— Motion in Straight line

Motion – Motion is change ~~o~~ in position of an object with time. Study of motion of material object is called Mechanics

Broad classification of Mechanics :-

(i) Statics – Study of objects at rest. i.e. when ~~iii~~ effect of forces on object is in equilibrium.

ii) Kinematics – Study of ~~objects~~ motion of material objects without taking into account the ~~factors~~ which cause motion.

iii) Dynamics – Branch of mechanics, deals with study of motion of objects taking in account factors which cause motion.

Rest and motion are relative :

Rest : object is at rest if it does not change its position w.r.t. ~~to~~ surrounding.

motion : if object changes its position with time, w.r.t. surrounding.

e.g. person sitting in moving bus is at rest w.r.t. fellow passenger, but in motion w.r.t. outside person.

Point mass object – if during motion in given time, object covers distance much greater than its own size. This concept is just to ~~make~~ simplify the problems.

e.g. Size of earth is negligible while studying revolution around sun. So earth can be considered Point mass.

⇒ Frame Of Reference - It is a system of coordinates axes → attached to an observer having a clock with him —— w.r.t. which, the observer can describe position, displacement acceleration, etc of a moving body.

a) Inertial F.o.R. - in which Newton's 1st law of motion holds good. eg. F.O.R. attached to person in bus moving with uniform velocity.

b) Non-Inertial Frame of Ref. - Newton's 1st law of motion does not hold good. F.oR. attached to person in bus moving with variable velocity or accelerated along path.

* Due to variable velocity of earth, Newton's 1st law does not hold good. ∴ for observing things outside earth, F.O.R. of person on earth is non-inertial.

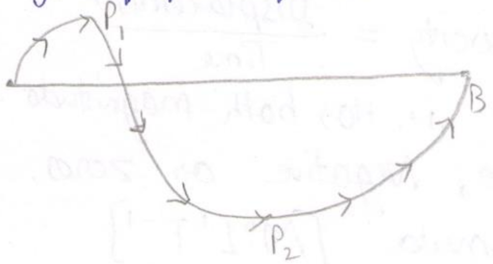
However if object is on earth, than earth can be taken at rest. So F.oR attached to person is inertial.

⇒ Scalar Quantities - only magnitude but no ~~friction~~ direction. eg. Mass, length, time, speed, work, etc

⇒ Vector Quantities - have magnitude as well as direction. eg. Displacement, velocity, acceleration, etc.

3.2

Path length (Distance)	Displacement
• It is length of actual path traversed	• It is shortest distance b/w initial & final position
• It is scalar	• It is vector quantity.
• Cannot be zero or negative. Alway negative.	• Can be positive, zero or negative.
• always equal to or greater than displacement.	• alway equal or less than distance.
• Distance b/w two pt. can have many value depending upon path.	• It is fixed b/w 2 point.
• Distance travelled b/w 2 pt. tells the type of path folled.	—



Here Distance is $AP_1 + P_1P_2 + P_2B$

& Displacement is AB.

Speed

Speed – Ratio of total path length and the corresponding time taken by the object.

Speed = Distance / time.

- Scaler Quantity, i.e. it gives no idea of direction of motion.
- Can be zero or positive but never negative.

Dimensional formula $[M^0L^1T^{-1}]$

– **uniform speed** – if object cover equal distance in equal interval of time.

– **Variable speed** – if object cover unequal distance in equal interval of time or vice-versa.

- **Average speed** ratio of. Total distance to time.

$$\text{Av. speed} = \frac{\text{Total distance travelled}}{\text{Total time taken.}}$$

- **instantaneous speed** Speed at given instant.
  - It is defined as limit of average speed as the time interval Δt at which given instant of time, becomes infinitesimally small.

$$\text{Instantaneous speed} = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = \frac{ds}{dt}$$

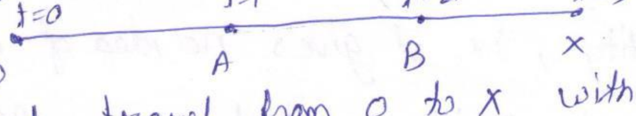
$S \to$ distance

- This speed is measured by speedometer.

- **Velocity** Ratio of displacement & corresponding time interval. $\text{velocity} = \frac{\text{Displacement}}{\text{Time}}$
  - Vector quantity. i.e. Has both magnitude & direction.
  - Can be positive, Negative or zero.
  - Dimensional formula. $[M^0 L^1 T^{-1}]$

⇒ **Uniform Motion in Straight line** if object undergoes equal displacement in equal time.

e.g.



let object travel from O to X with velocity $\vec{v}$

let $OA = \vec{x}_1$ & $OB = \vec{x}_2$

→ expression for velocity in AB:

$$\text{Displacement} = \vec{x}_2 - \vec{x}_1$$

$$\text{time interval} = t_2 - t_1$$

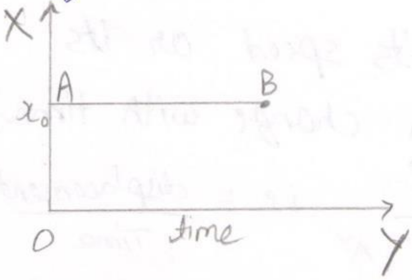
$$\vec{v} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1}$$

Important features of uniform motion

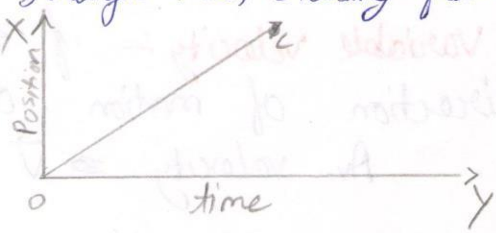
- No force is req. to be in uniform motion.
- Average and instantaneous velocities are same.
- For uniform motion along straight line, magnitude of displacement = distance covered.

⟹ Position time graph for moving object.

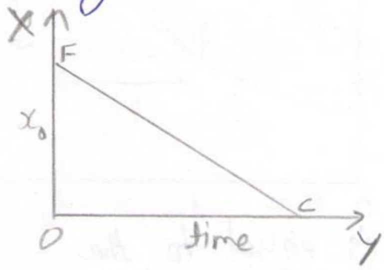
(i) object is at rest.



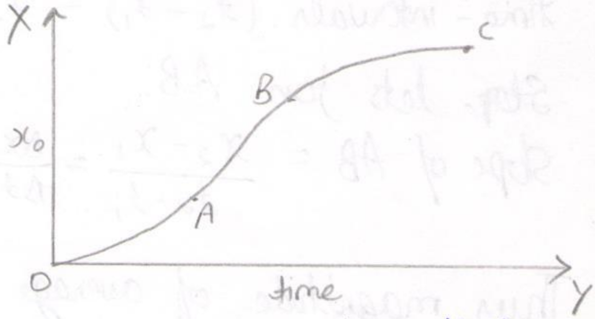
ii) object in uniform motion along straight line, starting from origin.



iii) object with constant negative velocity.



iv) object in non-uniform motion



Position time graph cannot be a straight line // to position axis, because it will indicate infine velocity.

* Velocity of an object in uniform motion is equal to the slope of position-time graph with time axis.

i.e. Slope of position-time graph gives velocity.

* Area enclosed in velocity-time graph give displacement (if sign considered) & distance (if sign not considered)

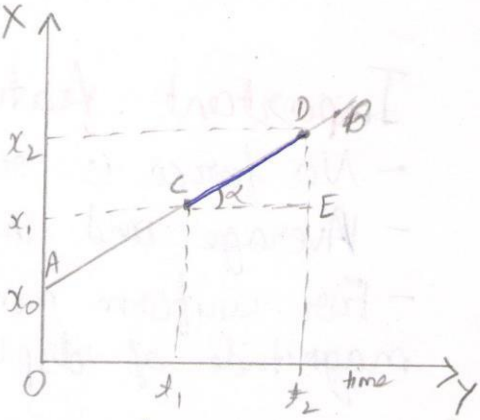
Considering motion of object from C to D

Displacement $= x_2 - x_1 = DE$

Similarly time $= t_2 - t_1 = CE$

$$\text{velocity} = \frac{DE}{CE} = \tan\alpha$$

= Slope of position-time graph.



⇒ Non-Uniform Motion

Variable velocity – if either its speed or its direction of motion or both change with time

$$\text{Av. velocity} = \vec{v} = \frac{\Delta\vec{x}}{\Delta t} \quad \text{i.e.} = \frac{\text{displacement}}{\text{Time}}$$

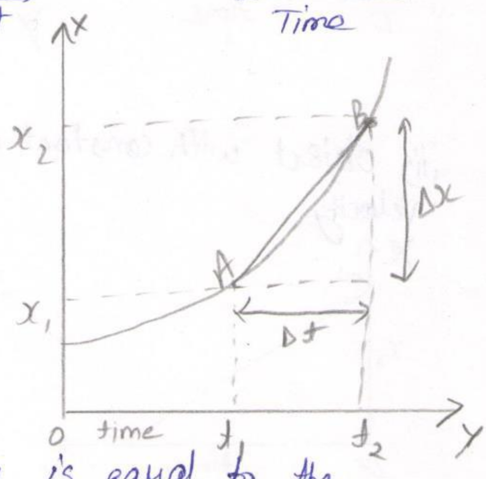
From the graph:

Change in position of object in time-intervals $(t_2 - t_1) = x_2 - x_1)$

~~Slop~~ lets join A.B.

$$\text{Slope of } AB = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = V_{av}$$

$= \tan\theta$



Thus magnitude of average velocity is equal to the slope of straight line joining initial & final point.

→ Quantitatively, ~~Vav~~ Average velocity is ratio of change in position or displacement $(\Delta\vec{x})$ to the time interval (Δt) in which displacement occur.

Instantaneous Velocity – limit of average velocity as time interval Δt, around time t, becomes infinitesimally small.

$$V_i = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

→ i.e. 1st derivative of displacement w.r.t. at that instant.

→ Relative velocity in one-dimensional motion

- The time rate of change of relative position of one object w.r.t. another.

let 2 object A & B be moving with velocity V_A & V_B. than as seen from A, B has a velocity $(V_B - V_A)$

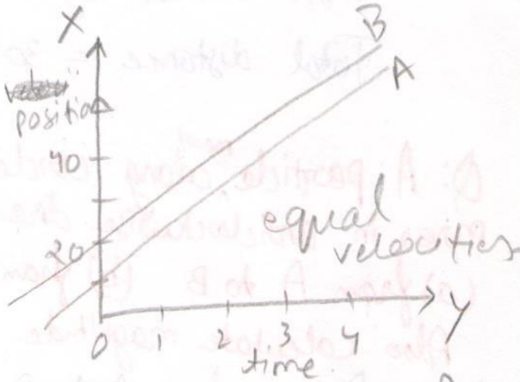
or. $V_{BA} = V_B - V_A$

$(\text{Velocity of B w.r.t. A}) = V_B - V_A$

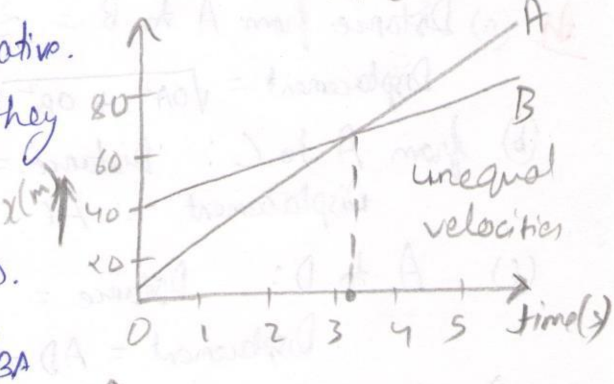
Special cases:

(i) if $V_A = V_B$

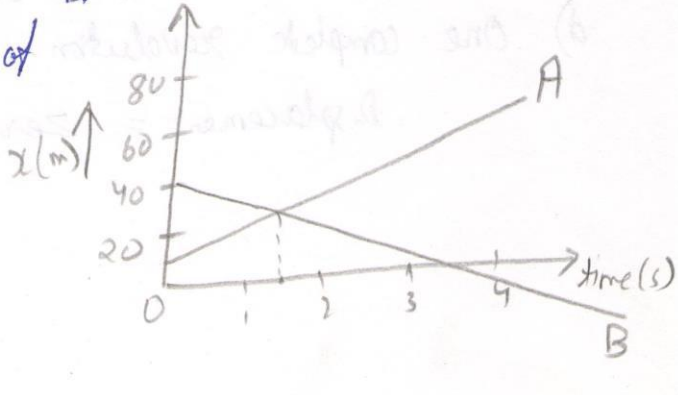
In such situation, 2 object stay at constant distant & their position time graph are st. lines parallel to each other.



ii) if $V_A > V_B \Rightarrow V_B - V_A$ is negative. One graph will be steeper & they meet at common point.



iii) if V_A & V_B are of opp. signs. Than, magnitude of V_{AB} or V_{BA} is greater than magnitude of individual velocity.



Numericals

Q- Delhi is 200 km from Ambala. A sets out from Ambala at speed of 30 km/h & B from Delhi at 20 km/h. When will they meet ?

Ans 1st method. Here $V_A = 30\,km/h$ $V_B = -20\,km/h$

Realtive speed. $V_{AB} = V_A - V_B = 30 - (-20) = 50\,km/h$.

Distance = 200 km/h.

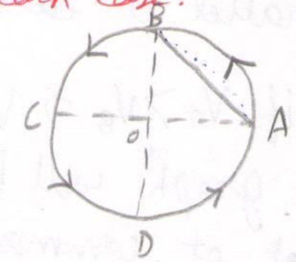
$Time = \frac{200}{50} = 4\,hr.$ Hence after 4 hours they meet

2nd - let two meet after t hours.

Dist. covered by A = $(30t)$ km, Distance by B = $(20t)$ km

Total distance = $30t + 20t = 200\,km$. $\Rightarrow$ $t = 4$ hrs.

Q: A particle moves along circle of radius R. It start from A & moves in anticlockwise direction. Calculate dist. travelled by it (a) from A to B (b) from A to C, (c) from A to D (d) in one revolution. Also calculate magnitude of displacement in each case.



Ans (a) Distance from A to B = $\frac{2\pi r}{4} = \frac{\pi r}{2}$

Displacement = $\sqrt{OA^2 + OB^2} = \sqrt{r^2 + r^2} = \sqrt{2}\,r$

(b) from A to C : Distance = πr

Displacement = AC = $2r$

(c) A to D : Distance = $2\pi r \times \frac{3}{4} = \frac{3\pi r}{2}$

Displacement = AD = $\sqrt{2}\,r$

d) One complete revolution : Distance = $2\pi r$

Displacement = zero (initial position = final position)

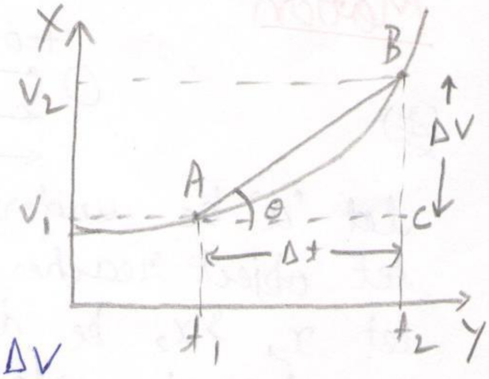
→ **Acceleration** – Ratio of change in Velocity and corresponding time taken.

$$Acc = \frac{\text{Change in Velocity}}{\text{time}}$$

• **Uniform Acc.** – if object's velocity changes by equal amounts in equal intervals of time.

• **Average Acc.** – ratio of total change in Velocity during motion to the time taken.

Thus, the slope of straight line joining 2 points on velocity - time graph gives average acceleration of object between these 2 points.



– From graph
Change in velocity in time $(t_2 - t_1)$
$= V_2 - V_1$

∴ Acc to formula.

$$\text{Av. Acceleration} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta V}{\Delta t}$$

$$\boxed{a_{av} = \frac{\Delta V}{\Delta t}}$$

In ΔACB $\boxed{\tan\theta = \frac{BC}{AC} = \frac{\Delta V}{\Delta t}}$

– Av. Acc. can be positive or negative or zero.

→ Instantaneous ~~Vector~~ Acceleration. –

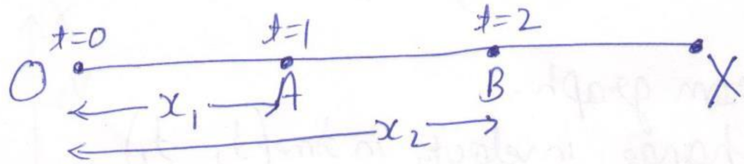
Acceleration at a given instant.

i.e. $\vec{a} = \lim_{\Delta t \to 0} \left(\frac{\Delta \vec{V}}{\Delta t} \right) = \frac{d\vec{V}}{dt}$

As $\boxed{\vec{V} = \frac{d\vec{x}}{dt}}$ → $\boxed{\therefore \vec{a} = \frac{d}{dt}\left(\frac{d\vec{x}}{dt}\right) = \frac{d^2\vec{x}}{dt^2}}$

* Instantaneous acceleration is also defined as slope of tangent to velocity – time graph corresponding to given instant of time.

⇒ Kinematic equations for Uniformly Accelerated Motion



let 'a' be uniform acceleration.
let object reaches point A & B at t_1 & t_2.
let x_1 & x_2 be displacement at t_1 & t_2 respectively
& V_1 & V_2 be velocities of object at A & B.

(i) Velocity – time relation

$$\text{Acceleration} = \frac{\text{change in Velocity}}{\text{time taken}}$$

$$\Rightarrow a = \frac{V_2 - V_1}{t_2 - t_1} \qquad \Rightarrow V_2 - V_1 = a(t_2 - t_1)$$

$$\Rightarrow \boxed{V_2 = V_1 + a(t_2 - t_1)}$$

if $V_2 \Rightarrow$ final velocity $= V$
$V_1 \Rightarrow$ initial velocity $= u$
& $t_2 - t_1 \Rightarrow$ time taken

than $\boxed{V = u + at}$

ii) Position - time relation

$$V_{av} = \frac{\text{displacement}}{\text{time taken}} = \frac{x_2 - x_1}{t_2 - t_1} \Rightarrow (x_2 - x_1) = V_{av}(t_2 - t_1)$$

But $V_{av} = \frac{\text{initial velocity} + \text{Final velocity}}{2} = \frac{V_1 + V_2}{2}$

$$\therefore (x_2 - x_1) = \left(\frac{V_1 + V_2}{2}\right)(t_2 - t_1)$$

from previous derivation we know $V_2 - V_1 = a(t_2 - t_1)$

$$\therefore x_2 - x_1 = \left[\frac{V_1 + V_1 + a(t_2 - t_1)}{2}\right](t_2 - t_1)$$

$$x_2 = x_1 + \frac{2V_1(t_2-t_1)}{2} + \frac{a}{2}(t_2 - t_1)^2$$

If $x_1 = 0$ at $t_1 = 0$, V_1 be initial velocity u

displacement $x_2 = x$ & time taken $(t_2 - t_1) = (t_2 - 0) = t$

our equation become:

$$\boxed{x = ut + \frac{1}{2}at^2}$$

(iii) Position - velocity relation

From 1st derivation: $t_2 - t_1 = \frac{V_2 - V_1}{a}$

" 2nd " : $x_2 - x_1 = \left(\frac{V_1 + V_2}{2}\right)(t_2 - t_1)$

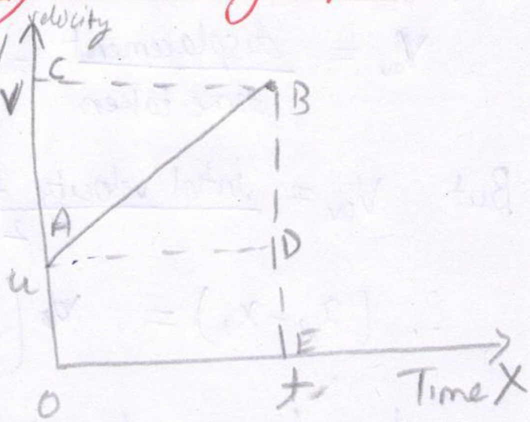
We get $(x_2 - x_1) = \left(\frac{V_1 + V_2}{2}\right)\left(\frac{V_2 - V_1}{a}\right)$

On replacing variables.

$$x = \left(\frac{u + v}{2}\right)\left(\frac{v - u}{a}\right)$$

$$\boxed{v^2 - u^2 = 2ax}$$

→ Derivation from Velocity - time graph



(i) ∵ Slope of v-t graph represent acceleration.

$$a = \frac{DB}{AD} = \frac{v-u}{t}$$

$$\Rightarrow \boxed{v = u + at}$$

ii) ∵ area under v-t graph represent distance covered.

∵ $a =$ Slope of v-t graph

$$a = \frac{DB}{AD} = \frac{DB}{t} \Rightarrow DB = at$$

Distance = area of ΔADB + area of rect. OADE

$$S = \frac{1}{2} \times t \times DB + (u \times t)$$

(∵ $DB = at$)

$$\boxed{S = ut + \frac{1}{2} at^2}$$

(iii) ∵ $a = \frac{DB}{AD} = \frac{v-u}{t} \Rightarrow t = \frac{v-u}{a}$ ← Because we do not need t in final equation

$S = \cancel{\frac{1}{2} \times t \times DB}$ + area of trapezium ABEO

$$S = \frac{1}{2}(AO + EB) \times OE = \frac{1}{2}(u+v)t$$

using value of t

$$S = \frac{1}{2}(u+v)\frac{(v-u)}{a}$$

$$\boxed{v^2 - u^2 = 2aS}$$

→ Derivation By Calculus Method

(i) v-t relation

let at an instant t, dv be change in velocity in time dt. Then $a = \frac{dv}{dt}$ or $dv = a\,dt$

integrating it w.r.t. time from $t=0$ to $t=t$ & velocity changes from u to v, we get.

$$\int_u^v dv = a\int_0^t dt$$

$$v - u = a(t-0)$$

$$\boxed{v = u + at}$$

ii) Distance-time relation

$\because$ $v = \frac{dx}{dt}$ or $dx = v\,dt = (u+at)\,dt$.

integrating: $$\int_{x_0}^{x} dx = u\int_0^t dt + a\int_0^t t^2\,dt$$

$$\boxed{x = ut + \frac{1}{2}at^2}$$

iii) Velocity – displacement relation

$\because$ $a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{dv}{dx} \times v$

$\Rightarrow$ $a\,dx = v\,dv$

on integrating: $$\int_{x_0}^{x} a\,dx = \int_u^v v\,dv$$

$$= a(x)_{x_0}^{x} = \left[\frac{v^2}{2}\right]_u^v \Rightarrow a(x - x_0) = \frac{v^2}{2} - \frac{u^2}{2}$$

$\because$ $x_0 = 0 \Rightarrow$ $$\boxed{v^2 - u^2 = 2ax}$$

→ V-t graph of Accelerated Motion

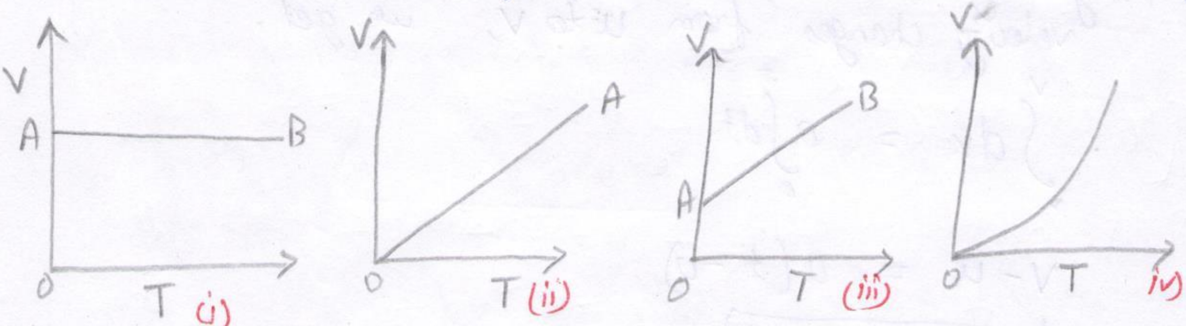
i) at $a = 0$

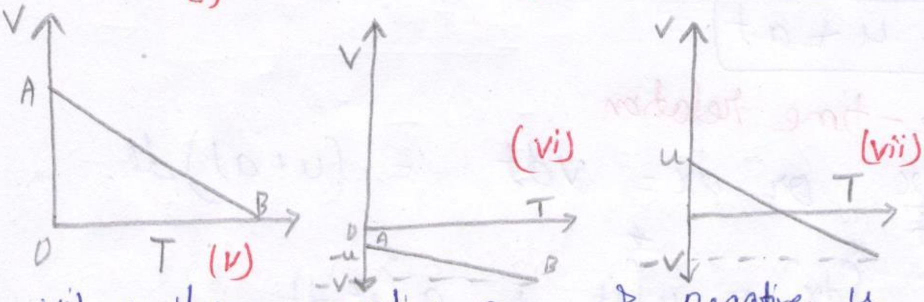
ii) $u = 0$ & constant a (>0)

iii) having some initial velocity & constant a.

iv) increasing a, $u = 0$

v) constant negative a., but positive initial velocity.





vi) uniform negative a, & negative u

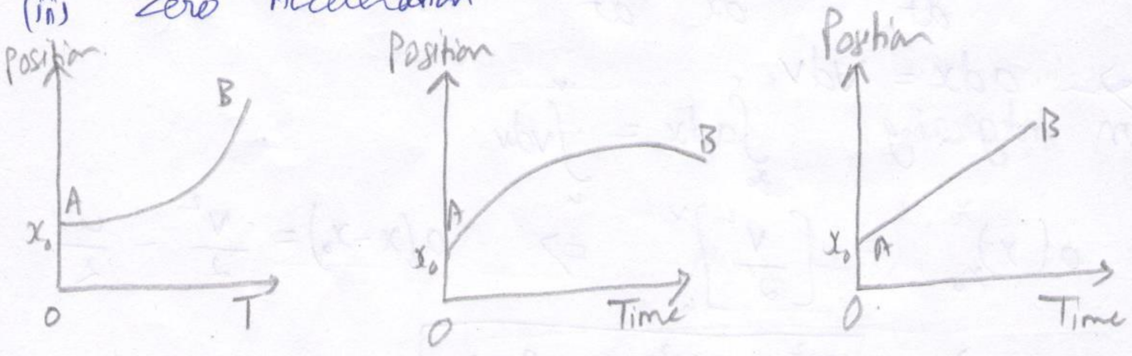
vii) uniform " a, & having positive u

→ Position – Time graph for Accelerated Motion

(i) object moving with uniform positive acceleration

(ii) " " " " negative "

(iii) Zero Acceleration



→ Distance travelled in n^{th} Second of Uniformly accelerated Motion :-

Let S_n & S_{n-1} be distance travelled in n^{th} & $(n-1)^{th}$ second.

∴ Distance travel in n^{th} sec ⇒ $D_n = S_n - S_{n-1}$

by equation of motion: $S_n = un + \frac{1}{2}an^2$

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

Putting values: $D_n = \left[un + \frac{1}{2}an^2\right] - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$

$$D_n = \cancel{un} + \cancel{\frac{1}{2}an^2} - \cancel{un} + u - \cancel{\frac{1}{2}an^2} - \frac{a}{2} + an$$

$$D_n = u + an - \frac{a}{2}$$

$$D_n = u + \frac{a}{2}(2n-1)$$

Q A particle moving with intial velocity 5 km/s & acc. $2 m/s^2$ for 10 sec. along straight line. Find. displacement in last second & total distance travelled.

Sol Here: $u = 5 m/s$ $\quad a = 2 m/s^2$ $\quad$ time $= 10$ sec

$$S = ut + \frac{1}{2}at^2 = 5 \times 10 + \frac{1}{2} \times 2 \times 10 \times 10$$

$$= 50 + 100 = 150 m$$

$$D_n = u + \frac{a}{2}(2n-1) = 5 + \frac{2}{2}(2 \times 10 - 1)$$

$$= 5 + 19 = 24 m$$

Q Ball thrown vertically up with velocity 20m/s from top of building. 25m ~~from~~ high. (a) How high will ball rise ? (b) How long it take to reach ground.

Sol (a) $u = 20 m/s$, $a = -10 m/s^2$, $V = 0$, $S = ?$, $t = ?$

As : $V^2 = u^2 + 2as$

$0 = (20)^2 + 2 \times (-10) S \Rightarrow S = 20m$

b) As, $V = u + at$

$0 = 20 + (-10)t \Rightarrow t = 2 sec$

b) from Top point to ground :

$u = 0$, $a = 10 m/s^2$, $S = 20 + 25 = 45 m$, $t = ?$

$S = ut + \frac{1}{2}at^2 \Rightarrow 45 = \frac{1}{2} \times 10 t^2$

$t = 3 sec$

Total time = $2 + 3 = 5$ Sec

Q Position of object moving along x axis is given by $x = a + bt^2$, where $a = 8.5m$ & $b = 2.5 m/s^2$. Find V at $t=0$ & $t = 2$ sec. find av. Velocity b/w $t = 2$ & 4 sec.

Sol: Velocity, $V = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 2bt$

$V = 2 \times 2.5 \times t = 5t$

at $t=0$; $V = 0$

$t = 2$; $V = 5 \times 2 = 10 m/s$.

at $t=0$; displacement ; $x_0 = a + b \times 0^2 = a$

at $t=2$; " $x_2 = a + b \times 2^2 = a + 4b$

at $t=4$; $x_4 = a + b \times 4^2 = a + 16b$

Displacement in time interval 2 to 4 sec = $x_4 - x_2$

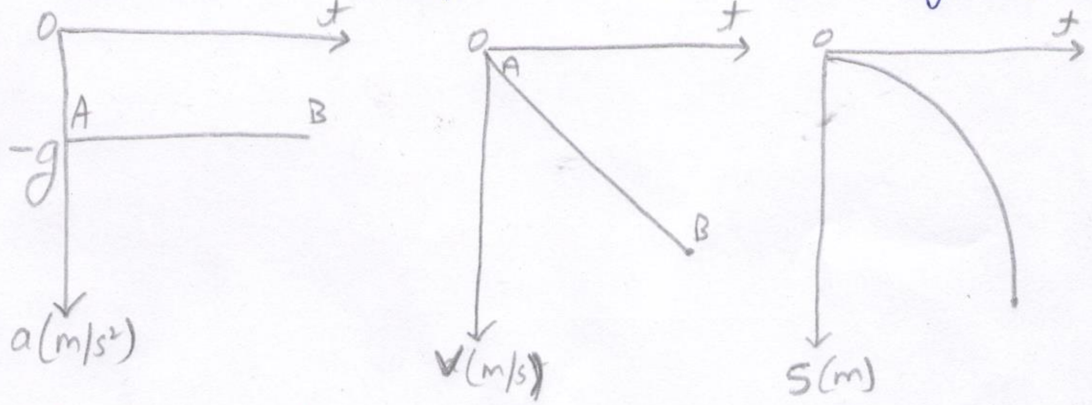
$= (a + 16b) - (a + 4b) = 12b$

$\therefore$ Average velocity = $\frac{\text{Displacement}}{\text{time taken}} = \frac{12b}{4-2} = \frac{12 \times 2.5}{2}$

$= 15 m/s$

⇒ Motion of an object under Free fall

* Free fall is case of motion of uniform acceleration under influence of gravity.
• If object is released from rest, in free fall,
$u = 0$, $a = 9.8 m/s^2$ & V is taken negative, than



⇒ **Reaction time** – time which a person takes to observe, think and act.

→ Droping a rod b/w finger & Catching it.

let distance d before catching be 0.21m.

$u = 0$, $a = 9.8 m/s^2$, $S = 0.21$, $t = ?$

Using: $S = ut + \frac{1}{2}at^2$

$$0.21 = \frac{1}{2}(9.8)t^2$$

$$t = 0.2 \text{ sec}$$

Depends on ① individual presence of mind
② complexity of situation