

Factorisation

Factorisation of algebraic expression

When we factorise an algebraic expression, we write it as a product of factors. These factors may be numbers, algebraic variables or algebraic expressions

The expression $6x(x - 2)$. It can be written as a product of factors.
 $2, 3, x$ and $(x - 2)$

$$6x(x - 2) = 2 \times 3 \times x \times (x - 2)$$

The factors $2, 3, x$ and $(x + 2)$ are irreducible factors of $6x(x + 2)$.

Method of Factorisation

Name	Working
Common factor method	1) We can look at each of the term in the algebraic expression, factorize each term 2) Then find common factors to factorize the expression Example $2x+4$ $=2(x+2)$
Factorisation by regrouping terms	1) First we see common factor across all the terms 2) we look at grouping the terms and check if we find binomial factor from both the groups. 3) Take the common Binomial factor out Example $2xy + 3x + 2y + 3$ $= 2 \times x \times y + 3 \times x + 2 \times y + 3$ $= x \times (2y + 3) + 1 \times (2y + 3)$ $= (2y + 3)(x + 1)$
Factorisation using identities	Use the below identities to factorise it $(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$ $(a + b)(a - b) = a^2 - b^2$

Factorisation of the form $(x+a)(x+b)$

Given $x^2 + px + q$,

1) we find two factors a and b of q (i.e., the constant term) such that

$$ab = q \text{ and } a + b = p$$

2) Now expression can be written

$$x^2 + (a + b)x + ab$$

$$\text{or } x^2 + ax + bx + ab$$

$$\text{or } x(x + a) + b(x + a)$$

or $(x + a)(x + b)$ which are the required factors.

Example

$$x^2 - 7x + 12$$

$$\text{Now } 12 = 3 \times 4 \text{ and } 3 + 4 = 7$$

$$= x^2 - 3x - 4x + 12$$

$$= x(x - 3) - 4(x - 3) = (x - 3)(x - 4)$$

Division of algebraic expression

Division of algebraic expression is performed by Factorisation of both the numerator and denominator and then cancelling the common factors.

Steps of Division

- 1) Identify the Numerator and denominator
- 2) Factorise both the Numerator and denominator by the technique of Factorisation using common factor, regrouping, identities and splitting
- 3) Identify the common factor between numerator and denominator
- 4) Cancel the common factors and finalize the result

Example

$$\begin{aligned} & 48(x^2yz + xy^2z + xyz^2) / 4xyz \\ &= 48xyz(x + y + z) / 4xyz \\ &= 4 \times 12 \times xyz(x + y + z) / 4xyz \\ &= 12(x + y + z) \end{aligned}$$

$$\text{Here Dividend} = 48(x^2yz + xy^2z + xyz^2)$$

$$\text{Divisor} = 4xyz$$

$$\text{Quotient} = 12(x + y + z)$$

So, we have

$$\text{Dividend} = \text{Divisor} \times \text{Quotient.}$$

In general, however, the relation is

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

When remainder is not zero