ICSE Board Class X Mathematics Board Paper Semester 2 - 2022

Time: 90 minutes Marks : 40

Maximum Marks: 40 Time allowed: **One and a half hours**

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during the first **10 minutes**. This time is to be spent reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

Attempt all questions from **Section** *A* and any three questions from **Section** *B*. The intended marks for questions or parts of questions are given in **brackets** [].

SECTION A (Attempt all questions from this section)

Question 1:

Choose the correct answers to the questions from the given options. (Do not copy the question, Write the correct answer only.)

(i) The probability of getting a number divisible by 3 in throwing a dice is:

(a)	$\frac{1}{6}$
(b)	<u>1</u> 3
(c)	<u>1</u> 2
(d)	$\frac{2}{3}$

Ans.

(b) $\frac{1}{3}$ Sample space of dice = 1, 2, 3, 4, 5, 6, = (6) Favourable out once which is divisible by 3 are -3, 6 = (2) $P(E) = \frac{Favourable \ out \ cone}{total \ out \ cone} = \frac{2}{6} = \frac{1}{3}$

(ii) The volume of a conical tent is 462 m^3 and the area of the base is 154 m^2 , the height of the cone is:

(a) 15 m(b) 12 m(c) 9 m(d) 24 m

Ans. (c) 9m

Ans. volume of tent = 462 m³ Area of base = 154 m² Height =? Rol of cone = $\frac{1}{3}\pi r^2 h$ $462 = \frac{1}{3} \times \left[\frac{22}{7} \times r^2\right] \times h$ $462 = \frac{1}{3} \times [154] \times h$ $\frac{462 \times 3}{154} = h$ 9m = h

(iii) The median class for the given distribution is:

Class Interval	0 - 10	10 - 20	20 - 30	30 - 40
Frequency	2	4	3	5

(a)	0 - 10
(b)	10 - 20
(c)	20 - 30
(d)	30 - 40

Ans. (c)20 - 30 Ans.

Class intorvel	Frequency	C.F
0 - 10	2	2
10 - 20	4	6
20 - 30	3	9
30 - 40	5	14

n = 14 which is even so Median $= \frac{n}{2} = \frac{14}{2}$ = 7th term therefore 20 - 30 is median class

-1

(iv) If two lines are perpendicular to one another then the relation between their slopes m_1 and m_2 is:

(a)
$$m_1 = m_2$$

(b) $m_1 = \frac{1}{m_2}$
(c) $m_1 = -m_2$
(d) $m_1 \times m_2 =$

Ans. (d) $m_1 \times m_2 = -1$

Ans. If perpendicular then slope m_1 and slope m_2 are related to $m_1 \times m_2 = -1$

(v) A lighthouse is 80 m high. The angle of elevation of its top from a point 80 m away from its foot along the same horizontal line is:

(a)	60°
(b)	45°
(c)	30°
(d)	90°

Ans. (b) 45°

Ans.



Height of the lighthouse = 80 m Angle of elevation = ? Dis from foot = 80 m

Let AB is height of height house and BC is distance from foot of light house Perpendicular

$$\Rightarrow \tan \theta = \frac{Perpendicula}{Base}$$
$$\Rightarrow \tan \theta = \frac{80}{80} = 1$$
$$\Rightarrow \tan 45^\circ = 1$$

Therefore, $\theta = 45^{\circ}$

(vi) The modal class of a given distribution always corresponds to the:

- (a) interval with highest frequency
- (b) Interval with lowest frequency
- (c) The first interval
- (d) The last interval

Ans. (a) interval with highest frequency

Ans. We know that modal class of any distribution is height frequency Therefore = interval with high frequency

(vii) The coordinates of the point P(-3, 5) on reflecting on the x-axis are:

(a)
$$(3,5)$$

(b) $(-3,-5)$

- (c) (3,-5)
- (d) (-3,5)

Ans. (b) (-3, -5)

Ans. On reflecting x axis value of y (ordinate) change

Therefore P(-3, 5) $\xrightarrow{x \text{ axis}}$ (-3, -5)

(viii) ABCD is a cyclic quadrilateral. If $\angle BAD = (2x + 5)^\circ$ and $\angle BCD = (x + 10)^\circ$ or then x is equal to:



(a)	65°
(b)	45°
(c)	55°
(d)	5°

Answer : (c) 55° Ans.



According to law of cyclic properties that sum of opposite angle is 180° Hence $\angle A + \angle C = 180$

$$\Rightarrow 2x + 5 + x = 10 = 180$$

$$\Rightarrow 3x + 15 = 180$$

$$\Rightarrow 3x = 180 - 15$$

$$\Rightarrow 3x = 165$$

$$\Rightarrow x = \frac{165}{3}$$

$$\Rightarrow x = 55^{\circ}$$

(ix) A (1,4), B (4,1) and C (x, 4) are the vertices of $\triangle ABC$. If the centroid of the triangle is G (4,3) then x is equal to:

(a) 2 (b) 1 (c) 7 (d) 4

Ans. (c) 7

(x) The radius of a roller 100 cm long is 14 cm. The curved surface area of the roller is

(Take
$$\pi = \frac{22}{7}$$
)
(a) 13200 cm²
(b) 15400 cm²

(c) $4400 \ cm^2$

(d) 8800 cm^2

Ans. (d) 8800 cm²

Ans.



radius = 14 cm

long / height = 100 cm

Attention - Read the question very carefully to understand which value is radius and which is height.

$$C S A = 2\pi r h$$

= $2 \times \frac{22}{7} \times 14 \times 100$
= $8800 \ cm^2$

SECTION B (Attempt all questions from this section)

Question 2:

(i) Prove that:

$$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$$
$$LHS = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$$
$$= \frac{1-\sin\theta+1+1\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$
$$= \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta(1-\sin\theta))}$$
$$= \frac{2}{1-\sin^2\theta}$$

Ans.

$$= \frac{2}{\cos^2 x}$$
$$= 2 \times \frac{1}{\cos^2 x}$$
$$= 2 \sec^2 x$$

(i) L. H. S

$$\begin{aligned} \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} \\ \Rightarrow \frac{(1-\sin\theta) + (1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)} \\ \Rightarrow \frac{1-\sin\theta + 1 + \sin\theta}{1-\sin^2\theta} \\ \Rightarrow \frac{2}{\cos\theta^2} \left[1-\sin^2\theta = \cos^2\theta \right] \\ \Rightarrow 2 \sec^2\theta \left[\frac{1}{\cos\theta} = \sec\right] \\ \Rightarrow R.H.S \end{aligned}$$

(ii) Find a if A (2a + 2, 3), B (7,4) and C (2a + 5,2) are collinear.

Ans.

$$\Rightarrow \frac{-2}{2a-2} = \frac{-2}{2(a-1)}$$
$$\Rightarrow \frac{-1}{(a-1)}$$

But slope AB and BC equal due to collinear point So $\frac{1}{5-2a} = \frac{-1}{a-1}$ $\Rightarrow a - 1 = -5 + \frac{1}{a-1}$

$$\Rightarrow a - 1 = -5 + 2a$$

$$\Rightarrow -1 + 5 = 2a - a$$

$$\Rightarrow 4 = a$$

Attention [do not use area of Δ formula for above it is CBSE method]

(iii) Calculate the mean of the following frequency distribution.

Class Interval	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55
Frequency	2	6	4	8	4

Ans.

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Class interval	Frequency(F)	Class mark (x)	fx
5 - 15	2	10	20
15 - 25	6	20	120
25 - 35	4	30	120
35 - 45	8	40	320
45 - 55	4	50	200

 $\sum f = 24 \sum fx = 780$ mean $\frac{\varepsilon x.f}{\varepsilon f} = \frac{780}{24} = 32.5$





∠QSR

Ans.

(b)



 $\angle Q = 90^{\circ}$ $\angle R = 90^{\circ} \text{ [straight line drawn from centre to tangent is 90^{\circ}]}$ $\angle O + 90^{\circ} + 90^{\circ} + 70^{\circ} = 360^{\circ} \text{ [sum of all angles of a quadrilateral is 360^{\circ}]}$

$$\angle QOR + 250^{\circ} = 360^{\circ}$$

 $\angle QOR = 360^{\circ} - 250^{\circ}$
 $= 110^{\circ}$



$$\angle QSR = ?$$

Draw a line QR which is chord.

$$\angle QSR = \frac{1}{2} [\angle QOR]$$
$$\angle QSR = \frac{1}{2} \times 110^{\circ}$$
$$\angle QSR = 55^{\circ}$$

[angle substend by a chord at centre is double then any other point on circle by same chord]

Question 3:

(i) A bag contains 5 white, 2 red and 3 black balls. A ball is drawn at random. What is the probability that the ball drawn is a red ball?

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Ans. white ball = 5
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Red ball = 2

Black ball = 3

Total ball = W = R + B

= 5 + 2 + 3

= 10

Favourable outcome = red ball

= 2

P(Event) = \frac{Fourable \ outcone}{total \ outcone} = \frac{2}{10} = \frac{1}{5}
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(ii) A Solid cone of radius 5 cm and height 9 cm is melted and made into small cylinders of radius of 0.5 cm and height 1.5 cm. Find the number of cylinders so formed.

Ans. radius of cone = 5 cm Hight = 9 cm First find vol of cone = ? Vol of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 9 = \pi \times 25 \times 3 = 75\pi cm^2$ Now radius of cylinder = 0.5 cm Height, h = 1.5 cm Vol of 1 cylinder $\pi r^2 h = \pi \times 0.5^2 \times 1.5 = \pi \times 0.25 \times 1.5 = \pi \times cm^3$ Let no of cylinder = n According law of conservation of mass vol of all cylinder formed = vol of 1 cone method $n \times 0.375 \pi = 75\pi$

$$n = \frac{75}{0.375} = 200$$

(iii) Two lamp posts AB and CD each of height 100 m are on either side of the road. P is a point on the road between the two lamp posts. The angles of elevation of the top of the lamp posts from the point P are 60° and 40° . Find the distances PB and PD.



Ans.



In $\triangle ABP$ Tan $40 = \frac{AB}{BP}$

$$0.84 = \frac{100}{BP}$$
$$BP = \frac{100}{0.84} = 119 m$$

So PB = 119 m

$$PD = \frac{100\sqrt{3}}{3}m$$

$$tan \ 60 = \frac{CD}{PD}$$

$$\sqrt{3} = \frac{100}{PD}$$

$$PD = \frac{100}{\sqrt{3}}$$

$$PD = \frac{100 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$PD = \frac{100\sqrt{3}}{3}$$

$$PD = \frac{100 \times 1.732}{3}$$

$$PD = \frac{173.2}{3}$$

$$= 57.73$$

(iv) Marks obtained by 100 students in an examination are given below.

Marks	0 - 10	10 - 20	20 -3 0	30 - 40	40 - 50	50 - 60
No. of students	5	15	20	28	20	12

Draw a histogram for the given data using a graph and find the mode.

Take 2 cm = 10 marks along one axis and 2 cm = 10 students along the other axis. Ans.



mode = 34 (Approx)

Question 4:

(i) Find a point P which divides internally the line segment joining the points A (-3,9) and B (1,-3) in the ratio 1:3.

(ii) A letter of the word SECONDARY is selected at random. What is the probability that the letter selected is not a vowel?

(iii) Use a graph paper for this question. Take 2cm-1 unit along both the axes.

(a) Plot the points A(0,4), B(2,2), C(5,2) and D(4,0). E (0,0) is the origin.

(b) Reflect B, C, D on the y-axis and name then as B', C' and D' respectively.

(c) Join the points ABCDD'C'B' and A in order and give a geometrical name to the closed figure

(iv) A solid wooden cylinder is of radius 6 cm and height 16 cm. Two cones each of radius 2 cm and height 6 cm are

drilled out of the cylinder. Find the volume of the remaining solid.

Take $\pi = 22/7$



Ans.

[i]

$$\begin{array}{ccc} & \underset{\uparrow}{\overset{\uparrow}{\uparrow}} & \underset{\uparrow}{\overset{\uparrow}{\uparrow}} \\ A \underbrace{\leftarrow 1 \longrightarrow P \leftarrow 3 \longrightarrow}_{(-3,9)} B \\ (-3,9) & (x,y) & (1,-3) \\ \downarrow & \downarrow \\ x_1 & y_1 & & x_2 & y_2 \end{array}$$

Let the co-ordinate of
$$P(x, y)$$

 $x = \frac{1 \times 1 + 3(-3)}{1+3}$

$$x = \frac{mx_2 + nx_1}{m + n}$$
$$x = \frac{1 - g}{4}$$
$$x = \frac{-8}{4}$$
$$x = -2$$

$$y = \frac{my_2 + ny_1}{m+n}$$

$$y = \frac{1 \times (-3) + 3 \times g}{1+3}$$
$$y = \frac{-3 + 2}{4}$$
$$y = \frac{24}{4}$$
$$y = 6$$

Therefore coordinate of p(x, y) = p(-2, 6)

[ii]

Total outcome = S, E, C, O, N, D, A, R, Y



[iv]

Radius of cylinder = 6 cm Height = 16 cm Volume of cylinder = $\pi r^2 h$

$$=\pi \times 6^2 \times 16$$

$$= 36 \ge 16\pi$$

Radius of each cone = 2 cm

Height = 6 cm

Vol of = $\frac{1}{3} \times \pi \times 2^2 \times 6$

Vol of 2 cone = 8π

Vol of 2 cone = $2 \times \text{vol of } 1$ cone

Vol of 2 cone =
$$2 \times 8\pi$$

Vol of 2 cone = 16π

Vol of remaining = vol of cylinder - vol of 2 cones solved



Question 5:

(i) Two chords AB and CD of a circle intersect externally at E. If EC = 2 cm, EA = 3 and AB = 5 cm cm, Find the length of CD.



(ii) Line AB is perpendicular to CD. Coordinate of B, C and D are respectively (4,0), (0, -1) and (4,3).



Find:

(a) Slope of CD

(b) Equation of AB

(iii) Prove that :

$$(1 + \sin\theta)^2 + (1 - \sin\theta)^2 / 2\cos^2\theta = \sec^2\theta + \tan^2\theta$$

(iv) The name of the following distribution is 50. Find the unknown frequency.

Class interval	Frequency
0 - 20	6
20 - 40	f
40 - 60	8
60 - 80	12
80 - 100	8

Ans.

[i]



If the two lines from external point intersect the circle at two points respectively, then:

$$EA \times EB = EC \times ED$$

$$\Rightarrow 3 \times (3+5) = 2 \times (2+x)$$

$$\Rightarrow g + 15 = 4 \times 2x$$

$$\Rightarrow 24 - 4 = 2x$$

$$\Rightarrow \frac{20}{2} = x$$

$$\Rightarrow 10 = x$$



Slope
$$CD = ?$$

C = (0, -1), D = (4, 3) $\downarrow \downarrow \qquad \downarrow \downarrow$ $x_1 \ y_1 \qquad x_2 \ y_2$ Slope CD = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-1)}{4 - 0} = \frac{3 + 1}{4} = \frac{4}{4} = 1$

Slope CD = 1

[b] Let slope of AB is m_2

According to law of perpendicularity

$$m_1 \times m_2 = -1$$

 $\Rightarrow 1 \times m_2 = -1$ $[m_1 = 1 from part a]$

$$m_2 = \frac{-1}{1} = -1$$

Now equation AB = ?

B = (4,0)

 $x_1 y_1$

equation of a line of one coordinate and slope given is

$$y - y_1 = m(x - x_1)$$

 $y - 0 = -1(x - 4)$
 $y = -x + 4$ [m = slope of AB = m₂]
 $x + y = 4$
 $x + y - 4 = 0$

[iii]

L.H.S

$$\frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{2\cos^2\theta}$$

$$= \frac{1+\sin^2\theta + 2\sin\theta + 1 + \sin^2\theta - 2\sin\theta}{2\cos^2\theta}$$

$$= \frac{2+2\sin^2\theta}{2\cos^2\theta}$$

$$= \frac{2(1+\sin^2\theta)}{2\cos^2\theta}$$

$$= \frac{1+\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1+\sin^2\theta}{\cos^2\theta}$$

$$[\frac{1}{\cos^2\theta} = \sec^2\theta, \frac{\sin\theta}{\cos\theta} = \tan\theta]$$

$$= \sec^2\theta + \tan^2\theta$$

$$= R.H.S$$

Class interval	Classwork	Frequency	fx
0 - 20	10	6	60
20 - 40	30	f	30f
40 - 60	50	8	400
60 - 80	70	12	840
80 - 100	90	8	720

$$\sum f = 34 + f$$
 $\sum f \cdot f = 2020 + 30f$

Mean = 50 (given)

 $mean = \frac{\sum fx}{\sum f}$ $50 = \frac{2020 + 30f}{34 + f}$

$$50 \times (34 + f) = 2020 + 30f$$
$$1700 + 50f = 2020 + 30f$$
$$50f - 30f = 2020 - 1700$$
$$20f = 320$$
$$f = \frac{320}{20}$$

Question 6 :

(i) Prove that:

$$1 + tan^2\theta/(1 + sec\theta) = sec\theta$$

(ii) In the given figure A, B, and D are points on the circle with centre O.



Given, $\angle ABC = 62^{\circ}$.

Find:

(a) ∠ADC

(b) ∠CAB

(iii) Find the equation of a line parallel to the line 2x + y - 7 = 0 and passing through the intersection of the line x + y - 4 = 0 and 2x - y = 8.

(iv) Marks obtained by students in an examination are given below.

Marks	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of Student	3	8	14	9	4	2

Using graph paper, draw an ogive and estimate the median marks. Take 2 cm = 10 marks along one axis and 2 cm = 5 students along the other axis.

Ans.

[i]



 $L.H.S = 1 + \frac{tan^2\theta}{1 + sec\theta}$

$$= \frac{1 + \sec\theta + \tan^2\theta}{1 + \sec\theta}$$
$$= \frac{1 + \sec\theta}{1 + \sec\theta}$$
$$[\tan^2\theta = \sec^2\theta - 1]$$
$$= \frac{\sec\theta + \sec^2\theta}{1 + \sec\theta}$$
$$= \frac{\sec\theta + (1 + \sec\theta)}{(1 + \sec\theta)}$$
$$= \sec\theta = R.H.S$$

[ii]

[a] $\angle ADC = ?$

 $\angle ADC = \angle ABC$

 $\angle ADC = 62^{\circ}$ [same chord subtend equal angles on the circle]

[b] $\angle CAB = ?$

 $\angle ACB = 90^{\circ}$ [angle on semi circle = 90°]

Now in ⊿ABC

 $\angle ACB + \angle CBA + \angle BAC = 180^{\circ}$

90° + 62° + ∠ACB = 180°

152° **+** ∠**ACB = 180**°

∠ACB = 180° - 152°

 $\angle ACB = 28^{\circ}$

[iii]

The equation 2x + y - 7 = 0 can be written as y = 7 - 2x

Comparing the above equation with y = mx + c, slope of the equation 2x + y - 7 = 0 will be:

Slope = m = -2

Let $m = m_1$ and a line parallel to any line has slope m_2 then,

$$m_1 = m_2 = -2$$

Therefore, slope of line parallel to 2x + y - 7 = 0 is -2

Now, it is given that x + y - 4 = 0 and 2x - y = 8 intersects, so

x + y - 4 = 02x - y = 8

3x = 12x = 4

Therefore, the parallel line cut at (4,0)

Putting value of x = 4 in the equation of line, we get:

$$x + y - 4 = 0$$
$$4 + y = 4$$
$$y = 4 - 4$$
$$y = 0$$

So now the equation of the line parallel to 2x + y - 7 = 0 and passing through the intersection of the line x + y - 4 = 0 and 2x - y = 8 will be:

$$y - y_1 = m (x - x_1)$$

 $y - 0 = -2 (x - 4)$
 $y = -2x + 8$
 $y + 2x - 8 = 0$

[iv]		
Marks	No of student	cf
10 - 20	3	3
20 - 30	8	11
30 - 40	14	25
40 - 50	9	34
50 - 60	4	38
60 - 70	2	40

Now we will plot the points (20,3), (30,11), (40,25), (50,34), (60,38) and (70,40) on graph

To get the median n = 40, which is even:

Median $=\frac{n}{2}=\frac{40}{2}=20$

Take a point 20 on y axis and through it draw a line parallel to x axis which meet current at A. Through A draw a perpendicular on x axis which meet on 3 > 5

So median is = 3 > 5

