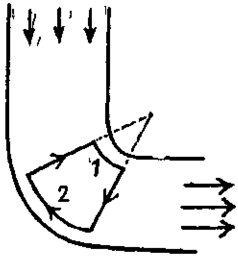
1.7 HYDRODYNAMICS

1.315 Between 1 and 2 fluid particles are in nearly circular motion and therefore have centripetal acceleration. The force for this acceleration, like for any other situation in an ideal fluid, can only come from the pressure variation along the line joining 1 and 2. This requires that pressure at 1 should be greater than the pressure at 2 i.e.

$$p_1 > p_2$$

so that the fluid particles can have required acceleration. If there is no turbulence, the motion can be taken as irrotational. Then by considering

$$\oint \vec{v} \cdot d\vec{l} = 0$$



along the circuit shown we infer that

$$v_2 > v_1$$

(The portion of the circuit near 1 and 2 are streamlines while the other two arms are at right angle to streamlines)

In an incompressible liquid we also have div $\vec{v} = 0$

By electrostatic analogy we then find that the density of streamlines is proportional to the velocity at that point.

1.316 From the conservation of mass

$$v_1 S_1 = v_2 S_2 \tag{1}$$

But $S_1 < S_2$ as shown in the figure of the problem, therefore

$$v_1 > v_2$$

As every streamline is horizontal between 1 & 2, Bernoull's theorem becomes

$$p + \frac{1}{2}\rho v^2 = \text{ constant, which gives}$$

$$p_1 < p_2 \text{ as } v_1 > v_2$$

As the difference in height of the water column is Δh, therefore

$$p_2 - p_1 = \rho g \Delta h \tag{2}$$

From Bernoull's theorem between points 1 and 2 of a streamline

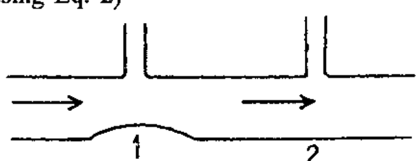
$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

or,
$$p_2 - p_1 = \frac{1}{2}\rho(v_1^2 - v_2^2)$$

or
$$\rho g \Delta h = \frac{1}{2}\rho(v_1^2 - v_2^2) \qquad (3) \text{ (using Eq. 2)}$$

using (1) in (3), we get

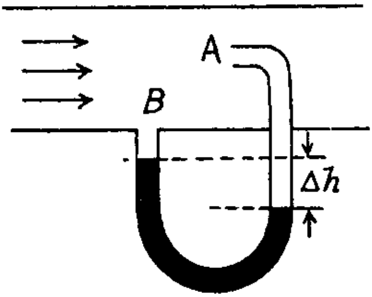
$$v_1 = S_2\sqrt{\frac{2 g \Delta h}{S_2^2 - S_1^2}}$$



Hence the sought volume of water flowing per sec

$$Q = v_1 S_1 = S_1 S_2\sqrt{\frac{2 g \Delta h}{S_2^2 - S_1^2}}$$

1.317 Applying Bernoulli's theorem for the point A and B,



$$p_A = p_B + \frac{1}{2}\rho v^2 \quad \text{as,} \quad v_A = 0$$

or, $$\frac{1}{2}\rho v^2 = p_A - p_B = \Delta h\, \rho_0\, g$$

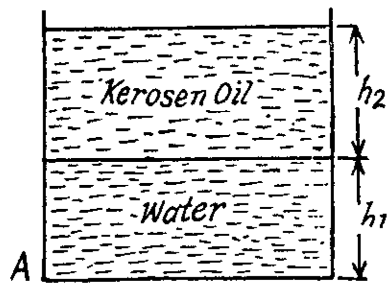
So, $$v = \sqrt{\frac{2\,\Delta h\, \rho_0\, g}{\rho}}$$

Thus, rate of flow of gas, $Q = S\,v = S\sqrt{\dfrac{2\,\Delta h\,\rho_0\, g}{\rho}}$

The gas flows over the tube past it at B. But at A the gas becomes stationary as the gas will move into the tube which already contains gas.

In applying Bernoulli's theorem we should remember that $\frac{p}{\rho} + \frac{1}{2}v^2 + gz$ is constant along a streamline. In the present case, we are really applying Bernoulli's theorem somewhat indirectly. The streamline at A is not the streamline at B. Nevertheless the result is correct. To be convinced of this, we need only apply Bernoull's theorem to the streamline that goes through A by comparing the situation at A with that above B on the same level. In steady conditions, this agrees with the result derived because there cannot be a transverse pressure differential.

1.318 Since, the density of water is greater than that of kerosene oil, it will collect at the bottom. Now, pressure due to water level equals $h_1\,\rho_1\, g$ and pressure due to kerosene oil level equals $h_2\,\rho_2\, g$. So, net pressure becomes $h_1\,\rho_1\, g + h_2\,\rho_2\, g$.

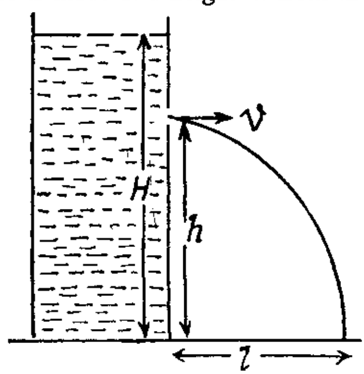


From Bernoulli's theorem, this pressure energy will be converted into kinetic energy while flowing through the whole A.

i.e. $$h_1\,\rho_1\, g + h_2\,\rho_2\, g = \frac{1}{2}\rho_1\, v^2$$

Hence $$v = \sqrt{2\left(h_1 + h_2\frac{\rho_2}{\rho_1}\right)g} = 3\text{ m/s}$$

1.319 Let, H be the total height of water column and the hole is made at a height h from the bottom.



Then from Bernoulli's theorem

$$\frac{1}{2}\rho\, v^2 = (H - h)\,\rho g$$

or, $v = \sqrt{(H-h)\;2g}$, which is directed horizontally.

For the horizontal range, $l = v\; t$

$$= \sqrt{2\,g\,(H-h)}\cdot\sqrt{\frac{2h}{g}} = 2\sqrt{(Hh - h^2)}$$

Now, for maximum l, $\frac{d\,(Hh - h^2)}{dh} = 0$

which yields $h = \frac{H}{2} = 25$ cm.

1.320 Let the velocity of the water jet, near the orifice be v', then applying Bernoullis theorem,

$$\frac{1}{2}\rho\, v^2 = h_0\, \rho\, g + \frac{1}{2}\rho\, v'^2$$

or,

$$v' = \sqrt{v^2 - 2g\,h_0} \qquad (1)$$

Here the pressure term on both sides is the same and equal to atmospheric pressure. (In the problem book Fig. should be more clear.)

Now, if it rises upto a height h, then at this height, whole of its kinetic energy will be converted into potential energy. So,

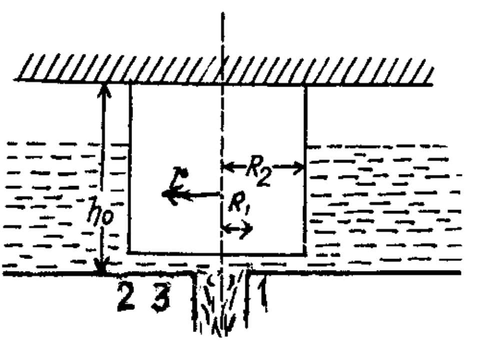
$$\frac{1}{2}\rho\, v'^2 = \rho\, gh \quad \text{or} \quad h = \frac{v'^2}{2g}$$

$$= \frac{v^2}{2g} - h_0 = 20 \text{ cm, [using Eq. (1)]}$$

1.321 Water flows through the small clearance into the orifice. Let d be the clearance. Then from the equation of continuity

$$(2\pi R_1 d)\, v_1 = (2\pi\, r\, d)\, v = (2\pi R_2 d)\, v_2$$

or $\quad v_1 R_1 = v\, r = v_2 R_2 \qquad (1)$



where v_1, v_2 and v are respectively the inward radial velocities of the fluid at 1, 2 and 3.

Now by Bernoulli's theorem just before 2 and just after it in the clearance

$$p_0 + h\,\rho g = p_2 + \frac{1}{2}\rho v_2^2 \qquad (2)$$

Applying the same theorem at 3 and 1 we find that this also equals

$$p + \frac{1}{2}\rho v^2 = p_0 + \frac{1}{2}\rho v_1^2 \qquad (3)$$

(since the pressure in the orifice is p_0)

From Eqs. (2) and (3) we also hence

$$v_1 = \sqrt{2gh} \qquad (4)$$

and

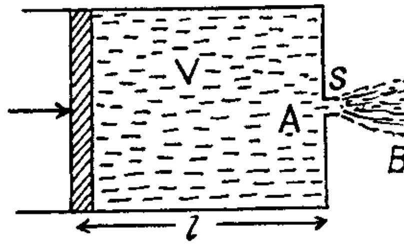
$$p = p_0 + \frac{1}{2}\rho v_1^2 \left(1 - \left(\frac{v}{v_1}\right)^2\right)$$

$$= p_0 + h\rho g \left(1 - \left(\frac{R_1}{r}\right)^2\right) \qquad \text{[Using (1) and (4)]}$$

1.322 Let the force acting on the piston be F and the length of the cylinder be l.

Then, work done $= Fl$ (1)

Applying Bernoulli's theorem for points A and B, $p = \frac{1}{2}\rho v^2$ where ρ is the density and v is the velocity at point B. Now, force on the piston,



$$F = pA = \frac{1}{2}\rho v^2 A \quad (2)$$

where A is the cross section area of piston.

Also, discharge through the orifice during time interval $t = Svt$ and this is equal to the volume of the cylinder, i.e.,

$$V = Svt \quad \text{or} \quad v = \frac{V}{St} \quad (3)$$

From Eq. (1), (2) and (3) work done

$$= \frac{1}{2}\rho v^2 A\, l = \frac{1}{2}\,\rho A \frac{V^2}{(St)^2} l = \frac{1}{2}\rho\, V^3/S^3 t^2 \quad (\text{as } Al = V)$$

1.323 Let at any moment of time, water level in the vessel be H then speed of flow of water through the orifice, at that moment will be

$$v = \sqrt{2gH} \quad (1)$$

In the time interval dt, the volume of water ejected through orifice,

$$dV = s\,v\,dt \quad (2)$$

On the other hand, the volume of water in the vessel at time t equals

$$V = SH$$

Differentiating (3) with respect to time,

$$\frac{dV}{dt} = S\frac{dH}{dt} \quad \text{or} \quad dV = S\,dH \quad (4)$$

Eqs. (2) and (4)

$$S\,dH = s\,v\,dt \quad \text{or} \quad dt = \frac{S}{s}\frac{dH}{\sqrt{2gH}}, \text{from (2)}$$

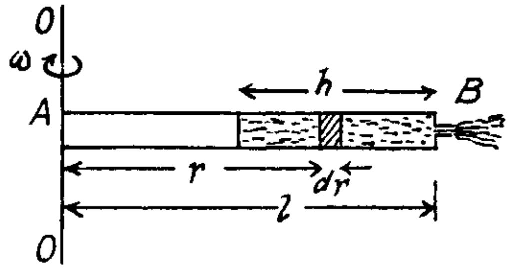
Integrating, $$\int_0^t dt = \frac{S}{s\sqrt{2h}}\int_h^0 \frac{dh}{\sqrt{H}}$$

Thus, $$t = \frac{S}{s}\sqrt{\frac{2h}{g}}$$

1.324 In a rotating frame (with constant angular velocity) the Eulerian equation is

$$-\vec{\nabla}p + \rho\vec{g} + 2\rho(\vec{v}' \times \vec{\omega}) + \rho\omega^2\vec{r} = \rho\frac{d\vec{v}'}{dt}$$

In the frame of rotating tube the liquid in the "column" is practically static because the orifice is sufficiently small. Thus the Eulerian Eq. in projection form along $\vec{r}$ (which is the position vector of an arbitrary liquid element of lenth dr relative to the rotation axis) reduces to



$$\frac{-dp}{dr} + \rho \omega^2 r = 0$$

or, $$dp = \rho \omega^2 r dr$$

so, $$\int_{p_0}^{p} dp = \rho\omega^2 \int_{(l-h)}^{r} r dr$$

Thus $$p(r) = p_0 + \frac{\rho\omega^2}{2}\left[r^2 - (l-h)^2\right] \tag{1}$$

Hence the pressure at the end B just before the orifice i.e.

$$p(l) = p_0 + \frac{\rho\,\omega^2}{2}(2\,l\,h - h^2) \tag{2}$$

Then applying Bernoull's theorem at the orifice for the points just inside and outside of the end B

$$p_0 + \frac{1}{2}\rho\,\omega^2\,(2\,l\,h - h^2) = p_0 + \frac{1}{2}\rho\,v^2$$ (where v is the sought velocity)

So, $$v = \omega h \sqrt{\frac{2l}{h} - 1}$$

1.325 The Euler's equation is $\rho \frac{d\vec{v}}{dt} = \vec{f} - \vec{\nabla} p = -\vec{\nabla}(p + \rho\, gz)$, where z is vertically upwards.

Now $$\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\,\vec{v} \tag{1}$$

But $$(\vec{v}\cdot\vec{\nabla})\,\vec{v} = \vec{\nabla}\left(\frac{1}{2}v^2\right) - \vec{v} \times \text{Curl}\,\vec{v} \tag{2}$$

we consider the steady (i.e. $\partial\vec{v}/\partial t = 0$) flow of an incompressible fluid then ρ = constant.

and as the motion is irrotational Curl $\vec{v} = 0$

So from (1) and (2) $$\rho\,\vec{\nabla}\left(\frac{1}{2}v^2\right) = -\vec{\nabla}(p + \rho\, gz)$$

or, $$\vec{\nabla}\left(p + \frac{1}{2}\rho\, v^2 + \rho gz\right) = 0$$

Hence $$p + \frac{1}{2}\rho\, v^2 + \rho\, gz = \text{constant.}$$

1.326 Let the velocity of water, flowing through A be v_A and that through B be v_B, then discharging rate through $A = Q_A = S\,v_A$ and similarly through $B = S\,v_B$.

Now, force of reaction at A,

$$F_A = \rho\, Q_A\, v_A = \rho\, S\, v_B^2$$

Hence, the net force,

$$F = \rho S (v_A^2 - v_B^2) \text{ as } \vec{F}_A \uparrow\downarrow \vec{F}_B \quad (1)$$

Applying Bernoulli's theorem to the liquid flowing out of A we get

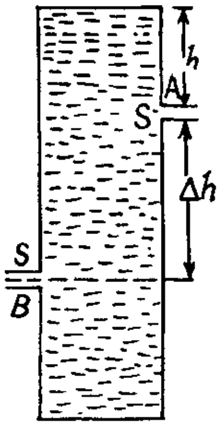
$$p_0 + \rho g h = p_0 + \frac{1}{2}\rho v_A^2$$

and similarly at B

$$p_0 + \rho g (h + \Delta h) = p_0 + \frac{1}{2}\rho v_B^2$$

Hence $$(v_B^2 - v_A^2)\frac{\rho}{2} = \Delta h \rho g$$

Thus $$F = 2\rho g S \Delta h = 0{\cdot}50\text{ N}$$



1.327 Consider an element of height dy at a distance y from the top. The velocity of the fluid coming out of the element is

$$v = \sqrt{2gy}$$

The force of reaction dF due to this is $dF = \rho\, dA\, v^2$, as in the previous problem, $= \rho\,(b\,dy)\,2gy$

Integrating $$F = \rho g b \int_{h-1}^{h} 2y\,dy$$

$$= \rho g b\,[h^2 - (h-l)^2] = \rho g b l\,(2h - l)$$

(The slit runs from a depth $h - l$ to a depth h from the top.)

1.328 Let the velocity of water flowing through the tube at a certain instant of time be u, then $u = \frac{Q}{\pi r^2}$, where Q is the rate of flow of water and πr^2 is the cross section area of the tube.

From impulse momentum theorem, for the stream of water striking the tube corner, in x–direction in the time interval dt,

$$F_x\,dt = -\rho Q u\,dt \quad\text{or}\quad F_x = -\rho Q u$$

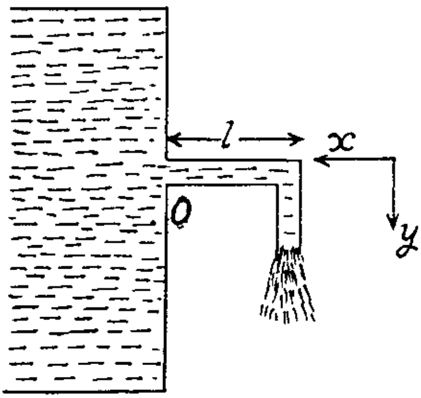
and similarly, $F_y = \rho Q u$

Therefore, the force exerted on the water stream by the tube,

$$\vec{F} = -\rho Q u\,\vec{i} + \rho Q u\,\vec{j}$$

According to third law, the reaction force on the tube's wall by the stream equals $(-\vec{F})$

$$= \rho Q u\,\vec{i} - \rho Q u\,\vec{j}.$$



Hence, the sought moment of force about 0 becomes

$$\vec{N} = l(-\vec{i}) \times (\rho Q u\,\vec{i} - \rho Q u\,\vec{j}) = \rho Q u\, l\,\vec{k} = \frac{\rho Q^2}{\pi r^2}\, l\,\vec{k}$$

and $$|\vec{N}| = \frac{\rho Q^2 l}{\pi r^2} = 0{\cdot}70\text{ N·m}$$

1.329 Suppose the radius at A is R and it decreases uniformaly to r at B where $S = \pi R^2$ and $s = \pi r^2$. Assume also that the semi vectical angle at 0 is α. Then

$$\frac{R}{L_2} = \frac{r}{L_1} = \frac{y}{x}$$

So
$$y = r + \frac{R-r}{L_2 - L_1}(x - L_1)$$

where y is the radius at the point P distant x from the vertex O. Suppose the velocity with which the liquid flows out is V at A, v at B and u at P. Then by the equation of continuity

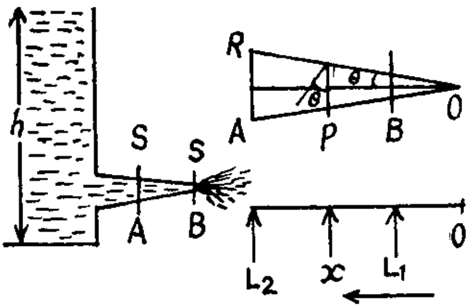
$$\pi R^2 V = \pi r^2 v = \pi y^2 u$$

The velocity v of efflux is given by

$$v = \sqrt{2gh}$$

and Bernoulli's theorem gives

$$p_p + \frac{1}{2}\rho u^2 = p_0 + \frac{1}{2}\rho v^2$$



where p_p is the pressure at P and p_0 is the atmospheric pressure which is the pressure just outside of B. The force on the nozzle tending to pull it out is then

$$F = \int (p_p - p_0)\sin\theta\; 2\pi y\, ds$$

We have subtracted p_0 which is the force due to atmosphenic pressure the factor $\sin\theta$ gives horizontal component of the force and ds is the length of the element of nozzle surface, $ds = dx \sec\theta$ and

$$\tan\theta = \frac{R-r}{L_2 - L_1}$$

Thus

$$F = \int_{L_1}^{L_2} \frac{1}{2}(v^2 - u^2)\rho\; 2\pi y\; \frac{R-r}{L_2 - L_1}\, dx$$

$$= \pi\rho \int_r^R v^2\left(1 - \frac{r^4}{y^4}\right) y\, dy$$

$$= \pi\rho v^2 \frac{1}{2}\left(R^2 - r^2 + \frac{r^4}{R^2} - r^2\right) = \rho g h\left(\frac{\pi(R^2 - r^2)^2}{R^2}\right)$$

$$= \rho g h\;(S - s)^2/S = 6{\cdot}02\,\text{N} \quad \text{on putting the values.}$$

Note : If we try to calculate F from the momentum change of the liquid flowing out w will be wrong even as regards the sign of the force.

There is of course the effect of pressure at S and s but quantitative derivation of F fron Newton's law is difficult.

1.330 The Euler's equation is $\rho \frac{d\vec{v}}{dt} = \vec{f} - \vec{\nabla} p$ in the space fixed frame where $\vec{f} = -\rho g \vec{k}$ downward. We assume incompressible fluid so ρ is constant.

Then $\vec{f} = -\vec{\nabla}(\rho g z)$ where z is the height vertically upwards from some fixed origin. We go to rotating frame where the equation becomes

$$\rho \frac{d\vec{v}'}{dt} = -\vec{\nabla}(p + \rho g z) + \rho \omega^2 \vec{r} + 2\rho (\vec{v}' \times \vec{\omega})$$

the additional terms on the right are the well known coriolis and centrifugal forces. In the frame rotating with the liquid $\vec{v}' = 0$ so

$$\vec{\nabla}\left(p + \rho g z - \frac{1}{2}\rho \omega^2 r^2\right) = 0$$

or

$$p + \rho g z - \frac{1}{2}\rho \omega^2 r^2 = \text{constant}$$

On the free surface $p =$ constant, thus

$$z = \frac{\omega^2}{2g} r^2 + \text{constant}$$

If we choose the origin at point $r = 0$ (i.e. the axis) of the free surface then "cosntant" = 0 and

$$z = \frac{\omega^2}{2g} r^2 \quad \text{(The paraboloid of revolution)}$$

At the bottom $z =$ constant

So

$$p = \frac{1}{2}\rho \omega^2 r^2 + \text{constant}$$

If $p = p_0$ on the axis at the bottom, then

$$p = p_0 + \frac{1}{2}\rho \omega^2 r^2.$$

1.331 When the disc rotates the fuild in contact with, corotates but the fluid in contact with the walls of the cavity does not rotate. A velocity gradient is then set up leading to viscous forces. At a distance r from the axis the linear velocity is ωr so there is a velocity gradient $\frac{\omega r}{h}$ both in the upper and lower clearance. The corresponding force on the element whose radial width is dr is

$$\eta\, 2\pi r dr \;\frac{\omega r}{h} \quad \left(\text{from the formular } F = \eta A \frac{dv}{dx}\right)$$

The torque due to this force is

$$\eta\; 2\pi r dr \;\frac{\omega r}{h}\; r$$

and the net torque considering both the upper and lower clearance is

$$2\int_0^R \eta 2\pi r^3\, dr \,\frac{\omega}{h}$$
$$= \pi R^4 \omega \eta / h$$

So power developed is

$$P = \pi R^4 \omega^2 \eta / h = 9 \cdot 05 \text{ W (on putting the values).}$$

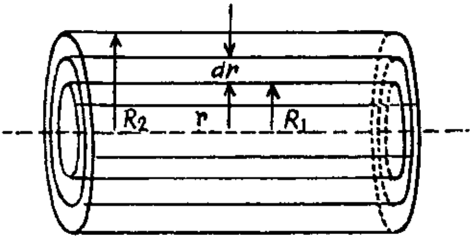
(As instructed end effects i.e. rotation of fluid in the clearance $r > R$ has been neglected.)

1.332 Let us consider a coaxial cylinder of radius r and thickness dr, then force of friction or viscous force on this elemental layer, $F = 2\pi r l \eta \frac{dv}{dr}$.

This force must be constant from layer to layer so that steady motion may be possible.

or, $$\frac{F\,dr}{r} = 2\pi l \eta\, dv. \qquad (1)$$

Integrating,



$$F \int_{R_2}^{r} \frac{dr}{r} = 2\pi l \eta \int_0^v dv$$

or, $$F \ln\left(\frac{r}{R_2}\right) = 2\pi l \eta\, v \qquad (2)$$

Putting $r = R_1$, we get

$$F \ln \frac{R_1}{R_2} = 2\pi l \eta\, v_0$$

From (2) by (3) we get,

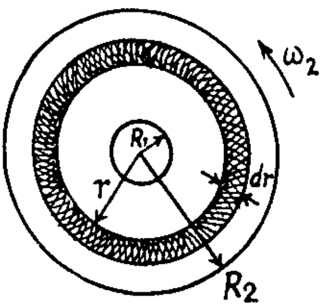
$$v = v_0 \frac{\ln r/R_2}{\ln R_1/R_2}$$

Note : The force F is supplied by the agency which tries to carry the inner cylinder with velocity v_0 .

1.333 (a) Let us consider an elemental cylinder of radius r and thickness dr then from Newton's formula

$$F = 2\pi r l \eta\, r \frac{d\omega}{dr} = 2\pi l \eta\, r^2 \frac{d\omega}{dr}$$

and moment of this force acting on the element,



$$N = 2\pi r^2 l \eta \frac{d\omega}{dr} r = 2\pi r^3 l \eta \frac{d\omega}{dr}$$

or, $$2\pi l \eta\, d\omega = N \frac{dr}{r^3} \qquad (2)$$

As in the previous problem N is constant when conditions are steady

Integrating, $$2\pi l \eta \int_0^{\omega} d\omega = N \int_{R_1}^{r} \frac{dr}{r^3}$$

or, $$2\pi l \eta\, \omega = \frac{N}{2}\left[\frac{1}{R_1^2} - \frac{1}{r^2}\right] \qquad (3)$$

Putting $r = R_2,\ \omega = \omega_2$, we get

$$2\pi l \eta\, \omega_2 = \frac{N}{2}\left[\frac{1}{R_1^2} - \frac{1}{R_2^2}\right] \qquad (4)$$

From (3) and (4),

$$\omega = \omega_2 \frac{R_1^2 R_2^2}{R_2^2 - R_1^2}\left[\frac{1}{R_1^2} - \frac{1}{r^2}\right]$$

(b) From Eq. (4),

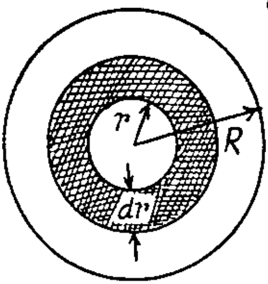
$$N_1 = \frac{N}{l} = 4\pi\eta\,\omega_2 \frac{R_1^2 R_2^2}{R_2^2 - R_1^2}$$

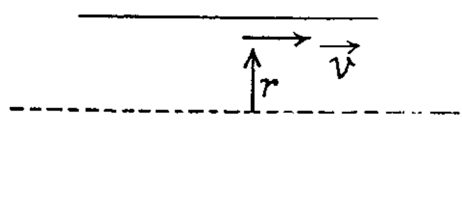
1.334 (a) Let dV be the volume flowing per second through the cylindrical shell of thickness dr then,

$$dV = -(2\pi r\,dr)\,v_0\left(1 - \frac{r^2}{R^2}\right) = 2\pi v_0\left(r - \frac{r^3}{R^2}\right)dr$$

and the total volume,

$$V = 2\pi v_0 \int_0^R \left(r - \frac{r^3}{R^2}\right)dr = 2\pi v_0 \frac{R^2}{4} = \frac{\pi}{2}R^2 v_0$$





(b) Let, dE be the kinetic energy, within the above cylindrical shell. Then

$$dT = \frac{1}{2}(dm)\,v^2 = \frac{1}{2}(2\pi r\,l\,d r\rho)\,v^2$$

$$= \frac{1}{2}(2\pi l\rho)\,r\,dr\,v_0^2\left(1 - \frac{r^2}{R^2}\right) = \pi l\rho v_0\left[r - \frac{2r^3}{R^2} + \frac{r^5}{R^4}\right]dr$$

Hence, total energy of the fluid,

$$T = \pi l\rho v_0^2 \int_0^R \left(r - \frac{2r^3}{R^2} + \frac{r^5}{R^4}\right)dr = \frac{\pi R^2 \rho\, l\, v_0^2}{6}$$

(c) Here frictional force is the shearing force on the tube, exerted by the fluid, which equals $-\eta S \frac{dv}{dt}$.

Given, $$v = v_0\left(1 - \frac{r^2}{R^2}\right)$$

So, $$\frac{dv}{dr} = -2v_0\frac{r}{R^2}$$

And at $$r = R,\ \frac{dv}{dr} = -\frac{2v_0}{R}$$

Then, viscous force is given by, $F = -\eta\,(2\pi Rl)\left(\frac{dv}{dr}\right)_{r=R}$

$$= -2\pi R\eta\, l\left(-\frac{2v_0}{R}\right) = 4\pi\eta\, v_0\, l$$

(d) Taking a cylindrical shell of thickness dr and radius r viscous force,

$$F = -\eta\,(2\pi\, rl)\frac{dv}{dr},$$

Let Δp be the pressure difference, then net force on the element $= \Delta p\,\pi\, r^2 + 2\,\pi\,\eta\, l\, r\,\frac{dv}{dr}$

But, since the flow is steady, $F_{net} = 0$

or, $$\Delta p = \frac{-2\,\pi\, l\eta\, r\,\frac{dv}{dr}}{\pi\, r^2} = \frac{-2\,\pi\, l\;\eta\, r\left(-2v_0\frac{r}{R^2}\right)}{\pi\, r^2} = 4\,\eta\, v_0\, l/R^2$$

1.335 The loss of pressure head in travelling a distance l is seen from the middle section to be $h_2 - h_1 = 10$ cm. Since $h_2 - h_1 = h_1$ in our problem and $h_3 - h_2 = 15\text{ cm} = 5 + h_2 - h_1$, we see that a pressure head of 5 cm remains incompensated and must be converted into kinetic energy, the liquid flowing out. Thus

$$\frac{\rho v^2}{2} = \rho g\,\Delta h \quad\text{where } \Delta h = h_3 - h_2$$

Thus $$v = \sqrt{2g\Delta h} = 1\text{ m/s}$$

1.336 We know that, Reynold's number (R_e) is defined as, $R_e = \rho\, v\, l/\eta$, where v is the velocity l is the characteristic length and η the coefficient of viscosity. In the case of circular cross section the chracteristic length is the diameter of cross-section d, and v is taken as average velocity of flow of liquid.

Now, R_{e_1} (Reynold's number at x_1 from the pipe end) $= \frac{\rho d_1 v_1}{m\,\eta}$ where v_1 is the velocity at distance x_1

and similarly, $$R_{e_2} = \frac{\rho\, d_2\, v_2}{\eta} \quad\text{so}\quad \frac{R_{e_1}}{R_{e_2}} = \frac{d_1 v_1}{d_2 v_2}$$

From equation of continuity, $A_1 v_1 = A_2 v_2$

or, $$\pi\, r_1^2\, v_1 = \pi\, r_2^2\, v_2 \quad\text{or}\quad d_1 v_1 r_1 = d_2 v_2 r_2$$

$$\frac{d_1 v_1}{d_2 v_2} = \frac{r_2}{r_1} = \frac{r_0\, e^{-\alpha x_2}}{r_0\, e^{-\alpha x_1}} = e^{-\alpha\Delta x} \quad (\text{as } x_2 - x_1 = \Delta x)$$

Thus $$\frac{R_{e_2}}{R_{e_1}} = e^{\alpha\Delta x} = 5$$

1.337 We know that Reynold's number for turbulent flow is greater than that on laminar flow.

Now, $$(R_e)_l = \frac{\rho v d}{\eta} = \frac{2\,\rho_1\, v_1\, r_1}{\eta_1} \quad\text{and}\quad (R_e)_t = \frac{2\,\rho_2\, v_2 r_2}{\eta}$$

But, $(R_e)_t \geq (R_e)_l$

so $$v_{2_{max}} = \frac{\rho_1 v_1 r_1 \eta_2}{\rho_2 r_2 \eta_1} = 5\ \mu\text{ m/s on putting the values.}$$

1.338 We have $R = \dfrac{v \rho_0 d}{\eta}$ and v is given by

$$6 \pi \eta r v = \frac{4\pi}{3} r^2 (\rho - \rho_0) g$$

(ρ = density of lead, ρ_0 = density of glycerine.)

$$v = \frac{2}{9\eta}(\rho - \rho_0) g r^2 = \frac{1}{18\eta}(\rho - \rho_0) g d^2$$

Thus $$\frac{1}{2} = \frac{1}{18\eta^2}(\rho - \rho_o) g \rho_0 d^3$$

and $d = [9\eta^2/\rho_0(\rho - \rho_0) g]^{1/3} = 5{\cdot}2$ mm on putting the values.

1.339 $$m\frac{dv}{dt} = mg - 6\pi\eta r v$$

or $$\frac{dv}{dt} + \frac{6\pi\eta r}{m} v = g$$

or $$\frac{dv}{dt} + kv = g, \ k = \frac{6\pi\eta r}{m}$$

or $$e^{kt}\frac{dv}{dt} + k e^{kt} v = g e^{kt} \quad \text{or} \quad \frac{d}{dt} e^{kt} v = g e^{kt}$$

or $$v e^{kt} = \frac{g}{k} e^{kt} + C \quad \text{or} \quad v = \frac{g}{k} + C e^{-kt} \quad \text{(where } C \text{ is const.)}$$

Since $$v = 0 \text{ for } t = 0, \ 0 = \frac{g}{k} + C$$

So $$C = -\frac{g}{k}$$

Thus $$v = \frac{g}{k}(1 - e^{-kt})$$

The steady state velocity is $\dfrac{g}{k}$.

v differs from $\dfrac{g}{k}$ by n where $e^{-kt} = n$

or $$t = \frac{1}{k}\ln n$$

Thus $$\frac{1}{k} = -\frac{\frac{4\pi}{3} r^3 P}{6\pi\eta r} = -\frac{4 r^2 \rho}{18\eta} = -\frac{d^2 \rho}{18\eta}$$

We have neglected buoyancy in olive oil.