JEE MAIN 2025

Sample Paper - 8

Time Allowed: 3 hours **Maximum Marks: 300**

General Instructions:

1. There are three subjects in the question paper consisting of Physics (Q. no. 1 to 25), Chemistry (Q, no. 26 to 50), and Mathematics (Q. no. 51 to 75).

2. Each subject is divided into two sections. Section A consists of 20 multiple-choice questions & Section B consists of 5 numerical value-type questions.

3. There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero marks will be awarded for not attempted questions.

4. For Section B questions, 4 marks will be awarded for correct answers and zero for unattempted and incorrect answers.

5. Any textual, printed, or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.

6. All calculations/written work should be done in the rough sheet is provided with the Question Paper.

PHYSICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

1. In a new system of units, if energy (E), velocity (V) and time (T) are chosen as fundamental quantities, then dimensional formula of force is $\left[E^{\alpha}V^{\beta}T^{\gamma}\right]$. The value of $\alpha+\beta-\gamma$ is

   1)2 2)-2 3)0 4)1

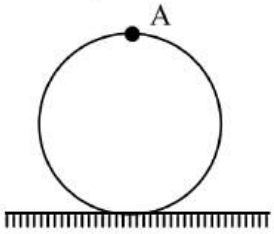
2. A bar magnet is held perpendicular to a uniform magnetic field. If the couple acting on the magnet is to be halved by rotating it, then the angle by which it is to be rotated is

   1) 30^0 2) 45^0 3) 60^0 4) 90^0

3. A block is moving on horizontal rough surface. It has velocity $v_0\hat{i}$ at $x=0.1$ m. The coefficient of friction varies with distance (x) from origin as $\mu=\frac{1}{2x^2}$. The minimum value of v_0 so that it never stops is

   1) 5 m/s 2) 7 m/s 3) 8.5 m/s 4) 10 m/s

4. A disc (m, R) is doing pure rolling motion on a horizontal surface. Radius of curvature of trajectory followed by point A at topmost point is

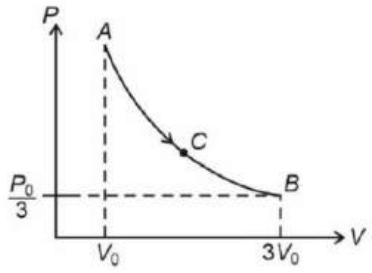


   1) R 2) 2R 3) 4R 4) $2\sqrt{2}R$

5. A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y, then $\frac{1}{Y}$ is equal to (g = gravitational acceleration)

   1) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$ 2) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ 3) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$ 4) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$

6. An ideal monatomic gas undergoes isothermal expansion from state A to B. Work done by the gas from $A \rightarrow B$ is double of work done by the gas from $A \rightarrow C$. Pressure at point C is



1) $\frac{P_0}{\sqrt{3}}$ 2) $\frac{P_0}{3}$ 3) $\frac{2P_0}{3}$ 4) $\frac{2P_0}{\sqrt{3}}$

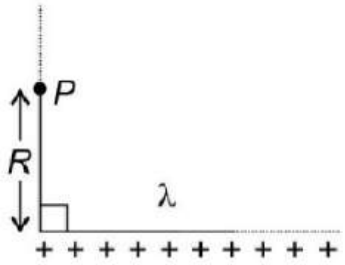
7. One mole of an ideal gas undergoes a process given by, $T = T_0 + \alpha V^2$ where T is the temperature and V is the volume of gas. Volume of gas when its pressure is least is [Where T_0 *and* α are positive constants].

1) $\sqrt{T_0 / 2\alpha}$ 2) $\sqrt{T_0 / \alpha}$ 3) $\sqrt{\frac{2T_0}{\alpha}}$ 4) $2\sqrt{\frac{T_0}{\alpha}}$

8. Two particle A and B are performing SHM with amplitude A_0, and time period T about the same mean position. At t = 0, A is at mean position and B is at distance $\frac{A_0}{2}$ from mean position and is going towards mean position. At what time they will be at maximum separation? (At t = 0, direction of velocities of A and B are same)

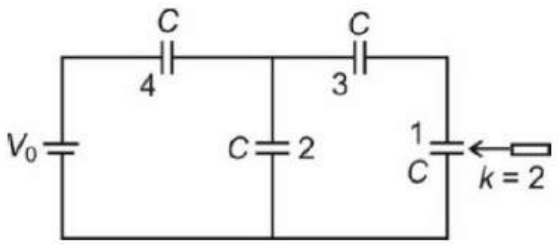
1) $\frac{T}{12}$ 2) $\frac{T}{8}$ 3) $\frac{T}{30}$ 4) $\frac{T}{24}$

9. Electric field at point 'P' due to long rod having uniform charge density, λ as shown is



1) $\frac{\lambda}{4\pi\varepsilon_0 R}$ 2) $\frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 R}$ 3) $\frac{\lambda}{2\pi\varepsilon_0 R}$ 4) $\frac{\lambda}{\sqrt{2}\pi\varepsilon_0 R}$

10. For the circuit shown in figure, dielectric slab of dielectric constant $k = 2$ is inserted in space between the plates of capacitor 1. As the dielectric is inserted



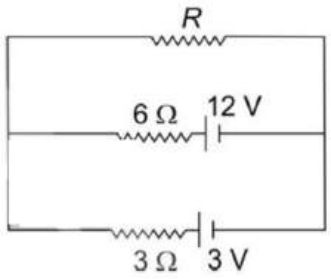
1) Potential across capacitor 1 increases

2) Charge on capacitor 3 decreases

3) Charge on capacitor 4 decreases

4) Charge on capacitor 2 decreases

11. For the circuit shown in figure, value of resistance R is adjusted so that power delivered to resistor, R is maximum and is equal to P_0. Value of P_0 is



1) 6.0W 2) 3.0W 3) 9.0W 4) 4.5W

12. STATEMENT-1 : Inductance plays same role in the electrical circuits as mass plays in the mechanical circuits.

STATEMENT-2 : Greater the value of inductance, harder it is to change the current in the circuit.

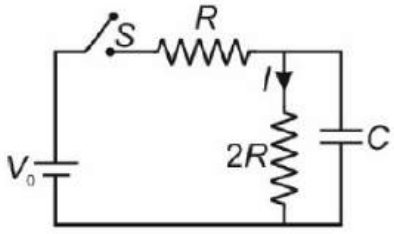
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

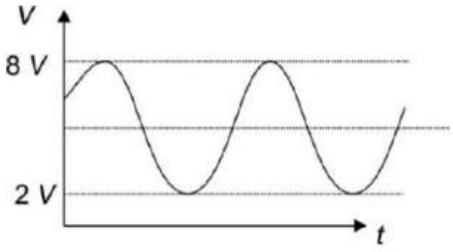
(4) Statement- 1 is False, Statement- 2 is True

13. For the RC circuit shown in figure, switch 'S' is closed at time t = 0. Current I at $t=\frac{2RC}{3}\ln 3$, is



1) $\frac{V_0}{9R}$ 2) $\frac{2V_0}{9R}$ 3) $\frac{2V_0}{R}$ 4) $\frac{V_0}{4R}$

14. A sinusoidally varying source voltage is given as a function of time as shown. RMS value of voltage is



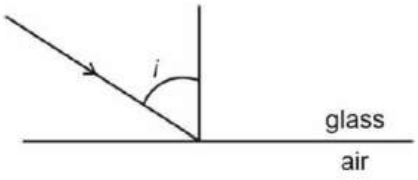
1) $\sqrt{\frac{59}{2}}V$ 2) $\frac{7}{\sqrt{2}}V$ 3) 4V 4) 6V

15. Magnetic field associated with electromagnetic wave whose electric field is given by $\vec{E}=2.1\sin\left(3\times10^8 t-1.8Z\right)\hat{i}\,N/C$, is

1) $\vec{B}=1.26\times10^{-8}\sin\left(3\times10^8 t-1.8Z\right)\hat{j}\,T$ 2) $\vec{B}=1.8\times10^{-8}\sin\left(3\times10^8 t-1.8Z\right)\hat{j}\,T$

3) $\vec{B}=0.7\times10^{-8}\sin\left(3\times10^8 t-1.8Z\right)\hat{j}\,T$ 4) $\vec{B}=-1.26\times10^{-8}\sin\left(3\times10^8 t-1.8Z\right)\hat{j}\,T$

16. A ray of light travelling from glass to air is incident at angle *i*. Maximum angle of deviation suffered for any angle of incidence is $\frac{\pi}{2}$, then refractive index of glass is



1) $\sqrt{2}$ 2) $\sqrt{3}$ 3) 2 4) 4/3

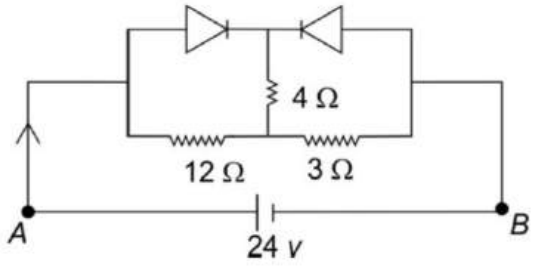
17. In a standard YDSE, the distance between the slits varies with time $(t)as(dt) = d_0 + a_0 \sin(\omega t)$. The difference between the largest fringe width and the smallest fringe width over time is given as (λ is wavelength of monochromatic light used, D is the screen distance from slits. $D >> d$)

1) $\frac{2\lambda D a_0}{d_0^2 - a_0^2}$ 2) $\frac{2\lambda D d_0}{d_0^2 - a_0^2}$ 3) $\frac{\lambda D}{d_0 + a_0}$ 4) $\frac{\lambda D a_0}{d_0^2 - a_0^2}$

18. In a hydrogen atom, electron jumps from 4th excited state to 2nd excited state. Wavelength of photon emitted is [R : Rydberg constant]

1) $\frac{225}{16R}$ 2) $\frac{225}{4R}$ 3) $\frac{100}{21R}$ 4) $\frac{100}{4R}$

19. In the circuit with ideal diodes as shown, current (in A) through battery is



1) 3 2) 5 3) 6 4) 4

20. In a Vernier calipers, one main scale division is 1mm and 9main scale divisions are equal to 10 vernier scale divisions. When nothing is put between jaws of the calipers, zero of the Vernier scale lies to the right side of zero of the main scale and the 2nd division of the Vernier scale coincides with a main scale division. While measuring inner diameter of a hollow cylinder the zero of Vernier scale lies between 1.7cm and 1.8cm of the main scale. Also, 8th division of Vernier scale coincides with a main scale division, inner diameter of the cylinder is

1) 17.2mm 2) 17.4mm 3) 17.6mm 4) 17.8mm

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**

Marking scheme: +4 for correct answer, -1 in all other cases.

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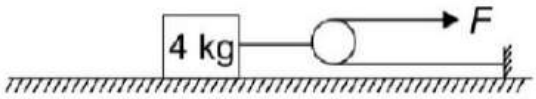
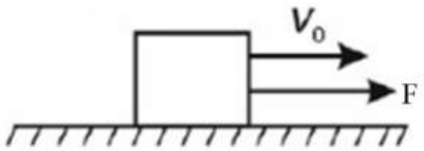
21. Pitch of a screw gauge is 1mm and its cap is divided into 100 divisions. When nothing is placed between studs of the screw gauge, zero of circular scale is 8 divisions above the reference line and zero of the main scale is not visible. Now, when a cylindrical wire is placed between its studs the main scale reading is 2 divisions and 15^{th} division of circular scale coincides with reference line. Diameter of the wire is x mm. Find the value of 100 x.

22. If coefficient of friction between all the surfaces is 0.50, then force, F (in N) required to move the block of mass 4 kg is [g = 10 m/s2]



23. A block of mass 'm' placed on smooth horizontal surface is acted upon by a horizontal force as shown, delivering constant power 'P'. If velocity of block changes from v_0 to $2v_0$, then time taken is $\frac{qmv_0^2}{nP}$,find the value of q x n is



24. Mass density of a disc is given by $\sigma = \sigma_0 \frac{r}{R}$, where σ_0 is constant, r is distance from centre and R is radius of disc. Moment of inertia of disc about an axis passing through centre and perpendicular to plane of disc is $\frac{n\pi\sigma_0 R^4}{m}$. Find the value of $n \times m$ is

25. The minimum and maximum distances of a planet from sun, revolving around the sun, are x_0 and $2x_0$. If the maximum speed is v_0, then minimum acceleration of planet during motion is $\frac{nv_0^2}{mx_0}$. Find $n \times m$

CHEMISTRY **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

26. Titanium oxide (TiO_2) is heated in stream of hydrogen to give water and new oxide Ti_xO_y. If 1.6gm TiO_2 produces 1.44 gm Ti_xO_y, (atomic mass Ti = 48, O = 16), The sum of 'x'&'y' is

1) 3 2) 5 3) 7 4) 8

27. The standard reduction potential for Zn^{+2}/Zn; Ni^{+2}/Ni and Fe^{+2}/Fe are –0.76V, –0.23V, –0.44V respectively. The reaction $X + Y^{+2} \rightarrow X^{+2} + Y$ will be non-spontaneous when :

	X	Y
(I)	Ni	Fe
(II)	Ni	Zn
(III)	Fe	Zn
(IV)	Zn	Ni

1) I, II, IV 2) I, II, III 3) II, III, IV 4) I,III,IV

28. Consider the following chemical reaction and the corresponding kinetic data showing the initial reaction rate as a function of the initial concentrations of the reactants:

$H_3AsO_4(aq) + 2H_3O^+(aq) + 3I^-(aq) \rightarrow HAsO_2(aq) + I_3^-(aq) + 4H_2O(liq)$

Initial Rate $\times 10^{-5}$ (M/sec)	$[H_3AsO_4]$	$[H_3O^+]$	$[I^-]$
3.7	0.001	0.01	0.10
7.4	0.001	0.01	0.20
7.4	0.002	0.01	0.10
3.7	0.002	0.005	0.20

Using the data, establish the correct reaction composite order.

1) 1 2) 2 3) 3 4) 4

29. A 0.4m aqueous solution of Na_xA has freezing point -3.72^0C. If $K_f(H_2O)$ is 1.86K kg mol^{-1}. The value of 'X" is (salt is 100% ionized).

1) 2 2) 3 3) 4 4) 6

30. Which of the following below electronic configuration of lanthanides is related to the formation of stable +2 oxidation state.

1) $[Xe]4f^7,5d^1,6s^2$ 2) $[Xe]4f^{14},5d^1,6s^2$ 3) $[Xe]4f^7,6s^2$ 4) $[Xe]4f^1,5d^1,6s^2$

31. $[Co(NH_3)_4(NO_2)_2]Cl$ exhibits:

1) linkage isomerism, geometrical isomerism and optical isomerism

2) linkage isomerism, ionization isomerism and optical isomerism

3) linkage isomerism, ionization isomerism and geometrical isomerism

4) ionization isomerism, geometrical isomerism and optical isomerism

32. The enthalpy of combustion of propane (C_3H_8) gas in terms of given data is :

Bond energy (kJ/mol)

ε_{C-H}	$\varepsilon_{O=O}$	$\varepsilon_{C=O}$	ε_{O-H}	ε_{C-C}
$+x_1$	$+x_2$	$+x_3$	$+x_4$	$+x_5$

Resonance energy of CO_2 is –z kJ/mol and $\Delta H_{vaporization}$ $[H_2O(l)]$ is y kJ/mol.

1) $8x_1 + 2x_5 + 5x_2 - 6x_3 - 8x_4 - 4y - 3z$

2) $6x_1 + x_5 + 5x_2 - 3x_3 - 4x_4 - 4y - 3z$

3) $8x_1 + 2x_5 + 5x_2 - 6x_3 - 8x_4 - y - z$

4) $8x_1 + x_5 + 5x_2 - 6x_3 - 8x_4 - 4y + 3z$

33. $\frac{N_0}{2}$ atoms of X (g) are converted into X^+ (g) by absorbing E_1 energy. $2N_0$ atoms of X (g) are converted into X^-(g) by releasing E_2 energy. Calculate ionisation enthalpy and electron gain enthalpy of X(g) per atom.

1) I.E. $= \frac{2E_1}{N_0}$, $\Delta_{eg}H = -\frac{E_2}{2N_0}$ 2) I.E. $= -\frac{E_2}{2N_0}$, $\Delta_{eg}H = \frac{2E_1}{N_0}$

3) I.E. $= \frac{E_1}{2N_0}$, $\Delta_{eg}H = -\frac{E_2}{2N_0}$ 4) I.E. $= \frac{N_0}{2E_1}$, $\Delta_{eg}H = -\frac{2N_0}{E_2}$

34. For an octahedral complex, which of the following d-electron configuration will give maximum CFSE?

1) high spin, d^6 2) low spin, d^5 3) low spin, d^4 4) high spin, d^7

35. In ICl_2^+, ICl_2^- and ICl_4^- sum of the bond pairs and lone pairs on eachiodine atom in the given ionic species are

1) 2,2 and 4 2) 2,3 and 2 3) 4,5 and 4 4) 4,5 and 6

36. 0.2 g of an organic compound was analysed by kjeldahl's method. Ammonia evolved was absorbed in 60mL $N/5 H_2SO_4$. Unused acid required 40 mL of N/10 NaOH for complete neutralisation. Find the percentage of nitrogen in the compound.

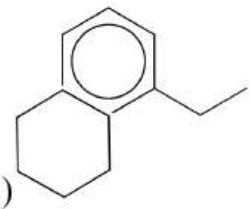
1) 70 % 2) 56 % 3) 46 % 4) 66 %

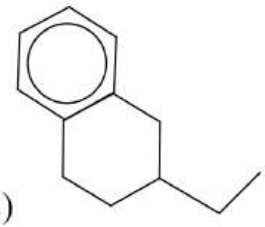
37. The correct order of increasing $C-O$ bond length of CO, CO_3^{-2}, CO_2 is

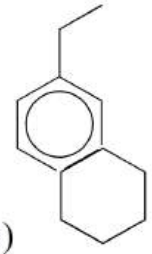
1) $CO_3^{-2} < CO_2 < CO$ 2) $CO_2 < CO_3^{-2} < CO$

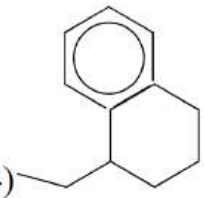
3) $CO < CO_3^{-2} < CO_2$ 4) $CO < CO_2 < CO_3^{-2}$

38. The major product in the following reaction.

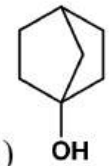
1) 

2) 

3) 

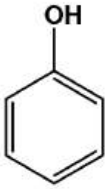
4) 

39. Which of the following can give immediate turbidity on treatment with Lucas Reagent?

1) 

2) $CH_3-CH_2-CH_2-OH$

3) $CH_2=CH-\underset{\displaystyle CH_3}{\underset{|}{CH}}-OH$

4) 

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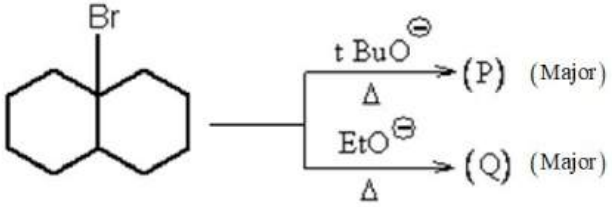
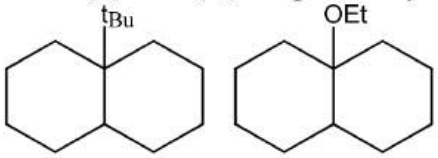
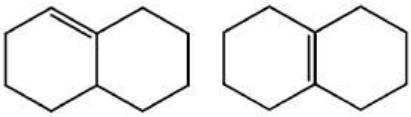
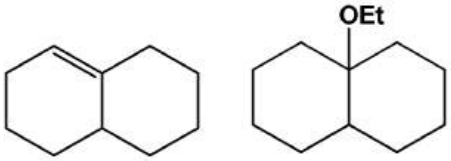
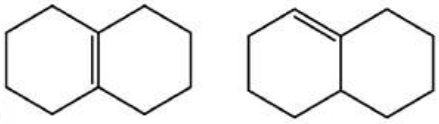


Product (P) and (Q) respectively

1) 

2) 

3) 

4) 

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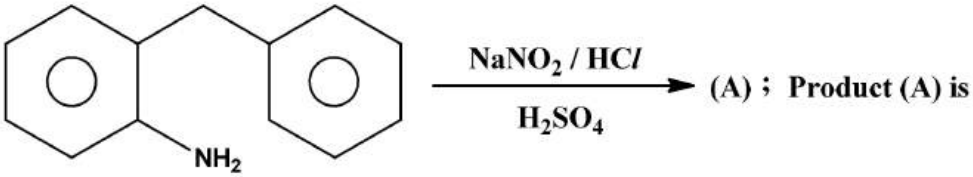
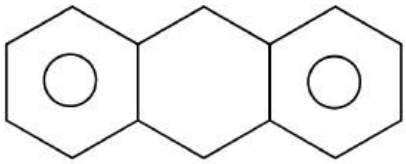
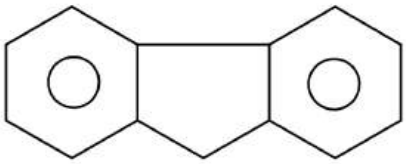
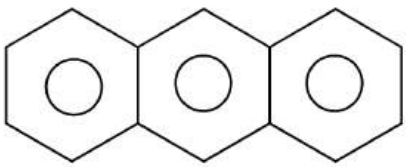
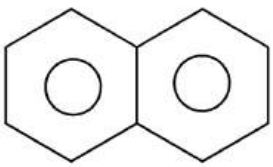


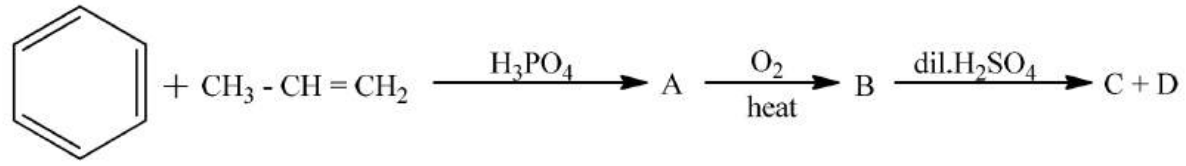
1) 

2) 

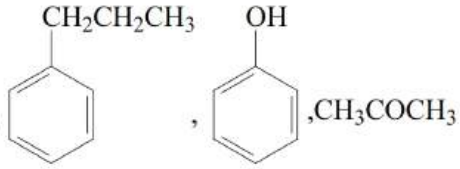
3) 

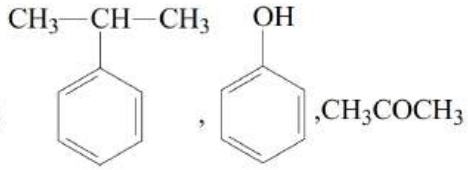
4) 

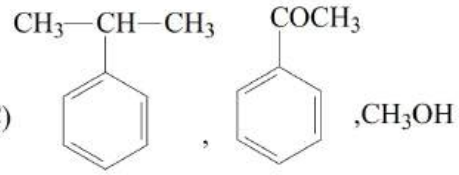
42.

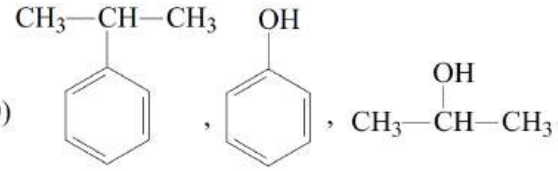


A, C, D are

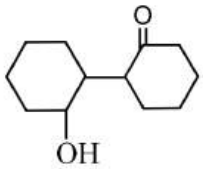
A) 

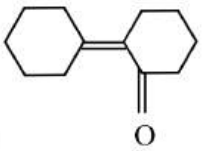
B) 

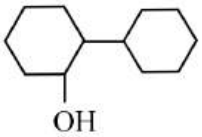
C) 

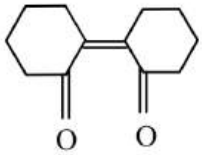
D) 

43. Which is the product formed when cyclohexanone undergoes aldol condensation followed by heating?

1) 

2) 

3) 

4) 

44. Which of the following reactions will not give propane?

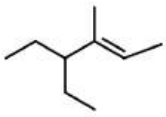
(A) $CH_3CH_2CH_2Cl \xrightarrow[H_2O]{Mg/ether}$

(B) $CH_3CH(OH)CH_3 \xrightarrow{Red\ P/HI}$

(C) $CH_3HC{=}CH_2 \xrightarrow[CH_3COOH]{B_2H_6}$

(D) $CH_3COCl \xrightarrow[H_2O]{CH_3MgX}$

45. The IUPAC name of the following compound is:



1) 3- ethyl – 4 – methylhex – 4 – ene
2) 4,4 – diethyl – 3 – methylbut – 2 – ene
3) 4-methyl – 3 – ethylhex -4-ene
4) 4-ethyl-3-methylhex-2-ene

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**
Marking scheme: +4 for correct answer, -1 in all other cases.

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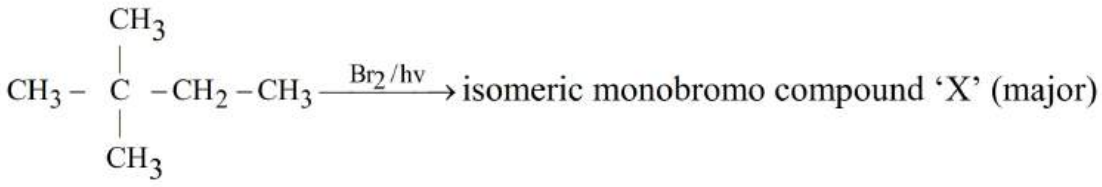
46. The angular velocity of an electron occupying the second Bohr orbit of He^+ ion is 2.067×10^x rad/sec, what is x_____

47. 
isomeric monobromo compound 'X' (major)

The position of the bromine atom in the major product 'X' is___ (as per IUPAC Nomenclature)

48. Number of dipeptides possible using alanine, glycine and tyrosine is 'X' then what is the value of 5x?

49. Sum of basicity of H_3PO_4, H_3PO_3 and H_3PO_2 is equal to

50. A buffer solution is formed by mixing 100 mL 0.01 M CH_3COOH with 200 mL 0.02 MCH_3COON_a. If this buffer solution is made to 1.0 L by adding 700 mL of water, pH will change by a factor of 'x' then what is the value of x+12?

MATHEMATICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

51. Statement-1: The point Z lies on the circle $|Z-2-i|=1$. The point Z_0 on the circle with maximum argument is given by $Z_0=2(\cos\theta+i\sin\theta)$ where $\tan\theta=\frac{4}{5}$ and

Statement-2: Point Z on the circle $|Z-2-i|=1$ nearest to the origin has modulus $(\sqrt{5}-1)$

1) Statement 1 is true, statement 2 is true, statement 2 is a correct explanation of statement 1

2) Statement 1 is true, statement 2 is true, statement 2 is not correct explanation of statement 1

3) Statement 1 is true, statement 2 is false

4) Statement 1 is false, statement 2 is true.

52. If $a=\cot 80^0$, $b=\cot 60^0$ and $c=\cot 40^0$, then the value of $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$ is equal to

1)2 2) 1 3)4 4)3

53. $f(x)$ is a cubic polynomial x^3+ax^2+bx+c such that $f(x)=0$ has three distinct integral roots and $f(g(x))=0$ does not have real roots, where $g(x)=x^2+2x-5$ then the minimum value of $a+b+c$ is

1)504 2) 532 3)719 4)764

54. In how many ways can 17 persons depart from railway station in 2 cars and 3 autos given that 2 particular persons depart by the same car (4 persons can sit in a car and 3 persons can sit in an auto) is

1) $\frac{15!}{2!4!(3!)^3}$ 2) $\frac{16!}{(2!)^2 4!(3!)^3}$ 3) $\frac{17!}{2!4!(3!)^3}$ 4) $\frac{15!}{4!(3!)^3}$

55. In a test an examinee either guess or copies or knows the answer to a multiple choice question with four choices of which only one option is correct. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6, the probability that his answer is correct given that he copied is 1/8, then the probability that he know the answer to the question given that he correctly answered it is

1)24/29 2)23/29 3)22/29 4)21/29

56. A random variable X has the probability distribution

X:	1	2	3	4	5	6	7	8
$P(X)$:	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events $E=\{X \text{ is a prime number}\}$ and $F=\{X<4\}$ then probability of $P(E\cup F)$

1)0.87 2)0.77 3)0.35 4)0.50

57. If $g(x)=\max\left|y^2-xy\right|$, $0\le y\le 1$ then minimum value of $g(x)$ for real x is $a-2\sqrt{b}$ (a, b are natural numbers) then a + b = _____

1) 5 2) 11 3) 24 4) 2

58. A lamp of negligible height is placed on the ground m_1 mt away from a wall. A man m_2 mt tall is walking at a speed of $\frac{m_1}{10} mt/\sec$ from the lamp to the nearest point on the wall. When he is midway between the lamp and the wall, the rate of change in the length of his shadow on the wall is

1) $\frac{-5m_2}{2} m/s$ 2) $\frac{-2m_2}{5} m/s$ 3) $\frac{-m_2}{2} m/s$ 4) $\frac{-m_2}{5} m/s$

59. If $f(x)=\prod_{k=1}^{\infty}\left(\frac{1+2\cos\left(\frac{2x}{3^k}\right)}{3}\right)$, then number of points where

$[xf(x)]+\left|xf(x)\right|+(x-1)\left|x^2-3x+2\right|$ is non-differentiable in $x\in(0,3\pi)$ is equal to

(where [.] denotes greatest integer function)

1) 5 2) 6 3) 4 4) 8

60. A flight of stairs has 10 steps. A person can go up the steps one at a time, two at a time or any combination of 1s and 2s. The total number of ways in which the person can go up the stairs is

1) 75 2) 79 5) 85 4) 89

61. Let $\vec{k},\vec{l},\vec{m},\vec{n}$ are four distinct unit vectors in a three-dimensional space such that $\vec{k}.\vec{l}=\vec{l}.\vec{m}=\vec{m}.\vec{k}=\vec{n}.\vec{l}=\vec{n}.\vec{m}=\frac{-1}{11}$. If the value of $\vec{k}.\vec{n}$ can be expressed as $\frac{-A}{B}$, where A, B are coprime positive integers, then the unit digit of the value of (A+B) is _____

1) 0 2) 4 5) 6 4) 8

62. The value of $\int_0^1 \frac{dx}{\left(5+2x-2x^2\right)\left(1+e^{2-4x}\right)}$ is

1) $\frac{1}{2\sqrt{11}}\ln\frac{\left(\sqrt{11}+1\right)^2}{10}$ 2) $\frac{1}{\sqrt{11}}\ln\frac{\left(\sqrt{11}+1\right)^2}{10}$ 3) $\frac{1}{2\sqrt{11}}\ln\frac{\left(\sqrt{11}-1\right)^2}{10}$ 4) $\frac{1}{\sqrt{11}}\ln\frac{\left(\sqrt{11}-1\right)^2}{10}$

63. The area bounded by the two branches of curve $(y-x)^2=x^3$ and the straight line $x=1$ is

1) 1/5 sq.unit 2) 3/5 sq.unit 3) 4/5 sq.unit 4) 8/5 sq.unit

64. Statement-1: Period of $f(x)=\sin 3x\cos[3x]-\cos 3x\sin[3x]$ where [ ] denotes the greatest integer function, is $\frac{2\pi}{3}$

Statement-2: Period of $\{x\}$ where {} denotes the fractional part of x, is 1

1) Statement-1is True, Statement-2 is True; Statement-2 is a correct explanation forStatement-1

2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

3) Statement-1 is True, Statement-2 is False

4) Statement-1 is False, Statement-2 is True

65. Let $a_1, a_2, a_3, ..., a_{101}$ are in GP with $a_{51} = 25 \, and \sum_{i=1}^{101} a_i = 125$, then the value of $\sum_{i=1}^{101} \left(\frac{1}{a_i} \right)$ equals to

1)5 2)1/5 3)1/25 4)1/125

66. The last term in the binomial expansion of $\left(\sqrt[3]{2} - \frac{1}{\sqrt{2}} \right)^n$ is $\left(\frac{1}{3\sqrt[3]{9}} \right)^{\log_3 8}$ then the 5th term from the beginning is

1) ${}^{10}C_6$ 2) $2.{}^{10}C_4$ 3) $\frac{1}{2}.{}^{10}C_4$ 4) $\frac{1}{3}.{}^{10}C_4$

67. The mean and variance of 7 observations are 8 and 16 respectively. If five observations out of them are 2, 4, 10, 12, 14 then the product of the two observations is

1)48 2)49 3)36 4)24

68. If $(a-b)\sin(\theta+\phi) = (a+b)\sin(\theta-\phi)$ *and* $a\tan\frac{\theta}{2} - b\tan\frac{\phi}{2} = c$ then the value of $\sin\phi$ is equal to

1) $\frac{2ab}{a^2-b^2-c^2}$ 2) $\frac{2bc}{a^2-b^2-c^2}$ 3) $\frac{2bc}{a^2-b^2+c^2}$ 4) $\frac{2ab}{a^2-b^2+c^2}$

69. The value of $\lim_{x\to 0^+} \frac{-1+\sqrt{(\tan x - \sin x)+\sqrt{(\tan x - \sin x)+\sqrt{(\tan x - \sin x)+...}}}}{-1+\sqrt{x^3+\sqrt{x^3+\sqrt{x^3+...}}}}$

1)1 2)1/2 3)1/4 4)-1

70. $\sin^{-1}\left(\frac{x^2}{4} + \frac{y^2}{9} \right) + \cos^{-1}\left(\frac{x}{2\sqrt{2}} + \frac{y}{3\sqrt{2}} - 2 \right)$ is equal to

1) $\frac{\pi}{2}$ 2) π 3) $\frac{\pi}{\sqrt{2}}$ 4) $\frac{3\pi}{2}$

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**
Marking scheme: +4 for correct answer, -1 in all other cases.

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71. Let $z_1 = 10+6i \, and \, z_2 = 4+6i$. If z is any complex number such that the argument of $(z-z_1)/(z-z_2)$ is $\frac{\pi}{4}$ and if $|z-7-9i|$ is k then k^2=

72. The value of $\tan 78^0 \tan 42^0 - \tan 12^0 \tan 48^0 =$

73. Consider a polynomial $p(x)$ of the least degree that has a maximum equal to 6 at x=1 and a minimum equal to 2 at x=3 then the value of $p(2)+p'(0)-7$ is

74. A line with direction ratios (2,1,2) intersects the lines $\bar{r}=-\bar{j}+\lambda\left(\bar{i}+\bar{j}+\bar{k}\right)$ and $\bar{r}=-\bar{i}+\mu\left(2\bar{i}+\bar{j}+\bar{k}\right)$ at A and B respectively then length of AB is

75. If the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ *and* $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is $\lambda\sqrt{30}$ units then the value of λ is

KEY SHEET

PHYSICS

1	**4**	2	**3**	3	**4**	4	**3**	5	**4**
6	**1**	7	**2**	8	**4**	9	**2**	10	**4**
11	**4**	12	**1**	13	**2**	14	**1**	15	**1**
16	**1**	17	**1**	18	**1**	19	**4**	20	**3**
21	**223**	22	**10**	23	**6**	24	**10**	25	**48**

CHEMISTRY

26	**2**	27	**2**	28	**4**	29	**3**	30	**3**
31	**3**	32	**1**	33	**1**	34	**2**	35	**4**
36	**2**	37	**4**	38	**1**	39	**3**	40	**2**
41	**2**	42	**2**	43	**2**	44	**4**	45	**4**
46	**16**	47	**3**	48	**45**	49	**6**	50	**12**

MATHEMATICS

51	**4**	52	**2**	53	**3**	54	**4**	55	**1**
56	**2**	57	**1**	58	**2**	59	**1**	60	**4**
61	**4**	62	**1**	63	**3**	64	**4**	65	**2**
66	**1**	67	**1**	68	**2**	69	**2**	70	**4**
71	**18**	72	**4**	73	**6**	74	**3**	75	**3**

SOLUTIONS
PHYSICS

1. $F = \dfrac{E}{VT} = \left[E^1 V^{-1} T^{-1}\right]$

   $\alpha = 1, \beta = -1, \gamma = -1$

2. $\tau = MB\sin\theta$

   $\tau\alpha \sin\theta$

   $\dfrac{\tau_1}{\tau_2} = \dfrac{\sin\theta_1}{\sin\theta_2} \Rightarrow \dfrac{\tau}{\tau/2} = \dfrac{\sin 90^0}{\sin\theta 2}$

   $\theta_2 = 30^0$

   Angle of rotation $= 90 - 30 = 60^0$

3. $\int_{0.1}^{\infty} -\dfrac{mg}{2x^2}dx = -\dfrac{1}{2}mv_0^2$

   $v_0 = 10\ m/s$

4. $a_A = \dfrac{v_0^2}{R}$

   $V_A = 2v_0$

   $R_C = \dfrac{v_A^2}{a_A} = 4R$

5. $T = 2\pi\sqrt{\dfrac{l}{g}}; T_M = 2\pi\sqrt{\dfrac{l}{g}}$

   $\Delta l = \dfrac{Mgl}{YA}$

   $l' - l = \dfrac{Mgl}{YA}$

   $\dfrac{1}{Y} = \left(l' - l\right)\dfrac{A}{Mgl}$

   $= \left[\dfrac{T_M^2}{T^2} - 1\right]\dfrac{A}{Mg}$

6. $W_{AC} = P_0 V_0 \left(\dfrac{V}{V_0}\right)$

   $2P_0V_0 \ln\left[\dfrac{P_0}{P}\right] = P_0V_0 \ln(3)$

   $P = \dfrac{P_0}{\sqrt{3}}$

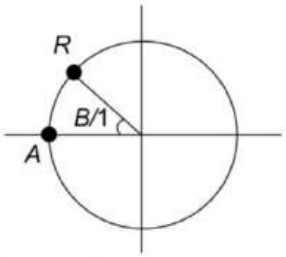
7. $PV = RT$

   $P = R\left[\dfrac{T_0 + \alpha V^2}{V}\right]$

   $\dfrac{dP}{dV} = 0$

   $\Rightarrow -\dfrac{T_0}{V^2} + \alpha = 0$

8.



$$\frac{2\pi}{T}t = \frac{\pi}{12}$$

$$t = \frac{T}{24}$$

9. $$\int dE = \int_0^{\pi/2} \frac{\lambda R d\theta \cos\theta}{4\pi\varepsilon_0 R^2}$$

$$E = \frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\Big|_0^{\frac{\pi}{2}}$$

$$E = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R} = \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 R}$$

10. Since equivalent capacitance increases, charge on capacitor 4 increases. By KVL, charge on capacitor 2 decreases.

11. $R_{eq} = 2\Omega$

For max power, $R = 2\Omega$

$$\frac{E_{eq}}{2} = 3 \Rightarrow E_{eq} = 6V$$

$$I = \frac{6}{4} = 1.5A$$

$$P_0 = (1.5)^2 \times 2 = 4.5W$$

12. Inductance can be set to be analogous to mass, as it poses inertia to current change in the electrical circuit.

13. $$V_C = \frac{2V_0}{3}\left(1 - e^{\frac{-3t}{2RC}}\right)$$

$$V_C = \frac{2V_0}{3}\left[1 - \frac{1}{3}\right]$$

$$V_C = \frac{4V_0}{9}$$

$$I = \frac{V_C}{2R} = \frac{2V_0}{9R}$$

14. $$V = 5 + 3\sin(\omega t)$$

$$V_{rms} = \sqrt{5^2 + \frac{3^2}{2}} = \sqrt{\frac{59}{2}}$$

15. $\vec{E} \times \vec{B}$ is along positive Z-direction. $\vec{B}$ is along positive $\hat{j}$ direction.

16. $\pi - 2\theta_c = \frac{\pi}{2} \Rightarrow \theta_c = \frac{\pi}{4}$

$\frac{1}{\mu} = \sin(\theta_c) \Rightarrow \mu = \sqrt{2}$

17. $\beta_{\max} = \frac{\lambda D}{d_0 - a_0}$

$\beta_{\min} = \frac{\lambda D}{d_0 + a_0}$

18. $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \Rightarrow \frac{1}{\lambda} = R\left[\frac{1}{9} - \frac{1}{25}\right] \Rightarrow \lambda = \frac{225}{16R}$

19. $R_{eq} = 6\Omega$

$I = \frac{v_0}{R_{eq}} = 4A$

20. Reading = 17+8×0.1-2×0.1=17.6 mm

21. L.C. =0.01 mm, zero error =-0.08 mm
Reading = 2 mm+15×0.01+0.08=2.23 mm

22. $f = T$
$2T = 0.50 \times 4 \times 10 \Rightarrow T = 10N$

23. $mv\frac{dv}{dt} = P$

$\frac{m}{t}\frac{v^2}{2}\Big|_{v_0}^{2v_0} = P$

24. $I = \int dI = \frac{2\pi}{R}\int_0^R \sigma_0 r^4 dr$

$T = \frac{2\pi\sigma_0 R^5}{5R} = \frac{2\pi\sigma_0 R^4}{5}$

25. $v_1 = \frac{v_0}{2}$

Radius of curvature at farthest position $= \frac{2(x_0)(2x_0)}{3} = \frac{4x_0}{3}$

$a_{\min} = \frac{\left[\frac{v_0}{2}\right]^2}{\frac{4x_0}{3}} = \frac{3v_0^2}{16x_0}$

CHEMISTRY

26. 1.6 gm oxide looses 0.16 gm
80 gm oxide looses = 8 gm 'O' = ½ mole of 'O'
$TiO_{\left(2-\frac{1}{2}\right)} = TiO_{\frac{3}{2}} = Ti_2O_3$

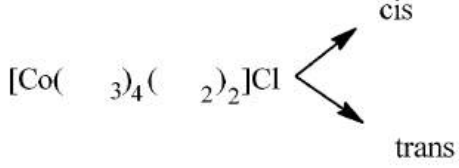
27. $Zn + Ni^{+2} \longrightarrow Zn^{+2} + Ni$
$E^o = E^o_{Ni^{+2}/Ni} - E^o_{Zn^{+2}/Zn}$
$= -0.23 - (-0.76) = +0.53$ V
Positive value shows that the process is spontaneous.
Rest of all (I) (II) (III) combination have negative E^o value.
(I) $E^o = -0.44 - (-0.23) = -0.21$ V
(II) $E^o = -0.76 - (-0.23) = -0.53$ V
(III) $E^o = -0.76 - (-0.44) = -0.32$ V

28. HINT: Assume rate law
$r = K[H_3AsO_4]^x [H_3O^+]^y [I^-]^z$
Solving by the help of various experiments
$x = 1$, $y = 2$ and $z = 1$
total order = 4

29. $\Delta Tf = i\ Kf\ m$
$3.72 = i \times 1.86 \times 0.4$
$i = 5 \quad \therefore n = 4$

30. Hint: Due to extra stability of half-filled f-subshell.

31. $\left[Co(NH_3)_4(ONO)_2\right]Cl$ =linkage isomers
$\left[Co(NH_3)_4(NO_2)(Cl)\right]NO_2$ = ionization isomers



32. $C_3H_8(g) + 5O_2 \square 3CO_2(g) + 4H_2O(\square)$

$$\square_C H = \begin{bmatrix} 8\times \text{B.E. } (C-H) \\ +2\times \text{B.E. } (C-C) \\ +5\times \text{B.E. } (O=O) \end{bmatrix} - \begin{bmatrix} 6\times \text{B.E. } (C=O) \\ +8\times \text{B.E.}(O-H) \\ +3\times |\text{R.E.}| \text{ of } CO_2 \\ +4\times \Delta_{vap} \ H(H_2O) \end{bmatrix}$$

33. $X(g) \rightarrow X^+(g) + e^-$

If I.E. is ionisation enthalpy, then

$\therefore \quad \frac{N_0}{2}(\text{I.E.}) = E_1 \qquad \therefore \quad \text{I.E.} = \frac{2E_1}{N_0}$

$X(g) + e^- \rightarrow X^-(g)$

If $\Delta_{eg}H$ is electron gain enthalpy, then

$\therefore \quad 2N_0(\text{E.A.}) = -E_2 \qquad \therefore \quad \Delta_{eg}H = -\frac{E_2}{2N_0}.$

34. (A) High spin, d^6 :CFSE$=\left(+0.6\times 2-0.4\times 4\right)\Delta_0 = -0.4\Delta_0$

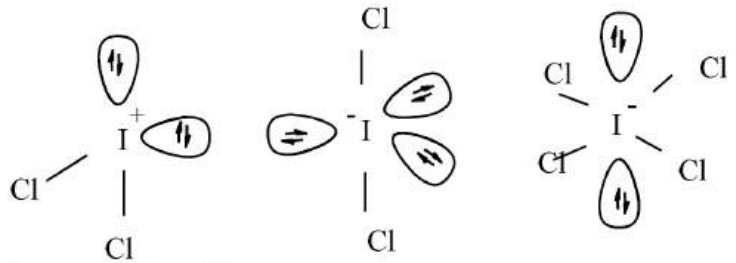
(A) Low spin d^5 ; CFSE $=\left(+0.6\times 2-0.4\times 5\right)\Delta_0 = -2.0\Delta_0$

(B) Low spin d^4 ; CFSE$=\left(+0.6\times 2-0.4\times 4\right)\Delta_0 = -1.6\Delta_0$

(C) High spin d^7 ; CFSE$=\left(+0.6\times 2-0.4\times 5\right)\Delta_0 = -0.8\Delta_0$

Magnitude of CFSE is maximum in (B)

35.



36. Mass of organic compound = 0.2 g

Unused acid required = 40 mL $\frac{N}{10}NaOH$

$40 \text{ mL}\frac{N}{10}NaOH = 40mL\frac{N}{10}H_2SO_4 = 20mL\frac{N}{5}H_2SO_4$

Acid used for absorption of ammonia= (60-20)mL $\frac{N}{5}H_2SO_4 = 40mL\frac{N}{5}H_2SO_4$

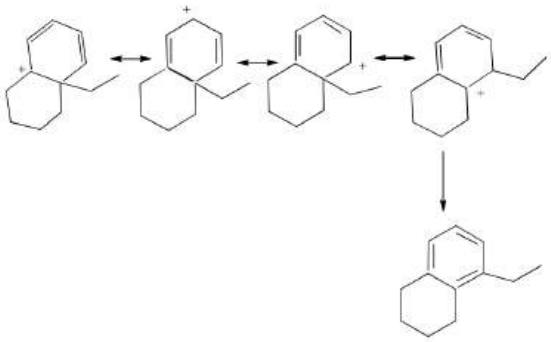
$$\text{Percentage of nitrogen} = \frac{1.4\times N_1\times V}{W} = \frac{1.4\times\frac{1}{5}\times 40}{0.2} = \frac{1.4\times 40}{0.2\times 5} = 56\%$$

37. Bond order of CO_2 (by resonance method)$=\frac{\text{no of bond's in between two atom}}{\text{no of resonating structures}}$

i.e., bond order in CO_3^{-2} (by resonance method)$=\frac{4}{3}=1.33$

$$\text{Bond length} \propto \frac{1}{Bond\ order}$$

38.

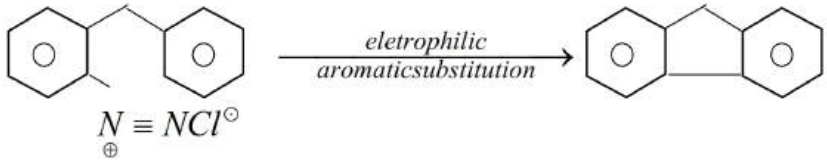


39. The compound A, despite a tertiary alcohol, cannot be readily converted into chloride because OH is present at bridge head. The compound C, allyl alcohol can be readily converted into allyl chloride, whose formation is responsible for white cloudiness.

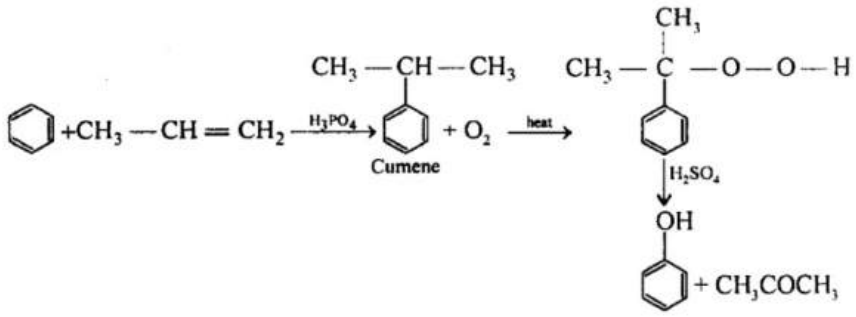
40. $t-BuO^{-} \rightarrow$ astearic ally hindered base will give Hofmann elimination as major product. Where as EtO^{-} will give Saytzeffs product.

41.

solution : B



42.

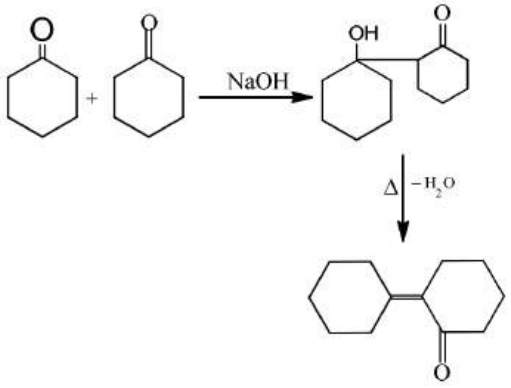


(A) is not possible because $CH_3-CH_2-\overset{\oplus}{C}H_2$ is less stable than $CH_3-\overset{\oplus}{C}H-CH_3$

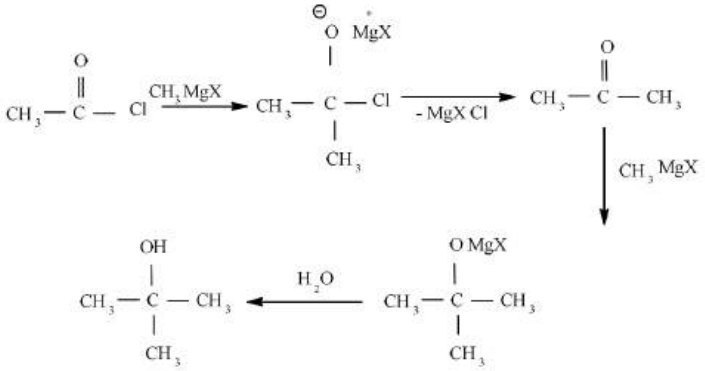
(C) is not possible because acetophenone and CH_3OH cannot be formed.

(D) is not possible because $CH_3-\underset{|}{\overset{OH}{CH}}-CH_3$

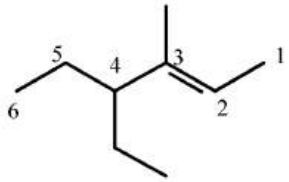
43.



44.



45.



4-Ethyl -3-methylhex-2-ene

46. Velocity of an electron in He^+ ion in an orbit = $\frac{2\pi Ze^2}{nh}$ .....(i)

Radius of He^+ ion in an orbit = $\frac{n^2h^2}{4\pi^2me^2Z}$ ....(ii)

By equations (i) and (ii),

Angular velocity (ω) = $\frac{u}{r} = \frac{8\pi^3Z^2me^4}{n^3h^3}$ ....(iii)

$= \frac{8\times(22/7)^3\times(2)^2\times(9.108\times10^{-28})\times(4.803\times10^{-10})^4}{(2)^3\times(6.626\times10^{-26})^3} = 2.067\times10^{16}\ sec^{-1}.$

47. $CH_3-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH_2-CH_3 \xrightarrow{Br_2/hv} CH_3-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-\underset{*}{\overset{\overset{Br}{|}}{CH}}-CH_3$ (One chiral carbon)

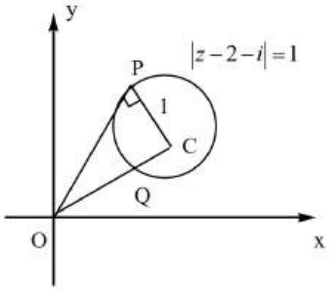
48. Number of possible dipeptides is $(3)^2 = 9$
5x9=45

49. Basicity of H_3PO_4, H_3PO_3 and H_3PO_2 are 3, 2, 1 respectively.
$\therefore$ sum of basicity = 6

50. On dilution (addition of water) pH of the buffer solution will not change therefore x=0 and x+12=12

MATHS

51. Consider statement-2



Q is the point on the circle which is nearest to the origin

$OQ = OC - CQ = \sqrt{5} - 1$

Statement-2 is true

Consider statement-1

Point on the circle with maximum argument is 1

If $\angle COP = \alpha \Rightarrow \sin\alpha = \frac{1}{\sqrt{5}} \Rightarrow \tan\alpha = \frac{1}{2}$

and $\arg(z_0) = 2\alpha = \tan^{-1}\left(\frac{4}{3}\right)$

Statement-1 is false.

53. Let α_1, α_2 *and* α_3 be the roots of $f(x) = 0$

Such that $\alpha_1 < \alpha_2 < \alpha_3$ and $g(x)$ takes all values from $[-6, \infty)$

$g(x) = (x+1)^2 - 6 \geq -6$

$\alpha_3 \leq -7, \alpha_2 \leq -8, \alpha_1 \leq -9$

$\therefore a + b + c \geq 719$

Minimum value of $a + b + c$ is 719

$\therefore \alpha_1 + \alpha_2 + \alpha_3 = -a \Rightarrow -a \leq -24 \Rightarrow a \geq 24$

$\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1=b\Rightarrow b=191$

$\alpha_1\alpha_2\alpha_3=-c$

$-c=-504$

$a+b+c\geq 719$

54. Make 1 group of 2 persons, 1 group of 4 persons and 3 groups of 3 persons among 15 persons (except 2 particular persons) hence the number of ways by grouping method is $\frac{15!}{2!4!(3!)^3\,3!}$

Now we add that 2 persons in the group of 2 persons and thus the number of arrangements of these groups into cars and autos is $\frac{15!}{2!4!(3!)^3\,3!}\times 2!\times 3!=\frac{15!}{4!(3!)^3}$

55. E_1 be the event that the answers is guessed
E_2 be the event that the answer is copied
E_3 be the event that the examinee knows the answer and
E be the event that the examinee answer correctly

$P(E_1)=\frac{1}{3},P(E_2)=\frac{1}{6}$

Assume that event E_1,E_2 and E_3 are exhaustive $P(E_1)+P(E_2)+P(E_3)=1$

$\Rightarrow P(E_3)=\frac{1}{2}$

$P\left(\frac{E}{E_1}\right)=\frac{1}{4},P\left(\frac{E}{E_2}\right)=\frac{1}{8},P\left(\frac{E}{E_3}\right)=1$ (Probability of answering correctly by knowing)

$$\therefore P\left(\frac{E_3}{E}\right)=\frac{P(E_3)P\left(\frac{E}{E_3}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)+P(E_3)P\left(\frac{E}{E_3}\right)}=\frac{24}{29}$$

56. $P(E)=P(2)+P(3)+P(5)+P(7)=0.62$

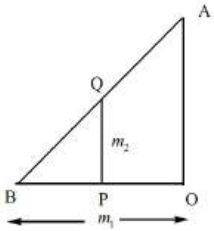
$P(F)=P(1)+P(2)+P(3)=0.50$

$P(E\cap F)=P(2)+P(3)=0.35$

$P(E\cup F)=P(E)+P(F)-P(E\cap F)$

$=0.62+0.50-0.35=0.77$

58. Let BP=x from the similar triangle properly



$\frac{AO}{m_1}=\frac{m_2}{x}\Rightarrow AO=\frac{m_1m_2}{x}$

$\frac{d(AO)}{dt}=\frac{-m_1m_2}{x^2}\frac{dx}{dt}$

When $x=\frac{m_1}{2}$

$\frac{d(AO)}{dt}=-\frac{2m_2}{5}m/s$

62. Let $I=\int_0^1 \frac{dx}{\left(5+2x-2x^2\right)\left(1+e^{2-4x}\right)} \to 1$

By kings rule

$I=\int_0^1 \frac{e^{2-4x}dx}{\left(5+2x-2x^2\right)\left(1+e^{2-4x}\right)} \to 2$

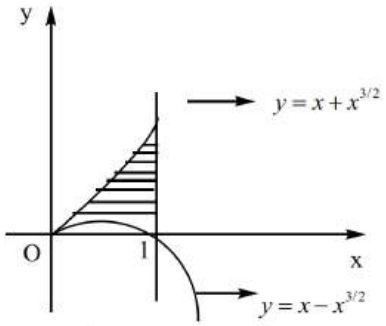
Adding 1 and 2

$$2I=\int_0^1 \frac{dx}{5+2x-2x^2}=\int_0^1 \frac{dx}{5-2\left(x^2-x\right)}=\int_0^1 \frac{dx}{\frac{11}{4}-\left(x-\frac{1}{2}\right)^2}=\frac{1}{2\sqrt{11}}\left|\ln\frac{\frac{\sqrt{11}}{2}+x-\frac{1}{2}}{\frac{\sqrt{11}}{2}-\left(x-\frac{1}{2}\right)}\right|_0^1$$

$$\frac{1}{2\sqrt{11}}\left|2\ln\left(\frac{\sqrt{11}+1}{\sqrt{11}-1}\right)\right|=\frac{1}{\sqrt{11}}\ln\left(\frac{\sqrt{11}+1}{\sqrt{11}-1}\right)$$

$$\Rightarrow I=\frac{1}{2\sqrt{11}}\ln\frac{\left(\sqrt{11}+1\right)^2}{10}$$

63.



$\left(y-x\right)^2=x^3 \Rightarrow y-x=\pm x^{3/2}$

$\Rightarrow y=x\pm x^{3/2}$

$y=x+x^{3/2} \to 1, y=x-x^{3/2} \to 2$

1 is an increasing function

2 meets x=axis at x=0,1

Required area $\int_0^1\left[\left(x+x^{3/2}\right)-\left(x-x^{3/2}\right)\right]dx$

$$=2\int_0^1 x^{3/2}dx=2\left[\frac{2}{5}x^{5/2}\right]_0^1=\frac{4}{5}sq.units$$

64. $\{x\}=x-[x]$ which is periodic with period 1.

Statement 2 is true.

Consider Statement 1.

$f(x)=\sin\left(3x-[3x]\right)=\sin\left(\{3x\}\right)$

Using Statement 2, period of $f(x)$ is $\frac{1}{3}$.

Statement 1 is false.

65. Let 1st term be 'a_1' and common ratio be 'r' then $a_1 + a_1 r + a_1 r^2 + ... + a_1 r^{100} = 125$

$$\Rightarrow \frac{a_1\left(1-r^{101}\right)}{1-r} = 125 \ \left(let\, 0 < r < 1\right)$$

$$\sum_{r=1}^{101}\left(\frac{1}{a_i}\right) = \frac{1}{a_1} + \frac{1}{a_1 r} + \frac{1}{a_1 r^2} + ... + \frac{1}{a_1 r^{100}} = \frac{\frac{1}{a_1}\left(\left(\frac{1}{r}\right)^{101} - 1\right)}{\frac{1}{r} - 1}\left(here \frac{1}{r} > 1\right)$$

$$= \frac{\left(1-r^{101}\right)}{a_1 r^{100}\left(1-r\right)} = \frac{1}{a_1 r^{100}} \times \frac{125}{a_1} = \frac{125}{\left(a_1 r^{50}\right)^2} = \frac{125}{\left(a_{51}\right)^2} = \frac{125}{\left(25\right)^2} = \frac{1}{5}$$

66. Last term expansion is ${}^n c_n \left(-\frac{1}{\sqrt{2}}\right)^n = \left(\frac{1}{3\sqrt[3]{9}}\right)^{\log_3^8}$

$$\Rightarrow (-1)^n \left(\frac{1}{2}^{n/2}\right) = \left(\frac{1}{3^{5/3}}\right)^{\log_3^8} = 3^{-\frac{5}{3}\times 3\times \log_3 2}$$

$$= 3^{-5\log_3 2} = 3^{\log_3 2^{-5}} = 2^{-5} = \left(\frac{1}{2}\right)^5 \Rightarrow n = 10$$

$\therefore 5^{\text{th}}$ term from the beginning ${}^{10}c_4 \left(\sqrt[3]{2}\right)^6 \left(-\frac{1}{\sqrt{2}}\right)^4 = {}^{10}c_4 2^2 . \frac{1}{2^2} = {}^{10}c_4$

67. Let the two numbers be a, b

$$\therefore \bar{x} = \frac{2+4+10+12+14+a+b}{7} = 8$$

$$\Rightarrow a + b = 14 \rightarrow 1$$

$$\sigma^2 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{n}\right)^2 = 16$$

$$\Rightarrow 460 + a^2 + b^2 = \left(16+64\right)\times 7 \Rightarrow a^2 + b^2 = 100 \rightarrow 2$$

From 1 and 2, $a - b = 2 \rightarrow 3$

$a = 8, b = 6$

68. $a\left\{\sin\left(\theta+\phi\right) - \sin\left(\theta-\phi\right)\right\} = b\left\{\sin\left(\theta-\phi\right) + \sin\left(\theta+\phi\right)\right\}$

$$\Rightarrow 2a\sin\phi\cos\theta = 2a\sin\theta\cos\phi$$

$$\Rightarrow a\tan\phi = b\tan\theta$$

$$\frac{2a\tan\frac{\phi}{2}}{1-\tan^2\frac{\phi}{2}} = \frac{2b\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} \rightarrow 1$$

$$a\tan\frac{\theta}{2} - b\tan\frac{\phi}{2} = c \Rightarrow \tan\frac{\theta}{2} = \frac{b\tan\frac{\phi}{2} + c}{a} \rightarrow 2$$

From 1 and 2 we get $\tan\frac{\phi}{2}\left(a^2 - b^2 - c^2\right) = bc\left(1 + \tan^2\frac{\phi}{2}\right)$

$$\Rightarrow \sin\phi = \frac{2\tan\frac{\phi}{2}}{1+\tan^2\left(\frac{\phi}{2}\right)} = \frac{2bc}{a^2 - b^2 - c^2}$$

69. Let $\sqrt{(\tan x-\sin x)+\sqrt{(\tan x-\sin x)+\sqrt{(\tan x-\sin x)+.....}}}$

$\Rightarrow y=\sqrt{(\tan x-\sin x)+y}$

$\Rightarrow y^2-y-(\tan x-\sin x)=0$

$\Rightarrow y=\dfrac{1+\sqrt{1+4(\tan x-\sin x)}}{2}$

Again $z=\sqrt{x^3+\sqrt{x^3+\sqrt{x^3+....}}}$

$=\sqrt{x^3+z}\Rightarrow z^2-z-x^3=0$

$\Rightarrow z=\dfrac{1+\sqrt{1+4x^3}}{2}$

$$\lim_{x\to 0^+}\frac{-1+\sqrt{1+4(\tan x-\sin x)}}{-1+\sqrt{1+4x^3}}=\lim_{x\to 0^+}\frac{4(\tan x-\sin x)+\left(1+\sqrt{1+4x^3}\right)}{4x^3\left(1+\sqrt{1+4(\tan x-\sin x)}\right)}$$

$$=\lim_{x\to 0^+}\frac{\left(\dfrac{\sin x}{\cos x}-\dfrac{\sin x}{1}\right)\left(1+\sqrt{1+4x^3}\right)}{x^3\left(1+\sqrt{1+4(\tan x-\sin x)}\right)}=\lim_{x\to 0^+}\frac{\sin x(1-\cos x)}{x^3\cos x}\times\frac{1+\sqrt{1+4x^3}}{1+\sqrt{1+4(\tan x-\sin x)}}$$

$$=\lim_{x\to 0^+}\frac{\sin x}{x}.\frac{2\sin^2 x/2}{4x^2/4}.\frac{1}{\cos x}.\frac{1+\sqrt{1+4x^3}}{1+\sqrt{1+4(\tan x-\sin x)}}=1.\frac{1}{2}.1.\frac{1+1}{1+1}=\frac{1}{2}$$

70. Put $x=2\cos\theta, y=3\sin\theta$

$\sin^{-1}(1)+\cos^{-1}\left(\dfrac{1}{\sqrt{2}}\cos\theta+\dfrac{1}{\sqrt{2}}\sin\theta-2\right)$

$\sin^{-1}(1)+\cos^{-1}\left(\cos\left(\theta-\dfrac{\pi}{4}\right)-2\right)$

$-1\le\cos\left(\theta-\dfrac{\pi}{4}\right)-2\le 1$

$1\le\cos\left(\theta-\dfrac{\pi}{4}\right)\le 1\Rightarrow\cos\left(\theta-\dfrac{\pi}{4}\right)=1$

$=\dfrac{\pi}{2}+\cos^{-1}(1-2)=\dfrac{\pi}{2}+\cos^{-1}(-1)\Rightarrow\dfrac{\pi}{2}+\pi=\dfrac{3\pi}{2}$

71. Let $z=x+iy$

$z-z_1=(x-10)+(y-6)i$

$z-z_2=(x-4)+(y-6)i$

$\arg\left(\dfrac{z-z_1}{z-z_2}\right)=\dfrac{\pi}{4}$

$\Rightarrow\tan^{-1}\left[\dfrac{6(y-6)}{(x-10)(x-4)+(y-6)^2}\right]=\dfrac{\pi}{4}$

$\Rightarrow x^2+y^2-14x-18y+112=0$

$|z-7-9i|^2=(x-7)^2+(y-9)^2\Rightarrow x^2-14x+y^2-18y+130=-112+130=18$

73. The polynomial is every where differentiable function

$\therefore$ The points of extremum can only be the no.of derivative

The derivative of polynomial is $P^1(x)=a(x-1)(x-3)=a(x^2-4x+3)$

$P(1)=6$

$P(x)=\int_1^x P^1(x)dx+6=a\left(\frac{x^3}{3}-2x^2+3x-\frac{4}{3}\right)+6$

Also P(3)=6 $\Rightarrow a=3$

$P(x)=x^3-6x^2+9x+2$

$\therefore P(2)=4, P^1(0)=9$

$\therefore P(2)+P^1(0)-7=6$

74. $L_1=\frac{x-0}{1}=\frac{y+1}{1}=\frac{z-0}{1}=\lambda$, $L_2=\frac{x+1}{2}=\frac{y-0}{1}=\frac{z-0}{1}=\mu$

Any point on L_1 and L_2 be $(\lambda,\lambda-1,\lambda)$ and $(2\mu-1,\mu,\mu)$ respectively.

$\frac{2\mu-1-\lambda}{2}=\frac{\mu-\lambda+1}{1}=\frac{\mu-\lambda}{2}$

On solving $\mu=1,\lambda=3$

$A=(3,2,3), B=(1,1,1)$

$AB=\sqrt{4+1+4}=3$

75. Shortest distance $=\frac{\left|\left[\bar{a}-\bar{c}\quad \bar{b}\quad \bar{d}\right]\right|}{\left|\bar{b}\times\bar{d}\right|}$

Here $\bar{a}=3\bar{i}+8\bar{j}+3\bar{k}, \bar{b}=3\bar{i}-\bar{j}+\bar{k}, \bar{c}=-3\bar{i}-7\bar{j}+6\bar{k}, \bar{d}=-3\bar{i}+2\bar{j}+4\bar{k}$

$\therefore$ Shortest distance $=\frac{270}{\sqrt{270}}=\sqrt{270}=3\sqrt{30}=\lambda\sqrt{30}\Rightarrow\lambda=3$