3, 1.5 - - - to 20 terms

3. If $\frac{3}{5}, K, \frac{13}{5}$ are in A.P., then find the value of K.

4. The first term of an AP is 3 and the common difference is -2. Find the 11^{th} term.

II. Two marks questions:

1. If the second term of an AP is 4 and tenth term is 20, find the 15^{th} term.
2. The third term of an AP is -11 and 14^{th} term is – 44. find the 20^{th} term.
3. The fifth term exceeds the third term by 10 and the sixth term is 35. Find the A.P.
4. How many terms of the AP 2, 3, 4, 5, 6…….. amount to 230?
5. How many terms of AP -3, -5, -7 amount to -120?
6. If a = 1, d = 7, T_n = 64, then find n & S_n.

III. Three marks questions:

1. The sum of three numbers in AP is 15 and their product is 105. Find the numbers.
2. The sum of three numbers in AP is -18 and sum of their squares is 140. Find the numbers.
3. Find the three numbers which all in AP whose sum is 12 and the sum of their cubes is 408.
4. Find the four numbers in AP whose sum is 20, and the product of whose extremes is 16.
5. The sum of n elements of an AP 21, 23, 25…….. is 384. Find the number of terms and the last term.

IV. Five marks questions:

1. Find the sum of all even integers from 40 to 160.
2. Find the sum of all integers between 100 and 300, which are divisible by 7.
3. Find the sum of all integers between 60 and 400, which are divisible by 13.

4. A person buys every year Bank's Cash Certificate of value exceeding the last year's purchase by ₹500. After 15 years, he finds that the total value of the certificates purchased by him is ₹82,500. Find the value of the certificates purchased by him a) in the first year and b) in the 10^{th} year.

5. Imrez buys a used car for ₹1,50,000 he pays ₹1,00,000 Cash and aggress to pay the balance in annual installments of ₹5,000 plus 8% interest on the unpaid amount. How much will the Car Cost for him?

ANSWERS : 5.1

I.

1. a) 34 b) 13/3 c) – 89 d) 2.3
2. a) 200 b) -210 c) 207/2 d) 60
3. 8/5
4. -17

II.

1. 30
2. -62
3. 10, 15, 20, 25 ……
4. 20
5. 10
6. n = 10, Sn = 325

III.

1. 3, 5, 7
2. - 10, -6, -2
3. 1, 4, 7
4. 2, 4, 6, 8
5. n = 12, T_{12} = 43

IV.

1. 6100
2. 5,586
3. 5,915
4. a) ₹2000 b) ₹6,500
5. ₹72,000

5.7 Geometric Progression

Definition:

A geometric progression is a sequence in which the ratio of a term and it's preceding term is a constant. The constant is known as **common ratio** and is denoted by 'r'.

Ex: 2,8, 32, 128……. form a G.P. where '2' is the first term and 4 is the common ratio.

5.8 n^{th} term of a Geometric progression:

Let the first term	$T_1 = a = ar^{1-1}$
second term	$T_2 = ar = ar^{2-1}$
third term	$T_3 = ar^2 = ar^{3-1}$
lll^{ly}	
Last term	$\mathbf{T_n = ar^{n-1}}$
Where	$T_n = n^{th}$ tern
	a = first term
	r = common ratio

The general form of a GP is given by **$a, ar, ar^2 \ldots\ldots\ldots ar^{n-1}$**

5.9 Sum of n terms of a geometric progression

Let S_n be the sum to n terms of a GP

i.e., $S_n = a + ar + ar^2 + \ldots\ldots\ldots\ldots\ldots\ldots + ar^{n-2} + ar^{n-1}$ ———————— (1)

multiply both sides by r

$\Rightarrow rS_n = ar + ar^2 + ar^3 + \ldots\ldots\ldots\ldots\ldots\ldots + ar^{n-1} + ar^n$ ———————— (2)

(1) – (2) = y

$S_n - rS_n = a - ar^n$

$(1-r)S_n = a(1-r^n)$

$$S_n = \frac{a(1-r^n)}{1-r}, \text{ when } r < 1$$

$$S_n = \frac{a(r^n-1)}{r-1}, \text{ when } r > 1$$

5.10 Sum of infinite terms of a G.P.

Consider $\boldsymbol{S_n = \dfrac{a(1-r^n)}{1-r}}$

If $r < 1$, r^n approaches zero as n approaches ∞

$$\therefore \mathbf{S_\infty} = \frac{\boldsymbol{a}}{\boldsymbol{1-r}}$$

WORKED EXAMPLES :

Example 1 Find the 4^{th} element of GP $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ ....

Solution : Given $a = 1$, $r = \frac{1}{2}$, $n=4$

Now, $T_n = ar^{n-1}$

$$T_4 = 1\left(\frac{1}{2}\right)^{4-1}$$

$$= \left(\frac{1}{2}\right)^3$$

$$= \frac{1}{8}$$

Example 2 Find the 9^{th} element of GP 0.3, 0.6, 1.2, ………..

Solution : Given $a = 0.3,\ r = \frac{0.6}{0.3} = 2\ \ n = 9$

$$\begin{aligned} \text{Now, } T_n &= ar^{n-1} \\ T_9 &= (0.3)\,(2)^{9-1} \\ &= (0.3)\,(2)^8 \\ &= (0.3)\,(256) \\ &= 76.8 \end{aligned}$$

Example 3 If the second term of the GP is 6 and fifth term is 162, then find the GP

Solution : Given $T_2 = 6 \quad \Rightarrow ar = 6$ …………. (1)

$T_5 = 162 \Rightarrow ar^4 = 162$ ……. (2)

Now,

$$\frac{1}{2} \Rightarrow \frac{ar^4}{ar} = \frac{162}{6}$$

$r^3 = 27$

$r^3 = 3^3$

$r = 3$

$(1) \Rightarrow ar = 6$

$$a = \frac{6}{r} = \frac{6}{3}$$

$\therefore a = 2$

Thus the G.P. is 2, 6, 18 ……………

Example 4 The fourth element is square the second term and third element is 27. Find the G.P.

Solution : Given : $T_4 = T_2^{\,2}$ ———————— (1)

$T_3 = 27$ ———————— (2)

(1) $\Rightarrow ar^3 = a^2r^2$

$\therefore\ a = r$

Now (2) $\Rightarrow ar^2 = 27$

$rr^2 = 27$ (since $a = r$)

$r^3 = 27$

$r^3 = 3^3$

$r = 3$

$\therefore$ GP is 3, 9, 27, 81 ....

Example 5 Find the three numbers in GP whose sum is $\frac{31}{5}$ and their product is 1

Solution : Let the three numbers in G.P. all $\frac{a}{r}, a, ar$

By data, $\frac{a}{r} \times a \times ar = 1$

$\Rightarrow a^3 = 1$

$\Rightarrow a = 1$

Now, $\frac{a}{r} + a + ar = \frac{31}{5}$

$\frac{a + ar + ar^2}{r} = \frac{31}{5}$ ($\because a = 1$)

$5 + 5r + 5r^2 = 31r$

$5r^2 + 5r - 31r + 5 = 0$

$5r^2 - 26r + 5 = 0$

$5r^2 - 25r - r + 5 = 0$

$5r(r-5) - 1(r-5) = 0$

$(r - 5)(5r - 1) = 0$

$r = 5$ or $r = \frac{1}{5}$

If $a = 1$, $r = 5$, the three numbers are $\frac{1}{5}$, 1, 5

If $a = 1$, $r = \frac{1}{5}$, the three numbers are 5, 1, $\frac{1}{5}$

Example 6 The three numbers whose sum is 12 are in A.P. If 1, 4, 11 are added to them respectively, then the resulting numbers are in G.P. Find the numbers.

Solution :

Let the three numbers be a – d, a, a + d

By data, a – d + a + a + d = 12

3a = 12

$a = \frac{12}{3} = 4$

Again by data, 4 – d + 1, 4 + 4, 4 + d + 11,

are in G.P. 5-d, 8, 15 + d are in GP

$$\therefore \frac{8}{5-d} = \frac{15+d}{8}$$

64 = (5 – d) (15 + d)

$64 = 75 + 5d - 15d - d^2$

$d^2 + 15d - 5d + 64 - 75 = 0$

$d^2 + 10d - 11 = 0$

$d^2 + 11d - d - 11 = 0$

d (d + 11) – 1 (d + 11) = 0

(d + 11) (d-1) = 0

d = 1 or d = -11

When a = 4, d = 1, the required numbers are 3, 4, 5

When a = 4, d = -11, the required numbers are 15, 4, -7

Example 7 Find the sum of 6 terms of the G.P. 1, 3, 9 - - -

Solution : Here, a = 1, r = 3, n = 6

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_6 = \frac{1(3^6 - 1)}{3 - 1}$$

$$= \frac{729 - 1}{2}$$

$$= \frac{728}{2} = 364$$

Example 8 How many terms of the G.P. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ - - - will make the sum $\frac{63}{64}$

Solution : Here, $a = \frac{1}{2}$, $r = \frac{1}{2}$, $S_n = \frac{63}{64}$

Now $S_n = \frac{a(1-r^n)}{1-r}$

$$\frac{63}{64} = \frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^n\right]}{\left(1-\frac{1}{2}\right)}$$

$$\frac{63}{64} = \frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^n\right]}{\frac{1}{2}}$$

$$\left(\frac{1}{2}\right)^n = \frac{1-63}{64}$$

$$\left(\frac{1}{2}\right)^n = \frac{64-63}{64}$$

$$\left(\frac{1}{2}\right)^n = \frac{1}{64}$$

$$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^6$$

$\therefore$ n = 6

Thus 6 terms are required to make the sum $\frac{63}{64}$

Example 9 The first and last term of the G.P. is 3 and 96 respectively, sum to n terms is 189. Find the common ratio and the number of terms.

Sol. By data $a = 3$, $T_n = 96$, $S_n = 189$

$T_n = 96$

$\Rightarrow ar^{n-1} = 96$

$\Rightarrow 3r^{n-1} = 96$

$$r^{n-1} = \frac{96}{3}$$

$r^{n-1} = 32$

Now, $S_n = 189$

$$\Rightarrow \frac{a(1-r^n)}{1-r} = 189$$

$$\frac{3(1-r.r^{n-1})}{1-r} = 189$$

$$\frac{3(1-32r)}{1-r} = 189 \qquad [\because r^{n-1} = 32]$$

$$\frac{1-32r}{1-r} = 63$$

$1 - 32r = 63 - 63r$

$63r - 32r = 63 - 1$

$31\,r = 62$

$$r = \frac{62}{31} = 2$$

Now, $r^{n-1} = 32$

$r^{n-1} = 2^5$

$n - 1 = 5$

$\Rightarrow n = 5 + 1$

$n = 6$

$\therefore$ The common ratio is 2 and the number of terms is 6.

Example 10 Find the sum to 'n' terms of the G.P. 7 + 77 + 777 - - - -

Solution : Let S_n = 7 + 77 + 777 - - - - n terms

$= 7\,[1 + 11 + 111 + \text{- - - - to n terms})$

$= \frac{7}{9}\,[9 + 99 + 999 + \text{- - - - to n terms}]$

$= \frac{7}{9}\,[(10\text{-}1) + (100 - 1) + (1000 - 1) + \text{— - - to n terms}]$

$= \frac{7}{9}\,[(10 + 10^2 + 10^3 + \text{- - to n terms}) - (1 + 1 + 1 \text{ - - to n terms}]$

$= \frac{7}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$ [$\because$ $10+10^2+10^3$——to n terms is in GP

Where a = 10, r = 10]

$$S_n = \frac{7}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$$

Example 11 Find the sum to n terms of the G.P. 0.6 + 0.66 + 0.666 + ...

Solution : Let S_n = 0.6 + 0.66 + 0.666 + …… to n terms

$= 6\,[0.1 + 0.11 + 0.111 + \text{- - - - to n terms}]$

$= \frac{6}{9}\,[0.9 + 0.99 + 0.999 + \text{. . . . to n terms})$

$$= \frac{6}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\left(1-\frac{1}{1000}\right)+\text{------to n terms}\right]$$

$$= \frac{6}{9}\left[(1+1+1+\text{----to n terms})-\left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\text{--to n terms}\right)\right]$$

$$= \frac{6}{9}\left[n-\frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right]$$

$$= \frac{6}{9}\left[n - \frac{1}{10} \times \frac{10}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

$$S_n = \frac{6}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right] = \frac{2}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

Example 12 Find the sum to infinity of the G.P. 3, -1, $\frac{1}{3}, \frac{-1}{9}$

Solution : Here a = 3, $r = \frac{-1}{3} < 1$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{3}{1+\frac{1}{3}}$$

$$= \frac{9}{4}$$

13. The sum to infinity of geometric series is 6 and the sum of first two terms is $\frac{9}{2}$. Find the first term and the common ratio.

Sol. By data, $S_\infty = 6$ and $a + ar = \frac{9}{2}$

Where a is the first term, r is the common ratio.

$$\Rightarrow a(1 + r) = \frac{9}{2}$$

$$a = \frac{9}{2(1+r)}$$

Now, $S_\infty = 6$

$$S_\infty = \frac{a}{1-r}$$

$$\Rightarrow \quad 6 = \frac{a}{1-r} \qquad \text{subtitle for 'a'}$$

$$\Rightarrow \quad 6 = \frac{9}{2(1+r)(1-r)}$$

$$12\ (1 - r^2) = 9$$

$$\Rightarrow \quad 1 - r^2 = \frac{9}{12}$$

$$\Rightarrow \quad 1 - r^2 = \frac{3}{4}$$

$$\Rightarrow \quad r^2 = 1 - \frac{3}{4}$$

$$r^2 = \frac{1}{4}$$

$$r = \pm \frac{1}{2}$$

Now,

When $r = \frac{1}{2}$, $a = \dfrac{9}{2\left(1+\frac{1}{2}\right)}$

$$= \frac{9}{2} \times \frac{2}{3}$$

$$= 3$$

When $r = \frac{-1}{2}$, $a = \dfrac{9}{2\left(1-\frac{1}{2}\right)}$

$$= \frac{9}{2} \times \frac{2}{1}$$

$$= 9$$

EXERCISE : 5.2

I. One mark questions:

1. Find the
   a) sixth element of the GP 3, 6, 12, - - -
   b) seventh element of the GP $\sqrt{2},\ 2,\ 2\sqrt{2}$ – – – –
   c) tenth element of the G.P. $\frac{1}{7}, \frac{3}{7}, \frac{9}{7}$ - - -
   d) twelth element of the G.P. 0.5, 1.5, 4.5 - - -

2. Find the sum of the following G.P.
   a) $1, \frac{1}{4}, \frac{1}{16}$ - - - to 5 elements
   b) $\sqrt{3}, 1, \frac{1}{\sqrt{3}}$ - - - - to 7 elements
   c) -2, 4, -8 - - - - to 8 elements
   d) 0.1, 0.3, 0.9 - - - to 10 elements

3. If $\frac{5}{2}$, K, 10 are in GP then find the value of K.

4. Find the sum to infinity of the G.P. a) $1, \frac{1}{2}, \frac{1}{4}$, - - -
   b) $3, 1, \frac{1}{3}$ - - -

II. Two marks questions:

1. Which element of the G.P. 4, 6, $\frac{18}{2}$ - - is 81/4?
2. Which element of the G.P. 5, 10, 20, - - - is 80?
3. How many terms of the G.P. 1, 3, 9 – - will amount to 364?.
4. How many terms of the G.P. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ - - - will amount to $\frac{31}{32}$?

III. Three marks questions:

1. The third and fifth element of G.P. are 3 and 27 respectively. Find the eighth element.
2. The third element of G.P. is twice the second element and the fifth element is 32. Find the G.P.
3. Find the three numbers in G.P. whose sum is 39 and their product is 729.
4. Find the three numbers in G.P. whose sum is $\frac{13}{3}$ & product of the extremes is 1
5. The sum of the first eight elements of G.P. is five times the sum of the first four terms. Find the common ratio.
6. The first term of GP exceeds the second term by ½ and the sum to infinity is 2. Find the G.P.

IV. Five marks questions:

1. Find the sum to n terms of the G.P.
   a) 5 + 55 + 555 + - - - -
   b) 4 + 44 + 444 + - - -
   c) 0.3 + 0.33 + 0.333 + - - - -
   d) 0.5 + 0.55 + 0.555 + - - - -
2. The first and the last elements of a GP are 4 and 128 respectively and the sum is 252. Find the common ratio and the number of terms.
3. The sum of an infinite G.P. whose common ratio is less than one, is 32 and the sum of the first two terms is 24. Find the G.P.

ANSWERS : 5.2

I. 1. a) 96, b) $8\sqrt{2}$ c) $\frac{19,683}{7}$ d) 88, 573.5

2. a) $\frac{341}{256}$ b) $\frac{3}{\sqrt{3}-1}\left(1-\frac{1}{27\sqrt{3}}\right)$ c) $\frac{510}{3}$ d) $\frac{1}{20}$ $(3^{10}-1)$

3. K = 5

4. a) 2 b) $\frac{9}{2}$

II. 1) 6 2) 5 3) 6 4) 5

III. 1) 729 2) 2, 4, 8, 16 - - - 3) 3, 9, 27

4) 3, 1, $\frac{1}{3}$ 5) $\pm\sqrt{2}$ or ± 1 6) 1, $\frac{1}{2},\frac{1}{4}$ - - -

IV. 1) a) $\frac{5}{9}\left[\frac{10}{9}(10^n-1)-n\right]$

b) $\frac{4}{9}\left[\frac{10}{9}(10^n-1)-n\right]$

c) $\frac{3}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

d) $\frac{5}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

2) Common ratio = 2, number of terms = 6

3) 16, 8, 4, 2 - - - - -

5.11 Harmonic Progression

Definition: A sequence is said to be in Harmonic Progression if the reciprocals of its terms form an A.P.

i.e. If a, a+d, a + 2d - - - +a + (n-1) d is in AP

Then $\frac{1}{a},\frac{1}{a+d},\frac{1}{a+2d}$ - - - - + $\frac{1}{a+(n-1)d}$ is in H.P.

Ex. I) 2, 4, 6, 8 - - is in A.P. $\Rightarrow \frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8}$ . . . is in H.P.

5.12 nth term of a H.P.

If T_n is the n^{th} term of H.P. then $\frac{1}{T_n}$ is the n^{th} term of A.P.

If $\frac{1}{T_n} = a + (n-1)\,d$ is in A.P. then

$$\mathbf{T_n = \frac{1}{a+(n-1)d}}$$ is the n^{th} term of H.P.

WORKED EXAMPLES :

Example 1 Find the 10^{th} term of H.P. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$

Solution : Given $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$ - are in H.P.

$\Rightarrow$ 2, 4, 6 are in A.P.

$$\begin{aligned} 10^{th} \text{ term of an A.P.} &= 2 + (10-1)\,2 \\ &= 2 + (9)\,(2) \\ &= 2 + 18 \\ &= 20 \end{aligned}$$

$\therefore$ 10^{th} term of H.P. $= \frac{1}{20}$

Example 2 If the third term of a H.P. is $\frac{1}{7}$ and fifth term is $\frac{1}{11}$, then find the seventh term

Solution : Third term of a H.P. is $\frac{1}{7}$

$\Rightarrow$ Third term of an A.P. is 7

Fifth term of a H.P. is $\frac{1}{11}$

$\Rightarrow$ Fifth term of an A.P. is 11

$$
\begin{array}{r}
a+2d=7 \\
\underset{(-)}{a}+\underset{(+)}{4d}\underset{(-)}{=}11 \\
\hline
-2d=-4
\end{array}
$$

d = 2

a = 7 – 2d

= 7 – 2(2)

= 7 – 4

= 3

$\therefore$ Seventh term in H.P. is $\dfrac{1}{3+6(2)}=\dfrac{1}{3+12}=\dfrac{1}{15}$

Example 3 If $\dfrac{2}{3}$, $x, \dfrac{1}{2}$ are in H.P. find x.

Solution : Given $\dfrac{2}{3}$, $x, \dfrac{1}{2}$ are in H.P.

$\Rightarrow$ $\dfrac{3}{2},\dfrac{1}{x}$, 2 are in A.P.

$$\Rightarrow \quad \frac{1}{x}-\frac{3}{2}=2-\frac{1}{x}$$

$$\frac{1}{x}+\frac{1}{x}=\frac{3}{2}+2$$

$$\frac{2}{x}=\frac{7}{2}$$

$$\frac{x}{2}=\frac{2}{7}$$

$$\therefore \quad x=\frac{4}{7}$$

Example 4 If b + c, c + a, a + b are in H.P. show that a^2, b^2, c^2 are in A.P.

Solution : Given b + c, c + a, a + b are in H.P.

$$\Rightarrow \quad \frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b} \text{ are in A.P.}$$

$$\Rightarrow \quad \frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\frac{b+c-c-a}{(c+a)(b+c)} = \frac{c+a-a-b}{(a+b)(c+a)}$$

$(b - a)(a + b) = (b + c)(c - b)$

$b^2 - a^2 = c^2 - b^2$

$2b^2 = a^2 + c^2$

$$b^2 = \frac{a^2 + c^2}{2}$$

$\Rightarrow$ a^2, b^2, c^2 are in A.P.

EXERCISE : 5.3

I. One mark questions:

1. Find the 5^{th} element of a H.P. $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$ - - - -

2 Find the 9^{th} element of a H.P. $\frac{1}{7,}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}$ - -

3. Find the 7^{th} element of a H.P. $\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{1}{3\sqrt{2}}$ ——

4. Find x, if $\frac{1}{3}, x, \frac{3}{2}$ are in H.P.

II. Two marks questions:

1. If fourth term of the H.P is $\frac{3}{4}$ and seventh term of the HP is $\frac{2}{3}$, find the 10^{th} term.
2. If 5^{th} term is $\frac{1}{5}$ and seventh term is $\frac{6}{5}$, find the 10^{th} term.
3. If a, b, c are in G.P. and $a^x=b^y=c^z$, show that x, y, z are in H.P.
4. If p^{th} element of an H.P is q and q^{th} element is p, show that $(pq)^{th}$ element is 1.

ANSWERS : 5.3

I.

1) $\frac{1}{11}$ 2) $\frac{1}{15}$ 3) $\frac{1}{7\sqrt{2}}$ 4) $\frac{6}{11}$

II.

1) $\frac{3}{5}$ 2) $\frac{-12}{65}$

5.13 Arthimetic, Geometric and Harmonic means:

Arthimetic Mean:

If a, A, b are in AP, then A is called the arithmetic mean between a and b.

Since a, A,b, are inb A.P.

$$A - a = b - A$$

$$\Rightarrow 2A = a + b$$

$$\Rightarrow \mathbf{A} = \frac{\boldsymbol{a+b}}{\mathbf{2}}$$

Thus, A.M of a and b is $\frac{1}{2}(a+b)$

Example: The A.M. of 2 and 6 is $\frac{2+6}{2}=4$

Geometric Mean:

If a, g, b are in G.P, then g is called the geometric mean between a and b

Since a, g, b are in G.P.

$$\frac{g}{a} = \frac{b}{g}$$

$\Rightarrow$ $g^2 = ab$

$\Rightarrow$ $\mathbf{g = \sqrt{ab}}$

Thus, G.M. of a and b is $\sqrt{ab}$

Example: G.M. between 3 and 12 is $\sqrt{3 \times 12} = \sqrt{36} = 6$

Harmonic Mean:

If a, H, b are in H.P. then H is called harmonic mean between a and b

Since a, H, b are in H.P, then $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A.P.

Thus $\frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H}$

$$\frac{1}{H} + \frac{1}{H} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{2}{H} = \frac{a+b}{ab}$$

$$\mathbf{H = \frac{2ab}{a+b}}$$

Thus Harmonic mean between a and b is $H = \frac{2ab}{a+b}$

Ex: H.M. of 1 and 2 is $\frac{2(1)\,(2)}{1+2} = \frac{4}{3}$

WORKED EXAMPLES:

Example 1 Insert 4 A.M. between 5 and 10

Solution : Let A_1, A_2, A_3, A_4 be the 4 A.M's between 5 and 10.

$\therefore 5, A_1, A_2, A_3, A_4, 10$ are in A.P.

Last element is 6^{th} element $a + 5d = 10$

$$5 + 5d = 10$$

$$5d = 10 - 5$$

$$5d = 5$$

$$d = \frac{5}{5} = 1$$

$\therefore$ A.M's are 6, 7, 8, 9

Example 2 Insert 3 G.M's between $\frac{1}{4}$ *and* $\frac{1}{64}$

let g_1, g_2, g_3 be the 3 G.M's

between $\frac{1}{4}$ *and* $\frac{1}{64}$

Solution: $\therefore \frac{1}{4}, g_1, g_2, g_3, \frac{1}{64}$ are in G.P.

Here $T_5 = \frac{1}{64}$

$$\Rightarrow ar^4 = \frac{1}{64}$$

$$\frac{1}{4} r^4 = \frac{1}{64}$$

$$r^4 = \frac{4}{64}$$

$$r^4 = \frac{1}{16}$$

$$r^4 = \frac{1}{24}$$

$$r = \frac{1}{2}$$

$\therefore$ The 3 G.M's are $\frac{1}{8}, \frac{1}{16}, \frac{1}{32}$

Example 3 Insert 4 H.M.'s between $\frac{1}{3}$ and $\frac{1}{13}$

Solution:

Let H_1, H_2, H_3, H_4 be the 4 H.M.'s between $\frac{1}{3}$ *and* $\frac{1}{13}$

$\therefore \frac{1}{3}, H_1, H_2, H_3, H_4, \frac{1}{13}$ are in HP

$\Rightarrow 3, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, 13$ are in AP

Here $T_6 = 13$

$a + 5d = 13$

$3 + 5d = 13$

$5d = 13 - 3$

$5d = 10$

$d = \frac{10}{5} = 2$

$\therefore$ The four harmonic means are $\frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}$

EXERCISE : 5.4

I. Two marks questions:

1. Insert 7 A.M.'s between 3 and 11
2. Insert 3 A.M.'s between – 2 and – 10
3. Insert 3 G.M.'s between $\frac{1}{3}$ *and* 27
4. Insert 3 G.M.'s between -4 and – 64
5. Insert 3 H.M.'s between $\frac{1}{4}$ *and* $\frac{1}{12}$

ANSWERS : 5.4

I.

1. 4, 5, 6, 7, 8, 9, 10
2. -4, -6, -8
3. 1, 3, 9
4. - 8, -16, -32
5. $\frac{1}{6}, \frac{1}{8}, \frac{1}{10}$

Chapter 6

THEORY OF EQUATIONS

6.1 Introduction:

In mathematics, the theory of equations comprises of major part of traditional algebra. This chapter includes polynomials, algebraic equations of the type of linear equations, quadratic, cubic equations, etc. It also includes a brief illustration of synthetic division. It is used in problem solving.

An equation is a statement that says two algebraic expressions are equal and is satisfied only for certain values of the variables.

In other words equations signify relation between two algebraic expressions symbolized by the sign of equality.

The two equal expressions are called members of the equation. While the left hand expression is called left hand side (L.H.S.) and the right hand expression is called right hand side (R.H.S.).

To solve an equation means to find the value of values of the variable (unknown quantities) satisfying the equation, these values are known as the roots of the equation. Solution in q the equation. The process of finding the root is called solving.

Examples:

i) The root of the equation $3x - 9 = 0$ is $x = 3$

ii) Take the equation $x^2 - x = 6$. When we give $x =- 2$ & $x = 3$, both sides becomes equal. Hence $x = 3$ and $x = -2$ are the solutions of the equation.

6.2 Degree of the equation:

The degree of the equation is the highest power of the variable involved in the equation.

For example:

- **Equations in one variable:-**

$2x + 3 = 0$ - The degree of equation is 1.
$x^2 + 3x = 4$ - The degree of equation is 2.
$x^7 - 3x^5 = x + 1$ - The degree of equation is 7.

- **Equations in two variables:-**
  $2x + 3y = 5$ - The degree of equation is 1.
  $2x^2 + 3y + 2 = 1$ - The degree of equation is 2.
  $7x^4 + 6y^4 + 5x^3 + 3y = 1$ - The degree of equation is 4.

6.3 Some General principles of solving simple linear equations.

1. The equations do not alter if:
   a) Same quantity is added or subtracted from both sides of the equation.
   b) If both sides of an equation are multiplied or divided by the same quantity except zero.
2) Any term of an equation may be transferred from one side to another side, while doing so, the sign is changed.
3) If fractions occur in the equation, then they are cleared by multiplying both sides by L.C.M. of the denominators. This process is also known as cross multiplication.
4) To multiply an equation by a quantity means that each term of the equation is multiplied by the same quantity.
5) To add two equations means the addition of the respective sides of those equations.
6) To solve a simple equation, bring all the terms containing known terms to one side and unknown to other sides. Finally divide the R.H.S. by the coefficient of the variable.

Examples:

1. Solve the equation: $2(x - 3) = 9 + 3(x - 9)$
   $2x - 6 = 9 + 3x - 27$
   $2x - 3x = 9 + 6 - 27$
   $-x = -12 \Rightarrow x = 12$

2. If $3(x + 5) - 25 = 9 + 2(x - 7)$ find x
   $3x + 15 - 25 = 9 + 2x - 14 \Rightarrow 3x - 10 = 2x - 5$
   $3x - 2x = -5 + 10$
   $\Rightarrow x = 5$

3. If $3(4x + 1) - (4x - 1) = 2(x + 5)$ solve for x

$\Rightarrow 12x + 3 - 4x + 1 = 2x + 10$

$\Rightarrow 8x + 4 = 2x + 10$

$\Rightarrow 8x - 2x = 10 - 4$

$\Rightarrow 6x = 6$

$\therefore x = 1$

4. Solve $\frac{x+2}{x-1} = \frac{5}{2}$

$\Rightarrow 2(x + 2) = 5(x-1)$

$2x + 4 = 5x - 5$

$4 + 5 = 5x - 2x$

$9 = 3x$

$\therefore x = 9/3 = 3$

5. $\frac{2x-7}{2x-1} = \frac{x-3}{x+3}$

$\Rightarrow (2x-7)(x + 3) = (2x - 1)(x - 3)$

$2x^2 + 6x - 7x - 21 = 2x^2 - 6x - x + 3$

$2x^2 - x - 21 = 2x^2 - 7x + 3$

$2x^2 - x - 2x^2 + 7x = 3 + 21$

$6x = 24 \Rightarrow x = \frac{24}{6} = 4$

6. Solve for x : $b(b + x) = a^2 - ax$

$\Rightarrow bx^2 + bx = a^2 - ax$

$ax + bx = a^2 - b^2$

$x(a + b) = (a + b)(a - b)$

$\therefore x = (a - b)$

7. $\frac{x}{2}+\frac{2x}{3}=\frac{7}{2}$

Multiply both sides by the L.C.M. of 2 & 3 (i.e.6)

$\Rightarrow \frac{6x}{2}+\frac{6\times 2x}{3}=\frac{6\times 7}{2}$

$3x + 4x = 21$

$7x = 21 \quad \Rightarrow \quad x = 3$

8. Solve for x : $3 - [3 +\{ x - (3+x)\}] = 5 + 2x$

$\Rightarrow 3x - \{3 + x - 3 -x) = 5 + 2x$

$3x - 2x = 5$

$x = 5$

$\Rightarrow \quad x = 5$

Problems leading to linear equations in one variable.

Simple linear equation useful in solving problems encountered in commerce and problems related with numbers, fractions, ages, speed etc.

The procedure with one variable is follows:-

i) Assume the unknown quantity as x

ii) Translate the verbal statement in to algebraic expression and/or equations.

iii) Solve the equation and find the value of x.

iv) If desired check your results.

Examples 1 The sum of two consecutive numbers is 23, find them.

Solution : Let the two consecutive numbers be x and $x + 1$

$x + x +1 = 23$

$2x = 23 - 1$

$2x = 22 \quad \Rightarrow x = 11$

$\therefore$ The numbers are 11, 12.

Example 2 The sum of three consecutive numbers is 183, find them.

Solution: Let the three consecutive numbers be x, $x + 1$, $x + 2$

Given $x + x + 1 + x + 2 = 183$

$$3x + 3 = 183 \Rightarrow 3x = 180 \Rightarrow x = \frac{180}{3} = 60$$

$\therefore$ The 3 numbers are 60, 61, and 62.

Example 3 The sum of four consecutive numbers is 366, find them.

Solution : Let the three consecutive numbers be x, $x + 1$, $x+2$, $x+3$

Given $x + x + 1 + x + 2 + x + 3 = 366$

$$4x + 6 = 366$$

$$4x = 366 - 6 = 360$$

$$x = \frac{360}{4} = 90$$

Hence the numbers are 90, 91, 92, and 93.

Example 4 Find two numbers whose sum is 64 and whose difference is 16.

Solution : Let the two numbers be x and $64-x$

Given: $(64-x) - x = 16$

$$64 - 2x = 16$$

$$-2x = 16 - 64$$

$$-2x = -48$$

$$x = 24$$

Thus the two numbers is 24 & $64-24 = 40$.

Example 5 Two numbers are in the ratio 5:8 their difference is 15, find the numbers.

Solution : Let the two numbers be $5x$ & $8x$

Given: $8x - 5x = 15$

$$3x = 15 \Rightarrow x = 5$$

Hence the Numbers are 5×5 and $5x8 = 25$ & 40.

Example 6 The sum of 6 times a number & 5 times the number is 55. Which is that number?

Solution: Let the number be x

$\therefore$ 6 times the number is $6x$ and similarly 5 times the number is $5x$

Given: $6x + 5x = 55$

$11x = 55 \Rightarrow x = 55/11 = 5$

Hence the number is 5.

Example 7 Divide ₹1,600/- between x, y & z. So that y may have ₹100 more than x and z ₹200 more than y.

Solution : Let x's share be ₹ a.

$\therefore$ y's share is ₹ 100 + a

& z's share is ₹ 200 + 100 + a

Given a + 100 + a + 200 + 100 + a = 1600

3a + 400 = 1600

3a = 1200 $\Rightarrow$ a = 400

$\therefore$ x, y & z's share is ₹400, ₹500 and ₹700 respectively.

Example 8 The age of the father is four times that of the son. 5 years ago, the age of the father was 7 times that of his son. Find their present ages.

Solution : Let the present age of the son be x years.

5 years ago. The age of the son was $x - 5$ & that of the father was $4x - 5$

Given: $4x - 5 = 7(x - 5)$

$4x - 5 = 7x - 35$

$-5 + 35 = 7x - 4x$

$3x = 30 \Rightarrow x = 10$ and $4x = 4 \times 10 = 40$ years

$\therefore$ The present age of the father is 40 years and that of his son is 10 years.

Example 9 The present ages of two brothers are in the ratio 3:4. Five years back their ages were in the ratio 5:7. Find this present ages.

Solution : Let the ages of two brothers be $3x$ and $4x$

5 years back their ages were $3x - 5$ and $4x - 5$

Given: $(3x - 5) : (4x - 5) = 5:7 \Rightarrow 7(3x - 5) = 5(4x - 5)$

$\Rightarrow 21x - 35 = 20x - 25.$

$x = 10$

Hence present ages are 3×10 & $4 \times 10 = 30$ & 40.

Example 10 A mother is 32 years older than her son. After 4 years, the mother's age will be 8 years more than twice that of her son. Find their present ages.

Solution : Let the present age of the son = x years

$\therefore$ The present age of the mother = $x + 32$ years

After 4 years age of the son = $x + 4$ years

After 4 years age of the mother = $x + 32 + 4 = x + 36$ years.

Given that: $x + 36 = 8 + 2(x + 4)$

$x + 36 = 8 + 2x + 8$

$36 - 16 = x$

$x = 20$

$\therefore$ The present age of the son is 20 years.

And the present age of the mother is $20 + 32 = 52$ years.

6.4 Simultaneous Linear Equations in two variables.

When two or more equations are satisfied by the same values of the unknown quantities. They are called simultaneous equations.

For the system of simultaneous linear equations. The number of equations should be equal to the number of variables. There are four methods of solving simultaneous linear equations in 2 variables.

I. **Method of comparison:** In this method variable say x from each of the equation is expressed in terms of the other variable. Then comparing those two equations, we get the value of y. Substituting the value of y in any of the equations given, we get the value of x.

Example: Consider: $x + 2y = 7$ ....(1)

& $2x - y = 4$ ....(2)

From equation (1) we get: $x = 7 - 2y$ (3)

From equation (2) we get: $2x = 4 + y \Rightarrow x = \dfrac{4+y}{2}$ (4)

Comparing equation 3 & 4 we get: $\dfrac{7-2y}{1} = \dfrac{4+y}{2}$

$2(7 - 2y) = 4 + y \Rightarrow 14 - 4y = 4 + y$

$14 - 4 = 4 + 4y$

$5y = 10$

$y = 2$

Substituting the value of given equation (3)

We get: $x = 7 - 2y = 7 - 2\times2 = 7 - 4 = 3 \Rightarrow x = 3$.

II. **Method of Substitution:** In this method the values of y (or x) is found in terms of x (or y) from an equation. And substituting this value in the other equation we get a linear equation of one variable. Solving this equation, we get the value of x (or y) putting this value in any of the equation given we get the value of y (or x).

Example: Consider $x + 2y = 1$

$3x - 2y = 5$

>Let $x + 2y = 1$ ....(1)

$3x - 2y = 5$ ....(2)

From equation (1) we have: $x = 1 - 2y$ ....(3)

Substitute the value of x in equation (2) we get:

$3(1 - 2y) - 2y = 5$

i.e. $3 - 6y - 2y = 5 \Rightarrow 3 - 8y = 5 \Rightarrow -8y = 5 - 3 = 2$

$\Rightarrow y = -1/4$

Substituting the value of y in equation (3) we get

$x = 1 - 2y$

$= 1 - 2\left(\frac{-1}{4}\right) = 1 + \frac{1}{2} = \frac{3}{2}$

$\therefore x = \frac{3}{2}$ and $y = \frac{-1}{4}$

III. **Method of Elimination:** In this method, the given equations are transformed to equivalent equations, so that co-efficient of any one of the variables in both the transformed equations became numerically equal. Solving the equations either by adding or subtracting, we get one of the variables.

Example consider the equations, $2x - 3y = 19$ and
$3x + 2y = 9$

Let $2x - 3y = 19$ .....(1)
$3x - 2y = 9$ .....(2)

Multiply equation (1) by 2 & equation (2) by 3, we get :

$4x - 6y = 38$
$9x - 6y = 27$

Adding the above equations: $13x = 65 \Rightarrow x = 65/13 = 5$

Now substituting the value of x in (1), we get:

$2x - 3y = 19$
$2\times5 - 3y = 19 \Rightarrow 10 - 19 = 3y \Rightarrow -9 = 3y \Rightarrow y = -3$
$y = 9/-3 = -3$
Hence $x = 5$, $y = -3$.

IV. **Method of cross multiplication:**

Consider two equations $a1\ x + b_1y + c_1 = 0$ ....(1)
$a_2x + b_2y + c_2 = 0$ ....(2)

Multiplying the 1st equation by a_2 and 2nd equation by a_1

We get: $a_1\ a_2x + a_2\ b_1y + a_2c_1 = 0$ ....(3)
$a_1a_2x + a_1\ b_2y + a_1c_2 = 0$ ....(4)

$(3) - (4) \Rightarrow \quad y\,(a_2\,b_1 - a_1\,b_2) + a_2c_1 - a_1c_2 = 0$

$$y = \frac{a_1c_2 - a_2c_1}{a_2b_1 - a_1b_2}$$

Similarly multiplying the 1st equation by b_2 and the 2nd equation by b_1 and subtracting the same we get:

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

This is called **rule of cross multiplication.** This can be easily performed in the following way

$$\begin{array}{cccccc} & x & y & 1 & & \\ a_1 & b_1 & c_1 & a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 & c_2 \end{array}$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$

Equating 1st and 3rd, we get: $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$

Equating 2nd and 3rd, we get: $y = \dfrac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$

Example Solve $x + 2y - 4 = 0$
$3x + y - 7 = 0$

$$\begin{array}{cccccc} & x & y & 1 & & \\ 1 & 2 & -4 & 1 & 2 & -4 \\ 3 & 1 & -7 & 3 & 1 & -7 \end{array}$$

$$\frac{x}{(2\times -7) - (1\times -4)} = \frac{y}{(-4\times 3) - (-7\times 1)} = \frac{1}{1\times 1 - 3\times 2}$$

$$\frac{x}{-10} = \frac{y}{-5} = \frac{1}{-5}$$

$$\frac{x}{-10} = \frac{1}{-5} \Rightarrow x = +2, \ \frac{y}{-5} = \frac{1}{-5} \Rightarrow y = +1$$

$\therefore x = 2$ and y = 1.

Example : Solve the following equations in all the four methods.

i) $x + 2y = 4$ ....(1)

$3x + y = 7$ ....(2)

I. Method of comparison

$x + 2y = 4$ ....(1)

$x = 4 - 2y$ ....(3)

$3x + y = 7$ ....(2)

$3x = 7 - y$

$x = \frac{7-y}{3}$ ....(4)

Comparing (3) and (4), we get:

$$\frac{4-2y}{1} = \frac{7-y}{3}$$

$3(4 - 2y) = 7 - y$

$12 - 6y = 7 - y$

$-6y + y = 7 - 12$

$-5y = -5 \Rightarrow y = \frac{-5}{-5} = 1 \Rightarrow y = 1$

Put y = 1 in equation (3) we get

$x = 4 - 2y = 4 - 2 = 2$

$\therefore x = 2$ and y = 1.

Verification: $x + 2y = 4$

$2 + 2x1 = 4$

$2 + 2 = 4 \Rightarrow$ L.H.S=R.H.S.

II. **Method of substitution:**

$x + 2y = 4 \quad (1) \quad 3x + y = 7 \quad (2)$

From equation (1) : $x = 4 - 2y \quad (3)$

Substitute the value of x in equation (2)

We get: $3x + y = 7$

$3(4 - 2y) + y = 7$

$12 - 6y + y = 7$

$12 - 7 = 6y - y \Rightarrow 5y = 5 \Rightarrow y = \frac{5}{5} = 1$

Substitute the value of y = 1 in equation (3), we get:

$x = 4 - 2y$

$x = 4 - 2x1= \quad 4 - 2 = 2$

$\therefore x = 2$ and $y = 1$.

III. **Method of Elimination:**

$x + 2y = 4 \quad (1)$

$3x + y = 7 \quad (2)$

Multiply equation (1) by 3, we get:

$3x + 6y = 12 \quad (3)$

$3x + y = 7 \quad (4)$

$(3) - (4) \quad \Rightarrow \quad 5y = 5 \Rightarrow \; y = 1$

Put y = 1 in equation (1), we get:

$x + 2y = 4$

$x + 2x1 = 4 \Rightarrow x = 4 - 2 = 2$

$\therefore x = 2$ and $y = 1$ is the solution.

IV. **Method of cross Multiplication:**

$x + 2y - 4 = 0$

$3x + y - 7 = 0$

		x	y	1		
1	2	-4	1		2	-4
3	1	-7	3		1	-7

$$\frac{x}{(2\times -7)-(-4\times 1)}=\frac{y}{(-4\times 3)-(-7\times 1)}=\frac{1}{(1\times 1)-(3\times 2)}$$

$$\frac{x}{-14+4}=\frac{y}{-12+7}=\frac{1}{1-6}$$

$$\frac{x}{-10}=\frac{y}{-5}=\frac{1}{-5}\Rightarrow \frac{x}{-10}=\frac{1}{-5}\Rightarrow x=\frac{-10}{-5}=2$$

$$\Rightarrow \frac{y}{-5}=\frac{1}{-5}\Rightarrow y=\frac{-5}{-5}=1$$

$\therefore$ $x = 2$ and $y = 1$ is the solution.

Application problems:

Example 1 Five years ago, father's age was 5 times as old as his son and after 3 years he will be 3 times as old as his son. Find their present ages.

Solution : Let the age of the father be x years and the age of the son be y years.

Given: $x - 5 = 5(y - 5)$

i.e. $x - 5 = 5y - 25$

$x - 5y + 20 = 0$ ....(1)

Also given: $x + 3 = 3(y + 3)$

$x + 3 = 3y + 9$

$x - 3y - 6 = 0$ ....(2)

Solving (1) and (2), we get: $x = 45$ and $y = 13$

Hence father's age is 45 years and son's age is 13 years.

Example 2 A father is 28 years older than the son, after 5 years the father's age will be 7 years more than twice that of the son. Find their present ages.

Solution: Let father's age be x years and son's age be y years.

Given : $x - y = 28$ ....(1)

After five years father's age is $x + 5$ and son's age is y+ 5.

Also given: $x + 5 = 2(y + 5)+7$

$x + 5 = 2y + 10+7$

Solving (1) and (2), we get: $x = 44$ and $y = 16$

This father's age is 44 years and son's age is 16 years.

Example 3 Nine tables and eight chairs cost ₹456/- Eight tables and nine chairs cost ₹462/-. Determine the cost of each table and of each chair.

Solution : Let x be the cost of a table and y be the cost of a chair.

Then the given data in terms of equations are:

$9x + 8y = 456$ ....(1) &

$8x + 9y = 462$ ....(2)

Solving the above equations we get $x = 24$ and $y = 30$

Hence the cost of each table = ₹24. And each chair is ₹30.

Example 4 Divide 25 into two parts that the sum of the reciprocals is 1/6.

Solution : Let the two parts be x and y

Given: $x + y = 25$ ....(1) and $\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$

$6x + 6y = xy$ ....(2)

Multiply equation (1) by 6, we get:

$6x + 6y = 150$ ....(3)

Solving 2 and 3 we get $0 = 150 - xy \Rightarrow xy = 150$ ....(4)

Also we have $(x - y)^2 = (x + y)^2 - 4xy$

$= (25)^2 - 4 .150$

$= 625 - 600$

$(x - y)^2 = 25$

$\Rightarrow x - y = 5$ ....(5)

Solving equations (1) and (5) , we get :

$x = 15$ and $y = 10$

Hence the two parts are 15 and 10.

Example 5 A number consists of two digits and whose sum is 3, if 9 is added to the number the digits get interchanged. Find the numbers.

Solution: Let the digit in the ten's place be x, and digit in units place be y.

$\therefore$ The number is $10x + y$ ....(1)

Given: $x + y = 3$ ....(2)

Also given $10x + y + 9 = 10y + x$

$9x - 9y + 9 = 0$ ....(3)

$x - y + 1 = 0$

Solving equations 2 and 3 we get $x = 1$, and $y = 2$

$\therefore$ The two digits number is $10x + y = 10\times1 + 2$

$= 10 + 2 = 12$.

Example 6 A certain two digits number is 2 times the sum of the digits, if 63 is added to the number the digits get interchanged. Find the number.

Solution : Let the digit in ten's place be and digit in units place to y.

$\therefore$ The number is $10x + y$

Given $10x + y = 2(x + y)$

$8x - y = 0$ ....(1)

Also given $10x + y + 63 = 10y + x$

$9x - 9y + 63 = 0 \Rightarrow x - y + 7 = 0$ ....(2)

Solving (1) and (2), we get: $x = 1$ and $y = 8$

$\therefore$ The required number is $10x + y = 10\ 1 + 8 = 10 + 8 = 18$.

Example 7 Divide ₹110 into two parts so that 5 times of one part together with 6 times of the other part will be equal to ₹610.

Solution : Let x & y be the two parts

Then the number will be $x + y$

Given: $x + y = 110$ ....(1)

Also given: $5x + 6y = 610$ ....(2)

Solving (1) and (2), we get: $x = 50$ and $y = 60$

Hence the two parts are ₹50 and ₹60.

Example 8 Two numbers are in the ratio of 4:5 and if 24 is subtracted from each of them, the resulting numbers are in the ratio of 2: 3. Find the numbers.

Solution : Let the two numbers be x and y.

Given $x: y = 4: 5 \Rightarrow 5x = 4y$

$5x - 4y = 0$ ....(1)

Also given $(x - 24) : (y - 24) = 2 : 3$

$3(x - 24) = 2(y - 24)$

$3x - 2y = 24$ ....(2)

Solving (1) and (2), we get: $x = 48$ and $y = 60$

Hence the two numbers are 48 and 60.

Example 9 Two brothers have their annual income as 8:5, while their annual expenditures are in the ratio 5:3, if they save ₹1200/- and ₹1000/- per annum. Find their incomes.

Solution : Let the income be x and expenditure be y.

So the income of two brothers would be $8x$ and $5x$ and expenditures would be 5y and 3y.

W.K.T. Income – Expenditure = Saving

$\therefore$ We get:

$8x - 5y = 1200$ ....(1)

$5x - 3y = 1000$ ....(2)

Solving (1) & (2), we get: $x = 1400$ & $y = 2000$

$\therefore$ Annual incomes of two brothers are $8x$ and $5x$

$= 8\times 1400$ & $5 \times 1400 = 11200$ and 7000.

Example 10 The incomes of three persons Anil, Akbar, and Antony as 6 : 5 : 4 and their expenditure are in the ratio 3 : 2 : 1. If Anil saves ₹120 out of his income of ₹1500. Find the savings of Akbhar and Antony.

Solution: Let the income be x and the expenditure by y.

$\therefore$ Incomes are $6x$, $5x$ and $4x$ and their expenditures are 3y, 2y and y.

Given: Anil's income = ₹1500/-

$\Rightarrow \quad 6x = 1500 \Rightarrow \quad x = 250$

Anil's expenditure = 3y = Income – Savings

$3y = 1500 - 120$

$3y = 1380 \Rightarrow y =$ ₹460

Thus Akbar's income = $5x = 5 \times 250 =$ ₹1250

Akbar's expenditure = $2y = 2 \times 460 =$ ₹920

Antony's income = $4x = 4 \times 250 =$ ₹1000

Antony's expenditure = y = ₹460

EXERCISE : 6.1

I. Solve the following: (1 mark each)

1 $2(7 + x) - 10 = 16 - 2(x - 24)$

2. $(x + 2)(x + 3) = (x - 2)(x - 4) + 20$

3. $x + a(x + b) = ax + b$

4. $7(x - 2) + 8(x - 3) - 22 = x + 10$

5. $3(x - 2) - (x - 1) = 7(x - 1) - 6(x - 2)$

6. $3(x + 5) - 25 = 9 + 2(x - 7)$

7. $8x + 17x - 51 = 16x - 36 + 12$

8. $7x - 5[x - \{7 - 6(x - 3)\}] = 3x + 1$

II. Solve the following: (2 marks each)

9. $\frac{x+2}{5} = \frac{x-1}{2}$

10. $\frac{x}{4} + \frac{x-5}{3} = 10$

11. $\frac{x+3}{7} - 2 = \frac{x-4}{8} - 1$

12. $\frac{x-5}{10} + \frac{x+5}{5} = 5$

13. $\frac{x+19}{5} - 3 = \frac{x}{4}$

14. $x + \frac{3x-5}{4} = 2 + \frac{6x-8}{5}$

15. $\frac{1}{3} - 5 + \frac{6}{2x} = \frac{2}{x}$

III. Solve the following equations by comparison method, substitution method. Elimination method and cross multiplication method: (3 marks each)

16. $x + 2y = 1$
    $3x - 2y = 5$

17. $x + 2y = 7$
    $2x - y = 4$

18. $2x - 3y = 19$
    $3x + 2y = 9$

19. $10x - 9y = 12$
    $3x - 9 = 17$

20. $x + 2y = 4$
    $3x + y = 7$

21. $2x + 3y = 8$
    $3x + 2y = 7$

22. $2x + y = 14$
    $3y = 33 + x$

23. $5x + 2y = 8$
    $9x - 5y = 23$

24. $5x - 2y + 25 = 0$
    $4y - 3x = 29$

25. $4x - y = 2$
    $-3x + 2y = 1$

IV. Statement problems: (3 marks each)

26. The sum of two consecutive numbers is 151. Find them
27. The sum of three consecutive numbers is 186 Find them.
28. The sum of two numbers is 107 and their difference is 17. Find the numbers.
29. Two numbers are in the ratio 7: 5 and their difference is 12. Find the numbers
30. Two numbers are in the ratio 5:6 and if 12 are subtracted from each of them. The resulting numbers are in the ratio 3: 4. Find the numbers.
31. Divide 36 into two parts such that the sum of the reciprocals is 1/8.
32. The sum of 4 times a number and 3 times the number is 70. Find the number.

V. Statement problems contd.: (5 marks each)

33. A number consists of 2 digits whose sum is 4, if 18 is added to the number, the digits get interchanged. Find the number.

34. A certain number is 4 times the sum of the digits. If 18 is added to the number, the digits get interchanged. Find the numbers.

35. A sum of two numbers is 21. If the larger is divided by the smaller, the quotient is 2 and the remainder is 3. Find the numbers.

36. The age of father is 5 times that of son. 3 years ago, the age of the father was 8 times that of his son. Find their present ages.

37. Three years ago father was 4 times as old as his son and after 5 years he will be three times as old as his son. Find their present ages.

38 12 Statistics books and 8 Mathematics books cost ₹204/- and 9 statistics books and 6 Mathematics books cost ₹153/-. Find the cost of each type of book.

39. The present ages of two brothers are in the ratio of 3:4. Four years ago, their ages were in the ratio of 2:3 find their present ages.

40 Two sisters have their monthly incomes in the ratio 7:5 and their monthly spending is in the ratio 5:3. If each saves ₹60/- per month, find their incomes.

ANSWERS : 6.1

I.	1) $x = 15$	2) 2	3) b-ab	4) $x = 5$
	5) $x = 10$	6) $x = 5$	7) $x = 3$	8) $x = 4$
II.	9) $x = 3$	10) $x = 20$	11) $x = 4$,	12) $x = 15$
	13) $x = 16$	14) $x = 3$	15) $x = 3/4$	
III.	16) $\frac{3}{2}, \frac{-1}{4}$	17) 3, 2	18) 5, -3	19) $\frac{-5}{7}, \frac{-134}{65}$
	20) 2, 1	21) 1, 2	22) $\frac{9}{7}, \frac{80}{7}$	23) 2, -1
	24) – 3, 5	25) 1, 2		
IV.	26) 75, 76	27) 61, 62, 63	28) 62, 45	29) 42, 30
	30) 10, 12	31) 24, 12	32) $x = 10$	
V.	33) 13,	34) 24	35) 6, 15	36) 35 & 5
	37) 65, 19	38) 9, 12	39) 12 & 16	40) 210, 150

6.5 Quadratic Equations:

The equation of the form ax^2 + bx + c = 0 (a ≠0) containing x^2 as the highest power of x is called quadratic equation or a second degree equation.

The quadratic equation has two and only two roots. These two roots may be equal or unequal. If α and β are the 2 roots. Then the equation will be of the form: (x - α) (x-β) =0.

Ex. Form the Quadratic – Equation whose roots are 2 & 3

$(x - 2)(x - 3) = 0$

There are two methods of finding the roots of the quadratic equation.

(1) Factorization method (2) Formula method.

Factorization Method: First, the quadratic equation is reduced to the standard form. Factorize the expression on the left side. Equate each factors to zero, solve the corresponding linear equations

Ex. 1)

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(x - 3) + 1(x - 3) = 0$

$(x + 1)(x - 3) = 0 \Rightarrow \quad x = -1$ or $x = 3$

2)

$2x^2 - 5x + 2 = 0$

$2x^2 - 4x - x + 2 = 0$

$2x(x - 2) - 1(x - 2) = 0$

$(2x - 1)(x - 2) = 0 \Rightarrow \quad x = 1/2$ or $x = 2$

Formula Method:

The solutions of the quadratic equation ax^2 + bx + c = 0 is obtained using the formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If α & β are the two root of the quadratic equation.

Then $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ & $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

Example 1 $x^2 - 4x + 3 = 0$

Compare with: $ax^2 + bx + c = 0$

a = 1, b = -4, c = 3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4.1.3}}{2.1}$$

$$= \frac{4 \pm \sqrt{16 - 12}}{2} = \frac{4 \pm \sqrt{4}}{2} = \frac{4 \pm 2}{2}$$

$$x = \frac{4+2}{2} \;\&\; \frac{4-2}{2} = 3,1$$

Example 2 $x^2 + 6x + 7 = 0$

a = 1, b = 6, c = 7

$$x = \frac{-6 \pm \sqrt{36 - 28}}{2} = \frac{-6 \pm \sqrt{8}}{2} = \frac{-6 \pm 2\sqrt{2}}{2} = -3 \pm \sqrt{2}$$

Thus the roots are. $-3 + \sqrt{2} \;\& -3 - \sqrt{2}$

6.6 Nature of the roots of $ax^2 + bx + c = 0$

The quantity $b^2 - 4ac$ on which the nature of the roots depend, is called the discriminant of the quadratic equation $ax^2 + bx + c = 0$ and is denoted by Δ or D.

1. If $b^2 - 4ac = 0$. Then the roots are real and equal.
2. If $b^2 - 4ac$ is positive. Then the roots are real and unequal.
3. If $b^2 - 4ac$ is negative. Then the two roots are unequal and imaginary.

Example 1 $4x^2 + 12x + 9 = 0$

A = 4, b = 12, c = 7

$\Delta = b^2 - 4ac = 12^2 - 2.9.4 = 144 - 144 = 0$

Thus the roots are real and equal.

Example 2 $3x^2 + 8x + 5 = 0$

a = 3, b = 8, c = 5

$\Delta = b^2 - 4ac = 8^2 - 4.3.5 = 64 - 60 = 4\ (> 0)$

Thus the roots are real and unequal

Example 3 $x^2 + x + 4 = 0$

a = 1, b = 1, c = 4

$\Delta = b^2 - 4ac = 1^2 - 4.4.1 = 1-16=-15 < 0$ (-ve)

The roots are unequal and imaginary.

Relation between the roots and the co-efficients of the quadratic equation $ax^2 + bx + c = 0$:

Let α and β are the roots of the equation $ax^2 + bx + c = 0$

Dividing throughout by a

$$\Rightarrow \quad x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \qquad \text{..........(1)}$$

$$x^2 - (\alpha + \beta)\, x + \alpha \,.\, \beta = 0 \qquad \text{..........(2)}$$

$(x -)\ (x - \beta) = 0$

Comparing (1) and (2), we get:

$$\text{Sum of the roots} = \alpha + \beta = \frac{-b}{a} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of the roots} = \alpha\beta = \frac{c}{a} = \frac{\text{constant}}{\text{coefficient of } x^2}$$

Example 1 If α & β are the roots of the equation $2x^2 + 3x + 7 = 0$

Find the value of i) $\alpha + \beta$ (ii) $\alpha\beta$ (iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ iv) $\alpha^2 \beta + \alpha\beta^2$

Solution : Comparing with the standard form: a = 2, b = 3, c = 7

We know that, (1) $\alpha + \beta = \frac{-b}{a} = \frac{-3}{2}$

$$(2)\ \alpha.\ \beta = \frac{c}{a} = \frac{7}{2}$$

$$(3)\ \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-3/2}{7/2} = \frac{-3}{7}$$

$$(4)\ \alpha^2\beta + \alpha\beta^2 = \alpha\beta\ (\alpha + \beta) = \frac{+7}{2} \times \frac{-3}{2} = \frac{-21}{4}$$

Example 2 If α & β are the roots of the equation $3x^2 - 6x + 4 = 0$.

Find the values of the following:

1) $\alpha^2 + \beta^2$ 2) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ 3) $\alpha^3 + \beta^3$

4) $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$ 5) $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} = \frac{\alpha^3 + \beta^3}{(\alpha\beta)^2}$

Solution: Comparing with the standard equation:

a = 3, b = -6, c = 4

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-6)}{3} = 2$$

$$\alpha\beta = \frac{c}{a} = \frac{4}{3}$$

1) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (2)^2 - 2.\frac{4}{3} == \frac{12-8}{3} = +\frac{4}{3}$

2) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{4/3}{4/3} = 1$

3) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 = 3\alpha\beta\ (\alpha + \beta) = 2^3 - \frac{3.4}{3}(2) = 8 - 8 = 0$

Example 3 If α and β are the roots of the equation $2x^2 - 10x + 5c = 0$

Find the value of : 1) $\alpha^2 + \beta^2$ 2) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

3) $^3 + \beta^3$ 4) $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$

5) $\alpha^3 \beta + \alpha\beta^3$ 6) $\alpha^4\beta + \alpha\beta^4$

7) $\frac{\alpha^3 + \beta^3}{\alpha^2 + \beta^2}$

Solution: a = 2, b = -10, c = 5

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-10)}{2} = 5$$

$$\alpha\beta = \frac{c}{\alpha} = \frac{5}{2}$$

1) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 5^2 - \frac{5}{2} = 25 - 5 = 20$

2) $$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{20}{\frac{5}{2}} = \frac{40}{5} = 8$$

3) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta)$

$$= 5^3 - 3.\frac{5}{2}(5)$$

$$= 125 - \frac{75}{2} = \frac{250 - 75}{2} = \frac{175}{2}$$

4) $$\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} = \frac{\alpha^3 + \beta^2}{(\alpha\beta)^2} = \frac{\frac{172}{2}}{\left(\frac{5}{2}\right)^2} = \frac{175}{2} \times \frac{4}{25} = 14$$

5) $\alpha\beta(\alpha^2 + \beta^2) = \frac{5}{2}.20 = 50$

6) $\alpha\beta\ (\alpha^3 + \beta^3) = \frac{5}{2} \times \frac{175}{2} = \frac{875}{4}$

7) $\frac{\alpha^3 + \beta^3}{\alpha^2 + \beta^2} = \frac{175/2}{20/1} = \frac{175}{2} \times \frac{1}{20} = \frac{35}{8}$

EXERCISE : 6.2

I. Form the quadratic equation whose roots are: (1 mark each)

(i) 1, 2 (ii) 2, -3

II. Solve the equations by Factorization method:

1. $2x^2 - 7x + 3 = 0$
2. $x^2 - 4x + 3 = 0$
3. $x^2 - 3x - 10 = 0$
4. $x^2 + x - 6 = 0$
5. $4x^2 + 4x = 3$
6. $9x^2 - 22x + 8 - 0$
7. $4x^2 + 4x - 15 = 0$
8. $6x^2 - 5x - 21 = 0$

III. Solve the following by Formula method: (2 marks each)

9. $3x^2 - 13x + 12 = 0$
10. $x^2 + 3x - 28 = 0$
11. $5x^2 - 7x - 12 = 0$
12. $2x^2 - 7x = - 3$
13. $x^2 + 6x + 8 = 0$
14. $3x^2 - x - 10 = 0$
15. $2x\ (4x - 1) = 15$
16. $12x^2 + 23x = 24$

IV. Find the nature of the roots without solving the equation: (1 mark each)

17. $2x^2 + 6x + 3 = 0$
18. $2x^2 - 5x + 6 = 0$
19. $36x^2 - 12x + 1 = 0$
20. $6x^2 - 5x + 2 = 0$
21. $x^2 - x + 1 = 0$
22. $2x^2 + 8x + 9= 0$

V. Solve the following: (3 marks each)

23. If α and β are the roots of the equation. $ax^2 + bx + c = 0$. Find the values of :

(1) $(1 + \alpha)(1 + \beta)$ 2) $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$

24. If α and β are the roots of the equation, $2x^2 + 4x + 1 = 0$. Find the value of :

1) $\alpha^2 \beta + \beta^2 \alpha$ 2) $\alpha^{-2} + \beta^{-2}$

25. If α and β are the roots of the equation $x^2 + 3x + 7 = 0$. Find the values of

1) $\alpha^3 + \beta^3$ 2) $\frac{\alpha}{\beta} + \frac{\beta^2}{\varepsilon}$

26. If α and β are the roots of the equation $2x^2 + 5x + 5 = 0$ then find the values of :

(1) $\frac{1}{\alpha} + \frac{1}{\beta}$ 2) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ 3) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

27. If α and β are the roots of the equation $2x^2 - 5x + 7 = 0$. Find the values of

(1) $\alpha^2 + \beta^2$ 2) $\alpha + \beta + \alpha\beta$ 3) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

28. If α and β are the roots of $3x^2 - 6x + 4 = 0$. Find the value of:

$$\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \left(2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3(\alpha + \beta)\right)$$

29. If α & β are the roots of $2x^2 - 10x + 5 = 0$. Find the value of :

$$\left(\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}\right) + 2\left(\frac{\beta}{\alpha} + \frac{\alpha}{\beta}\right) - 12\alpha\beta$$

30. If α & β are the roots of the equations $3x^2 + 2x + 1 = 0$. Find the values of

i) $\alpha\beta^2 + \alpha^2\beta$ ii) $\dfrac{1}{\alpha+2\beta}+\dfrac{1}{\beta+2\alpha}$

ANSWERS : 6.2

I. (i) $(x-1)(x-2)=0$ ii) $(x-2)(x+3)=0$

II. 1) $\frac{1}{2}, 3$ 2) 3, 1, 3) 5, -2 4) 3, -2,

5) $\frac{1}{2}, \frac{-3}{2}$ 6) 2, $\frac{4}{9}$ 7) $\frac{3}{2}, \frac{-5}{2}$ 8) $\left(\frac{1}{6}, \frac{1}{6}\right)$

III. 9) $(3, \frac{4}{3})$ 10) 4, -7 11) 2.4, -1 12) $\frac{1}{2}, 3$

13) (- 2, -4) 14) $\left(2, \frac{-5}{3}\right)$ 15) $\frac{-5}{4}, \frac{3}{2}$ 16) $\left(\frac{-8}{3}, \frac{-3}{4}\right)$

IV 17) Real and Unequal 18) Unequal and Imaginary

19) Real and Equal 20) Unequal and Imaginary

21) Unequal and Imaginary 22) Unequal and Imaginary

V. 23. $\left(\frac{a-b+c}{a}, \frac{3abc-b^3}{ac^2}\right)$

24. (-1, 12) 25) $\left(36, \frac{36}{7}\right)$ 26) $\frac{-3}{5}, \frac{-11}{25}, \frac{-11}{10}$

27. $\left(\frac{-3}{4}, 6, \frac{-85}{28}\right)$ 28) 10 29) 0 30) $\left(\frac{-2}{9}, \frac{-12}{11}\right)$

6.7 Cubic Equations:

An equation of the form $ax^3 + bx^2 + cx + d = 0$ $(a \neq 0)$ where a, b, c, d are real constants is called a cubic equation or third degree equation. It has 3 roots.

If α, β, γ are the roots of the equation then the equation will be of the form $(x - \alpha)(x - \beta)(x - \gamma) = 0$.

Example 1 Form the cubic equation whose roots are 3, 4, 5 .
The required solution is $(x - 3)(x - 4)(x - 5) = 0$.

Example 2 Form the cubic equation whose roots are 2, -3, 1 .
The required solution is $(x - 2)(x + 3)(x - 1) = 0$.

6.8 Synthetic Division:

Consider a polynomial F(x) of degree 3, where

$$f(x) = a_0 x^3 + a_1 x^2 + a_2 x + a_3$$

Supposing we divide f(x) by a linear expression say $(x - h)$ then we get the quotient as $Q(x) = b_0 x^2 + b_1 x + b_2$ is of 2^{nd} degree and the remainder R as a constant.

The successive coefficients b_0, b_1 and b_2 of the quotient polynomial, Q(x) and the remainder R can be obtained by means of simple procedure known as **synthetic division**. This process is explained below.

$$\begin{array}{lc|cccc}
1^{st}\text{ row} \rightarrow & h & a_0 & a_1 & a_2 & a_3 \\
2^{nd}\text{ row} \rightarrow & & & b_0 h & b_1 h & b_2 h \\
\hline
3^{rd}\text{ row} \rightarrow & & b_0(-a_0) & b_1 & b_2 & \multicolumn{1}{|c}{R}
\end{array}$$

Explanation:

Write all the coefficients of different powers of x of the given polynomial in the first row. (If any term is absent in the given polynomial, corresponding to that we write zero for its coefficient)

In the extreme left corner of the 1^{st} row, write 'h' which is called the <u>Multiplier or operator</u> (this is the value of x when the divisor is equated to zero)

Now first term of the third row is $b_0 - a_0$ multiply $(b_0 - a_0)$ with h and write it below a_1 and add to a_1 we get $b_1 = (a_1 + b_o h)$. This will be second element in the third row.

Again multiply b_1 with h and write it below a_2 add to get b_2. This will be the third element in the third row. Now multiply b_2 with h and write it below a_3 and add to get R_1 the remainder, which is the last sum obtained in the third row is the reminder R.

Note: If the remainder R = 0. Then we say $(x - h)$ is a factor of the polynomial $f(x)$ under consideration i.e. h is a root of the equation $f(x) = 0$ and conversely.

Examples:

1. Find the quotient and the remainder obtained by dividing $x^3 + 4x^2 - 7x - 10 = 0$ by $(x + 1)$

Solution: Here the multiplier is -1 $(\because x + 1 = 0 \Rightarrow x = -1)$

Let us remove the root -1 by synthetic division.

$x = -1$	1	4	-7	-10
		-1	-3	+10
	1	3	-10	0

Quotient = $x^2 + 3x - 10 = 0$ and Remainder = 0 .

2. Find the quotient and remainder obtained by dividing $4x^3 + 3x^2 - 2x - 1$ by $(x + 1)$

Solution: Here the multiplier is –1

$x = -1$	4	3	-2	-1
		-4	+4	+1
	+4	-1	-1	0

Quotient = $4x^2 - x - 1 = 0$ and Remainder = 0.

3. Find the quotient and remainder when $x^4 + 10x^3 + 39x^2 + 76x + 65$ is divided by $x + 4$.

Solution: $x = -4$ is the multiplier.

$x = -4$	1	10	39	76	65	
		-4	-24	-60	-64	
	1	6	15	16	1	= R

$\therefore$ Quotient $= x^3 + 6x^2 + 15x + 16$ and Remainder $= 1$

4. Find an integral root between - 3 & 3 by inspection and then using synthetic division. Solve the equation $x^3 - 2x^2 - 5^x + 6$

Solution: Let $f(x) = x^3 - 2x^2 - 5^x + 6$

$f(1) = 1 - 2 - 5 + 6 = 7 - 7 = 0$

$\therefore$ $x = 1$ is a root of the given equation. Let us remove this root by Synthetic division.

$x = 1$	1	-2	-5	6	
		1	-1	-6	
	1	-1	-6	0	= R

$\therefore$The result and equation is $x^2 - x - 6 = 0$ is the quotient and remainder $= 0$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or $- 2$

Thus $x = 1, -2, 3$ are the roots of the given equation.

5. Obtain a root of the equation $x^3 - 2x^2 - 2^x + 3 = 0$ by inspection and hence solve the equation.

Solution: Let $f(x) = x^3 - 2x^2 - 2^x + 3$

$f(1) = 1 - 2 - 2 + 3 = 4 - 4 = 0$

$\therefore$ $x = 1$ is a root of the given equation. Let us remove the root +1 by synthetic division.

$$\begin{array}{c|cccc}
x = 1 & 1 & -2 & -2 & 3 \\
 & & 1 & -1 & -3 \\
\hline
 & 1 & -1 & -1 & \boxed{0}
\end{array}$$

$\therefore$ The resultant equation is $x^2 - x - 3 = 0$ and R = 0

$$x = \frac{1 \pm \sqrt{1+12}}{2} = \frac{1 \pm \sqrt{13}}{2}$$

$\therefore$ The roots are 1, $\frac{1+\sqrt{13}}{2}, \frac{1-\sqrt{13}}{2}$

EXERCISE : 6.7

I. Form cubic equation whose roots are (1) 3, 5, 7 (2) (-1,4,6)

II. (3) By Synthetic Division, find the quotient & remainder when $2x^3$-$5x^2$- $32x + 6$

(4) Find the quotient & the remainder obtained by dividing $3x^3$-$4x^2$+$2x$+1 by x - 3.

III. Find an integral root between - 3 and 3 by inspection & then using synthetic division solve the following equations:

(5) $x^3 + 6x^2 + 9x + 4 = 0$

(6) $x^3 + 15x^2 - 72x + 76 = 0$

(7) $x^3 - 10^2 + 29x - 20 = 0$

(8) $x^3 + 2x^2 - 11x - 12 = 0$

(9) $x^3 - 3x^2 - 28x + 60 = -$

(10) $x^3 - 2x^2 - 29x - 42 = 0$

ANSWERS : 6.7

I. (1) $x^3 - 15x^2 - 17x - 105 = 0$ (2) $x^3 - 9x^2 + 14x + 24 = 0$

II. (3) $2x^2 + x - 29$, -81 (4) $3x^2 + 5x + 17$, 52

III. (5) -1, -1, -4 (6) 2, 2, -19, (7) 1, 4, 5

(8) -1, 3, -4 (9) (2, -5, 6) (10) (-2, -3, 7)

Chapter 7

LINEAR INEQUALITIES

7.1 Introduction:

"Larl Friedrich Gauh" a French Mathematician developed the theory of numbers. In this chapter we consider the expression, which involve signs such as '<' (less than), '>' (greater than) '$\leq$' (less than or equal) and $\geq$ (greater than or equal) called Inequalities and such expressions is generally called **"Inequations'.**

In this chapter we will study linear inequalities in one and two variable. The study of inequalities is very useful in solving problems in the field of science, mathematics, statistics, optimization problems, economics etc., It plays a very important role in Linear programming problems.

7.2 Definition of Inequality:

If two real numbers or two algebraic expression related by the symbol '<', '>', '$\leq$' or '$\geq$' form an Inequality

Examples:

$3< 5, 10>8$,	$5x<6$,	$4x - 3 < 8$,	$3x - 7>0$,	$4x\leq 7$,
$7x - 5 \geq 0$,	$y \leq 4$,	$2x + 4y \leq 7$,	$x - 4y \geq 10$	etc.,

(1) The Inequality in the form $3< 5$, $7> 5$, are the example of **numerical inequalities.**

(2) The Inequality in the form $x < 5$, $y > x \geq 3$, $y \leq 7$ etc. are called **literal inequalities.**

(3) The inequality in the form $3 < 5 < 7$ (read as 5 is greater than 3 and less than 7)

$3 \leq x < 5$ (read x is greater than or equal to 3 and less than 5)

$2 < y \leq 4$ (y is greater than 2 and less than or equal to 4)

are the example of **double inequalities.**

4. Inequalities in the form: $ax + b < 0$
$ax + b > 0$
$ax + by < c$
$ax - by > c$ are **strict inequalities**

5. Inequalities in the form: $ax + b \leq 0$
$ax - b \leq 0$
$ax + by \geq c$
$ax - by \leq c$, are **slack inequalities**

6. inequalities in the form: $ax + b < 0$
$ax - b \leq 0$
$ax + b > 0$
$ax + b \geq 0 \, (a \neq 0)$ are, inequalities in **one variable** 'x'

7. Inequalities in the form: $ax + by < c$
$ax + by > c$
$ax + by \leq c$
$ax - by \geq c$, $(a \neq 0, \ b \neq 0)$
are inequalities in **two variable** x & y

8. Inequalities in the form: $ax^2 + bx + c \leq 0 \quad (a \neq 0, \ b \neq 0)$ and $ax^2 + bx + c > 0$ are **quadratic inequalities** in one variable x

SOME IMPORTANT SETS:

N	=	Set of Natural numbers	=	{1, 2, 3, ...}
W	=	Set of whole numbers	=	{0, 1, 2, 3 ....}
I or Z	=	Set of Integers	=	$\{0, \pm 1, \pm 2, \pm 3 \, ...\}$
I^+	=	Set of +ve Integers	=	{1, 2, 3 ....}
I^-	=	Set of –ve integers	=	{-1, -2, -3, ...}
Q	=	set of rational numbers	=	$\{p/q, \ p,q \in I, \ q \neq 0\}$
R	=	Set of real numbers	=	{all Natural, whole, Integer, rational Irrational numbers}
R^+	=	Set of +ve Real Numbers		
R^-	=	Set of –ve Real numbers		

7.3 Linear Inequalities in one variable:

A linear inequality in one variable is an expression of the form $ax + b < c$, $ax+b>c$, $ax + b \leq c$ or $ax + b \geq c$ where a, b, c $\in$ R (set of Real numbers and $a \neq 0$).

Example $5x + 3 < 4$, $4x - 5 \geq 7$, $8x - 1 \leq 3$, $3x + 1 < 7$, are linear inequations.

In the above examples we can find the values of 'x' which makes the above in equality a statement. True values of x are called solutions of inequality.

Solution set : A solution set of inequality is the set of all real numbers (R) that satisfies the inequality the method of finding the solution set of the inequality is known as solving the inequation

Rules for solving the linear inequation:

Rule 1 : The inequality does not change if the same number is added on both the side of the inequality.

Rule 2: (**Multiplication or Division Rule**) The inequality does not change if we multiply or divide both the side of inequality by the same +ve real number.

Rule 3: The inequality reverses its direction if we multiply or divide both the side of the inequality by the same –ve real number.

i.e. If $a < b \Rightarrow a \pm K < b \pm K$ $(K \in R)$

$a > b \Rightarrow a \pm K > b \pm K$ $(K \in R)$

$a > b \Rightarrow aK < bK$ ($K < 0$ i.e. K is –ve)

NOTE:

'O' Represent Solution Exclude

'•' Represent include

∞ Infinity

( ) both side excludes

( ] left excludes and right include

[ ) Left Include and Right exclude

[ ] both side include

Example 1

Solve linear inequalities in one variable and represent the solution on the number line.

a) $2x + 6 < 0, x € Z$

$2x \leq - 6 \quad \therefore x \leq -3$

$\therefore$ Solution set $(- \infty, -3]$

b) $7x + 3 < 5x + 9, x € R$

$7x – 5x < 9 – 3$

$2x < 6$

$x < 3 \quad \therefore$ Solution set $(- \infty, 3)$

c) $$\frac{3x-4}{2} \geq \frac{x+1}{4} - 1, x € R$$

$$\frac{3x-4}{2} \geq \frac{x+1-4}{4} \text{ (taking the LCM on R.H.S)}$$

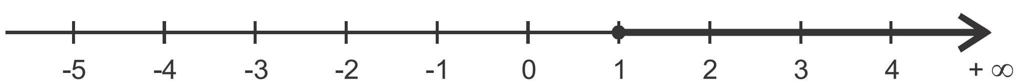
$$\geq$$

$4 (3x – 4) \geq 2 (x – 3)$

$12x – 16 \geq 2x – 6$

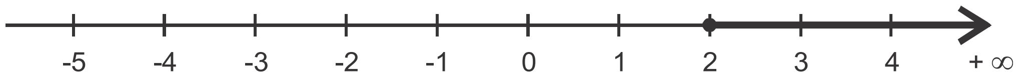
$12x – 2x \geq -6 + 16$

$10x \geq 10 \quad \therefore x \geq 1 \qquad \therefore$ solution set $[1,\infty)$



d) $5x - 10 \geq 0, x \in R$

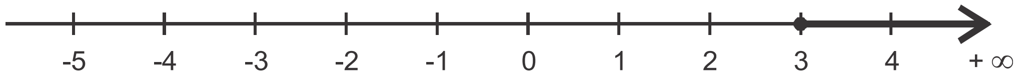
$5x \geq 10 \quad \therefore x \geq 2$ Solution set $[2, \infty)$



e) $3x - 9 > 0, x \in R$

$3x > 9$

$\therefore x > 3$ Solution set $(3, \infty)$



f) $5x - 3 < 3x + 1, x \in R$

$5x - 3x < 3 + 1$

$2x < 4$

$\therefore x < 2,$ Solution set $(-\infty, 2)$

g) $\dfrac{5-2x}{3} \leq \dfrac{x}{6} - 5, x \in R$

$\dfrac{5-2x}{3} \leq \dfrac{x-30}{6}$

$6(5 - 2x) \leq 3(x - 30)$

$30 - 12x \leq 3x - 90$

$30 + 90 \leq 12x + 3x$

$120 \leq 15x$

$15x \geq 120 \quad \therefore x \geq \dfrac{120}{15}$

$\therefore x \geq 8,$ Solution set $[8, +\infty)$

Example 2

Solve the following Inequalities:

i) $x+\frac{2}{15}\leq\frac{-8}{15}$, x € R

Solution: $\frac{15x+2}{15}\leq\frac{-8}{15}$

$15x + 2 \leq - 8$

$15x \leq - 8 - 2$

$15x \leq - 10$

$x \leq - \frac{10}{15}$

$\therefore x \leq -\frac{2}{3} \in R$

ii) $\frac{1}{2}\left[\frac{3x}{2}+4\right]\geq\frac{1}{3}(x-6)$, x € R

Solution: $\frac{1}{2}\left[\frac{3x+8}{2}\right]\geq\frac{x-6}{3}$

$\frac{3x+8}{4}\geq\frac{x-6}{3}$

$3(3x + 8) \geq 4(x - 6)$

$9x + 24 \geq 4x - 24$

$9x - 4x \geq -24 - 24$

$5x \geq - 48 \quad \therefore x \geq -48/5$

iii) $\frac{(2x+1)}{2} + 2(3 - x) \geq 7$, x € R

Solution: $2x + 1 + 12 - 4x \geq 14$

$- 2x + 13 \geq 14$

$\therefore -2x \geq 14 - 13$

$- 2x \geq 1$

÷ by – 2 $\quad \therefore x \leq - \frac{1}{2}$

iv) $3\ (2 - x) \geq 2\ (1 - x),\ x \in R$

Solution : $6 - 3x \geq 2 - 2x$

$6 - 2 \geq 3x - 2x$

$4 \geq x$

$\therefore x \leq 4$

v) $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$, $x \in R$

Solution: $\frac{x}{4} < \frac{5(5x-2)-3(7x-3)}{15}$ LCM

$\frac{x}{4} < \frac{25x-10-21x+9}{15}$

$\frac{x}{4} < \frac{4x-1}{15}$

$15x < 4\ (4x - 1)$

$15x < 16x - 4$

$4 < 16x - 15x$

$4 < x$OR $x > 4$

vi) $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$, $x \in R$

Solution: $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

$3\ (3x - 6) \leq 5\ (10 - 5x)$

$9x - 16 \leq 50 - 25x$

$25x + 9x \leq 50 + 18$

$34x \leq 68$

$x \leq \frac{68}{34}$

$\therefore x \leq 2$

vii) $x+\frac{x}{2}+\frac{x}{3}<11$

Solution: $\frac{x}{1}+\frac{x}{2}+\frac{x}{3} < 11$

$\frac{6x+3x+2x}{6}<11$

$11x < 66$

$x < \frac{66}{11} \therefore x < 6$

viii) $\frac{x}{14} < \frac{5x-2}{3}-\frac{7x-3}{5}, x € R$

Solution: $\frac{x}{14}<\frac{5x-2}{3}-\frac{(7x-3)}{5}$

$\frac{x}{14}<\frac{5(5x-2)-3(7x-3)}{15}$

$\frac{x}{14}<\frac{25x-10-21x+9}{15}$

$\frac{x}{14}<\frac{x-1}{15}$

$4(x - 1) < 15x$

$14x - 4 < 15x$

$-4 < 15x - 14x$

$-4 < x$

or $x > -4$

ix) $\frac{x+2}{x+3} \geq -1, x$ €R

Solution: $\frac{x+2}{x+3} \geq -1$

$$\frac{x+2}{x+3} + \frac{1}{1} \geq 0$$

$$\frac{(x+2)+(x+3)}{x+3} \geq 0$$

$$\frac{2x+5}{x+3} \geq 0$$

$$2x + 5 \geq 0$$

$$x \geq -\frac{5}{2}$$

x) The marks obtained by a student of class in first and second term exam are 62 and 48 respectively. Find the minimum marks he should get in the annual examination to have an average of at least 60 marks?

Solution Let 'x' be the marks obtained by student in the annual exam, then,

$$\frac{62+48+x}{3} \geq 60 \quad \text{(atleast)}$$

$$110 + x \geq 60 \times 3$$

$$110 + x \geq 180$$

$$x \geq 180 - 110$$

$$x \geq 70$$

$\therefore$ The student must obtain a minimum of 70 marks to get an average of atleast 60 mark.

xi) Find all pair of consecutive add natural numbers, both of which are larger than 10, such that their sum is less than 40?

Solution Let 'x' be the smaller of the two consecutive add natural numbers. So that the other one is $(x + 2)$. Thus we have $x > 10$ ....(1) and $x + (x + 2) < 40$ ....(2)

xii) Find all the pair of consecutive even pair of number both of which are larger than 5, such that their sum is less than 28.

Solution Let 'x' be the smaller of the two consecutive even natural number, so that the other is $(x + 2)$.

Thus we have $\quad x > 5 \quad$ ....(1)

and $\quad x + (x + 2) < 28$ ....(2)

From (2) $\quad x + (x + 2) < 28$

$2x + 2 < 28$

$2x < 28 - 2$

$2x < 26$

$$x < \frac{26}{2}$$

$\therefore x < 13$ .... (3)

From (1) and (2) $\quad 5 < x < 13$

$\therefore$ The possible pair will be (6, 8), (8, 10), (10, 12)

xiii) The longest side of a triangle is 3 times the shorter side and the third side in 2cm shorter than the largest side. If the perimeter of the triangle is at least 61 cm. Find the minimum length of the shortest side.

Solution Let the length of the shortest side $= x$ cm

The length of the largest side $= 3x$ cm

And the length of the 3rd side $= (3x - 2)$ cm

Given the perimeter of the triangle is atleast 61 cm

$\therefore x + (3x) + (3x - 2) \geq 61$

$7x - 2 \geq 61$

$7x \geq 63$

$x \geq 63/7=9$

$\therefore$ the minimum length of the shortest side = 9cm

7.4 System of Linear Inequalities in one variable

Two or more linear inequalities in one variable together form a system of linear inequalities in one variable. The solution set of the system is the set of all values which satisfy all the inequation involved in the system.

i.e., the Intersection of the solution set of each is the solution of the system of linear inequation.

Example

1. **Solve :** $\frac{5x}{4} \geq \frac{5}{4}, -6 \leq x < 6, x \in \text{I}$

Solution:

$\frac{5x}{4} > \frac{5}{4}$

Multiply by 4 $\quad \therefore 5x > 5$

$\therefore x > 1$ ....(1)

Given $-6 < x < 6$ ....(2)

From (1) & (2) $\quad x = \{2, 3, 4, 5\}$ i.e., $1 < x < 6$

2. **Solve :** $2 \leq 2x - 3 \leq 5, \quad x \in \text{R}$

Solution: Give $2 \leq 2x - 3 \leq 5$

Add 3 through out

$\therefore 2 + 3 \leq 2x - 3 \leq 5 + 3$

$5 \leq 2x \leq 8$

$\div$ by 2 we get

$\frac{5}{2} \leq x \leq 4$

$\therefore 2.5 \leq x \leq 4$

3. **Solve:** $-2\frac{2}{3} \leq x + \frac{1}{3} < 3\frac{1}{3}, x \in \text{R}$

Solution: $-\frac{8}{3} \leq x + \frac{1}{3} < \frac{10}{3}$

By subtracting $\frac{1}{3}$,

$$\frac{-8}{3} - \frac{1}{3} \leq x + \frac{1}{3} - \frac{1}{3} < \frac{10}{3} - \frac{1}{3}$$

$$\frac{-9}{3} \leq x < \frac{9}{3}$$

$\therefore -3 \leq x < 3$

4. **Solve :** $3x - 7 > 2(x - 6)$ and $6 - x > 11 - 2x$, $x \in R$

Solution:

Give $3x - 7 > 2(x - 6)$

$3x - 7 > 2x - 12$

$3x - 2x > -12 + 7$

$x > -5$ .....(1)

again

$6 - x > 11 - 2x$

$2x - x > 11 - 6$

$x > 5$ .....(2)

From (1) & (2)

$\therefore x > 5$

5. **Solve :** $-2\frac{3}{4} \leq x + \frac{1}{4} < 4\frac{1}{4}$, $x \in R$

Solution:

The given inequalities can be split as

$-2\frac{3}{4} \leq x + \frac{1}{4}$ and $x + \frac{1}{4} < 4\frac{1}{4}$

$-\frac{11}{4} \leq x + \frac{1}{4}$ and $x + \frac{1}{4} < \frac{17}{4}$

$-\frac{11}{4} - \frac{1}{4} \leq x$ and $x < \frac{17}{4} - \frac{1}{4}$

$-\frac{12}{4} \leq x$ and $x < \frac{16}{4}$

$-3 \leq x$ and $x < 4$

$\therefore$ The combined (Intersection) solution is $-3 \leq x < 4$

6. **Solve:** $3 \geq \dfrac{x-4}{2} + \dfrac{x}{3} \geq 2,\ x \in I$

Solution:

Given $3 \geq \dfrac{x-4}{2} + \dfrac{x}{3} \geq 2$

$$3 \geq \frac{3(x-4)+2x}{6} \geq 2$$

$$3 \geq \frac{5x-12}{6} \geq 2$$

Multiples by 6 weget, $18 \geq 5x - 12 \geq 12$

Adding by 12 weget, $18 + 12 \geq 5x - 12 + 12 \geq, 12 + 12$

$30 \geq 5x \geq 24$

Dividing by 5 weget, $6 \geq x \geq 4.8$

$\because x \in I$ the solution set is $x = \{5, 6\}$

7. **Solve :** $2x + 3 \geq 18$ and $3x + 1 \leq 12$, x €R

Solution:

Given $2x + 3 \geq 18$ and $3x + 1 \leq 12$

$\therefore 2x \geq 18 - 3$ | $3x \leq 12 - 1$

$2x \geq 15$ | $3x \leq 11$

$x \geq \dfrac{15}{2}$ | $x \leq \dfrac{11}{3}$

$\therefore x \geq 7.5$ .....(1) | $\therefore x \leq 3.66$ .....(2)

8. **Solve:** $2(x-1) < x + 5$ and $3(x + 2) > 2 - x$

Solution:

$2x - 2 < x + 5$ and $3(x + 2) > 2 - x$

$2x - x < 5 + 2$ | $4x > 2 - 6$

$x < 7$ .....(1) | $4x > -4$

$x > \dfrac{-4}{4}$

$\therefore x > -1$ .....(2)

From (1) and (2)

$-1 < x < 7$

9. **Solve :** $5(2x\text{-}7) - 3\ (2x + 3) \leq 0$ and $2x + 19 \leq 6x + 47,\ x$ €I

Solution:

$5(2x\text{-}7) - 3\ (2x + 3) \leq 0$ and	$2x + 19 \leq 6x + 47$
$10\ x - 35 - 6x - 9 \leq 0$	$19 - 47 \leq 6x - 2\ x$
$4x - 44 \leq 0$	$-28 \leq 4x$
$4x < 44$	$-\dfrac{28}{4} \leq x$
$x \leq \dfrac{44}{4}$	$-7 \leq x$
$\therefore\ x \leq 11$ .....(1)	or $x \geq -7$ .....(2)

From (1) & (2) $-7 \leq x \leq 11$

10. **Solve:** $-3 \leq 4 - \dfrac{7x}{2} \leq 18,\ x$ € I

Solution:

$$-3 \leq \frac{8-7x}{2} \leq 18$$

Multiply by 2, $-6 \leq 8 - 7\ x \leq 36$

Subtract 8 $-6 - 8 \leq 8 - 7\ x - 8 \leq 36 - 8$

$-14 \leq -7x \leq 28$

Divide by -7 we get

$2 \geq x \geq -4$ or $-4 \leq x \leq 2$

EXERCISE : 7.3

Solve the following Inequalities in one variable.

1. **One mark/two mark**

1. $3x - 4 > 7 - 2\ x\ (x$ € R)
2. $5x - 3 < 7\ (x \in R)$
3. $4x - 2 < 8\ (x \in R)$

4. $5x - 3 < 3x + 1$ $(x \in R)$

5. $3(x - 1) \leq 2(x + 3)$ $(x \in R)$

6. $3x + 8 > 2$ when $x \in I$ and $x \in R$

7. $\frac{x}{3} > \frac{x}{2} + 1$ $(x \in R)$

8. Solve $3x - 2 < 2x + 1$, $(x \in R)$ Represent on number line

9. Solve $3(1 - x) < 2(x + 4)$, $(x \in R)$ Represent on number line

10. $\frac{x+5}{x-2} \leq 0, x \in I$

II. 3 mark question Solve the following Inequalities in one variable.

1. $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1, x \in R$

2. $\frac{5x-2}{3} - \frac{7x-3}{5} > \frac{x}{4}$

3. $\frac{x+1}{2} \geq \frac{2-x}{-3}$ $x \in I$

4. $\frac{2x+3}{4} - 4 < \frac{x-4}{3} - 2$ $, x \in R$

5. $37 - (3x + 5) \geq 9 - 8(x - 3), x \in I$

6. $2(2x + 3) - 10 < 6(x - 2), x \in I$

7. $\frac{x}{2} \leq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

ANSWERS : 7.3

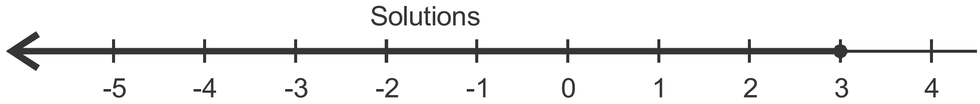
I.

1. $x > 11/5$ or $x \in (11/5, \infty)$
2. $\{.....-2, -1, 0, 1\}$ or $(-\infty, 2)$
3. $x < 5/2$ or $(-\infty, 5/2)$
4. $x < 2$ or $x \in (-\infty, 2)$
5. $x \in (-\infty, 9]$

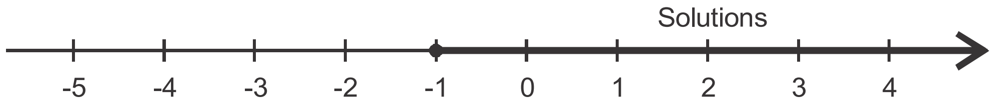
6. (i) $\{-1, 0, 1, 2, 3 .....\}$ (ii) $(-2, \infty)$

7. $x \in (-\infty, -6)$

8. $x < 3$



9. $x > -1$



10. $x \leq -5$ or $x \in [-5, \infty)$

II.

1.	$x \in [1, \infty)$	or	$x \geq 1$
2.	$x \in (4, \infty)$	or	$x > 4$
3.	$x \in [-7, \infty)$	or	$x \geq -7$
4.	$x \in (-\infty, \frac{-1}{2})$	or	$x < \frac{-1}{2}$
5.	$(-\infty, 2)$	or	$x < 2$
6.	$(4, \infty)$	or	$x > 4$
7.	$x < -2/7$	or	$x \in (-\infty, -2/7)$

EXERCISE : 7.4

Two marks & Three marks:

Solve the following system of linear Inequalities in one variable if $x \in R$

1. $\frac{1-7x}{2} > 3$ and $\frac{3x+8}{5} < -11$

2. $5x - 7 < 3(x + 3)$ and $1 - \frac{3x}{2} \geq x - 4$

3. $\frac{2x+3}{4} > 3$ and $\frac{x-4}{-3} < 2$

4. $\dfrac{2x+1}{7x-1} > 5$ and $\dfrac{x+7}{x-8} > 2$

5. $-15 < \dfrac{3(x-2)}{3} \leq 0$

6. $-12 < 4 - \dfrac{3x}{-5} \leq 2$

7. $2x - 5 \geq 7$ and $\dfrac{2x-1}{1+2x} < 3$

8. $\dfrac{7x-1}{2} < -3$ and $\dfrac{3x+8}{5} + 11 < 0$

9. $-5 \leq \dfrac{5-3x}{2} \leq 8$

10. $3x - 7 < 5 + x$ and $11 - 5x \leq 1$

ANSWERS : 7.4

1. $x \in (-\infty, -21)$

2. $x \in (-\infty, 2]$

3. $x \in (-\infty, 21)\ (9/2, \infty)$

4. $x \in \{\phi\}$ (null set)

5. $-23 < x \leq 2$ or $x \in (-23, 2]$

6. $\dfrac{-80}{3} \leq x \leq 10/3$ *or* $x \in \left[\dfrac{-80}{3}, \dfrac{-10}{3}\right)$

7. $x \in [6, \infty)$

8. $x \in (-\infty, -21)$

9. $\dfrac{-11}{3} \leq x \leq 5$ or $x \in \left[\dfrac{-11}{3}, 5\right)$

10. $-2 \leq x < 6$ or $x \in [-2, 6)$

7.5 Application of Linear Inequalities:

(Statement problem related to commerce, Economics, etc.)

Example 1 A man want to cut three length from a single piece of board of length 91 cm. The second length is to be 3 cm larger than the shortest and the 3^{rd} length is to be twice as long as the shortest. What are the possible length of the shortest board if the 3^{rd} piece is to be at least 5 cm larger than the 2^{nd} piece?

Solution : Let the length of the shortest Board $=$ x cm

Length of the 2^{nd} piece $=$ $(x + 3)$ cm

Length of the 3^{rd} piece $=$ $2x$ cm

Given, length of (shortest + 2^{nd} + 3^{rd}) $\leq$ 91 cm

$\therefore\ x + (x + 3) + 2x \leq 91$

$4x + 3 \leq 91$

$4x \leq 91 - 3$

$4x \leq 88 \qquad \therefore x \leq 22 \qquad \ldots.\ (1)$

Again given 3^{rd} piece $\geq$ 2^{nd} piece + 5

$\therefore 2x \geq (x + 3) + 5$

$2x \geq x + 8$

$(2x - x) \geq 8$

$x \geq 8$ …… (2) From (1) and (2) $-8 \leq x \leq 22$

$\therefore$ The length of the shortest Board must be greater than or equal to 8 cm but less than or equal to 22 cm.

Example 2 In an experiment, a solution of Hydrochloric acid is kept between 30^0 and 35^0 celsius. What is the range of temperature in degree Fahrenheit if Conversion Formula is given by $C = \frac{5}{9}(F - 32)$

(where c = Celsius, F – Fahrenheit)

Solution: Given : $30^0 < C < 35^0$ substitute in

$C = \frac{5}{9}(F - 32)$ we get

$30^0 < \frac{5}{9}(F - 32) < 35^0$ [Linear inequalities in one variable F)

Multiply by $\frac{9}{5}$ $\therefore$ $\frac{9}{5} \times 30 < \frac{9}{5} \times \frac{5}{9}$ $(F - 32) < \frac{9}{5} \times 35$

$\therefore$ $54 < (F - 32) < 63$

Add 32 on both sides,

$$54 + 32 < F < 63 + 32$$

$$86 < F < 95$$

$\therefore$ the required range of temperature is between 80^0F and 95^0F

Example 3 Find all pair of consecutive odd positive integer which are smaller than 10 such that their sum is greater than 11.

Solution: Let the odd consecutive Integers are x and $x + 2$

Given $x < 10$ and $x + 2 < 10$

Also given $x + (x + 2) > 11$ $\quad$ $x < 10$ and $x + 2 < 10$ $\therefore$ $x < 8$

$2x + 2 > 11$ $\quad$ $x < 10$ ….. (1)

$2x > 11 - 2$

$2x > 9$

$x > 9/2$

or $x > 4\frac{1}{2}$ …. (2)

From (1) and (2) $\quad$ $4\frac{1}{2} < x < 10$

$\therefore$ the possible pair of odd Integers are (5, 7), (7, 9)

Example 4 A manufacturer has 600 Litres of a 12% solution of acid. How many liters of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?

Solution : Let 30% acid solution added $=$ x litres

$\therefore$ Total mixture $=$ $(x + 600)$ litres

$\therefore$ $30\% \, x + 12\% \text{ of } 600 > 15\% \, (x + 600)$ ....(1)

$30\% \, x + 12\% \text{ of } 600 < 18\% \, (x + 600)$ ....(2)

From (1) and (2)

$$\frac{30x}{100}+\frac{12\times 600}{100} > \frac{15}{100}(x+600)$$

$$\frac{30x}{100}+\frac{12\times 600}{100} < \frac{18}{100}(x+600)$$

Multiply by 100 for both the inequalities we get

$30x + 7200 > 15\ (x + 600)$ and $30x + 7200 < 18\ (x + 600)$

$\Rightarrow 30x + 7200 > 15x + 9000$ and $30x + 7200 < 18x + 10{,}800$

$\therefore\ 30x - 15x > 9000 - 7200$

$15x > 1800$

$x > 120$ ....(3) and

$30x + 7200 < 18x + 10{,}800$

$12x < 10{,}800 - 7{,}200 \therefore\ 12x < 3600$

$x < 300$ ....(4)

$\therefore$ from (3) and (4) $\quad 120 < x < 300$

The number of litres of 30% acid solution to be added is more than 120 litres but less than 300 litres.

Example 5 Suresh obtained 70 and 75 marks in first two unit tests. Find the minimum marks he should get in the 3rd test to have an average of atleast 60 marks.

Solution: Let the marks obtained by Suresh in the third test be ‘x’ .

Given the average of the marks of all the tests is atleast 60

i.e. $\frac{70+75+x}{3} \geq 60$

$\therefore\ 70 + 75 + x \geq 60 \times 3$

$145 + x \geq 180$

$x \geq 180 - 145$

$x \geq 35$

$\therefore$ Suresh should get greater or equal to 35 mark.

Example 6 To receive grade 'A' in a course, one must obtain an average of 90 marks or more in 5 exams. (Each of 100 marks), If Gopal's marks in the First June examination are 87, 92, 94 and 95. Find the minimum marks obtained by Gopal in 5^{th} exam to get 'A' grade in the course?

Solution : Let the minimum marks required in the 5^{th} exam = x

$\therefore$ Average marks in all the 5 exams ≥ 90

$$\therefore \frac{87+92+94+95+x}{5} \geq 90$$

$$\frac{368+x}{5} \geq 90$$

$368 + x \geq 90 \quad 5$

$368 + x \geq 450$

$\therefore x \geq 450 - 368$

$x \geq 82$

$\therefore$ the required marks in the 5^{th} exam is greater than or equal to 82.

EXERCISE 7.5 (Statement problem in one variable)

3 to 5 marks questions.

1. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% of acid content? (water contains 0% of acid)
2. A solution is to be kept between 60^0F and 77^0F. What is the range in temperature in degree celcius (C) if the celcius /Fahrenheit conversion formula is given by F = 9/5 C + 32 ?
3. Find all pair of consecutive even Integers which are greater than 5 and their sum must be less than 23.
4. The cost and Revenue junction of a product are given by $C(x) = 2x + 400$ and $R(x) = 6x + 20$ respectively. Where 'x' is the number of item produced by the manufacturer. How many items the manufacturer must sell to realize some profit. (Hint : $R(x) - C(x) > 0$)

ANSWER : 7.5

1. The number of litres of water added must be more than 562.5 litres but less than 6.900 litres.
2. $20^0 < C < 25^0$
3. $x < 10\frac{1}{2}$,(6, 8), (8, 10)
4. More than 95 items ($x > 95$)

7.6 Linear Inequation in two variable and their graphical representation:

A linear inequality in two variable is an expression in the form $ax + by < c$, $ax + by > c$, $ax + by \leq c$ or $ax + by \geq c$ (a, b, c, $\in$ R and a $\neq$ b $\neq$ 0)

Example: $4x - y < 3$, $2x + y > 5$, $x + 3y \leq 3$, $2x - y \geq 1$

Solution Set: The solution set for a linear inequality in two variable is the set of all values of (x, y) which satisfy the given linear inequality.

Geometrically this represents a section of co-ordinate plane described by a set of co-ordinate system with x and y axes

Working rule to finding the solution set of linear inequalities in two variables

Step 1: Replace the inequality sign involving in the given statement by Equality sign. The resulting equation represents a straight line, which acts as the boundary of the solution.

Step 2: Draw the line graph of the equation from step 1 using convenient point on the line and join them. If the inequality in the given equation is

(i) < or > then the boundary line is not included in solution set and represent the boundary by **broken line.**

(ii) '$\leq$' or '$\geq$' then the boundary line is included in solution set and represent the boundary by **solid line.**

Step 3: The line in step 2 divides the co-ordinate place into two region one above the boundary and the other below. (or is the boundary line is vertical then one region is to left and the other is right). To find the required region choose any convenient point which is not on the boundary and verify whether the coordinate of this point satisfy the given inequality or not. If it satisfy the inequality then the solution set is the set of all point which lie on the same side of the boundary as the chosen point. If the inequality does not satisfy the co-ordinates of the chosen point then the solution set is the set of all points on the opposite side of the boundary.

Example 1 Solve the following in equation graphically.

a) $x > -2$

Solution : The boundary of $x > -2$ is lying in the line $x=-2$ parallel to the Yaxis and 2 units to the left of it. Since it is the inequality '>' the boundary is represented by **broken line** as shown below. Consider the origin as the test point. Substituting for x as x=0 in $x > -2$ we find $0 > -2$ and hence the solution set is the region which includes **O(0,0)** i.e the origin.

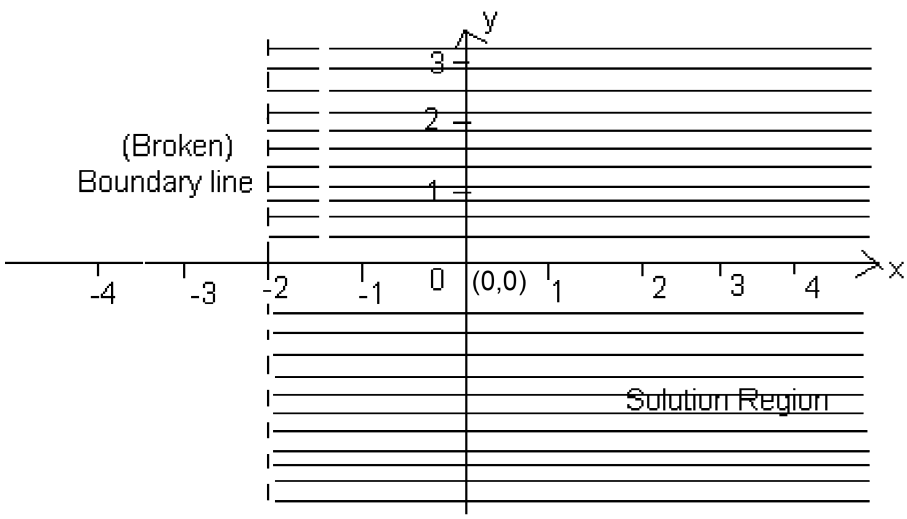


Fig. (a)

b) y < -3

Solution: The boundary of y < - 3 is the line y = -2. This is a line parallel to x-axis 3 unit below. Since the inequality sign is '<' the boundary is represented by Broken line as in Fig.(b). Consider the point 0 (0, 0), 0 < -3 which is not true. Therefore the solution region which does not contains the origin.

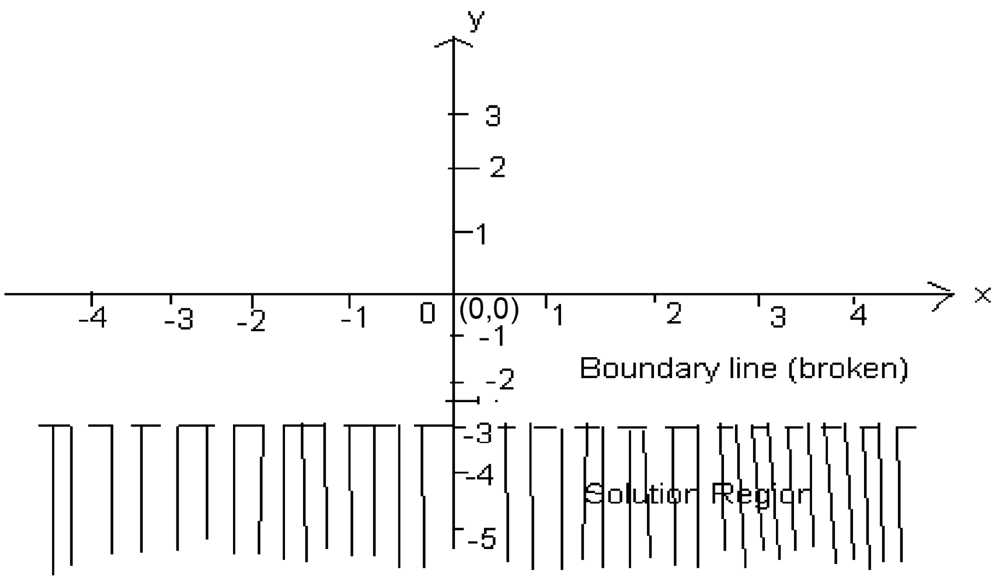


Fig. (b)

c) x + y < 3

Solution: The boundary of the inequality is x + y = 3.

If x = 0, then y = 3 and if y = 0, then x=3

$\therefore$ A (0, 3) and B (3, 0) is the point on the boundary line

$\because$ the inequality involves '<' sign the boundary line is represented by **Broken line.**

Consider O (0,0) as a test point is x = 0, and y = 0, 0 < 5 (which is true) hence the solution set contain (0,0) it is shown by shaded region in fig (c)

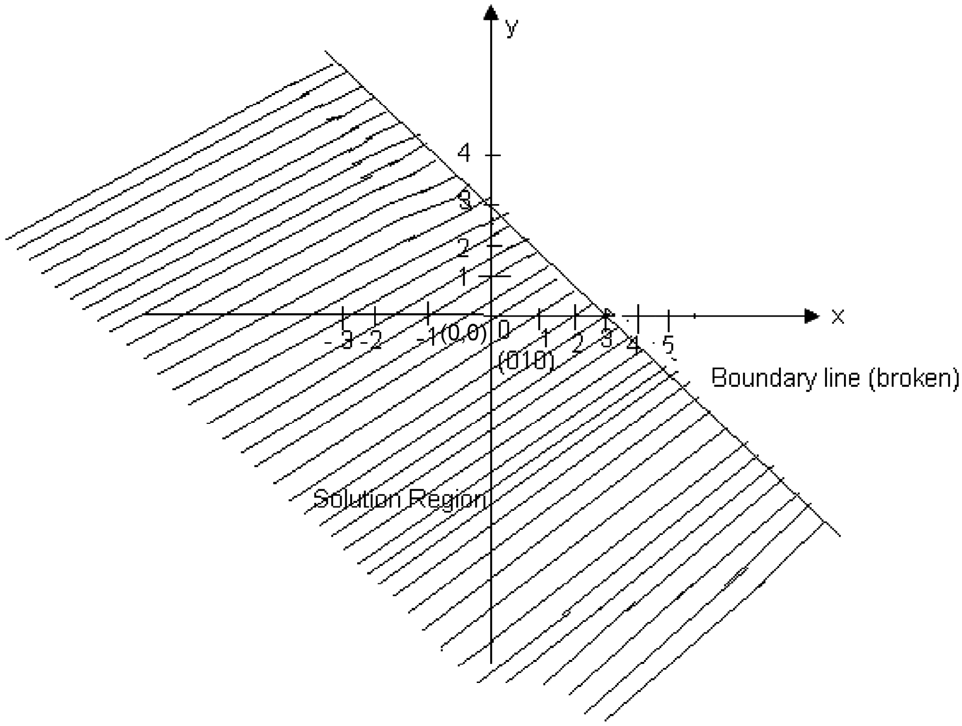


Fig. (c)

(d) $x - y \leq 2$

Solution :

The boundary of $x - y \leq 2$ in the line $x - y = 2$

If $x = 0$ then y = -2.Also if y = 0 then $x = 2$.

$\therefore$ A (0,-2) and B (2,0)are the points on the boundary line.

Since the inequality involves '$\leq$' sign the boundary line is represented by **Solid line.**

Consider (0,0) as a test point.

Putting $x = 0$, y = 0 in $x - y \leq 2$ we find $0 - 0 = 0 < 2$, which is true hence the solution in the region. Includes origin O (0,0) and shown by shaded region in fig (d)

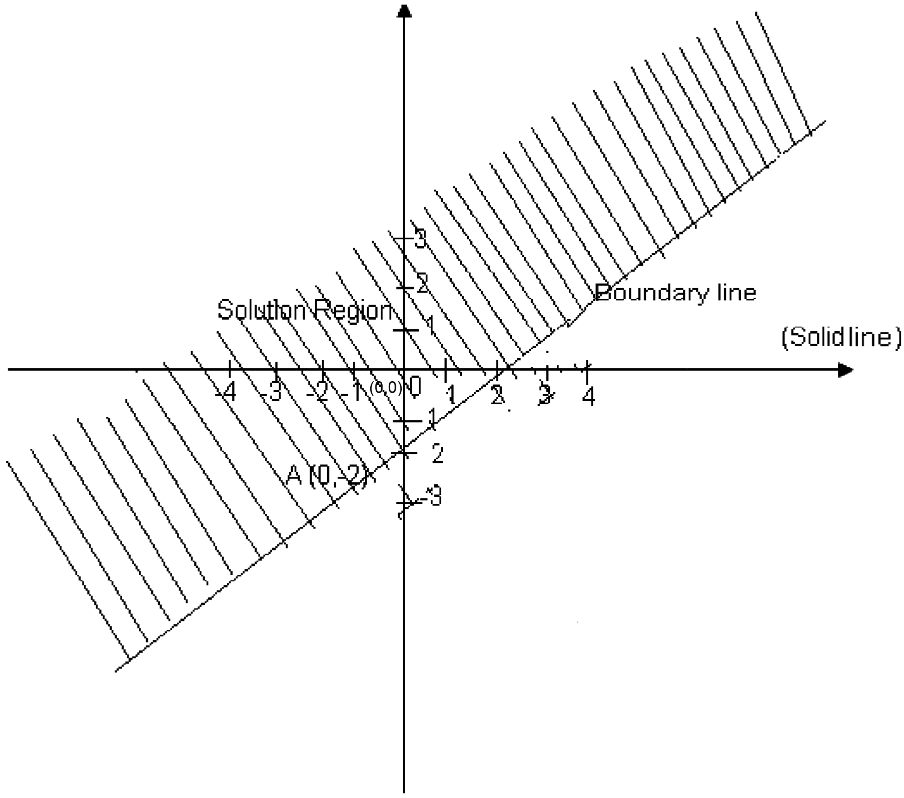


Fig. (d)

Example 2 Solve the following Inequalities graphically

(a) $3x + 4y \leq 12$ (b) $3y - 5x < 30$ (c) $-3x + 2y \geq -6$
(d) $2x - y \geq 6$ (e) $3x + 2y > 6$ (f) $2x - 30 < 6$

(a) $3x + 4y \leq 12$

Solution:

The boundary of $3x + 4y \leq 12$ is the line $3x + 4y = 12$

If $x = 0$ then $y = 3$ and if $y = 0$ then $x = 4$

$\therefore$ A (0,3) and B (4,0) are the points

On the boundary line since the inequality in values '$\leq$' sign the boundary line is **Solid line.** Consider O (0,0) as the test point. Putting x = 0, y = 0 in the inequality $3(0) + 4(0) = 0 \leq 12$ (which is true) Therefore the solution region contains (0,0) is shown by shaded region in fig (a)

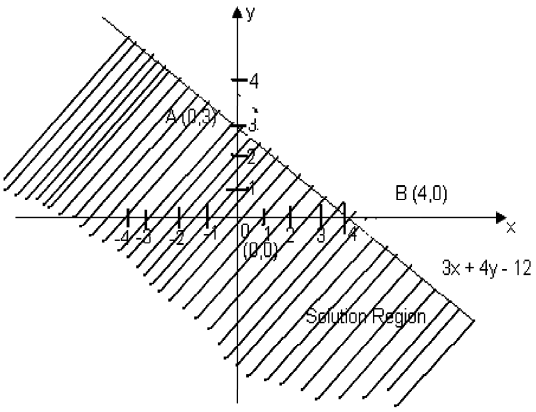


Fig. (a)

(b) $3y - 5x < 30$

Solution: The boundary $3y - 5x < 30$ in the line $3y - 5x = 30$

If $x = 0$ then $y = 6$.Also if $y = 0$ then $x = 6$

$\therefore$ A (0,6) and B (6,0) the points on the boundary line.

since the equation involves '<' sign the boundary line is represented by **broken line.**

Consider (0,0) as a test point . Putting $x = 0$, $y = 0$ in the given inequality

We find $3(0) - 5(0) < 30$

$0 - 0 = 0 < 30$

$\therefore 0 < 30$ which is true. Therefore 0 (0,0) lies in the solution region as shown by shaded area in fig (b).

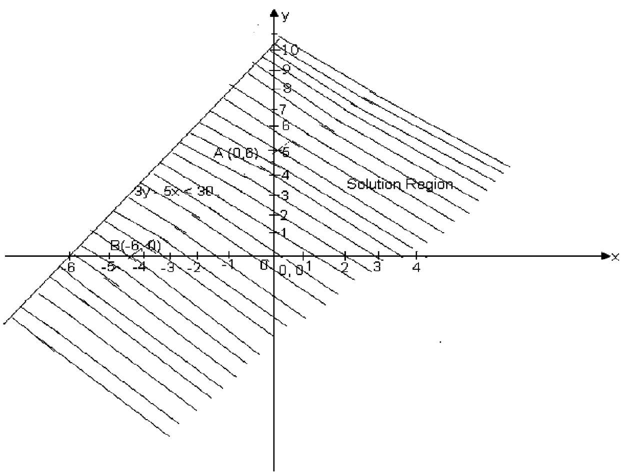


Fig. (b)

(c) $3x - 2y \geq 6$

Solution:

The boundary for the inequality is the line $-3x + 2y = -6$ or $3x - 2y = 6$.

If $x = 0$ then $y = 3$ and if $y=0$ then $x=2$

$\therefore$ A (0,3) and B (2,0) are the points on the boundary line. Since the inequality is '$\geq$' sign the boundary is represented by **Solid line.**

Consider a test point 0 (0,0).Putting $x = 0$, $y = 0$ in the inequality

$-3(0) + 2(0) = 0 \geq -6$

which is true hence the solution region contain 0 (0,0) as shown by the shaded region in figure.

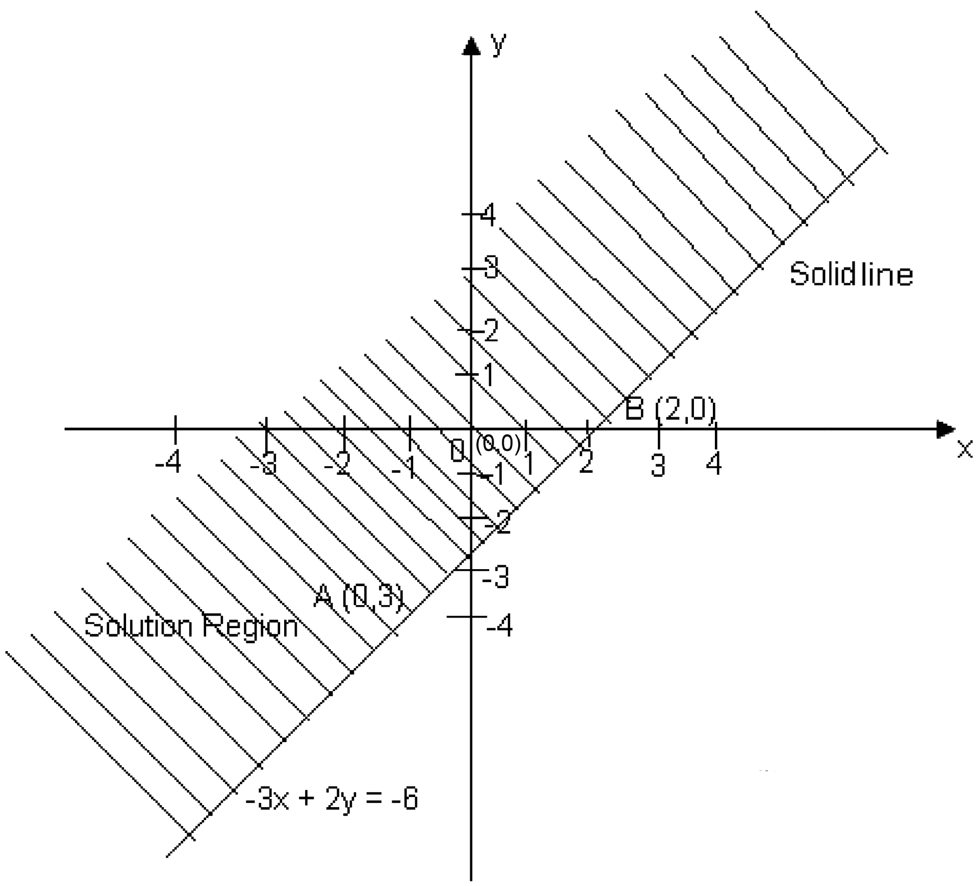


Fig. (c)

(d) $2x + y \geq 6$

Solution:

The boundary of $2x + y = \geq 6$ in a line $2x + y = 6$

If $x = 0$, $y = 6$ and if $y = 0$, $x = 3$

$\therefore$ A (0,6) and B (3,0) are the points boundary line. Since the inequation involves '$\geq$' sign the boundary line in represented by a **solid line**. Consider a point 0 (0,0) as a test point, $x = 0$, $y = 0$, then

$2(0) + 0 \geq 6$

$0 + 0 \geq 6$

$0 \geq 6$ which is not true, hence the solution region does not contain (0,0) as a point as shown by a shaded region in fig (d)

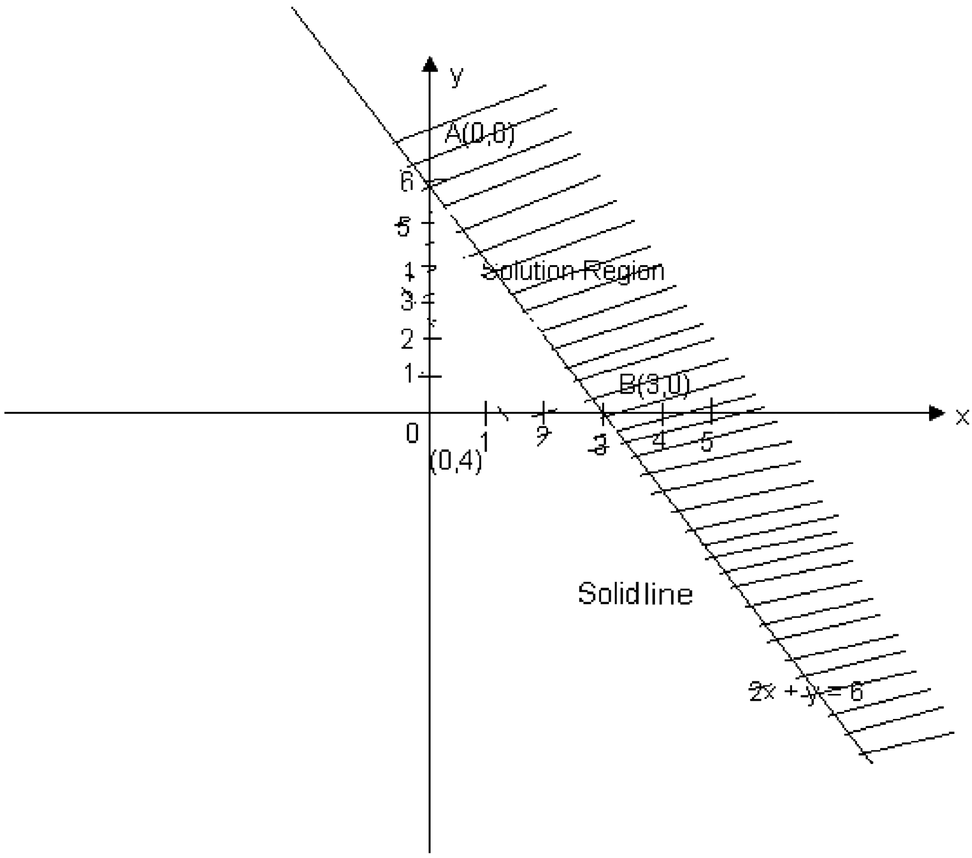


Fig. (d)

(e) $3x + 2y > 6$

Solution: The boundary of the inequality $3x + 2y > 6$ is a line $3x + 2y = 6$.

If $x = 0$, $y = 3$ and if $y = 0$, $x = 2$

$\therefore$ A (0.3) and B (2.0) are the points on the boundary line represented by a **broken line**. Consider (0, 0) as a test point. $x = 0$, $y = 0$ in the equation

$$3(0) + 2(0) > 6$$

$0 > 6$ which is not true hence the solution region does not contains (0,0) as shown by shaded region in fig. (0)

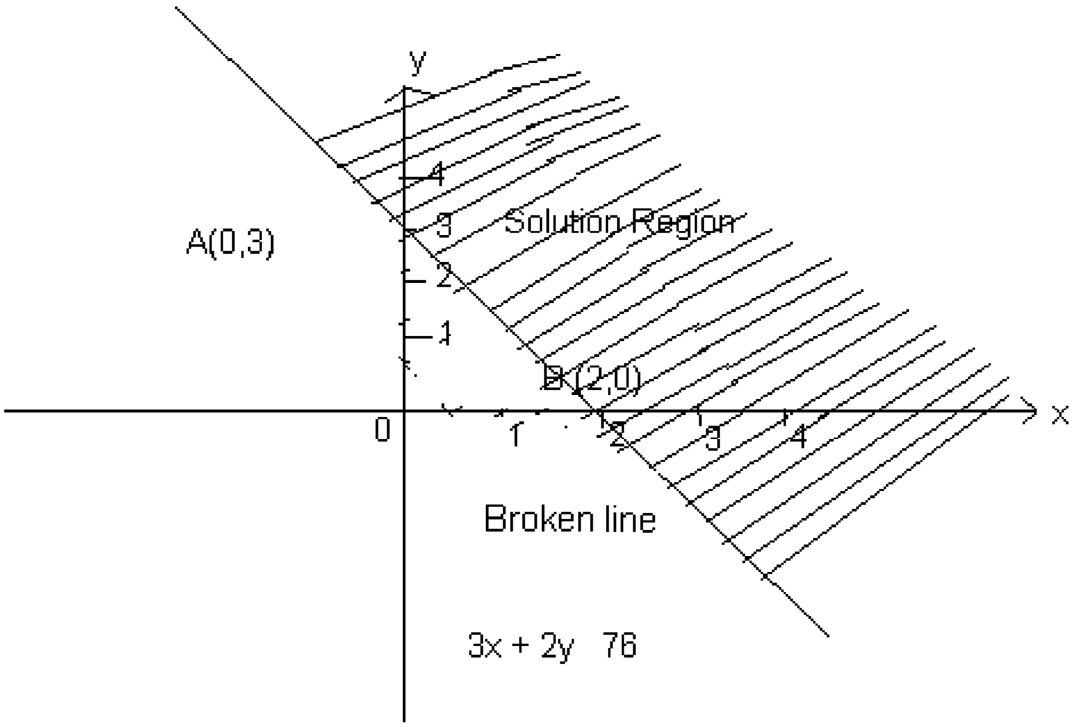


Fig. (e)

(f) $2x - 3y < 6$

Solution: The boundary of the in equation $2x - 3y < 6$ is a line $2x - 3y = 6$

If $x = 0$, $y = -2$ and if $y = 0$, $x = 3$

$\therefore$ A (0, -2) and B (3, 0) are the points as the boundary line. Since the inequation involves '<' sign the boundary line is represented by a **broken line**. Consider (0, 0) as a test point $x = 0$, $y = 0$ in the in equation.

$$2(0) - 3(0) < 6$$

$0 < 6$ which is true hence the shaded region contain (0, 0) as shown by a shaded region its Fig. (f)

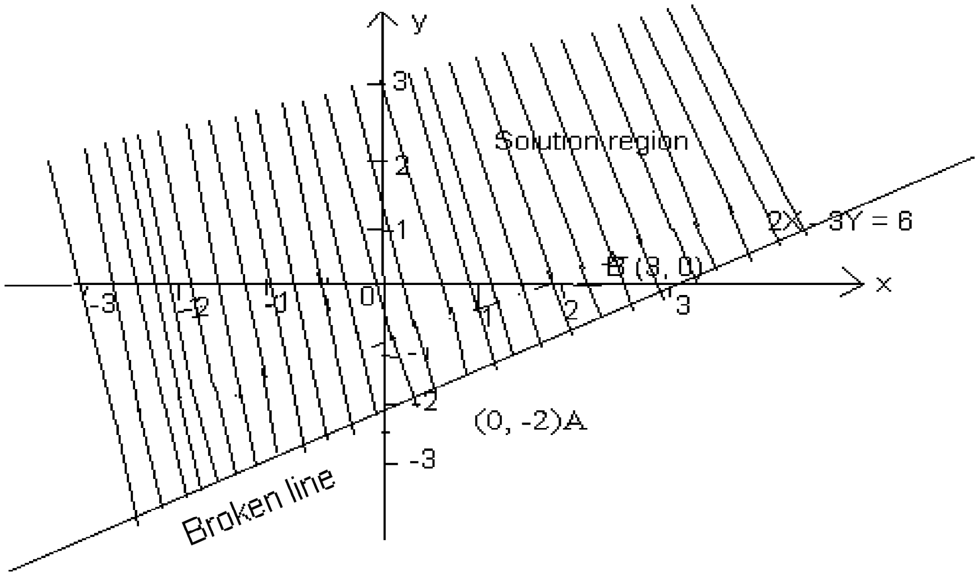


Fig. (f)

EXERCISE: 7.6

1. Solve the following in equality graphically (2 Marks)

1 $x > -3$

2. $y < -4$

3 $4x \leq 4$

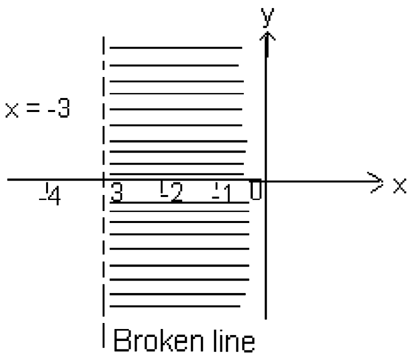
4. $3y \geq -6$

3 Marks question:

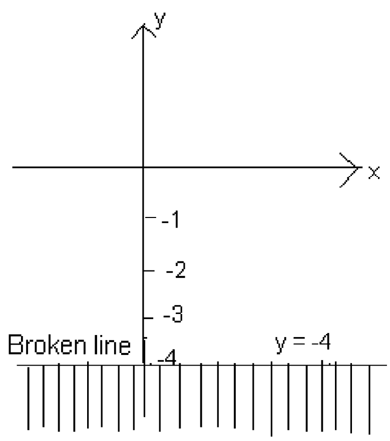
5. $x + y < 5$
6. $x - y \leq 3$
7. $3x - 4y \leq 12$
8. $x - 2y > 6$
9. $2x - 3y \geq 6$

Answers (2 Marks)

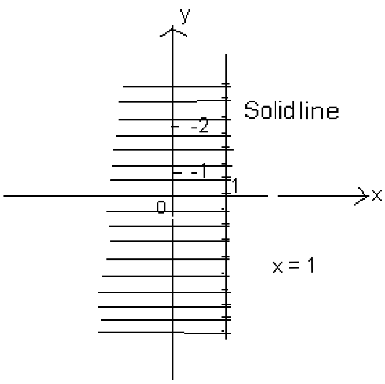
(1)



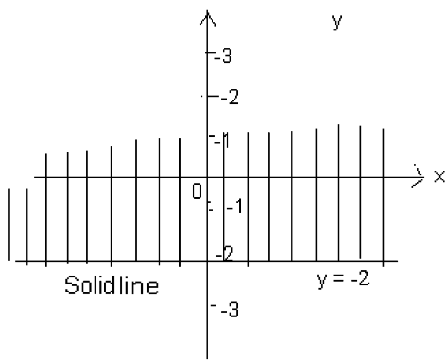
(2)



(3)

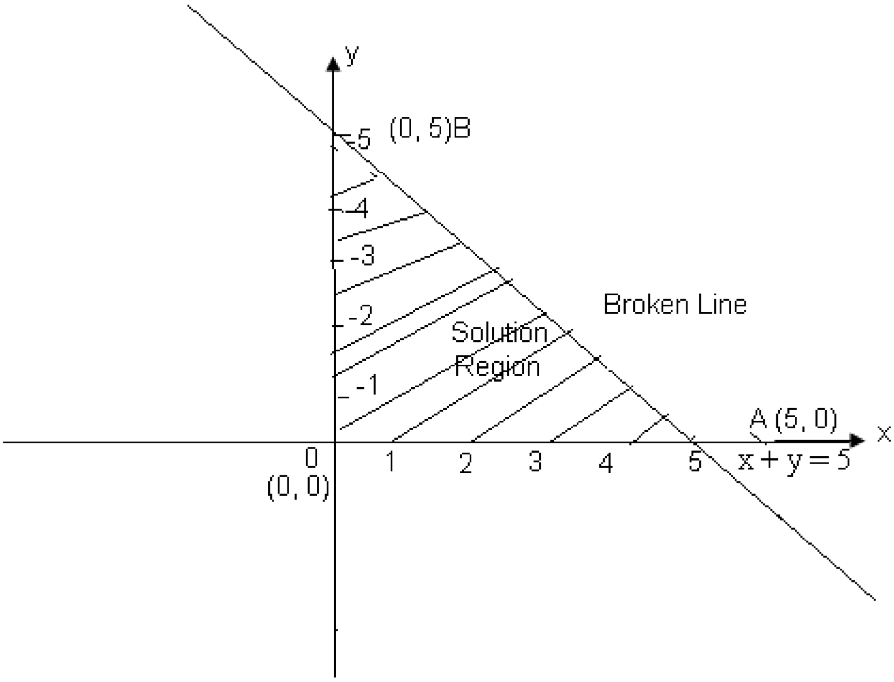


(4)

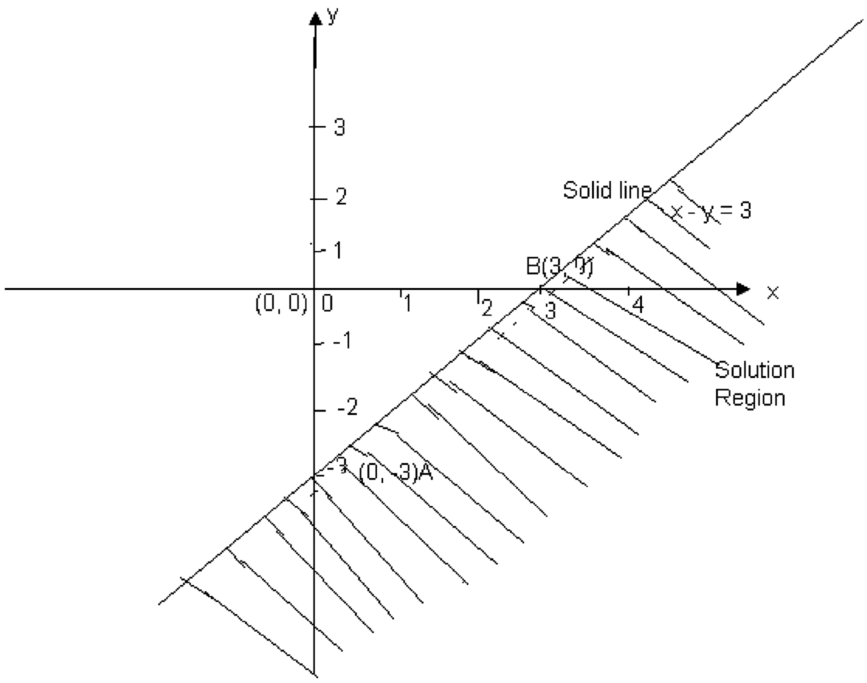


Answer :7.6 (3 Marks)

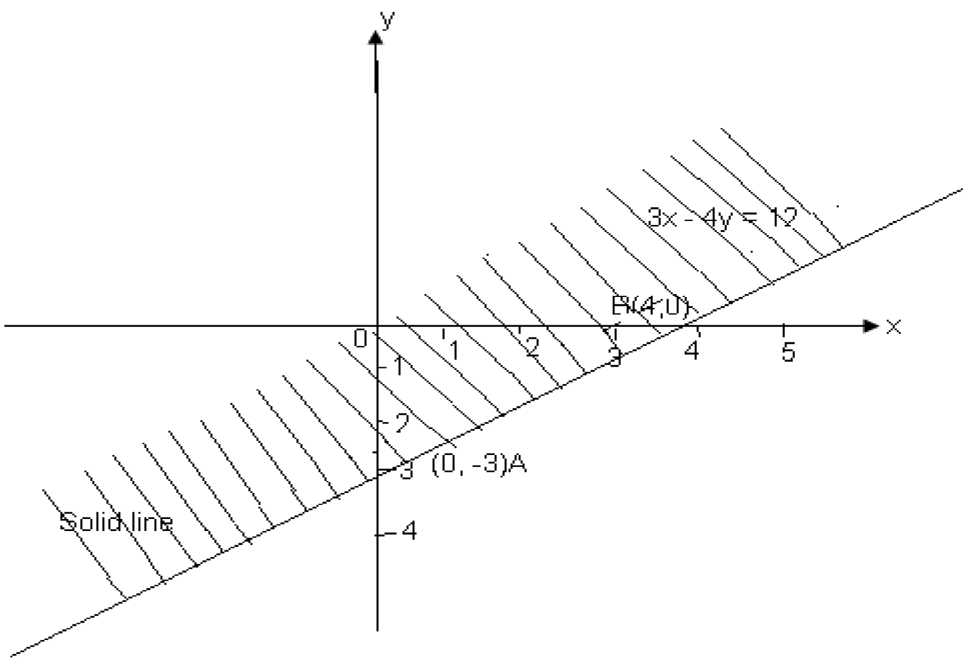
(5)



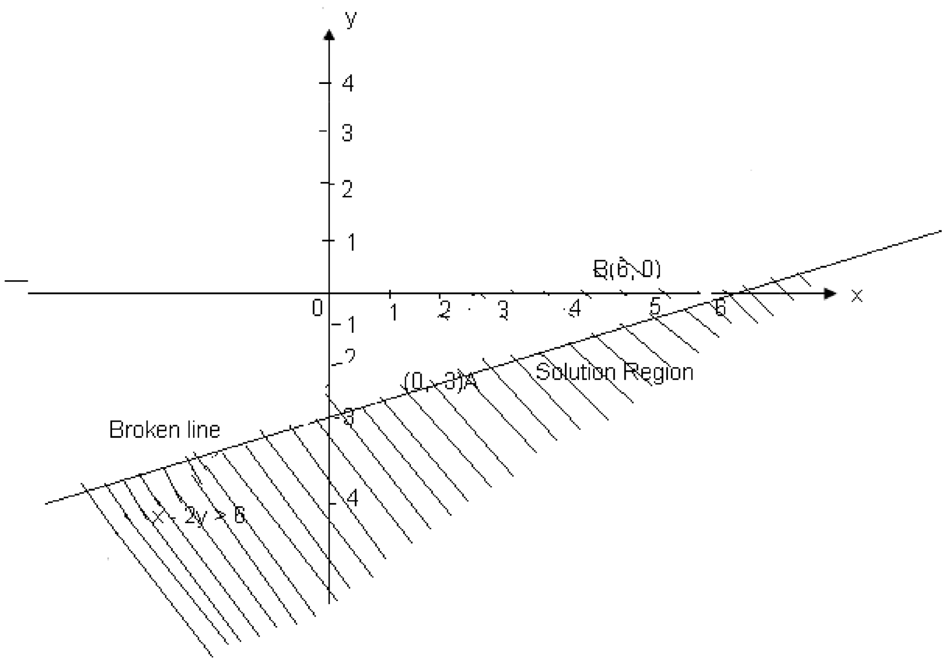
(6)



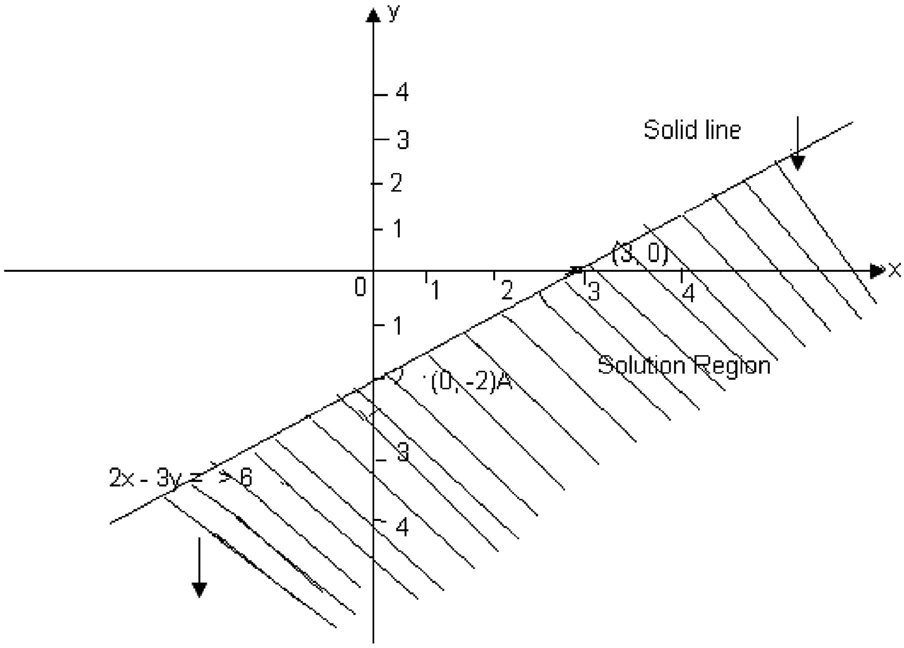
(7)


y
x
0
1
2
3
4
5
-1
-2
-3
-4
3x - 4y = 12
B(4,0)
(0, -3)A
Solid line

(8)


y
x
0
1
2
3
4
5
6
4
3
2
1
-1
-2
-3
-4
B(6, 0)
(0, -3)A
Solution Region
Broken line
x - 2y > 6

(9)



7.7 System of Linear Inequation in two variable and their graphical solution.

Two or more linear equalities in two variable together forms a system of linear inequalities in two variable. The solution set is the set which satisfies both the linear inequation.

Example: i) $x \geq 3$, $y \geq 2$

ii) $2x + y \geq 6$, $3x + 4y \leq 12$

iii) $3x + 4y \leq 60$, $x + 3y \leq 30$, $x \geq 0$, $y \geq 0$

iv) $x + 2y \leq 10$, $x + y \leq 1$, $x - y \leq 0$, $x \geq 0$, $y \geq 0$

here $x \geq 0$, $y \geq 0$ are condition to the system of inequalities called non-negative condition. The solution set of the about system of linear inequalities in two variable is the set of all ordered pain (x, y) which satisfies each inequality of the system simultaneously.

Feasible Solution: A point (x, y) on the co-ordinate plane is called Feasible solution of the system of inequalities in two variable is it satisfy all the in equation of the system.

The set of all feasible solution form a feasible region.

Example (1) Solve the following system of inequalities graphically.

a) $x \geq 2$, $y \geq 3$

Solution: $x \geq 2$, $y \geq 3$ have the boundary line $x = 2$ & $y = 3$

$x = 2$ is the line parallel to y – axis 2 unit right of y – axis.

$y = 3$ is the line parallel to x-axis 3 unit above the x – axis.

Since the inequalities involues '$\geq$' sign the boundary line is a solid line.
Consider (0, 0) test point $x = 0$, $\quad 0 \geq 2$

and $y = 0$, $\quad 0 \geq 3$

which is not true hence the feasible solution does not contain (0, 0) and represented by shaded region in Fig. (a)

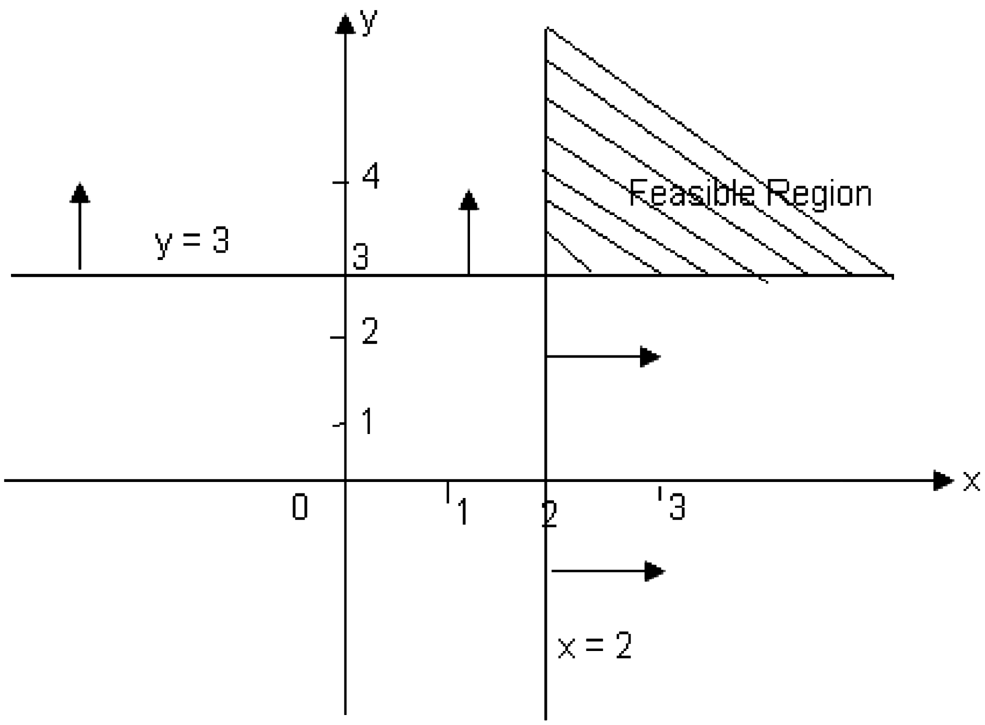


Fig. (a)

b) Solve $x + y \geq 5$

$x - y \leq 3$

Solution: We have $x + y \geq 5$ and $x - y \leq 3$. The boundary of the feasible Region are determined by the lines $x + y = 5$ and $x - y = 3$

Solid Line for $x + y = 5$, If $x = 0$, $y = 5$ $\therefore$ A (0, 5)

(AB) $y = 0$, $x = 5$ B (5, 0)

Solid line for $x - y = 3$, If $x = 0$, $y = -3$ C (0, -3)

(CD) $y = 0$, $x = 3$ D (3, 0)

The shaded region is the feasible Region which is the intersection of the solution set of each in equalities as shown in Figure. (b)

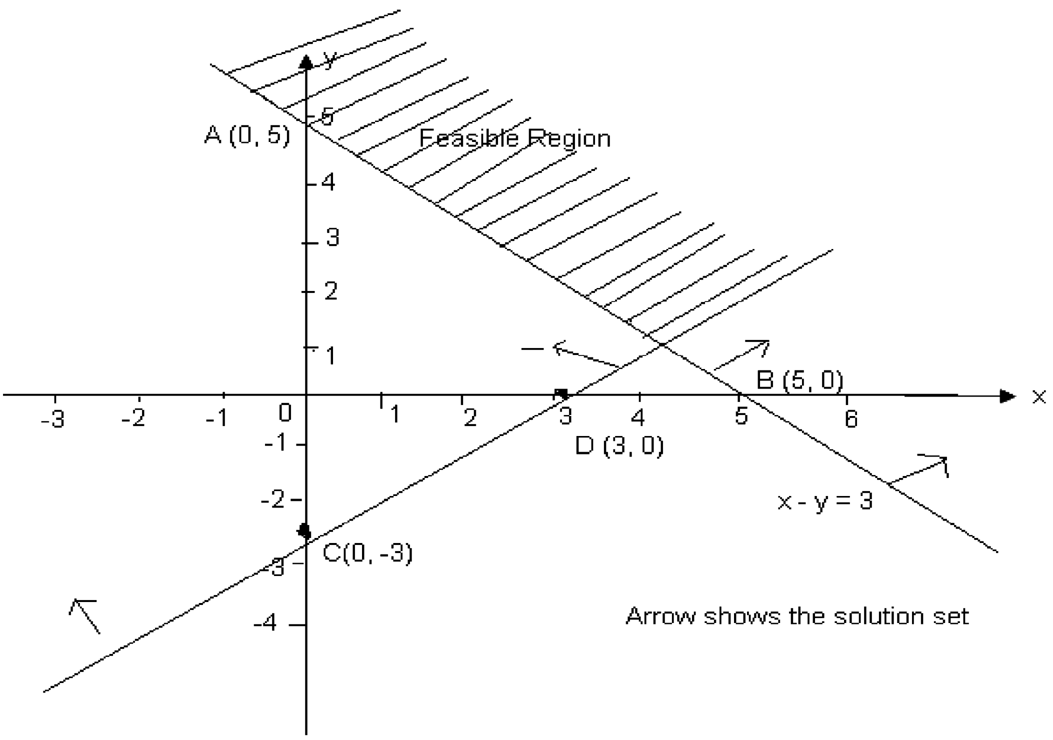


Fig. (b)

c) Solve $x + 2y \le 8$

$2x + y \le 8$

$x \ge 0$

$y \ge 0$ graphically.

Solution: We have $x + 2y \le 8$ and $2x + y \le 8$. The boundary of the Feasible Region are determined by the line $x + 2y = 8$ and $2x + y = 8$

$\therefore$ Solid line (AB) for $x + 2y = 8$, If $x = 0$, $y = 4$ A (0, 4)

Y = 0, $x = 8$ B (8, 0)

Solid line (CD) for $2x + y = 8$, if $x = 0$, $y = 8$ C (0, 8)

Y = 0, $x = 4$ D (4, 0)

$x \ge 0$, $y \ge 0$ are non negative in equalities given the condition that every point in the shaded region must lies in the First Quadrant of the contain plane. The shaded region is the feasible region which is the intersection of the solution set of each inequation on shown in Fig. (c)

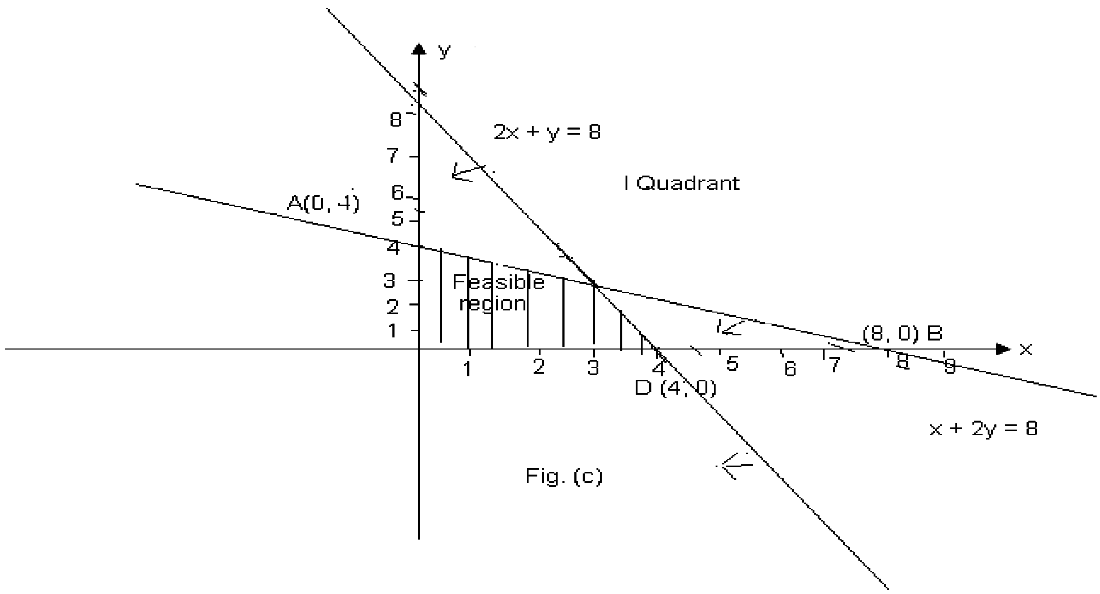


Fig. (c)

d) **Solve** $2x - y < 1$

$x - 2y < -1$ graphically

Solution: We have $2x - y < 1$ and $x - 2y < -1$. The boundaries of the feasible region are determined by the line.

$2x - y = 1$ and $x - 2y = -1$.

Broken line (AB) for $2x - y = 1$, $x = 0$, $y = -1$ A $(0, -1)$

$y = 0$, $x = ½$ B $(1/2, 0)$ OR $(0.5, 0)$

Broken line (CD) for $x - 2y = -1$ $x = 0$, $y = 1$ C $(0, ½) = (0, 0.5)$

$y = 0$, $x = -1$ D $(-1, 0)$

The solution set of inequality is shown by arrow the intersection of the solution set by the feasible regionas shaded area in fig. (d)

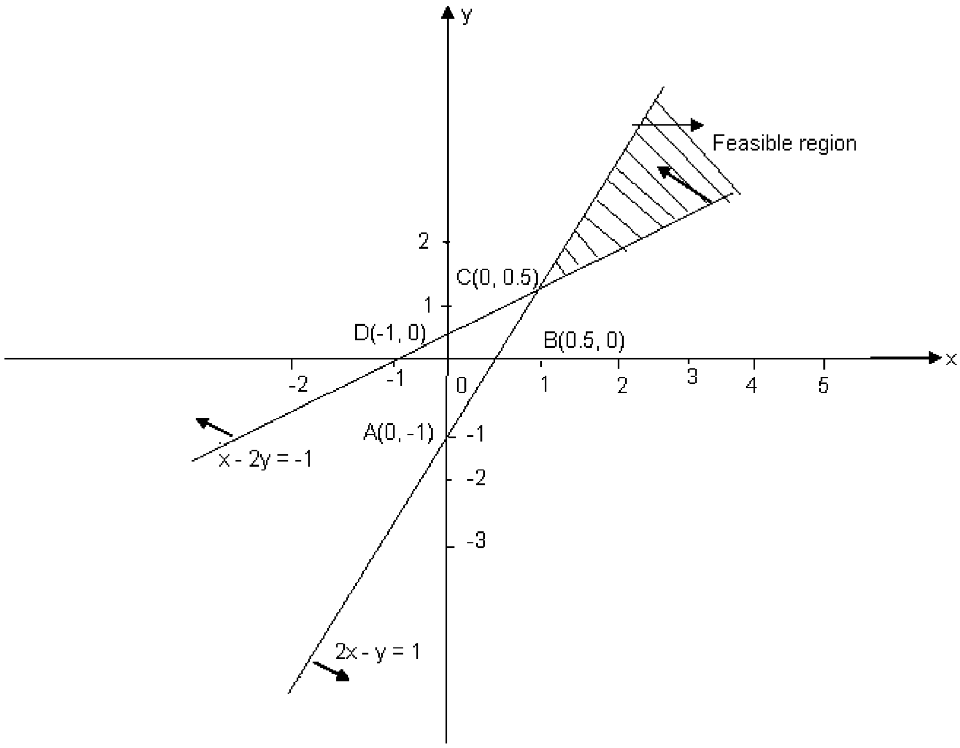


Fig. (d)

(e) **Solve** $3x + 4y \leq 12$, $2x + y \geq 6$ graphically.

Solution: We have $3x + 4y \leq 12$, and $2x + y \geq 6$. The boundaries of the feasible regions are determined by the lines $3x + 4y = 12$ and $2x + y = 6$

Solid line AB for $3x + 4y = 12$ $x = 0$, $y = 3$ A(0, 3)

$y = 0$, $x = 4$ B (4, 0)

Solid line CD for $2x + y = 6$, $x = 0$, $y = 6$, C (0, 6)

$y = 0$ $x = 3$ D (3, 0)

The solution set of each inequation is indicated by arrow mark in the figure. The intersection of these solution set in the feasible region – shaded region in the fig. (e)

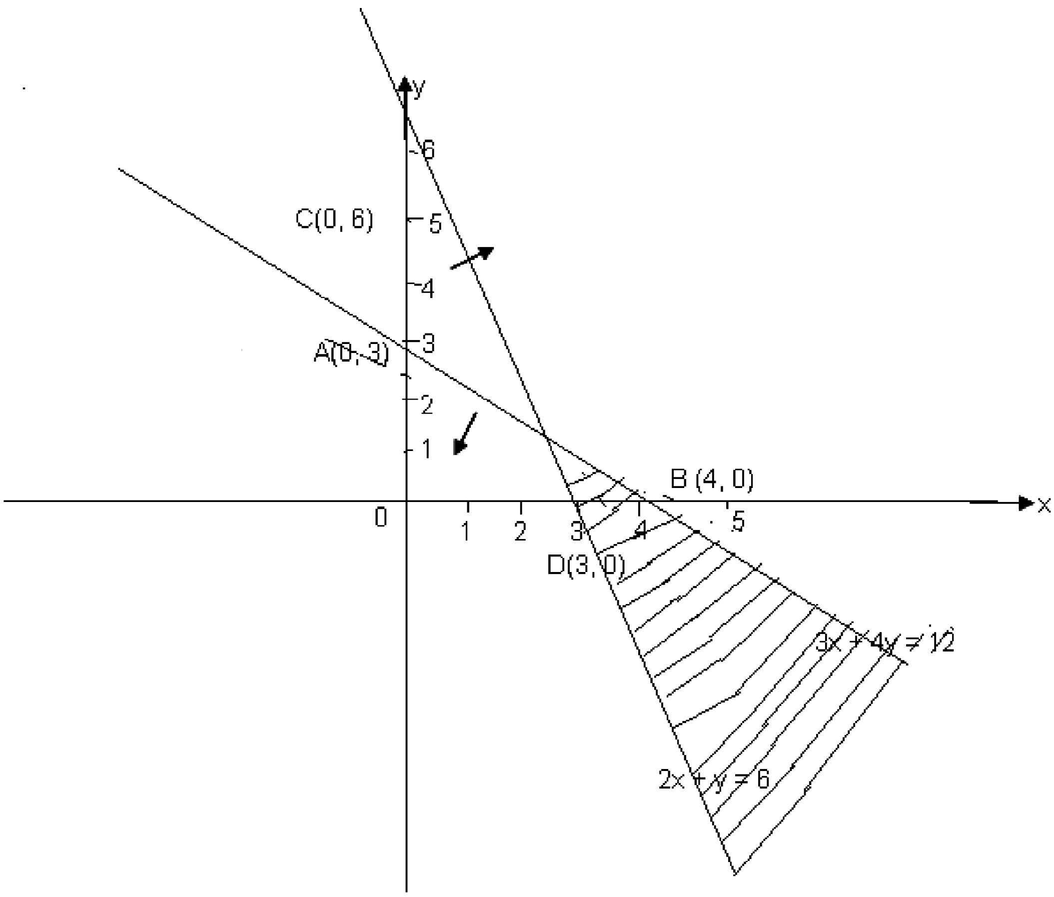


Fig. (e)

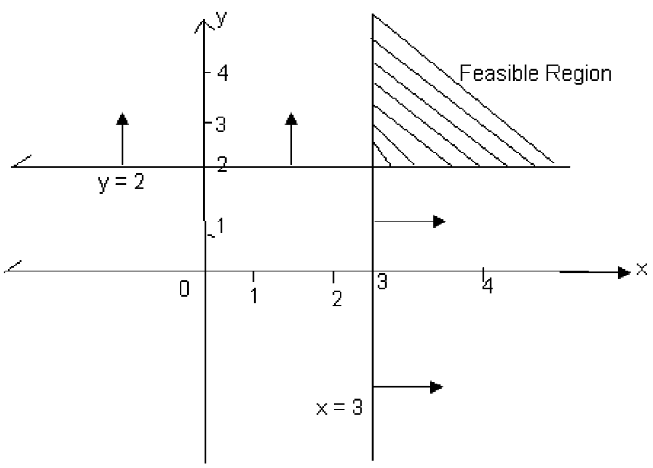
EXERCISE : 7.7 (e and 5 Marks question)

Solve the follow system of linear inequation in 2 variable graphically.

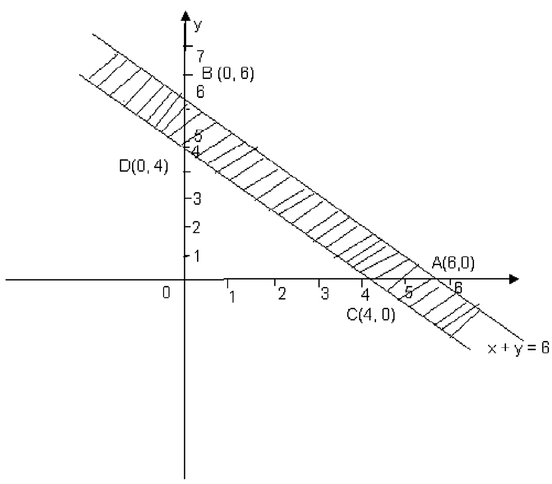
1. $x \geq 3y \geq 2$
2. $x + y \leq 6, \quad x + y \leq 4$
3. $2x + y \geq 8, \quad x + y \geq 10$
4. $x + 3y \geq 3, \quad 2x + y \geq 2, \quad x \geq 0, y \geq 0$
5. $3x + 3y \leq 6, \quad x + 4y \leq 4, \quad x \geq 0, y \geq 0$
6. $3x + 4y \geq 12, \quad 4x + y \geq 8$

ANSWERS : 7.7

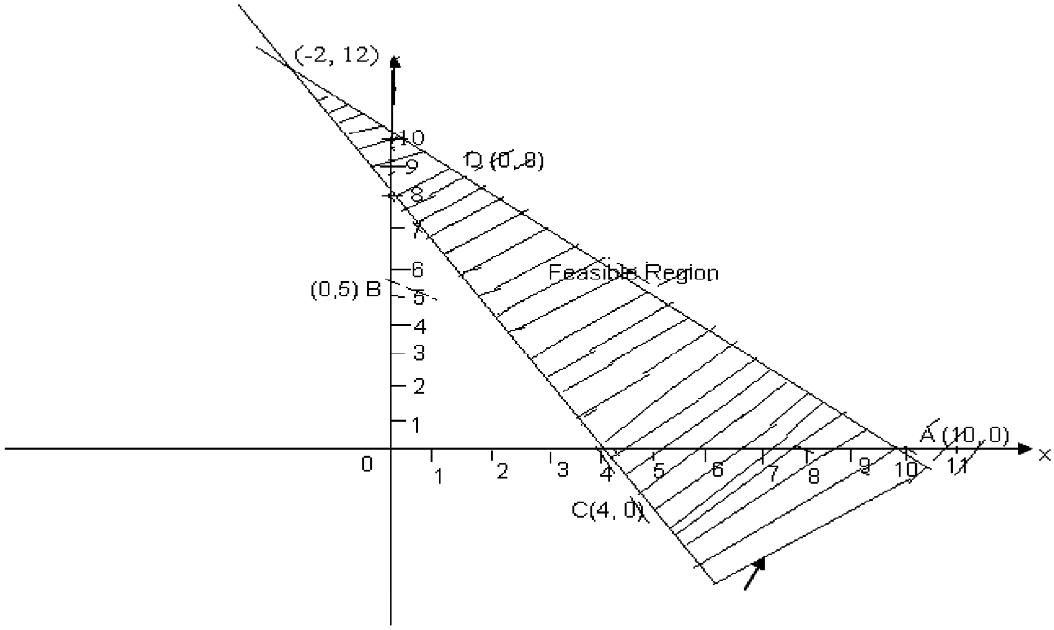
(1)



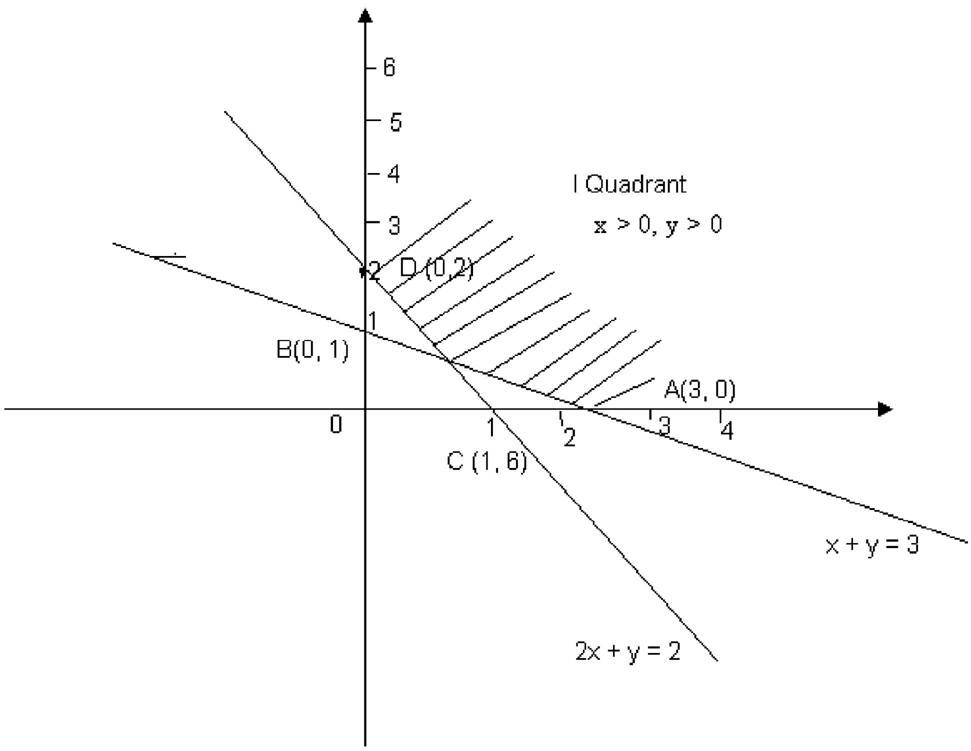
(2)



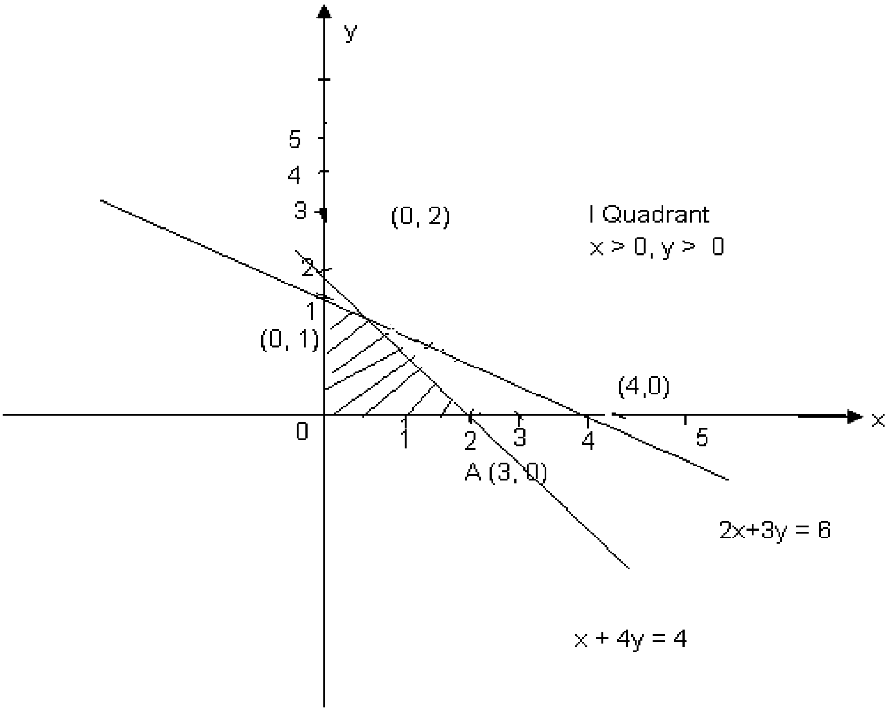
(3)


(-2, 12)
D (0, 8)
Feasible Region
(0,5) B
A (10, 0)
C(4, 0)
0
1
2
3
4
5
6
7
8
9
10
11
x

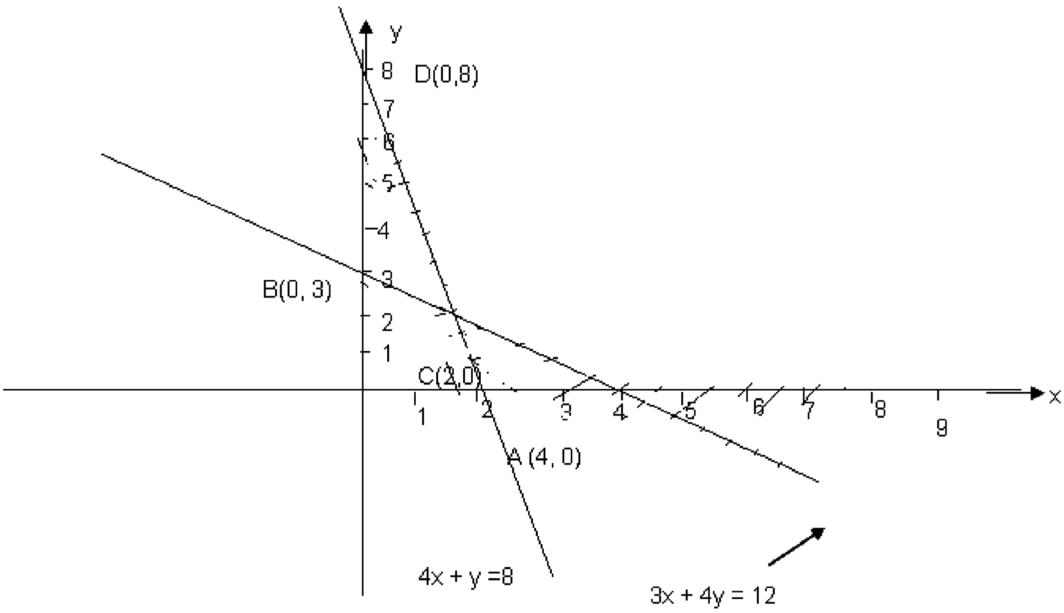
(4)


6
5
4
3
I Quadrant
x > 0, y > 0
D (0,2)
B(0, 1)
A(3, 0)
0
1
2
3
4
C (1, 6)
x + y = 3
2x + y = 2

(5)


y
5
4
3
2
1
(0, 2)
I Quadrant
x > 0, y > 0
(0, 1)
(4,0)
0
1
2
3
4
5
x
A (3, 0)
2x+3y = 6
x + 4y = 4

(6)


y
8
D(0,8)
7
6
5
4
3
B(0, 3)
2
1
C(2,0)
1
2
3
4
5
6
7
8
9
x
A (4, 0)
4x + y =8
3x + 4y = 12

UNIT - II

COMMERCIAL ARITHMETIC

CHAPTER	NAME OF THE CHAPTER	TEACHING HOURS
8	SIMPLE AND COMPOUND INTEREST	08
9	ANNUITIES	06
10	AVERAGES	04
11	PERCENTAGE, PROFIT AND LOSS	06
12	LINEAR FUNCTION	04
	TOTAL	28 Hours

CHAPTER 8

SIMPLE INTEREST AND COMPOUND INTEREST

8.1 All money dealings are associated with three factors i.e. principal, rate of interest and time. Computation of simple and compound interest are directly based on these factors:

Principal (P): It is the sum of money deposited / borrowed etc. also known as capital.

Time (T/n): Duration for which the money is lent / borrowed.

Rate of Interest (R/r): It is the rate at which interest is charged on the principal.

Amount (A) = Principal + Interest

There are 2 kinds of Interest:

a) Simple Interest **b) Compound Interest**

8.2 Simple Interest : when interest is calculated every year (or every time period) on the original principal i.e., the sum at the beginning of first year such interest is called simple Interest. Here year after year even though the interest gets accumulated and is due to the lender, this accumulated interest is not taken into account for the purpose of Calculating interest for latter years.

Simple Interest, SI = $\frac{PTR}{100}$

P = Principal

T = Time in years

R = Rate of interest in %

A = $P + I$

$$= P + \frac{PTR}{100}$$

$$= P\left(1 + \frac{TR}{100}\right)$$

Simple Interest

Example 1 Veena deposited ₹10,000 in a finance Company which pays 15% interest per year. Find the interest and amount she is expected to get after 5 years and 3 months.

Solution: P = 10,000 $t = 5 + \frac{3}{12} = 5\frac{1}{4} = 5.25$ R= 15%

$$SI = \frac{PTR}{100} = \frac{10{,}000 \times 5.25 \times 15}{100} = 7875 ₹$$

A = P + I

= 10,000 + 7875

= 17, 875₹

Example 2 Find the interest on ₹1500 at 4% p.a. for 145 days

Solution : P = 1500 $t = \frac{145}{365}$ R = 4%

$$SI = \frac{PTR}{100} = \frac{1500 \times \frac{145}{365} \times 4}{100}$$

$$= 23.835$$

$$\approx 23.84$$

Example 3 What principal will amount to ₹46000 in 7 years at 12% p.a.?

Solution : A = 46000T = 7 R = 12

$$A = P + I$$

$$= P\left(1 + \frac{TR}{100}\right)$$

$$46000 = P\left(1 + \frac{7 \times 12}{100}\right)$$

$$46000 = 1.84P.$$

$$\frac{46000}{1.84} = P$$

$$\therefore P = 25000$$

Example 4 Priya invested ₹6000 for 3 years and received ₹1080 as interest. Find the rate of interest.

Solution : P = 6000 T = 3 SI = 1080 R = ?

$$SI = \frac{6000 \times 3 \times R}{100}$$

$$1080 = 180 R$$

$$\frac{1080}{180} = R$$

$$R = 6 \%$$

Example 5 If the simple interest on a certain sum of money for 2 years is one fifth of the sum. Find the rate of interest.

Solution : Let P = x SI = $\frac{1}{5}x$ T = 2 R = ?

$$SI = \frac{PTR}{100}$$

$$\frac{1}{5}x = \frac{x \times 2 \times R}{100}$$

$$\frac{100}{5 \times 2} = R \Rightarrow R = 10\%$$

Example 6 Sowmya invested ₹1500 for 8 years and Anisha invested ₹7500 for 3 years at the same rate of interest. If altogether they received ₹1725 as interest find the rate of simple Interest charged.

Solution : Sowmya,

$P_1 = 1500$, $T_1 = 8$, $R_1 = R$

$$I_1 = \frac{1500 \times 8 \times R}{100}$$

$$= 120 R$$

Anisha,

$P_2 = 7500 \qquad T_2 = 3 \qquad R_2 = R$

$$I_2 = \frac{7500 \times 3 \times R}{100}$$

$$= \quad 225\ \text{R}$$

$$I_1 + I_2 \quad = \quad 1725$$

$$120\ \text{R} + 225\ \text{R} = 1725$$

$$345\ \text{R} = 1725$$

$$R = \frac{1725}{345}$$

$$= 5\%$$

Example 7 If a certain sum of money is doubled in 8 years at a given simple interest. In how many years will it be four times?

Solution : Let P = x A = 2x I = A – P = 2x – x = x T = 8

$$\text{SI} = \frac{PTR}{100}$$

$$x = \frac{x \times 8 \times R}{100}$$

$$R = \frac{100}{8} = 12.5\%$$

A = 4x , T = ?, I = A – P= 4x - x = 3x, R = 12.5

$$3x = \frac{x \times T \times 12.5}{100}$$

$$\therefore \text{T} = 24 \ \text{years}$$

Example 8 If 500 amounts to ₹725 at 9% simple interest in some time, what will ₹600 amount to at 11% in the same time.

Solution : P = 500 A = 725 I = 225 R = 9 T = ?

$$I = \frac{PTR}{100}$$

$$225 = \frac{500 \times T \times 9}{100}$$

$$\frac{225}{45} = T$$

$$T = 5 years$$

P = 600, R = 11, T = 5, I = ?

$$I = \frac{600 \times 5 \times 11}{100} = 330$$

$$\begin{aligned} A &= P + I \\ &= 600+330 \\ &= 930 \text{ ₹} \end{aligned}$$

Example 9 Reshma invested a part of ₹6,500 at 10% simple interest and rest of it at 12% simple interest. At the end of three years she got a total amount of 8,690. How much money she had invested at different rates?

Solution : Investment at 10%,

P = x, R = 10, T = 3

$$I_1 = \frac{P \times T \times R}{100}$$

$$= \frac{x \times 3 \times 10}{100}$$

$$= 0.3\, x$$

Investment at 12%,

P = 6500 – x , R = 12 , T = 3

$$I_2 = \frac{(6500 - x) \times 3 \times 12}{100} = 2340 - 0.36x$$

$$I_1 + I_2 = 8690 - 6500$$
$$0.3x + 2340 - 0.36x = 2190$$
$$2340 - 2190 = 0.06x$$
$$x = 2500$$

$\therefore$ Investment at 10% = x =2500

Investment at 12% = 6500- 2500 = 4000

Example 10 Three equal principals amount to ₹3720 after 3, 4 and 5 years at simple interest 6% p. a. Find the principal.

Solution : Amount after 3 years,

P = x, T =3, R = 6

$$I_1 = \frac{x \times 3 \times 6}{100}$$

$$A_1 = P + I = x + 0.18x = 1.18x$$

Amount after 4 years,

P = x, T = 4 , R = 6

$$I_2 = \frac{x \times 4 \times 6}{100} = 0.24x$$

$$A_2 = x + 0.24x = 1.24x$$

Amount after 5 years,

P = x T =5 , R = 6

$$I_3 = \frac{x \times 5 \times 6}{100}$$

$$A_3 = x + 0.3x = 1.3x$$

Given $A_1 + A_2 + A_3 = 3720$

$1.18x + 1.24\,x + 1.3\,x = 3720$

$3.72\,x = 3720$

$$x = \frac{3720}{3.72} = 1000$$

Each principal = 1000₹

Total principal = 3000₹

Example 11 If the interest on 800 be more than the interest on ₹400 by Rs 40 in 2 years. Find the rate of interest.

Solution : P = 800 , T = 2 , R = R

$$SI = \frac{800 \times 2 \times R}{100}$$

$I_1 = 16R$

P= 400, T = 2, R = R

$$SI = \frac{400 \times 2 \times R}{100}$$

$I_2 = 8R$

$I_1 - I_2 = 40$

$16R - 8R = 40$

$$R = \frac{40}{8} = 5\%$$

Example 12 Nicole got a certain sum of money as prize in a lottery and deposited the same in a bank. It amounted to ₹1,624 in four years and amounts to 1736 in 6 years. Find the prize and rate of simple interest allowed by the bank.

Solution: Let P = P , T = 4 , R = R , A = 1624

$$A = P\left(1 + \frac{TR}{100}\right)$$

$$1624 = P\left(1 + \frac{4R}{100}\right) \qquad ....(1)$$

P = P , T = 6 , R = R , A = 1736

$$\frac{1736}{1624} = \frac{P(1+0.06R)}{P(1+0.04R)} \quad 1736 = P\left(1 + \frac{6R}{100}\right) \quad ....(2)$$

(2) ÷ (1)

$$1.068 = \frac{1+0.06R}{1+0.04R}$$

$$1.068 \quad (1+0.04R) = 1 + 0.06R$$

0.068 = 0.017 R (approximate value)

R =4 %

Substituting in (1)

1624 = P(1+ 0.04 × 4)

$$\frac{1624}{1.16} = P$$

P =1400

EXERCISE : 8.1

I One Mark Questions.

1. Calculate the simple interest on ₹18000 for 4 years at 12½% p. a simple interest.
2. Find the simple interest on ₹600 for 3 years 3 months at 4% p.a simple interest.
3. What is the simple interest for ₹245 days for ₹6000 at 8% p. a simple interest?
4. What is the simple interest on ₹650 for 14 weeks at 6% p.a.?
5. Calculate the simple interest on ₹4000 at 4% from June 27 to Aug 29 in the same year.

II Two marks questions.

1. Sanjay deposits ₹25000 at 8% simple interest. What is the amount he will get after 5 years?
2. How much should Bhavya invest in a finance company which offers 12% p. a. simple interest, so that she may have ₹80000 after 5 years?
3. In what time will the simple interest on ₹500 at 6% be equal to the interest on 540 for 8 years at 5% ?
4. Rochelle wishes to borrow a certain sum of money for 120 days. She goes to bank whose rate of interest is 6%. The bank charges ₹360 as interest. How much does Rochelle borrow?
5. In what time will ₹4000 at 3% interest produce the same income as ₹5000 in 5 years at 4%?
6. In what time will ₹35000 amount to ₹45,500 at 7.5% p.a.?
7. In how many years will a sum be double of itself at 10% simple interest?
8. If the simple interest on a certain sum of money after 6¼ years is 3/8 of principal, what is the rate of interest p.a. ?

III Three marks questions

1. The simple interest on a certain sum of money is $\frac{4}{25}th$ of the sum and the rate percent equals the number of years. Find the rate of interest.
2. Sandhya invested a certain amount in a bank. When the rate of interest changed from 10% to 12½ % her annual income increased by ₹1250. How much did Sandhya invest?
3. The difference between simple interest received from two different sources on ₹2500 for 3 years is 375. find the difference in their rate of interest [Hint: Find $R_1 \sim R_2$]
4. Find the rate at which a sum becomes four times of itself in 15 years.
5. A sum of money doubles itself in 12 years 6 months. In how many years will it triple itself?

IV Five marks questions.

1. Jason invested an amount of ₹12000 at the rate of 10% p.a. simple interest and another amount at the rate of 20% p.a. simple interest. The total interest earned at the end of on year one the total amount invested became 14% p.a. Find the total amount invested.

2. ₹1200 becomes 1536 in 4 years at a certain rate of simple interest. If the rate of interest is increased by 3%, what will ₹2000 amount to in 2 years?

3. If the difference between the simple interest at a certain sum for 4 years at 2.5% per annum and the simple interest on the same sum for the same period at 3% p.a. is ₹60. Find the sum.

4. A sum was put at simple interest at a certain rate for 4 years. Had it been put at 2% higher rate, it would have fetched ₹56 more. Find the sum.

5. A sum of money amounts to ₹19500 in 5 years and 22,200 after 8 years, at the same rate of interest. Find the rate of interest. Also find the principal.

6. A lent ₹5000 to B for 2 years and ₹3000 to C for 4 years on simple interest at the same rate and received ₹2200 in all from both of them as interest. Find the rate of interest.

7. Satwik obtained a loan of ₹4000 at an interest rate of 6% per year. He immediately lent ₹2500 at an interest rate of 9% per year to Jason and balance at 12% per year to Vivek. After 3 years he collected the amount due to him and repaid his loan. Find his gain.

8. Privil invested a part of ₹6500 at 10% annual interest and rest of it 12% annual interest. At the end of three years he got a total amount of ₹8,690. How much money he has invested at different rates?

9. Sujith borrowed ₹1,00,000 from money lender and a bank. If the rate of interest are 18% p.a. and 16% p.a. respectively and Sujith pays ₹16,600 as interest for one year, find the amount borrowed from money lender and bank.

10. A man left ₹1,30,000 for his two sons aged 10 years and 16 years with the direction that the sum should be divided in such a way that the two sons get the same amount when they attain the age of 18 years. Assuming the rate of simple interest as 12 ½ % p.a., calculate how much the elder son got in the beginning.

ANSWERS : 8.1

I	1. 9000	2. 78	3. 322.19	4. 10.5	5. 27.61
II	1. 35000	2. 50000	3. 7.2 yrs	4. 18250	5. 8.33 yrs
	6. 4 yrs	7. 10 Yrs	8. 6%		
III	1. 4%	2. ₹50000	3. 5%	4.20%	5. 25 yrs
IV	1. 20000	2. 2400	3. 3000	4. 700	5. 6% , 15000
	6. 10%	7. 495	8. 4000 at 12% , 2500 at 10%		
	9. 30000, 70000		10. 80000		

8.3 Compound Interest:-

Under compound interest, the interest is added to the principal at the end of each period to arrive at the new principal for the next period.

In other words, the amount at the end of first year (or period) will become the principal for the second year (or period); the amount at the end of second year (or period) becomes the principal for the third year (or period) and so on.

Important formulae in compound interest:-

1) To find amount

$$\mathbf{A = P(1 + i)^n}$$

$$\mathbf{CI = A - P}$$

P = Principal

n = time in years

i = rate of interest in decimal = $\frac{R}{100}$

A = Amount

2) To find principal

$$P = \frac{A}{(1+i)^n}$$

3) To find the value of r

$$i = \text{Antilog}\left[\frac{\log A - \log P}{n}\right] - 1$$

To convert to % multiply by 100. i.e., R= i × 100

4) To find n:

$$n = \frac{\log A - \log P}{\log(1+i)}$$

Example 1 Find the compound interest on ₹13,000 at 6% p.a. for 4 years.

Solution : P = 13,000 $\quad i = \frac{6}{100} = 0.06 \quad n = 4$

$A = P(1+i)^n$

$=13000(1+0.06)^4$

= 16412.20 ₹

CI = A – P

= 16412.2 – 13000

= 3412.2 ₹

Example 2 A certain sum of money amounts to ₹24,200 in 2 years at 10% compound interest. Find the sum.

Solution : A = 24,200 $\quad n = 2 \quad i = \frac{10}{100} = 0.1$

$$P = \frac{A}{(1+i)^n}$$

$$= \frac{24200}{(1+0.1)^2}$$

= 20,000₹

Example 3 In what time a sum of ₹1,200 will earn ₹573 as compound interest at the rate of 5% p.a, if the interest is added annually?

Solution: P = 1200 CI = 573 A = 1200 + 573 = 1773

$i = \frac{5}{100} = 0.05$ $n = ?$

$$A = P(1+i)^n$$

$$1773 = 1200(1+0.05)^n$$

$$\frac{1773}{1200} = (1.05)^n$$

$$1.4775 = (1.05)^n$$

$$\Rightarrow n = 8$$

Note: n can be calculated with the help of log or directly with calculator as follows: 1.05 when multiplied how many times the result is 1.4775. You will get the result when you multiply 8 times. Hence n = 8

Example 4 ₹9000 amounts to 10,418.625 in 3 years. Find the compound interest rate percent.

Solution: P = 9000 A = 10,418. 625 n = 3r = ?

$$i = \text{Antilog}\left[\frac{\log A - \log P}{n}\right] - 1$$

$$= \text{Antilog}\left[\frac{\log 10418.625 - \log 9000}{3}\right] - 1$$

$$= \text{Antilog}\left[\frac{4.0174 - 3.9542}{3}\right] - 1$$

$$= \text{Antilog}\ [\ 0.021067\ ] - 1$$

$$= 1.050 - 1$$

$$= 0.05$$

$$R = i \times 100$$

$$= 0.05 \times 100 = 5\%$$

Example 5 A sum of money was invested at compound interest. At the end of the first year the interest was ₹125 at the end of the second year, it was ₹130 Find the sum invested and the rate of interest.

Solution : The increase in the interest by 5₹ for the second year is due to the increase in principal by 125.

$\therefore$ Interest for 125₹ for one year is 5.

The interest for 100 ₹ is I $= \dfrac{100 \times 5}{125}$ 125 -5 100 $-?$

$= 4\%$

Compound interest and simple interest for one year is same

$\therefore$ If principal is 100 interest is 4 ₹

If interest is ₹125 then principal is,

$$P = \frac{100 \times 125}{4} = 3125 \qquad \begin{matrix} 100 & -4 \\ ? & -125 \end{matrix}$$

Example 6 A sum of money placed at compound interest doubles itself in 4 years. In how many years will it amount to eight times.

Solution : $A = 2P$, $P = P$, $n = 4$

$A = P\,(1+i)^{n}$

$2P = P\,(1+i)^4$

$2 = (1+i)^4$

Taking log,

$$\text{Log}2 = 4\ \log(1+i)$$

$$\frac{\log 2}{4} = \log(1+i)$$

$$\frac{0.3010}{4} = \log(1+i)$$

$$0.07525 = \log(1+i)$$

Taking Antilog,

$$1.19 = 1 + i$$

$$i = 0.19$$

A = 8P, P = P, i = 0.19 n = ?

$$8P = P\ (1+0.19)^n$$

$$8 = (1.19)^n$$

$$n = 12.$$

Or

This particular sum could be done without log as follows.

A = 2P , P =P , n =P

$$2P = P(1+i)^4$$

$$2 = (1+i)^4$$

$$8P = P(1+i)^n$$

$$8 = 91+i)^n$$

$$2^3 = (1+1)^n$$

$$((1+i)^4)^3 = (1+1)^n (\because (1+i)^4 = 2)$$

$$(1+i)^{12} = (1+i)^n$$

$$\therefore n = 12$$

Note: The second method may not be applicable for all other sums of the same type.

To compute compound interest when n is not an integer

When 'n' is not an integer write 'n' in the form of whole number + proper fraction.

For example, if n = 4 ½ years write n = 4 + ½

In general, write n = k + t

Where k is an integer and t is a proper fraction. Then the formula for amount will be

$A = P(1+i)^k (1+i.t)$

Example 7 Find the compound interest on ₹22000 for 2½ years at 6% p.a

Solution : P = 22000 $i = \frac{6}{100} = 0.06$ $n = 2 + \frac{1}{2}$

$$A = 22000\,(1+0.06)^2\,(1+0.06\times\frac{1}{2})$$

$= 25460.78$ ₹

CI = A – P

= 3460.78

Doubling period: Investors commonly ask the question: How long would it take to double the amount at a given rate of interest. If you invest some amount at 8% and want to know in how many years the amount will double itself you can calculate as follows:

A = 2P, P = P, r = 0.08, n = ?

$$2P = P(1 + 0.08)^n$$

$$2 = (1 + 0.08)^n$$

$$\log 2 = n \log (1.08)$$

$$\frac{\log 2}{\log 1.08} = n$$

$$\frac{0.3010}{0.0334} = 9.012 \text{ years}$$

Rule of 72 : According to rule of 72, the doubling period is obtained by dividing 72 by the interest rate. Then for above example, doubling period

$$= \frac{72}{8} = 9 \text{ years.}$$

Rule of 69: According to rule of 69, the doubling period is equal to 0.35 + $\frac{69}{Interest\ rate}$.

EXERCISE : 8.2

I Two marks questions

1. When Geetha retired at 58, she deposited ₹1,00,000 in the bank which pays 18% p.a Compound interest. How much amount will she receive when she is 70?
2. What sum will amount to ₹6525 at 10% p.a compounded annually for 13 years?
3. Joanna invested ₹8000 for 3 years at 5% Compound interest in a post office what sum will she receive after 3 years?
4. On what sum will the Compound interest at 5% p.a for two years compounded annually be ₹1640?

II Three marks questions.

1. Find the Compound interest on ₹7,000 at 5% p.a for 8 years.
2. In how many years will ₹30,000 give ₹4347 compound interest at 7% p.a.?
3. In how many years a sum of ₹2000 becomes ₹2205 at the rate of 5% p.a. compound interest?
4. After how many years will ₹2000 earn a compound interest of ₹662 at the rate of 10% p.a. ?
5. How much compound interest can be obtained if ₹6500 is invested for 15 years at 12% p.a.?
6. If ₹500 amounts to ₹583. 2 in two years compounded annually, find the rate of interest p.a.
7. Calculate the Compound interest on ₹600 for 4 ½ years at 7% p.a.
8. Calculate the amount received after 15 months if ₹8500 is invested at 8% p.a compound interest .

   [Hint: n = 1 ¼]
9. Poornima and Raj deposited 50000 in a bank on the day their first child was born. If the bank offers 16% p.a. compound interest, how

much money will be accumulated in the bank on the childs tenth birthday?

[Hint: The day the child is born is the first birthday and hence on the child's tenth birthday child will be 9 years old n=9]

10. If the interest rate is 12%, what are the doubling periods as per rule of 72 and rule of 69 respectively?

III Five marks questions

1. If ₹600 amounts to ₹1,510.9 at compound interest in 12 years. Find the rate of compound interest.

2. If ₹2000 amounts to ₹2315.25 at compound interest in 3 years, find the rate of interest.

3. A sum of money put at compound interest amounts to ₹672 in two years and ₹714 in 3 years. Find the rate of interest p.a Also find the principal.

4. A sum of money put out at compound interest amounts in 2 years to ₹4410 and in 3 years to ₹4630.5. Find the rate of interest and the original sum.

5. At what rate percent compound interest a sum of ₹2,550 will amount to ₹2952 in 3 years ?

6. At what rate percent of compound interest a sum of money will double itself in 12 years?

7. A sum triples itself in 4 years under compound interest at a certain rate of interest. Find the time it would take to become 9 times itself.

8. Laxmi decided to purchase a hair dressing machine. For this purpose, she took a loan of 5,000 at 5% p.a. for 3 years at Compound interest. How much amount did she return after 3 years? Also find the amount paid as interest.

ANSWERS : 8.2

I.	1) 7,28,759.26	2) ₹1890.06	3) 9261₹	4) 16000
II.	1) 3342.188	2) 2 years	3) 2 years	4) 3 years
	5) 29078.17	6) 8%	7) 214	8) 9363.6
	9) 190148.06	10) 6 years, 6.1 years		

III.	1) 8%	2) 5%	3) 6.25%, 595.27₹
	4) 5%, 4000₹	5) 5%	6) 5.95% ≈ 6%
	7) 7.99 ≈ 8 years	8) 5788.12₹, 788.12₹	

8.4 Nominal Rate and effective rate of interest.

The annual compound interest rate is called the nominal interest rate. But if the interest rate is compounded more than once i.e. twice (for half yearly) or four times (for quarterly), or more during a year then, the actual percentage of interest per year will be called the effective rate of interest. Effective rate of interest is always grater than nominal rate of interest.

The effective rate of interest can be calculated as follows.

$$r = \left[1 + \frac{i}{q}\right]^q - 1$$

i = Rate of interest in decimal form $i = \dfrac{R}{100}$

r = effective rate of interest

q =number of times interest is computed in a year.

Example 1 Find the effective rate of interest when a sum lent at 18% p.a. is computed quarterly.

Solution: i = 0.18 q = 4

$$r = \left(1 + \frac{i}{q}\right)^q - 1$$

$$= \left(1 + \frac{0.18}{4}\right)^4 - 1$$

$$= 0.1925$$

$$\therefore r = 19.25\%$$

Example 2 Find the effective Rate of interest if ₹6000 is invested at 6% p. a. Compound interest calculated monthly.

Solution : i =0 .06 q = 12

$$r = \left(1 + \frac{i}{q}\right)^q - 1$$

$$= \left(1 + \frac{0.06}{12}\right)^{12} - 1$$

= 0.0616

R = 6. 16%

Example 3 A certain sum invested at 4% p.a. compounded semi annually amounts to 78030 at the end of one year. Find the sum.

Solution : A = 78030 i = 0.04 n =1

$$A = P(1 + \frac{i}{2})^{n \times 2}$$

$$78030 = P\left(1 + \frac{0.04}{2}\right)^2$$

$$\frac{78030}{1.0404} = P$$

P = 75000

Note: when interest is calculated more than once in a year instead of Calculating effective rate of interest the CI formula for amount can be used dividing i by q and multiplying n by q.

Example 4 When a child is born ₹25000 is put into an account which pays at the rate of 6% compounded monthly. If the account is not disturbed what amount will be there in the account when the child is 20 years.

Solution : P = 25000 $i = \frac{6}{100} = 0.06$ n =20 q =12

$$A = P\left(1 + \frac{i}{q}\right)^{n \times q}$$

$$= 25000 \left(1 + \frac{0.06}{12}\right)^{20 \times 12}$$

$$= 25000\ (1.005)^{120}$$

Log A = log 25000+120 log (1.005)

= 4.3979+120 × 0.0021

= 4.6499

A = Antilog (4.6499)

= 44660₹

EXERCISE : 8.3

I Two marks questions

1. Find the effective rate of interest when a sum lent at 12% is computed half yearly.
2. Find the effective rate of interest at 6% computed once in 3 months.
3. Find the effective rate of interest when a sum lent at 9% is computed quarterly.
4. If the nominal rate is 13% and frequency of Computing interest is once in 4 months. Find the effective rate of interest.

II Three marks questions

1. A sum of money lent at compound interest for 2 years at 20% p.a. would fetch ₹482 more, if the interest was payable half yearly than if it was payable annually. Find the sum

   [Hint : Difference in two Amounts is 482]

2. ₹16000 invested at 10% p.a. compounded semi annually amounts to ₹18522. Find the time period of investment.
3. In what time 800 will amount to ₹882 at 10% p.a. interest compounded half yearly.
4. Find the compound interest on ₹6,000 for 3 years at 5% p.a. if interest is calculated half yearly.
5. Find compound interest on ₹8000 for 8 years at 16% p.a. if interest is calculated quarterly.

III Five marks questions :

1. Find the Compound interest on ₹6950 at 12% for 1 year 9 months while interest is calculated quarterly.
2. Find the compound interest on ₹7500 at 14% for 4½ years while interest is calculated half yearly.
3. In what time will a sum of money double itself at 10% p.a. compound interest payable half yearly.
4. Find rate percent p.a. if ₹200000 amounts to ₹231525 in 1.5 years interest being compounded half yearly.

ANSWERS : 8.3

I	1) 12.36%	2) 6.14%	3) 9.3%	4) 13.57%
II	1) ₹20,000	2) 1.5 years	3) 1 years	4) 958.16
	5) 20064.46			
III	1) ₹1,597.6	2) 6288.4	3) 7.09 years	4) 10%

8.5 Varying rate of interest

It is possible that the interest rate may vary during the period of loan or investment. Then if P is principal, rate of interest for a period of n_1 is i_1 ; for n_2 is i_2 ; for n_3 is i_3 and so on then Amount formula of compound interest will be as follows.

$A = P\,(1 + i_1)^{n_1}\,(1+i_2)^{n_2}\,(1+i_3)^{n_3}\ldots\ldots\ldots$

Example A person deposits ₹10,000 in a bank. If the bank offers compound interest at the rate of 5% for the first 3 years, 7% for the next 2 years and 8% from the sixth year onwards. What is the amount after 10 years?

Solution : $P = 10{,}000 \quad i_1 = 0.05 \quad n_1 = 3 \quad i_2 = 0.07 \quad n_2 = 2$

$i_3 = 0.08 \quad n_3 = 5$

$A = P\,(1 + i_1)^{n_1}\,(1+i_2)^{n_2}\,(1+i_3)^{n_3}$

$= 10{,}000\,(1 + .05)^3(1+0.07)^2(1+0.08)^5$

$= 19473.95$₹

8.6 Depreciation

All fixed assets such as machinery, building, furniture etc.. gradually diminish in value as they get older and become worn out by constant use . In business depreciation is the term used to describe this decrease in book value of an asset. There are a number of methods of calculating depreciation. However the most common method which is also approved by income tax authorities is the "diminishing Balance method". Here each year's depreciation is calculated on the book value (i.e. depreciated value) of the asset at the beginning of the year rather than original cost. This method is also called 'Reducing installment method.' Since the same concept is used in computation of compound interest the same formula can be used for depreciation. However as the book value decreases every year minus sign is used instead of plus sign. Hence if Initial value of asset is P and future value of asset is A, rate of depreciation is i. then future value or Book value of assets can be computed using the formula.

$\mathbf{A=P(1-i)^n}$

Example 1 Evergreen Suppliers buy a machine for 20,000. The rate of depreciation is 10%. Find the depreciated value of the machine after 3 years. Also find the amount of depreciation. What is the average rate of depreciation?

Solution : P=20,000 i=0.1 n=3

$$A = 20,000\ (1-0.1)^3$$
$$= 20,000 \times (0.9)^3$$
$$= 14,580$$

Amount of depreciation in 3 years is

$$= 20,000 - 14,580$$
$$= 5,420$$

Average rate of depreciation in 3 yrs is $= \dfrac{\dfrac{5420}{20,000} \times 100}{3} = 9.033\%$

Note: Average rate of depreciation can be calculated using simple Interest as follows:

$$5420 = \frac{20,000 \times 3 \times R}{100}$$

$$R = \frac{5420 \times 100}{20000 \times 3}$$

$$= 9.033\%$$

8.7 Application of Compound interest formula in growth rate

Compound interest formula finds its application in growth rate problems also. The most popular problem is that of population which can be best explained with the help of an example.

Example 1 If the current population of Mangalore city is 4,84,785 and the average growth rate of population in Mangalore city is 2.1%p.a. What can city planners expect the population of Mangalore to be in 10 years.

Solution : $p = 4,84,785$ $\quad i = \frac{2.1}{100} = 0.021$ $\quad n=10$

$A=4,84,785(1+0.021)^{10}$

$\approx 5,96,769$

EXERCISE : 8.4

I Three marks Problems

1. A person deposited ₹5000 in a bank. The bank offers 6% p.a. Compound interest for the first two years 10% for the next 4 years, 12% for the next 3 years and 14% from 10th year onwards. How much will he get after 15 years?
2. Samarth bought a walkman for ₹1800. If it depreciates at the rate of 15% per year how much is it worth after 3 years?
3. Preritha bought a Car for ₹4,00,000. If it depreciates at the rate of 12% per year how much will it be worth after 10 years?
4. Prateeksha bought a sound system for ₹22,000. If it depreciates at the rate of 11% per year, what is its worth after 3 years?
5. The Cost of a refrigerator is ₹27,000. If it depreciates at the rate of 8%, find its value after 4 years.
6. The present population of a town is 80000. If it increases at the rate of 5% per anum, what will be its population after 3 years?
7. The population of a town is 40,000. If the annual birth rate is 8% and death rate is 2%, calculate the population after 4 years.
8. In 2010 the population of a town was 2,70,000. If the rate of increase is 45 per thousand of the population, find the estimated population for the year 2025.
9. The value of a machinery depreciates every year by 20%. What would be the value of the machinery bought for ₹6250 at the end of 3 years?
10. A machine depreciates in value each year at 10% of its previous value and at the end of 4th year its value is 1,31,220. Find the original value.

II Five marks problems

1. The population of a town was 2,50,000 three years ago. If the population increased by 4%, 3.5% and 6% respectively in the last three years find its present population.
2. A rare species of tigers were kept in an wild life sanctuary and reared. Its population increased by 10% in the fist year and 5% in the second year but decreased by 20% in the third year. If the animals in the beginning were 500 find their number at the end of third year.
3. If the population of a town increases every year by 2% of the population at the beginning of that year, in how many years will the total increase of population be 40%?
4. The bacteria in a culture grew by 8% in the first hour, decreases by 8% in the second hour, and again increases by 7% in the third hour. If at the end of the third hour the count of bacteria is 12170 thousands, find the original Count (in thousands) of bacteria in sample.
5. A machine depreciates at 10% of its value at the beginning of a year. The Cost and scrap value realized at the time of sale being 23240 and 9000 respectively. For how many years the machine was put to use?
6. The population of a town increases annually by 25%. If the present population is one crore then what is the difference between the population 3 years ago and 2 years ago?
7. The population of a town increased by 4% in the first year and diminished by 4% in the second year. If the population of the town at the end of second year is 39936, find the population of the town at the beginning of the year.
8. A machine worth of ₹12000 is depreciated at the rate of 10% a year. It was sold eventually as waste metal for ₹200. Find the number of years the machine was in use.
9. A machine was purchased for ₹30,000. It depreciates at the rate of 5% p.a. for the first two years and then depreciates at 8% p.a. from the third year. Find value of the machine after 10 years. What is the total depreciated value? What is the average depreciation?

10. The annual birth and death rate per 1500 are 30.5 and 12.5 respectively. If the present population is 2,45,000, find the population after 10 years.

11. The scrap value obtained by a selling a machine after 10 years of purchase is 19660.8. If the machine depreciated at the rate of 20% p.a. find the cost at which the machine was purchased 10 years ago.

12. A factory owner wants to replace his old machine by another machine with new technology which will cost him ₹75000. So he decides to sell his old machine which he purchased 10 years back for 60,000. If the rate of depreciation is 10% find how much more money he needs to buy the new machine?

ANSWERS : 8.4

I	1) 25365.04	2) 1105.425	3) 111400.39
	4) 15509.32	5) 19342.61	6) 92610
	7) 50499.08	8) 522526.26	9) 3200 10) 2,00,000
II	1) 285246	2) 462	3) 17 years
	4) 11447.09	5) 9 years	6) 12,80,000
	7) 40,000	8) 38.8	9) 13,895.4, 16104.6, 5.3%
	10) 276039.4	11) 183105.46	12) 54079.29

8.8 Problems related to simple Interest and Compound interest

Example 1 Find the difference between simple Interest and Compound Interest on 18,000 invested for 4 years at 8%p.a. where Compound interest is compounded annually.

Solution : **Simple Interest**

P =18,000 T = 4 R = 8

$$SI = \frac{PTR}{100}$$

$$= \frac{18,000 \times 4 \times 8}{100}$$

$$= ₹5,760$$

Compound Interest

P=18,000   n = 4   I =0.08

$A= P(1+i)^n$

$=18,000(1+0.08)^4$

$=24,488.8$

$CI = A — P$

$= 24,488.8 - 18,000$

$CI = 6488.8$

Difference between CI and SI is

$CI - SI = 6488.8,-5760$

$= 728.8$

Example 2 The difference between simple Interest and Compound interest on a certain sum of money invested for 3 years at 6% p.a. is 110.16 Find the sum.

Solution : Let the sum = x

$$\text{Simple Interest} = \frac{x \times 3 \times 6}{100}$$

$$= 0.18\,x$$

Compound interest

$$A = P(1 + i)^n$$

$$= x\,(1 + 0.06)^3$$

$$= 1.191016\,x$$

$$CI = A - P$$

$$= 1.191016\,x - x$$

$$= 0.191016\,x$$

Given, $CI – SI = 110.16$

$$0.191016x – 0.18x = 110.16$$

$$x = \frac{110.16}{0.011016}$$

$$= 10,000 \text{ Rs}$$

Example 3 A father wishes to divide 50,000 amongst his two daughters who are respectively 12 and 15 years old in such a way that the sum invested at 5% p.a compound interest will give the same amount to both of them when they attain the age of 18. How is the sum divided?

Solution : Let the share of the older daughter = x then share of young one = 50,000 – x

For older daughter,

$A = A_1$ $\quad$ $P = x$ $\quad$ $i = 0.05$ $\quad$ $n = 3$

$A_1 = x(1 + 0.05)^3$

For younger daughter,

$A_1 = A_2$ $\quad$ $P = 50{,}000 - x$ $\quad$ $i = 0.05$ $\quad$ $n = 6$

Given $A_1 = A_2$

$x(1.05)^3 = (50{,}000 - x)(1.05)^6$

$x = (50{,}000 - x)(1.05)^3$

$x = 57881.25 - 1.157625\,x$

$2.157625\,x = 57881.25$

$x = 26826$ (Approx.)

$\therefore$ Share of older daughter = 26826₹

Share of younger daughter = 50,000 – 26826

= 23174₹

EXERCISE : 8.5

Five Marks Questions

1) Suraj borrowed ₹18000 at 5½% p.a. simple Interest for 3 years and lent it to Akash at 5½% p.a. compound Interest for 3 years. What interest was paid and received by Suraj and what gain did he make in this transaction.

2) Find the difference between simple Interest and compound interest on ₹6,400 at 9% p.a. for 5 years.

3) Lavina borrowed ₹2,400 at 6% p.a. simple Interest and invested this at 7½ p.a. compound interest. If the transaction was for two years , what was her gain?.

4) A person borrows a certain sum of money at 3% p.a. simple Interest and invests the same at 5% p.a. compound interest compounded annually. After 3 years he makes a profit of ₹1,082. Find the amount he borrowed.

5) If an amount is invested on 4% compound interest ₹1,352 will be received at the end of second year. If the same amount at the same rate of simple interest is invested how much less will be received? Also find the principal.

6) Pradeep invested equal amounts one at 6% SI and other at 5% CI. If the former earns ₹437.5 more as interest at the end of two years, find the total amount invested.

7) A sum of 75,000 is to be divided between 2 persons aged 16 and 19 years in such a way that if their shares are invested at 6% p.a. compound interest they shall receive equal amounts on attaining the age of 21 years. How the sum will be divided and how much will they receive when they are 21 years old?

8) If the difference between simple interest and compound interest for 3 years at 2.5% p.a. is ₹625, find the sum invested.

9) The difference in compound interest and simple interest on a sum for 2 years at 10% p.a. when compound interest is computed annually is 16. Find the difference in the difference in compound interest and simple interest if compound interest is computed half yearly.

10) A person borrowed 65,000 at 8% p.a. simple Interest for 4 years and lent out the money for 10% compound interest for 4 years. How much did the person gain?

ANSWERS : 8.5

1) paid 2970, received 3136.34, gain 166.34 2) 567.19

3) 85.5₹ 4) ₹16000 5) ₹2 less, 1250 is principal

6) 50,000 (25000 + 25000) 7) 34230.69, 40769.31

8) 330578.5 9) ₹24.81 10) 9366.5

CHAPTER 9

ANNUITIES

9.1 Meaning and definition of Annuity

An annuity is a fixed sum paid at regular intervals of time under certain conditions. These equal intervals may be either a year or a half year or a quarter year or month etc. If nothing is mentioned about the interval of time, it is always taken as one year. For example; repayment installments of loan, LIC premiums, deposits into a recurring account etc. are all examples of annuities.

Types of Annuity

According to the time of payment there are two kinds of annuity

1. **Annuity immediate**: If the payments are made at the end of each interval of time, the annuity is called annuity immediate. If nothing is mentioned, then the annuity is considered as annuity immediate.

2. **Annuity Due:** If the payments are made at the beginning of each interval of time, the annuity is called annuity due.

Some terms related with annuity:

1. **Present value of an annuity:** It is the sum of the present values of all the installments. In simple words if the lump-sum amount is obtained before and the annuity (equal installments) is paid later then the lump-sum amount is called present value i.e. Today's value is called as present value.

2. **Future value of an annuity:** It is the sum of the future values of all the installments. In simple words if the lump-sum amount is obtained at the end and annuity (equal installments) is paid before then the lump-sum amount is called future value. It is denoted by F.

   Example: LIC Policy, RD amount.

3 **Annuity**: The equal amount of money is called annuity. It is denoted by 'a'

9.2 Future value of Annuity Immediate :

Future value F = $\dfrac{a\left[(1+i)^n - 1\right]}{i}$

Where F = Future value

a = annuity or equal installment

i = rate of interest in decimal.

n = no of installments.

Example 1 Find the future value of an annuity of ₹500 at 5% p.a payable for 5 years

Solution : a = 500 i = 0.05 n = 5

$$F = a\frac{\left[(1+i)^n - 1\right]}{i}$$

$$= 500\frac{\left[(1+0.05)^5 - 1\right]}{0.05} = 500\frac{\left[1.2763 - 1\right]}{0.05}$$

$$= 2763₹$$

[**Note:** Calculate the complete answer directly with calculator instead of step by step calculation to get a more accurate result, however slight variation in the answer due to rounding off during calculation is still permissible. For example, in the above problem if direct calculator answer is taken then the answer is 2762.81 which is almost equal to 2763 and hence small variations in the answer can be ignored.]

Example 2 Suppose you have decided to deposit 10,000 per year in your Public Provident fund account for 25 years. What will be the accumulated amount in your Public provident fund at the end of 25 years if the interest rate is 11%

Solution : a = 10,000 n = 25 i = 0.11 F= ?

$$F = \frac{a[(1+i)^n - 1}{i}$$

$$= 10{,}000\frac{[(1+0.11)^{25} - 1}{0.11}$$

$$= 10{,}000\,\frac{[(13.585 - 1]}{0.11}$$

$$= 144090.9₹$$

(by direct calculation with calculator 11,44,133.07₹)

Example 3 Preritha wants to buy a house after 5 years when it is expected cost 50 lakhs. How much should she save annually if her savings earn a compound interest of 12 percent.

Solution : F = 50,00,000 n = 5 i = 0.12 a = ?

$$F = \frac{a\left[(1+i)^n - 1\right]}{i}$$

$$50{,}00{,}000 = \frac{a\left[(1+0.12)^5 - 1\right]}{0.12}$$

50,00,000 = 6.3528 a

$$\frac{50{,}00{,}000}{6.3528} = a$$

a = 787054.5₹

Example 4 A company has an obligation to redeem 2 lakhs bonds 6 years hence. How much should the company deposit every six months in a sinking find account which offers 14% p.a interest compounded half yearly to accumulate 2 lakhs in 6 years.

Solution : F = 2,00,000 n = 6 × 2 =12 $i = \frac{0.14}{2} = 0.07$

$$F = \frac{a\left[(1+i)^n - 1\right]}{i}$$

$$2{,}00{,}000 = \frac{a\left[(1+0.07)^{12} - 1\right]}{0.07}$$

2,00,000 = 17.888 a

$$a = \frac{2{,}00{,}000}{17.888}$$

$$= 11180.679 ₹$$

Note: If the interest is compounded half yearly and the payment also is made half yearly then divide i by 2.

Example 5 If you want to have 80,000 after 5 years, how much should you deposit every year if the banks offers 12 % p.a interest compounded quarterly.

Solution: F = 80,000 n= 5 $i = \left(1+\frac{0.12}{4}\right)^4 - 1$

$= 0.125$

$$F = \frac{a[(1+i)^n - 1]}{i}$$

$$80{,}000 = \frac{a\left[(1+0.125)^5 - 1\right]}{0.125}$$

80,000= 6.416 a

$$\frac{80{,}000}{6.416} = a$$

A = 12,468.8₹

Note : Here annuity is paid yearly but compound interest is computed quarterly, there fore first find effective rate of interest.

EXERCISE : 9.1

I Five marks questions

1. A company needs ₹1,50,000 at the end of 10 years from now. It would like to set a side an equal amount each year from its profit. If the present market rate is 15%. How much should be the annual amount to be invested?
2. A company establishes a sinking fund to provide for the payment of 5,00,000 debt maturing in 20 years. Contribution to the fund to be made at the end of every year. Find the amount of each annual deposit if interest is 8% p.a
3. Find the future value of an annuity of ₹5000 at 12% p.a for 6 years.
4. Mr. Ashok has 20 more years to retire. He decides to save some money for his retirement. If he saves ₹9000 every year, how much will he have when he retires if the interest rate is 18% p.a?

5. Sukanya made a LIC policy of 12 years. If the annual premium is 2000₹, how much will she get when policy matures after 12 years if the interest rate is 13% ?

6. A father wants to send his child for higher studies after 15 years. He expects the cost of higher studies to be 1,00,000. How much should he save annually to have 1,00,000 after15 years if interest rate is 12% p.a?

7. In how many years an annuity of ₹100 amounts to ₹3137.12 and 4.5% p.a compound interest?

8. Mr. Shashank needs ₹60,000 for his child's education after 10 years, how much should he set aside each year for 10 years to accumulate this amount at the rate of 8% compounded quarterly?

9. Vani decides to save ₹10,000 every six months for the next 5 years and deposit it in a bank which offers 7% p.a interest compounded half yearly. How much will Vani have in her account after 5 years?

10. A Company advertises that it will pay a lumpsum of ₹8000 at the end of 6 years. If the interest rate is 12%, how much is the company demanding as annual deposit every year for the next 6 years?.

11. Romeo aged 40 wishes his wife Juliet to have ₹40 lakhs at his death. If the expectation of his life is another 30 years and he starts making equal annual investments at 8% p.a compound interest computed semi annually. How much should Romeo invest every year in an annuity?.

12. Find the future value of an annuity of ₹200 payable every month at 12% p.a compound interest computed every month for the next two years

ANSWERS : 9.1

1. 7387.8	2. 10926.1	3. 40575.9	4. 1319651.73
5. 51300.35	6. 2682.42	7. 20 yrs	8. 4094.8
9. 117313.93	10. 985.8	11. 34287.05	12. 5394.6

9.3 Present value of Annuity Immediate:

$$P = \frac{a[(1+i)^n - 1}{i(1+i)^n}$$

P = Present value

a = annuity

i = rate of interest in decimal

n = number of installments

Example 1 Find the present value of an annuity of ₹400 for 3 years at 16% p.a compound interest

Solution : a = 400 i = 0.16 n = 3 p = ?

$$P = 400\ \frac{\left[(1+0.16)^3 - 1\right]}{0.16(1+0.16)^3}$$

$$= \frac{400[1.5609 - 1]}{0.16 \times 1.5609}$$

= 898.35₹

Example 2 After reviewing their budget, Krishna and Harsha determined that they can afford to pay ₹12,000 per month for 3 years towards a new car. They call a finance Company and learn that the going rate of interest on car finance is 1.5% per month for 36 months. How much amount can they borrow.

Solution : a = 12000 n = 36 $i = \frac{1.5}{100} = 0.015$ P = ?

$$P = a\frac{\left[(i+i)^n - 1\right]}{i\ (1+i)^n}$$

$$= \frac{12000\left[(1+0.015)^{36} - 1\right]}{0.015 \times (1+0.015)^{36}}$$

$$= \frac{12000\left[1.7091 - 1\right]}{0.015 \times 1.7091}$$

= 3,31,917.38 ₹

Note: Here rate of interest is given per month instead of per anum but since payments also are made monthly value of i remains the same

Example 3 Mrs. Arpan deposits 4, 00, 000 on retirement in a bank which pays 10% p.a interest. How much can be drawn annually for a period of 10 years?

Solution: P = 4,00,000 i = 0.10 n = 10 a = ?

$$P = \frac{a\left[(1+i)^n - 1\right]}{i(1+i)^n}$$

$$4{,}00{,}000 = \frac{a\left[(1+0.10)^{10} - 1\right]}{0.1(1+0.1)^{10}}$$

$$4{,}00{,}000 = \frac{a[2.5937 - 1]}{0.1 \times 2.5937}$$

$$4{,}00{,}000 = a \times 6.1445$$

$$\frac{4{,}00{,}000}{6.1445} = a$$

A = 65,098.8 Rs.

Example 4 Find the present value of an annuity of ₹2500 payable at the end of each 6 months for 5 years if money is worth 10% converted semi annually.

Solution : a = 2500 $i = \frac{0.10}{2} = 0.05$ $n = 5 \times 2 = 10$

p = ?

$$P = \frac{a\left[(1+i)^n - 1\right]}{i(1+i)^n}$$

$$= 2500 \frac{\left[(1+0.05)^{10} - 1\right]}{0.05(1+0.05)^{10}}$$

$$= \frac{2500[1.6289 - 1]}{0.05 \times 1.6289}$$

= 19304.44₹

Example 5 Find the present value of an annuity of ₹3000 for 12 years at 6% p.a computed half yearly.

Solution : effective rate of interest

$$r = \left(1+\frac{i}{q}\right)^q - 1$$

$$= \left(1+\frac{.06}{2}\right)^2 - 1$$

$$= 1.0609 - 1$$
$$= 0.0609$$

A = 3000 i = 0.0609 n =12

$$\text{P} = \frac{a\left[(1+i)^n - 1\right]}{i \times (1+i)^n}$$

$$= \frac{3000\left[(1+0.0609)^{12} - 1\right]}{0.0609\,(1+0.0609)^{12}}$$

$$= \frac{3000(2.0328-1)}{.0609 \times 2.0328}$$

= 25027.96₹

(by direct calculation(without rounding in between values) with calculator, answer is 25027.89)

Example 6 Machine A costs 10,000₹ and has useful life of 8 years. Machine B costs 8000₹ and has useful life of 6 years. Suppose machine A generates an annual savings of 2000 while machine B generates an annual saving of ₹1,800. Assuming the time value of money is 10% p.a which machine is preferable.

Solution : calculate the present value of both the machines and compare with the cost

For machine A

a=2000 n = 8 i = 0.1

$$P = \frac{a\left[(1+i)^n - 1\right]}{i \times (1+i)^n}$$

$$P = \frac{2000\left[(1+0.1)^8 - 1\right]}{0.1(1.1)^8}$$

$$P = \frac{2000\left[(2.1436) - 1\right]}{0.1(2.1436)}$$

= 10,669.9 is the actual cost of the machine but if we are getting it at a cost of 10000 then there is a profit

Profit = 10,669.9-10000=669.9

For machine B

a=1800 n = 6 i = 0.1

$$P = \frac{a\left[(1+i)^n - 1\right]}{i \times (1+i)^n}$$

$$P = \frac{1800\left[(1+0.1)^6 - 1\right]}{0.1(1.1)^6}$$

$$P = \frac{1800\left[(1.7716) - 1\right]}{0.1(1.7716)}$$

= 7839.7 ₹ is the actual cost of machine B but if we are charged 8000 then there is a loss

Loss = 8000-7839.7

= 160.3₹

Hence Machine A is preferred

EXERCISE : 9.2

Five Marks questions

1. Find the present value of an annuity of ₹500 at 6% p.a for 7 years.
2. A man borrows 20,000 and agrees to pay the borrowed amount in 10 equal installments at the rate of 6% p.a. Find the amount of each installment.
3. Uma bought a TV costing ₹21000 by making a down payment of 5000 and agreeing to repay the balance amount in equal annual payments for five years. How much would be each payment if the interest rate is 14% p.a.?
4. What is the annual income that can be obtained for the next 12 years from an initial payment of ₹50,000 if the interest rate is 15% ?
5. Raj wants to invest a lump-sum amount in the bank so that he can get an annual income of ₹15,000 every year for the next 10 years. If the bank offers 16% p.a compound interest, what is amount he should invest today?.
6. Ayush purchases a car for ₹5,50,000. He gets a loan of ₹5,00,000 at 15% p.a from a bank and balance amount he pays as down payment. He has to pay whole amount of loan in 12 equal monthly installments.

   Find the money he has to pay at the end of every month. $(h\text{int } i = \frac{0.15}{12})$
7. Sanjana sold to Reema a machine, the cash price of which is ₹10,000 payment will be made in three equal annual installments at 10% p.a interest Compounded quarterly. Each installment is payable at the end of each year. Calculate the amount of annual installment.
8. Vani borrowed 20,000 at 6% Compound interest Compounded quarterly promising to repay the money in 4 equal annual instalemnts. Find the amount of each installment.
9. How much should you invest today at 8% p.a. Compound interest computed quarterly so that you get 3000₹ every 3 months for the next 7 years.

   (Hint: i=0.08/4 n=28)

10. Find the present value of an annuity of ₹500 payable for 10 years hence when interest of 10% is Compounded half yearly.

11. A Company is considering a proposal of purchasing a machine either by making full payment of ₹5000 or by leasing it for 5 years at an annual rent of ₹1200. Which course of actions preferable if the Company can borrow money at 12% p.a?

    [Hint : Find Present value and compare with 5000. the lesser value among two is preferable]

ANSWERS : 9.2

1. 2791.08	2. 2717.46	3. 4660.5	4. 9224.07
5. 72498.2	6. 45130.4	7. 4048.07	8. 5790.3
9. 63843.8	10. 3039.5	11. 4325.7, Better to lease	

Annuity due (payment made at beginning of each period)

9.4 Future value of annuity due

$$F = \frac{a\left[(1+i)^n - 1\right]}{i}(1+i)$$

F = Future value

a = annuity or equal installment

i = rate of interest in decimal

n = number of installments

9.5 Present value of annuity due

$$P = \frac{a\left[(1+i)^n - 1\right]}{i(1+i)^n}(1+i)$$

P = Present value

a = annuity or equal installment

i = rate of interest in decimal

n = number of installments

Example 1 Find the present value of an annuity due of 8000 for 5 years at 5% p.a

Solution:

$$P = \frac{a\left[(1+i)^n - 1\right]}{i(1+i)^n}(1+i)$$

$$= \frac{8000\left[(1+0.05)^5 - 1\right](1+0.05)}{(0.05)(1+0.05)^5}$$

$$= \frac{8000[1..2763-1](1.05)}{0.05 \times 1.2763}$$

= 36369.50 ₹

Example 2 A person repaid his loan in 10 equal annual installments starting from the beginning of the first year. If each installment was ₹6000 and compound interest charged was 12% p.a. What was the amount borrowed.

Solution : a = 6000 n = 10 i = 0.12

$$P = \frac{a\left[(1+i)^n - 1\right]}{i(1+i)^n}(1+i)$$

$$= \frac{6000\left[(1+0.12)^{10} - 1\right](1+0.12)}{0.12(1+0.12)^{10}}$$

$$= \frac{6000[3.1058-1](1.12)}{(0.12)(3.1058)}$$

= 37969.2₹

Example 3 Sharan borrows a sum of ₹2,00,000 and promises to repay in 20 equal annual installments at the beginning of each year. What is the annual installment to be paid if the interest rate is 16% computed quarterly.

Solution : P = 2,00,000 n = 20

Effective rate of interest = $\left(1+\frac{i}{4}\right)^4-1$

$$\left(1+\frac{0.16}{4}\right)^4-1$$

$$= 0.1698$$

$$2,00,000 = \frac{a\left[(1+0.1698)^{20}-1\right]}{0.1698(1+0.1698)^{20}}(1+0.1698)$$

$$2,00,000 = \frac{a[23.027-1](1.1698)}{(0.1698)(23.027)}$$

$$2,00,000 = 6.590\ a$$

$$\frac{2,00,000}{6.590} = a$$

$$A = 30349₹$$

Example 4 If Poornima deposits ₹600 at the beginning of every year for the next 15 years. Then how much will be accumulated at the end of 15 years if interest rate is 7% p.a

Solution : a = 600 n = 15 i = 0.07

$$F = \frac{a\left[(1+i)^n-1\right]}{i}(1+i)$$

$$= \frac{600\left[(1.07)^{15}-1\right]}{0.07}(1+0.07)$$

$$\frac{6000[2.759-1](1.07)}{0.07}$$

$$= 161325.428₹$$

EXERCISE : 9.3

Five marks questions

1) Find the future value of an annuity of ₹2000 for 6 years, if the payment is made at the beginning of each year, interest rate being 10% p.a.

2) Find the future value of an annuity due of ₹1,500 for 17 years at 8% p.a

3) If person wants ₹25,000 after 8 years, how much should he invest in an annuity due each year at 6% p.a.?

4) How much amount is required to be invested at the beginning of every year so as to accumulate ₹3,00,000 10 years hence,if interest is compounded annually at 10% ?

5) Find the present value of an annuity due of ₹1000 for 3 years if the payment is made at the beginning of each year, interest rate being 4% p.a

6) Shreya purchased a mobile paying ₹5000 as down payment. And promising to pay ₹200 every three months for the next 4 years. The seller charges interest at 8% p. a compounded quarterly. What is the cash price of the mobile if the payments are made at the beginning of each 3 months [Hint cash price= Present value +5000]

7) Rohan repaid his housing loan in 15 equal annual installments starting from the beginning of the first year. If each installment was ₹50,000 and the compound interest charged was 12% p.a. What was the amount borrowed by Rohan.

8) How much should you invest if you want to receive ₹5000 at the beginning of each year for the next 5 years if the compound interest is 16% p.a Compounded quarterly.

9) Find the present value of an annuity of ₹2000 payable at the beginning of each quarter for the next 3 years if the rate of interest is 4% p.a compounded quarterly.

   [$n = 12$, $i = 0.01$]

10) A company needs ₹1,00,000 , 7 years from now. It would like to set aside an equal amount at the beginning of each year out of its profits. If the interest rate is 16% compounded semi - annually, how much should be invested annually.

ANSWERS : 9.3

1. 16974.34	2. 54675.37	3. 2382.9	4. 17112.35
5. 2886.09	6. 7769.9	7. 381408.4	8. 18721.58
9. 22735.2	10. 7364.32		

9.6 Perpetuity: An annuity which is payable forever (infinite number of year) is called a perpetuity. In other words a perpetuity is an annuity whose payments continue forever.

Future value of a perpetuity does not exist.

Present value can be found using the formula $P_{\infty} = \frac{a}{i}$

Example 1 What is the present value of an income of 3000 to be received forever if the interest rate is 14% p.a.

Solution : Since the income is to be received forever such an annuity is called perpetuity.

Hence Present Value $P_{\infty} = \frac{a}{i}$

$$= \frac{3000}{0.14}$$

$$= 21428.5₹$$

Example 2 A Maths professor while retiring wants to institute a scholarship of ₹5000 every year to the student scoring highest marks in I Basic Maths annual exam. How much should he deposit if bank offer 5% p.a.

Solution : Since the scholarship is forever it is an perpetuity

$$P_{\infty} = \frac{a}{i}$$

$$= \frac{5000}{0.05} = 1{,}00{,}000\,Rs$$

9.7 Deferred Annuity:

A deferred annuity is an annuity in which the periodic payments start only after a certain specified period equivalent to a certain number of payment period.

If the annuity is deferred for n years the first payment will become due at the end of $(n + 1)^{th}$ year in annuity immediate.

For example, when a person takes a home loan from his employer, generally the repayment in equal installments does not start immediately but lets say begins at the end of 3years. then installment was not paid for the first two years and the annuity was delayed by two years or we say the annuity is deferred for two years.

Generally, we come across deferred annuity cases in loans and hence in case of deferred annuity to find present value divide by factor $(1 + i)^d$

Present value in case of annuity immediate $P = \dfrac{a[(1+i)^n - 1]}{i(1+i)^n (1+i)^d}$

$$= \frac{a[(1+i)^n - 1]}{i(1+i)^{n+d}}$$

Example 1 A man borrowed ₹20,000 and agrees to pay the borrowed amount in 10 equal installments at the rate of 6% p.a. Find the amount of each installment the first being paid at the end of the 2^{nd} year after the money was borrowed.

Solution : Paid at the end of 2^{nd} year means in the end of first year installment was not paid then it is deferred for 1 year.

P = 20,000 i = 0.06 n = 10 d = 1

$$P = a\left[\frac{(1+i)^n - 1}{i(1+i)^n (1+i)^d}\right]$$

$$20{,}000 = a\left[\frac{[(1+0.06)^{10} - 1]}{0.06(1+.06)^{10}\ (1+.06)^1}\right]$$

$$20,000 = a\left[\frac{[1.7908-1]}{0.06(1.7908)(1.06)}\right]$$

$$20,000 = a \times 6.9432$$

$$a = \frac{20,000}{6.9432} = 2880.52 \ ₹$$

EXERCISE : 9.4

I Two marks questions

1) Find the present value of a perpetuity of ₹3000 to be received forever at 4% p.a.

2) A scholarship of ₹2000 every year has to be instituted. How much should be invested today if the interest rate is 8%p.a?

3) A person endows a bed in a hospital at the cost of ₹7000 p.a If the interest rate is 14% p.a how much should he provide in perpetuity?.

II Five marks questions

4) A person purchases a house for ₹25 lakhs with ₹5 lakhs as down payment. The Rest of the amount he loans from a bank which offers 16% p.a compound interest and has to repay the loan in 20 equal annual installments. If the first installment is paid at the end of the third year, find how much he has to pay each year?

   [Hint: The annuity is deferred for two years]

5) Find the present value of an annuity of ₹500 at 7% p.a for 15 years if the annuity is deffered for 3 years .

   [Hint: n =15, d = 3]

6) What is the present value of an annuity of ₹6000 payable from the end of 11^{th} year and for 15 years thereafter. The interest rate is 9% p.a?.

   [Hint: d =10, n =15]

7) What is the present value of ₹2000 receivable for 20 years, If the annuity is deferred for 10 years if the interest rate is 10%?

8) What is the present value of an perpetuity of ₹5000 to be received forever if the first receipt occurs at the end of the sixth year from now. Interest rate being 8% p.a?

[Hint: d = 5 use perpetuity formula]

ANSWERS : 9.4

1. 75000	2. 25000	3. 50000	4. 453916.7
5. 3717.39	6. 20429.56	7. 6564.8	8. 42536.4

CHAPTER 10

AVERAGES

10.1 Introduction: Average or Average value is widely used in the field of Business examines and grievances for studying the behavioral of a result in a given time. For example to determine the performance of students in a class, price and weight of the commodities, height of the plants or students. In this chapter we are learning the definitions different types of Average and various applications. We need an average when we are dealing with a large number of quantities it is difficult to describe the entire data set individually. By average involved we can find a single value that is very close to all or a majority of individual value in a data set. The single value or typical value is called Average.

Definition: Average or average value of a set of quantity is the sum of quantities divided by number of quantities in the set. The average is also called the 'mean of the quantities'.

$$\text{Average} = \text{Mean} = \frac{sum\ of\ the\ quantities}{Number\ of\ quantities}$$

An average will be efficient if and only if the following two conditions are fulfilled.

i) The individual values in the data set must be homogeneous.

ii) The value in the data set sum must fall within a normal range.

10.2 Types of Averages: The different kinds of Averages are

1) Simple Average:
2) Weighted Average
3) Combined Average

10.3 Simple Average: When an arithmetic average of a certain number of quantities (or values) which are all of equal weightage is required a simple Average is computed.

If $x_{1,}$ x_2, x_3 ..... x_n are the values in a data set containing 'n' number of quantities.

Simple Average $= \bar{x} = \dfrac{x_1 + x_2 + x_3 .... + x_n}{n}$

Example1 In a class of 10 students, the marks obtained in Mathematics, are 88, 71, 35, 30, 46, 92, 67, 53, 76 and 28. What is the average marks?

Solution : Average Marks $= \dfrac{\text{Total Marks of all 10 students}}{\text{Number of student}}$

$$= \frac{88 + 71 + 35 + 30 + 46 + 92 + 67 + 53 + 76 + 28}{10}$$

$\therefore$ Average Marks $= 58.6$

Example 2 The rainfall in a week in Bangalore are 18mm, 25mm, 20mm, 9mm, 30mm, 10mm, 15mm. Find the Average rainfall

Solution : Average Rainfall $= \dfrac{\text{Total Rainfall in a week}}{\text{No.of days}}$

$$= \frac{18 + 25 + 20 + 9 + 30 + 10 + 15}{7}$$

$= 18.14$ mm

Example 3 The weight 6 men are 90kg, 70.5 kg, 56 kg, 45.5 kg., 85 kg, and 78 kg, Find average weight.

Solution : Average weight of 7 men $= \dfrac{\text{Total weight of 7 men}}{\text{Number of men}}$

$$= \frac{90 + 70.5 + 56 + 45.5 + 85 + 78}{6} = \frac{425}{6}$$

Average weight $=$ 70.83 Kgs.

10.4 Weighted Average: If all the quantities are not of equal weightage or importance in such cases it is appropriate to complete a 'weighted average of the value in the data set.

If $x_1, x_2, x_3 \ldots\ldots x_n$ represented the values of 'n' quantities

$w_1, w_2, w_3, \ldots\ldots w_n$ represented the weight assigned to 'n' values. Respectively

Then the weighted Average = $(\overline{X}) = \dfrac{x_1w_1 + x_2w_2 + \ldots\ldots + x_nw_n}{w_1 + w_2 + \ldots + w_n}$

Example 1 Ramesh has 4 Kannada, 5 English, 6 Maths books. Each Kannada book cost ₹8.50, English ₹10.50, and Maths ₹15.00. Find the average cost per book of all 3 subjects.

Solution : Average cost of a Book $= \dfrac{\text{Total cost of all the books}}{\text{Number of books}}$

$$= \frac{(4\times8.50)+(5\times10.50)+(6\times15)}{4+5+6}$$

$$= \frac{34+52.5+90}{15} = \frac{176.5}{15} = ₹\,11.76$$

Example 2 5 Kg. of sugar at the rate ₹15/Kg., 8 kg. of wheat at the ₹22/kg., 7 kg. of rice at ₹35/kg. and 4 kg. of oil at ₹85/ kg. What is the average price/kg. of all the commodity.

Solution :

Commodity	Weight	Price / Kg.
	w_1 = 5 kg.	n_1 = Rs.15
	w_2 = 8 kg.	n_2 = Rs.22
	w_3 = 7 kg.	n_3 = Rs.35
	w_4 = 4 kg.	n_4 = Rs.85

$\therefore$ Average price /kg $(\overline{X}w) = \dfrac{\text{Total cost of all the commodity}}{\text{Total weight of all the commodity}}$

$$= \frac{n_1 w_1 + n_2 w_2 + n_3 w_3 + n_4 w_4}{w_1 + w_2 + w_3 + w_4}$$

$$= \frac{(15 \times 5) + (22 \times 8) \times (35 \times 7) + (85 \times 4)}{5 + 8 + 7 + 4}$$

$$= \frac{75 + 176 + 245 + 340}{24}$$

$\therefore$ Average price/kg $= \frac{836}{24} =$ ₹ 34.83

10.5 Combined Average: This is very similar to a weighted average. It is computed when the data set consist of different group and for each group an average has already been computed.

If $\overline{x_1}, \overline{x_2}, \& \overline{x_3}$ are the average of the number of values n_1, n_2 and n_3 then the combined average of all the three group is given by

$$\bar{X}_{123} = \frac{\bar{x}_1 n_1 + \bar{x}_2 n_2 + \bar{x}_3 n_3}{n_1 + n_2 + n_3}$$

Example 1 The average weight of 12 girls in a class in 4 feet and the average height of 8 boys in 5 feet. Find the combined average height of both girls and boys.

Solution : Given $\overline{x_1} = 4,\ \overline{x_2} = 5$

$n_1 = 12,\quad n_{2=}\ 8$

$\therefore$ the combined average height $= \frac{\bar{x}_1 n_1 + \bar{x}_2 n_2}{n_1 + n_2}$

$$= \frac{(4 \times 12) + (5 \times 8)}{12 + 8}$$

$$= \frac{48 + 40}{20}$$

$$= \frac{88}{20}$$

$\therefore$ the combined average height of girls and boys = 4.4 feet

Example 2 The average score of 35 girls is 80 and the average score of 25 boys is 68. Find the average score of both boys and girls together.

Solution : given $\overline{x_1} = 80,\ \overline{x_2} = 68$

$n_1 = 35,\ n_2\ \ 25$

The combined average Score $= \dfrac{\bar{x}_1 n_1 + \bar{x}_2 n_2}{n_1 + n_2}$

$= \dfrac{(80 \times 35) + (68 \times 25)}{35 \times 25}$

$= \dfrac{2800 + 1700}{60}$

$= \dfrac{4500}{60}$

$\therefore$ the combined average score = 75

Example 3 A Survey in a village shows the following results.

Number of mens	Average weight in kg.
1^{st} Batch 15	50
2^{nd} Batch 20	55
3^{rd} Batch 25	60
4^{th} Batch x	65
5^{th} Batch 10	70

If the combined average of all the batch is 60 kg. Find the value of 'x' ?

Solution: Combined average weight of all the 5 batches

$$60 = \frac{\text{Total weight of all the men of 5 batches}}{\text{Total no. of men}}$$

$$60 = \frac{(50 \times 15) + (55 \times 20) + (60 + 25) + (65 \times x) + (70 + 10)}{15 + 20 + 25 + x + 10}$$

$$\therefore 60 = \frac{750 + 1100 + 1500 + 65x + 700}{70 + x}$$

$$60 = \frac{4050 + 65x}{70 + x}$$

$\therefore 4050 + 65x = 60\ (70 + x)$

$4050 + 65\ x = 4200 + 60x$

$65x - 60x = 4200 - 4050$

$5x = 150$

$\therefore x = \dfrac{150}{5}$

$x = 30$

WORKED EXAMPLE:

I. One and two Marks question:

Example 1 Thirty five boys and sixty five girls are tested for their numerical abilities. The boys have an average score of 80% and the girls score an average of 90%. Calculate the average score of boys and girls combined.

Solution :

$$\text{Combined average score} = \frac{\text{Total score of boys and girls}}{\text{Number of student}}$$

$$= \frac{(35 \times 80) + (65 \times 90)}{35 + 65}$$

$$= \frac{2800 + 5850}{100}$$

$$\text{Combined average score} = \frac{8650}{100} = 86.50\%$$

Example 2 A Train runs at a speed of 28 kmph for 4 hours at 30 kmph for 5 hours and the remaining 40 kms in 1 hour. What is the total distance and what is the average speed/hr.

Solution : Given

$n_1 = 28$ kmph $\quad t_1 = 4$ hrs.

$n_2 = 30$ kmph $\quad t_2 = 5$ hrs.

$n_3 = 40$ kmph $\quad t_3 = 1$ hrs.

Combined Average speed $= \dfrac{n_1t_1 + n_2t_2 + n_3t_3}{t_1 + t_2 + t_3}$

$= \dfrac{(28 \times 4) + (30 \times 5) + (40 \times 1)}{4 + 5 + 1}$

$= \dfrac{112 + 150 + 40}{10} = \dfrac{302}{10} = 30.2$ kmph.

$\therefore$ Total distance covered = 302 kms

Average speed = 30.2 kmph

Example 3 Rekha purchased 3 varieties of cooking oil, 5 kg. of oil at ₹100/Kg, 6 kg of oil at ₹110/Kg. and 9 kg. of oil at ₹120/kg. What is the average price of the oil/kg.

Solution :

Cost	Weight
n_1 = ₹100	w_1 = 5 kgs
n_2 = ₹110	w_2 = 6 kgs
n_3 = ₹120	w_3 = 9 kgs.

$\therefore$ Average price of the oil = $\overline{X}w = \dfrac{n_1w_1 + n_2w_2 + n_3w_3}{w1 + w_2 + w_3}$

$= \dfrac{(100 \times 5) + (110 \times 6) + (120 \times 9)}{5 + 6 + 9}$

$= \dfrac{500 + 660 + 1080}{20}$

$= \dfrac{2240}{20}$

Average price of the oil = ₹112/kg.

Example 4 The profit of a business firm for the 5 years are ₹17,598, ₹20,703, ₹10,085, ₹25,375 and ₹16,315. Find the average profit?

Solution :

$$= \frac{\text{Total profit of all the 5 years}}{\text{No.of years}}$$

$$= \frac{(17{,}598 + 20{,}703 + 15{,}085 + 25{,}375 + 16{,}315)}{5}$$

Average profit $= ₹19015.20$

Example 5 The average marks of a group of student is 50. Another group of 15 students have an average marks of 60. What is the average marks of 80 students.

Solution : Given that $\bar{x}_1 = 60$ $\quad n_1 = 15$ *students*

$\bar{x}_2 = 50$ $\quad n_2 = (80 - 50) = 65$ *students*

Combined Average marks $= \dfrac{(60 \times 15) + (50 + 65)}{80}$

$$= \frac{900 + 3250}{80}$$

$$= \frac{4150}{80}$$

Average marks of 80 student $=$ 51,875

Example 6 The average weight of 10 boys is 30 kg. If a 11^{th} boy is added the average weight increased by 2 kg. Find the weight of 11^{th} boy.

Solution : Total weight of 10 boys = 30 × 10= 300 Kg.

Total weight of (10+1) boys = (30 + 2) 11

= 32 × 11

= 352 Kgs.

The weight of 11^{th} boy added = (352 – 300) kg.

= 52 Kgs.

Example 7 A train travels at an average speed of 50 mph. For 40 minutes and then travels at an average of 80 mph for the next 30 minutes. Find the average speed of the entire distance travelled.

Solution : Given the average speed 50 mph → 40 minutes

$\therefore$ Distance traveled in 40 minutes $= \frac{50}{60} \times 40$

$= \left[\frac{100}{3}\right]$ mile

Again 80 mph → 30 minutes.

$\therefore$ Distance travelled in 30 minute $= \frac{80}{60} \times 30 = (40) mile$

$\therefore$ Average speed for the full distance travelled $= \frac{Total\ distance}{Total\ time}$

$$= \frac{\left(\frac{100}{3} + 40\right)}{40 + 30}$$

$$= \frac{\frac{[220]}{3}}{70}$$

$$= \frac{220}{210} = \left[\frac{22}{21}\right] \text{ miles/min}$$

$\therefore$ Average speed for entire distance travelled $= \left[\frac{22}{21} \times 60\right]$ mph

$= 66.67$ mph

Example 8 A farmer walks from village A to village B at the speed 10 kmph and returned back in 15 kmph. Find his average speed of the entire journey.

Solution : Let the distance from A to B $= x$ km.

$\therefore$ Time taken from A to B $= \frac{x}{10}$ hr.

Again on returning from B to A distance = x km.

Time taken from B to A $= \frac{x}{15}$ hr.

$\therefore$ Average speed of the entire journey $= \frac{Total\ distance}{Total\ time}$

$$= \frac{x+x}{\frac{x}{10}+\frac{x}{15}}$$

$$= \frac{2x}{\left[\frac{3x+2x}{30}\right]}$$

$$= \frac{60x}{5x}$$

$$= 12 \text{ kmph}$$

Example 9 The average age of 10 students in 14 years. Among them the average age of 4 student is12 years. Find the average of the remaining students.

Solution : Given $\bar{x}_1 = 12\,yr.$ $\quad n_1 = 4$

$\bar{x}_2 = ?$ $\quad n_2 = 6$

And Average of 10 students is 14 years.

$$\therefore \quad 14 = \frac{\bar{x}_1 n_1 + \bar{x}_2 n_2}{n_1 + n_2}$$

$$14 = \frac{(12 \times 4) + (\bar{x}_2 \times 6)}{4+6}$$

$$14 = \frac{48 + 6\bar{x}_2}{10}$$

$$\therefore \quad 48 + 6\bar{x}_2 = 140$$

$$6\bar{x}_2 = 140 - 48$$

$$6\bar{x}_2 = 92$$

$$\therefore \quad \bar{x}_2 = \frac{92}{6} = 15.33 \text{ yrs.}$$

Example 10 The average weight of a group of 35 people is 47.5 kg. If 36[th] person is added to the group. The average weight increased by 0.5 kg. What is the weight of the 36[th] person?

Solution : Let the weight of the 36[th] person = x kg.

$$\therefore \text{Average weight of 36 person} = \frac{Total\ weight\ of\ 36\ persons}{No.\ of\ persons}$$

$$(47.5 + 0.5) = \frac{(35 \times 47.5) + (1 \times x)}{36}$$

$$48 = \frac{1662.50 + x}{36}$$

$$1662.50 + x = 1728$$

$$\therefore x = 1728 - 1662.50$$

Weight of the 36[th] person $x = 65.50$ kg.

WORKED EXAMPLE:

II. 3 and 5 Marks question:

Example 1 A merchant buys two types of chalk Powder A and B at ₹5.70 and ₹6.40 per kg. respectively. He mixes them in the proportion 4:3 and sells the mixture at ₹7.20/kg. What is his Profit and Profit %?

Solution : Given p_1 = ₹5.70 weight $w_1 = 4$ kg.

p_2 = ₹6.40 $w_2 = 3$ kg.

$$\text{Average price of the mixture} = \frac{total\ price\ of\ the\ two\ types\ of\ chalk\ powder}{total\ weight}$$

$$= \frac{(5.70 \times 4) + (6.40 \times 3)}{4 + 3}$$

$$= \frac{22.80 + 19.20}{7}$$

$$= \frac{42}{7} = ₹6$$

Average price of the mixture = ₹6/Kg.

Profit = Selling price – cost price

= 7.20 – 6.00

= ₹1.20/Kg.

$$\text{Profit \%} = \frac{\text{Profit}}{\text{cost price}} \times 100$$

$$= \frac{1.20}{6.00} \times 100 = 20\%$$

Example 2 The average weight of a group containing 25 persons is 70 kg. 5 persons with an average weight 63 kg leave the group and 4 persons with weight 72 kg, 78 kg, 70 kg and 73 kg joins the group. Find the average weight of the new group.

Solution : Total weight of 25 persons= 25 × 70 = 1750 Kg.

Total weight of 5 persons leave the group= 5 × 63 = 315 Kgs.

Total weight of 20 persons = 1750 – 315 = 1435 Kg.

Total weight of 4 persons who have joined the group

= (72+78+70+73)

= 293 Kgs.

Total weight of 24 persons = (1435 +293)

= 1728 Kg.

$$\text{Average weight of 24 persons} = \frac{\textit{Total weight of 24 person}}{24}$$

$$= \frac{1728}{24}$$

= 72 Kgs.

Example 3 The average weight of 40 student is 163 cms. On a particular day 3 students A, B and C, were absent and the average of the 37 students was found to be 162 cms. If A and B have equal heights and the height of 'C' be 2cms less than that of A. Find the heights of A, B, and C.

Solution: Total height of 40 students = 163 × 40 = 6520 cm.

Total height of 37 students = 162 × 37 = 5994 cm

Total height of 3 students absent = 6529-5994 = 526 cm.

Total height of A, B, and C = 526 cm

Let the height of A = B = x cms.

Given the height of C is 2 cm less than A = $(x - 2)$ cm.

Sum of their height

$$x + x + (x - 2) = 526 \text{ cm}$$

$$3x - 2 = 526$$

$$3x = 528$$

$$x = \frac{528}{3} = 176 cms$$

The height of A and B = x = 176 cm.

The height of C = $(x - 2)$ = (176-2) = 174 cms.

Example 4 A Dental Clinic purchased a certain number of chairs at an average price of ₹190 each. The average price of 30 chairs was ₹175 and that of the remaining chairs was ₹200/-. Find the total number of chairs the clinic purchased.

Solution : Assume that the number of chair the clinic purchased = x.

Total average price = ₹190

<u>Average price</u>	<u>No.of chair</u>
$\overline{x_1}$ = ₹175	N_1 = 10
$\overline{x_2}$ = ₹200	$N_2 = (x - 10)$

$$\text{Combined average price} = \frac{\overline{x}_1 N_1 + \overline{x}_2 N_2}{(10) + (x - 10)}$$

$$190 = \frac{(175 \times 10) + (200 \times (x - 10))}{x}$$

$$190 = \frac{1750 + 200x - 2000}{x}$$

$\therefore 190x = \quad 200x - 250$

$250 = 200x - 190x$

$250 = \quad 10x$

$$\therefore x = \frac{250}{10} = ₹25$$

Total No. of chairs purchased by the clinic=25

Example 5 Ramesh bought 4 shirts in a discount sale. The average price of the shirt being ₹150/-, the average price of two polyster shirts is Rs.170. If the price of the remaining 2 cotton shirts is in the ratio 7:6. Find the price of the cheapest cotton shirt.

Solution : Total price of 4 shirts $= 150 \times 4 = ₹600$

Total price of 2 polyster shirts $= 170 \times 2 = ₹340$

Total price of 2 cotton shirts $= ₹260$

Given the price of 2 cotton shirts are in the ratio 7:6

1^{st} Cotton shirt price $= 7x$

2^{nd} cotton shirt price $= 6x$

Total price of 2 cotton shirt $= 7x + 6x = ₹260/-$

$13x = ₹260$

$$\therefore x = \frac{260}{13} = 20$$

$\therefore$ The price of 1^{st} cotton shirt $= 7x = 7 \times 20 = ₹140$

2^{nd} cotton shirt $= 6x = 6 \times 20 = ₹120$

The cheapest cotton shirt price $= ₹120/-$

Example 6 Calculate the arithmetic average mark from the following data.

Marks	45	75	60	55	93
No.of students	11	10	15	12	2

$$\text{Average Marks of student} = \frac{\textit{Total Marks of all student}}{\textit{Number of students}}$$

$$= \frac{(45\times 11)+(75\times 10)+(160\times 15)+(55\times 12)+(93\times 2)}{11+10+15+12+2}$$

$$= \frac{495+750+900+660+186}{50}$$

$$= \frac{2991}{50} = 59.82$$

Average Marks of 50 student = 59.82.

Example 7 The average height of a group of boys and girls is 38 kg. The average weight of the boys in 42 kgs and that of the girls is 33 kgs. If the numbers of girls is 20. Find the number of boys.

Solution:

Let the number of boys $= x$

Combined average weight $= x_w = 38$ kg.

$\bar{x}_B = 42$ kg No.pof boys $= x = n_B$

$\bar{x}_G = 33$ kg. No.of girls $= 20 = n_G$

$$\therefore \text{Combined Average height} = \frac{\bar{x}_B n_B + \bar{x}_G n_G}{n_B + n_G}$$

$$38 = \frac{(42\times x)+(33\times 20)}{x+20}$$

$$38 = \frac{42x+660}{x+20}$$

$\therefore 38\,(x + 20) = 42x + 660$

$38x + 760 = 42x + 660$

$760 - 660 = 42x - 38x$

$100 = 4x$

$$\therefore x = \frac{100}{4}$$

$\therefore$ No. of boys $= x = 25$

Example 8 Mr. Raju purchased 17 English books in a discount sale, the average price of the book being ₹53. The average price of 11 English books is ₹71. If the average price of 6 different English story books form an increasing arithmetic progression with last book price being ₹25. Find the price of the cheapest English Story book?

Solution:

The total price of the 17 books = (17 × 53) = ₹901

The total price of the 11 English books = 11 × 71 = ₹781

Total price of 6 story books = ₹(901) - ₹781 = ₹120

Given the price of 6 English story books form are A.P.

With last book price is ₹25

∴ n = 6, Sn = ₹120, Tn = ₹25, a = ?

Formula Sum = Sn = $\frac{n}{2}[a + Tn]$

$$120 = \frac{6}{2}[a + 25]$$

120 = 3 (a + 25)

120 = 3a + 75

120 – 75= 3a

45 = 3a

∴ a = 15

∴ The price of the cheapest English story book = ₹15

Example 9 3 test in Economics, 2 in Kannada, 4 in Accounts, and 5 in English are conducted. The average mark scored by Mr. Suresh in Economics in 60, in Kannada 56 and that of account is 45. If the average marks of all the test taken together is 48. Find the average marks scored by him in English?

Solution : Assured the Average marks in English = x

The total marks scored in English = $5x$

Total marks scored in Economics = 3 × 60 = 180

Total marks scored in Kannada = 2 × 56 = 112

Total Marks scored in Accounts = 4 × 45 = 180

Total No. of test in all 4 subject = 3 + 2 + 4 + 5 = 14

$\therefore$ Combined Average Marks in all subject in all test

$$= \frac{Total\ marks\ in\ all\ subject\ and\ in\ all\ test}{Total\ No.of\ test}$$

$$48 = \frac{5x + 180 + 112 + 180}{14}$$

$\therefore$ $5x + 180 + 112 + 180 = 48 \times 14$

$5x + 472 = 672$

$5x = 62 - 472$

$5x = 200$

$$\therefore x = \frac{200}{5}$$

$x = 40$

$\therefore$ The average marks scored by Suresh in English = 40

Example 10 The average age of A and B are 18 years that of B & C. is 17 years. And that of C and A is 20 years. What is the ages of A, B and C?

Solution :

The total age of A and B = 2 × 18 = 36 years = A

The total age of B and C = 2 × 17 = 34 years

The total age of C and A = 2 × 20 = 40 years

$\therefore$ A + B = 36 ....(1)

B + C = 34 ....(2)

C + A = 40 years ....(3)

(1) + (2) + (3) = 2A + 2B, 2C = 36 + 34 + 40 = 110

$\therefore$ 2 (A + B + C) = 110

$$\therefore A + B + C = \frac{110}{2} = 55 \text{ years} \quad ....(4)$$

$\therefore$ (4) – (1)

$$\begin{array}{lrcl} & A + B + C & = & 55 \\ (-) & \underline{A + B} & = & \underline{36} \\ & \therefore + C & = & 19 \end{array}$$

Age of C = 19 years.

(4) – (2) $\quad A + B + C = 55$

$B + C = 34$

$\therefore A = 21$ years

Again (4) – (3) $\quad A + B + C = 55$

$A + C = 40$

$\therefore B = 15$ years

Age of A = 21 years, B = 15 years, C = 19 years

Example 11 A batsman finds that by scoring a century in the 11th innings of his test matches he has bettered his average of the previous ten innings by 5 run. What is the average after the 11th Innings.

Solution : Let us assume that the average runs in 10th innings = x

Total runs in 10 innings = $10x$ ....(1)

Average runs after 11 innings = $(x + 5)$

$\therefore$ Total runs after 11 innings = $11 (x + 5)$ ....(2)

$\therefore$ Total runs in 11th innings = (2) – (1)

A century = $11 (x + 5) - 10x$

$100 = 11x + 55 - 10x$

$100 = x + 55$

$\therefore$ Average runs in 10th innings = 45

$\therefore$ Average runs after 11th innings = $x + 5 = 45 + 5$

= 50 runs

Example 12 Ten years ago the average age of the family of 4 members were 24 years. Two children have been born. The average age of the family is same as today.What is the present age of the two children assuming that the children's age differ by 2 years?.

Solution : 10 years ago the total age of 4 members of the family = 4 x 24 = 96 years.

Let the age of the two children are 'x' years and $(x + 2)$ years.

After 10 years

The total age of 4 members in increased to = 96 + (4×10) = 136yrs

$\therefore$ Total age of 4 members + 2 children born

$= 136 + (x) + (x + 2) = (138 + 2x)$

Today the No.of family members = 4 + 2 = 6

The present average Age = 24 years.

The present average age = $\dfrac{\textit{total present age of 6 members}}{\textit{No.of members}}$

$$24 = \frac{138 + 2x}{6}$$

$\therefore 138 + 2x = 144$

$2x = 144 - 138$

$2x = 6$

$\therefore x = 3$

$\therefore$ The age of the 1st child = $x = 3$ years

The age of the 2nd child = $x + 2 = (3 + 2) = 5$ years

Example 13 An exporter of coffee powder mixes 60 kg. of superior grade coffee at ₹300/kg. and 40 kg.of low grade at ₹180/Kg. He would like to add a profit margin of 20% on his cost. What will be the price of the mixed coffee powder.

Solution:

		weight
Given	p_1 = ₹300/Kg.	w_1 = 60 kg.
	p_2 = ₹180/-kg.	w_2 = 40 kg.

Average cost of the mixed coffee powder

$$= \frac{\text{Total price (both superior and low grade)}}{\text{Total weight}}$$

$$= \frac{(300 \times 60) + (180 \times 40)}{60 + 40}$$

$$= \frac{1800 + 7200}{100} = ₹252/\text{Kg.}$$

Price of mixed coffee powder = cost price + profit

$= 252 + 20\%\ 252$

$= 252 + \left(\frac{252 \times 20}{100}\right)$

$= 252 + 50.40$

$= ₹302.40$ Kg.

Example 14 The average weight of a group of boys and girls is 38 kg. The average weight of the boys in 42 kg. and that of the girls is 33 kg. If the number of boys is 25. Find the number of girls.

Solution : Let the number of girls = x

Girls $\bar{x}_1$ = 42 kg. No.of boys = 25

$\bar{x}_2$ = 33 kg. No.of girls = x

$\therefore$ Average weight of group of boys and girls

$$= \frac{\textit{Total weight of all boys and girls}}{\textit{No. of boys and girls.}}$$

$$38 = \frac{(42 \times 25) + (33 \times x)}{25 + x}$$

$$38 = \frac{1050 + 33x}{25 + x}$$

$\therefore 38\ (25 + x) = 1050 + 33x$

$950 + 38x = 1050 - 33x$

$38x - 33x = 1050 - 950$

$5x = 100$

$$x = \frac{100}{5}$$

$x = 20.$ No.of girls = 20

Example 15 The average age of 10 students in a class increases by 4.8 months. When a boy of age 6 years is replaced by a new boy. What is the age of the new boy?

Solution :

Let the Average age of 10 boys = x years.

$\therefore$ Total age of 10 boys = $10x$

After replacement of a boy of 6 years age and inclusion of new boys if his age is assumed to be 'y' years.

Then the total age of 10 boys in a new group = $(10x - 6) + y$

Given the new average of 10 boys = (x + 4.8 months) or

$$= \quad x + \frac{4.8}{12} \text{ years}$$

$$= \quad (x + 0.4) \text{ years.}$$

$$\therefore \text{ Average of new group of 10 boys} \quad = \quad \frac{\textit{Total age of 10 boys}}{\textit{No of boys}}$$

$$(x + 0.4) \quad = \quad \frac{(10x - 6) + y}{10}$$

$$10\,(x + 0.4) \quad = \quad (10x - 6) + y$$

$$10x + 4 = 10x - 6 + y \qquad \therefore y = 10$$

$\therefore$ Age of the new boy replaced = 10 years.

Example 16 A batsman's average score for a number of innings was 21.75 runs per innings. In the next three inning he scored 28, 34 and 37 runs. And his average for all the inning was revised by 1.125 runs. How many inning did he play?

Solution:

Let the number of inning he played = x

$\therefore$ Total runs in 'x' innings = $21.75\,x$

Again total runs in next 3 innings = 28 + 34 + 37 = 99 runs.

Number of inning raised to = $(x + 3)$

New Average after 3 inning raised to = 21.75 + 1.125

= 22.875

$\therefore$ Average run after (x+3) innings $= \dfrac{\textit{Total runs of all the innings}}{\textit{Total no. of innings}}$

$$22.875 = \frac{21.75x + 99}{x+3}$$

$\therefore$ (22.875) (x + 3) = 21.75x + 99

22.875x + 68.625 = 21.75x + 99

(22.875x – 21.75x) = 99 – 68.625

1.125x = 30.375 $\qquad \therefore\ x = \dfrac{30.375}{1.125} = 27$

$\therefore$ Total innings played = x + 3

= 27 + 3

= 30.

EXERCISE : 10.1

I. 1 & 2 Mark question:

1. The height of 10 girls in Dance class are 90 cm, 95cm, 100cm, 98 cm, 102cm, 110cm, 105 cm, 97 cm, 102cm, 99 cm. Find the average height.
2. The age of 10 boys in a class are 4.3, 4.4, 4, 4.2, 4.3, 4.5, 4.7, 4.6, 4.5 and 4.8 years. What is the average age?
3. The average age of 10 boys in a class is 13 years. What is sum of their ages?
4. The average age of 7 member of a family is 18 years. If the head of the family is excluded the average age of the rest of the members would fall to 13 years. What is the age of the head of the family?
5. The average marks of 15 students of a class is 45. A student also has secured 50 marks leaves the class room. Find the average marks of the remaining 14 students.?
6. The average age of 10 students is 6 years. The sum of the ages of 9 of them is 52 years. Find the age of 10 students.
7. The average age of 12 boys is 8 years. Another boy 21 years. Join the group. Find the average of the new group.

8. The average score of 20 boys is 60% and average score of 30 girls is 70%. Find the combined average score.

9. The average height of a group of people is 6 ft. 10 more people are added with an average height of 5 ft. find the average height of the group of people consisting of 60 people.

10. Ram and Rahim went up a hill at a speed of 20 kmph. And both of them came tumbling down the same distance at a speed of 30 kmph. Find the average speed for the round trip.

II. 3 & 5 Marks Questions:

1. A batsman find that by getting out for a duck (0 runs) in the 11^{th} inning of his test matches. His average of the previous 10 inning decreased by 5 runs. What is his average after the 11^{th} innings?

2. A schools runs in morning and afternoon shift and employees 40 teachers. The average salary of 25 teachers working in the morning shift is ₹2800/- and the average salary of teachers working the afternoon shift is ₹3000/- find

   i) the average salary of the teachers in the school.

   ii) the average salary is 5 teachers shifted from morning to afternoon shift.

3. Find the total wage earned per month by 564 workers in a factory given the following information; 38 workers get ₹8.5 to 12.5, 46 workers get ₹12.5 to 16.5, 120 workers get ₹16.5 to 20.5, 360 workers get ₹20.5 to 24.5 daily wages (assuming that a month has 30 days and all the day they work) ?

   (Hint: take the value $\frac{8.5+12.5}{2}=10.5$ ₹/day for 38 workers and so on)

4. The average age of Ashok and Abdul is 45 years, the average age of Abdul and Anthony is 50 years and the average age of Anthony and Ashok is 35 years. Find the age of Abdul, Ashok and Anthony

5. Calculate the average daily way earned by 100 workers in a factory using the following data.

Daily wage	70 - 80	80 - 90	90 - 100	110-110	110-120
No of workers	18	7	23	44	8

(Take the value $\frac{70+80}{2} = 75$ and so on)

6. A book seller bought 228 note books at an average price of ₹8.50 in which 80 books he bought at ₹7.50, each and 84 books at ₹10.50 each. Find the price of the remaining books per unit.

7. A painter works 8 hrs. on Monday, 9 hrs on Tuesday, 7½ hrs on Wednesday 7½ hrs on Thursday, 6 hrs. on Friday and 10 hrs on Saturday. He is paid on hourly wages at the rate ₹8.50. What is his average daily earning?

8. An aeroplane flies once round a square whose side is 100 km long taking the first at 100 kmph., second at 200kmph,third at 300kmph and the fourth at 400kmph. Find the average speed of the plane in its flight along the square.

9. Of a number of persons donating to a charity 10 persons gave ₹99 each, 25 gave ₹50 each, 33 gave ₹25 each, 46 gave ₹10 each and the rest gave ₹5 each. It was found that the average donation is ₹20/- How many donors are there?

10. Govind bought 11 bags in the whole sale market at an average price of ₹450 each. In which the price of 7 leather bags was ₹575 each. The price of the remaining 4 cotton bags all in the increasing Arithmetic progression having the price of the costliest cotton bag was ₹300/-. Find the price of the cheapest cotton bag.

11. Rajhamsa bus covers the distance of 360 km between Bengaluru and Chennai in 5 hrs. 45 minutes with a stoppage of 10 minutes for coffee and tiffen and a stoppage of 3 minutes at Bannerghatta Bus stop and 2 stoppages of 5 minutes each at Hosur and Dharmapuri bus stop respectively. What is the average speed of the bus?

ANSWERS : 10.1

I. 1) 99.8 cm 2) $\bar{x}$ = 4.43 yrs. 3) 130 yrs. 4) 41 years

5) 44.64 6) 8 years, 7) 9 years 8) 66%

9) 5.85 feet 10) 24 kmph.

II. 1. Average run after 11^{th} innings = 50 run

2. ₹2875, ₹2900

3. Total daily wage = ₹11,386.00

   Total monthly wage = ₹605.63

4. Ashok 25 years, Abdul = 65 years, Anthony = 35 years

5. ₹96.70

6. ₹7.10

7. ₹68

8. 192 Kmph.

9. 432

10. 162.50 = Cheapest cotton bag price

11. 58.69kmph

Chapter 11

PERCENTAGE, PROFIT AND LOSS

11.1 Percentage: In mathematics a percentage is a number or ratio as a fraction of 100. It is denoted by symbol %. The word percent consists of two words 'per' and 'cent'. Per means each and cent means hundred. Thus percent means 'on each hundred' or 'for each hundred'.

We frequently come across statements like "The price of petrol is liked by 3%", "Our college has got 97% results ", "60% of the staff are women". In all the above statements comparison is being made. Percentage is mainly used as a tool of comparison and uniformity. Percentage are widely used in Commercial applications. Percentages are used in Calculation of interest, brokerage, dividend, population, depreciation etc.

11.2 Conversion Of A Fraction Into Percentage:

To convert a fraction into percentage just multiply by 100 and put % symbol.

Example 1 Convert the following fraction to percentages.

1) $\frac{2}{5}$ 2) $\frac{5}{8}$ 3) $\frac{7}{4}$ 4) $\frac{1}{3}$

Solution : 1) $\frac{2}{5} \times 100 = 40\%$ 2) $\frac{5}{8} \times 100 = 62.5\%$

3) $\frac{7}{4} \times 100 = 175\%$ 4) $\frac{1}{3} \times 100 = 33.33\%$

Conversion Of A Percentage Into A Fraction:

To convert percentage to fraction divide by 100 and reduce the fraction to its simple form.

Example 2 Convert the following percentages to fractions

1) 25% 2) 45% 3) 60%

Solution : 1) $\frac{25}{100} = \frac{1}{4}$ 2) $\frac{45}{100} = \frac{9}{20}$ 3) $\frac{60}{100} = \frac{3}{5}$

Conversion Of Percentage To Ratio

To convert percentage to Ratio first convert it to fraction and then write as ratio.

Example 3 Convert the following percentages to ratios.

1) 30% 2) 20% 3) 80%

Solution : 1) $\frac{30}{100}=\frac{3}{10}=3:10$

2) $\frac{20}{100}=\frac{1}{5}=1:5$

3) $\frac{80}{100}=\frac{4}{5}=4:5$

Conversion Of Ratio To Percentage

To convert ratio to percentage first convert ratio to fraction and then multiply by 100.

Example 4 express the following ratios as percentage.

1) 1: 5 2) 2 : 3 3) 4: 7

Solution : 1) $1 : 5 = \frac{1}{5}=\frac{1}{5}\times 100=20\%$

2) $2 : 3 = \frac{2}{3}=\frac{2}{3}\times 100=66.66\%$

3) $4 : 7 = \frac{4}{7}=\frac{4}{7}\times 100=57.14\%$

Conversion Of Percentage To Decimal

To convert percentage to decimal divide by 100 and write the result in decimal form

Example 5 Convert the following percentages to decimal.

1) 35% 2) 42% 3) 120%

Solution : 1) $\frac{35}{100} = 0.35$

2) $\frac{42}{100} = 0.42$

3) $\frac{120}{100} = 1.2$

Conversion Of Decimal To Percentage

To convert decimal to percentage multiply by 100.

Example 6 Convert the following decimal to percentage.

1) 0.12 2) 0.05 3) 1.25

Solution : 1) $0.12 \times 100 = 12\%$

2) $0.05 \times 100 = 5\%$

3) $1.25 \times 100 = 125\%$

To Find Percentage Decrease Of A Number :

$$\% \text{ increase} = \frac{\text{Increase}}{\text{Initial value}} \times 100$$

Example 7 Karthik received a scholarship of ₹5000 in 2011 and ₹8000 in 2012. Find the percentage increase.

Solution : increase = 8000 – 5000 = 3000.

$$\therefore \% \text{ increase} = \frac{\text{Increase}}{\text{Initial value}} \times 100$$

$$= \frac{3000}{5000} \times 100$$

$$= 60\%.$$

To Find Percentage Decrease Of A Number:

$$\% \text{ decrease} = \frac{\text{Decrease}}{\text{Initial value}} \times 100$$

Example 8 The sales of a company was ₹35000 in June and ₹30000 in July. Find the percentage decrease.

Solution : $\% \text{ decrease} = \dfrac{\text{Decrease}}{\text{Initial value}} \times 100$

Decree $= 35,000 - 30,000 = 5000$

$$\% \text{ decrease} = \frac{\text{decrease}}{\text{Initial Value}} \times 100$$

$$= \frac{5000}{35000} \times 100$$
$$= 14.285$$
$$\approx 14.29\%$$

To Increase A Number By A Given Percentage

Multiply the number by the factor $\dfrac{100 + Rate}{100}$

Example 9 Increase 30 by 10 %

Solution : $30 \times \dfrac{(100+10)}{100} = 30 \times \dfrac{110}{100} = 33$

OR $30 + \dfrac{10}{100} \times 30 = 30 + 3 = 33$

To Decrease A Number By A Given Percentage

Multiply the number by the factor $\dfrac{100 - Rate}{100}$

Example 10 Decrease 200 by 40 %

Solution : $200 \times \left(\dfrac{100 - 40}{100} \right) = 120$

Or

$$200 - \frac{40}{100} \times 200 = 200 - 80 = 120$$

11.3 Application Problems Involving Percentages

Example 1 In an election the winning candidate got 4,800 votes which is 80% of the total votes. Calculate the total number of votes.

Solution : Let total no of votes = x.

$$\frac{80}{100} \times x = 4{,}800$$

$$x = \frac{4{,}800 \times 100}{80}$$

= 6000 votes.

Example 2 A's income is 10% more than B's How much is B's income less than A's

Solution : Let B's income be 100

Then A's income = 110

$$\% \text{ decrease} = \frac{\text{decrease}}{\text{Initial value}} \times 100$$

$$= \frac{10}{110} \times 100$$

= 9.09%

Example 3 The original price of a shirt was ₹700. It was decreased by ₹50. What is the percentage decrease of the price of the shirt?

Solution : $$\% \text{ decrease} = \frac{\text{decrease}}{\text{Initial value}} \times 100$$

$$= \frac{50}{700} \times 100$$

= 7.14%

Example 4 Sanjana has a monthly salary of ₹20,000. She spends ₹4000 per month on cosmetics. What percent of her monthly salary does she spend on Cosmetics?

Solution : $\frac{\text{Amount spent}}{\text{Total Amount}} \times 100$

$$= \frac{4{,}000}{20{,}000} \times 100 = 20\%$$

Example 5 The price of a pair of trousers was decreased by 22% to ₹390. What was the original price of the trousers?

Solution : Let the original price be ₹ x.

Then $x - \frac{22}{100} \times x = 390$

$$x\left(1 - \frac{22}{100}\right) = 390$$

$$x\left(\frac{78}{100}\right) = 390$$

$$x = \frac{390 \times 100}{78} = 500Rs.$$

Or

If Total amount is 100%, then Balance Amount is 78% which is equal to 390

Hence total amount $= \frac{100 \times 390}{78}$ 78% - 390

= 500 Rs 100% - ?

Example 6 In the annual budget of a certain college, the annual income was estimated at ₹250 lakhs and annual expenditure was estimated at ₹210 lakhs. Actually that year the annual income increased by 5 percent than estimated income and expenditure increased by 10 percent than estimated. Find the difference between the actual income and actual expenditure.

Solution : Actual income $= 250 + \frac{5}{100} \times 250$

$= 262.5$ lakhs.

Actual expenditure $= 210 + \frac{10}{100} \times 210$

$= 231$ lakhs

$\therefore$ Required difference $= 262.5 - 231 = 31.5$ lakhs

Example 7 There were two candidates in an election. 20% of the members in the voters list did not cast their votes and 50 votes were declared invalid. The successful candidate secured 300 votes more than his rival. If 45% of the total members voted in favour of successful candidate then find the votes secured by each candidate.

Solution : Let the total number of votes $= x$

No. of people who did not vote $= \frac{20}{100} \times x$

$= 0.2x$

No. of people who voted for successful candidate $= \frac{45}{100} \times x = 0.45x$

Remaining votes $= \frac{35}{100} \times x = 0.35x$ (100 - (20+45) =35)

Invalid votes + Rival candidates votes $= 0.35\,x$

Rival Candidate votes $= 0.35x$ – Invalid votes

$= 0.35\,x - 50$

Given diff in votes $= 300$

$0.45x - (0.35x - 50) = 300$

$0.45x - 0.35\,x + 50 = 300$

$0.1x = 250$

$x = 2500$

votes secured by winning candidate $= 0.45\,x = 0.45 \times 2500 = 1125$

votes secured by rival candidate $= 0.35x = 50 = 0.35 \times 2500 - 50 = 875\text{-}50$

$= 825$

Example 8 In a dance competition 70% of the participants were girls. 35% of the boys and 65% of the girls got qualified for the next round. If 49 girls were eliminated find the number of boys who were eliminated.

Solution : Let total number of participants $= x$

Total number of girls $= \frac{70}{100}x = 0.7x$

Total number of boys $= \frac{30}{100} \times x = 0.3x$

If 65% of girls got qualified then 35% of girls got eliminated

$\therefore$ total girls eliminated $= \frac{35}{100} \times 0.7x$

$= 0.245x$

Hence $0.245x = 49$

$x = \frac{49}{0.245}$

$= 200$

Hence total number of boys eliminated $= \frac{35}{100} \times 0.3 \times 200$

$= 21$

Example 9 By how many percent should the use of tea be increased if the price of tea is decreased by 10% so that the expenditure remains unchanged.

Solution : Let Price be 100

Let Quantity be 100

Total expenditure $= 100 \times 100 = 10{,}000$

Then New Price $= 100 - 10 = 90$

Let New Qty $=$ y

Total expenditure $=$ 90 y

90 y $=$ 10,000

$$y = \frac{10,000}{90} = 111.11$$

$$\therefore \% \text{ increase} = \frac{\text{increase}}{\text{Original value}} \times 100$$

$$= \frac{11.11}{100} \times 100 = 11.11\%$$

EXERCISE : 11.1

I . One Mark questions.

1. Convert the following fractions to percentages.

   1) $\frac{4}{5}$ b) $\frac{3}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{5}$

2. Convert the following percentages into fractions

   a) 30% b) 40% c) 20% d) 75%

3. Convert the following percentage to ratio

   a) 25% b) 18% c) 36% d) 10%

4. Convert the following ratio to percentage

   a) 1 : 2 b) 3.5 c) 2 : 3 d) 1 : 4

5. Convert the following percentages to decimal

   a) 30% b) 12% c) 64% d) 8%

6. Convert the following decimal to percentage.

   a) 0.32 2) 0.06 c) 0.51 d) 0.28

7. Find 12 ½% of 1 hour 40 minutes

8. What percent is is 7 paise of ₹75?

9. What percent is 64 m of 12 km?

10. What percent is $\frac{4}{5}$ of 125?

II. Two marks questions.

1. While taking measurement a tailor writes 34 instead of 24. What is the percentage error?
2. When 40% of a number is added to 42, the result is the number itself. Find the number.
3. A student has to score 50% marks to pass. He gets 100 marks and yet fails by 50 marks. Find the maximum marks.
4. Shreya and Sanju scored 78% and 72% in an examination. If the difference in their marks is 36, find the maximum marks.
5. Ayush gets ₹33000 after getting an increase of 10% in his salary. What was his original income?
6. After revaluation a students mark was changed from 80 to 92. Find the percentage increase in marks.

III. Three marks questions.

1. Monthly income of Shreya, Akashatha and Pooja was increased from 25000, 22000 and 15000 to 35000, 26000 and 18000 respectively whose gain percent is maximum.
2. Ritu's salary was increased by 10% and then again by 5%. If the present salary is ₹9,240. What was Ritu's previous salary.
3. The cost of T.V. increased by 20% and then decreased by 5%. Find the percentage increase in the original cost.
4. A number x is mistakenly divided by 10 instead of being multiplied by 10 what is the percentage error in the result.
5. By how much percent should the use of milk be increased if the price of the milk is decreased by 20% so that the expenditure remains unchanged.
6. The rate of a movie ticket was ₹150. This was reduced by 20% Due to the discount in price the revenue increased by 20%, what was the percentage increase in the number of viewers.?

   [Hint: Take the number of viewers as 100]

IV. Five marks questions.

1. The total number of students in a Arts and Science college is 4200. If the number of arts students is increased by 40% and the number of science students is decreased by 30% the total strength remains unchanged. Find the number of arts and commerce students.

2. Venu gives 50% of his salary to his wife 40% of the remaining he spends on recreation 20% of the remaining he gives to his daughter as pocket money and still saves ₹12,000. What is Venu's income? Also find the amount he gives his wife and daughter.

3. A person spent 30% of his wealth and thereafter ₹20,000 and further 10% of the remainder. If 29,250₹ is still remaining, what was his total wealth?

4. In a school there are 1800 students. Last day except 4% of the boys all the students were present in the school. Today except 5% of the girls all the students are present in the school, but in both the days number of students present in the school were same. Find the number of boys and girls in the school.

5. Namma TV is a very popular TV Channel. It telecasts the programs from 8.00 AM to 12.00 p.m. It telecasts 60 advertisements each of 8 seconds and 16 advertisement each of 30 seconds. What is the percentage of time devoted in a day for the advertisement?

6. Due to increase in the price of sugar by 5% ,a man reduces his consumption by 5%.. Find the percentage increase or decrease in expenditure. What difference would it make if the price decreases by 5% and the consumption increases by 5%?

ANSWERS : 11.1

I. 1. a) 80% b) 150% c) 25% d) 20%

2.	a) $\frac{3}{10}$	b) $\frac{2}{5}$	c) $\frac{1}{5}$	d) $3/4$	
3.	a) 1:4	b) 9:50	c) 9:25	d) 1:10	
4.	a) 50%	b)60%	c) 66.67%	d) 25%	
5.	a) 0.3	b) 0.12	c) 0.64	d) 0.08	
6.	a) 32%	b) 6%	c) 51%	d) 28%	
7.	12.5 minutes	8. 0.0933%	9. 0.533%	10. 0.64%	

II. 1) 41.66% 2) 70 3) 300 4) 600
5) 30,000 6) 15%

III. 1) 40%, 18.18%, 20%, % increase is max for Shreya
2) 8000 3) 14% 4) 99% 5) 25% 6) 50%

IV. 1) Arts 1800, Commerce 2400
2) 50,000 , 25,000- wife , 3000 daughter
3) 75000 4) 800 girls, 1000 boys.
5) 1.66 % 6) both case decrease in expenditure is 0.25%

11.4 Profit and loss

Cost Price (CP): The amount used in manufacturing an article or the price at which article is bought is called cost price.

Selling Price (SP): The price at which an article is sold is called selling price.

Profit: When an article is sold for more than what it costs we say there is a profit or gain. Profit = SP – CP.

Loss: When an article is sold for less than what it costs we say there is loss

Loss = CP – SP

Formulae in Profit and loss

1) $\%\text{ profit} = \frac{Profit}{CP} \times 100$

2) $\%\text{ Loss} = \frac{Loss}{CP} \times 100$

3) $\text{Profit} = \frac{Profit\% \times CP}{100}$

4) $\text{Loss} = \frac{loss\% \times CP}{100}$

5) $SP = \frac{100 + Profit\%}{100} \times CP$

6) $\text{SP} = \frac{100 - Loss\%}{100} \times CP$

7) $\text{CP} = \text{SP} \times \frac{100}{(100 + \text{Profit}\%)}$

8) $\text{CP} = SP \times \frac{100}{(100 - loss\%)}$

9) By using false weight if a substance is sold at cost price, the overall gain % is given by

$$\frac{100 + gain\%}{100} = \frac{\text{True scale or weight}}{\text{False scale or weight}}$$

11.5 Application problems

Example 1 There is a profit of 20% when an article is sold at ₹96. What will be the gain percent if the article is sold for ₹100?

Solution : If the CP is 100 then selling price will be SP = CP + Profit = 100 + 20 = 120

Then if SP is 96

$\text{CP} = \frac{100 \times 96}{120} = 80$

CP	SP
100	120
?	96

If article is sold at 100 Profit = 100 – 80

= 20

Profit % = $\frac{profit}{CP} \times 100 = \frac{20}{80} \times 100 = 25\%$

Note: CP can also be calculated using Formula CP = SP $\frac{100}{100 + profit\%}$

$$= 96 \times \frac{100}{100+20} = \frac{96 \times 100}{120} = 80$$

Example 2 Savitha sold her bag at a loss of 7%. Had she been able to sell it at a gain of 9% it would have fetched ₹64 more than it did. What was the cost price of the bag?

Solution : Let C.P. = 100

Loss at 7% $\Rightarrow$ SP = 93

Gain of 9% $\Rightarrow$ SP = 109

Diff = 109-93 = 16

If CP is 100 difference is 16

But if the actual diff is 64

CP	Diff
100	16
?	64

Then cost price of the bag is = $\frac{100 \times 64}{16} = 400.$ ₹

Example 3 A person gets ₹1216 more when selling a product at a profit of 15% instead of a loss at 4%. What would be the percentage profit or loss if it is sold for ₹7552?

Solution : $\therefore$ Let the C.P. be 100

S.P. making profit of 15% = 115 s

S.P. While making loss of 4% = 96

Diff = 19

If the CP is 100, difference is 19		CP	Diff
but if the diff is 1216 then CP	$= \dfrac{100\times1216}{19}$	100	19
		?	1216
	$= 6400$		

If SP $= 7552$,

Profit $= 7552 - 6400 = 1152$

$$\text{Profit }\% = \frac{\text{Profit}}{CP}\times100$$

$$= \frac{1152}{6400}\times100$$

$$= 18\%$$

Example 4 A man buys an article at ¾ of its cost value and sells it for 20% more than its cost value. What is his profit percentage?

Solution : Let the Actual price $= 100$

Then purchase price (CP) $= \dfrac{3}{4}\times100 = 75$

At profit of 20% SP $= 120$

Profit $= 120 - 75 = 45$

$\therefore$ Profit % $= \dfrac{45}{75}\times100 = 60\%$

Example 5 A watch is sold for ₹150, at a profit of 25% At what price should it be sold in order to have 50% profit.

Solution :

	C.P.	S.P.
Let CP be 100		
Then S.P is 125	100	125
If SP is 150 then CP $= \dfrac{100\times150}{125} = 120$	?	150
SP to get a profit of 50% is	CP	SP
$= \dfrac{120\times150}{100}$	100	150
$= 180$ Rs.	120	?

Example 6 A dealer buys 200 quintals of wheat at ₹1200 a quintals. He spends ₹10,000 on transportation and storage. Then he sells the wheat at ₹13 per kg. Find his profit or loss. Also calculate it as a percentage.

Solution :

C.P. = 1200×200 = 2,40,000

Transportation and storage cost = Rs. 10,000

Total CP = 2,40,000 + 10,000

= 2,50,000

Total SP = $13 \times 200 \times 100$

= 2,60,000

Profit = 2,60,000 – 2,50,000 = 10,000

Profit % = $\frac{Profit}{CP} \times 100$

= $\frac{10,000}{2,50,000} \times 100 = 4\%$

Example 7 Ram Singh purchased two camels for ₹18000 and ₹15000 respectively. He sold them at a loss of 15% and a gain of 19% respectively Find the selling price of each of the camels. Also find the overall loss or gain percent in the transaction.

Solution :

C.P. of camel = 18000

S. P while sold at loss of 15% = $\frac{85}{100} \times 18000$

= 15300

C.P. of second camel = 15000

S.P. while sold at gain of 19% = $\frac{119}{100} \times 15000$

= 17,850

Total CP = 18000+15000

= 33000

Total S.P. = 15,300 + 17,850

= 33150

$\therefore$ Profit = 33,150 – 33000 = 150

Profit % = $\frac{150}{33000} \times 100 = 0.45\%$

Example 8 By selling 8 erasers a trader gains the selling price of 1 eraser. Calculate the gain percent.

Solution : Let the Selling Price of one eraser be x then SP of 8 erasers $= 8x$

$$\text{Profit} = x$$

$$\text{C.P.} = \text{S.P.} - \text{profit}$$

$$= 8x - x = 7x$$

$$\text{Profit \%} = \frac{\text{Profit}}{C.P} \times 100$$

$$= \frac{x}{7x} \times 100 = 14\frac{2}{7}\%$$

EXERCISE : 11.2

I. One mark questions.

1. If a company makes a profit of 10,000 by selling goods worth ₹25000. Find the profit percentage.
2. By selling a book at ₹250 the profit made is ₹50. What is the cost price of the Book?
3. Find the value of a house in the purchase of which the broker was paid 2% brokerage which amounted to ₹80000.
4. Nihal bought a cycle for ₹3000. For what price should he sell it to gain 10%?
5. The cost of an article is 80₹. If a profit of ₹20 is made by selling the article, find the profit percentage.

II. Two marks questions

1. A seller bought a colour T.V. set for ₹10,000. He marked the selling price as ₹25,000. If he sells the TV after giving a discount of 30% from the marked price Find the profit percentage.
2. A dishonest dealer professes to sell his goods at cost price, but he uses a weight of 960gm for the kg weight. Find his gain percent. [Hint: profit=40 ; CP=960]

3. The cost price of 10 articles is equal to the selling price of 9 articles. Find the profit percent. [Hint: Let cost price of 1 article be x, then selling price of one article will be $10x/9$]

4. A person sold his watch for ₹75 and got a percentage profit equal to the cost price. Find the cost price of the watch. [Hint :let cost price and profit $\%=x$]

5. Find the cost price of an article which is sold for ₹220 at a loss of 12%

6. The S.P. of an article is 3680 and profit percent is 15%. Find the cost price.

7. A dealer by selling 10 oranges get the cost price of 15 oranges. What is the profit percentage?

   [Hint let SP = $15x$ CP = $10x$]

8. By selling 16 rings a shopkeeper loses the selling price of 4 rings. Find the loss percent [Hint: SP=$16x$ CP=$20x$]

9. The difference between cost price and selling price is 225. If the profit percentage is 15%, find the selling price.

10. Abhinav saves ₹75 by getting a discount of 15% on a text book. How much did he pay for the book?

III Three marks questions.

1. A sells a bicycle to B at a profit of 20% and B sells to C at a profit of 25%. If C pays ₹225 for the bicycle what did A pay for it. [Hint: let CP of A = 100]

2. 1 Kg of salt and 4Kg of sugar cost ₹160. But if the cost of sugar rose by 20% and that of salt by 10% the same quantity of salt and sugar would cost ₹190. Find the prices per kg of salt and sugar.

3. A bookseller sells a book at a profit of 10%. If he had brought it at 4% less and sold it for ₹6 more, he would have gained 18 ¾ percent. What did it cost him.

4. A pressure Cooker is sold for ₹1,200/- in which sales tax amounts for 20% of this and profit 1/3 of the remainder. Find the cost price of the pressure cooker. Calculate the sale than % profit %.

5. Crystal electronics sold a calculator at a profit of 5% instead of a loss of 5% and got ₹11 more. Find the cost price of the calculator.
6. If an article is sold at ₹24, there is a profit of ₹4. If it is sold at a loss of 10%. Find out its selling price.
7. Nihal refused to sell his book for ₹726 because there was a loss of 12%. If he sold the book at a profit of 5%. Find selling price.

IV Five marks questions.

1. A man purchased a certain number of oranges at 25 a rupee and the same number at 20 a rupee. He mixes them together and sells them at 45 for two rupees. How much percent does he lose or gain in the transaction.
2. Rohan sold to Rakshith cinema tickets at a profit of 10%. Rakshith sold it back to Rohan at a loss of 10%, In the whole process Rohan gained ₹55 in all. Find the price at which Rohan originally bought the tickets.
3. A dealer sold 3 T.V. sets at ₹11,500 each. He sold one at a profit of 15% and the other two at a loss of 8%. Find his gain or loss percentage.
4. A merchant purchased 25 baskets of fruits at Rs.2 per basket. The fruits of 5 baskets turned out bad and were thrown off. Find the selling price per basket if he wants 12 ½ % profit.
5. Four percent more is gained by selling a table for ₹180 than by selling for 175. Find the cost price of the table.
6. A radio is sold at a profit of 25%. Cost price and selling price both are increased by ₹100. If the new profit is at the rate of 20%, find the original cost of the radio.
7. A Shopkeeper sold a watch at 5% loss. Had he purchased it at 10% less cost and sold it for ₹140 more, his gain would have been one fourth of the original cost price. Find the Cost Price of the watch.
8. A business man sells an article for ₹720 and earns a profit of 20%. Find the a) Cost Price b) Profit percentage at selling price.

   [Hint: for b) profit%=profit/SP x 100]

9. If a Commodity is sold for ₹1,500 there is a profit of 25%. If the Cost Price is increased by ₹100. Find the profit percentage.

10. A wholesale dealer sold a machine to a shopkeeper at 20% profit. The shopkeeper sold it to a customer so as to get 25% profit for himself. The difference between the selling price of the shopkeeper and that of the whole sale dealer was found to be ₹129. Find the initial price of the machine.

ANSWERS : 11.2

I.	1) 66.67%	2) 200	3) 41,00,000	4) 3300	5) 25%
II.	1) 75%	2) 4.16%	3) $11\frac{1}{9}\%$	4) 50	5) Rs.250
	6) 3200	7) 50%	8) 20%	9) 1725	10) 425

III.	1) 150	2) salt 20, sugar 35	3) ₹150
	4) ₹640 excluding tax	5) 110₹	6) ₹18
	7) ₹866.25		

IV.	1) 1.23% Loss	2) ₹500	3) 1.43% loss	4) ₹2.8
	5) ₹125	6) 400	7) 700	8) ₹600, 16.66%
	9) 15. 38%	10) 430		

Chapter 12

LINEAR FUNCTION

12.1 INTRODUCTION: Linear function has greater importance in the field of Economics, Commerce and other business activities. Linear Function are very useful in representing situation where one variable increases or decreases proportionally with another.

12.2 DEFINITION:

Linear Function is defined by y = f(x) where y = f(x) = ax + b , a ≠ 0 and a, b ∈R. It is called linear because the graph of such a function is a straight line where 'a' is called the slope of the line and 'b' is called the intercept.

Eg. Consider the function y = f(x) = 2x + 1

When $x = 0, y = 2(0) + 1 = 1$, (0, 1)

$x = 1, y = 2 (1) + 1 = 3$ (1, 3)

hence the graph of f(x) is shown below as a straight line.

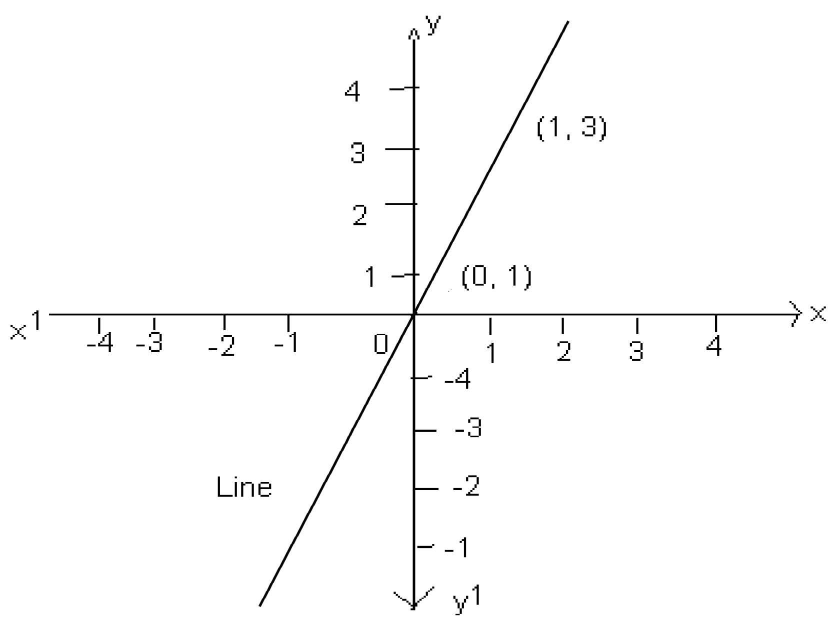


Fig. 1

12.3 Linear cost, Revenue and Profit Function:

1. **Linear Cost Function:** C(x) Function in the form

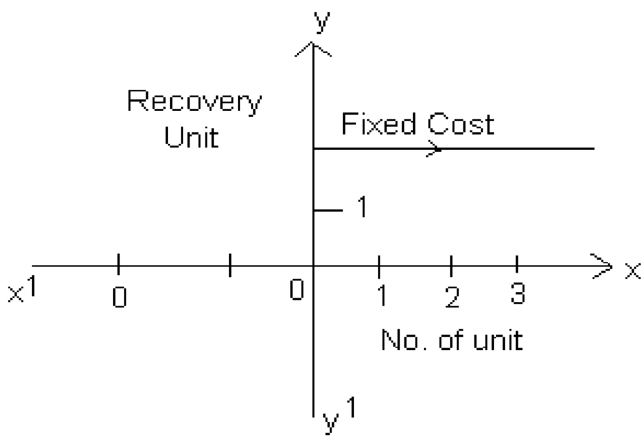
   C (x) = ax + b where C(x) denotes the total cost. 'x' denote the quantity or Number .of unit or output of a commodity or a product.

   The total cost of a product can be split into two components as total fixed cost and total variable cost.

   $\therefore$ Total cost = Total fixed + Total variable cost

   $$C(x) = TFC + TVC$$

2. **Total fixed cost (TFC)** Defined as that cost which does not vary with the output. It is the cost which a Firm or a company has to bear. Even when the output 'x' is zero. Eg. Rent, Insurance and other initial capital. It can be represented graphically by a line parallel to the x-axis. Its slope is zero.



In $C(x) = ax + b$

If $x = 0$

$C(x) = b = TFC$

Fig. 2

3. **Total Variable cost : (TVC)** Defined as that cost which varies with the output 'x'. As the production increases they increases and if the production decreases they decreases proportionately. Eg. Labour cost, Transport cost of Fuel, and operating cost etc. It can be represented graphically by a straight line passing through origin (0, 0) i.e. TVC = ax $(\because b = 0)$.

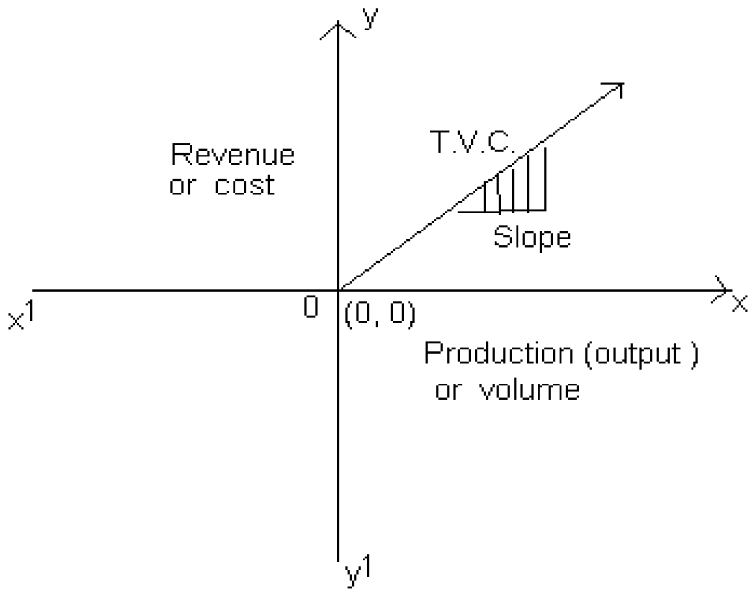


Fig. 3

Hence the **total cost C(*x*) function** can be represented graphically as below.

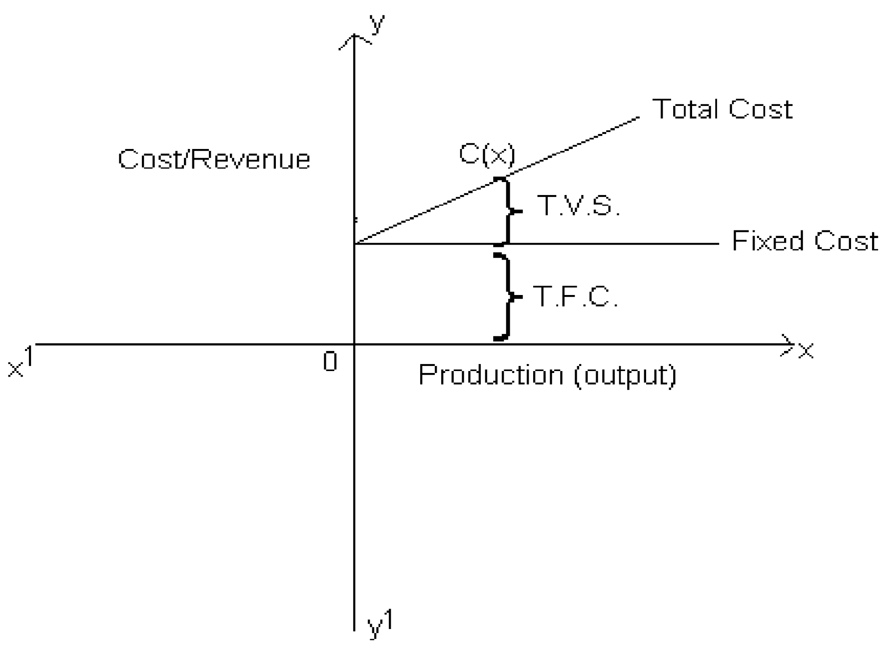


Fig. 4

4. **Linear Revenue Function:** R(x) Defined as the relation between Total Revenue (T.R.) and the output (x) denoted by R(x)

$$R(x) = ax + b, \qquad (a \neq 0)$$

Total Revenue = TR = (Selling price / Unit) × (No. of unit sold)

$$\therefore \ TR = P \times x$$

The linear revenue function R(x) can be graphically represented as below

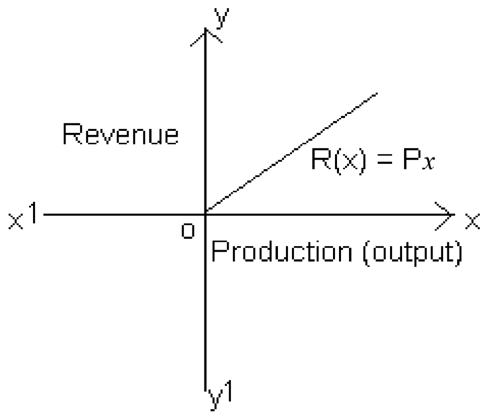


Fig. 5

5. **Linear Profit Function:** P(x) defined as the difference of total revenue function R(x) and the total cost function C(x)

ie. $P(x) = R(x) - C(x)$

Total profit = Total Revenue – Total cost

12.4 Break Even analysis:

12.4 Break Even analysis: Break even analysis is a very important technique used to trace and establish the relationship between cost production and sales volume while exhibiting the probable profit. Hence this analysis is also known as cost-volume – profit analysis (C.V.P analysis)

Break – Even Point (BEP) : It is the point of intersection of total cost and total revenue. Hence it is defined as volume of production at which the **total revenue(TR)** equal to **total cost (TC).** It is the point x_1 where $R(x_1) = C(x_1)$

Note: At BEP the manufacturer (or sellers) does not make any profit or loss.

Total Revenue (TR) , the Total cost (TC), the profit /Loss. TFC and BFP are represented graphically below in the form of Break-Even chart.

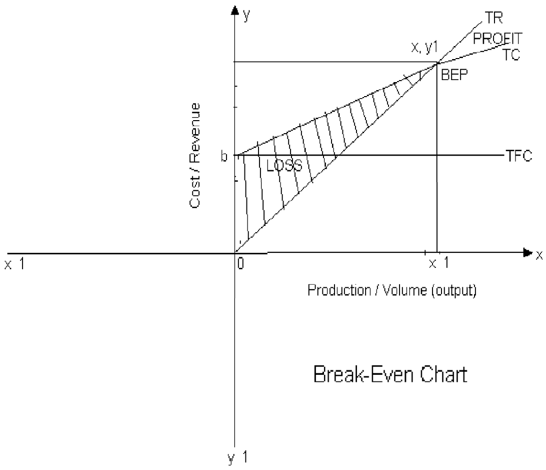


Fig. 6

The shaded area below BEP (Left) is called **loss zone** and the blank area above BEP (Right) is called the **profit zone**.

Limitation Break –Even Analysis:

i) The selling price assumed to be a constant but in practically price is rarely constant as production increases the price/unit will be decreases.

ii) It is limited only for short term period. It is a static picture in which the profit is influenced solely by the level output.

iii) It assumed that the Fixed cost is a constant and the variable cost does not change at each level of operation. This is not true in reality.

iv) The risks and uncertainties are neglected in this analysis but a business operation are full of risks and uncertainties.

WORKED EXAMPLES:

Example a) A manufacture produce and sells bags at ₹8/ unit. His Fixed cost is ₹5550 and the variable cost per bag is ₹2.45. Find

i) Revenue Function

ii) Cost Function

iii) Profit Function

iv) BEP in unit

Given TFC = ₹5550 P = ₹8/unit, VC = ₹2.45/unit.

Solution: i) Revenue function : $R(x)$ = Price × Quantity

$= (8)(x)$

$R(x) = 8x$

ii) Cost function : $C(x) = ax + b$

$= TVC + TFC$

$\therefore C(x) = 2.45x + 5550$

iii) Profit Function: $P(x) = R(x) - C(x)$

$= 8x - (2.45x + 5550)$

$= 8x - 2.45x - 5550$

$\therefore P(x) = 5.55x - 5550$

iv) BEP in Unit

At BEP : Total Revenue = Total cost

$R(x) = C(x)$

$8x = 2.45x + 5550$

$8x - 2.45x = 5550$

$5.55x = 5550$

$$\therefore x = \frac{5550}{5.5}$$

$x = 1000$ units

Example 2 The Fixed cost of a Firm and variable cost / unit of the product are ₹5,000/- years and ₹5 respectively. If the selling price is ₹15/- unit. Find the

i) BEP in unit

ii) Prove that Total Revenue is equal to the Total cost at BEP.

Solution : Given TFC = ₹5,000 VC = ₹5 and P = ₹15

i) Total cost = $C(x)$ = T.F.C. + T.V.C.

$= 5{,}000 + 5x \quad (x = \text{output})$

Total Revenue = $R(x)$ = Price x quantity output

$R(x) = 15x$

At BEP $R(x) = C(x)$

$$15x = 5{,}000 + 5x$$
$$10x = 5{,}000$$
$$\therefore x = \frac{5000}{10}$$
$$x = 500 \text{ units}$$

ii) Prove that TR = TC

Price × output = TFC + TVC

$15x = 5000 + 5x$

Put $x = 500$ then we get $15\ 500 = 5000 + 5\ (500)$

i.e $7500 = 5000 + 2000$

TR = TC = 7500 hence proved

Example 3 Find the Break-Even Points in units if the total cost and the Revenue function are $C(x) = 450 + 1.5x$ and $R(x) = 3x$ (x = output) ?

Solution : At Break Even point , Total Revenue = Total cost

$$\therefore \quad R(x) = C(x)$$
$$3x = 450 + 1.5x$$
$$3x - 1.5x = 450$$
$$1.5x = 450$$
$$\therefore \quad x = \frac{450}{1.5}$$
$$x = 300 \text{ unit}$$

Example 4 A company sells x Box of chalk powder each day at ₹20/Box. The cost of manufacturing and selling these Boxes is ₹15/box. Plus a fixed dailies overhead cost of ₹900. Find the profit if 1000 boxes are manufactured and sold/day?

Solution : Total Revenue by selling x Boxes $R(x)$ = Price × quantity sold

$$= 20x \text{ ....(1)}$$

The total cost of manufactured Boxes in a day

$$C(x) = TVC + TFC$$
$$= 15x + 900 \text{ ....(2)}$$

Hence the profit / day $= P(x) = R(x) - C(x)$

$$= 20x - (15x + 900)$$
$$= 20x - 15x - 900 = 5x - 900$$

Put $x = 1000$ Boxes / day

$$\therefore \text{Profit/Day} = 5(1000) - 900$$
$$= 5000 - 900$$
$$= 4{,}100 \text{ ₹}$$

Example 5 A manufacturer of steel vessels finds that his cost function is linear. He calculates that the total cost for 250 units is ₹8000 and for 350 unit the total cost is ₹10,000. What are his fixed cost and variable cost/unit.

Solution : Cost function is given by $C(x) = ax + b$

If $x = 250$ unit than, $C(250) = a(250) + b$

$$8000 = 250a + b. \qquad \text{......(1)}$$

If $x = 350$ unit, $C(350) = a(350) + b$

$$10{,}000 = 350a + b. \qquad \text{......(2)}$$

(2) – (1)

$$10{,}000 = 350a + b$$
$$8{,}000 = 250a + b$$
$$2{,}000 = 100a$$

$$\therefore a = \frac{2000}{100}$$

$a = 20$ Substitute in (1)

$$8{,}000 = 250(20) + b$$
$$8{,}000 = 5000 + b$$
$$\therefore b = 3000$$

Hence fixed cost is ₹3,000/- and the variable cost is ₹20.

Example 6 The Daily cost of production C for x units of a manufactured product is given by $C(x) = 3.5x + 15000$

(i) If each unit is sold for ₹5. determine the minimum number of units that should be produced and sold to ensure no loss.

(ii) If the selling price in increased by half a Rupee / Unit what should be the Break-Even point.

(iii) If 5000 units are sold daily what price/unit should be charged to guarantee no loss.

Solution :

(i) Given $C(x) = 3.5x + 15000$, No.of units $= x$

Total Revenue (Selling price) $R(x) = 5x$

For BEP $C(x) = R(x)$

$3.5x + 15000 = 5x$

$5x - 3.5x = 15000$

$1.5x = 15000$

$$x = \frac{15000}{1.5}$$

$x = 10{,}000$ units

(ii) If the selling price is increased by half a Rupee/unit.

Total Revenue $= R(x) = 5.5x$

Again For BEP $C(x) = R(x)$

$$3.5x + 15000 = 5.5x$$

$$5.5x - 3.5x = 15000$$

$$2x = 15{,}000$$

$$\therefore x = \frac{15000}{2} = 7{,}500 \text{ unit}$$

(iii) If x = 5,000 units

Total Revenue = $5000 \times P$ (P = Price / Unit)

$C(x) = 3.5\,(5000) + 15{,}000$

$= ₹32{,}500$

For No Loss

$C(x) = R(x)$

$32{,}500 = 5000\,P$

$$\therefore P = \frac{32500}{5000}$$

$P = 6.5.$ ₹

$\therefore$ Selling price per unit = ₹6.5

Example 7 A manufacture produced and sells balloons at ₹8 per unit. His fixed cost is ₹6500 and the variable cost per Balloon is ₹3.50 calculate.

(i) Revenue Function (ii) Cost Function

(iii) Profit Function (iv) Break Even point.

Solution :

i) Revenue function : $R(x)$ = Price × quantity.

$= 8x$

ii) Cost function : $C(x) = ax + b$ = TVC + TFC

$= 3.5x + 6500$

iii) Profit Function : $P(x) = R(x) - C\,(x)$

$= 8x - (3.5x + 6500)$

$P(x) = 4.5x - 6500$

iv) At Break – Even point : $R(x) = C(x)$

$R(x) - C(x) = 0$

$\therefore P(x) = 0$

$$\therefore\ 4.5x - 6500 = 0$$
$$4.5x = 6500$$
$$x = \frac{6500}{4.5}$$
$$x = 1445 \text{ Unit}$$

Break-Even Revenue in ₹ = No. of unit X price / Units
= 1445 x 8
= ₹11,560

Example 8 A watch manufacturer produced 100 watches for a total cost of ₹20,000 and when the production is increased to 200 watches. The total cost increases to ₹30,000. Assuming that the costs and outputs are linearly related. Find the cost of equation and find the cost of manufacturing 150 watches?

Solution : Total cost = T.V.C. + T.F.C.

$C(x) = ax + b$

When x = 100 watches

T.C. = a (100) + b

i.e. 20,000 = 100 a + b ....(1)

Similarly when x = 200 watches

30,000 = 200a + b ....(2)

(2) – (1) 30,000 = 200a+b

- 20,000 = 100a +b

10,000 = 100a

$$\therefore\ a = \frac{10,000}{100} = 100 \text{ substitute in (1)}$$

20,000 = 100 (100) + b

20,000 = 10,000 + b ∴b= 10,000

∴ When x = 150 ,TC = (100) (150) + 10,000

= 15,000 + 10,000

TC = ₹ 25,000

Example 9 A two – wheeler spare parts manufacturing company introduces production bond to the employees that increases the cost of the spare parts. The daily cost of production C(x) for 'x' numbers of spare parts is given by C(x) = 25x + 550

(i) If each spare part is sold for ₹3, then find the minimum number that must be produced and sold daily to ensure no loss?

(ii) If the selling price is increased by 50 paise / spare part what would be the break – even point?

(iii) If it is known that at least 500 spare part can be sold daily. What price should the company charge / sparepart to guarantee no loss?

Solution:

(i) Given total Cost : C(x) = 25x + 550 .....(1)

Total Revenue R(x) = price × quantity

If x = output, $\therefore$R(x) = 3 x .....(2)

For Break even point

T(x) = R (x)

25x + 550 = 3x

550 = 3x – 25x

550 = 0.5x

$\therefore$x = 1100 units

(ii) If the selling price is increased by 30 paise then

R(x) = 3.30 x

Again for BEP C(x) = R (x)

$\therefore$2.5x + 550 = 3.50x

550 = 3.50x – 2.5x

550 = 1x

$\therefore$x = 550 Units

(iii) If at least 500 spare parts can be sold daily the price / unit needed to guarantee

$C(x) = 2.5x + 500$

If $x = 500$

$C(x) = 2.5(500) + 500$

$= 2.5\ (500) + 500$

$C(x) = 1750$

$R(x) = \text{price} \times \text{quantity}$

$R(x) = 500\ P$

For no loss $\quad R(x) = c\ (x)$

$500p = 1750$

$$\therefore P = \frac{1750}{500}$$

$P = ₹3.5$

Example 10 Tata Automobile Ltd., estimates that when they manufacture 1,00,000 cars per year their total cost will be ₹1700 Crore rupees. If they increases their production to 1,50,000 cars per year. They expect their total cost to increases to ₹2,450 crores. If the selling price of each car is ₹2,75,000/

(i) What will be the Break – Even production

(ii) What is the linear function relationship between the total cost and production value.

Solution:

Let $C\ (x_1)$ be the cost function when $X_1 = 1{,}00{,}000$ Cars

Let $C\ (x_2)$ be the cost function when $X_2 = 1{,}50{,}000$ Cars

$\therefore C(x_1) = a\ (1{,}00{,}000) + b = 1700$ Crores ....(1)

$C\ (x_2) = a\ (1{,}50{,}000) + b = 2450$ Crores ....(2)

$\therefore 1 - 2 \qquad 50{,}000\ a = -\ 750$ crore rupees

$$\therefore a = \frac{750\ Crore}{50{,}00}$$

$\therefore$ Total variable cost = a = ₹1,50,000

Substituting in (1)

a (1,00,000) + b = 1700 Crore

(1,50,000) (1,00,000) + b = 1700 Crore

$\therefore$ b = ₹200 Crore = Total fixed cost

$\therefore$ The total cost function

C (x)= (1,50,000) x + 200

(i) At break Even point

$C(x) = R(x)$

$1,50,000\ x + 200$	=	Selling price x unit produced
$1,50,000\ x + 200$	=	$(2,75,000)\ x$
200	=	$2,75,000\ x - 1,50,000x$
200	=	$1,25,000\ x$

$$\therefore\ x = \frac{200\,Crore}{1,25,000} = 16,000$$

$\therefore$ To reach the BFP the Tata Automobile must produce atleast 16,000 Cars

Example 11 (1) If $R(x) = 1.05x$, $C(x) = 0.85x$, Total fixed cost = 600 (x = the volume of output) find the rupees sales and quantity sold at break – even point. If a profit of ₹5,000 is required how much rupees sales and volume of output are required?

Solution:

At Break – Even Point

$C\ (x) = R\ (x)$

$0.85\ x + 600 = (1.05)\ x$

$600 = (1.05 - 0.85)\ x$

$$\therefore x = \frac{600}{0.20}$$

x = 3,000 Units are produced and rupees sale will be

3000.05 = ₹3150

If a profit of ₹5,000 is required the volume of output (x) will be calculated as follows.

$R(x) = C(x) + p\ (x)$

$1.05\ x = (0.85\ x + 600) + 5{,}000$

$1.05\ x - 0.85\ x = 600 + 5000$

$0.20\ x = 5{,}000$

$$\therefore x = \frac{5600}{0.20} = 28{,}000 \text{ Unit}$$

To produce a profit of ₹5,000, 28,000 Unit should be produced and sold = 28,000 × 1.05

Profit = ₹29,400

Example 12 A manufactures of Transistor finds that his cost function is linear. The total cost for 200 units is ₹6,000 and for 300 units the total cost is ₹8,000. what are the fixed cost and variable cost permit?

Solution:

Given Total Cost	$C(x) = ax + b$	Where	a = TVC = ?
			b = TFC = ?
at x_1 = 200 Unit	$C(x_1) = 200\ a + b$	= 6000 ....(1)	
at x_1 = 300 Unit	$C(x_2) = 300\ a + b$	= 8000 ....(2)	
(1) – (2)	- 100 a = - 2000		

$$\therefore a = \frac{\cancel{-2000}}{\cancel{-100}} = 20$$

∴ Total variable cost – TVC = ₹20 subsitute in (1)

200 a + b = 6000

200 (20) + b = 6000

400 + b = 6000

∴ b = ₹2000 Total fixed cost

Example 13 The daily cost of production 'C' in ₹ And 'x' unit of an assembly in $C(x) = 12.5x + 6400$. If each unit is sold for ₹25 then find the minimum number of unit that should be produced and sold to ensure no. loss. If the selling price is reduced by ₹By 2.5/unit. What would be the Break-Even Point.

Solution : Given $C(x) = 12.5x + 6400$

Selling price = Total Revenue = $R(x) = 25x$

For Break Even point $C(x) = R(x)$

$$12.5x + 6400 = 25x$$

$$6400 = 25x - 12.5x$$

$$6400 = 12.5x$$

$$\therefore \quad x = \frac{6400}{12.5}$$

$$x = 512 \text{ Units}$$

If the selling price is reduced by ₹2.5 /unit.

Total Revenue $R(x) = (25 - 2.5)\,x = 22.5x$

At Break Even point, $C(x) = R(x)$

$$12.5x + 6400 = 22.5x$$

$$6400 = 22.5x - 12.5x$$

$$6400 = 10x$$

$$\therefore \quad x = \frac{6400}{10}$$

$$\therefore x - 640 \text{ unit}$$

Example 14 A school bag manufacture company starts production of a new variety of cotton bag. For the 1^{st} year the Fixed cost for selling up the infrastructure is ₹1,40,000 and variable cost for production of each bag is ₹75. But the company given production bonus to its employees so the variable cost further increases by 50 paise/bag. Each bag is sold at ₹250.50 What is the profit $P(x)$ for x bags. Calculate the profit is 1000 bags are produced and sold. Also find the Break Even Point.

Solution : Given total fixed cost = ₹1,40,000

Total variable cost = (₹75.50)x (x = No.of bags produced)

Total selling price = $(250.50)x$

(Total Revenue)

$\therefore$ Profit $P(x) = R(x) - C(x)$

$= R(x) - (\text{T.V.C.} + \text{T.F.C.})$

$= 250.50x - (75.50x + 1,40,000)$

$= 250.50x - 75.50x - 1,40,000$

$\therefore P(x) = 175x - 1,40,000$

If $x = 1000$ bag profit $= P(x) = 175(1000) - 1,40,000$

$= 1,75,000 - 1,40,000$

$=$ ₹35,000

For Break Even point. $C(x) = R(x)$

OR

$P(x) = 0$

$175x - 1,40,000$

$175x = 1,40,000$

$$x = \frac{1,40,000}{175}$$

$x = 800$ bags has to produced by company for no loss/no profit.

Example 15 If the Sale price per unit is ₹3/-, the variable cost per unit is ₹2/- and the total Fixed cost is ₹4,500 find the

i) Break Even quantity

ii) Total Revenue function and total cost function at BEP

iii) If a profit of ₹10,000 is desired the volume of output to be produced and sold.

iv) Sketch the Break Even chart.

Solution : (i) Total cost $= C(x) = $ T.V.C. + T.F.C. [x= output]

$= 2x + 4500$

Total Revenue $= R(x) =$ Selling price X Quantity

$= 3x$

For BEP; $C(x) = R(x)$

$2x + 4500 = 3x$

$4500 = x \quad \therefore\ x = 4500$ unit

ii) Total Revenue at 4500 unit

$R\ (4500) = 3\ (4500)$

$= ₹13,500$

Total cost at 4,500 unit

$C\ (4500) = 2\ (4500) + 4500$

$= 9000 + 4500$

$= ₹13,500$

iii) If the desired Profit is Rs.10,000

Then $P(x) = R(x) - C(x)$

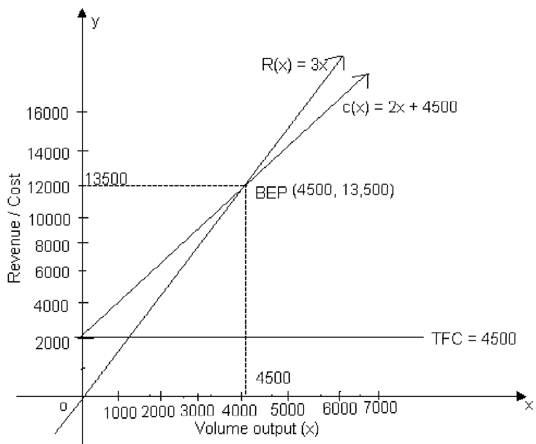
$10,000 = 3x - [2x + 4500]$

$10,000 = 3x - 2x - 4500$

$10,000 = x - 4500$

$\therefore\ x = 14,500$ units

iv) Break Even chart :



Break Ench Chart

Example 16 If 'x' represents number of units produced, selling price per unit is ₹14, variable cost /unit is ₹7.33 and the Fixed cost is ₹1200. Find the Break-Even point and quantity of sales. What is the slope of the total cost line?

Solution :

Total Revenue $= R(x) =$ Selling price X Quantity

$= 14x$

Total cost $= C(x) = TVC + TFC$

$= 6.67x + 1200$

For BEP $C(x) = R(x)$

$6.67x + 1200 = 14x$

$1200 = 14x - 6.67x$

$1200 = 7.33x$

$$\therefore x = \frac{1200}{7.33} = 180 \text{ unit}$$

$\therefore$ Break Even Quantity $=$ 180 units

Break Even Sale $= R(180)$

$= 14x$

$= 14(180)$

$= 2{,}520$

The slope of the total cost line is the variable cost/unit = 7.33

Example 17 A confectioner make and sells biscuit. He sells one pack of biscuit at ₹80. His cost of manufacturing is ₹40/- pack as variable cost and Rs.3000 as fixed cost find

a) His Revenue Function

b) His cost function

c) His profit function

d) If he limits his production to 100 packets can he make profit?.

e) What will be number of boxes he must sell to make a profit so that he does not incur any loss?

Solution : a) Revenue function = $R(x)$ = Selling price X quantity

$= 80\,x \quad (x = \text{No.of pack of biscuit})$

TC= TVC + TFC

b) Cost function $= C(x) = ax + b$

$= 40x + 3000$

c) Profit function $= P(x)$

$= R(x) - C(x)$

$= 80x - (40x + 3000)$

$= 80x - 40x - 3000$

$= 40x - 3000$

d) If $x = 100$ packets

$\therefore \quad P(x) = 40x - 3000$

$P(100) = 40\,(100) - 3000$

$= ₹1{,}000$

e) No.of packets required to ensure no loss (at BEP)

$C(x) = R(x)$

$40x + 3000 = 80x$

$3000 = 80x - 40x$

$3000 = 40x$

$$x = \frac{3000}{40}$$

$x = 75$ packet

If he makes 75 packets, the confectioner will not incur loss .If it is less than 75 packets he incurs a loss.

Example 18 The Philips Light Co. a manufacturing of light bulbs will break even at a Sales Volume of ₹2,00,000. The Fixed Cost is ₹40,000 and the selling price/bulb is ₹5/- What is the average variable cost/bulb?

Solution : Given Total Cost = $C(x) = TVC + TFC$, a = variable cost

$= ax + 40{,}000$, x = No.of bulbs produced

Total Revenue $= R(x) =$ Selling price X Quantity.

$R(x) = 5x$

Total Revenue $= 5x$

$2{,}00{,}000 = 5x$

$$x = \frac{2{,}00{,}000}{5}$$

$x = 40{,}000$ bulbs

For Break Even Sale

$C(x) = R(x)$

$ax + 40{,}000 = 2{,}00{,}000$

$ax = 2{,}00{,}000 - 40{,}000$

$ax = 1{,}60{,}000$

at $x = 40{,}000$ unit $\therefore$ $a(40{,}000) = 1{,}60{,}000$

variable cost a = ₹4

Example 19 If the cost function $C(x)$ of producing 'x' unit of a product is given by $C(x) = 500x^2 + 2500x + 5000$ and if each unit of the product in sold at ₹6000. then find BEP.

Solution: Given $C(x) = 500x^2 + 2500x + 5000$ (x = output)

Total Revenue $= R(x) =$ Selling price X Quantity.

$= 6000x$

For BEP : $C(x) = R(x)$

$500x^2 + 2500x + 5000 = 6000x$

$500x^2 + 2500x - 6000x + 5000 = 0$

$500x^2 - 3500x + 5000 = 0$

$\div 500$ $x^2 - 7x + 10 = 0$

$(x - 5)(x - 2) = 0$

$x = 5$ or 2 units

Example 20 The cottage toy Industry has 29 workers. The cost of producing a unit of toy is ₹2.07 and the Fixed Cost including the production of bonus is ₹30/worker.

i) If each toy is sold for ₹6/- determine the No. of toys to be produced and sold daily to ensure no loss.

ii) If a promote sale price is reduced by 50 paise/toy what would be BEP and is at this rate 500 toys are sold daily what would be the profit?

Solution : Total cost $=$ C(x) $=$ TVC + TFC

TFC = 29 x 30 = ₹870.

TVC = 2.07x (x = No.of unit of toys)

$\therefore$ C(x) = 2.07x + 870 ....(1)

Total Revenue = Selling price X Quantity

R(x) = 6x ....(2)

For No. loss (BEP) C(x) = R(x)

$2.07x + 870 = 6x$

$870 = 6x - 2.07x$

$870 = 3.93x$

$$x = \frac{870}{393} = 221 \text{ Unit}$$

iii) BEP at reduced selling price = 6 – 0.50 = ₹5.50

$\therefore C(x) = R(x)$

$2.07x + 870 = 5.50x$

$870 = 5.50x - 2.07x$

$870 = 3.43x$ $\therefore x \frac{870}{3.43} = 253$ unit

Profit at Reduces price P(x) = R(x) – C(x)

$= 5.50x - [2.07x + 870]$

$= 5.50x - 2.07x - 870$

$= 3.43x - 870$

Profit at x = 500 toys sold

$= 3.43 (500) - 870$

$\therefore$Profit = ₹845

EXERCISE : 12.1

2 Mark questions :

1. A confectioner makes and sells Chocolates. He sells one box of Chocolates at ₹180. The cost of manufacturing is ₹60/box as variable cost and ₹2000 as Fixed Cost. Find (i) Revenue function (ii) the cost function

2 If the total fixed cost are ₹60,000 total variable cost is ₹80,000 and the sales are ₹1,20,000 Find the (i) cost function and the (ii) Slope (variable cost)

3. A publishing house finds that the production cost directly attributed to each book is ₹30 and the Fixed cost is ₹15,000. If each book can be sold for ₹45. Then find (i) cost function (ii) Revenue function.

4. For a manufacturer of dry cell the daily cost of production of 'x' cells is given by $(x) = 2.05x + 550$

   Find i) The fixed cost

   ii) The variable cost

5. The fixed cost and the variable cost of 'x' units of a product for a company are ₹40,000 and ₹80,000 respectively. If each unit is sold for ₹250. Find the (i) cost furniture (ii) Revenue (selling) function

6. A pen manufacturer determine that the production cost associated with each pen is ₹5 and the fixed cost is ₹7000. If each pen can be sold at ₹7/-. Find the profit function.

7. The daily cost of production 'C' for x unit of a manufactured product is given by $C(x) = 3.5x + 12000$. If the total cost of production is ₹82,000 Find the No.of unit produced.

8. If $R(x) = 0.25x$ and $C(x) = 0.16x + 360$ [x = No.of unit produced and sold). Find the break even quantity?

9. If $R(x) = 0.5x$ and $C(x) = 0.85x + 600$ can there be a break even point? Why?

10. The selling price of a product is ₹16/unit the variable cost is ₹8/unit, the fixed cost are ₹10,000. Find the Break even quantity.

ANSWER (1 or 2 Marks)

1. $R(x) = 180x$, $C(x) = 60x + 2000$

2. $C(x) = 0.6x + 60{,}000$, slope $= \dfrac{\text{TVC}}{\text{Sale}} = 0.6$

3. $C(x) = 30x + 15000$, $R(x) = 45x$

4. FC = ₹550
   VC = ₹2.05

5, $C(x) = 80x + 40{,}000$, $R(x) = 250x$

6. $P(x) = R(x) - C(x) = 2x - 700$

7. $x = 20{,}000$ unit

8. $x = 4000$ unit

9. No. BEP $\because$ $x = -750$ unit (negative output)

10. $x = 1250$ unit

II 3 and 5 Mark question:

1. A manufacturers sells his product at ₹8.35/unit, he is able to sell his entire production. His fixed cost is ₹2,116 and his variable cost/unit is ₹7.20. Find
   i) The level of production at which he can make a profit of ₹4,600.
   ii) The level of output at which he will incur a loss of ₹1150 and
   iii) The Break Even level of production.

2. M/s. Chandana and Co., Bombay finds that the production cost directly attributed to each book is ₹25 and the fixed cost are ₹10,000. If each book can be sold for ₹35 find (i) cost function, (ii) Revenue Function, (iii) Profit function (iv) Break Even Point

3. For the 1st year the Fixed cost for setting up a new electronic pocket calculators company is ₹3,00,000. The variable cost for producing a calculator is ₹70. The company expect the revenue from the sales of the calculators to be ₹270/-

i) Construct the revenue function.

ii) Construct the cost function.

iii) Find the Break Even output

iv) Find the number of Calculator produced for which the company will suffer loss.

4. The Co. decides to set up a small production plant for manufacturing electronic clock. The total cost/unit for initial set up (fixed cost) is ₹9 lakh. The additional cost (variable cost) for producing each clock are produced and sold.

i) Find the cost function C(x) for 'x' No.of clock.

ii) Find the Revenue function R(x) for total revenue from the sale of 'x' clock.

iii) Find the profit function p(x) for the total revenue from the sale of 'x' clock

iv) Determine the B.E.P.

v) What profit or loss the company incur during the first month when all the 1500 clocks are sold?

5. The daily cost of production C for 'x' unit of an assembly is given by n C(x) = 17.5x + 7000

i) If each unit is sold for ₹30. Then determine the minimum of unit that should be produced and sold to ensure no loss.

ii) If the selling price is reduced by ₹3/unit then what would be the BEP?

iii) If it is known that 500 unit can be sold daily. What price/unit should be charged to guarantee no loss?

6. A company sells 'x' tins of talcum powder /day at ₹10/tin the cost of manufacturing is ₹6/tin and the distributor charge ₹1/tin. Besides the daily overhead cost comes to ₹600/ (i) Determine the profit function.

ii) What is the profit if 500 tins are manufactured and sold/day.

iii) How do you interpret the situation is the company manufacturer and sells 100 tns/day.

iv) What is the B.E.P.?

7. Suppose the total daily cost in Rupees of producing 'x' chair is given by y_c=2.5x+ 300
   i) If each chair sells for ₹4/-. What is the BEP? Represent it graphically.
   ii) If the selling price is increased to ₹5/ chair. What is the new BEP
   iii) Find the Fixed and variable cost.

8. A manufacturer of a product sells his entire output (x). His total revenue R(x) = 7x and C(x) = 6x + 800 find the
   i) BEP
   ii) Write the Break Even chart
   iii) The BEP output of the total cost increased by 5%

9. A watch manufacturer produce 100 watches for a total cost of ₹20,000 and when the production is increased to 200 watches the total cost increased to ₹30,000. Assuming the cost and output to be in each related. Find the variable cost/unit, Fixed cost and the slope of the line y = a + bx (y = total cost) If the selling and sold what is the profit/watch. At a selling price of ₹200. What will be the BEP output?

10. A shoe manufacturer is planning production of new varieties of shoes. For the first year the fixed cost of setting up the new production line are ₹1.25 lakh Variable cost for producing each pair of shoes are ₹35. The sales department project that 1500 pair can be sold in the first year at the rate of ₹160/pair
    i) Find the cost function
    ii) For Revenue function
    iii) Find the profit function for the product for the sale of 'x' pair of shoes.
    iv) If 1500 pairs are actually sold then what profit or loss does the company incur?
    v) Determine the BEP

ANSWERS : 12.1 (3/5 marks)

1. 5840 unit, 540 unit, BEP Output 1840 unit

2. i) $25x + 1000$ ii) $35x$ iii) $10x - 10{,}000$ iv) 1000 unit

3. $R(x) = 210x$, BEP = 1500 unit $C(x) = 70x + 3{,}00{,}000$
   Less than $x = 1500$ unit the Co. will suffer loss

4. $C(x) = 300x + 9{,}00{,}000$
   $R(x) = 750x$, $p(x) = 450x - 90{,}00{,}000$, BEP $x = 2000$ clock,
   Profit = ₹2,25,000

5. $x = 5000$ unit, BEP $x = 737$, Selling price / unit = ₹31

6. $R(x) = 3x - 600$, ₹900, then company incur loss, BEP $x = 200$ tins

7. BEP at selling price ₹4 = 200 chair
   BEP at selling priced ₹5 =120 chair
   Fixed cost = ₹300, V.C. = ₹2.5/chair.

8. i) $x = 1800$ unit (at BEP)
   ii) $x = 840$ unit

9. V.C. = ₹100 F.C. = ₹10,000, Profit/watch ₹50/- BEP = 100 unit

10. $C(x) = 35x + 1{,}25{,}000$, $R(x) = 160x$, $P(x) = 125x -$ ₹1,25,000
    ₹62,500, BEP $x = 1000$ pair.

UNIT - III

TRIGONOMETRY

CHAPTER	NAME OF THE CHAPTER	TEACHING HOURS
13	ANGLES AND TRIGONOMETRIC RATIOS	06
14	STANDARD ANGLES AND ALLIED ANGLES	06
	TOTAL	12 Hours

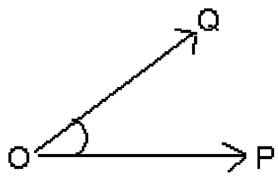
CHAPTER 13

ANGLES AND TRIGONOMETRIC RATIOS

13.1 Introduction: Trigonometry is a branch of Mathematics. The word trigonometry is derived from Greek words tri-gono & metron (tri-three, gono=angle, metron=measurement.) Thus the word trigonometry, literally means measurement of triangles. Primarily the subject deals with the relations between the sides, angles and area of a triangle. But trigonometry has a much wider scope and is used in geometrical and algebraic investigation in pure and applied mathematics. Hipparchus, a Greek astronomer, is known as a founder of trigonometry and others, who contributed to this branch include Euler, John-Bernolli, Fourier, and Gauss. The Foremost Indian Mathematicians who have worked on this branch of mathematics are Aryabhatta, Brahmagupta and Bhaskaracharya.

13.2 Measurement of angles:

Angle: When a line revolves about a point O from one position P to another position Q. We say an angle is formed and this angle is denoted by $\angle POQ$. The point O is called the vertex of the angle.



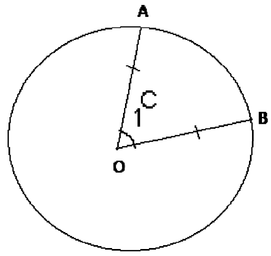
In General we have two common system to measure an angle, These are.

I. **Sexagesimal system :** In this system, we divide a right angle into 90 equal parts and call the measure of each part as "One degree" denoted by 1^0. Then we divide one degree in to 60 equal parts, each part is called as one minute, denoted by 1' and again we divide one minute into 60 equal parts, each part is called as one second, denoted by 1".

1 Right angle $= 90^0$, $1^0 = 60$ minute $= 60^0$, 1 minute $= 1' = 60$ sec $= 60^0$

II. **Circular system of Radian Measure:** In this system, we call the unit of measurement of an angle as a "Radian" and the measure of the angle as Radian measure or circular measure.

Definition: A radian is an angle subtended at the centre of the circle, by an arc, whose length is equal to radius of the circle. From the figure



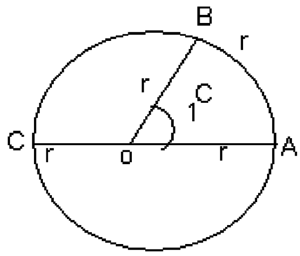
OA = O B = r =arc AB then

/AOB = 1^C (One Radian is denoted by 1^C)

Now, we shall show that a radian, constructed according to above definition is of constant magnitude.

1. Theorem: P.T. Radian is a constant angle

Proof : Consider a circle with centre O and radius r. Let AB be an arc of length r units. Then /AOB = 1^C. Produce AO to meet the circle at C. We know that he angles at the centre of a circle are proportional to the arcs on which they stand.



$\therefore \dfrac{\angle AOB}{\angle AOC} = \dfrac{arc\ AB}{arc\ ABC}$ (arc ABC = $\dfrac{1}{2}$ circumference of the circle)

$\therefore \dfrac{1^C}{180^0} = \dfrac{r}{\pi r}$ = (½ x 2π r = πr)

$1^C = \dfrac{180^0}{\pi} \Rightarrow \pi^C = 180°$

Thus 1^c is a constant angle and its value is given by

1^C = 57° 162 223 .

Note: From the above Theorem, we have a relation between degrees and radians. i.e. $\pi^C = 180^0$

$\Rightarrow 1^0 = \dfrac{\pi^C}{180}$

Note: To convert the angles (i) from degrees to radians, multiply the given angle by π/180 (ii) From radians to degrees, multiply the angle by $\dfrac{180}{\pi}$

1. Convert the following angles measured in degrees into radians.

1) $15^0 = 15\times\frac{\pi}{180} = \frac{\pi}{12}$ radians

2) $75^0 = 75\times\frac{\pi}{180} = \frac{5\pi}{12}$ radians

3) $270^0 = 270\times\frac{\pi}{180} = \frac{3\pi}{2}$ radians

4) $120^0 = 120\times\frac{\pi}{180} = 2\frac{\pi}{3}$ radians

Standard Angles:

Degrees	30^0	45^0	60^0	90^0	180^0	360^0
Radius	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	2π

2. Convert the following angles measured in radians to degrees.

- $\frac{2\pi^c}{3} = \frac{2\pi}{3} \times \frac{180}{\pi} = 120^0$
- $\frac{3\pi^c}{2} = \frac{3\pi}{2} \times \frac{180}{\pi} = 270^0$
- $\frac{3\pi^c}{5} = \frac{3\pi}{5} \times \frac{180}{\pi} = 108^0$
- $\frac{3^c}{4} = \frac{3}{4} \times \frac{180}{\pi} = \frac{3}{4} \times \frac{180}{22/7} = 42.96^0 \quad (\because \pi = \frac{22}{7})$
- $4.8^o = 4.8\times\frac{180}{\pi} = 274.91^0$

3. The angles of a triangle are in the ratio 2:3:4 Express them in radians & as well as in degrees.

Let the angles of the triangle be $2L^0$, $3L^0$ & $4L^0$

We know that sum of the 3 angles of a triangle = 180^0

$\therefore 2L^0 + 3L^0 + 4L^0 = 180^0$

$9L^0 = 180^0 \Rightarrow L = 20^0$

$\therefore$ The angles of the triangle are 40^0, 60^0, 80^0

The angles of the triangle in radians are $2\pi/9$, $\pi/3, 4\pi/9$.

4. The angles of a triangle are in A.P & the greatest is double the least. Express the angles in degrees & radians

Let the angles of the triangle be

$L - \beta$, L, $L + \beta$

We know that. Sum of the $L^{les} = 180^0$

$L - \beta + L + L + \beta = 180^0$

$3L = 180 \Rightarrow L = 60^0$

Given greatest angle is double to least

$L + \beta = 2(L - \beta) \quad \Rightarrow 2L - 2\beta = L + \beta$

$\Rightarrow L - 3\beta = 0$

$\Rightarrow 60 - 3\beta = 0 \Rightarrow \beta = 20^0$

$\therefore$ The angles of the triangle in degrees are $(60\text{-}20)^0$, 60^0, $(60 + 20)^0$

i.e 40^0, 60^0, 80^0.

In radians: $\frac{2\pi}{9}, \frac{\pi}{3}, \frac{4\pi}{9}$.

5. In a right angle triangle, the difference between the two acute angle is $\frac{\pi}{9}$ radians. Express the angles in degrees.

Let the 2 Angles = α & β

Given $\alpha - \beta = \frac{\pi}{9} = 20^0$ ----------------(1)

But in a right Angle triangle

$$90^0 + \alpha + \beta = 180^0$$

$$\alpha + \beta = 90^0 \text{ -----------------------(2)}$$

Solving (1) & (2) we get :

$$\alpha = 55^0, \quad \beta = 35^0$$

6. The angles of a quadrilateral are in the ratio 2:3:5:8. Find them in radians & as well as in degrees.

Let the angles be $2\alpha, 3\alpha, 5\alpha, 8\alpha$.

We know that the sum of the Angles of a Quadrilateral = 360^0

$$2\alpha + 3\alpha + 5\alpha + 8\alpha = 360^0$$

$$18\alpha = 360^0 \Rightarrow \alpha = 20^0$$

$\therefore$ The angles in degrees are 2×20^0, 3×20^0, 5×20^0, 8×20^0

$= 40^0, 60^0, 100^0, 160^0$

& angles in radians = $\frac{2\pi}{9}, \frac{\pi}{3}, \frac{5\pi}{9}, \frac{8\pi}{9}$ radians.

EXERCISE : 13.1

(1 MARK QUESTIONS)

1. Express the following in radian measure.

a) 25^0 b) 95^0 c) 105^0 d) 135^0

e) 210^0 f) $22\frac{1}{2}^0$ g) 315^0 h) 720^0

i) 18^0 j) 36^0 k) 144^0 l) $67\frac{1}{2}^0$

2. Express the following in Sexagesimal measure. (degrees)

a) $\frac{7\pi}{3}$, b) $\frac{17\pi}{9}$, c) $\frac{7\pi}{8}$ d) $\frac{\pi}{24}$

e) $\frac{9\pi}{5}$ f) $\frac{2\pi}{12}$ g) $\frac{3\pi}{5}$ h) 2π

III. a) The angles of a triangle are in the ratio 3:4:5. Find them in radians and in degrees.

b) The angles of a triangle are in the ratio

i) 1:3:5 ii) 2:3:5 iii) 4:5:6 Find them in degrees and radians.

c) The difference between the acute angles of a right angled triangle is $\frac{2\pi}{5}$. Express the angles in degrees.

d) The difference of two angles is 45^0 and their sum is 90^0. Find the angles in degrees and in radians .

e) The angles of a triangle are in A.P. The least being 36^0. Find the angles in degrees and in radians

f) The angles of a quadrilateral are in A.P and the greatest of which is double the least. Express the angles in degrees and in radians .

g) The greatest angle of a cyclic quadrilateral is double the least and the difference of the other two angles is 30^0. Find the angles in radians.

h) The angles of a quadrilateral are in A.P such that the greatest is double the least. Express the least angle in radians.

i) The angles of a triangle are in A.P & the ratio of number of degrees in the least to the number of radians in the greatest is 60: π. Find the angles of the triangle in radians.

j) The angles of a triangle are in A.P & the greatest angle is 84^0. Find all the three angles in radians.

k) The angles of a triangle are in A.P & the greatest is 5 times The least. Find the angles in radians.

ANSWERS : 13.1

I. a) $\frac{5\pi}{36}$ b) $\frac{19\pi}{36}$ c) $\frac{7\pi}{12}$ d) $\frac{3\pi}{4}$ e) $\frac{7\pi}{6}$ f) $\frac{\pi}{8}$

g) $\frac{7\pi}{4}$ h) 4π i) $\frac{\pi}{10}$ j) $\frac{\pi}{5}$ k) $\frac{4\pi}{5}$ l) $\frac{3\pi}{8}$

II. a) 420^0 b) 340^0 c) 157.5^0 d) 7.5^0
e) 324^0 f) 30^0 g) 135^0 h) 360^0

III. a) 45^0, 60^0, 75^0, $\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{12}$

b) (i) 20^0, 60^0, 100^0, $\frac{\pi}{9}, \frac{\pi}{3}, \frac{5\pi}{9}$

(ii) 36^0, 54^0, 90^0, $\frac{\pi}{5}, \frac{3\pi}{10}, \frac{\pi}{2}$

(iii) 48^0, 60^0, 72^0, $\frac{4\pi}{15}, \frac{\pi}{3}, \frac{2\pi}{5}$

c) $9^0, 81^0$,

d) $22\frac{1}{2}, 67\frac{1}{2}^0$, $\frac{\pi}{8}, \frac{3\pi}{8}$

e) 36^0, 60^0, 84^0, $\frac{\pi}{5}, \frac{\pi}{3}, \frac{7\pi}{15}$

f) 60^0, 80^0, 100^0, 120^0, $\frac{\pi}{3}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{2\pi}{3}$

g) $\frac{2\pi}{3}, \frac{7\pi}{12}, \frac{5\pi}{12}, \frac{\pi}{3}$

h) $60^0 = \frac{\pi}{3}$ = least angle

i) $(30^0, 60^0, 90^0)$ $\left(\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\right)$

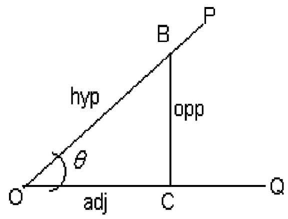
j) $\left(\frac{\pi}{5}, \frac{\pi}{3}, \frac{7\pi}{15}\right)$

k) $\left(\frac{\pi}{9}, \frac{\pi}{3}, \frac{5\pi}{9}\right)$

13.3 TRIGONOMETRIC RATIOS OF ACUTE ANGLE:

INTRODUCTION: In this chapter we shall introduce six trigonometric ratios for an acute angle and find relations between them.

Trigonometic Ratios:



Let $\angle BOC = \theta$, be any acute angle, B be any point on OC, one of the boundary lines and BC is perpendicular to OC. Thus we have a right angled triangle BOC.

With respect to the angle θ, the side BC is called the opposite side, OC is called adjacent side and OB is the hypotenuse of the triangle. With respect to the angle θ, we define the following six ratios called trignomotric ratios.

Let us now define the 6 trigonometric ratios as follows.

1. Sine of θ written as sin θ $= \dfrac{BC}{OB} = \dfrac{\textit{Opposite side}}{\textit{Hypotenuse side}}$

2. Cosine of θ written as cos θ $= \dfrac{OC}{OB} = \dfrac{\textit{Adjacent side}}{\textit{Hypotenuse side}}$

3. tangent of θ written as tanθ $= \dfrac{BC}{OC} = \dfrac{\textit{Opposite side}}{\textit{Adjacent side}}$

4. Cosecant of θ written as cosecθ $= \dfrac{OB}{BC} = \dfrac{\textit{Hypotenuse side}}{\textit{Opposite side}}$

5. Secant of θ written as secθ $= \dfrac{OB}{OC} = \dfrac{\textit{Hypotenuse side}}{\textit{Adjacent .side}}$

6. Cotangent of θ written as cotθ $= \dfrac{OC}{BC} = \dfrac{\textit{Adjacent. side}}{\textit{Opposite side}}$

The above definitions of trigonometric ratios remains unaltered as long as the angle remains the same, i.e. trigonometric ratios depends on the angle but not on the lengths of the sides of the right angled triangle.

Note: We know that in every right angled triangle the hypotenuse is the greatest side, thus it follows from the definition of trigonometric ratios that those ratios which have the htpotenuse in the denominator can never be greater than unity, while those which have hypotenuse in the numerator can never be less than unity. Further those ratios which do not involve the hypotenuse may have any numerical value. Thus we have the following results.

- The sin θ and cos θ of an angle can never be greater than 1.
- The cosec θ and sec θ of an angle can never be less than 1.
- The tan θ and cot θ of an angle may have any numerical value.

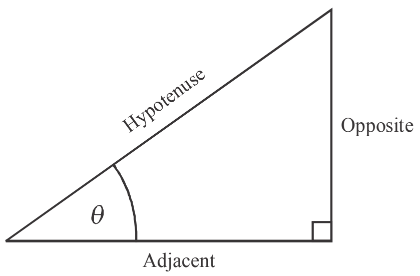
13.4 Relation between the trigonometric ratios.

In the right angled triangle as shown, consider:

Hypotenuse = r
Adjacent side = x
Opposite side = y

1. Prove that sin θ . cosec θ = 1

From a right angled triangle shown below we have:



$\text{Sin } \theta = \frac{y}{r} \text{cosec } \theta = \frac{r}{y}$

$\text{L.H.S} = \sin \theta \,.\, \text{cosec } \theta = \frac{y}{r} . \frac{r}{y} = 1$

Therefore, sin θ . cosec θ = 1

$\Rightarrow \text{cosec}\theta = \frac{1}{\sin\theta}$ or $\sin \theta = \frac{1}{\cos ec\theta}$

Similarly we can prove the following results:

2. cos θ . sec θ = 1
3. tan θ . cot θ = 1
4. Prove that $\tan \theta = \frac{\sin \theta}{\cos \theta}$

$> \text{R.H.S} = \frac{\sin \theta}{\cos \theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{x} = \tan \theta = \text{L.H.S}$

Therefore $\tan \theta = \frac{\sin \theta}{\cos \theta}$.

5. Similarly we can prove $\cot\theta = \dfrac{\cos\ \theta}{\sin\ \theta}$

Note : We write different powers of trigonometric ratios as
$(\sin\theta)^2 = \sin^2\theta$,
$(\cos\theta)^3 = \cos^3\theta$ and so on.

Basic Identities:

6. Prove that $\sin^2\theta + \cos^2\theta = 1$.

$$\text{L.H.S} = \sin^2\theta + \cos^2\theta = \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = \frac{x^2+y^2}{r^2} = \frac{r^2}{r^2} = 1 = \text{R.H.S.}$$

Therefore, $\sin^2\theta + \cos^2\theta = 1$.
$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$ or $\sin^2\theta = 1 - \cos^2\theta$.

7. Prove that $1 + \tan^2\theta = \sec^2\theta$.

$$\text{L.H.S} = 1 + \tan^2\theta = 1 + \left(\frac{y}{x}\right)^2 = \frac{x^2+y^2}{x^2} = \frac{r^2}{x^2} = \sec^2\theta = \text{R.H.S.}$$

Therefore $1 + \tan^2\theta = \sec^2\theta$.
$\Rightarrow \tan^2\theta = \sec^2\theta - 1$ or $\sec^2\theta - \tan^2\theta = 1$.

8. Prove that $1 + \cot^2\theta = \text{cosec}^2\theta$.

$$\text{L.H.S} = 1 + \cot^2\theta = 1 + \left(\frac{x}{y}\right)^2 = \frac{x^2+y^2}{y^2} = \frac{r^2}{y^2} = \text{cosec}^2\theta = \text{R.H.S.}$$

Therefore $1 + \cot^2\theta = \text{cosec}^2\theta$.
$\Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$ or $\text{cosec}^2\theta - \cot^2\theta = 1$.

Worked Examples:

Prove the following :

I. 1. $(\sin^2\theta \,.\, \cot^2\theta) + (\cos^2\theta \,.\, \tan^2\theta) = 1$

L.H.S $= (\sin^2\theta \,.\, \cot^2\theta) + (\cos^2\theta \,.\, \tan^2\theta)$

$$= \sin^2\theta \,.\, \frac{\cos^2\theta}{\sin^2\theta} + \cos^2\theta \,.\, \frac{\sin^2\theta}{\cos^2\theta} = \sin^2\theta + \cos^2\theta = 1 = \text{R.H.S.}$$

2. $(1 - \sin^2\theta) . \sec^2\theta = 1$

L.H.S $= (1 - \sin^2\theta) . \sec^2\theta$

$= \cos^2\theta . \sec^2\theta = 1 =$ R.H.S.

3. $(1 - \sin^2\theta) . (1 + \cot^2\theta) = \cot^2\theta$

L.H.S $= (1 - \sin^2\theta) . (1 + \cot^2\theta)$

$$= \cos^2\theta . \text{cosec}^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta = \text{R.H.S.}$$

4. $\cos^4 A - \sin^4 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

Consider $\cos^4 A - \sin^4 A$

$= (\cos^2 A)^2 - (\sin^2 A)^2$

$= (\cos^2 A - \sin^2 A) . (\cos^2 A + \sin^2 A)$

$= (\cos^2 A - \sin^2 A) . (1)$

$= (1 - \sin^2 A - \sin^2 A)$

$= (1 - 2\sin^2 A)$ ....(1)

Again consider $\cos^4 A - \sin^4 A$

$= (\cos^2 A)^2 - (\sin^2 A)^2$

$= (\cos^2 A - \sin^2 A) . (\cos^2 A + \sin^2 A)$

$= (\cos^2 A - \sin^2 A) . (1)$

$= \cos^2 A - (1 - \cos^2 A)$

$= \cos^2 A - 1 + \cos^2 A$

$= 2\cos^2 A - 1$ ....(2)

5. $\sec^2 A + \text{cosec}^2 A = \sec^2 A . \text{cosec}^2 A$

L.H.S $= \sec^2 A + \text{cosec}^2 A$

$$= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} = \frac{\sin^2 A + \cos^2 A}{\cos^2 A . \sin^2 A} = \frac{1}{\cos^2 A . \sin^2 A}$$

$= \sec^2 A . \text{cosec}^2 A =$ R.H.S.

6. $\dfrac{1}{\cos^2 A} - \dfrac{1}{\cot^2 A} = 1$.

L.H.S. $= \dfrac{1}{\cos^2 A} - \dfrac{1}{\cot^2 A} = \text{Sec}^2 A - \tan^2 A = 1 =$ R.H.S.

7. Sec.A $\sqrt{1-\cos^2 A}$ = tan A

L.H.S. = sec.A . $\sqrt{\sin^2 A} = \frac{1}{\cos A}.\sin A = \frac{\sin A}{\cos A}$ = tan A = R.H.S.

8. $\tan^2 A\ (1 - \sin^2 A) = \sin^2 A$.

L.H.S. $= \frac{\sin^2 A}{\cos^2 A}(1 - \sin^2 A)$

$= \frac{\sin^2 A}{\cos^2 A} . \cos^2 A = \sin^2 A$

9. cosA. cosec. A. $\sqrt{\sec^2 A - 1}$ = 1

L.H.S. = cos A. $\frac{1}{\sin A}$. tanA = cot A.tan A = 1 = R.H.S.

10. $(\text{cosec}^2 A\ \tan^2 A) - 1 = \tan^2 A \Rightarrow (\text{cosec}^2 A.\ \tan^2 A - 1) = \tan^2 A$ ‘

L.H.S. = $(\text{cosec}^2 A\ .\ \tan^2 A) - 1 = \left(\text{cosec}^2 A.\frac{\sin^2 A}{\cos^2 A}\right)-1 = \sec^2 A - 1$

$= \tan^2 A$ = R.H.S.

II. 1) Express all the trigonometric ratios of the angle θ in term of

(i) sinθ (ii) tanθ

i. 1) sinθ = sinθ

2) $\cos\theta = \sqrt{1-\sin^2\theta}$

3) $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$

4) $\text{cosec}\,\theta = \frac{1}{\sin\theta}$

5) $\text{Sec}\,\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{1-\sin^2\theta}}$

6) $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta}$

ii. 1) $\sin\theta = \cos\theta \,.\, \tan\theta = \dfrac{\tan\theta}{\sec\theta} = \dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

2) $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{\sqrt{1+\tan^2\theta}}$

3) $\tan\theta = \tan\theta$

4) $\text{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{\sqrt{1+\tan^2\theta}}{\tan\theta}$

5) $\sec\theta = \sqrt{1+\tan^2\theta}$

6) $\cot\theta = \dfrac{1}{\tan\theta}$

III. 1) If x = a $\sec\theta$; y = $b\tan\theta$. P.T. $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

Consider $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = \dfrac{(a\sec\theta)^2}{a^2} - \dfrac{(b\tan\theta)}{b^2}$

$$= \frac{a^2\sec^2\theta}{a^2} - \frac{b^2\tan^2\theta}{b^2}$$

$= \sec^2\theta - \tan^2\theta = 1$ = R.H.S.

2) P.T $\sqrt{\dfrac{1-\sin A}{1+\sin A}}$ = Sec A - tan A

L.H.S. = $\sqrt{\dfrac{1-\sin A}{1+\sin A}} = \sqrt{\dfrac{(1-\sin A)(1-SinA)}{(1+\sin A)(1-\sin A)}}$

$$= \sqrt{\frac{(1-\sin A)^2}{(1-\sin^2 A)}} \quad = \quad \sqrt{\frac{(1-\sin A)^2}{\cos^2 A}}$$

$$= \quad \frac{1}{\cos A} - \frac{\sin A}{\cos A}$$

$=$ SecA - tan A = R.H.S.

3. P.T. $\sqrt{\frac{SecA + \tan A}{\sec A - \tan A}} = \frac{1+\sin A}{\cos A}$

L.H.S. $= \sqrt{\frac{\sec A + \tan A}{\sec A - \tan A} \times \frac{\sec A + \tan A}{\sec A + \tan A}}$

$$= \sqrt{\frac{(\sec A + \tan A)^2}{\sec^2 A - \tan^2 A}}$$

$$= \sqrt{\frac{(SecA + \tan A)^2}{1}} \quad = \quad \text{SecA + tanA}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \frac{1+\sin A}{\cos A} \quad = \quad \text{R.H.S.}$$

4. P.T. $\frac{SinA + SinB}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A - \sin B} = 0$

L.H.S. $+ = \frac{(SinA + \sin B)(\sin A - \sin B) + (\cos A - \cos B)(\cos + \cos B)}{(\cos A + \cos B)(\sin A - \sin B)}$

$$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A - \sin B)} = \frac{1-1}{(\cos A + \cos B)(\sin A - \sin B)} = 0 = \text{R.H.S.}$$

5. If $\cos\theta = \frac{a}{b}$ S.T. $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 - b^2}$

$\tan\theta = \frac{a}{b} = \frac{Opp}{adj}$, Hypotenuse = $\sqrt{a^2+b^2}$,

$\text{Sin }\theta = \frac{a}{\sqrt{a^2+b^2}}$, $\cos\theta = \frac{b}{\sqrt{a^2+b^2}}$

$$\text{L.H.S.} = \frac{a.\sin\theta - b.\cos\theta}{a.\sin\theta + b.\cos\theta} = \frac{\frac{a.a.}{\sqrt{a^2+b^2}} - \frac{b.b.}{a^2+b^2}}{\frac{a.a.}{\sqrt{a^2+b^2}} + \frac{b.b.}{a^2+b^2}}$$

$$= \frac{a^2-b^2}{\sqrt{a^2+b^2}} / \frac{a^2+b^2}{\sqrt{a^2+b^2}} = \frac{a^2-b^2}{a^2+b^2} = \text{R.H.S.}$$

6. If sin+ x $\sin^2 x = 1$ S.T. $\cos^2 x + \cos^4 x = 1$

Given $\sin x + \sin^2 x = 1$

$\sin x = 1 - \sin^2 x = \cos^2 x$

$\cos^2 x = \sin x \Rightarrow \cos^4 x = \sin^2 x$

Consider $\cos^2 x + \cos^4 x$

$\cos^2 x + \sin^2 x = 1 =$ R.H.S.

7. P.T. $\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

$$\text{L.H.S.} = \frac{1-\cos\theta}{\sin\theta} = \frac{1-\cos\theta}{\sin\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$$

$$= \frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)} = \frac{\sin^2\theta}{\sin\theta(1+\cos\theta)} = \frac{\sin\theta}{1+\cos\theta} = \text{R.H.S.}$$

8. P.T. $(1 + \sin\theta + \cos\theta)^2 = 2(1 + \sin\theta)(1 + \cos\theta)$

L.H.S. $= \{(1 + \sin\theta) + (\cos\theta)\}^2$

$= (1+\sin\theta)^2 + \cos^2\theta + 2(1 + \sin\theta)\cos\theta$

$= 1 + \sin^2\theta + 2\sin\theta + \cos^2\theta + 2\cos\theta(1 + \sin\theta)$

$= 2 + 2\sin\theta + 2\cos\theta(1 + \sin\theta)$

$= 2(1+\sin\theta) + 2\cos\theta(1 + \sin\theta)$

$= (1 + \sin\theta)(2 + 2\cos\theta)$

$= 2(1 + \sin\theta)(1 + \cos\theta)$

9. P.T. $\dfrac{\cos\theta}{1-\tan\theta} + \dfrac{\sin\theta}{1-\cot\theta} = \cos\theta + \sin\theta$

$$\text{L.H.S.} = \frac{\cos\theta}{1-\dfrac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{1-\dfrac{\cos\theta}{\sin\theta}}$$

$$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} + \frac{\sin^2\theta}{\sin\theta-\cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} + \frac{\sin^2\theta}{\sin\theta-\cos\theta}$$

$$= \frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$$

$= \cos\theta + \sin\theta$ = R.H.S.

10. P.T. $\dfrac{\tan A}{1-\cot A} + \dfrac{\cot A}{1-\tan A} = 1 + \text{Sec A . Cosec A}$

$$\text{LHS} = \frac{\dfrac{\sin A}{\cos A}}{1-\dfrac{\cos A}{\sin A}} + \frac{\dfrac{\cos A}{\sin A}}{1-\dfrac{\sin A}{\cos A}}$$

$$= \frac{\frac{\sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}} + \frac{\frac{\cos A}{\sin A}}{\frac{\cos A - \sin A}{\cos A}}$$

$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A(\cos A - \sin A)}$$

$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} - \frac{\cos^2 A}{\sin A(\sin A - \cos A)}$$

$$= \frac{\sin^3 A - \cos^3 A}{\cos A.\sin A(\sin A - \cos A)} \quad = \frac{(\sin A - \cos A)(1 + \sin A.\cos A)}{\cos A.\sin A(\sin A - \cos A)}$$

$$= \frac{1 + \sin A.\cos A}{\cos A.\sin A} = \frac{1}{\cos A.\sin A} + 1$$

= sec A . cosec A + 1 = R.H.S.

11. If tan A + sin A = m and tan A – sin A = n.

S.T. $m^2 - n^2 = 4\sqrt{mn}$

Consider m.n = (tanA + sinA) (tan A – sinA)

= $\tan^2 A - \sin^2 A$

$$mn = \frac{\sin^2 A}{\cos^2 A} - \sin^2 A = \sin^2 A\left(\frac{1-\cos^2 A}{\cos^2 A}\right) = \sin^2 A.\frac{\sin^2 A}{\cos^2 A}$$

mn = $\sin^2 A. \tan^2 A$

$\therefore$ sinA . tan A = $\sqrt{mn}$ ....(1)

Again consider $m^2 - n^2 = (\tan A + \sin A)^2 - (\tan A - \sin A)^2$

= $\tan^2 A + \sin^2 A + 2 . \tan A . \sin A - \tan^2 A - \sin^2 A + 2 \tan A . \sin A$

= 4 tanA. Sin A

$m^2 - n^2 = 4.\sqrt{mn}$ from equation (1)

12. If $\cot\theta = \frac{5}{2}$ and θ is acute. Then P.T. $\frac{3\cos\theta + 2\sin\theta}{3\cos\theta - 4\sin\theta} = \frac{19}{7}$

Given $\cot\theta = \frac{5}{2} = \frac{adj}{opp}$, Hypotenuse $= \sqrt{29}$

$\cos\theta = \frac{5}{\sqrt{29}}$, $\sin\theta = \frac{2}{\sqrt{29}}$

LHS $= \frac{3\cos\theta + 2\sin\theta}{3\cos\theta - 4\sin\theta} = 3.\frac{\frac{5}{\sqrt{29}} + 2\frac{2}{\sqrt{29}}}{3\frac{5}{\sqrt{29}} - 4.\frac{2}{\sqrt{29}}} = \frac{\frac{15+4}{\sqrt{29}}}{\frac{15-8}{\sqrt{29}}} = \frac{19}{7} =$ RHS

13. If $\tan\theta + \sec\theta = \frac{5}{2}$ then find $\sin\theta$.

Given : $\tan\theta + \sec\theta = \frac{5}{2}$

$\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = \frac{5}{2} \Rightarrow \frac{\sin\theta + 1}{\cos\theta} = \frac{5}{2} \Rightarrow$

$2(\sin\theta + 1) = 5(\cos\theta)$

Squaring on both sides, we get :

$4(\sin^2\theta + 1 + 2\sin\theta) = 25\cos^2\theta$

$4(\sin^2\theta + 1 + 2\sin\theta) = 25(1 - \sin^2\theta)$

$4\sin^2\theta + 4 + 8\sin\theta = 25 - 25\sin^2\theta$

$29\sin^2\theta + 8\sin\theta - 21 = 0$

$29\sin^2\theta + 29\sin\theta - 21\sin\theta - 21 = 0$

$29\sin^2\theta(\sin\theta + 1) - 21(\sin\theta + 1)$

$(\sin\theta + 1)(29\sin\theta - 21) = 0$

$\therefore \sin\theta = 1$, $\sin\theta = \frac{21}{29}$

14. P.T. $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

L.H.S. $= (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$

$$= \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$

$$= \left(\frac{\sin A+\cos A-1}{\sin A}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$$

$$= \frac{(Cos A+\sin A)^2-1}{\sin A\cos A}$$

$$= \frac{\cos^2 A+\sin^2 A+2\cos A\sin A-1}{\sin A.\cos A}$$

$$= \frac{1+2\cos A.\sin A-1}{\sin A.\cos A}$$

$$= \frac{2\cos A.\sin A}{\sin A.\cos A} = 2 = R.H.S.$$

15. It $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$ then P.T. $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Given $\sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$\sin\theta = \cos\theta(\sqrt{2}-1)$

$\Rightarrow \quad \cos\theta = \dfrac{\sin\theta}{\sqrt{2}-1}$

$\Rightarrow \quad \cos\theta = (\sqrt{2}+1)\sin\theta \qquad \ldots \quad \ldots \quad \ldots \qquad (1)$

Consider L.H.S. $= \cos\theta - \sin\theta = (\sqrt{2}+1)\sin\theta - \sin\theta$

$= \sqrt{2}\sin\theta + \sin\theta - \sin\theta$

$= \sqrt{2}\sin\theta =$ R.H.S.

$\therefore \quad \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

16. P.T. $\dfrac{1}{1+\sin^2\theta} + \dfrac{1}{1+\cos^2\theta} + \dfrac{1}{1+\sec^2\theta} + \dfrac{1}{1+\text{cosec}^2\theta}$

$$\text{L.H.S.} = \frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\dfrac{1}{\cos^2\theta}}+\frac{1}{1+\dfrac{1}{\sin^2\theta}}$$

$$= \frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{\cos^2\theta}{\cos^2+1}+\frac{\sin^2\theta}{\sin^2\theta+1}$$

$$= \frac{1+\sin^2\theta}{\sin^2\theta+1} + \frac{1+\cos^2\theta}{\cos^2\theta+1}$$

$$= 1 + 1 = 2 = \text{R.H.S.}$$

17. P.T. $\dfrac{\sec A+\tan A+1}{\sec A-\tan A+1} = \text{SecA} + \text{tanA}$

$$\text{L.H.S.} = \frac{(\sec A+\tan A)+1}{\sec A-\tan A+1} = \frac{\sec A+\tan A+(\sec^2 A-\tan^2 A)}{\sec A-\tan A+1}$$

$$= \frac{(\sec A+\tan A)(1+\sec A-\tan A)}{(\sec A-\tan A+1)} = \text{SecA} + \text{tanA} = \text{R.H.S.}$$

18. If x = a sin α cosβ, y = b sin α sin β , z = cos α

S.T. $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$

$$\text{L.H.S.} = \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = \sin^2\alpha.\cos^2\beta + \sin^2\alpha.\ \text{Sin}^2\beta + \cos^2\alpha$$

$$= \sin^2\alpha\,(\cos^2\beta + \sin^2\beta) + \cos^2\alpha$$

$$= \sin^2\alpha + \cos^2\alpha = 1 = \text{R.H.S.}$$

19., If sin A + cos A = $\sqrt{2}$ sinA . S.T. sin A – cos A = $\sqrt{2}$ cos A

Given : sin A + cos A = $\sqrt{2}$ sinA

$\therefore$ cosA = $(\sqrt{2}-1)\sin A \Rightarrow \sin A = \frac{\cos A}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} \Rightarrow \cos A(\sqrt{2}+1)$

$\therefore$ sinA = $\sqrt{2}$ cosA + cosA

$\Rightarrow$ sinA – cos.A = $\sqrt{2}$ cosA

20. Eliminate θ between x = a $\cos^4\theta$ & y = a $\sin^4\theta$

Given x = a $\cos^4$ & y = a $\sin^4\theta$

$\therefore \sqrt{x} = \sqrt{a}\cos^2\theta \quad , \sqrt{y} = \sqrt{a}.\sin^2\theta$

On Adding we get

$\Rightarrow \sqrt{x} + \sqrt{y} = \sqrt{a}$

EXERCISE : 13.2

1 If sin θ = $\frac{3}{5}$ & θ is acute, then find all the other Trigonometric functions of θ.

2. If tanθ = $\frac{4}{3}$ & θ is acute, then find cosec θ and sec θ .

3. If sec A = $\frac{2}{\sqrt{3}}$ & A is acute, then find sin A & tan A .

4. If sec A = $\frac{13}{12}$ & A is an acute angle, then find sin A + cos A.

5. If cos A = $\frac{12}{13}$ & A is acute, find other five trigonometric ratios.

6. If cosA = $\frac{4}{5}$ & A is acute, find other five trigonometric ratios.

7. If x = a cos θ , y = a sinθ . S.T. $x^2+y^2 = a^2$
8. If x = a $\cos^3\theta$, y = a $\sin^3\theta$. S.T. $x^{2/3} + y^{2/3} = a^{2/3}$
9. If x = a cosθ, y = b sinθ P.T. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

II. Prove the following (SHORT ANSWER) - One Mark.

1. cos A. tan A = sin A
2. sin A. Sec A = tan A
3. $(1 - \cos^2 A) \operatorname{cosec}^2 A = 1$
4. $\tan^2 A (1 - \sin^2 A) = 1$
5 $(\sec^2 A - 1) . \cot^2 A = 1$
6. $\cos^2 A . \tan^2 A + \cos^2 A = 1$
7. cos A . cosec A = cot A
8. tan A. cosec A. cos A = 1
9. $(1 - \sin^2 A) . \sec^2 A = 1$
10. $\cos^2 A (1 + \tan^2 A) = 1$
11. $(1-\sin^2 A) . (1 + \tan^2 A) = 1$
12. $\sin^2\theta . \sec^2\theta = \sec^2\theta - 1$

Prove the following - Two marks :

13. P.T. $(1 + \tan^2\theta) . (1 - \sin^2\theta) = 1$
14. P.T. sec A – cos A = tan A. sin A
15. P.T. $(\cos A + \sin A)^2 + (\cos A - \sin A)^2 = 2$
16. P.T. (cosec A – sin A) = cot A. cos A
17. P.T. $\tan^2 A + \sec^2 B = \sec^2 A + \tan^2 B$
18. P.T. $(1 + \cot A)^2 + (1 - \cot A)^2 = 2 \operatorname{cosec}^2 A$
19. P.T. $\frac{1}{\sec A + \tan A} = \sec A - \tan A$
20. P.T. tan A + cot A = sec A. cosec A.
21. Express all the trigonometric ratios in terms of
    1) sin θ 2) cos θ 3) tan θ 4) sec θ 5) cosec θ 6) cot θ

III. Prove the following (Essay type questions) - Three marks :

1. P.T. (1 + tan A – sec A) (1 + cot A + cosec A) = 2
2. P.T. $\frac{\cot A + \tan B}{\cot B + \tan A} = \frac{\cot A}{\cot B}$
3. P.T.(1 + cosec A) . (1 + cos A) . (1 – cosec A) . (1 – sec A) = cosA

4. P.T. $(1 + \cot A - \operatorname{cosec} A) . (1 + \tan A + \sec A) = 2$
5. P.T. $(\sec^4 A - \sec^2 A) = \tan^2 A + \tan^4 A$
6. P.T. $\sin^3\theta - \cos^3\theta = (\sin\theta + \cos\theta)(1 - \sin\theta . \cos\theta)$
7. P.T. $(1 - \tan A)^2 + (1 - \cot A)^2 = (\sec A - \operatorname{cosec} A)^2$
8. P.T. $\sin^4 A - \cos^4 A = 1 - 2\cos^2 A$
9. P.T. $\cos^4 A - \sin^4 A = 1 - 2\sin^2 A$
10. P.T. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$
11. P.T. $(1 - \sin A + \cos A)^2 = 2(1 - \sin A)(1 + \cos A)$
12. P.T. $\sec^6 A - \tan^6 A = \; = 1 + 3\tan^2 A . \sec^2 A$
13. P.T. $\cot^2 A + \cot^4 A = \operatorname{cosec}^4 A - \operatorname{cosec}^2 A$
14. P.T. $\sqrt{\sec^2 A + \operatorname{cosec}^2 A} = \tan A + \cot A$
15. P.T. $\sqrt{\dfrac{1+\cos A}{1-\cos A}} = \operatorname{cosec} A + \cot A$
16. P.T. $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$
17. P.T. $\sqrt{\dfrac{\sec\theta+1}{\sec\theta-1}} = \cot\theta + \operatorname{cosec}\theta$
18. P.T. $\dfrac{1}{1+\sin A} + \dfrac{1}{1-\sin A} = 2\sec^2 A$
19. P.T. $\dfrac{1}{1+\sin^2 A} + \dfrac{1}{1+\operatorname{cosec}^2 A} = 1$
20. P.T. $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$
21. P.T. $\dfrac{1+\sin A}{1-\sin A} - \dfrac{1-\sin A}{1+\sin A} = 4\sec A . \tan A$
22. P.T. $\dfrac{1}{1+\cos A} + \dfrac{1}{1-\cos A} = 2\operatorname{cosec}^2 A$

23. If cot A = $\frac{5}{12}$ & θ is acute, S.T. 2 cosecθ - 4 sec θ = $\frac{247}{30}$

24. If tan θ = $\frac{5}{12}$ & θ is acute, S.T. 3 sin θ - 4 cosθ = $\frac{-33}{13}$

IV. Essay type questions - Five marks :

25. P.T. $\frac{\cos^3 A+\sin^3 A}{\cos A+\sin A} + \frac{\cos^3 A-\sin^3 A}{\cos A-\sin A} = 2$

26. If x = r cos A cos B, y = r cos A sin B and z = r sin A
Then P.T. $x^2 + y^2 + z^2 = r^2$

27. If x = a r sin A cos B, y = b r sin A sin B, and z = c r cos A
Then P.T. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = r^2$

28. If tan A = $\frac{p}{q}$. Then P.T. $\frac{p\sin A-q\cos A}{p\sin A+q\cos A}=\frac{p^2-q^2}{p^2+q^2}$

29. If tan A + sin A = m & tan A – sin A = n , then P.T. $(m^2 - n^2)^2 = 16$ mn .

30. If cot θ = $\frac{5}{2}$ & θ is acute , S.T. $\frac{5\cos\theta+2\sin\theta}{5\cos\theta+2\sin\theta}=\frac{29}{21}$

31. If sin θ = $\frac{a-b}{a+b}$ S.T. tan θ + sec θ = $\sqrt{\frac{a}{b}}$

32. If cos θ = $\frac{4}{5}$ and θ is acute S.T. $\frac{2\tan\theta}{1-\tan^2\theta}=\frac{24}{7}$

33. P.T. $\frac{\tan A-\sin A}{\sin^2 A}=\frac{\tan A}{1+\cos A}$

34. P.T. $\frac{\tan A}{\sec .A-1}+\frac{\tan A}{\sec A+1}$ = 2 cosecA

35. If $\sin x + \sin^2 x = 1$. Then P.T. $\cos^2 + \cos^4 = 1$

36. If $= a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$, S.T. $^2 + y^2 = a^2 + b^2$

ANSWERS : 13.2

1. $\cos \theta = \frac{4}{5}$, $\sec \theta = \frac{5}{4}$, $\tan \theta = \frac{3}{4}$, $\cot \theta = \frac{4}{3}$, $\operatorname{cosec} \theta = \frac{3}{5}$.

2. $\operatorname{cosec} \theta = \frac{5}{4}$, $\sec \theta = \frac{5}{3}$

3. $\sin A = \frac{1}{2}$, $\tan A = \frac{1}{\sqrt{3}}$

4. $\sin A + \cos A = \frac{17}{13}$

5. $\sin A = \frac{5}{13}$, $\tan A = \frac{5}{12}$, $\operatorname{cosec} A = \frac{13}{5}$, $\sec A = \frac{13}{12}$, $\cot A = \frac{12}{5}$.

6. $\sin A = \frac{3}{5}$, $\tan A = \frac{3}{4}$, $\operatorname{cosec} A = \frac{5}{3}$, $\sec A = \frac{5}{4}$, $\cot A = \frac{4}{3}$.

Chapter 14

STANDARD ANGLES AND ALLIED ANGLES

14.1 Trignometric ratios of standard angles

The following table is helpful in remembering the trigonometric ratios of the angles 0^o, 30^o. 45^o, 60^o and 90^o, called standard angles.

	0^o	30^o	45^o	60^o	90^o
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta = \frac{\sin\theta}{\cos\theta}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	is not defined ∞
$\text{Cosec} = \frac{1}{\sin\theta}$	is not defined ∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\text{Sec}.\theta = \frac{1}{\cos\theta}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	is not defined ∞
$\text{Cot}.\theta = \frac{1}{\tan\theta}$	is not defined ∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

The entries of the above table are obtained as follows:

I. Step write the numbers 0, 1, 2, 3 & 4 under 0^o, 30^o, 45^o, 60^o & 90^o

II. Step Divide each of them by 4 to get $\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}$ & $\frac{4}{4}$.

III. Step take square root to each of them & there by we get the final numbers as

$$\sqrt{\frac{0}{4}},\ \sqrt{\frac{1}{4}},\ \sqrt{\frac{2}{4}},\ \sqrt{\frac{3}{4}}\ \&\ \sqrt{\frac{4}{4}}$$

$$= \quad 0, \quad \frac{1}{2}, \quad \frac{1}{\sqrt{2}}, \quad \frac{\sqrt{3}}{2}, \quad 1$$

These are the sin θ ratios of the angles 0°, 30°. 45°, 60° and 90° respectively. The cos θ ratios can be obtained by reversing sin θ ratios.

While the tan θ ratios can be obtained by using $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Cosec θ ratios can be obtained by $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

Sec θ ratios can be obtained by $\sec\theta = \frac{1}{\cos\theta}$

Cot θ ratios can be obtained by $\cot\theta = \frac{\cos\theta}{\sin\theta}$

WORKED EXAMPLES:

I. 1. P.T. Sin 30° . cos 60° + cos 30°. sin 60° = 1

L.H.S. $= \frac{1}{2}.\frac{1}{2}+\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}$

$= \frac{1}{4}+\frac{3}{4}=\frac{1+3}{4}=\frac{4}{4}=1=R.H.S.$

2. P.T. sin 30°. cos 60° + cos 30°. cos 60° = $\frac{1+\sqrt{3}}{4}$

L.H.S. $= \frac{1}{2}.\frac{\sqrt{1}}{2}. + \quad \frac{\sqrt{3}}{2}.\frac{1}{2}$

$= \frac{1}{4}+\frac{\sqrt{3}}{4}=\frac{1+\sqrt{3}}{4}$

3. S.T. $4\sec^2 45^\circ + 4\sin^2 30^\circ - 2\cot^2 60^\circ = \dfrac{25}{3}$

$$\text{L.H.S.} = 4.2 + 4.\frac{1}{4} - 2.\frac{1}{3}$$

$$= 9\text{-}\ \frac{2}{3} = \frac{27-2}{3} = \frac{25}{3} = \text{R.H.S.}$$

4. If $A = 45^\circ$ then Show that $\sin 2A = 2\sin A.\cos A$

L.H.S. = $\sin 2(45) = \sin 90 = 1$

R.H.S. = $2\sin 45 \cos 45 = \dfrac{2.\times 1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}} = 1$

L.H.S. = R.H.S.

5. If $A = 30^\circ$ S.T. $\cos 2A = \cos^2 A - \sin^2 A$

L.H.S. = $\cos 2.30 = \cos 60 = \dfrac{1}{2}$

R.H.S. = $\cos^2 30 - \sin^2 30 = \dfrac{3}{4} - \dfrac{1}{4} = \dfrac{2}{4} = \dfrac{1}{2}$

L.H.S. = R.H.S.

6. Find the value of $\sin^2\dfrac{\pi}{6} + \cos^2\dfrac{\pi}{3} - \tan^2\dfrac{\pi}{4}$

L.H.S. = $\left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2 - 1 = \dfrac{1}{4} + \dfrac{1}{4} - 1 = \dfrac{2}{4} - \dfrac{1}{2} = \dfrac{1}{2}$

$$= \frac{2}{4} - 1 = \frac{-1}{2}$$

7. P.T. $\sec^2\dfrac{\pi}{6} + \operatorname{cosec}^2\dfrac{\pi}{4} + \cot^2\dfrac{\pi}{3} + \sin^2\dfrac{\pi}{2} = \dfrac{14}{3}$

L.H.S. = $\left(\dfrac{2}{\sqrt{3}}\right)^2 + \left(\sqrt{2}\right)^2 + \left(\dfrac{1}{\sqrt{3}}\right)^2 + (1)^2$

$$= \frac{4}{3} + 2 + \frac{1}{3} + 1 = \frac{4}{3} + \frac{1}{3} + 3 = \frac{4+1+9}{3} = \frac{14}{3} = \text{RHS.}$$

8. Find the ratio of

$Cos^2 45^o - cos^4(30^o) + sin^4(30^o) + sin^4(60^o)$

$$= (1/\sqrt{2})^2 - (\sqrt{3}/2)^4 + (1/2)^4 + (\sqrt{3}/2)^4$$

$$\frac{1}{2} + \frac{1}{16} = \frac{8+1}{16} = \frac{9}{16}$$

EXERCISE : 14.1 (ONE MARK)

I. If A = 60^o verify the following.

1 $\sin 2A = 2 \sin A \cdot \cos A$

2. $\cos 2A = \cos^2 A - \sin^2 A$

3. $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

4. $\sin 2A = \dfrac{2\tan A}{1 + \tan^2 A}$

5. $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

II. Find the values of the following (two marks)

1 $\cos^2 30^o + \cos^2 45^o + \cos^2 60^o$

2. $\cot^2 60^o + \sin^2 45^o + \sin^2 30^o + \cos^2 90^o$

3 $\sin^2\dfrac{\pi}{6} + \cos^2\dfrac{\pi}{3} - \tan^2\dfrac{\pi}{4} + \cot^2\dfrac{\pi}{4}$

4. $\cos 60^o - \sin 30^0 - \cot^3 45^o$

5. $3.\tan^2\dfrac{\pi}{6} + \dfrac{4}{3}\cos^2\dfrac{\pi}{6} - \dfrac{1}{2}\sec^2\dfrac{\pi}{4} - \dfrac{1}{3}\sin^2\dfrac{\pi}{3}$

III. Find the values of : (three marks)

1. $3\tan^2 30o + \dfrac{4}{3}\cos^2 30^o - \dfrac{1}{2}\cot^2 45^o - \dfrac{2}{3}\sin^2 60^o + \dfrac{1}{8}\sec^4 60^o$

2. $\sin^3 60^o.\cot 30^o - 2\sec^2 45^o + 3\cot 60^o \tan^2 60^o$

3. $$\frac{\sin\frac{\pi}{2}.\cos^2\frac{\pi}{6}.\sec^2\frac{\pi}{4}}{\tan\frac{\pi}{3}+\cot\frac{\pi}{3}}$$

4. $$\frac{\sin^2 60^o.\cos^3 60^o.\sec^2 30^o}{2\cos ec^2 30^o - \frac{1}{2}\sin^2 60^o.\tan^2 30^o}$$

5. S.T. $$\left(\frac{1-\cot\frac{\pi}{3}}{1+\cot\frac{\pi}{3}}\right)^2 = \frac{1-\cos\frac{\pi}{6}}{1+\cos\frac{\pi}{6}}$$

IV. Find x from the following :

1. $$x.\sin 30.\cos^2 45^o = \frac{\cot^2 30^o.\sec 60^o.\tan 45^o}{\cos ec^2 45^o.\cos ec 30^o}$$

2. $$\frac{x.\cos ec^2 30^o.\sec^2 45^o}{8\cos 45^o.\sin 60^o} = \tan^2 60^o - \tan^2 30^o$$

3. $$x.\sin .30^o.\cos ec^2 60^o = \frac{\cos^2 45^o.\tan 60^o}{\cot^2 30^o. \sec^2 0^o}$$

4. $$\frac{x.\cos ec^2 30^o.\sec^2 45^o}{\sin^2 45^\circ.\cos^2 60^o} = \tan^2 60^o - \cot^2 60^o$$

5. $$x.\sin 45^o.\cos^2 60^o = \frac{\tan^2 60^o \cos ec 30^o}{\sec 45^o.\cot^2 30^o}$$

V.

1. If $x^2-\left(2+\operatorname{cosec}^2\frac{\pi}{4}\right)x-2\sec^2\frac{\pi}{3}=\cos 90^o$ S.T. $x=4,-2$

2. If $x.\sin 30^o.\cos^2 45^o=\dfrac{\cos^3 30^o.\sec 60^o.\tan 45^0}{\operatorname{cosec}^2 45^o.\operatorname{cosec} 30^o}$

S.T. $x=\dfrac{3\sqrt{3}}{4}$

3. If $x.\sin 45^o.\tan 60^o=\dfrac{\sin 30^o.\cot 30^o}{3\cos 60^o.\operatorname{cosec} 45^o}$ $S.T. x=\frac{1}{3}$

4. If $x^2(\operatorname{cosec}^2 45^o-\cos^2 90^o)-5(\cos 0°+\tan 0°)x+\sec^2 45^o=0$

S.T. $x=2,\frac{1}{2}$

5. S.T. x = 7/2
If x = sec.30°. tan 60°. + sin 45°. cosec 45° + cos 30° cot 60°

6. Find the value of $\cot^2\frac{\pi}{6}-2\cos^2\frac{\pi}{3}-\frac{3}{4}\sec^2\frac{\pi}{4}-4\sin^2\frac{\pi}{6}$

7. Find x from the equation.

$$x.\sin 30^o.\cos^2 45^o=\frac{\cot^2 30^o.\sec 60^o-\tan 45^o}{\operatorname{cosec}^2 45^o.\operatorname{cosec} 30^o.\cos 60^o}$$

ANSWERS : 14.1

II. 1) $\frac{3}{2}$ 2) $\frac{13}{12}$ 3) $\frac{1}{2}$ 4) -1 5) $\frac{3}{4}$

III. 1) 3 2) $\frac{13}{18}$ 3) $\frac{3\sqrt{3}}{8}$ 4) $\frac{1}{63}$

IV. 1) x = 6 2) $x=\frac{4}{\sqrt{6}}$ 3) x = 4) x = 5) x = 8

V. 6) 0 7) x = 3

14.2 Signs of Trigonometric ratios:

The trigonometric functions of any angle are already defined as

$\sin\theta = \frac{y}{r}, \cos\theta = \frac{x}{r}$ *and* $\tan\theta = \frac{y}{x}$ and the others are reciprocals of these, since r is always positive. The algebraic sign of these numbers depend on x & y only. Now let us consider the different positions of the terminal ray of angle .

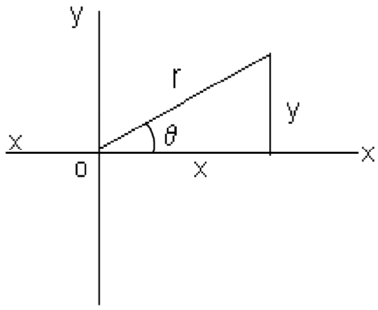
1 If the terminal ray of angle θ lies in I quadrant :

$$\sin\theta = \frac{y}{r} = \frac{+ve}{+ve} = +ve$$

$$\cos\theta = \frac{x}{r} = \frac{+ve}{+ve} = +ve$$

$$\tan\theta = \frac{y}{x} = \frac{+ve}{+ve} = +ve$$

The reciprocal are also positive.

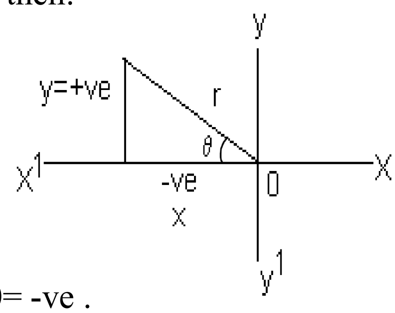


2 If the terminal ray of angle θ lies in II quadrant then:

$$\sin\theta = \frac{y}{r} = \frac{+ve}{+ve} = +ve$$

$$\cos\theta = \frac{x}{r} = \frac{-ve}{+ve} = -ve$$

$$\tan\theta = \frac{y}{x} = \frac{+ve}{-ve} = -ve$$



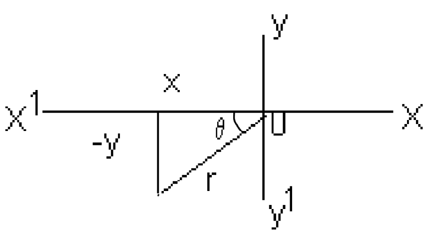
The reciprocal cosec θ = +ve , sec.θ = -ve, cotθ= -ve .

3 If the terminal ray of angle θ lies in III quadrant, then:

$$\sin\theta = \frac{y}{r} = \frac{-ve}{+ve} = -ve$$

$$\cos\theta = \frac{x}{r} = \frac{-ve}{+ve} = -ve$$

$$\tan\theta = \frac{y}{x} = \frac{-ve}{-ve} = +ve$$



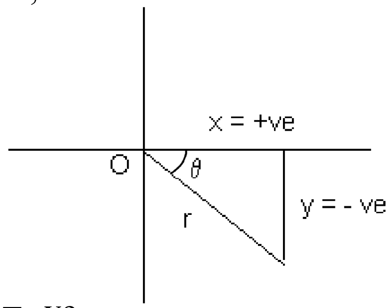
The reciprocals cosecθ= -ve , secθ = -ve , cot θ = +ve .

IV. If the terminal ray of angle θ lies in IV quadrant, then.

$$\sin\theta = \frac{y}{r} = \frac{-\,ve}{+ve} = -\,ve$$

$$\cos\theta = \frac{x}{r} = \frac{+ve}{+ve} = +ve$$

$$\tan\theta = \frac{y}{x} = \frac{-\,ve}{+ve} = -\,ve$$



The reciprocal cosecθ = -ve, secθ = +ve, cotθ = -ve.

Thus, if the terminal ray of the angle θ lies I Quadrant, all the trigonometric ratios are positive. (All)

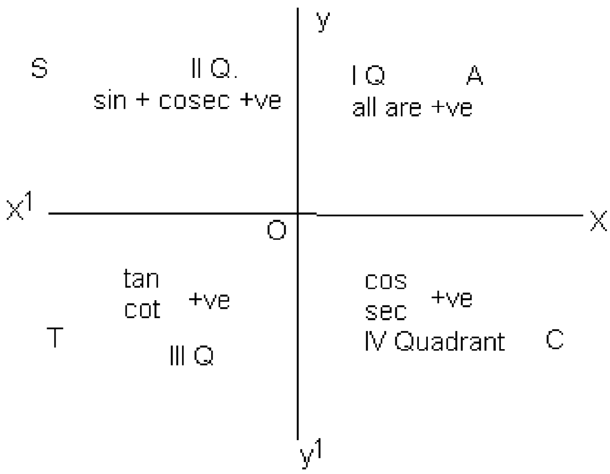
If the terminal ray lies in II Quadrant, only sinθ and cosecθ are positive whereas others are negative. (sinθ)

If the terminal ray lies in III Quadrant, only tanθ and cotθ are positive whereas others are negative. (tanθ)

And if the terminal ray lies in IV Quadrant, only cosθ and sec θ are positive & others are negative. (cosθ)

This rule is called the ASTC Rule. This can be easily remembered by the following statements.

"**A**ll **S**tudents **T**ake **C**offee" or "**A**ll **S**ilver **T**ea **C**ups"



14.3 Allied Angles:

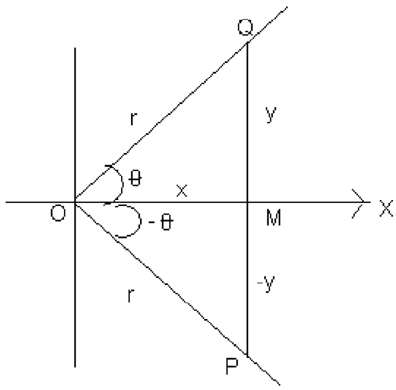
If θ is any angle, which take angles of the form $\left(\frac{n\pi}{2}\pm\theta\right)$, where n is any integer are called allied angles.

Hence -θ, (90 ± θ), (180 ± θ), (270 ± θ), (360± θ) etc. are all called allied angles.

a) Trigonometric functions of (-θ) or (360-θ) :

Let P(x, y) be any point on the terminal ray of the angle (-θ) .Construct the angle in I Quadrant. Choose the point Q on the terminal ray of θ such that OQ = OP.

Then from the figure we have : ΔOPM is congruent to Δ OQM. OP = OQ = r , and MP = - MQ



$$\sin(-\theta) = \frac{MP}{OP} = \frac{-MQ}{OQ} = -\sin\theta$$

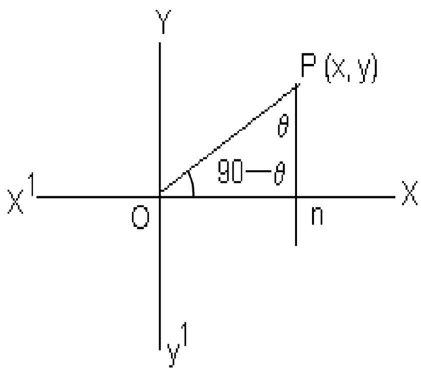
$$\cos(-\theta) = \frac{OM}{OP} = \frac{OM}{OQ} = \cos\theta$$

$$\tan(-\theta) = \frac{MP}{OM} = \frac{-MQ}{OM} = -\tan\theta$$

Similarly cot (-θ) = - cot θ , Sec(-θ) = sec θ , and cosec(-θ) = - cosec θ.

b) Trigonometric functions of (90 - θ) :

Let P (*x*, y) be any point on the terminal ray OP. Let OPM = θ∴ POM = 90 - θ.



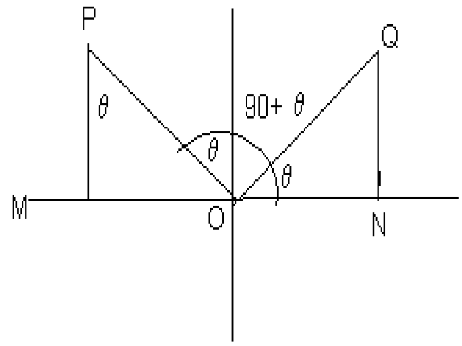
$$\text{Sin}(90-\theta) = \frac{PM}{OP} = \cos\theta$$

$$\text{Cos}(90-\theta) = \frac{OM}{OP} = -\sin\theta$$

$$\text{Tan}(90-\theta) = \frac{PM}{OM} - \cot\theta$$

Similarly cosec (90-θ) = sec θ , sec (90 - θ) = cosec θ , cot (90-θ) = tanθ.

c) Trigonometric functions of (90 + θ) :



From the figure OP = OQ, PM = ON, OM = QN .

$\therefore$ sin (90 + θ) = $\frac{PM}{OP} = \frac{QN}{OQ} = \cos\theta$

cos (90 + θ) = $\frac{OM}{OP} = \frac{QN}{OQ}$ = - sin θ

tan (90 + θ) = $\frac{PM}{OM} = \frac{ON}{-QN}$ = - cot θ

Similarly cosec (90 + θ) = sec θ , sec (90+θ) = -cosec θ, cot (90+θ)=- tan θ

d) Trigonometric functions of (180 - θ) :

(180 - θ) lies in II Quadrant where sin θ and cosec θ are positive others are negative.

As done above, we can prove :

sin (180 - θ) = sin θ , cos (180 - θ) = - cos θ , tan (180 - θ) = - tan θ

cot (180 -θ) = - cot θ , sec (180 - θ) = - sec θ

e) Trigonometric functions of (180 + θ):

(180 + θ) lies in III Quadrant where tan θ and cot θ are positive and others are negative.

As done above, we can prove :

Sin (180 + θ) = - sin θ	cosec (180 + θ) = - cosec θ
Cos (180 + θ) = - cosθ	sec (180 + θ) = - sec θ
Tan (180 + θ) = tan θ	cot (180 + θ) = cot θ

f) Trigonometric functions of (270 - θ) :

As (270-θ) lies in III Quadrant, both tan θ and cot θ are positive others are negative.

As done above, we can prove :

Sin (270 - θ) = - cos θ	cos (270 - θ) = - sinθ	tan (270 - θ) = cot θ
cosec (270 - θ) = - secθ	sec (270 - θ) = - cosec θ	cot (270 - θ) = tan θ

g) Trigonometric ratios of (270 + θ) :

(270 + θ) lies in IV Quadrant where both cos θ and sec θ are positive and others are negative.

As done above, we can prove :

$\sin(270+\theta) = -\cos\theta$	$\cos(270+\theta) = \sin\theta$	$\tan(270+\theta) = -\cot\theta$
$\text{cosec}(270+\theta) = -\sec\theta$	$\sec(270+\theta) = \text{cosec}\,\theta$	$\cot(270+\theta) = -\tan\theta$

h) Trigonometric ratios of (360 + θ) :

As (360 + θ) lies in I Quadrant , where all ratios are positive.

$\text{Sin}(360+\theta) = \sin\theta$	$\text{cosec}(360+\theta) = \text{cosec}\,\theta$
$\text{Cos}(360+\theta) = \cos\theta$	$\sec(360+\theta) = \sec\theta$
$\text{Tan}(360+\theta) = \tan\theta$	$\cot(360+\theta) = \cot\theta$

Note: The trigonometric ratios of allied angles can be remembered by the following procedure.

1) The trigonometric ratios of angles $(90 \pm \theta)$ and $(270 \pm \theta)$ changes from

$\sin\theta \to \cos\theta$	$\cos\theta \to \sin\theta$
$\tan\theta \to \cot\theta$	$\text{cosec}\,\theta \to \sec\theta$,
$\sec\theta \to \text{cosec}\theta$	$\cot\theta \to \tan\theta$

2) Whereas the trigonometric ratios of angles $180 \pm \theta$, $360 \pm \theta$ remains same.

3) In both 1) & 2) the sign changes according to ASTC rule or Quadrant rule.

WORKED EXAMPLES:

I. Find the values of the following:

1. $\cos 120^\circ = \cos(180 - 60) = -\cos 60 = \frac{-1}{2}$

2. $\sin 150^\circ = \sin(90 + 60) = \cos 60^\circ = \frac{1}{2}$

3. $\tan 225^\circ = \tan(180 + 45^\circ) = \tan 45^\circ = 1$

4. $\sin 960^o = \sin(2 \times 360 + 240) = \sin 240^o = \sin(180 + 60^o) = -\sin 60 = -\frac{\sqrt{3}}{2}$

5. $\cos(-870^o) = \cos 870 = \cos(2 \times 360 + 150^0) = \cos 150^o = \cos(180-30)$

$= -\cos 30 = -\frac{\sqrt{3}}{2}$

6. $\sin(-780^o) = -\sin 780 = -\sin(2 \times 360 + 60) = -\sin 60^0 = -\frac{\sqrt{3}}{2}$

7 $\sec(-1585^o) = \sec 1575^o = \sec(4 \times 360^o + 135^o) = \sec 135^o$

$= \sec(90 + 45^o) = \text{Cosec } 45^o = -\sqrt{2}$

8. $\text{cosec } 1305^o = \text{cosec}(3 \times 360 + 225) = \text{cosec } 225$

$= \text{cosec}(180+45) = -\text{cosec } 45 = -\sqrt{2}$

9. $\cot 840^0 = \cot(2 \times 360 + 120) = \cot(120^0) = \cot(180-60^o) = -\cot 60^o$

$= -\frac{1}{\sqrt{3}}$

10. $\tan(-660^o) = -\tan(660^o) = -\tan(360 + 300^o) = -\tan 300°$

$= -\tan(360-60)° = \tan 60° = \sqrt{3}$

11. $\sin\frac{15\pi}{4} = \sin\left(2 \times 2\pi - \frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

12. $\sec\frac{7\pi}{3} = \sec\left(2\pi + \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$

13. $\text{cosec}\left(\frac{11\pi}{4}\right) = \text{cosec}\left(2\pi + \frac{3\pi}{4}\right) = \text{cosec}\left(\frac{3\pi}{4}\right) = \text{cosec}\left(\pi - \frac{\pi}{4}\right) = \text{cosec}\frac{\pi}{4} = \sqrt{2}$

14. $\tan\left(\frac{16\pi}{3}\right) = \tan\left(4\pi + \frac{4\pi}{3}\right) = \tan\frac{4\pi}{3} = \tan\left(\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

II. Prove the following:

1. $\cos(A - 180^0) == -\cos A$

L.H.S. $= \cos(-(180-A))$

$= +\cos(180-A) = -\cos A$

2. $\tan A + \tan(180 - A) + \cot(90 + A) = -\tan A$

L.H.S. $= \tan A + \tan(180 - A) + \cot(90 + A)$

$= \tan A - \tan A - \tan A = -\tan A =$ R.H.S.

3. P.T $\dfrac{\sin(180 - A).\cot(90 + A).\cos(360 - A)}{\tan(180 + A).\tan(90 + A).\sin(-A)} = \sin A$

L.H.S. $= \dfrac{\sin(180 - A).\cot(90 + A).\cos(360 - A)}{\tan\ 180 + A\ .\tan\ 90 + A\ .\sin(-A)}$

$= \dfrac{\sin A.\tan A.\cos A}{\tan A.(-\cot A)(-\sin A)} = \dfrac{\cos A}{\dfrac{\cos A}{\sin A}} = \sin A = R.H.S.$

4. $\dfrac{\sin(90 + \theta).\cos\ 180 - \theta\ .\cot(270 + \theta)}{\sin(90 - \theta).\sin\ 270 - \theta\ .\cot(90 + \theta)} = 1$

L.H.S. $= \dfrac{\sin(90 + \theta).\cos\ 180 - \theta\ .\cot(270 + \theta)}{\sin(90 - \theta).\sin\ 270 - \theta\ .\cot(90 + \theta)}$

$= \dfrac{\cos\theta(-\cos\theta)(-\tan\theta)}{\cos\theta(-\cos\theta)(-\tan\theta)} = 1 =$ R.H.S.

III.

1. Simplify $\dfrac{\cos(360 + A).\sec(-A)\tan(180 - A)}{\sec(360 + A).\sin(180 + A).\cot(90 - A)}$

$= \dfrac{(\cos A)(\sec A).(-\tan A)}{(\sec.A).(-\sin A)(\tan A)} = \tan A$

2. $$\frac{\sin 150^0 - 5\cos 300° + 7\tan 2250°}{\tan 135^o + 3.\sin 210^{\circ}}$$

$$= \frac{\sin(180-30^0) - 5\cos(360-60^o\ ) + 7\tan(180^o + 45^o)}{\tan(180-45^o) + 3\sin(180^o + 30^o)}$$

$$= \frac{\sin 30^o - 5\cos 60^o + 7\tan 45^o}{-\tan 45^o - 3\sin 30^o}$$

$$= \frac{\frac{1}{2} - 5\frac{1}{2} + 7.1}{-1-3.\frac{1}{2}} = \frac{5}{\frac{-5}{2}} = -2$$

3. $$\frac{\cos 120^o + \sin 135^o}{\sin 135^o - \cos 120^o} = \frac{\sin(180-45^o) + \cos(180-60)}{\sin(180-45^o) - \cos(180-60)}$$

$$= \frac{\sin 45 - \cos 60}{\sin 45 + \cos 60} = \frac{\frac{1}{\sqrt{2}} - \frac{1}{2}}{\frac{1}{\sqrt{2}} + \frac{1}{2}} = \frac{2-\sqrt{2}}{2+\sqrt{2}}$$

4. Find x if $\dfrac{x.\sin^2 300.\sec^2 240}{\cos^2 225.\operatorname{cosec}^2 240} = \cot^2 315^o.\ \tan^2 300$

sin 300 = sin (360 – 60) = - sin 60 = $\dfrac{-\sqrt{3}}{2}$

sec 240 = sec (180 + 60) = - sec 60^o = - 2

cos 225 = cos (180 + 45^o) = - cos 45^o = - $\dfrac{1}{\sqrt{2}}$

cosec 240 = cosec (180 + 60^o) = - cosec 60 = - $\dfrac{2}{\sqrt{3}}$

cot 315^o = cot (360 – 45^o) = - cot 45^o = -1

tan 300 = tan (360 – 60^o) = - tan 60^o = - $\sqrt{3}$

$\therefore$ The given equation becomes :

$$\frac{x.\frac{3}{4}.4}{\frac{1}{2}.\frac{4}{3}} = 1\ .\ 3$$

$$\Rightarrow x = \frac{6}{9} = \frac{2}{3}$$

5. If $\sin\theta = \frac{-3}{5}$ and θ lies in IV Quadrant.

Then P.T. $\frac{3\tan\theta - 4\cos\theta}{4\tan\theta + 3\cos\theta} = \frac{109}{12}$

Since θ is in the IV Quadrant : $\tan\theta = -\frac{3}{4}$ $\cos\theta = \frac{4}{5}$.

L.H.S. $= \frac{3.\frac{-3}{4} - 4.\frac{4}{5}}{4.\frac{-3}{4} + 3.\frac{4}{5}} = 0$

$= \frac{\frac{-9}{4} - \frac{16}{5}}{-3 + \frac{12}{5}} = \frac{109}{12} =$ R.H.S.

IV. If A, B, C are the angles of a triangle then P.T.

1) sin (A + B) = sin C 2) $\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$ 3) $\cos(B + C) = -\cos A$

4) $\tan\frac{c}{2} = \cot\left(\frac{B+A}{2}\right)$

Solution: In any triangle ABC, A + B + C = 180°
Therefore, A + B = 180 – C

i) sin (A + B) = sin (180 – C) = sin C = R.H.S.

ii) $\frac{A+B}{2} = 90 - \frac{C}{2}$

$$\cos\left(\frac{A+B}{2}\right) = \cos(90 - \frac{c}{2}) = \sin\frac{C}{2}$$

iii) B + C = 180-A

cos (B + C) = cos (180 – A) = - cos A

iv) $\frac{C}{2} = 90 - \frac{B+A}{2}$

$\tan\frac{c}{2} = \tan(90 - \frac{B+A}{2}) = \cot\frac{B+A}{2}$

EXERCISE : 14.2

I. Find the values of the following :

1) cos 480^0, 2) cos 1125^0 3) sin 855^0 4) sin 840

5) tan (- 855^o) 6) cos (-780^0). 7) $\sec\left(\frac{11\pi}{6}\right)$ 8) $\tan\frac{11\pi}{4}$

9) $\text{cosec}\left(\frac{-7\pi}{4}\right)$ 10) $\cot\left(\frac{-13\pi}{4}\right)$

II. Prove the following :

1. cos (189^o) + cos 9^0 = 0
2. cos (287^o) – sin (17^0)= 0
3. tan (225^0) . cot (405^0) + tan(765^0). cot (675^0) = 0
4. cos (570^0) . sin (510^0) – sin (330^0) . cos (390^0) = 0
5. sin (480^o) . cos (690^0) + cos (780^0) .sin (1050^0) = ½

III. Simplify the following :

1. cos A + sin (270 + A) – sin (270 – A) + cos (180 + A)
2. $\sec\left(\frac{3\pi}{2} - A\right)\sec\left(\frac{\pi}{2} - A\right) - \tan\left(\frac{3\pi}{2} - A\right).\tan\left(\frac{\pi}{2} + A\right)$
3. $\frac{\tan(180+A).\sec(180+A).\text{cosec}(90+A)}{\sec(360-A).\cot(90+A)}$
4. $\frac{\cos(270-A).\tan(90+A).\sin(180+A)}{\sin(270+A).\cos(180+A).\sin(360+A)}$

5. $\dfrac{\cos ec.(180+\theta).\sin(360-\theta).\tan(360+\theta)}{\sin(90+\theta).\cos(180-\theta).\tan(-\theta)}$

IV.

1. If $\sin\theta = \dfrac{11}{61}$ & $90 < \theta < 180^0$. Find the values of $\cos\theta$, $\tan\theta$, $\sec\theta$.

2. If $\operatorname{cosec}\theta = \dfrac{5}{3}$ and $90 < \theta < 180$. S.T. $\dfrac{4\sin\theta - 7\cos\theta}{3\sin\theta + 2\cos\theta} = 40$

3. If $\tan A = \dfrac{12}{13}$ and $270 < A < 360^0$. Find the value of $\dfrac{3\sin A - 2\cos A}{9\cos A + 4\sin A}$

4. If $\sin\theta = \dfrac{-8}{17}$ & $\pi < \theta < \dfrac{3\pi}{2}$. Find the value of $\dfrac{\tan\theta - \cos\theta}{\sec\theta + \cos ec\theta}$

5. Find the values of θ such that ($0 \le \theta < 360$) :

a) $\sin\theta = \dfrac{1}{\sqrt{2}}$ b) $\cos\theta = \dfrac{-\sqrt{3}}{2}$

c) cot = $\theta = \dfrac{1}{\sqrt{3}}$ d) $2\sin\theta = \sqrt{3}\tan\theta$

ANSWERS : 14.2

I 1) $\dfrac{-1}{2}$ 2) $\dfrac{1}{\sqrt{2}}$ 3) $\dfrac{1}{\sqrt{2}}$ 4) $\dfrac{\sqrt{3}}{2}$ 5) 1 6) $\dfrac{+1}{2}$

7) $\dfrac{2}{\sqrt{3}}$ 8) -1 9) $\sqrt{2}$ 10) – 1

III. 1) 0 2) -1 3) Sec A 4) –sec A 5) $\sec^2\theta$

IV. 1) $\dfrac{-60}{61}, \dfrac{-11}{60}, \dfrac{-61}{60}$ 2) 40 3) $\dfrac{46}{3}$ 4) $\dfrac{7}{17}$

5) a) 45^0, 135^0 b) 150^0, 210^0 c) 120^0 300^0 d) 0, 180^0, 30^0, 330^0

UNIT - IV

ANALYTICAL GEOMETRY

CHAPTER	NAME OF THE CHAPTER	TEACHING HOURS
15	CO-ORDINATE SYSTEM IN A PLANE THEORY	05
16	LOCUS AND ITS EQUATIONS	03
17	STRAIGHT LINE	10
	TOTAL	18 Hours

Chapter 15

CO-ORDINATE SYSTEM IN A PLANE

15.1 INTRODUCTION:

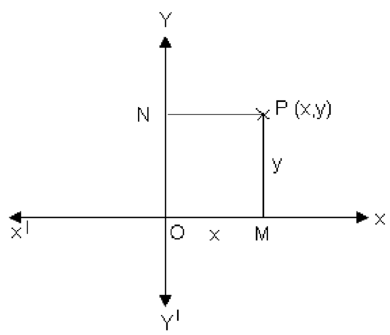
Analytical Geometry or Co-ordinate geometry is the branch of Mathematics which deals with the study of geometry through algebra. It was **Rene Descartes** (1596-1650) a famous French Mathematician, who introduced algebraic methods to solve geometrical problems. The entire subject is a progressive development of the basic idea of a "point". Descartes established a relationship between the basic geometric concept of **Point** with basic algebraic entity **Number.** This relationship is called as **system of co-ordinates.** The plane in which the points are represented by an ordered pair (x,y) of real numbers uniquely and conversely is called the **Cartesian Plane**.

Axes of reference: Every ordered pair (x, y) of real numbers x and y can be represented by a point in a plane with reference to two fixed lines called **reference axes** which may be mutually perpendicular or may be not perpendicular. If the two lines are perpendicular then the axes are called Rectangular, otherwise they are called Oblique.

15.2 RECTANGULAR CARTESIAN COORDINATE SYSTEM

Draw two straight lines X'OB and Y′OY intersecting each other at right angles and let O be their point of intersection. Now X'OB is called the x-axis (axis of x) and Y′OY is called the y-axis (axis of y). O is called the origin.

The two mutually perpendicular lines taken together are called as rectangular axes or co-ordinate axes or the axes of co-ordinates. Let P be any point in the plane. Through P draw PM parallel to the y axis cutting the x axis at M and draw PN parallel to the x axis cutting the y axis at N as shown in the figure.



Then OM is called the x **co-ordinate or abscissa** of the point P denoted by 'x' and MP is called the the **y co-ordinate or ordinate** of the point P denoted by 'y'. Then the point P is completely determined by the ordered pair (x,y) of real numbers.

This ordered pair (x,y) is the co-ordinates of P and this is written as P (x,y).

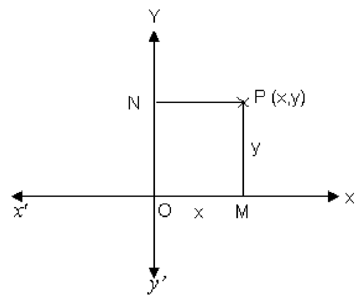
The above system of co-ordinating an ordered pair (x,y) with every point in a plane is called the **Rectangular Cartesian Co ordinate system .**

Note:

- The co-ordinates of origin is taken as O(0,0)
- Any point on x axis can be taken as (x,0) since for any point on the x axis, the y co-ordinate is zero.
- Any point on y axis can be taken as (0,y) since for any point on the y axis, the x co-ordinate is zero
- The abscissa of any point is numerically equal to the distance of the point from the y axis and the ordinate of any point is numerically equal to the distance of the point from the x axis.

Quadrants:

Two mutually perpendicular lines $X'OX$ and $Y'OY$ divide the plane into four parts called as the **Quadrants**. The region XOY is called **the first Quadrant**, the region YOX' is called **the second quadrant,** the region $X'OY'$ is called **the third quadrant** and the region $Y'OX$ is called the **fourth quadrant.**



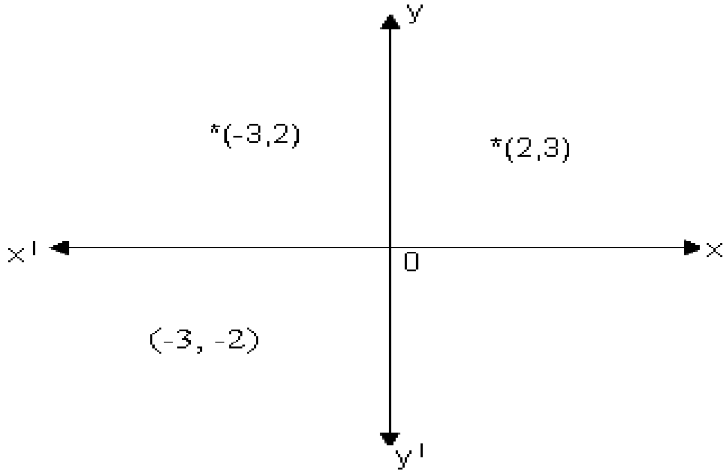
According to our convention of signs of measurement of distances along the X and Y axes the following table gives the sign of x and y co-ordinates of the points in different quadrants.

Quadrant	x co-ordinate	y co-ordinate
First	+	+
Second	-	+
Third	-	-
Fourth	+	-

WORKED EXAMPLES

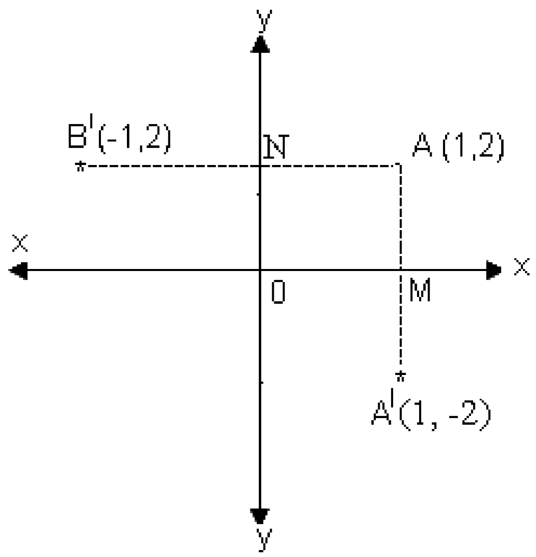
1. Mark the points (2,3), (-3,2) and (-3,-2) in a rectangular co ordinate system

Solution :



Reflection / Image of a point with respect to the *x* axis and y axis

To find the image of a point A(1,2) in *x* axis , produce AM to A′ such that MA′ =2 units. We arrive at the point A′(1,-2) which is the image of A in the *x* axis. Similarly to find the image of A(1,2) in y axis, produce AN to B′ such that NB′=1 unit. Then B′(-1,2) is the reflection or the image of the point A in the y axis



Note: When **a point is reflected in the x axis, the sign of its ordinate changes**. For eg, the image of the point A(1,2) in the x axis is A(1,-2).Thus the image of the point S(x,y) in x axis is S′(x,-y).**When a point is reflected in the y axis the sign of its abscissa changes.** For example, the image of the point B(2,3) in the y axis is given by B′(-2,3)Thus the Image of the point A (x,y) in y axis is A′(-x,y) .

Reflection of a point in the origin

When a point P(x,y) is reflected in the origin the signs of its abscissa and ordinate both changes. Thus the reflection or the image of the point P(2,3) in the origin is given by P′(-2,-3)

WORKED EXAMPLES

Find the co ordinates of the images of the point (4,5), (-7,8), (-5,-5), (4,-8) with respect to x axis , y axis and origin

Solution:

Point P	Reflection in x axis	Reflection in y axis	Reflection in the origin
(4, 5)	(4,-5)	(-4, 5)	(-4,-5)
(-7, 8)	(-7,-8)	(7, 8)	(7,-8)
(-5,-5)	(-5, 5)	(5,-5)	(5, 5)
(4,-8)	(4, 8)	(-4,-8)	(-4, 8)

EXERCISE : 15.1

1. Plot the points A(2,3), B(3,-4), C(0,5) and D(-1,4) on the co-ordinate plane.

2. Find in which quadrants do the following lie?
   a. (9,8) b. (3,-2) c. (-4,-5)

3. What are the distances of the following points from co-ordinate axes.
   a. (3,4) b. (8,-7) c. (-6,-6)

4. Find the co-ordinates of the reflection of the points
   a. (2,4) b. (-3,5) with respect to x axis, y axis and the origin.

ANSWERS : 15.1

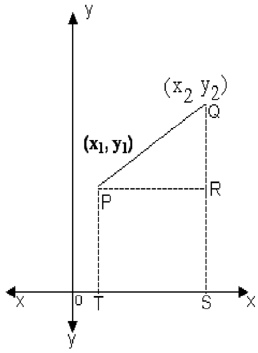
2. a) First b) Fourth c) Third

3. a) distance of (3,4) from x and y axes are 4 and 3 respectively
   b) distance of (8,-7) from x and y axes are 7 and 8 respectively
   c) distance of (-6,-6) from x and y axes are 6 and 6 respectively

4.

Point P	Reflection in x axis	Reflection in y axis	Reflection in the origin
(2, 4)	(2,-4)	(-2, 4)	(-2,-4)
(-3, 5)	(-3,-5)	(3, 5)	(3,-5)

15.3 DISTANCE BETWEEN TWO POINTS (DISTANCE FORMULA)



The distance between any two points in the plane is the length of the line segment joining them. The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Hence the distance PQ is given by

$$\sqrt{(difference\ of\ x\ coordinates)^2 + (difference\ of\ y\ coordinates)^2}$$

Note:

- The distance of the point $P(x,y)$ from the origin O(0,0) is given by $\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$
- When the line PQ is parallel to the y axis , the x co ordinates or the abscissa of the points P and Q will be equal and so the distance PQ is given by (y_2-y_1) or (y_1-y_2) and it is taken to be positive always.
- When the line PQ is parallel to the x axis, the y co ordinates of the points P and Q will be equal and so the distance PQ is given by (x_2-x_1) or (x_1-x_2) and taken to be positive always.

Application of distance formula in geometrical problems

1. If A, B and C are any three given points in the plane, we have the following results

   a. If the sum of the distances between two line segment is equal to the length of the 3rd line segment, then the three points are said to be collinear

   b. If any two sides of a triangle are equal then the three points form the vertices of an isosceles triangle

   c. If all the three sides are equal then the three points form the vertices of an equilateral triangle

   d. If the sum of the squares of lengths of any two sides of the triangle is equal to the square of the third side length (Pythagoras theorem) then the points form a right angled triangle.

2. If A,B,C and D are four points ,no three of which are collinear , the type of quadrilateral formed by these points is determined by using the distance formula based on the properties:

   i) Square$\rightarrow$, prove that the four sides are equal and the diagonals are equal

   ii) Rhombus$\rightarrow$ prove that the four sides are equal and the diagonals are not equal

   iii) Rectangle$\rightarrow$ prove that opposite sides are equal and the diagonals are also equal

   iv) Parallelogram$\rightarrow$ prove that opposite sides are equal and the diagonals are not equal

WORKED EXAMPLES

Example 1 Find the distance between the points A (5,8) and B (9,9)

Solution : $distance\ AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(9-5)^2+(9-8)^2}$

$= \sqrt{16+1} = \sqrt{17}\ units$

Example 2 Show that the points A(7,9),B(3,-7), C(-3,3) are the vertices of the right angled isosceles triangle

Solution: $AB^2 = (7-3)^2 + (9+7)^2 = 272$

$BC^2 = (-3-3)^2 + (3+7)^2 = 136$

$AC^2 = (-7-3)^2 + (9-3)^2 = 136$

Clearly $AB^2 = AC^2 + BC^2$, Thus ABC is a right angled triangle .Also $BC^2= AC^2$ which implies that BC=AC and so ABC is an isosceles triangle. Hence the given three points form an right angled isosceles triangle.

Example 3 If the distance between the points (3,-2) and (-1,a) is 5 units , find the values of a

Solution : Let A be (3,-2) and B be (-1,a). Given AB =5

$AB=\sqrt{(-1-3)^2+(a+2)^2} = 5$

Squaring on both sides we get $(-1-3)^2+(a+2)^2 = 25$

Thus $(a+2)^2 = 9 \Rightarrow (a+2) = \pm 3$ then a=1 or a = -5

Example 4 Give the relation that must exist between x and y so that (x,y) is equidistant from (6,-1) and (2,3)

Solution : Let P the point (x,y) and A be (6,-1) and B be (2,3) .

Given that PA= PB

i.e $PA^2 = PB^2$

$i.e\ (x-6)^2+(y+1)^2 = (x-2)^2+(y-3)^2$

On expanding we get, $x^2-12x+36 +y^2+2y +1=x^2-4x+4 +y^2-6y +9$

i.e $8x-8y=24$ or $x-y=3$

Example 5 What point on the x axis is equidistant from (7,6) and (-3,4)?

Solution : Let P (x,0) be the point on the x axis.

Given PA =PB wher A is (7,6) and B(-3,4)

i.e $PA^2 = PB^2$

$(x-7)^2 + 36 = (x+3)^2 + 16,$

$x^2-14x+49 +36 = x^2+6x+ 9 +16$

$-20x = -60$

$\Rightarrow\ x=3$

therefore the required point is (3,0)

Example 6 Show that the points (-2 -1), (1, 0), (4, 3) and (1, 2) are the vertices of a parallelogram.

Solution : Let A = (-2 -1), B = (1, 0), C (4, 3) and D = (1, 2)

Clearly AB = $\sqrt{(1+2)^2+(0+1)^2} = \sqrt{9+1} = \sqrt{10}$

BC = $\sqrt{(4-1)^2+(3-0)^2} = \sqrt{9+9} = \sqrt{18}$

CD = $\sqrt{(1-4)^2+(2-3)^2} = \sqrt{9+1} = \sqrt{10}$

DA = $\sqrt{(1+2)^2+(2-3)^2} = \sqrt{9+9} = \sqrt{18}$

We have AB = CD and BC = DA i.e. opposite sides are equal

AC = $\sqrt{(4+2)^2+(3+1)^2} = \sqrt{36+16} = \sqrt{52}$

BD = $\sqrt{(1-1)^2+(2-0)^2} = \sqrt{0+4} = 2$

Also the diagonals AC and BD are not equal and so ABCD is a parallelogram.

Example 7 Show that the points (2, -2), (8, 4), (5, 7) and (-1, 1) are the vertices of a rectangle .

Solution: Let A (2, -2) B = (8, 4), C = (5, 7) and D = (-1, 1)

Consider AB = $\sqrt{(8-2)^2+(4+2)^2} = \sqrt{36+36} = \sqrt{72}$

BC = $\sqrt{(5-8)^2+(7-4)^2} = \sqrt{9+9)} = \sqrt{18}$

CD = $\sqrt{(-1-5)^2+(1-7)^2} = \sqrt{36+36} = \sqrt{72}$

DA = $\sqrt{(-1-2)^2+(1+2)^2} = \sqrt{9+9} = \sqrt{18}$

Thus AB = CD and BC = DA. That is opposite sides are equal.

AC = $\sqrt{(5-2)^2+(7+2)^2} = \sqrt{9+81} = \sqrt{90}$

BD = $\sqrt{(8+1)^2+(4-1)^2} = \sqrt{81+9} = \sqrt{90}$

Clearly the diagonals are equal. Therefore ABCD is a rectangle.

Example 8 Show that the points (2, -1), (3, 4), (-2, 3) and (-3, -2) form a rhombus

Solution : Let A (2, -1), B (3, 4), C = (-2, 3) and D = (-3, -2)

Consider $AB = \sqrt{(3-2)^2+(4+1)^2} = \sqrt{1+25} = \sqrt{26}$

$BC = \sqrt{(2-3)^2+(3-4)^2} = \sqrt{25+1} = \sqrt{26}$

$CD = \sqrt{(-3+2)^2+(-2-3)^2} = \sqrt{1+25} = \sqrt{26}$

$DA = \sqrt{(-3-2)^2+(-2+3)^2} = \sqrt{1+25} = \sqrt{26}$

The AB = BC = CD = AD. That is all the sides are equal to show that ABCD is a rhombus, also we have to show that the diagonals are not equal.

Consider $AC = \sqrt{(-2,-2^2+(3+1)^2} = \sqrt{16+16} = \sqrt{32}$

$BD = \sqrt{(-3-3)^2+(-2-4)^2} = \sqrt{36+36} = \sqrt{12}$

Thus $AC \neq BD$ $\therefore$ ABCD is a rhombus

Example 9 Find the co ordinates of the circumcentre of the triangle so formed by the points (1,1), (2,-1) and (3,2)

Solution: Let A = (1, 1), B = (2, -1), C = (3, 2)

Let S(x,y) be the circumcentre of the triangle ABC**.**

Then SA= SB = SC.

$\Rightarrow$ $SA^2 = SB^2 = SC^2$

Consider $SA^2 = SB^2$

i.e. $(x-1)^2+(y-1)^2 = (x-2)^2+(y+1)^2$

i.e. $x^2 - 2x + 1 + y^2 - 2y + 1 = x^2 - 4x + 4 + y^2 + 2y + 1$

$\Rightarrow$ $2x - 4y = 3$ (1)

Again consider $SA^2 = SC^2$

$(x - 1)^2 + (y - 1)^2 = (x - 3)^2 + (y - 2)^2$

$x^2 - 2x + 1 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 4y + 4$

$\Rightarrow 4x + 2y = 11$ (2)

Solving (1) and (2) we get $x = 5/2$ and y = 1/2

Thus the circumcentre is given by $\left(\frac{5}{2}, \frac{1}{2}\right)$

Example 10 Show that the points (1, -1), (5, 2) and (9, 5) are collinear.

Solution : We know that the three points A, B and C taken in this order are collinear if and only if AB + BC = AC

Let A = (1, -1), B = (5, 2) C = (9, 5)

Consider

$$AB = \sqrt{(5-1)^2+(2+1)^2} = \sqrt{16+9} = 5$$

$$BC = \sqrt{(9-5)^2+(5-2)^2} = \sqrt{16+9} = 5$$

$$AC = \sqrt{(-9-1)^2+(5+1)^2} = \sqrt{64+36} = 10$$

Clearly AB + BC = AC

$\Rightarrow$ The points, A, B and C are collinear

Example 11 Show that the points (1, -1), (-1, 1) and $(-\sqrt{3}, -\sqrt{3})$ are the vertices of an equilateral triangle.

Solution: Let A = (1, -1), B = (-1, 1) C = $(-\sqrt{3}, -\sqrt{3})$

Consider AB = $\sqrt{(-1-1)^2+(1+1)^2} = \sqrt{4+4} = \sqrt{8}$

BC = $\sqrt{(-\sqrt{3}+1)^2+(-\sqrt{3}-1)^2} = \sqrt{4-2\sqrt{3}+4+2\sqrt{3}} = \sqrt{8}$

AC = $\sqrt{(-\sqrt{3}-1)^2+(-\sqrt{3}+1)^2} = \sqrt{4+2\sqrt{3}+4-2\sqrt{3}} = \sqrt{8}$

Thus AB = AC = CA. Hence the points A, B and C are the vertices of an equilateral triangle.

EXERCISE : 15.2

I. 1. Find the distance between the following pair of points

a) (3, 2) and (7, 5) b) (5, 4) and (0, 0)

c) (4, 5) and (-3, 2) d) (-3, -6) and (-14, -8)

2. Find the value of k if the distance between (2k, 5) and (-k, -4) is $\sqrt{90}$

3. Find the value of x such that |PQ| = |QR| where P, Q and R are (6, -1), (1, 3) and (x, 8) respectively.

4. Find a point on the y axis which is equidistant from (7, 6) and (-3, 4)?

5. Find a point on the x axis which is equidistant from (-6, 4) and (2, -4)?

6. Find the distance of the following points from the origin.

   a) (4, -2), b) (-3, 1) c) (a + b, a – b)

7. Find the distance of the following points from the x axis

   a) (-4, 11) b) $\left(\frac{-3}{4}, 4\right)$ c) (5, 6)

8. Find the distance of the following points from the y axis

   a) (8, 6) b) (-4, 9) c) (0, 5)

9 Find the lengths of the sides of the triangle whose vertices are A(2, -2), B(-1, 2) and C(3, 5)

II. 1. Find the abscissae of points whose ordinate is 4 and which are at a distance of 5 units from (7, 8)?

2. Find a point in the II quadrant which is at a distance of 2 units from the x axis and 5 units from the y axis.

3. Prove that (4, 3) is the centre of the circle which passes through the points (1, 7), (7, -1) and (8, -6)

4. Find the perimeter of the triangle formed by the points (3, -1), (5, 2) and (-1, 2)

5. Prove that the following points are collinear.

   a) (3, -2), (5, 2) and (8, 8)
   b) (4, 2), (7, 5) and (9, 7)
   c) (4, -2), (2, -4) and (7, 1)

6. Find a relation between x and y if (x, y) is equidistant from (6, -1) and (2, 3)

7. If the distance of the point (x, y) from the points (1, 2) and (-3, 0) are equal shown that $2x + y + 1 = 0$.

III. 1. Show that the following points are the vertices of the right angled triangle.

a) (10, 4), (-4, 6) and (2, -2)

b) (4, 4), (3, 5) and (-1, -1)

c) (- 9, -16), (2, 6), (-6, 10)

2. Show that each of the triangles whose vertices given below are isosceles.

a) (8, 2), (5, -3), (0, 0)

b) (0, 6), (-5, 3), (3, 1)

c) (4, 2) (3, 1), (2, 3)

3. Show that the following points form the vertices of an equilateral triangle. Also find the area

a) (2, 4), (2, 6) $(2+\sqrt{3},5)$

b) (0, 0) $(3,\sqrt{3}),(3,-\sqrt{3})$

4. Show that the following points are the vertices of a square and hence find the areas.

a) (1, 1), (4, 1), (4, 4) and (1, 4)

b) (10, -9) (4, -5) (14, -3) and (8, 1)

c) (3, 2), (0, 5), (-3, 2) and (0, -1)

5. Prove that the following sets of points form a rhombus. Also find the area.

a) (-3, 6), (-2, 11), (3, 12) and (2, 7)

b) (2, -1), (3, 4), (-2, 3) and (-3, -2)

6. Show that the following points are the vertices of a rectangle.

a) (8, 4), (4, 7), (-1, 1) and (2, -2)

b) (3, -2), (3, 1), (5, 1) and (5, -2)

c) (1, 6) , (-1, -2), (4, 1) and (-4, 3)

7. Show that the following points form a parallelogram.

a) (-3, 1), (-6, -7), (3, -9) and (6, -1)

b) (2, 4), (4, -2), (10, 6) and (8, 12)

c) (3, 2), (6, 6), (7, 3) and (4, -1)

d) (0, 1), (-3, 7), (6, -9) and (9, -1)

8. Find a point which is equidistant from the points (1, 2), (5, -6) and (3, 4)

9. Find the circumcentre of the triangle whose vertices are (1, 2), (2, 1) and (2, 3). Also find the circumradius.

10. Find the circumcentre of the triangle whose vertices are (4, 4), (-3, 3) and (6, 0)

11. Find the centre of the circle passing through the points (0, 0), (-3, 3) and (5,4)

ANSWERS : 15.2

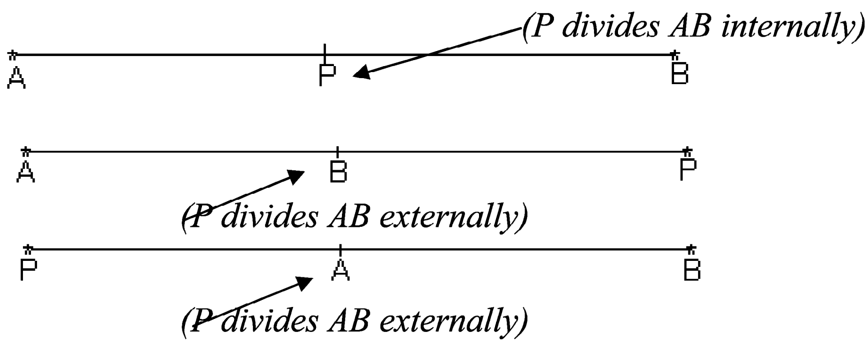
I.	1)	a) 5	b) $\sqrt{41}$	c) $\sqrt{58}$	d) $\sqrt{125}$
	2)	K = ± 1	3) x = 5, -3	4) (0, 15)	5) (-2, 0)
	6)	a) $2\sqrt{5}$	b) $\sqrt{10}$	c) $2\sqrt{ab}$	
	7)	a) 11	b) 4	c) 6	
	8)	a) 8	b) 4	c) 0	
	9)	5, 5, $5\sqrt{2}$			

II.
1. 10, 4
2. (-5, 2)
4. $5 + \sqrt{13}$
5 $x - y = 3$

III.
8. (11, 2)
9. (2, 2), radius = 1 unit
10. (1, 0)
11. $\left(\frac{17}{18}, \frac{71}{18}\right)$

15.4 SECTION FORMULA

In this section we shall note a formula to find the co-ordinates of the point which divides the line joining the two points in the given ratio internally or externally. Let AB be a straight line. If P is a point on the straight line AB, then AP/PB is called the position ratio of P on AB. If P lies between A and B then P is said to divide AB internally in the given ratio. In this case as both AP and PB are measured in the same direction , they are of the same sign and hence the ratio AP:PB is positive. On the other hand, if P lies on AB produced as shown in the figure, then the division is said to be external and here AP and PB are measured in the opposite directions and so they are of different signs and hence the ratio AP:PB is negative.



1. To find the co-ordinates of the point $P(x,y)$ which divides the line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio m:n

   The co-ordinates of P is given by

$$P(x,y) \quad = \quad \left(\frac{mx_2 + nx_1}{m+n}, \ \frac{my_2 + ny_1}{m+n} \right)$$

Remarks:

1. If a point $P(x,y)$ divides the line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio k:1 then the

   The co-ordinates of P is given by

$$P(x,y) \quad = \quad \left(\frac{kx_2 + x_1}{k+1}, \ \frac{ky_2 + y_1}{k+1} \right)$$

2. To find the co-ordinates of the point P(x,y) which divides the line joining the points A(x_1,y_1) and B(x_2,y_2) externally in the ratio m:n

The co-ordinates of P is given by

$$P(x,y) = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

15.5 MID POINT FORMULA

The co-ordinates of the mid point M of the line joining the points A(x_1,y_1) and B(x_2,y_2) is obtained by putting m=1 and n=1 in the section formula since M divides AB in the ratio 1:1.and so the co ordinates of M is given by

$$M(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

CENTROID FORMULA

Let G be the centroid of the triangle ABC. G is the point of intersection of the medians of the triangle; The centroid G divides the median AD (D is the mid point of BC) in the ratio 2:1 internally. i.e AG:GD =2:1

The co ordinates of the centroid of the triangle whose vertices are given by A(x_1,y_1), B(x_2,y_2) and C(x_3,y_3) is given by

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

WORKED EXAMPLES

Example 1 Find the co-ordinates of the point P which divides the line joining the points.

a) (1, -3) and (-3, 9) internally in the ratio 1: 3

b) (3, 5) and (2, 4) internally in the ratio 1:6

c) (3, -5) and (2, 4) internally in the ratio 3:2

d) (-1, 8) and (-2, 4) externally in the ratio 3:5

e) (3, 5) and (2, 4) externally in the ratio 4:3

Solution :

a) Let A (1, -3) and B (-3, 9)

$$P = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$= \left(\frac{1(-3)+3(1)}{1+3}, \frac{1(9)+3(-3)}{1+3}\right) = (0, 0)$$

b) Let A = (3, 5) and B = (2, 4)

$$P = \left(\frac{1(2)+6(3)}{1+6}, \frac{1(4)+6(5)}{1+6}\right) = \left(\frac{20}{7}, \frac{34}{7}\right)$$

c) Let A (3, -5) and B (2, 4)

$$P = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$= \left(\frac{3(2)+2(3)}{3+2}, \frac{3(4)+2(-5)}{3+2}\right)$$

$$= \left(\frac{12}{5}, \frac{2}{5}\right)$$

d) Let A = (-1, 8) and B = (- 2, 4)

$$P = \left(\frac{3(-2)-5(-1)}{3-5}, \frac{3(4)-5(8)}{3-5}\right)$$

$$= \left(\frac{-6+5}{2}, \frac{12-40}{-2}\right) = \left(\frac{1}{2}, 14\right)$$

e) Let A = (3, 5) and B = (2, 4)

$$P = \left(\frac{4(2)-3(3)}{4-3}\right), \frac{4(4)-3(5)}{4-3} = (-1, 1)$$

Example 2 Find the points of trisection of the line joining (3, 4) and (5, -2)

Solution : Let A ≡ (3, 4) and B ≡ (5, -2). Let P and Q be the points of trisection of AB. Then P divides AB internally in the ratio 1:2 and Q is the mid point of PB.

$$P = \left(\frac{1(5)+2(3)}{1+2}\right),\left(\frac{1(-2)+2(4)}{1+2}\right) = \left(\frac{11}{3},2\right)$$

$$Q = \left(\frac{\frac{11}{3}+5}{2},\frac{2+(-2)}{2}\right) = \left(\frac{13}{3},0\right)$$

Example 3 Find the ratio in which the line joining the points (3, 5) and (-7, 9) is divided by the point

Solution Let A $(x_1, y_1) = (3, 5)$ and B $= (x_2, y_2) = (-7, 9)$ and P $(x, y) =$ $\left(\frac{1}{2},6\right)$

The ratio m:n on which P divides AB is given by

$$\frac{x-x_1}{x_2-x} = \frac{m}{n} \quad \text{or} \quad \frac{y-y_1}{y_2-y} = \frac{m}{n}$$

$$\frac{\frac{1}{2}-3}{-7-\frac{1}{2}} = \frac{m}{n} \quad \text{or} \quad \frac{6-5}{9-6} = \frac{6-5}{9-6}$$

$$\frac{1}{3} = \frac{m}{n} \quad \text{or} \quad \frac{1}{3} = \frac{m}{n}$$

The ratio is m:n = 1:3

P divides AB in the ratio 1:3 internally.

Alternate Method:

Let P divide AB in the ratio r : 1

Then

$$P = \left(\frac{-7r+3}{r+1}, \frac{9r+5}{r+1}\right)$$

But $P = \left(\frac{1}{2}, 6\right)$

$$\Rightarrow \left(\frac{1}{2}, 6\right) = \left(\frac{-7r+3}{r+1}, \frac{9r+5}{r+1}\right)$$

$\frac{-7r+3}{r+1} = \frac{1}{2}$: $\frac{9r+5}{r+1} = 6$

15r = 5 : 3r = 1

r = 1/3 : r = 1/3

The required ratio is r: 1 = 1/3: 1 = 1:3

Example 4 Find the ratio in which the co-ordinate axes, divide the line joining the points (2, 5) and (1, 9). Find also the co-ordinates of the points of division.

Solution : The co-ordinate of the point dividing the join of A (2, 5) and B (1, 9) in the ratio K : 1 are $\left(\frac{K+2}{K+1}, \frac{9K+5}{K+1}\right)$ (1)

i) If this point lies on x axis then its y co-ordinate is zero

i.e. $\frac{9K+5}{K+1} = 0$

$\Rightarrow$ K = $\frac{-5}{9}$

$\therefore$ x axis divides AB externally in the ratio 5:9

Substituting K $= \frac{-5}{9}$ in (1) the co-ordinates of the point

of division is $\left(\frac{K+2}{K+1}, \frac{9K+5}{K+1}\right) = \left(\frac{\frac{-5}{9}+2}{\frac{-5}{9}+1}, \; 0\right)$

$= \left(\frac{13}{4}, 0\right)$

ii) If the point lies on y axis then its abscissa (x coordinate) = 0

i.e. $\frac{K+2}{K+1} = 0$

$\Rightarrow$ K = -2

$\therefore$ y axis divides AB externally in the ratio 2:1

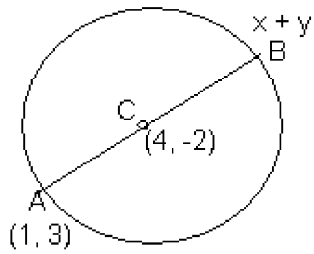
Coordinates of the point of division (K = -2)

$= \left(\frac{K+2}{K+1}, \frac{9K+5}{K+1}\right) = \left(0, \frac{9(-2)+5}{-2+1}\right) = (0,13)$

Example 5 One end of a diameter of a circle is (1, 3) and its centre is (4, -2). Find the coordinates of the other end of this diameter.

Solution : Let the co-ordinates of the other end be B (x, y) Mid point of AB is the centre C (4, -2)

$\therefore \frac{x+1}{2} = 4 \text{ and } \frac{y+3}{2} = -2$



x= 7 and y = -7

The other end B ≡ (7, -7)

Aliter:

B divides AC in the ratio 2:1 externally

B $= \left(\frac{2(4)-1(1)}{2-1}\right), \frac{2(-2)-1(3)}{2-1}$

= (7, -7)

Example 6 Three corners of a parallelogram ABCD taken in order are A (-1, -6), B(2, -5) and C(7, 2) Find the fourth vertex.

Solution : Let the fourth vertex be D (x, y)

Since ABCD is a parallelogram, the midpoint of the diagonal BD = midpoint of the diagonal AC.

$$\left(\frac{2+x}{2}, \frac{-5+y}{2}\right) = \left(\frac{-1+7}{2}, \frac{-6+2}{2}\right)$$

$\Rightarrow$ $2 + x = -1 + 7$ and $-5 + y = -6 + 2$

$\Rightarrow$ $x = 4$ and $y = 1$

$\therefore$ The fourth vertex is (4, 1)

Example 7 Find the co-ordinates of the centroid of the triangle whose vertices are (2, 3), (-5, 2) and (1, 7)

Solution : Let A (2, 3), B (-5, 2) , C (1, 7) be the vertices of the triangle

The centroid G is given by

$$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) = \left(\frac{2+-5+1}{3}, \frac{3+2+7}{3}\right)$$

$$= \left(\frac{-2}{3}, 4\right)$$

Example 8 Find the third vertex of a triangle if two of its vertices are at (-2, 4) and (7, -3) and the centroid at (3, -2)

Solution : Given A = (- 2, 4) B = (7, -3), G = (3, -2) we have

$$x = \frac{x_1+x_2+x_3}{4} \quad \Rightarrow \quad 3 = \frac{-2+7+x_3}{3}$$

$$\Rightarrow \quad x_3 = 4$$

$$y = \frac{y_1+y_2+y_3}{3} \quad \Rightarrow \quad -2 = \frac{4-3+y_3}{3}$$

$$\Rightarrow \quad -6 = 1 + y_3$$

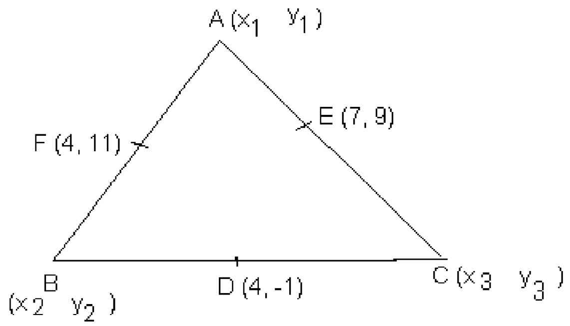
$$\Rightarrow \quad y_3 = -7$$

$\therefore$ Third vertex $(x_3, y_3) = (4, -7)$

Example 9 Find the co-ordinates of the vertices of the triangle given the mid points of the sides as (4, -1), (7, 9), (4, 11)

Solution : Let A, B and C be the vertices of the triangle and D, E and F be the mid points of the sides BC, CA and AB respectively.

Let A = (x_1, y_1) B = (x_2, y_2) and C = (x_3, y_3)



Now D = midpoint of BC

$$= (4, -1) = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right) \quad \left(\begin{array}{l} x_2 + x_3 = 8 \ldots\ldots\ldots(1) \\ y_2 + y_3 = -2 \ldots\ldots\ldots(2) \end{array}\right.$$

E = mid point of CA

$$(7, 9) = \left(\frac{x_3 + x_1}{2}, \frac{y_3 + y_1}{2}\right) \rightarrow \left(\begin{array}{l} x_3 + x_1 = 14 \ldots\ldots\ldots(3) \\ y_3 + y_1 = 18 \ldots\ldots\ldots(4) \end{array}\right.$$

F = mid point of AB

$$(4, 11) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \rightarrow \left(\begin{array}{l} x_1 + x_2 = 8 \ldots\ldots\ldots(5) \\ y_1 + y_2 = 22 \ldots\ldots(6) \end{array}\right.$$

Solving (1), (3), (5) we get x_1, x_2 and x_3

Consider (1) + (3) + (5) we get

$$2\,(x_1 + x_2 + x_3) = 30$$

$$(x_1 + x_2 + x_3) = 15$$

$x_1 + x_2 = 8$ and $x_1 + x_2 + x_3 = 15 \Rightarrow x_3 = 7$

$x_2 + x_3 = 8$ and $x_1 + x_2 + x_3 = 15 \Rightarrow x_1 = 7$

$x_3 + x_1 = 14$ and $x_1 + x_2 + x_3 = 15 \Rightarrow x_2 = 1$

Consider (2) + (4) + (6) we get

$$2(y_1 + y_2 + y_3) = 38$$
$$(y_1 + y_2 + y_3) = 19$$

Now $y_2 + y_3 = -2$ and $(y_1 + y_2 + y_3) = 19 \Rightarrow y_1 = 21$

$y_3 + y_1 = 18$ and $(y_1 + y_2 + y_3) = 19 \Rightarrow y_2 = 1$

$y_1 + y_2 = 22$ and $(y_1 + y_2 + y_3) = 19 \Rightarrow y_3 = -3$

Thus A = (7, 21) B = (1, 1) and C (7, -3)

EXERCISE : 15.3

1. Find the co-ordinates of the point which divides the line joining the following pair of points , in the given ratio
   a. (3,2) and (1,4) in the ratio 5:6 internally
   b. (2,4) and (-3,10) in the ratio 1:4 internally
   c. (-1,2) and (5,-7) in the ratio 3:4 internally
   d. (0,4) and (-3,7) in the ratio 5:4 externally
   e. (3,5) and (-2,7) in the ratio 2:3 externally
   f. (1,3) and (2,7) in the ratio 3:4 externally
2. The line joining the points (1,-2) and (-3,4) is trisected. Find the coordinates of the point of trisection
3. Find the coordinates of the point of trisection of the line joining (5,-6) and (-3,4)
4. Find the coordinates of the point of trisection of the line joining (15,-18) and (-9,12)
5. Find the coordinates of the centre of the circle having (-3,4) and (5,8) as ends of the diameter.
6. Find the other end of the diameter of the circle whose centre is the origin and one end of the diameter is (-3,6).
7. (-3,4) is a point on a circle whose centre is the origin. Find the coordinates of the extremity of the diameter through (-3,4). Also find the radius of the circle.

8. Find the mid point of the line segment joining the points
   a. (1,3) ,(5,7)
   b. (-2,6),(3,-1)
   c. (1/2,3/2),(2,5)
9. Find the centroid of the triangle with vertices
   a. (7,-3),(4,6),(-11,-3)
   b. (a,0), (0,b), (x,y)
   c. (-3,2),(1,-4),(-4,5)
10. The centroid of the triangle ABC is the point (2,3). The co ordinates of A are (5,6) and B(-1,4).Find the co ordinates of C.
11. Find the distance of the centroid of the triangle formed by the points (7,1),(1,5) and (1,6) from the origin.
12. Find the co ordinates of the point of trisection of the medians of the triangle whose vertices are (-2,-3),(-1,7) and (5,2)
13. Find the ratio in which the point (-5,2) divide the join of (-7,1) and (3,6) internally ?
14. In what ratio does the point (2,3) divides the line joining the points (-1,0) and (4,5) externally ?
15. Find the ratio in which the line segment joining (2,-3) and (5,6) is divided by the x axis?
16. Find the ratio in which the line segment joining the points (4,5) and (1,2) is divided by the x axis? Also find the Coordinates of the point of division.
17. In what ratio is the line segment joining the points (4,5) and (1,2) divided by the y axis? Find also the Co ordinates of the point of division.
18. Find the ratio in which the line segment joining (2,-3) and (5,6) is divided by the y axis?
19. Find the length of the medians of the triangle with vertices (-3,6), (5,4), (1,-2)
20. The three vertices of a rhombus taken in order are (2,-1),(3,4),(-2,3). Find the fourth vertex?
21. Three consecutive vertices of a parallelogram are A(3,0), B(5,2), C(-2,6). Find the fourth vertex D.

22. If the mid points of the sides of the triangle are (2,6),(4,6) and (3,5) then find the vertices of the triangle?

23. The mid points of the sides of the triangle ABC is given by P (3,1), Q (5,6) and R(-3,2). Find the coordinates of the vertices of the triangle

15.6 AREA OF A TRIANGLE

The area of the triangle whose vertices are A(x_1,y_1) , B(x_2,y_2) and C(x_3,y_3) is given by

$$\Delta ABC = \frac{1}{2}\left(x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right)$$

Note:

1. The area of the triangle ABC can also be written in the summation notation

$$\Delta ABC = \frac{1}{2}\sum \left(x_1\left(y_2 - y_3\right)\right)$$

2. It is a convention that the area of the triangle is taken to be positive, when the vertices A,B,C of the triangle taken in the anticlockwise direction, during the calculation of the area.

3. If the three points A(x_1,y_1) , B(x_2,y_2) and C(x_3,y_3) are collinear (i.e., lie on the same line) then the area of the triangle ABC is zero and conversely.Thus the condition for the three points A,B,C to be collinear is that the area of the triangle A,B,C is zero.

Example 1 Find the area of the triangle whose vertices are A(3,4) , B(2,-1) and C(4,-6)

Solution

$$\Delta ABC = \frac{1}{2}\left(x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right)$$

$$= \frac{1}{2}\left(3(-1+6) + 2(-6-4) + 4(4+1)\right)$$

$$= \frac{15}{2}$$

Area of the triangle is 15/2 sq.units.

Example 2 Show that the points (-5,1),(5,5) and (10,7) are collinear

Solution : Let A=(-5,1),B=(5,5) and C= (10,7)

we shall show that area of the triangle ABC =0

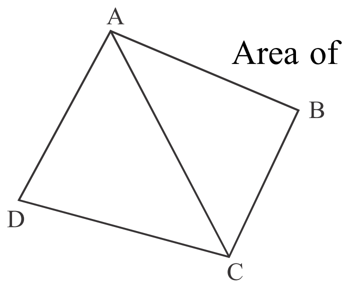
$$\Delta ABC = \frac{1}{2}\left(-5(5-7)+5(7-1)+10(1-5)\right)$$

$$= 0$$

Thus the points A,C and B are collinear

Example 3 Find the area of the quadrilateral whose vertices are A(1,1), B(7,-3) and C(12,2) and D(7,21)

Solution: **The required area ABCD= area of the triangle ABC + area of triangle ACD**



Area of the triangle ABC $= \frac{1}{2}\left(1(-3-2)+7(2-1)+12(1+3)\right)$

= 25 sq.units

Similarly

area of the triangle ACD =**107** sq.units

Area of the triangle ABCD=25+107=132sq.units

Example 4 For what value of 'a' the points (1,4),(a,-2),(-3,16) are collinear

Solution : Let A=(1,4) B=(a,-2) C=(-3,16)

Given these points are collinear.

then the area of the triangle ABC=0

i.e. $\frac{1}{2}\left(1(-2-16)+a(16-4)-3(4+2)\right)=0$

i.e. $\left(1(-2-16)+a(16-4)-3(4+2)\right)=0$

i.e. 12a-36=0

i.e. a=3

EXERCISE

I. Find the area of the triangle whose vertices are

a) A(6,3), B(-3,5) and C(4,-2)

c) A(0,0), B(-2,3) and C(10,7)

b) A(a,0) , B(0,b) and C(x,y)

d) A(5,2) , B(-9,-3) and C(-3,-5)

II. If the area of the triangle whose vertices are A(x,y) , B(1,2) and C(2,1) is 6, show that x+y=15

III Show that the following points are collinear

a) A(1,-1) , B(2,1) and C(4,5)

b) A(a,b+c) , B(b,c+a) and C(c,a+b)

c) A(-5,-4) , B(1,2) and C(3,4)

d) A(4,-5) , B(1,1) and C(-2,7)

IV Find the area of the quadrilateral whose vertices are

a) (1,2), (6,2), (5,3), (3,4)

b) (-3,2), (7,-6), (-5,-4), (5,4)

c) (1,1), (3,4), (5,-2), (4,-7)

ANSWERS

I a) 49/2

b) (bx+ay-ab)/2

c) 22

d) 29

II a) 11/2

b) 60

c) 41/2

Chapter 16

LOCUS AND ITS EQUATIONS

16.1 INTRODUCTION

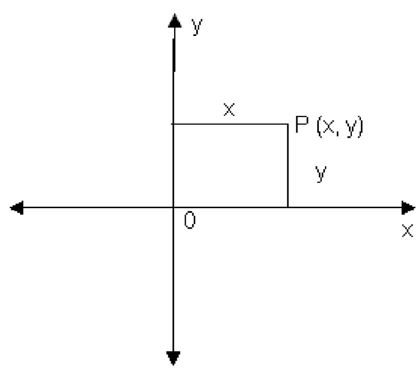
The most important aspect in analytical geometry is to express the plane curve in terms of algebraic equation, called the **equation of curve.** For this purpose, we define the curve as the path traced by a moving point under a given geometrical conditions. This is known as the **locus of a point**. For example, the path traced by a point which moves such that the distance from the fixed point is always the same is a curve called a circle. Locus of a point which moves so that it is equidistant from two fixed points is the perpendicular bisector of the line joining the fixed points.

16.2 EQUATION OF THE LOCUS OF A POINT

A relation between x and y when the point P(x,y) moves is called the equation of the locus of the point P (x, y)

Worked Examples:

Example 1 Find the equation of the locus of a point which moves so that its distance from the year is 3 times its distance from the x axis.



Solution: Let P (x, y) be a point on the locus

Distance from the y axis = x

Distance from the x axis = y

$\therefore\ x = 3y$ or $x - 3y = 0$

Example 2 Find the equation of the locus of the point equidistant from (-1, 1) and (4,2)

Solution : Let P(x, y) be a point which is equidistant from A (-1, 1) and B (4, 2) Then |PA| = |PB|

$$\therefore \sqrt{(x+1)^2 + (y+1)^2} = \sqrt{(x-4)^2 + (y-2)^2}$$

Squaring

$$(x + 1)^2 + (y + 1)^2 = (x - 4)^2 + (y - 2)^2$$

i.e. $x^2 + 2x + 1 + y^2 + 2y + 1 = x^2 - 8x + 16 + y^2 - 4y + 4$

i.e. $2x + 2y + 2 = - 8x - 4y + 20$

i.e. $5x + 3y = 9$

Hence the locus of P is the straight line $5x + 3y = 9$

Example 3 Find the equation of the locus of points such that the sum of its distance from (0, 3) and (0, -3) is 8

Solution : Let P (x, y) be any point of (0, -3) respectively. We are given that $|PA| + |PB| = 8$

$$\therefore \sqrt{(x-0)^2+(y-3)^2}+\sqrt{(x-0)^2+(y+3)^2} = 8$$

i.e. $\sqrt{x^2+y^2-6y+9}+\sqrt{x^2+y^2+6y+9} = 8$

$$\sqrt{x^2+y^2-6y+9} = 8 - \sqrt{x^2+y^2+6y+9}$$

Squaring $x^2 + y^2 - 6y + 9 = 64 - 16\sqrt{x^2+y^2+6y+9}+x^2+y^2+6y+9$

$$12y + 64 = 16$$

$$3y + 16 = 4$$

Squaring again $(3y + 16)^2 = 16\,(x^2 + y^2 + 6y + 9)$

i.e. $16x^2 + 7y^2 = 112$

which is the required equation of the lcus.

Example 4 Find the locus of a point which moves so that its distances from the point A(3,1) and B(1, 3) are in the ratio 2:3.

Solution : Let P (x, y) be a point on the locus so that

$$\frac{PA}{PB} = \frac{2}{3}$$

$\therefore$ 3PA = 2PB

i.e. $3\sqrt{(x-3)^2+(y-1)^2} = 2\sqrt{(x-1)^2+(y-3)^2}$

Squaring on both sides we get

$$9\,(x^2 - 6x + 9 + y^2 - 2y + 1) \;=\; 4\,(x^2 - 2x + 1 + y^2 - 6y + 9)$$

i.e. $5x^2 + 5y^2 - 46x + 6y + 50 = 0$ is the required locus.

Example 5 Find the equation of the locus of the point which moves such that it is equidistant from (4, 2) and the x axis.

Solution : Let A = (4, 2) and P (x, y)be any point on the locus.

Thus by data PA = distance of P from x axis.

We know that distance of any point from x axis is its y co-ordinate.

$$\sqrt{(x-4)^2 + (y-2)^2} \;=\; y$$

$\Rightarrow \quad (x - 4)^2 + (y - 2)^2 = y^2$

$\Rightarrow \quad x^2 - 8x + 16 + y^2 - 4y + 4 = y^2$

$\Rightarrow \quad x^2 - 8x - 4y + 20 = 0$

This is the equation of the locus.

Example 6 A point P moves such that $PA^2 = 3PB^2$. If A = (5, 0) and B = (-5, 0) Find the equation of the locus of P.

Solution : Let P = (x, y)

By data $PA^2 = 3\,PB^2$

$(x - 5)^2 + y^2 = 3\,\{(x + 5)^2 + y^2\}$

$x^2 - 10x + 25 + y^2 = 3\,(x^2 + 10x + 25 + y^2)$

$2x^2 + 2y^2 + 40x + 50 = 0$

$x^2 + y^2 + 20x + 25 = 0$

This is the equation of the locus.

Example 7 Find the equation of the perpendicular bisector of the line joining A(3, -2) and B (4, 1)

Solution: The perpendicular bisector of the line joining A and B is the locus of the point which moves such that it is equidistant from A and B

Now we have A (3, -2) and B (4, 1)

Let P (x, y) be any point on the perpendicular bisector. Thus we have.

$$PA = PB$$

$$PA^2 = PB^2$$

$$(x-3)^2 + (y+2)^2 = (x-4)^2 + (y-1)^2$$

$$\Rightarrow \quad x^2 - 6x + 9 + y^2 + 4y + 4 = x^2 - 8x + 16 + y^2 - 2y + 1$$

$$\Rightarrow \quad 2x + 6y - 4 = 0$$

$$\Rightarrow \quad x + 3y - 2 = 0$$

This is the equation of the locus.

EXERCISE : 16.1

I Find the equation of the locus of the point which moves such that

a) its distance from (1, 2) is 3

b) its distance from the co-ordinate axes which is in the rate 5:3

c) the sum of its distance from the co-ordinate axes is 5

d) the square of its distance from (2, 3) is 3

e) the sum of the square of its distances from the co-ordinate axes is 2

II. Find the equation of the locus of the point which moves such that

a) its distance from the point (-4, 0) is 4 times its distance from (0, -2)

b) it forms a right angled triangle with the points (2, 3) and (3,4)

c) it is collinear with the points (2, 4) and (5, 9)

d) it lies on the perpendicular bisector of the line joining the points (-1, 5) and (2,4)

III. 1. Find the equation of the locus of all points equidistant from the point (4, 2) and x axis.

2. Find the equation of locus of a point such that the sum of its distance from (0, 2) and (0, -2) is 6.

3. Find the equation of the locus of all points equidistant from the point (2, 4) and y axis.

4. Find the equation of the locus of the point P(x, y) such that its distance from (1, -2) is greater than 3.

5. Find the equation of the locus of points twice as far from (-a, 0) as from (a, 0)

ANSWERS : 16.1

I.
a. $x^2 + y^2 - 2x + 4y - 4 = 0$
b. $5x = 3y$
c. $x + y = 5$
d. $x^2 + y^2 - 4x - 6y + 10 = 0$
e. $x^2 + y^2 = 2$

II
a. $15x^2 + 15y^2 - 8x + 64y + 48 = 0$
b. $x^2 + y^2 - 5x - 7y + 18 = 0$
c. $5x - 3y + 2 = 0$
d. $3x - y + 3 = 0$

III.
1) $x^2 - 8x - 4y + 20 = 0$
2) $9x^2 + 5y^2 = 45$
3) $y^2 - 8y - 4x + 20 = 0$
4) $x^2 + y^2 + 2x - 4y - 4 > 0$
5) $3x^2 + 3y^2 - 10ax + 3a^2 = 0$

CHAPTER 17

STRAIGHT LINE

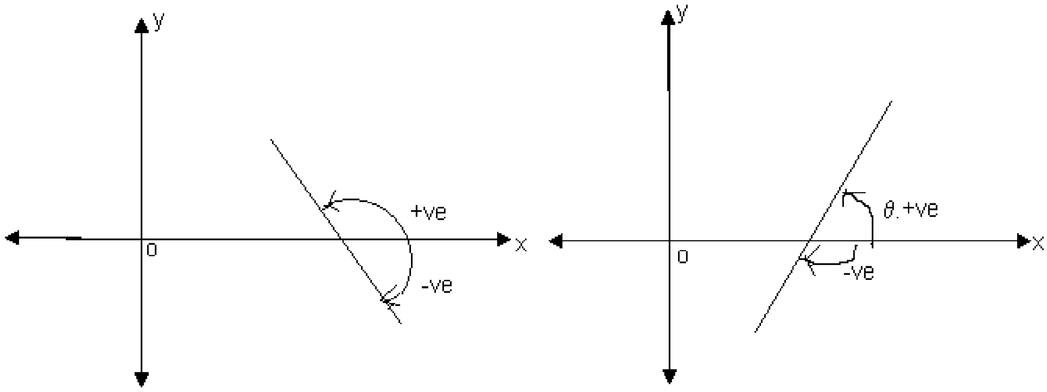
17.1 INTRODUCTION

In this chapter we learn some very important basic terminologies about a straight line, conditions for the two lines to be parallel and perpendicular.

Also we study different forms of equation of straight line. We shall conclude this chapter with some important derivations like angle between two lines, length of the perpendicular from the origin and from any point to a line etc.

17.2 SLOPE OR GRADIENT OF A LINE

In the co-ordinate plane, a line L which is not parallel to the x axis intersects it such a way that it makes two angles which are supplementary. To be definite we select an angle which is made going anticlockwise direction from the x axis. This angle θ will have values between 0^0 and 180^0 and is called angle of inclination or simply inclination of the line L. all lines parallel to the x axis or coincident with the x-axis have inclination 0°. Note that when two lines are parallel they have same inclination. In analytical geometry we associates a number with the inclination of a line called its slope.



Definition: The slope m of a line with an inclination θ (not perpendicular to x axis) is defined to be the tangent of the angle made by the line with the x axis in the positive direction i.e. **m = tan** θ

The slope of a line perpendicular to the x axis is not defined since the value of tan 90^0 is undefined.

Following observation follows directly from the definition of the slope of a line

1. Let θ be the angle made by the line with the x axis in the positive direction. Now

   Slope of a line is positive $\Leftrightarrow$ Tan θ is positive

   $\Leftrightarrow$ θ is lies between 0^0 and 90^0

   $\Leftrightarrow$ θ is acute

   Slope of a line is negative $\Leftrightarrow$ Tan θ is negative

   $\Leftrightarrow$ θ lies between 90^0 and 180^0

   $\Leftrightarrow$ θ is obtuse

2. From the above observation we have:

   a) If the slope of the line is positive, as we move from left to right along the line, then the line will "rise"

   b) If the slope of the line is negative, as we move from left to right along the line, then the line will "fall"

3. If a line is parallel to x axis (called as horizontal line) then the angle made by it with the x axis is zero. Hence the slope is zero. In particular.

   Slope of x axis is zero.

4. The slope of a line parallel to y axis (called as vertical line) is not defined since the value of tan 90^0 is undefined and here we take the slope as '∞' (infinity).

5. If three points A, B and C are collinear then the slope of AB = slope of BC and conversely. This fact can be used to establish that the given points are collinear.

17.3 SLOPE OF PARALLEL AND PERPENDICULAR LINES:

a) If two lines are parallel then their slopes are equal and conversely.

b) If two lines are perpendicular to each other, then the product of their slopes is -1 and conversely. i.e. If m_1 and m_2 are the slopes of two lines then we have the condition for parallel lines given by $\mathbf{m_1 = m_2}$. And if the lines are perpendicular then $\mathbf{m_1\ m_2 = -1}$.

NOTE: If the slope of the line is 'm' then the slope of any line parallel to it is given by 'm'. Also the slope of any line perpendicular to it is given $\frac{-1}{m}$ (negative reciprocal of m)

17.4 SLOPE OF THE LINE JOINING TWO POINTS

The slope of the line joining the points (x_1, y_1) and (x_2, y_2) is given by

$$\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1}$$

Worked out Examples:

Example 1 Find the slope of the line joining the points A and B where A (3, -2) and B(-2, 1)

Solution: Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-2)}{-2-3} = \frac{3}{-5} = \frac{-3}{5}$

Example 2 Find the value of x so that the slope of the line joining the points (2, 5) and $(x, 13)$ is 20

Solution : Let A = (2, 5) B = $(x, 13)$

Slope m = 20

$\Rightarrow \quad \frac{13-5}{x-2} = 20$

$\Rightarrow \quad 8 = 20(x - 2)$

$\Rightarrow \quad 8 = 20x - 40$

$\Rightarrow \quad 48 = 20x$

$\therefore \quad x = \frac{48}{20} = \frac{12}{5}$

Example 3 Show that the points (-4, -5) , (1, -1) and (6, 3) are collinear

Solution: Let A (-4, -5) B (1, -1) and C (6, 3) be the given points. If the points are collinear then they lie on the same line.

Then the slope of AB = slope of AC

Slope of AB $= \dfrac{-1-(-5)}{1-(-4)} = \dfrac{4}{5}$

Slope of AC $= \dfrac{3-(-5)}{6-(-4)} = \dfrac{8}{10} = \dfrac{4}{5}$

A, B, C are collinear.

Example 4 Find the value of K if the points A (2, 3) B (-1, -2) and C (5, K) are collinear.

Solution : If A, B and C are collinear then

Slope of AB = slope of BC = slope of AC

Slope of AB $= \dfrac{-2-3}{-1-2} = \dfrac{-5}{-3} = \dfrac{5}{3}$

Slope of AC $= \dfrac{K-3}{5-2} = \dfrac{K-3}{3}$

$\dfrac{5}{3} = \dfrac{K-3}{3} \Rightarrow 5 = K - 3 \quad \Rightarrow \quad K = 8$

Example 5 Show that the line joining the points (3, -4) and (-4, 0) is

a) Parallel to the line joining (7, -1) and (0, 3)

b) Perpendicular to the line joining (4, 5) and (0, -2)

Solution : Let A $\equiv$ (3, -4), B $\equiv$ (-4, 0)

Slope of AB $= m_1 = \dfrac{0-(-4)}{-4-3} = \dfrac{-4}{7}$

a) Let C $\equiv$ (7, -1) and D $\equiv$ (0, 3)

Slope of CD $= m_2 = \dfrac{3-(-1)}{0-7} = \dfrac{-4}{7}$

$m_1 = m_2 \Rightarrow$ AB || CD

b) Let E $\equiv$ (4, 5) and F $\equiv$ (0, -2)

Slope of EF $= m_3 = \dfrac{-2-5}{0-4} = \dfrac{7}{4}$

$m_1\, m_2 = \dfrac{-4}{7} \times \dfrac{7}{4} = -1 \Rightarrow$ AB $\perp$ EF

Example 6 If the line joining the points (2, 3) and (5, 4) is parallel to the line joining the points (3, K) and (4, 2) find the value of K.

Solution Let A ≡ (2, 3) B ≡ (5, 4) C ≡ (3, K) D ≡ (4, 2)

By data AB || CD

Then slope of AB = slope of CD

$$\Rightarrow \frac{4-3}{5-2} = \frac{2-K}{4-3}$$

$$\Rightarrow \frac{1}{3} = 2 - K$$

$$\Rightarrow K = \frac{5}{3}$$

Example 7 If A (2, 3), B (-3, -2), C (6, K) and D (5, 2) are four points such that the line AB is perpendicular to the line CD then find K.

Solution If AB || CD then slope of AB × slope of CD = -1

$$\left(\frac{-2-3}{-3-2}\right) \times \left(\frac{2-K}{5-6}\right) = -1$$

$$\left(\frac{-5}{-5}\right)\left(\frac{2-K}{-1}\right) = -1$$

$$2 - K = 1$$

$$\therefore K = 1$$

Example 8 Show that the points (8, 6), (3, 4) and (7, -6) form a right angled triangle.

Solution Let A = (8, 6) B = (3, 4) C = (7, -6)

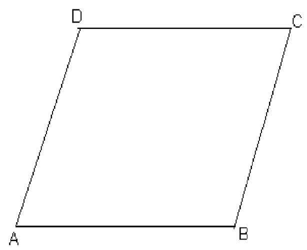
$$\text{Slope of AB} = \frac{4-6}{3-8} = \frac{2}{5} = m_1$$

$$\text{Slope of BC} = \frac{-6-4}{7-3} = \frac{-10}{4} = \frac{-5}{2} = m_2$$

$$m_1\, m_2 = \frac{2}{5} \times \frac{-5}{2} = -1. \text{ So } AB \perp BC$$

∴ ABC is a right angled triangle.

Example 9 Using the concept of the slope, show that the points A(-3, 2), B (3, 4), C (5, -2) and D (-1, -4) are the vertices of a parallelogram.



$$\text{Slope of AB} = \frac{4-2}{3-(-3)} = \frac{2}{6} = \frac{1}{3}$$

$$\text{Slope of CD} = \frac{-4+2}{-1-5} = \frac{-2}{-6} = \frac{1}{3}$$

$$\text{Slope of BC} = \frac{-2-4}{5-3} = \frac{-6}{2} = 3$$

$$\text{Slope of DA} = \frac{-4-2}{-1+3} = \frac{-6}{2} = -3$$

Solution Slope of AB = Slope of CD & Slope of AD = Slope of BC = We find AB $\|$ CD since their slopes are equal. Also AD $\|$ BC. $\therefore$ A, B, C, D form the vertices of a parallelogram.

EXERCISE: 17.1

I.

1. Find the slope of the line joining the points
   a) (3, 2) and (-1, 5) b) (2, 0) and (0, 2)
2. If the slope of the line joining (3, a) and (4, 3) is 7/2, find 'a'
3. If the line joining the points (6, 4) and (8, K) makes an angle of 45° with the x axis find K.
4. Find the slopes of the lines with the following inclinations
   a) $\frac{\pi}{4}$ b) 60^0 c) $\frac{\pi}{2}$
5. If the slope of the line AB is $\frac{3}{2}$ and the line CD is perpendicular to AB, then find the slope of CD

II.

1. Using slopes, prove that the points (1, 1), (-2, 4) and (3, -1) are collinear.
2. Find K if AB is parallel to CD where A (K,1), B (4, 3), C (1, 5) and D (-1, 1)
3. If (3,a) lies on the line joining (1, -4) and (-2, 5) find a
4. Show that (3, 2) (0, 4) (-4, -2) and (-1, -4) are the vertices of a parallelogram.
5. Show that the following points form a right angled triangle
   a) A (5, 4), B (-6, 2), C (2, -2)
   a) A (6, 8), B (1, 3), C (4, 2)
6. What is the value of y so that the line joining (4,1) and (y, 3) is perpendicular to the line joining (1, 6) and (-1, -2)
7. State whether the two lines AB and CD are parallel, perpendicular or neither
   a) A (5, 6), B (2, 3) and C (9, 2), D (6, -5)
   b) A (8, 2),B (-5, 3) and (C 16, 6), D (3, 15)
   c) A (2, -5), B (-2, 5) and C (6, 3), D (1, 1)
8. Show that the line joining (2, -3) and (-5, 1) is
   a) Parallel to the line joining (7, -1) and (0, 3)
   b) Perpendicular to the line joining (4, 5) and (0, -2)
9. What is the value of x so that the line joining (x, 2) and (11, 8) is parallel to the line through (8, 12) and (0, 6)

ANSWERS : 17.1

I.

1. a) $\frac{-3}{4}$ b) – 1
2. $\frac{-1}{2}$
3. 6

4. a) 1 b) $\sqrt{3}$ c) ∞

5. $\frac{-2}{3}$

II.

2. 3
3. a = -10
6. y = - 4
7. a) parallel b) neither c) perpendicular
9. $x = 3$

17.5 STANDARD FORMS OF EQUATION OF STRAIGHT LINES

Equation of the coordinate axes and the lines parallel to the co-ordinate axes

a) **Equation of x axis**

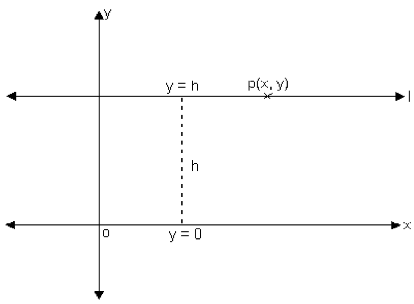
We know that the y co-ordinate of every point on the x-axis is zero. This is the common property that every point on the x-axis will satisfy. Thus, the equation of x axis is y = 0

b) **Equation of y axis**

The x coordinate of each point on the y axis is zero. Thus the equation of y axis is $x = 0$

To Find the equation of the St. line parallel to x axis

Let 'l' be a straight line parallel to x axis and at a directed distance 'h' from it. Let P (x, y) be a general point on the line 'l' $\therefore$ y = h This is the required equation of the line



Note: If a line is parallel to x axis at a distance of 'a' units from the x axis (lying above the x axis) the equation of the line is y = a provided 'a' is positive. If 'a' is negative, then the line lies below the x axis and so the equation becomes y = -a

For Example, equation of the line parallel to x axis which is at a distance of 15 units above the x axis is y = 15 or y – 15 = 0

The equation y + 3 = 0 or y = -3 represent the line parallel to x axis, which is 3 units below the x-axis.

Example 1 Find the equation of the line which is parallel to x axis and at a distance of 5 units below the x axis

Solution : Equation of the line parallel to x axis and is 5 units below the x axis is given by y = -5

Example 2 Find the equation of the line which is parallel to the x axis and at a distance of $\frac{5}{2}$ units above x axis.

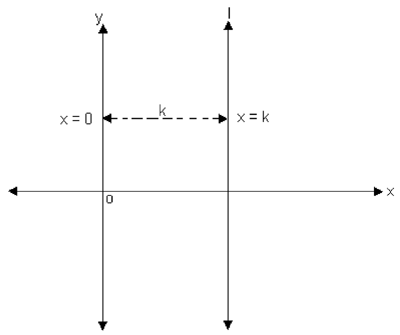
Solution : Since the line is above the x axis and parallel to x axis equation is given by

$$y = \frac{+5}{2}$$

$\Rightarrow 2y = 5$ or $2y - 5 = 0$

To Find the equation of the Straight line which is parallel to y axis

Let 'l' b a straight line parallel to y axis and at a directed distance 'k' from it Let P (x, y) be a general point on the line. $\therefore x$ = k which is the required equation of the line.



Note: Equation of y axis is x = 0. If k > 0, then the line lies on the right of the y axis and if k < 0 then the line lies on the left of the y axis.

Example 1 Find the equation of the line parallel to y axis and is at a distance of

a) 2 units to the right of it

b) 4/7 units to the left of it

Solution : Equation of the line parallel to y axis and is at a

a) distance of 2 units to the right of it is given by $x = 2$

b) Equation of the required line is $x = \frac{-4}{7}$ i.e. $7x + 4 = 0$

Example 2 Find the equation of the line which is

a) Parallel to x axis and passing through (2, -3)

b) Parallel to y axis and passing through (3, -4)

Solution : Any line parallel to x axis is of the form y = k. By data

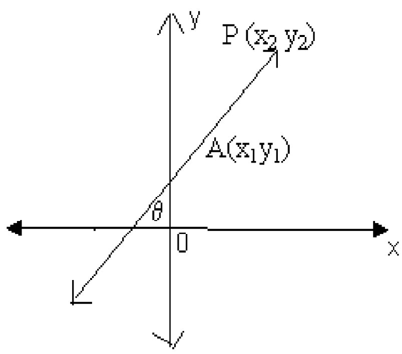
a) this should pass through (2, -3). So K = -3.
Thus the equation of the line is y = -3 i.e. y + 3 = 0

b) Any line parallel to y axis is of the form x = k. By data.
This should pass through (3, -4). Thus k = 3
Thus equation of the line is x = 3.

Equation of the line in slope point form.

To find the equation of the line whose slope is m and passing through the point $A(x_1, y_1)$

Let $A = (x_1, y_1)$. Let P (x, y) be any point on the line, Now slope of AP = $\frac{y - y_1}{x - x_1}$ By data, slope of the line is m = $\frac{y - y_1}{x - x_1}$



This relation is true for any point on the line.

Thus, the equation of the line whose slope is m and passing through (x_1, y_1) is given by

$$\mathbf{y - y_1 = m (x - x_1)}$$

This form of the equation of a line is called **slope point form.**

Note: Equation of the line passing through the origin and having slope 'm' is given by **y=mx.**

Example 1 Determine the equation of the line passing through (-1, -2) and with slope $\frac{4}{7}$

Solution $(x_1, y_1) = (-1, -2)$

$m = \frac{4}{7}$

Equation of the line is given by $y - y_1 = m(x - x_1)$

$$y - (-2) = \frac{4}{7}(x - 1)$$

$$y + 2 = \frac{4}{7}(x + 1)$$

$$7(y + 2) = 4(x + 1)$$

i.e. $4x - 7y = 10$

Example 2 Find the equation of the line passing through (0, -4) and making an angle of 30^0 with the x axis

Solution : $m = \tan 30^0 = \frac{1}{\sqrt{3}}$

$(x_1, y_1) = (0, -4)$

Equation of the line is given by $y - y_1 = m(x - x_1)$

$$y + 4 = \frac{1}{\sqrt{3}}(x - 0)$$

$$\sqrt{3}\,(y + 4) = x$$

i.e. $x - \sqrt{3y} = 4\sqrt{3}$

Example 3 Find the equation of the line passing through (-1, -1) and perpendicular to the line whose slope is $\frac{-2}{5}$

Solution : Let $m_1 = \frac{-2}{5}$. Let m_2 be the slope of the required line.

By data $m_1 . m_2 = -1$

$$\frac{-2}{5} m_2 = -1 \qquad \Rightarrow m_2 = \frac{5}{2}$$

Let $(x_1\ y_1) = (-1, -1)$

Equation of the required line is

$y - y_1 = m (x - x_1)$

$$y + 1 = \frac{5}{2} (x + 1)$$

$\Rightarrow \quad 2y + 2 = 5x + 5$

$\Rightarrow \quad 5x - 2y + 3 = 0$

Example 4 Find the equation of the line passing through (3, -2), and parallel to the line whose slope is $\frac{-5}{7}$

Solution : Let $m_1 = \frac{-5}{7}$. Then the slope of required line m_2 will be the same since the lines are parallel.

So $m_2 = \frac{-5}{7}$

Let $(x_1, y_1) = (3, -2)$

Equation of the required line is given by $y - y_1 = m (x - x_1)$

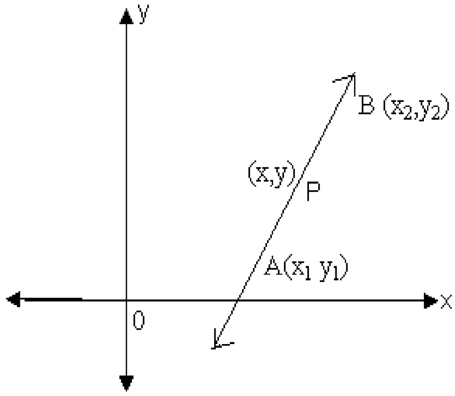
$$y + 2 = \frac{-5}{7} (x - 3)$$

$7 (y + 2) = -5 (x - 3)$

i.e. $5x + 7y = 1$

Equation of the line in two point form

Let A = (x_1, y_1) and B = (x_2, y_2). Let P(x,y) be any point on the line joining A and B.Now we observe A, B and P are collinear So slope of AP = slope of AB.



$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

This relation is true for all points on the line.

Thus the equation of the line joining the points (x_1 y_1) and (x_2 y_2) is given by

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

This equation can also be written as

$$\frac{y-y_1}{y_1-y_2}=\frac{x-x_1}{x_1-x_2}$$

This form of equation of a line is called two point form.

Example 1 Find the equation of the line passing through (0, -4) and (-6, 2)

Solution : $x_1 = 0$, $y_1 = -4$,$x_2 = -6$, $y_2 = 2$

Equation of the required line is

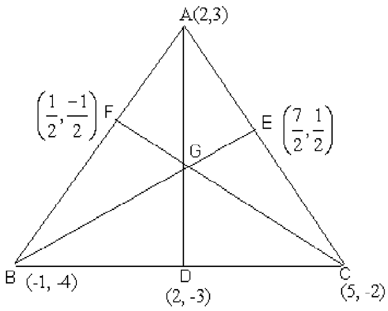
$$\frac{y-y_1}{y_1-y_2}=\frac{x-x_1}{x_1-x_2} \quad \text{i.e.} \quad \frac{y+4}{-4-2}=\frac{x-0}{0-^{-}6}$$

i.e. $$\frac{y+4}{-6}=\frac{x}{6}$$

$$\Rightarrow \quad x + y + 4 = 0$$

Example 2 Find the equation of the medians of the triangle whose vertices are A (2, 3), B (-1, -4) and C (5, -2)

Solution : D is the mid point of



$$BCD = \left(\frac{-1+5}{2}, \frac{4-2}{2}\right) = (2, -3)$$

Equation of the median AD is

$$\frac{y-3}{x-2} = \frac{-3-3}{2-2} = \frac{-6}{0}$$

$\Rightarrow x - 2 = 0$E is the midpoint of AC

$$E = \left(\frac{5+2}{2}, \frac{-2+3}{2}\right) = \left(\frac{7}{2}, \frac{1}{2}\right)$$

Equation of the median BE is

$$\frac{y-(-4)}{x-(-1)} = \frac{\frac{1}{2}-(-4)}{\frac{7}{2}-(-1)} \Rightarrow \frac{y+4}{x+1} = 1$$

$\Rightarrow$ y + 4 = x + 1

$\Rightarrow x$ – y = 3

$$\text{Mid point of AB} = F = \left(\frac{-1+2}{2}, \frac{-4+3}{2}\right)$$

$$= \left(\frac{1}{2}, \frac{-1}{2}\right)$$

Equation of CF is

$$\frac{y-(-2)}{x-5} = \frac{\frac{-1}{2}-(-2)}{\frac{1}{2}-5}$$

$$\Rightarrow \frac{y+2}{x-5} = \frac{-1}{3}$$

$\Rightarrow x$ + 3y + 1 = 0

Example 3 The vertices of a triangle are the points (0,0), (2,4) and (6,4). Find the equations of its sides

Solution : Let A (0,0), B (2, 4) and C (6, 4) be the vertices of the triangle. The equation of the side AB is

$$\frac{x-0}{2-0} = \frac{y-0}{4-0}$$

$$\frac{x}{2} = \frac{y}{4}$$

$\Rightarrow 2x - y = 0$

The equation of the side BC is

$$\frac{x-2}{6-2} = \frac{y-4}{4-4}$$

$$\Rightarrow \frac{x-2}{4} = \frac{y-4}{0} \qquad \Rightarrow y=4$$

The equation of the side AC is

$$\frac{x-6}{0-6} = \frac{y-4}{0-4} \qquad i.e., \frac{x-6}{-6} = \frac{y-4}{-4} \qquad i.e., \ 2x-3y=0$$

Example 4 Find the ratio in which the line joining (1, 2) and (4, 3) is divided by the line joining the points (2, 3) and (4, 1)

Solution Let A = (2,3) and B (4,1). The equation of the line AB is given by

$$\frac{y-3}{x-2} = \frac{1-3}{4-2} \Rightarrow \text{ y} - 3 = x + 2$$

$\Rightarrow x + \text{y} = 5$

Let the line joining the pints C (1, 2) and D (4, 3) cut the line AB at P(x, y) in the ratio r:1. Then

$$\text{P} = \left(\frac{4r+1}{r+1}, \frac{3r+2}{r+1}\right)$$

This lies on the line $x + y = 5$

Thus the co-ordinates of P must satisfy this equation.

$$\therefore \quad \frac{4r+1}{r+1} + \frac{3r+2}{r+1} = 5$$

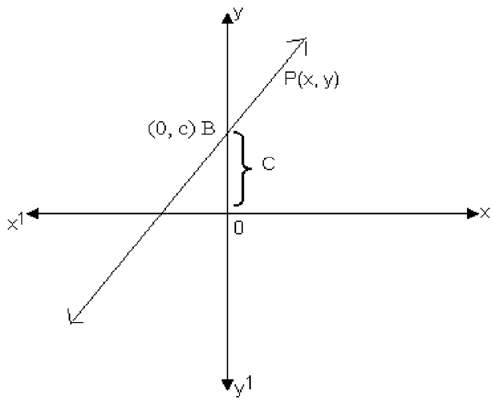
$4r + 1 + 3r + 2 = 5(r + 1)$

$7r + 3 = 5r + 5 \Rightarrow \; r = 1$

Thus the required ratio is 1:1

Equation of a line in slope intercept form

To find the equation of the line whose, slope is 'm' which cuts off intercept 'c' on y axis.



Solution.: Given y intercept = OB = c$\Rightarrow$ B = (0, c) By data slope of the line = mLet P (x, y) be any point on the line Slope of

$$BP = \frac{y-c}{x-0} = m \text{ (given)}$$

$\therefore \; y - c = m(x - 0)$

$y = mx + c$ is the equation of the line. This form is called **slope intercept form.**

Remarks: If the equation of the straight line is put in the slope intercept form $y = mx + c$ then the x coefficient gives the slope of the line.

Note: Equation of the line passing through the origin and having slope m is $y = mx$.

Example 1 Find the equation of the line whose y intercept is -2 and slope is $\frac{3}{2}$

Solution : Let $m = \frac{3}{2}$ and C = -2

Equation of the required line is of the form $y = mx + c$

$$y = \frac{3}{2}x - 2$$

$\Rightarrow \; 3x - 2y = 4$

Example 2 Find the equation of the straight line which cuts off y intercept -3 and inclined at 60^0 to x axis.

Solution : Slope of the line m = tan 60^0 = $\sqrt{3}$

Y intercept C = -3

Equation of the line y = m x+c

$$y = \sqrt{3}x - 3$$

ie. $\sqrt{3}x - y = 3$

Example 3 Find the equation of the straight line which is parallel to the line whose slope is 10 and with y intercept 2.

Solution: Slope of the required line = slope of the given line

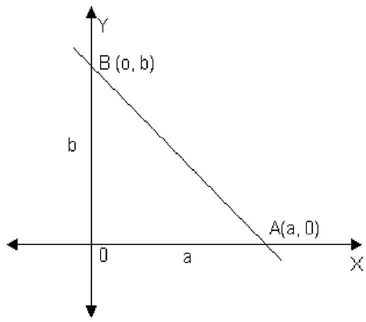
[$\because$ the two lines are parallel $m_1 = m_2$]

$\because$ slope of required line m = 10 and C = 2 (given)

Equation of the required line is of the form y = mx + c.

y = 10x + 2

Equation of the line in the intercept form



Let a line cut the x axis at A and y axis at B. Let OA = a and OB = b. Then the length OA = a is called the x intercept of the line and the length OB = b is called the y intercept of the line. If the line passes the origin. Then both the x intercept and y intercepts are zero.

Consider the XOY plane. Let the line cut the x axis at A and y axis at B. By data OA = a and OB = b. Thus A = (0, 0) and B = (0, b). Hence the required line is the line joining A and B, Its equation can be obtained by using two point form of the equation of a line.

Then the equation of AB is

$$\frac{y-o}{x-a} = \frac{0-b}{a-0}$$

$$\Rightarrow \quad \frac{y}{x-a} = \frac{-b}{a}$$

$\Rightarrow$ ay = -bx + ab

$\Rightarrow$ bx + ay = ab

Dividing throughout by 'ab' we have

$$\frac{bx}{ab}+\frac{ay}{ab}=\frac{ab}{ab}$$

$$\Rightarrow \quad \frac{x}{a}+\frac{y}{b} = 1$$

This form of equation of the line is called **Intercept form.**

Example 1 Find the equation of the line whose intercepts on the x axis and y axis are 3 and -4.

Solution : Let a = 3, b = -4. The equation of the line is given by

$$\frac{x}{a}+\frac{y}{b} = 1$$

$$\Rightarrow \quad \frac{x}{3}+\frac{y}{-4}=1 \quad or \quad \frac{x}{3}-\frac{y}{4}=1$$

$$\Rightarrow \quad 4x - 3y = 12$$

Example 2 Find the equation to the straight line cutting of equal intercepts and passing through (-2, 5)

Solution : Let the straight line AB cut off equal intercept with both the axes. Let OA = OB = a

Hence equation of AB is given by $\frac{x}{a}+\frac{y}{a}=1$

i.e. x + y = a

Since this line passes through (-2, 5) we get -2 + 5 = a i.e. a = 3

Hence the required equation of AB is x + y = 3

Example 3 Find the equation of the line passing through (2, -3) which cuts off intercepts on X and Y axes which are in the ratio 3:4

Solution : Let the intercepts be 3K and 4K (because they are in 3:4)

Equation of line is given by $\frac{x}{3K}+\frac{y}{4K}=1$

i.e. $4x + 3y = 12K$ ……. (1)

This line passes through (2, -3)

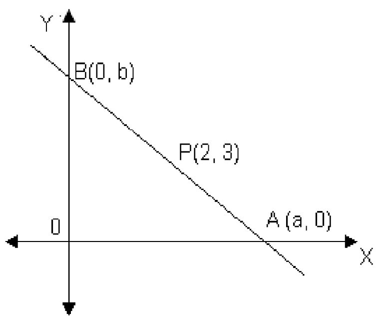
$\therefore\ 4(2) + 3(-3) = 12K$

$-1 = 12K,\ K = -1/12$

Substituting the value of K in (1) we get

$4x + 3y + 1 = 0$ as the required equation.

Example 4 A line passes through (2, 3) and this point bisects the portion of the line intercepted between the co-ordinate axis. Find the equation of the line.



Solution : Let the required line cut the axis at A(a, 0) and the y axis at B(0, b) Then x intercept = a and y intercept = b.Thus the equation of the line is given by

$$\frac{x}{a}+\frac{y}{b}=1$$

Let P = (2, 3) by data P is the mid point of AB

$$\Rightarrow \quad (2, 3) = \left(\frac{a}{2}, \frac{b}{2}\right)$$

$$\frac{a}{2} = 2 \text{ and } \frac{b}{2} = 3$$

a = 4 and b = 6

Thus the equation of the line is

$$\frac{x}{4}+\frac{y}{6} = 1 \quad \Rightarrow \quad 3x + 2y = 12$$

Note: In general, the equation of the line, such that its portion between the axes is bisected at the point (x_1, y_1) is given by

$$\frac{x}{2x_1}+\frac{y}{2y_1} = \mathbf{1}$$

Example 5 Find the equation of a straight line passing through the point (2,2) such that the sum of its intercepts on the axes = 9

Solution : Let a = x intercept and b = y intercept of the line.

By data $a + b = 9$

$\Rightarrow$ $b = 9 - a$

Now the equation of the line is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \quad \frac{x}{a} + \frac{y}{9-a} = 1 \qquad \text{..... (1)}$$

$$\Rightarrow \quad (9 - a)\,x + ay = a(9 - a) \qquad \text{..... (2)}$$

By data this must pass through (2, 2)

Equation (2) must satisfy $x = 2$ and $y = 2$

$\Rightarrow$ $(9 - a)\,2 + a \times 2 = 9a - a^2$

$\Rightarrow$ $a^2 - 9a + 18 = 0$

$\Rightarrow$ $(a - 6)(a - 3) = 0$

$\Rightarrow$ a = 6 or 3 and so b = 3 or 6 respectively.

Putting these values in (1) we get

$$\frac{x}{6} + \frac{y}{3} = 1 \text{ and } \frac{x}{3} + \frac{y}{6} = 1$$

$x + 2y = 6$ and $2x + y = 6$

Example 6 Find the equation of a line which passes through the point (-4, 5) and whose intercepts are equal in magnitude but opposite in sign.

Solution : Let a = x intercept. Then y intercept = -a

Equation of the line is $\frac{x}{a} + \frac{y}{-a} = 1$

$\Rightarrow x - y = a$ ....(1)

$\Rightarrow -4 -5 = a \Rightarrow a = -9.$

Putting these value of a in (1) we get the equation of the line

$x - y = -9 \Rightarrow \quad x - y + 9 = 0$

Example 7 Find the equation of a line which passes through (-4, 1) and portion of it between the axes is divided by the point in the ratio 1:2

Solution : Let A = (a, 0) B = (0, b). Then x intercept = a and y intercept = b.

Thus the equation of the line will be of the form $\frac{x}{a}+\frac{y}{b}=1$ ..(1)

Now by data P (-4, 1) divides AB in the ratio 1:2

$$P = \left(\frac{1(0)+2(a)}{1+2}, \frac{2.(b)+2.(0)}{1+2}\right)$$

$$(-4,\ 1) = \left(\frac{2a}{3}, \frac{b}{3}\right)$$

$\Rightarrow \quad a = -6$ and $b = 3$

Putting these values in (1) we get $\frac{x}{-6}+\frac{y}{3}=1$

$$\Rightarrow \quad x - 2y + 6 = 0$$

This is the required equation.

EXERCISE : 17.2

I. 1) Find the equation of the line.

a) Parallel to the x axis and at distance of +7 from it.

b) Parallel to the x axis and passing through (3, -4)

c) Parallel to the y axis and at a distance of 5 units to the left of it.

d) Parallel to the y axis and passing through (- 8, 6)

2. Find the equation of the line in each of the following

a) Passing through (4, 3) and with slope 2

b) Passing through (0, -2) with slope – 4 .

c) Passing through (-2, 2) with slope -3.

d) Passing through (-3, -1) with slope 5/6

3. Find the equation of the line.

a) Passing through (3, 5) and making an angle 45^0 with the positive direction of x axis.

b) Passing through (1, 2) and parallel to the join of (3, 1) and (4, -5)

c) Passing through (3, -1) and perpendicular to the join of (0, 0) and (7, 2)

d) Passing through (-5, 2) and parallel to x axis.

e) Passing through (-5, 12) and parallel to y axis.

4. Find the equation of the line joining the points

a) (0, -3), (5, 0) (b) (-1, -2), (-5, -2) (c) (-1, -3) (6, 11)

5. Write down the equation of the line which

a) has x intercept = 3, y intercept = 5

b) has x intercept = -7, y intercept = 2

c) Passing through $\left(\frac{-2}{5}, 0\right)$ and has y intercept = 4

d) Passes through $\left(0, \frac{5}{7}\right)$ and has x intercept $\frac{-5}{4}$

II. 1. Find the equation of the line which passes through (5, 2) and cutting off intercepts which are equal in magnitude but opposite in sign.

2. Find the equation of a straight line which passes through the point (3, 4) and have intercepts on the axes such that their sum is 14.

3. If a straight line cuts the co-ordinate axes at A and B and if (3, 2) is the mid point of AB, find the equation of AB.

4. A line passes through (3, 4) and the portion of the line between the axes is divided at that point in the ratio 1:2. Find the equation.
5. Find the equation of the line whose x and y intercepts are equal and passes through (2, -3)
6. The intercepts of a straight line on the x and y axes are in the ratio 2:1 and it bisects the join of (3, -4) and (5, 2). Find the equation of the straight line.

III.

1. Find the equation of the straight line which passes through the point (-5, 4) and is such that the portion of it between the co-ordinate axes is divided by this point in the ratio 3:2.
2. In what ratio is the line joining the points (2, 3) and (4, -5) is divided by the line joining (6, 8) and (-3, 2)
3. Find the equation of the straight line given a + b = 1 and ab = 6 where a and b are the x intercept and y intercept respectively.
4. Find the equations of the medians of the triangle formed by the points (-1, 3) (-3, 5) and (7, -9)
5. If the straight line $\frac{x}{a}+\frac{y}{b}=1$ passes through (8, -9) and (12, -15) find the values of a and b.

ANSWERS : 17.2

I. 1. a) y = 7 b) y = -4
c) x = -5 or $x + 5 = 0$ d) $x + 8 = 0$

2. a) $2x - y = 5$, b) $4x + y + 2 = 0$
c) $3x + y + 4 = 0$ d) $5x - 6y + 9 = 0$

3. a) $x - y + 2 = 0$ b) $6x + y = 8$
c) $7x + 2y = 19$ d) y = 2, e) $x + 5 = 0$

4. a) $3x - 5y = 15$ b) $y + 2 = 0$ c) $2x - y = 1$

5. a) $5x + 3y = 15$, b) $2x - 7y + 14 = 0$
c) $10x - y + 4 = 0$ d) $4x - 7y + 5 = 0$

II. 1) $x - y = 3$ 2) $x + y = 7$; $4x + 3y = 24$ 3) $2x + 3y = 12$
4) $8x + 3y = 36$ 5) $x + y + 1 = 0$ 6) $x + 2y = 2$

III. 1) $8x - 15y + 100 = 0$
2) 1 : 5 externally
3) $2x - 3y = 6$, $3x - 2y + 6 = 0$
4) $5x + 3y = 4$, $4x + 3y = 3$, $13x + 9y - 10 = 0$
5) a = 2 b = 3.

17.6. EQUATION OF A LINE IN GENERAL FORM.

A general linear equation in x and y of the form $ax + by + c = 0$ provided 'a' and 'b' are not both zero always represents a straight line.

An equation of this form $ax + by + c = 0$ can be reduced to any of the forms.

1. Reduction to slope intercept form

We have $ax + by + c = 0$

$\Rightarrow by = -ax - c$

Dividing throughout by 'b' we have

$$y = \frac{-ax}{b} - \frac{c}{b} \quad (b \neq 0)$$

$$y = \left(\frac{-a}{b}\right)x + \left(\frac{-c}{b}\right)$$ which is of the form $y = mx + c$.

Where slope $m = \dfrac{-a}{b} = \dfrac{-\textit{coefficient of } x}{\textit{coefficient of } y}$

And y intercept $c = \dfrac{-c}{b} = \dfrac{-\textit{constant term}}{\textit{coefficient of } y}$

Note: Slope of the straight line $ax + by + c =$ is $\dfrac{-a}{b}$

2. Reduction to intercept form

We have ax + by + c = 0 $\Rightarrow$ ax + by = -c

$$\Rightarrow \frac{ax}{-c}+\frac{by}{-c}=\frac{-c}{-c} \quad \text{(dividing by } -c, \text{ provided } c \neq 0)$$

$$\Rightarrow \frac{x}{(-c/a)}+\frac{y}{(-c/b)}= 1 \text{ which is of the form } \frac{x}{a}+\frac{y}{b}=1$$

Thus we have the following important results.

For the equation ax + by + c = 0 of a line, we have

$$\text{Slope} = \frac{-a}{b}=\frac{-coefficient\ of\ x}{coefficient\ of\ y}$$

$$x \text{ intercept} = \frac{-c}{a}=\frac{-\ constant\ term}{coefficient\ of\ x}$$

$$y \text{ intercept} = \frac{-c}{b}=\frac{-\ constant\ term}{coefficient\ of\ y}$$

Also the area of the triangle formed by A (a, 0), B (0, b) and the origin 0 (0,0) is given by Area of Δ OAB = $\left|\frac{1}{2}\ OA..OB\right| = \frac{1}{2}\left|\frac{c^2}{ab}\right|$

WORKED EXAMPLES:

Example 1 Find the slopes of the following lines. Also find the x intercept and y intercept.

a) 5x–7y +11 = 0 (b) $\sqrt{3}\,x$+y+2 = 0 c) 4y – 3 = 0

Solution : a) 5x – 7y + 11 = 0, a = 5 b = 7 c = 11

$$\text{Then slope} = \frac{-a}{b}=\frac{-5}{-7}=\frac{5}{7}$$

x intercept $= \dfrac{-c}{a} = \dfrac{-11}{5}$

y intercept $= \dfrac{-c}{b} = \dfrac{-11}{-7} = \dfrac{11}{7}$

b) $\sqrt{3}\, x + y + 2 = 0$

Slope $= \dfrac{-a}{b} = -\sqrt{3}$

x intercept $= \dfrac{-c}{a} = \dfrac{-2}{\sqrt{3}}$

y intercept $= \dfrac{-c}{b} = \dfrac{-2}{1} = -2$

c) $4y - 3 = 0$; $y = \dfrac{3}{4}$

This equation is of the form of y = K. Thus it is a line parallel to the x axis. And $\dfrac{3}{4}$ units above it.

$\therefore$ slope = 0, x intercept $= \dfrac{-c}{a}$ does not exist.

y intercept $= \dfrac{3}{4}$

Example 2 Reduce the following equation to (i) slope intercept form (ii) the intercept form

(a) $2x + 3y = 7$ (b) $\sqrt{3}x + y + 8 = 0$

Solution:

a) $2x + 3y = 7$

$\Rightarrow$ $3y = -2x + 7$

$\Rightarrow$ $y = \dfrac{-2}{3}x + \dfrac{7}{3}$ $(y = mx + c)$

This is the equation of the line in the slope intercept form with

Slope m = $\frac{-2}{3}$ and c, the y-intercept as $\frac{7}{3}$

For the intercept form, $2x + 3y = 7$ has to be divided by 7

We get $\frac{2x}{7}+\frac{3y}{7}=1$

$\Rightarrow \quad \frac{x}{7/2}+\frac{y}{7/3}=\left(\frac{x}{a}+\frac{y}{b}=1\right)$

Thus the equation of the line is intercept form is $\frac{x}{(7/2)}+\frac{y}{(7/3)} = 1$

b) $\sqrt{3}\, x + y + 8 = 0$

In slope intercept form, we have $y = -\sqrt{3}x - 8$ where $m = -\sqrt{3}$ and y intercept is -8

In the intercept form $\sqrt{3}\, x + y + 8=0 \Rightarrow \sqrt{3}x + y = -8$

$$\frac{\sqrt{3x}}{-8}+\frac{y}{-8}=1$$

$$\Rightarrow \quad \frac{x}{\left(\frac{-8}{\sqrt{3}}\right)}+\frac{y}{(-8)}=1 \qquad \left(\frac{x}{a}+\frac{y}{b}=1\right)$$

Example 3 Show that the lines i) $3x + 4y + 1 = 0$ and $6x + 8y + 3 = 0$ are parallel and (ii) $5x + y + 2 = 0$ and $x - 5y + 1 = 0$ are perpendicular.

Solution : 1) Slope of $3x + 4y + 1 = 0$ is $\frac{-3}{4} = m$,

Slope of $6x + 8y + 3 = 0$ is $\frac{-6}{8} = \frac{-3}{4} = m_2$

Since $m_1 = m_2 \Rightarrow$ the lines are parallel

ii) Slope of $5x + y + 2 = 0$ is $\frac{-5}{1} = m_1$

Slope of $x - 5x + 1 = 0$ is $\frac{-1}{-5} \Rightarrow m_2 = \frac{1}{5}$

Since $m_1 \times m_2 = -5 \times \frac{1}{5} = -1$

The lines are perpendicular.

Example 4 Find the value of K such that the line.

$(K-2)x + (K+3)y - 5 = 0$ is perpendicular to the line $2x - y + 7 = 0$

Solution : $(K-2)x + (K+3)y - 5 = 0$ ……… (1)

$2x - y + 7 = 0$ …….(2)

Slope of (1) $= \frac{-(k-2)}{K+3}$ m, slope of (2) $= -\frac{2}{-1} = 2 = m_2$

By data (1) and (2) are perpendicular

$m_1 m_2 = -1$

$\Rightarrow \quad -\left(\frac{K-2}{K+3}\right)(2) = -1$

$\Rightarrow \quad 2K - 4 = K + 3$

$\Rightarrow \quad K = 7$

Example 5 Find the ratio in which the line $2x - 3y + 4 = 0$ divides the line joining the points (-1, 2) and (3, -4).

Solution : If the line joining $(x_1\ y_1)$ and $(x_2\ y_2)$ is divided by the line $ax + by + c = 0$, then the ratio of division is given by

$$K : 1 = \frac{K}{1} = -\left(\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}\right)$$

If $(-1, 2) = (x_1\ y_1)$ and $(3, -4) = (x_2\ y_2)$ then

$$K = -\left(\frac{2(-1) - 3(2) + 4}{2(3) - 3(-4) + 4}\right)$$

$$= \frac{2}{11}$$

$$\therefore K : 1 = \frac{2}{11} : 1 = 2 : 11$$

Example 6 Find the ratio in which the line $x + y + 1 = 0$ divides the line joining the points (2, 3) and (-1, 4). Also find the point of division.

Solution : Let A = (2, 3) B (-1, 4). Let the line $x + y + 1 = 0$ …….. (1)
Cut the segment AB at P in the ratio K : 1

$$\therefore \ P = \left(\frac{-K+2}{K+1}, \frac{4K+3}{K+1}\right)$$

But this lies on the line (1)

$$\Rightarrow \quad \left(\frac{-K+2}{K+1}\right) + \frac{4K+3}{K+1} = 1 = 0$$

$$\Rightarrow \quad -K + 2 + 4K + 3 + K + 1 = 0$$

$$\Rightarrow \quad K = \frac{-3}{2}$$

Thus the line AB is divided by the point P in the ratio 3:2 externally.

Point of division is given by

$$P = \left(\frac{\frac{3}{2}+2}{\frac{-3}{2}+1}, \frac{4(\frac{-3}{2})+3}{\frac{-3}{2}+1}\right)$$

$$= (-7, 6)$$

Example 7 Find the equation of the line parallel to

a) $2x + 3y + 1 = 0$ and passing through (-1, 1)

b) $x = y$ and passing through (-1, 4)

Solution : (a) Equation of a line passing through $(x_1\ y_1)$ and with slope m is given by $y - y_1 = m(x - x_1)$

Slope of the required line parallel to $2x + 3y + 1 = 0$ is the same as the slope of the given line.

$$\text{So } m = \frac{-\textit{coefficient of } x}{\textit{coefficient of } y} = \frac{-2}{3}$$

$(x_1\ y_1) = (-1, 1)$

$\therefore$ Equation of the required line is $y - y_1 = m(x - x_1)$

$$y - 1 = \frac{-2}{3}(x + 1)$$

$$3y - 3 = -2x - 2$$

$$2x + 3y - 1 = 0$$

b) Slope of the required line parallel to $x = y$ i.e. $x - y = 0$ is the same as the slope of the given line.

$$\text{Slope m} = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-1}{-1} = 1$$

$(x_1, y_1) = (-1, 4)$

Equation of the required line is

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 1(x + 1)$$
$$x - y + 5 = 0$$

Example 8 Find the equation of the line perpendicular to $3x - 2y + 1 = 0$ and passing through (1, -2)

Solution : Slope of the given line $3x - 2y + 1 = 0$ is $\frac{3}{2} = m_1$

Since the two lines are perpendicular $m_1 m_2 = -1$

$$\therefore m_2 = \frac{-1}{m_1} = \frac{-1}{3/2} = \frac{-2}{3}$$

Thus the equation of the line with slope $m = \frac{-2}{3}$ and passing through $(x_1\ y_1) = (1, -2)$ is given by

$$\Rightarrow \quad y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y + 2 = \frac{-2}{3}(x - 1)$$

$$\Rightarrow \quad 3y + 6 = -2x + 2$$

$$\Rightarrow \quad 2x + 3y + 4 = 0$$

Example 9 If the line $2x + 3y - 1 = 0$ cuts the x and y axis at A and B respectively. Find the area of the triangle OAB.

Solution: Equation of the line which cuts the x axis and y axis at A and B is given by $\frac{x}{a} + \frac{y}{b} = 1$ where

a = x intercept and b = y interceptThen area of the triangle OAB

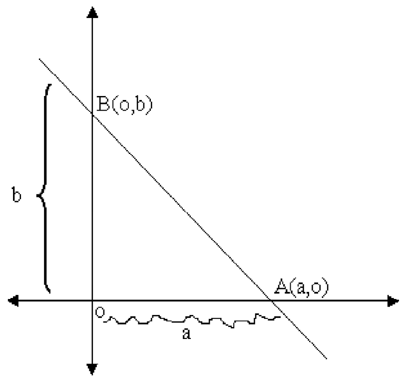
$= \frac{1}{2}ab$ From the given equation $2x + 3y - 1 = 0$

$a = x \text{ intercept} = \frac{-c}{a} = \frac{1}{2}$

$b = y \text{ intercept} = \frac{-c}{b} = \frac{1}{3}$

$\therefore \text{Area of } \Delta OAB = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{12}$ Sq. units



EXERCISE : 17.3

I.

1. Write the slope x intercept and y intercept of the lines
   a) $3x - 2y + 1 = 0$ b) $y = 3x - 4$
   c) $y + 7 = 0$ d) $2y - 3x + 7 = 0$
2. Find k if the line $(k + 1)x + (2k + 3)y + 3 = 0$ and $2x - 5y + 1 = 0$ are perpendicular to each other.
3. Find k if the lines $2x - 3y + 4 = 0$ and $x + ky = 3$ are (i) parallel (ii) perpendicular
4. Show that the lines are parallel
   a) $2x - 3y + 7 = 0$ and $12y = 8x - 5$
   b) $\frac{x}{4} + \frac{y}{7} = 1$ and $7x = 4(7 - y)$

5. Show that the lines are perpendicular
   a) $4x - 7y = 2$ and $7x + 4y = 5$
   b) $3y = 2x + 7$ and $6x = 13 - 8y$

6. If the line $x - y + 2 = 0$ cuts the x and y axes at P and Q respectively, find the area of the triangle OPQ.

7. Find the equation of the line which has y intercept as $\frac{-2}{3}$ units and is perpendicular to $2x + y = 1$

8. Find the equation of the line which has x intercept $\frac{3}{2}$ units and is parallel to $3x - y + 1 = 0$

9. Find the ratio in which the line segment joining (2, 3) and (4, 1) is divided by the line $x - 3y + 5 = 0$

10. Reduce the following equations to slope intercept form
    a) $4x + 3y - 9 = 0$ b) $8x + y - 4 = 0$
    c) $3x - 2y + 1 = 0$

ANSWERS : 17.3

I.

1.

	Slope	x intercept	y intercept
a)	$\frac{3}{2}$	$\frac{-1}{3}$	$\frac{1}{2}$
b)	3	$\frac{4}{3}$	-4
c)	0	does not exist	-7
d)	$\frac{3}{2}$	$\frac{7}{3}$	$\frac{-7}{2}$

2. $\frac{-13}{8}$

3. $\frac{-3}{2}$ and $\frac{2}{3}$

6. 2 Sq. units

7. $3x - 6y = 4$

8. $6x - 2y = 9$

9. 1 : 3

10. Slope intercept form

a) $y = \frac{-4}{3}x + 3$

b) $y = -8x + 4$

c) $y = \frac{3}{2}x + \frac{1}{2}$

Conditions for parallelism of lines (Equations of parallel lines)

Any line parallel to $ax + by + c = 0$ is of the form $ax + by + k = 0$

Note: Keep the terms containing x and y unaltered, change the constant. The constant k is determined from an additional condition given in the problem.

Example 1 Find the equation of the line passing through (1, -1) and parallel to $2x - 3y + 1 = 0$

Solution : Any line parallel to $2x - 3y + 1 = 0$ is of the form $2x - 3y + k = 0$. This line passes through (1, -1). So put $x = 1$ and $y = -1$ in the new equation. We get $2 + 3 + k = 0 \Rightarrow k = -5$

$\therefore$ Required equation of the line $2x - 3y - 5 = 0$

Condition for perpendicularity of lines (Equation of perpendicular lines)

Any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$

Rule:Interchange the coefficient of x and y and change the sign of one of them and change the constant. The constant k can be determined from an additional condition given in the problem.

Example 1 Find the equation of a line perpendicular to $5x + 3y + 8 = 0$ and passing through (1,2)

Solution : Any line perpendicular to $5x + 3y + 8 = 0$ of the form $3x - 5y + k = 0$. This is passing through (1,2). Therefore put $x = 1$, $y = 2$ we get $3 - 5 \times 2 + k = 0 \Rightarrow k = 7$ $\therefore$ Equation of the required line is $3x - 5y + 7 = 0$

EXERCISE : 17.3

1. Find the equation of the line
   i) Parallel to the $4x + 3y + 2 = 0$ and passing through (4, 1)
   ii) Passing through (2, 2) and parallel to $2x - 3y + 1 = 0$
   iii) Passing through (-2, 4) and parallel to $3x - 4y + 1 = 0$

2. Find the equation of the line perpendicular to
   i) $3x - 2y + 1 = 0$ and passing through (1, - 2)
   ii) $3x + 2y - 1 = 0$ and passing through (-2, 1)
   iii) $3x - 4y + 7 = 0$ and passing through (5, -1)

ANSWERS : 17.4

1.
   i) $4x + 3y = 19$
   ii) $2x - 3y + 2 = 0$
   iii) $3x - 4y + 14 = 0$

2.
   i) $2x + 3y + 4 = 0$
   ii) $2x - 3y + 7 = 0$
   iii) $4x + 3y - 17 = 0$

17.7 POINT OF INTERSECTION OF TWO LINES

Let $a_1x + b_1 y + c_1 = 0$ and $a_2x + b_2 y + c_2 = 0$ be two straight lines intersecting each other at a point $(x_{1,}y_1)$

Then $a_1x_1 + b_1y_1 + c_1 = 0$ (1)

And $a_2x_2 + b_2y_2 + c_2 = 0$ (2)

Solving (1) and (2) we get the point of intersection of two lines as

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$$

Remember that $a_1b_2 - b_1a_2 \neq 0$.

Note: Method of cross multiplication can also be used to obtain the point of intersection. Thus, the find the co-ordinates of point of intersection of two non parallel lines we solve the equations of the lines simultaneously and the values of x and y so obtained are the co-ordinates of the point of intersection.

Example 1 Find the point of intersection of the straight lines

$3x - 4y = 1$ and $5x - 7y = 1$

Solution We have $3x - 4y = 1$ and $5x - 7y = 1$

Solving by the method of cross multiplication

	x	y	1	
3	-4	-1	3	-4
5	-7	-1	5	-7

$$\Rightarrow \quad \frac{x}{4-7} = \frac{y}{-5+3} = \frac{1}{-21+20}$$

$$\Rightarrow \quad \frac{x}{-3} = \frac{y}{-2} = \frac{1}{-1}$$

$$\Rightarrow \quad \frac{x}{-3} = \frac{1}{-1} \Rightarrow x = 3$$

and $\dfrac{y}{-2} = \dfrac{1}{-1} \Rightarrow y = 2$

Point of intersection is (3, 2)

Example 2 Find the point of intersection of the lines $3x + 2y - 5 = 0$ and $4x - y - 3 = 0$

Solution : We have

$3x + 2y - 5 = 0$ (1)

$4x + y - 3 = 0$ (2)

Solving the above equation using simultaneous method we get

$3x + 2y - 5 = 0$ (1)

$8x - 2y - 6 = 0$ (3) (Multiplying eqn. (2) by 2)

(1) + (3) $\Rightarrow 11x - 11 = 0$

$x = 1$

Substituting $x = 1$ in $3x + 2y - 5 = 0$ we get $3 + 2y - 5 = 0$

$2y = 2$

$y = 1$

Point of intersection is (1, 1)

Example 3 Find the equation of the straight line which passes through the point of intersection of the lines $3x + y = 10$ and $x + 7y = 10$ and parallel to the line $4x\text{-}3y\text{+}1\text{=}0$

Solution : The point of intersection of the given lines is obtained by solving the given equations by simultaneous method or by the method of cross multiplication.

Solving the above given equation we get

$(x, y) = (3, 1)$

Now equation of the required line which is parallel to $4x - 3y + 1 = 0$ is given by $4x - 3y + k = 0$ …. (1)

Since this is passing through (3, 1)

Therefore $4(3) - 3(1) + K = 0 \quad \Rightarrow \quad K = -9$

Hence required equation of the straight line is $4x - 3y = 9$

Example 4 Find the equation of the straight line which passes through the point of intersection of $2x - 3y = 4$ and $2x + 2y = 1$ and perpendicular to the $x + 4y = 8$

Solution : Solving $2x - 3y = 4$ (1)

And $2x + 2y = 1$ (2)

$(1) - (2) \Rightarrow -5y = 3 \qquad y = \frac{-3}{5}$

But $y = \frac{-3}{5}$ in $2x - 3y = 4$, we get $x = \frac{11}{10}$

Point of intersection is $(x, y) = \left(\frac{11}{10}, \frac{-3}{5}\right)$

Now any line perpendicular to $x + 4y = 8$ is of the form $4x - y + k = 0$. Since the line is passing through $\left(\frac{11}{10}, \frac{-3}{5}\right)$. Therefore $4\left(\frac{11}{10}\right) - \left(\frac{-3}{5}\right) + K = 0$

$\Rightarrow K = -5$

Required line is $4x - y - 5 = 0$

Example 5 Find the equation of the line passing through the point of intersection of the lines $2x + 3y - 7 = 0$ and $5x - 6y + 8 = 0$ and the point $(4, 3)$

Solution:

Note: Equation of the straight line passing through the point of intersection of the lines L=0 and L = 0 is $L + \lambda L' = 0$. λ is determined by using an additional condition provided in the problem. This method can also be employed.

Equation of any line through the intersection of the lines $2x + 3y - 7 = 0$ and $5x - 6y + 8 = 0$ is of the form

$2x + 3y - 7 + \lambda(5x - 6y + 8) = 0$

$(2 + 5\lambda)x + (3 - 6\lambda)y - 7 + 8\lambda = 0$ (1)

By data this passes through (4, 3). Thus we have

$(2 + 5\lambda)(4) + (3 - 6\lambda) 3 - 7 + 8\lambda = 0$

$10\lambda + 10 = 0$

$\lambda = -1$

Putting λ in (1) we get the required line as

$(2 - 5)x + (3 - 6) y - 7 - 8 = 0$

$\Rightarrow \quad -3x + 9y - 15 = 0$

$\Rightarrow \quad 3x - 9y + 15 = 0$

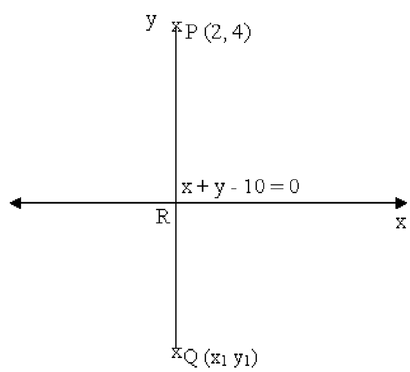
OR

$x - 3y + 5 = 0$

Example 6 Find the image of the point (2, 4) on the line $x + y - 10 = 0$

Solution : Let P be the point (2, 4) and Q (x_1, y_1) be its reflection on the line $x + y - 10 = 0$. Let PQ cut the line at R. Then R is the mid point of PQ and PQ is perpendicular to the given line.

Now $R = \left(\dfrac{x_1 + 2}{2}, \dfrac{y_1 + 4}{2}\right)$



Since R lies on $x + y - 10 = 0$,
We have

$$\left(\frac{x_1 + 2}{2} + \frac{y_1 + 4}{2}\right) - 10 = 0$$

i.e $\quad x_1 + y_1 - 14 = 0 \quad (1)$

Slope of PQ $= \dfrac{y_1 - 4}{x_1 - 2}$

Slope of the given line $x + y - 10 = 0$ is -1

Since PQ is perpendicular to the line $x + y - 10 = 0$

$m_1 m_2 = -1$

So $\left(\dfrac{y_1 - 4}{x_1 - 2}\right) \times (-1) = -1$

$\Rightarrow \quad x_1 - y_1 + 2 = 0 \quad (2)$

Solving (1) and (2) we get the point of intersection as $x_1 = 6$ and $y_1 = 8$

$\therefore$ Image = Q (6, 8)

Alter method : The image point Q (h, k) of the point P (x_1 y_1) along the line ax + by + c = 0 is given by

$$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}$$

So take P (x_1,y_1) as P(2, 4) and the line is x + y – 10 = 0 where a = 1, b = 1 and c = -10

$$\frac{h-2}{1} = \frac{k-4}{1} = \frac{-2(1\times 2+1\times 4-10)}{1^2+1^2}$$

$$\left(\frac{h-2}{1}\right) = \left(\frac{K-4}{1}\right) = \frac{-2(-4)}{2}$$

$$\Rightarrow \quad \frac{h-2}{1} = \frac{-2(-4)}{2}, \quad \frac{k-4}{1} = \frac{-2(-4)}{2}$$

$\Rightarrow$ h – 2 = 4 $\qquad\qquad$ K – 4 = 4

$\Rightarrow$ h = 2 + 4 = 6, $\qquad\qquad$ K = 8

$\therefore$ image of the point is given by Q (h, k) = (6, 8)

Example 7 Find the foot of the perpendicular drawn from the point (-2, -1) on the line 3x + 2y – 5 = 0.

Solution : Let P (x_1 ,y_1) = (-2, -1) and Q (h, k) be the foot of the perpendicular n the 3x + 2y – 5 = 0

Note: If Q (h, k) is the foot of the perpendicular drawn from P (x_1 ,y_1) into ax + by + c = 0, then the co-ordinates of Q are given by

$$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-(ax_1+by_1+c)}{a^2+b^2}$$

So here a = 3, b = 2, c = 5 and (x_1 y_1) = (-2, -1)

Substituting all these values in the above expression we get

$$\frac{h-(-2)}{3}=\frac{K-^{-}1}{2}=-\left(\frac{3(-2)+2(-1)-5}{3^2+2^2}\right)$$

$$\frac{h+2}{3}=\frac{K+1}{2}= -(-1)=1$$

$$\Rightarrow \quad \frac{h+2}{3}=1,\frac{K+1}{2}=1$$

$\Rightarrow$ h + 2 = 3 K + 1 = 2

$\Rightarrow$ h = 1 and K = 1

$\therefore$ foot of the perpendicular is given by Q(1, 1)

EXERCISE: 17.5

I. 1. Find the points of intersection of the lines

a) $x + 5y + 4 = 0$ and $3x - 4y - 7 = 0$

b) $2x + y + 2 = 0$ and $-x + y - 1 = 0$

c) $3x + 2y - 9 = 0$ and $x - y + 2 = 0$

2. Find the equation of the line through the point of intersection of $2x$-$5y$=1 and $3x - 2y = 8$ and parallel to the line $2x + y = 3$.

3. Find the equation of the line through the point of intersection of x-$8y$+11=0 and $4x$ -$7y + 3 = 0$ and perpendicular to the line $3x$+$2y$+5=0.

4. Find the equation of the line through the intersection of the lines x -$2y$ + $5 = 0$ and $2x$= $3y - 8$ and the intersection of the lines $4x$-$9y$+13=0 and $y = 8x - 3$

5. Find the coordinate of the foot of the perpendicular from (-6, 2) on the line $3x - 4y + 1 = 0$

6. Find the image of the point (2, 5) on the line $4x - 3y + 1 = 0$

7. Find the image of the point (2, 3) on the line $3x + 5y + 4 = 0$

8. Find the equation of the straight line passing through the point of intersection x +$2y + 3 = 0$ and $3x + 4y + 7 = 0$ and has a slope $\frac{-3}{2}$

ANSWERS : 17.5

I. 1. a) (1, -1) b) (-1, 0) c) (1, 3)

2. $38x + 19y - 97 = 0$

3. $50x - 75y + 17 = 0$

4. $5x + 27y - 49 = 0$

5. (-3, -2)

6. $\left(\frac{98}{25}, \frac{89}{25}\right)$

7. (-1, -2)

8. $3x + 2y = 5$

17.8 CONDITION FOR CONCURRENCY OF THREE LINES:

Let $a_1x + b_1y + c_1 = 0$ (1)

$a_2x + b_2y + c_2 = 0$ (2)

$a_3x + b_3y + c_3 = 0$ (3)

be the three given lines. The above three lines are said to be concurrent if one of the lines passes through the point of intersection of the other two lines. Hence solving (2) and (3) for x and y. We get the point of intersection given by

$$\left(\frac{b_2c_3 - b_3c_2}{a_2b_3 - a_3b_2}, \frac{c_2a_3 - c_3a_2}{a_2b_3 - a_3b_2}\right)$$

If (1), (2) and (3) are concurrent, then the line (1) must pass through this above point.

$$\therefore\ a_1 \frac{b_2c_3 - b_3c_2}{a_2b_3 - a_3b_2} + b_1 \frac{c_2a_3 - c_3a_2}{a_2b_3 - a_3b_2} + c_1 = 0$$

i.e. $a_1 (b_2 c_3 - b_3c_2) + b_1 (c_2 a_3 - c_3 a_2) + c_1 (a_2 b_3 - a_3 b_2) = 0$ is the required condition.

Note: Problems on concurrency can be worked out using the above formula or directly.

Example 1 Show that the lines $3x - 4y + 5 = 0$, $4x + 5y - 45 = 0$ and $7x - 8y + 5=0$ are concurrence.

Solution : Solving the first two lines we get the point of intersection

$3x - 4y + 5 = 0$ (1)

$4x + 5y - 45 = 0$ (2)

$x = 5$ and $y = 5$ and so $(x, y) = (5, 5)$

Substituting $x = 5$ and $y = 5$ in the third line $7x - 8y + 5 = 0$

we find $7(5) - 8(5) + 5 = 0$

i.e. $0 = 0$

Hence the three given lines are concurrent and the point of concurrency is (5, 5)

Example 2 Find 'a' so that the lines $x - 6y + a = 0$, $2x + 3y + 4 = 0$ and $x + 4y + 1 = 0$ are concurrent.

Solution : Consider $x - 6y + a = 0$ (1)

$2x + 3y + 4 = 0$ (2)

$x + 4y + 1 = 0$ (3)

We know that the condition for 3 lines may be concurrent is

$a_1 (b_2 c_3 - b_3 c_2) + b_1 (c_2a_3 - c_3 a_2) + c_1 (a_2 b_3 - a_3 b_2) = 0$

i.e. $1 (3(1) - 4(4)) + (-6) (4(1) - 1(2)) + a (2(4) - 1(3)) = 0$

i.e. $- 13 - 12 + 5a = 0$

$5a = 25$

$a = 5$

Example 3 If the lines $2x - y = 5$, $Kx - y = 6$ and $4x - y = 7$ are concurrent, find K.

Solution : We have $2x - y = 5$ (1)

$Kx - y = 6$ (2)

$4x - y = 7$ (3)

Solving (1) and (3) we get $x = 1$ and $y = - 3$

Since the lines are concurrent put $x = 1$ and $y = -3$ in (2)

We get $K (1) - (- 3) = 6 \Rightarrow K + 3 = 6$

$\Rightarrow K = 3$

EXERCISE : 17.6

1. Show that the straight lines given by the following equations are concurrent. Also find the point of concurrence.
   a) $2x - 3y = 7$, $3x - 4y = 13$, $8x - 11y = 33$
   b) $x - y - 1 = 0$, $4x + 3y = 25$, $2x - 3y + 1 = 0$
   c) $3x - y + 4 = 0$, $2x + 7y - 5 = 0$, $5x + 6y - 1 = 0$
   d) $4x + 7y - 9 = 0$, $5x - 8y + 15 = 0$, $9x - y + 6 = 0$

2. For what values of k are the three lines $x - 2y + 1 = 0$, $2x - 5y + 3 = 0$ and $5x - 9y + k = 0$ concurrent?

3. Find the value of a if the lines $x - 2y = 1$, $2x + y = 7$ and $ax - 5y = 4$ are concurrent.

4. If the three lines $ax + by + c = 0$, $bx + cy + a = 0$, $cx + ay + b = 0$ are concurrent, show that $a + b + c = 0$

ANSWERS : 17.6

2) $k = 4$
3) $a = 3$

Sign of $ax + by + c$ (position of two points with respect to a line)

Two points (x_1, y_1) and (x_2, y_2) lies on

(i) Same side of the line $ax + by + c = 0$, if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the same sign.

(ii) opposite side of the line $ax + by + c = 0$, if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the opposite signs.

Example 1 Are the points (3, -4) and (2, 6) are on the same side or opposite sides of the line $3x - 4y = 8$

Solution Let $ax + by + c = 0$ be $3x - 4y - 8 = 0$ and $(x_1, y_1) = (3, -4)$ and $(x_2, y_2) = (2, 6)$

Now consider $as_1 + by_1 + c = 9 + 16 - 8 = 17 > 0$

$ax_2 + by_2 + c = 6 - 24 - 8 = - 26 < 0$

Since they are of opposite signs, the points lie on opposite sides of the line.

Example 2 Determine the position of the points (0, 0) and (1, -1) w.r.t the line $2x + 4y - 1 = 0$

Solution : $ax + by + c = 0$ be $2x + 4y - 1 = 0$

$(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (1, -1)$

$ax_1 + by_1 + c = 2(0) + 4(0) - 1 = -1 < 0$

$ax_2 + by_2 + c = 2(1) + 4(-1) - 1 = -3 < 0$

since the two are of the same signs, the two points lie on the same side of the line $2x + 4y - 1 = 0$

EXERCISE : 17.7

a) Show that the points (5, -1) and (-3, 4) lie on either side of the line $6x - 5y + 1 = 0$

b) Show that the points (3, -1) and (-2, 5) lie on either side of the line $3x - 7y + 8 = 0$

c) Show that the points (3, 1) and (2, -4) lie on either side of the line $2x + 3y - 1 = 0$

d) Prove that the points (0, 0) and (1, -1) lie on the same side of the line $4x - 7y + 1 = 0$.

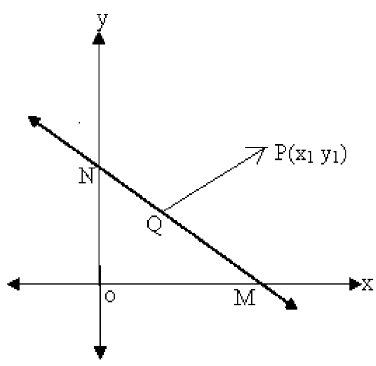
e) Show that the points (0, 0) and (2, 1) lie on the same side of the line $2x - 3y = 5$

17.9 Length of the perpendicular drawn from the point $P(x_1\ y_1)$ to the line $ax + by + c = 0$

Length of the perpendicular from $P(x_1\ y_1)$ to the line $ax+by+c=0$ us

$$PQ = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

Particular caseThe length of the perpendicular from the origin by (0,0) to the line $ax + by + c = 0$ is given by $P = \left|\frac{C}{\sqrt{a^2 + b^2}}\right|$



The absolute value is taken to show that the length of the perpendicular is measured positively.

17.10 DISTANCE BETWEEN PARALLEL LINES:

Definition: The distance between two parallel lines is the length of the perpendicular from any point on one line on to the other line.

Distance between the parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by

$$P = \left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right|$$

Example 1 Find the length of the perpendicular from (-3, 2) to the line $12x-5y+7= 0$

Solution : Length of the perpendicular $= \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

$$= \left|\frac{12(-3) - 5(2) + 7}{\sqrt{(12)^2 + (-5)^2}}\right|$$

$= 3$ units

Example 2 Find the distance between the line $3x - 4y + 12 = 0$ and the point (4, 1)

Solution : Perpendicular distance from (4, 1) to $3x - 4y + 12 = 0$

$$= \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{3(4) + (-4)(1) + 12}{\sqrt{3^2 + (-4)^2}}\right|$$

$= 4$ units

Example 3 Find the distance between the following pair of parallel lines

a) $5x + 12y + 7 = 0$ and $5x + 12y - 19 = 0$

b) $2x - 3y + 4 = 0$ and $4x - 6y - 5 = 0$

Solution:

a) Distance between parallel lines is given by $\left|\dfrac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

$$= \left|\frac{-19-7}{\sqrt{5^2+12^2}}\right| = \left|\frac{-26}{13}\right| = |-2| = 2$$

b) Equations are $2x - 3y + 4 = 0$ and $4x - 6y - 5 = 0$.

Note: The coefficients of x and y are not equal and so we shall rewrite the equations by multiplying the first equation by 2.

i.e. $4x - 6y + 8 = 0$ and $4x - 6y - 5 = 0$

Now $a = 4$, $b = -6$ $c_{1-} = 8$ $c_2 = -5$

$$\therefore \text{ distance between the lines } \left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right| = \left|\frac{8 -^{-} 5}{\sqrt{4^2+(-6)^2}}\right|$$

$$= \left|\frac{13}{\sqrt{52}}\right| = \left|\frac{13}{2\sqrt{13}}\right| = \left|\frac{\sqrt{13}}{2}\right|$$

Example 4 Find the equation of the locus of the point which moves such that its distance from $3x - 4y + 1 = 0$ is equal to its distance from $(1, -1)$

Solution: Let $A \equiv (1, -1)$ and $P(x, y)$ be any point on the locus.

Then PA = distance of P from the line $3x - 4y + 1 = 0$

$$\sqrt{(x-1)^2 + (y+1)^2} = \left|\frac{3x - 4y + 1}{\sqrt{9^2 + 16^2}}\right|$$

$$\Rightarrow (x-1)^2 + (y+1)^2 = \frac{(3x-4y+1)^2}{25}$$

$$\Rightarrow 25(x^2 - 2x + 1 + y^2 + 2y + 1) = 9x^2 + 16y^2 + 1 - 24xy - 8y + 6x$$

$$\Rightarrow 16x^2 + 24xy + 9y^2 - 56x + 58y + 49 = 0$$

which is the equation of the locus.

EXERCISE : 17.8

I. 1. Find the length of the perpendicular drawn from the point.

a) (-2, -1) to the line $4x + 3y – 5 = 0$

b) (3, 0) to the line $5x + 12y -41 = 0$

c) (2, 3) to the line $y – 4 = 0$

d) (-2, -3) to the line $x – 2y + 6 = 0$

2. Find 'K' so that the distance from (2, 3) to the line $8x + 15y + K = 0$ may be equal to 4 units.

3. Find the lengths of the altitudes of the triangle whose vertices are (5, 2), (3, -3) and (-4, 3)

4. Show that the point (1, 2) is equi distant from the lines $4x – 3y + 7 = 0$ and $5x + 12y = 16$

5. Find the co-ordinates of a point on the line $x + y + 3 = 0$ whose distance from $x + 2y + 2 = 0$ is 5 units..

II. 1. Find the distance between the parallel lines

a) $x + 2y + 3 = 0$ and $x + 2y – 7 = 0$

b) $3x + 4y – 7 = 0$ and $3x + 4y + 13 = 0$

c) $x + 7y – 3 = 0$ and $2x + 14y – 7 = 0$

d) $4x – 3y – 6 = 0$ and $4x – 3y – 2 = 0$

e) $3x + 4y – 7 = 0$ and $6x + 8y – 9 = 0$

2. Find the equation of the lines parallel to $4x + 3y = 10$ at a distance of 3 units.

3. Find the co-ordinates of a point on the x axis whose distance from $5x + 12y – 12=0$ is 1 unit.

4. Find the equation of the locus of a point which moves so that its distance from (3, 2) is equal to its distance from $2x + y = 3$

5. Show that the line $4x – 3y + 15 = 0$ and $8x + 6y – 30 = 0$ are tangents to the circle of diameter 6cm and centre at the origin.

6. Find the equation of the locus of the point which moves such that its distance from $x – y + 1 = 0$ is twice its distance from $x + y + 6 = 0$

7. Find the distance of the point of intersection of the lines $2x + 3y = 21$ and $3x - 4y + 11 = 0$ from the line $8x + 6y + 5 = 0$

8. Find the equation of the locus of the point which moves such that its distance from (a, 0) is equal to its distance from the line $x + a = 0$

9. Find the locus of the point which moves so that it is equidistant from the lines $x + y + 4 = 0$ and $7x + y + 20 = 0$

ANSWERS : 17.8

I. 1. a) $\frac{16}{5}$ b) 2 c) 1 d) $2\sqrt{5}$

2. K = 7 or - 129

3. $\frac{47}{\sqrt{85}}, \frac{47}{\sqrt{82}}, \frac{47}{\sqrt{29}}$

5. (-9, 6) and (1, -4)

II. 1. a) $2\sqrt{5}$ b) 4 c) $1/10\sqrt{2}$ d) 4/5 e) ½

2. $4x + 3y = 25$ or $4x + 3y + 5 = 0$

3. (5, 0) , $\left(\frac{-1}{5}, 0\right)$

4. $2x^2 - 4xy + 4y^2 - 18 - 14y + 56 = 0$

5. $x + 3y + 11 = 0$

7. $\frac{59}{10}$

8. $y^2 = 4ax$

9. $x = 2y$

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BASIC MATHEMATICS

[NEW NCERT SYLLABUS 2013-14 ONWARDS]

UNIT CHAPTER	NAME OF THE CHAPTERS	NO.OF TEACHING HOURS	1M	2M	3M	4M	5M	Total Marks
Unit I ALGEBRA (64 Hours)								
1.	Number Theory	08	1	3	1	-	-	10
2.	Sets, Relation and Functions	16	1	1	1	1	1	15
3.	Theory of Indices	04	1	-	1	-	-	04
4.	Logarithms	06	1	-	1	-	1	09
5.	Progressions	12	1	1	1	1	1	15
6.	Theory of equations	12	1	2	1	-	1	13
7.	Linear inequalities	06	-	1	-	1	-	06
Unit II Commercial Arithmetic (28 hours)								
8.	Simple interest and Compound interest	08	1	-	1	-	1	09
9.	Annuities	06	1	-	-	1	1	10
10.	Averages	04	-	2	-	-	-	04
11.	Percentage, profit and loss	06	1	-	1	-	1	09
12.	Linear functions	04	-	-	2	-	-	06
Unit III Trigonometry (10 hours)								
13.	Angles and Trigonometric ratios	06	1	-	1	-	1	09
14.	Standard and Allied angles	04	1	1	1	-	-	06
Unit IV Analytical Geometry (18 hours)								
15	Co-ordinate system in a plane	05	-	1	-	-	1	07
16	Locus and its Equation	03	1	1	-	-	-	03
17	Straight Line	10	-	2	1	-	1	12
		120	12	30	39	16	50	147

Model Question Paper I

BASIC MATHEMATICS

Time: 3 hrs 15min   Marks :100

Instructions :
1) The question paper consists of five parts A,B,C,D and E.
2) Part A carries 10 marks, Part B carries 20 marks, Part C carries 30 marks, Part D carries 30 marks and Part E carries 10 marks.
3) Write the question numbers properly as indicated in the question paper.

PART – A

I. Answer any Ten questions. **$10 \times 1 = 10$**

1. Define an imaginary number.
2. If A= {1,2} and B= {a,b} then find B×A.
3. If f:RR is defined by f(x)=2x+3 then find f $\left(\frac{1}{2}\right)$?
4. Simplify $\left(\frac{81}{256}\right)^{\frac{1}{4}}$
5. Find the value of $\log_{0.1}10$
6. Find the nature of the roots without solving the equation x^2-x+1=0?
7. What is the present value of an income of ₹1000 a year to be received for ever? Assume the discount rate to be 12%.
8. Convert $\frac{1}{4}$ into percentage
9. Define a radian
10. Convert 315° into radians
11. If the slope of the line AB is $\frac{3}{4}$ and AB ⊥CD then find the slope of CD
12. Find the centroid of the triangle formed by the points (2,4),(5,3) and (8,3)

PART – B

II Answer any Ten Questions. 10x2=20

13. Find the number of divisors of 960

14. If A= {1,3,5,7,9} and B= {2,4,6,8,10,12} define a function f:A→B by f(x)=x+1 $\forall$ x$\in$A, is the function one- one and onto?

15. Simplify $\left(\frac{x^a}{x^b}\right)^{a+b}\left(\frac{x^b}{x^c}\right)^{b+c}\left(\frac{x^c}{x^a}\right)^{c+a}$

16. Prove that $\log_2\left[\log_2\left[\log_2 16\right]\right]=1$

17. Is -300 a term of the A.P.10,7,4……..?

18. If $k+9, k-6, 4$ are in GP then find the value of K.

19. If α and β are the roots of the equation $x^2 - x + 2 = 0$ then show that $\alpha^2\beta + \beta^2\alpha = 2$

20. Solve 5x-3 < 3x +1 when x is an integer and x is a real number

21. Sowmya invested ₹1500 for 8 years and Anisha invested ₹7500 for 3 years at the same rate of interest. If altogether they received ₹1725 as interest find the rate of simple Interest charged

22. If the cost price of a machine is ₹150 and selling price is ₹100 fimd the loss percentage?

23. The average score of 20 boys is 60% and average score of 30 girls is 70%. Find the combined average of boys and girls?

24. Derive the equation of the line in one point form. i.e.$y-y_1 = m(x - x_1)$ geometrically where m is the slope and P(x_1,y_1) is the given point.

25. Find the value of x if the distance between (x,3) and (4,5) is "5 units

PART – C

III Answer any ten questions. **10x3=30**

26. Prove that $\sqrt{2}$ is an irrational number

27. Define an equivalence relation with an example. Also give an example of a relation which is only symmetric

28. If $a^x = b^y = c^z$ and $b^2 = ac$. Show that $\frac{1}{x}+\frac{1}{z}=\frac{2}{y}$

29. Find the number of zeroes in $(0.2)^{100}$ after the decimal point and the first significant figure.

30. Find the sum of the following series : 7+77+777+7777+…….n terms

31. A Father is 28 years older than the son, After 5 years the father's age will be 7 years more than twice that of the son. Find their present ages.

32. Solve the linear inequalities graphically:
$x + 3y \geq 3$, $2x + y \geq 2$, $x \geq 0$, $y \geq 0$

33. In what time a sum of ₹500 will earn ₹975 at the rate of 6% p.a if the compound interest is payable half yearly?

34. Calculate the arithmetic mean for the following data.

Class Interval	0 -10	10 - 20	20 - 30	30 - 40
No of Items	5	6	7	2

35. By how many percent should the use of tea be increased if the price of tea is decreased by 10% so that the expenditure remains unchanged

36. If $\tan A = \frac{12}{13}$ and $270 < A < 360^0$. Find the value of $\frac{3\sin - 2\cos A}{9\cos A + 4\sin A}$

37. Show that the straight lines $2x - 3y = 7$, $3x - 4y = 13$, $8x - 11y = 33$ are concurrent. Also find the point of concurrence.

38. Find the distance between two parallel lines $3x+4y+5=0$ and $6x+8y+20=0$

PART – D

IV Answer any six questions . **6x5=30**

39. In a group of 150 people, 70 like Cricket, 30 like hockey and cricket both. How many like Cricket only and not hockey? How many like hockey?Show the result using venn diagram

40. Evaluate using log. tables $\frac{42.15 \times 0.2713}{0.8932}$

41. Find the sum of the following series: 1.3.5 + 3.5.7 + 5.7.9 +….n terms

42. The cost of 2 kgs of sugar and 5 bags of dhal ios ₹90. The cost of 5 kgs of sugar and 2 kgs of dhal is ₹120. Find the cost of sugar and dhal per kg

43. Govind bought 51 bags in the whole sale market at an average price of ₹318 each. In which the price of 33 leather bags was ₹426 each. Find the price of the remaining cotton bags all in the increasing Arithmetic progression having the price of the costliest cotton bag was ₹150/-. Find the price of the cheapest cotton bag.

44. The daily cost of production 'C' in Rs. And 'x' unit of an assembly in $C(x) = 3.5x + 1200$. It each unit is sold for ₹6 then find the minimum number of units that should be produced and sold to ensure no loss. If the selling price is increased by half a rupee a unit then what would be the Break-Even Point.

45. P.T. $\sec^2 \frac{5\pi}{4} \operatorname{cosec}^2 \frac{5\pi}{4} - \sin^2 \frac{3\pi}{4} \cos^2 \frac{4\pi}{3} = 31/8$

46. Derive the section formula for internal division if $A(x_1,y_1)$ and $B(x_2,y_2)$ are the two given points and $P(x,y)$ be the point which divide the join of AB in the ratio m:n

47. Find the equation of the locus of the point which moves such that the ratio of its distances (2,-3) and (4,-2) is 2:3

48. Find the equation of the line which passes through the intersection of the lines $x-2y+4=0$ and $4x-3y+1=0$ and is inclined at an angle 135° with the x axis

PART – E

V Answer any one question. 1x10=10

49. a) Find the domain and Range of the function.

$$f(x) = \frac{x^2+2x+1}{x^2-8x-12} \text{ where } x \varepsilon R \quad (4)$$

b) what is the future value of ₹1000 deposited annually years for 12 gathering compound interest at 16%? (4)

c) Form the cubic equation whose roots are 3,5 and 7 (2)

50. a) Find the value of x

if $\frac{x.\sin^2 300.\sec^2 240}{\cos^2 225.\operatorname{cosec}^2 240} = \cot^2 315^o.\ \tan^2 300$ (4)

b) Find the equation of the line perpendicular to $3x$-2y+1=0 and passing through the point (1,-2) (4)

c) Insert two geometric means between 1 and 1/27 (2)

Model Question Paper II

BASIC MATHEMATICS

Time: 3 hrs 15min   Marks :100

Instructions : 1) The question paper consists of five parts A,B,C,D and E.

2) Part A carries 10 marks, Part B carries 20 marks, Part C carries 30 marks, Part D carries 30 marks and Part E carries 10 marks.

3) Write the question numbers properly as indicated in the question paper.

PART – A

I. Answer any Ten questions. **10 x 1 = 10**

1. Write the imaginary part of 4-5i.
2. If A= {1,2,3,4} and B= {1,2,3,4,5,6,7} find A- B.
3. If f:R→R is defined by f(x)=3x+5 then find f (–1) ?
4. Simplify $(5^0) + (5)^{2^0}$
5. Find the value of $\log_{10} .01$
6. Find the 8^{th} term of the progression -2,-4,-6 ……?
7. Solve for x: 2(7+x)-10=16-2(x-24).
8. Convert the ratio 3:5 into percentage
9. Define perpetuity
10. Convert 450° into radians
11. The average age of 10 boys in a class is 13 years.What is the sum of their ages?
12. Find the sslope of the line 2x+5y-11=0

PART – B

II Answer any Ten Questions. 10x2=20

13. Find the number which when divided by 36,40 and 48 leaves the same remainder

14. If A = {1, 2, 3, 4} and B = {3, 4, 5, 6} and U= {1, 2, 3, 4, 5, 6, 7, 8} verify $(A\cup B)'=A'\cap B'$

15. Find the number of positive divisors and the sum of the divisors of 960

16. Simplify $\dfrac{2^{n+1}+2^{n-1}}{2^{n}+2^{n+2}}$

17. If a,b,c are in G.P and $a^x=b^y=c^z$ show that x, y and z are in H.P?

18. The sum of two numbers is 107 and their difference is 17. Find the numbers

19. Determine the principal which will amount to ₹15000 in 8 years at 11% per annum simple interest?

20. Solve $3x-2 < 2x+1$ when x is an integer and x is a real number.Also represent on a number line

21. The average score of 20 boys is 60% and the average score of 30 girls is 70%. Find the combined average.

22. If the cost price of 10 articles is equal to the selling price of 9 articles, find the gain percent?

23. Find the value of $\sin^2\frac{\pi}{6} \quad \cos^2\frac{\pi}{3} \quad - \tan^2\frac{\pi}{4} \quad + \cot^2\frac{\pi}{3}$

24. Prove that $(\text{Sin}A + \text{Cos}\,A)^2 + (\text{Sin}\,A - \text{Cos}A)^2 = 2$

25. Find the equation of the straight line passing through (2,3) and (3,4)

PART – C

III Answer any ten questions. 10x3=30

26. In a group of 600 people, 150 students were found to be taking tea, 225 like Coffee, 100 like both tea and coffee. Find out how many were taking neither tea nor coffee? Represent using venn diagram

27. If $R^{-1} = \{(2,4), (1,2)\ (3,1), (3,2)\}$ Find R. Also find its domain and range.

28. Prove $\sqrt{2}$ is an irrational number.

OR

An electronic device makes a beep after every 60 sec. Another device makes a beep after every 62 sec,. They beeped together at 10 am.Find the time when they will next make a beep together at the earliest.

29. Prove that $\frac{1}{\log_2 4} + \frac{1}{\log_8 4} + \frac{1}{\log_{16} 4} = 4$

30. The sum of three numbers in an A.P is 15 and their product is 105.Find the numbers.

31. Find the difference between the compound interest and the simple interest on ₹5000 invested for 4 years at 8% p.a

32. Solve the linear inequalities graphically:
$5x + 4y$ d” 40, $x \geq 2$, $y \geq 3$

33. A batsman finds by getting out for a duck (0 runs) in the 11^{th} innings of his test matches, his average of the previous innings is decreased by 5 runs. What is his average after the 11^{th} innings?

34. Find the ratio in which the line joining the points (2,5) and (1,9) is divided by the x axis. Also find the point of division.

35. If an article is sold at ₹24 therre is a profit of ₹4 and if it is sold at a loss of 10% find the selling price of the article

36 . If Cot A = $\frac{5}{12}$ and A is acute, show that 2 cosec A- 4secA= $\frac{247}{50}$

37. Show that the points A(2,2), B(6,3) and C(4,11) form a right angled triangle.

38. If a train travels 15km/hr faster it would take 1 hour less to travel 180 km, find the original speed of the train.

OR

If α and β are the roots of the equation $2x^2 + 5x + 5 = 0$ then find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

PART – D

IV Answer any six questions **6x5=30**

39. Let f = { (1,1), (2,3), (0,–1) } be a function from Z to Z defined by $f(x)=ax+b$ $\forall$ some integers a and b. i) Determine a and b ii) If $f(x)=2x+1$, $g(x)=x^2+2x+1$ find i) fog(2) ii) gof(3)

40. Find the sum of all integers between 60 and 400 which are divisible by 13

41. The age of father is 5 times that of his son.Three years ago, the age of the father was 8 times that of his son. Find the present ages

42. A machine depreciates at 10% of its value at the beginning of the year.Thhe cost and the scrap value realized at the time of sale being 23240 and 9000 respectively. For how many years the machine was put to use?

43. Calculate the future value of the annuity immediate of ₹1000p.a for 12 years at 16% p.a. compounded quarterly?

44. Find the equation of the straight line passing through (-2,6) and the sum of the intercepts on the co ordinate axes is 5.

45 a) Form a quadratic equation whose roots are 2+”3, 2-”3

b) Evaluate using log. tables $\frac{0.5634 \times 0.0635}{2.563}$.

46. Find the reflection of the point P(2,1) in the line x+y=5

47. If x=rcosAcosB, y=rcosAsinB and z=rsinA prove that $x^2+y^2+z^2=r^2$

48. Find the locus of a point equidistant from (2,0) and (-2,0)

PART – E

V Answer any one question. 1x10=10

49. a) Find the equation of the straight line passing through the intersection of the line 2x+3y=5, 7x-y=6 is perpendicular to the line 3x+4y+1=0. (4)

b) If tan A+ sin A=m and tanA- sin A=n show that m^2-n^2=4(“mn) (4)

c) Find the number of digits in 3^{20}? (2)

50. a) Find the sum to n terms of the series:
7+77+777+…..n terms (4)

b) A confectioner makes and sells biscuit.He sells one pack of biscuit at ₹80. His cost of manufacturing is ₹40 per packet as variable cost and ₹3000 as fixed cost.Find the

i) Revenue function

ii) Cost function

iii) Profit function

iv) If he limits his production to 100 packets can he make profit?

v) What will be the number of boxes he must sell to make a profit so that he does not incur loss? (4)

c) If the product of two numbers is 216 and their LCM is 36. Find the HC F (2)

Model Question Paper III

BASIC MATHEMATICS

Time: 3 hrs 15min | Marks :100

Instructions :

1) The question paper consists of five parts A,B,C,D and E.
2) Part A carries 10 marks, Part B carries 20 marks, Part C carries 30 marks, Part D carries 30 marks and Part E carries 10 marks.
3) Write the question numbers properly as indicated in the question paper.

PART – A

I. Answer any Ten questions. **10 x 1 = 10**

1. Give the canonical representation of 156.
2. If A= (1,2,3,4,5} B= {1,2,3,4,5,6,7} find a relation from A to B defined by R= $\{(x,y) / x>y\}$
3. If f(x)=x+1 and g(x)= x^2 +1 find fog(1)?
4. Simplify $\left(\frac{5x^3}{2y}\right)^2$
5. Evaluate x if $\log_x 625=4$
6. Find the 6^{th} of 3,6,12 ……?
7. Solve for x if $(x+2)(x+3)=(x-2)(x-4)+20$.
8. What percent is 64m of 12km?
9. Define annuity
10. Express $3\pi/4$ in degrees
11. The average age of 10 boys in a class is 6 years.The sum of the ages of 9 of them is 52 years.Find the age of the 10^{th} student?
12. Find the slope of the line joining the points (1,2) and (-1,-2)?

PART – B

II Answer any Ten Questions. 10x2=20

13. Find the greatest number which when divides 989 and 1327 leaves the remainder of 5 and 7 respectively.

14. If A= $\{x : x \in N \ and \ x < 3\}$ and B= $\{x : x^2 - 16 = 0 \ and \ x < 0\}$ find B×A

15. Find the number of positive divisors and the sum of the divisors of 825

16. Simplify $\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}$

17. Find the value of k so that 2/3,k, (5/3)k are three consecutive terms of A.P

18. Determine the nature of the roots of $2x^2$-9x+7=0

19. Determine the amount on ₹500 for 10 years at the rate of 15% compound interest?

20. Solve for x if 4x-5 $\leq$ 27 and represent on a number line

21. The average score of 65 boys is 60 and the average score of 15 girls is 65. Find the combined average.

22. A shopkeeper buys 50 pencils for Rs.80 and sells them at 40 pencils for ₹90. Find his gain or loss percent?

23. Prove that $\frac{1}{1+\cos A} + \frac{1}{1-\cos A} = 2\text{cosec}^2 A$

24. Prove that if θ=45 show that $\frac{\tan\theta}{1+\tan\theta} - \frac{1+\tan\theta}{\tan\theta} = \frac{-3}{2}$

25. Derive the equation of the straight line in the slope point form Show that $\frac{1}{x} + \frac{1}{z} = \frac{2}{y}$

PART – C

III Answer any ten questions. **10x3=30**

26. In a group of 65 people, 40 were found to like hockey, 10 like both tennis and hockey. How many like only tennis but not hockey? How many like tennis? Represent using venn diagram

27. Given A= {2,4,6,8} and R= {(2,4), (4,2), (4,6), (6,4)} Show that R is not reflexive, symmetric and not transitive.

28. The sum of two number s is 528 and their HCF is 33. Find the number of pairs satisfying the given condition?

29. Prove that xyz=1 if $\frac{\log x}{a-b} = \frac{\log y}{b-c} = \frac{\log z}{c-a}$

30. If the first term of G.P is 729 and the 7^{th} term is 64 find the sum of first seven terms of the G.P

31. Find the present value of the annuity immediate for ₹3000 for 5 years at 10% p.a

32. Solve the linear inequalities graphically:
$3x + 2y \leq 6, \quad 4x - y \leq 6$

33. The average temperature for the first four days of the week was 39°C. The average of the whole week was 40°C.What was the average temperature during the last three days of the wweek?

34. Find the circumcentre of the triangle with vertices A(-3,4),B(3,4) and C(-4,-3). Also find the circumradius and the area of the circle

35. By selling an article for ₹825 a man loses equal to $1/3^{rd}$ of its selling price

    Find the i) cost price of the article ii) the gain % or the loss% if the same article is sold for ₹1265

36 . Find the value of x if $x \sin 45°.\tan 60° = \frac{\sin 30^0 \ \cot 30^0}{\cos 60^0 \ \operatorname{cosec} 45}$

37. Find the third vertex of the triangle if two of its vertices are A(-2,4) and B(7,-3) and the centroid at (3,2).

38. A number which when decreased by 20 is equal to 69 times the reciprocal of the number. Find the number.

PART – D

IV Answer any six questions. 6x5=30

39. In a survey it was found that 31 people liked a product A,36 liked a product B and 39 liked the product C. If 24 people liked products A and B,22 people liked product C andA, 24 people liked products B and C, 18 liked all the three products, then find how many people liked product C only?

40. The sum of three numbers which are in G.P is 30 and their product is 216. Find the numbers

41. If α and β are the roots of the equation $3x^2 - 4x + 15 = 0$ then find the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

42. If the present price of a bike is ₹32,290.If its value decreases every year by 10% then find its value before 3 years.?

43. In what time will a sum of ₹2000 becomes ₹3900 at 5% p.a compound interest payable half yearly?

44. Find the equation of the medians of the triangle formed by the points (-1,3),(-3,5) and (7,-9)

45. a) Form a quadratic equation whose roots are -2 and 5

    b) Evaluate using log. tables $\frac{25.36^2 \times 0.4569}{847.5}$

46. Find the equation of the straight line passing through the point of intersection of $2x+4y=3$ and $x+5y=1$ and making equal positive intercepts on the coordinate axes.

47. Find the locus of a point equidistant from (1,0) and (-1,0)

48. Find the sum of the following series:1+(1+2)+(1+2+3)……n terms

PART - E

V Answer any one question **1x10=10**

49 a) Prove that the lines x+y+4=0, 2x=3y+7 and 3x+y+6=0 are concurrent. Also find the point of concurrency. (4)

b) If 12 $\cot^2$A-31cosecA +32=0, find the value of sinA (4)

c) Find the number of digits in 2^{64}? (2)

50 a) Find the sum to n terms of the series: .3+.33+.333+.3333+…..n terms (4)

b) A publishing house finds that the production of each book and the cost of the book are directly attributed. If the cost of each book is ₹30 and the fixed costs are ₹15000, selling price of each book is ₹45 then determine

i) Revenue function

ii) Cost function

iii) Break even point function (4)

c) If n(U)=700, n(A)=200, n(B)=300 and n(A∩B)=100 find n (A′∩B′) (2)