Class-X Session 2022-23

Subject - Mathematics (Standard)

Sample Question Paper - 42

With Solution

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Ch. No.	Chapter Name	Per Unit Marks	Section-A (1 Mark)		Section-B (2 Marks)	Section-C (3 Marks)	Section-D (5 Marks)	Section-E (4 Marks)	Total Marks
			MCQ	A/R	VSA	SA	LA	Case-Study	
1	Real Number	6	1(Q5)	1(Q19)				1(Q36)	6
2	Polynomials	20	1(Q2)			1(Q28)			4
3	Pair of Linear Equations in Two Variables		1(Q1)		1(Q21)	1(Q27)			6
4	Quadratic Equations		1(Q4)				1(Q32)		6
5	Arithmetic Progression		1(Q3)			1(Q26)			4
6	Triangles	15	1(Q8)		1(Q22)		1(Q33)		8
7	Circles		2(Q13, 17)		1(Q24)	1(Q30)			6
8	Coordinate Geometry	6	4(Q6, 8, 11, 15)		1(Q25)				6
9	Introduction to Trigonometry	12	2(Q7, 10)		1(Q23)	1(Q29)			7
10	Some Applications of Trigonometry		1(Q14)					1(Q37)	5
11	Areas Related to Circles	10	1(Q12)						1
12	Surface Areas and Volumes						1(Q34)	1(Q38)	9
13	Statistics	11	2(Q16, 18)				1(Q35)		7
14	Probability			1(Q20)		1(Q31)			4
Total Marks (Total Questions)		80	18(18)	2(2)	10(5)	18(6)	20(4)	12(3)	80(38)

Note : The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

Time : 3 Hours **Max. Marks : 80**

General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 case based integrated units of assessment (4 marks each) with sub parts of values of 1, 1 and 2 marks each respectively.*

SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. For which value of p, will the lines represented by the following pair of linear equations be parallel

$3x - y - 5 = 0$ $\quad$ $6x - 2y - p = 0$

(a) all real values except 10 (b) 10 (c) 5/2 (d) 1/2

2. If one zero of the quadratic polynomial

$2x^2 - 8x - m$ is $\frac{5}{2}$, then the other zero is

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-15}{2}$

3. The n^{th} term of the A.P. a, $3a$, $5a$, ..., is

(a) na (b) $(2n - 1)a$ (c) $(2n + 1)a$ (d) $2na$

4. The condition for one root of the quadratic equation $ax^2 + bx + c = 0$ to be twice the other, is

(a) $b^2 = 4ac$ (b) $2b^2 = 9ac$ (c) $c^2 = 4a + b^2$ (d) $c^2 = 9a - b^2$

5. The rational number of the form $\frac{p}{q}$, $q \neq 0$, p and q are positive integers , which represents $0.1\overline{34}$ i.e., (0.1343434....) is

(a) $\frac{134}{999}$ (b) $\frac{134}{990}$ (c) $\frac{133}{999}$ (d) $\frac{133}{990}$

6. Ratio in which the line $3x + 4y = 7$ divides the line segment joining the points (1, 2) and (–2, 1) is

(a) 3 : 5 (b) 4 : 6 (c) 4 : 9 (d) None of these

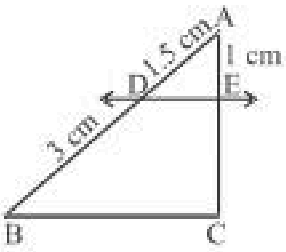
7. If $x = p \sec\theta$ and $y = q \tan\theta$, then

(a) $x^2 - y^2 = p^2q^2$ (b) $x^2q^2 - y^2p^2 = pq$ (c) $x^2q^2 - y^2p^2 = \frac{1}{p^2q^2}$ (d) $x^2q^2 - y^2p^2 = p^2q^2$

8. If the mid point of the line joining (3, 4) and (k, 7) is (x, y) and 2x + 2y + 1 = 0. Find the value of k.

(a) 10 (b) –15 (c) 15 (d) –10

9. In the given figure, $DE \parallel BC$. The value of EC is



(a) 1.5 cm (b) 3 cm (c) 2 cm (d) 1 cm

10. $(\cos^4 A - \sin^4 A)$ is equal to

(a) $1 - 2\cos^2 A$ (b) $2\sin^2 A - 1$ (c) $\sin^2 A - \cos^2 A$ (d) $2\cos^2 A - 1$

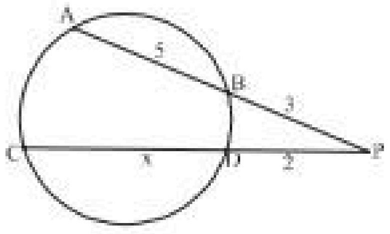
11. In what ratio is the line segment joining the points (3, 5) & (–4, 2) divided by y–axis?

(a) 3 : 2 (b) 3 : 4 (c) 2 : 3 (d) 4 : 3

12. The perimeter of a sector of a circle with central angle 90° is 25 cm. Then the area of the minor segment of the circle is.

(a) 14 cm² (b) 16 cm² (c) 18 cm² (d) 24 cm²

13. Two chords AB and CD of a circle intersect each other at P outside the circle. If AB = 5 cm, BP = 3 cm and PD = 2 cm, find CD.



(a) 4 cm
(b) 5 cm
(c) 8 cm
(d) 10 cm

14. The angles of elevation of the top of a tower, as seen from two points A and B situated in the same line and at distance x and y respectively. from the foot of the tower, are complementary. Find the height of the tower.

(a) $\sqrt{x+y}$ (b) $\sqrt{xy}$ (c) xy (d) $\sqrt{x}+\sqrt{y}$

15. ABCD is a rectangle whose three vertices are B (4, 0), C (4, 3) and D (0, 3). The length of one of its diagonals is

(a) 5 (b) 4 (c) 3 (d) 25

16. What is the arithmetic mean of 20 fours, 40 fives, 30 sixes and 10 tens?

(a) 50 (b) 25 (c) 5.6 (d) 33

17. If radii of two concentric circles are 4 cm and 5 cm, then length of each chord of one circle which is tangent to the other circle, is

(a) 3 cm (b) 6 cm (c) 9 cm (d) 1 cm

18. The mean of discrete observations $y_1, y_2, ..., y_n$ is given by

(a) $\dfrac{\sum_{i=1}^{n} y_i}{n}$ (b) $\dfrac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} i}$ (c) $\dfrac{\sum_{i=1}^{n} y_i f_i}{n}$ (d) $\dfrac{\sum_{i=1}^{n} y_i f_i}{\sum_{i=1}^{n} f_i}$

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***
(b) ***Both A and R are true but R is not the correct explanation of A.***
(c) ***A is true but R is false.***
(d) ***A is false but R is true.***

19. **Assertion :** 2 is a rational number.

Reason : The square roots of all positive integers are irrationals.

20. **Assertion :** If the probability of an event is P then probability of its complementary event will be $1 - P$.

Reason : When E and $\overline{E}$ are complementary events, then $P(E) + P(\overline{E}) = 1$

SECTION-B

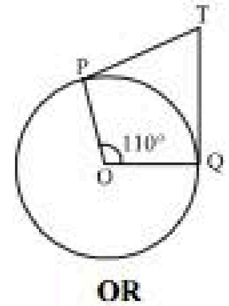
This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their respective ages.

22. S and T are points on sides PR and QR of ΔPQR such that ∠ P = ∠ RTS. Show that ΔRPQ ~ ΔRTS.

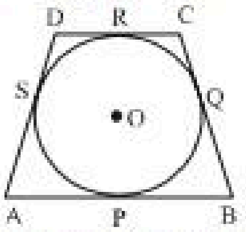
23. Find out the value of $\sqrt{\sec^2\theta + \text{cosec}^2\theta}$

24. In Fig. if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then find the measure of $\angle PTQ$.



OR

In figure, a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that AB + CD = BC + DA.



25. Find the coordinates of the point which divides the line segment joining the points (6, 3) and (– 4, 5) in the ratio 3 : 2 internally.

OR

Find the coordinates of points which trisect the line segment joining (1, – 2) and (– 3, 4).

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses fallowing it. Find this value of x. [Hint : $S_{x-1} = S_{49} - S_x$]

27. Find the sum of values of a and b for which the following system of linear equations has infinite number of solutions:
$2x + 3y = 7$
$(a + b + 1)x + (a + 2b + 2)y = 4(a + b) + 1$

28. Draw the graph of the quadratic polynomial $f(x) = 3 - 2x - x^2$. Also find its zeroes.

29. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that $(m^2 + n^2)\cos^2\beta = n^2$

OR

Find the value of 'x' such that $2\operatorname{cosec}^2 30° + x\sin^2 60° - \frac{3}{4}\tan^2 30° = 10$

30. A quadrilateral ABCD is drawn to circumscribe a circle, Prove that AB + CD = AD + BC.

31. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is a prime number?

OR

Cards marked with numbers 13, 14, 15, ..., 60 are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that the number on drawn card is
(i) divisible by (ii) a number which is a perfect square.

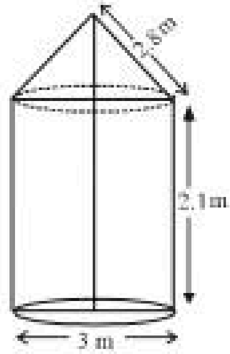
SECTION-D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Find the roots of the equations $x^2 - 3x + 2 = 0$

33. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

34. In figure, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of ₹ 500/sq. metre. $\left(\text{Use } \pi = \frac{22}{7}\right)$



OR

A the largest possible sphere is carved out from a wooden solid cube of side 7 cm. Find the volume of the wood left. $\left(\text{Use } \pi = \frac{22}{7}\right)$.

35. The following table shows marks secures by 140 students in an examination :

Marks	0-10	10-20	20-30	30-40	40-50
No. of Student	20	24	40	36	20

Calculation of mean by Step-deviation method.

OR

The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

SECTION-E

This section comprises of 3 case study/passage - based questions of 4 marks each with three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively.

36. Case - Study 1: Read the following passage and answer the questions given below.
To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections-section A and section B of grade X. There are 32 students in section A and 36 students in section B.

(i) What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of Section A or Section B?

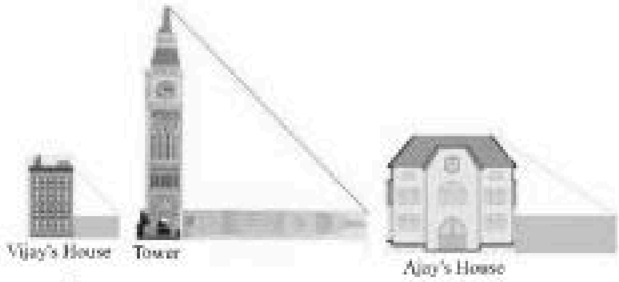
(ii) If the product of two positive integers is equal to the product of their HCF and LCM is true then, the HCF (32, 36) is

(iii) $7\times 11 \times 13 \times 15 + 15$ is a ______.

OR

If p and q are positive integers such that $p = ab^2$ and $q = a^2b$, where a, b are prime numbers, then the LCM (p, q) is ______.

37. **Case - Study 2:** Read the following passage and answer the questions given below.
Vijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Vijay's house if 20m when Vijay's house casts a shadow 10m long on the ground. At the same time, the tower casts a shadow 50m long on the ground and the house of Ajay casts 20m shadow on the ground.

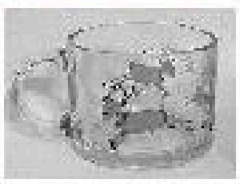


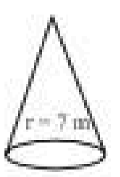
(i) What is the height of the tower?
(ii) What will be the length of the shadow of the tower when Vijay's house casts a shadow of 12m?
(iii) What is the height of Ajay's house?

OR

When the tower casts a shadow of 40m, same time what will be the length of the shadow of Ajay's house?

38. **Case - Study 3:** Read the following passage and answer the questions given below.
Adventure camps are the perfect place for the children to practice decision making for themselves without parents and teachers guiding their every move. Some students of a school reached for adventure at Sakleshpur. At the camp, the waiters served some students with a welcome drink in a cylindrical glass and some students in a hemispherical cup whose dimensions are shown below. After that they went for a jungle trek. The jungle trek was enjoyable but tiring. As dusk fell, it was time to take shelter. Each group of four students was given a canvas of area $551 m^2$. Each group had to make a conical tent to accommodate all the four students. Assuming that all the stitching and wasting incurred while cutting, would amount to $1m^2$, the students put the tents. The radius of the tent is 7 m.



(i) The volume of cylindrical cup is ______.
(ii) The volume of hemispherical cup is ______.
(iii) The height of the conical tent prepared to accommodate four students is ______.

OR

How much space on the ground is occupied by each student in the conical tent ______.

Solution

SAMPLE PAPER-10

1. (a) If the lines are parallel, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Here, $a_1 = 3, b_1 = -1, c_1 = -5,$
$a_2 = 6, b_2 = -2, c_2 = -p$

$$\Rightarrow \frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{-p} \quad \text{...(i)}$$

Taking II and III part of equation (i), we get

$$\Rightarrow \frac{1}{2} \neq \frac{-5}{-p} \Rightarrow -p \neq -10 \Rightarrow p \neq 10$$

So, option (a) is correct.

2. (c) Let α, β be two zeroes of $2x^2 - 8x - m$, where $a = \frac{5}{2}$.

$$\therefore a + b = \frac{(-\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\Rightarrow \frac{5}{2} + b = \frac{8}{2}$$

$$\Rightarrow b = \frac{8}{2} - \frac{5}{2} = \frac{3}{2}.$$

3. (b) $a_n = a + (n-1)d = a + (n-1)2a$
$[\because d = 3a - a = 2a]$
$= a + 2an - 2a = 2an - a = (2n-1)a$

4. (b) $\alpha + 2\alpha = -\frac{b}{a}$ and $\alpha \times 2\alpha = \frac{c}{a} \Rightarrow 3\alpha = -\frac{b}{a}$

$$\Rightarrow \alpha = -\frac{b}{3a} \text{ and } 2\alpha^2 = \frac{c}{a} \Rightarrow 2\left(\frac{-b}{3a}\right)^2 = \frac{c}{a}$$

$$\Rightarrow \frac{2b^2}{9a^2} = \frac{c}{a} \Rightarrow 2ab^2 - 9a^2c = 0 \Rightarrow a(2b^2 - 9ac) = 0$$

Since $a \neq 0$, $\therefore 2b^2 = 9ac$

Hence, the required condition is $2b^2 = 9ac$

5. (d) $0.1\overline{34} = \frac{134-1}{990} = \frac{133}{990}$

6. (c) $-\frac{3(1)+4(2)-7}{3(-2)+4(1)-7} = -\frac{4}{-9} = \frac{4}{9}$

7. (d) We know that $\sec^2\theta - \tan^2\theta = 1$ and $\sec\theta = \frac{x}{p}$,

$$\tan\theta = \frac{y}{q}$$

$\therefore \quad x^2q^2 - p^2y^2 = p^2q^2$

8. (b) Since (x, y) is midpoint of (3, 4) and (k, 7)

$$\therefore x = \frac{3+k}{2} \text{ and } y = \frac{4+7}{2}$$

Also $2x + 2y + 1 = 0$ putting values we get
$3 + k + 4 + 7 + 1 = 0$
$\Rightarrow \quad k + 15 = 0 \Rightarrow k = -15$

9. (c) Since, $DE \parallel BC$ $\therefore \Delta ADE \sim \Delta ABC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{1.5}{3} = \frac{1}{EC} \Rightarrow EC = 2 \text{ cm}$$

10. (d) $(\cos^4 A - \sin^4 A) = (\cos^2 A)^2 - (\sin^2 A)^2$

$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$

$= (\cos^2 A - \sin^2 A)(1) = \cos^2 A - (1 - \cos^2 A)$

$= 2\cos^2 A - 1$

11. (b) Let the required ratio be K : 1

∴ The coordinates of the required point on the y-axis is

$$x = \frac{K(-4)+3(1)}{K+1}; y = \frac{K(2)+5(1)}{K+1}$$

Since, it lies on y-axis
∴ Its x-cordinates = 0

$$\therefore \frac{-4K+3}{K+1} = 0 \Rightarrow -4K + 3 = 0$$

$$\Rightarrow K = \frac{3}{4}$$

$\Rightarrow$ Required ratio $= \frac{3}{4} : 1$ $\quad \therefore$ ratio = 3 : 4

12. (a) Perimeter of sector = 25 cm

$$\Rightarrow 2r + \frac{\theta}{360^\circ} \times 2\pi r = 25$$

$$\Rightarrow 2r + \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times r = 25$$

$$\Rightarrow 2r + \frac{11}{7} r = 25 \Rightarrow \frac{25}{7} r = 25 \Rightarrow r = 7$$

Area of minor segment $= \left(\frac{\pi\theta}{360^\circ} - \frac{\sin\theta}{2}\right)r^2$

$= \left(\frac{22}{7} \times \frac{90^\circ}{360^\circ} - \frac{\sin 90^\circ}{2}\right)(7)^2$

$= \left(\frac{11}{14} - \frac{1}{2}\right) \times 49 = \frac{4}{14} \times 49 = 14 \text{ cm}^2.$

13. **(d)** Since, two chords AB and CD of the circle are intersecting at P, when produced.

$\therefore$ PA. PB = PC.PD

[Each = (length of the tangent from P)^2]

$\Rightarrow$ (AB + PB). (PB) = (PD + DC) $\cdot$ PD

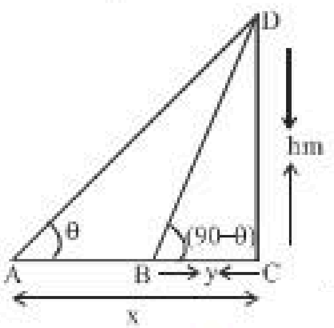
$\Rightarrow$ $(5 + 3)(3) = (2 + x)2$

$\Rightarrow$ $24 = (2 + x)(2) \Rightarrow 12 = 2 + x$

$\Rightarrow$ $x = 10 \Rightarrow$ CD = 10 cm

14. **(b)** Let DC be the tower of height 'h' metres.

In rt. ΔACD, $\tan\theta = \frac{h}{x}$ ...(i)



In rt. ΔBDC, $\tan(90 - \theta) = \frac{h}{y}$

$\Rightarrow \cot\theta = \frac{h}{y}$ ...(ii)

Multiplying (i) by (ii), we get

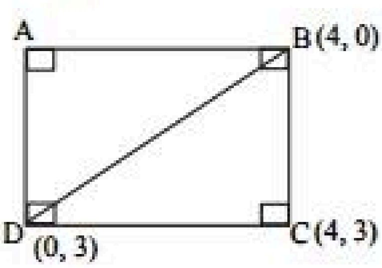
$\tan\theta \times \cot\theta = \frac{h}{x} \times \frac{h}{y}$

$\Rightarrow \tan\theta \times \frac{1}{\tan\theta} = \frac{h^2}{xy}$

$\Rightarrow 1 = \frac{h^2}{xy} \Rightarrow h^2 = xy$

$\Rightarrow h = \sqrt{xy}$

15. **(a)**



According to the figure

BD is the diagonal of ABCD.

By distance formula

$BD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

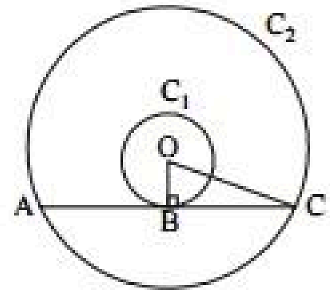
$= \sqrt{(4 - 0)^2 + (0 - 3)^2}$

$= \sqrt{25} = 5$ units

16. **(c)** $\bar{x} = \frac{20(4) + 40(5) + 30(6) + 10(10)}{20 + 40 + 30 + 10}$

$= \frac{80 + 200 + 180 + 100}{100} = \frac{560}{100} = 5.6$

17. **(b)** Suppose O be the centre of two concentric circles C_1 and C_2, whose radii are $r_1 = 4$ cm and $r_2 = 5$ cm we draw a chord AC to circle C_2, which touches the circle C_1 at B. Then, join OB, which is perpendicular to AC.



Now, in right angled ΔOBC,

$OC^2 = BC^2 + BO^2$

by using Pythagoras theorem

$\Rightarrow$ $5^2 = BC^2 + 4^2$

$\Rightarrow$ $BC^2 = 25 - 16 = 9$

$\Rightarrow$ BC = 3 cm

So, length of chord AC = 2 BC = 2 × 3 = 6 cm

18. **(a)**

19. **(c)** Here, reason is not true.

$\because \sqrt{4} = \pm 2$, which is not an irrational number.

$\therefore$ Reason does not hold. Clearly, assertion is true.

20. **(a)** Both statements are correct and Reason is the correct for Assertion.

21. Let ages of father and son be x and y respectively.

$x + y = 40$ ...(i)

$x = 3y$ ...(ii) [1 Mark]

By solving eqs. (i) and (ii)

$x = 30$ and $y = 10$

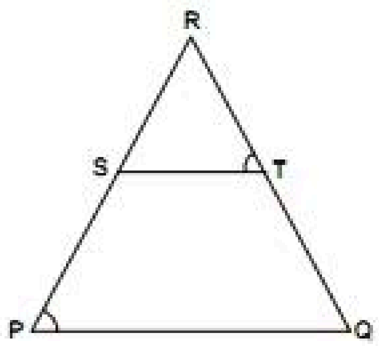
Ages are 30 years and 10 years. [1 Mark]

22. In figure,

We have ΔRPQ and ΔRTS in which

$\angle RPQ = \angle RTS$ (Given)

$\angle PRQ = \angle SRT$ (Each $= \angle R$) [1 Mark]



Then by AA similarity criterion, we have

$\Delta RPQ \sim \Delta RTS$ [1 Mark]

23. Consider $\sqrt{\sec^2\theta + \text{cosec}^2\theta} = \sqrt{\sec^2\theta + \text{cosec}^2\theta + 2 - 2}$

[1 Mark]

$= \sqrt{\sec^2\theta - 1 + \text{cosec}^2\theta - 1 + 2} = \sqrt{\tan^2\theta + \cot^2\theta + 2}$

$= \sqrt{(\tan\theta + \cot\theta)^2} = \tan\theta + \cot\theta$ [1 Mark]

24. In figure, TPOQ is a quadrilateral. Here, $\angle OPT = \angle OQT = 90°$

[$\because$ radius is perpendicular to tangent]

$\Rightarrow \angle PTQ + \angle POQ = 180°$ [1 Mark]

[$\because$ sum of all angles of quadrilateral is 360°]

$\Rightarrow \angle PTQ + \angle 110° = 180° \Rightarrow \angle PTQ = 70°$

OR

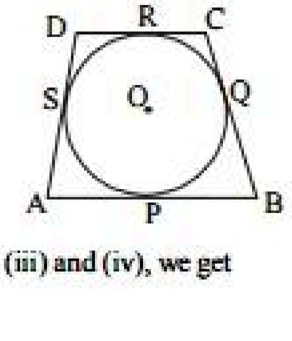
As, the tangents drawn from the exterior point to a circle are equal in length.

So, DR = DS ...(i)

AP = AS ...(ii)

BP = BQ ...(iii)

CR = CQ ...(iv) [1 Mark]



Adding (i), (ii), (iii) and (iv), we get

DR + AP + BP + CR = DS + AS + BQ + CQ

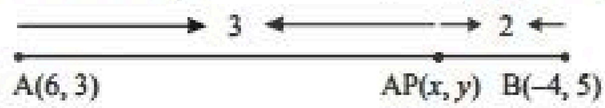
$\Rightarrow$ (DR + CR) + (AP + BP) = (DS + AS) + (BQ + CQ)

$\Rightarrow$ CD + AB = DA + BC $\Rightarrow$ AB + CD = BC + DA **(Hence Proved).** [1 Mark]

25. Let $P(x, y)$ be the required point. Then,

$x = \frac{3\times -4 + 2\times 6}{3+2}$ and $y = \frac{3\times 5 + 2\times 3}{3+2}$

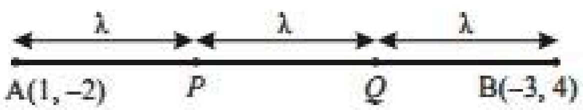
$\Rightarrow$ $x = 0$ and $y = \frac{21}{5}$ [1 Mark]



So, the coordinates of P are (0, 21/5) [1 Mark]

OR

Let A (1, – 2) and B (– 3, 4) be the given points. Let the points of trisection be P and Q. Then, AP = PQ = QB = λ (say).



$\therefore$ PB = PQ + QB = 2λ and AQ = AP + PQ = 2λ [1 Mark]

$\Rightarrow$ AP : PB = $\lambda : 2\lambda = 1 : 2$ and AQ : QB = $2\lambda : \lambda = 2 : 1$

So, P divides AB internally in the ratio 1 : 2 while Q divides internally in the ratio 2 : 1. Thus, the coordinates of P and Q are

$P\left(\frac{1\times(-3) + 2\times 1}{1+2}, \frac{1\times 4 + 2\times(-2)}{1+2}\right) = P\left(\frac{-1}{3}, 0\right)$ [1 Mark]

$Q\left(\frac{2\times(-3) + 1\times 1}{2+1}, \frac{2\times 4 + 1\times(-2)}{2+1}\right) = Q\left(\frac{-5}{3}, 2\right)$

respectively

Hence, the two points of trisection are (–1/3,0) and (–5/3,2)

[1 Mark]

26. Here, $a = 1$, and $d = 1$

$\therefore$ $S_{n-1} = \frac{x-1}{2}[2\times 1 + (x-1-1)\times 1]$

$= \frac{x-1}{2}(2 + x - 2) = \frac{(x-1)(x)}{2} = \frac{x^2 - x}{2}$

$S_n = \frac{x}{2}[2\times 1 + (x-1)\times 1] = \frac{x}{2}(x+1) = \frac{x^2 + x}{2}$ [1 Mark] and, $S_{49} = \frac{49}{2}[2\times 1+(49-1)\times 1]$

$$= \frac{49}{2}[2+48] = \frac{49}{2}\times 50 = 49\times 25$$

According to the question, [1 Mark]

$S_{n-1} = S_{49} - S_x$

i.e., $\frac{x^2-x}{2} = 49\times 25 - \frac{x^2+x}{2}$

$\Rightarrow \frac{x^2-x}{2} + \frac{x^2+x}{2} = 49\times 25$

$\Rightarrow \frac{x^2-x+x^2+x}{2} = 49\times 25$

$\Rightarrow x^2 = 49\times 25 \quad \Rightarrow x = \pm 7\times 5$ [1 Mark]

$\because$ x is a counting number, so taking positive square root, $x = 7\times 5 = 35$.

27. Here, $\frac{a_1}{a_2} = \frac{2}{a+b+1}$; $\frac{b_1}{b_2} = \frac{3}{a+2b+2}$;

$\frac{c_1}{c_2} = \frac{7}{4(a+b)+1}$

For Infinite number of solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ or $\frac{2}{a+b+1} = \frac{3}{a+2b+2} = \frac{7}{4(a+b)+1}$

[½ Mark]

(I) (II) (III)

Taking I and II & taking II and III

$\frac{2}{a+b+1} = \frac{3}{a+2b+2}$ and $\frac{3}{a+2b+2} = \frac{7}{4(a+b)+1}$

[½ Mark]

$3a+3b+3 = 2a+4b+4$ and $12a+12b+3 = 7a+14b+14$

$a-b=1$ ...(i) [1 Mark]

and $5a-2b=11$ ...(ii)

Multiplying (i) by 2 and subtracting (ii) from (i)

$2a-2b=2$

$5a-2b=11$

$-3a=-9 \Rightarrow a=3$

Putting the value of a in (i), we get

$a-b=1 \Rightarrow 3-b=1 \quad \Rightarrow b=2$ [1 Mark]

28. Let $y = f(x)$ or $y = 3-2x-x^2$

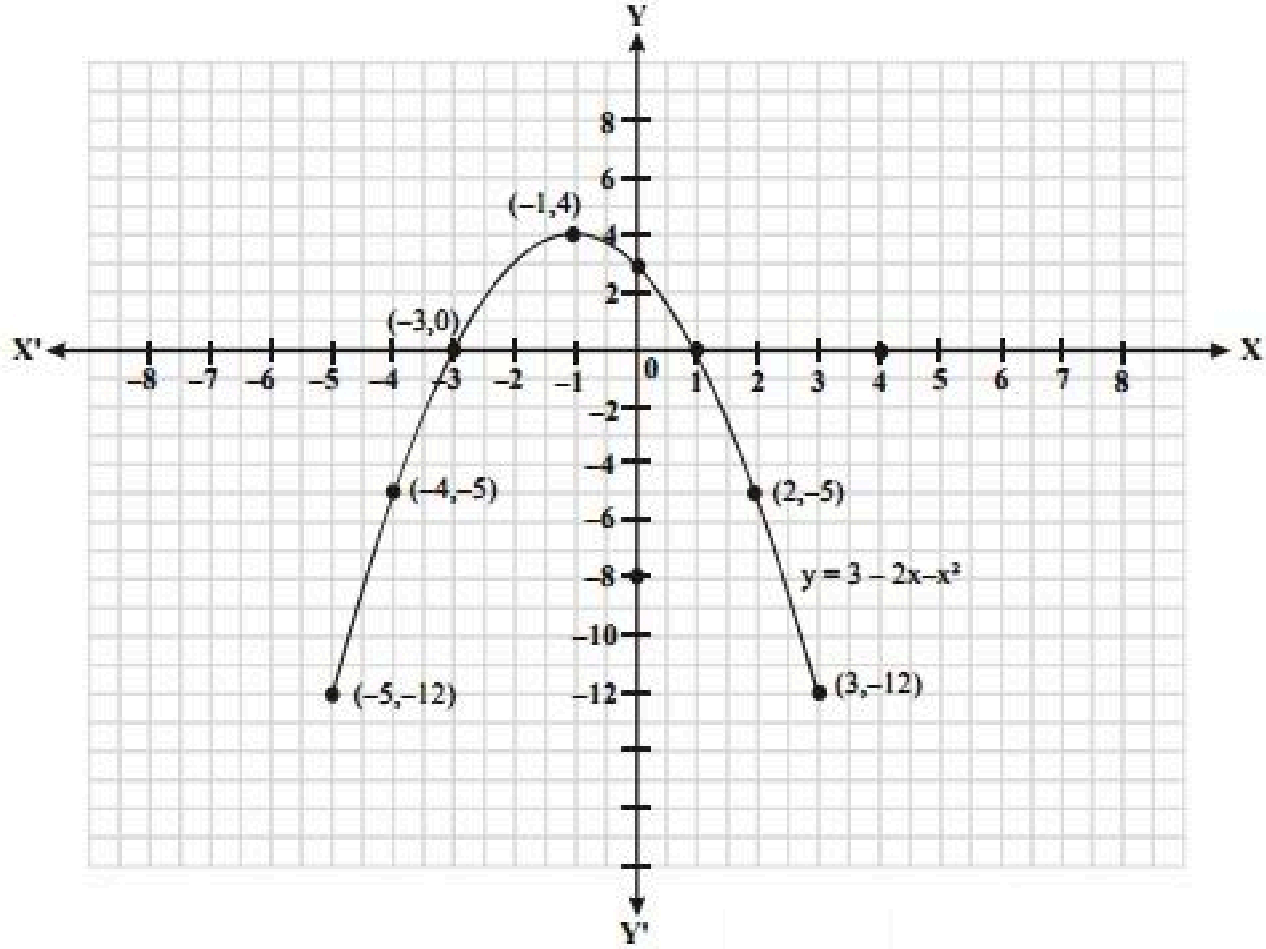
Let us list a few values of $y = 3-2x-x^2$ corresponding to a few values of x as follows :

x	–5	–4	–3	–2	–1	0	1	2	3
$y=3-2x-x^2$	–12	–5	0	3	4	3	0	–5	–12

Thus, the following points lie on the graph of polynomial $y = 3-2x-x^2$:

(–5, –12), (–4, –5), (–3, 0), (–2, 3), (–1, 4), (0, 3), (1, 0), (2, –5), and (3, – 12)

Let us plot these points on a graph paper and draw a smooth free hand curve passing through these points to obtain the graph of $y = 3-2x-x^2$. The curve thus obtained is a parabola. [2 Mark]



[1 Mark]

The parabola intersects *X*-axis at $x = -3$ and 1. Therefore, zeroes or roots of the polynomial are –3 and 1.

29. LHS = $(m^2 + n^2)\cos^2\beta = \left(\dfrac{\cos^2\alpha}{\cos^2\beta} + \dfrac{\cos^2\alpha}{\sin^2\beta}\right)\cos^2\beta$ [1 Mark]

$$= \left(\frac{\cos^2\alpha\sin^2\beta + \cos^2\alpha\cos^2\beta}{\sin^2\beta\cos^2\beta}\right)\cos^2\beta$$

$$= \left(\frac{\cos^2\alpha\left(\sin^2\beta + \cos^2\beta\right)}{\sin^2\beta\cos^2\beta}\right)\cos^2\beta$$

[1 Mark]

$$= \left(\frac{\cos^2\alpha\cos^2\beta}{\sin^2\beta\cos^2\beta}\right) = \frac{\cos^2\alpha}{\sin^2\beta} = (n)^2 = \text{RHS}$$

$\left(\because \dfrac{\cos\alpha}{\sin\beta} = n\right)$ [1 Mark]

Hence Proved.

OR

$$2\operatorname{cosec}^2 30° + x\sin^2 60° - \frac{3}{4}\tan^2 30° = 10$$

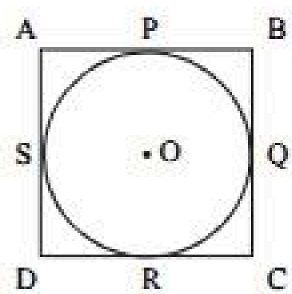
$\Rightarrow 2(2)^2 + x\left(\dfrac{\sqrt{3}}{2}\right)^2 - \dfrac{3}{4}\left(\dfrac{1}{\sqrt{3}}\right)^2 = 10$ [1 Mark]

$\Rightarrow 8 + \dfrac{3}{4}x - \dfrac{3}{4}\times\dfrac{1}{3} = 10$ [½ Mark]

$\Rightarrow \dfrac{3}{4}x = 10 - 8 + \dfrac{1}{4} \Rightarrow \dfrac{3}{4}x = \dfrac{40 - 32 + 1}{4}$ [½ Mark]

$\Rightarrow \dfrac{3}{4}x = \dfrac{9}{4} \Rightarrow 3x = 9 \Rightarrow x = 3$ [1 Mark]

30. **Given :** A quadrilateral ABCD circumscribes a circle with centre O.



To prove : AB + CD = AD + BC

Proof : Since, tangents drawn to a circle from an exterior point are equal

AP = AS I

BP = BQ II

CR = CQ III

DR = DS .....IV [1 Mark]

By adding I, II, III and IV we get,

AP + BP + CR + DR = AS + BQ + CQ + DS

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ) [2 Marks]

AB + CD = AD + BC

Hence, proved.

31. Total number of possible outcomes when two dice are thrown simultaneously = 36 [1 Mark]

Sum of the numbers appearing on the dice is a prime number i.e., 2, 3, 5, 7 and 11

So, the possible outcomes are (1, 1), (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (5, 6) and (6, 5).

Number of possible outcomes = 15 [1 Mark]

$\therefore$ required probability $= \frac{15}{36} = \frac{5}{12}$ [1 Mark]

OR

Total no. of cards = 60 – 12 = 48

$\Rightarrow$ Total no. of outcomes = 48

Numbers are 13, 14, 15, 16, ..., 60.

(i) Numbers divisible by 5 are 15, 20, 25, 30, 35, 40, 45, 50, 55, 60. [1 Mark]

$\therefore$ Favourable outcomes = 10

$\therefore$ P(no. is divisible by 5) $= \frac{10}{48} = \frac{5}{24}$ [½ Mark]

(ii) Perfect square numbers are 16, 25, 36, 49 [½ Mark]

$\therefore$ Favourable outcomes = 4

$\therefore$ P(perfect square) $= \frac{4}{48} = \frac{1}{12}$ [1 Mark]

32. Given equation is $x^2 - 3x + 2 = 0$ [1 Mark]

On comparing with $ax^2 + bx + c = 0$, we get $a = 1, b = -3, c = 2$

Now, Apply discriminant

$D = b^2 - 4ac = (-3)^2 - 4(1)(2) = 1 \Rightarrow \sqrt{D} = 1$ [2 Marks]

The two roots are given by $\frac{-b \pm \sqrt{D}}{2a}$, *i.e.*, $\frac{3 \pm 1}{2} = \frac{4}{2}$ and $\frac{2}{2}$

[2 Marks]

Hence, the two roots are 1 and 2.

33. $\Delta ABC \sim \Delta PQR$

(Given)

$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$;

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ ...(1)

Now, $BD = CD = \frac{1}{2} BC$ and $QM = RM = \frac{1}{2} QR$ ...(2)

($\because$ D is mid-point of BC and M is mid-point of QR) [1 Mark]

From (1), $\frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM}$ (By (2))

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$ [2 Marks]

Thus, we have $\frac{AB}{PQ} = \frac{BD}{QM}$

and $\angle ABD = \angle PQM$ ($\because \angle B = \angle Q$)

$\Rightarrow \Delta ABD \sim \Delta PQM$

(By SAS similarity critetrion)

$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$ [2 Marks]

34. Canvas needed to make the tent = C.S.A of the conical part + C.S.A of the cylindrical part

Given that

Radius of the conical part = Radius of the cylindrical part

$= \frac{3}{2}$ m

Slant height of the conical part = l = 2.8 m

Height of the cylindrical part = h = 2.1m

C.S.A of the conical part $= \pi r l = \frac{22}{7} \times \frac{3}{2} \times 2.8 \text{ m}^2$ [1 Mark]

C.S.A of the cylindrical part $= 2\pi r h = 2 \times \frac{22}{7} \times \frac{3}{2} \times 2.1 \text{ m}^2$

[2 Marks]

$\therefore$ Total area of the canvas needed to make the tent

$= \frac{22}{7} \times \frac{3}{2} \times 2.8 + 2 \times \frac{22}{7} \times \frac{3}{2} \times 2.1$

$= \frac{22}{7} \times \frac{3}{2} \times (2.8 + 4.2) = \frac{22}{7} \times \frac{3}{2} \times 7 = 33 \text{m}^2$ [2 Marks]

Cost of the canvas = ₹ 500/m^2

So, total cost of the canvas needed to make the tent = ₹ 500 × 33 = ₹16, 500 [1 Mark]

OR

Side of cube = 7 cm

Largest sphere carved out from cube with radius $= \frac{7}{2}$ cm

[1 Mark]

Vol. of wooden left = Vol. of cube – Vol. of sphere [2 Marks]

$= (\text{side})^3 - \frac{4}{3}\pi r^3 = 7^3 - \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$

$= 343 - \frac{539}{3} = \frac{1029 - 539}{3} = \frac{490}{3} \text{cm}^3$ [2 Marks]

35. Let a = 25 (assumed mean) and h = 10 (class interval)

Marks	f_i	Mid - Point (x_i)	Deviation $\left(u_i = \frac{x_i - 25}{10}\right)$	f_iu_i
0 – 10	20	5	–2	–40
10 – 20	24	15	–1	–24
20 – 30	40	25	0	0
30 – 40	36	35	1	36
40 – 50	20	45	2	40
Total	140			12

[2 Marks]

Since, mean $= a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times h$ [1 Mark]

$\Rightarrow$ mean $= 25 + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times 10 = 25 + \left(\frac{12}{140}\right) \times 10$

= 25.86 (Approximate) [2 Marks]

OR

From the given data, we have the modal class 35-40.

{$\because$ It has largest frequency among the given classes of the data}

So, $l = 35, f_m = 23, f_1 = 21, f_2 = 14$ and $h = 10$.

Mode $= l + \left\{\frac{f_m - f_1}{2f_m - f_1 - f_2}\right\} \times h$

$= 35 + \left\{\frac{23-21}{46-21-14}\right\} \times 10 = 36.8$ years

Now, let us find mean of the data : [2 Marks]

Age (in years)	Number of patients f_i	Class mark x_i	$u_i \frac{x_i - 30}{10}$	$f_i \times u_i$
5-15	6	10	– 2	– 12
15-25	11	20	– 1	– 11
25-35	21	**30 = a**	0	0
35-45	23	40	1	23
45-55	14	50	2	28
55-65	5	60	3	15
Total	$n = 80$			43

$a = 30, h = 10, n = 80$ and $\sum f_iu_i = 43$ [2 Marks]

Mean $= a + h \times \frac{1}{n} \times \sum f_iu_i =$

$30 + 10 \times \frac{1}{80} \times 43 = 30 + 5.37 = 35.37$ years.

Thus, mode = 36.8 years and mean = 35.37 years.

[1 Mark]

So, we conclude that the maximum number of patients admitted in the hospital are of the age 36.8 years (apporx.), whereas on an average the age of a patient admitted to the hospital is 35.37 years.

36. (i) For getting least number of books, taking LCM of 32, 36

4	32, 36
8	8, 9
9	1, 9
	1, 1

$\Rightarrow 4 \times 8 \times 9 = 288$ [1 Mark]

(ii) HCF of 32, 36 is

4	32, 36
	8, 9

[1 Mark]

= 4

(iii) $7 \times 11 \times 13 \times 15 + 15$

$\Rightarrow 15(7 \times 11 \times 13 + 1)$

so given no. is a composite number. [2 Marks]

OR

Given a, b are prime number. So

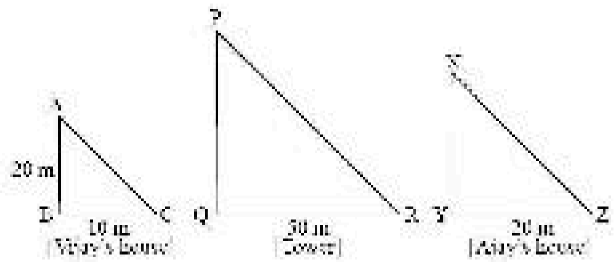
LCM of p, q, where $p = ab^2, q = a^2b$

$p = a \times b \times b$

$q = a \times b \times a$

$a \times b \times b \times a \Rightarrow a^2b^2$ [2 Marks]

37.



(i) $\because$ $\Delta ABC \sim \Delta PQR$

$\therefore$ $\frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{20}{PQ} = \frac{10}{50}$

$\Rightarrow$ $PQ = 100$

$\therefore$ Height of the tower = 100 m [1 Mark]

(ii) Let $BC = 12$ m and $PQ = 100$ m

$\frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{20}{100} = \frac{12}{QR}$

$\Rightarrow$ $QR = 60$ [1 Mark]

(iii) $\because$ $\Delta ABC \sim \Delta XYZ$

$\therefore$ $\frac{AB}{XY} = \frac{BC}{YZ} \Rightarrow \frac{20}{XY} = \frac{10}{20}$

$\Rightarrow$ $XY = 40$ [2 Marks]

OR

Let $QR = 40$ m, PQ = 100 m and $XY = 40$ m

$\therefore$ $\frac{PQ}{XY} = \frac{QR}{YZ} \Rightarrow \frac{100}{40} = \frac{40}{YZ}$ $\Rightarrow$ $YZ = 16$ m. [2 Marks]

38. (i) Volume of cylindrical cup = $\pi r^2 h$

$\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10.5 = 404.25 \text{ cm}^3$ [1 Mark]

(ii) Volume of hemispherical cup

$= \frac{2}{3}\pi r^3 = \frac{3}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3 = 89.83 \text{ cm}^3$

(iii) Curved surface area of cone = 551 $\Rightarrow \pi r l = 441$

$\Rightarrow$ $\frac{22}{7} \times 7 \times l = 551 \Rightarrow l = 25.045$

$\therefore$ $h = \sqrt{l^2 - r^2} = 24$ m [2 Marks]

OR

Space occupied by each student = $\frac{\pi r^2}{4} = 38.5 \text{ m}^2$

[2 Marks]