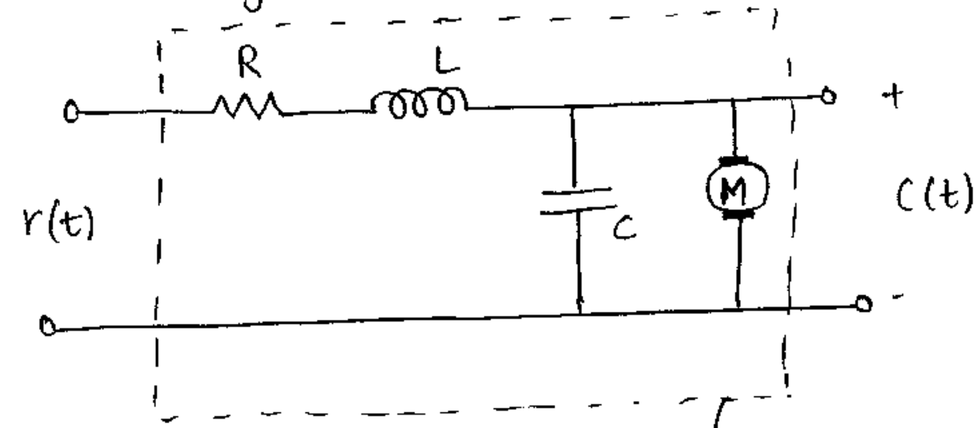
Ch-2 Block diagram and Signal Flow Graph (SFG)

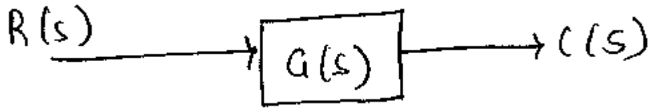
What is purpose?

- Purpose of block diagram is to find the overall transfer function of the system between input and output.

- Block diagram is short hand pictorial representation of the system between input and output.



→ short hand pictorial representation of system betⁿ i/p & o/p.

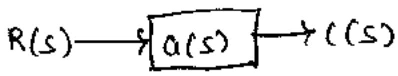


- The system can be represented in two ways

(1) open loop form

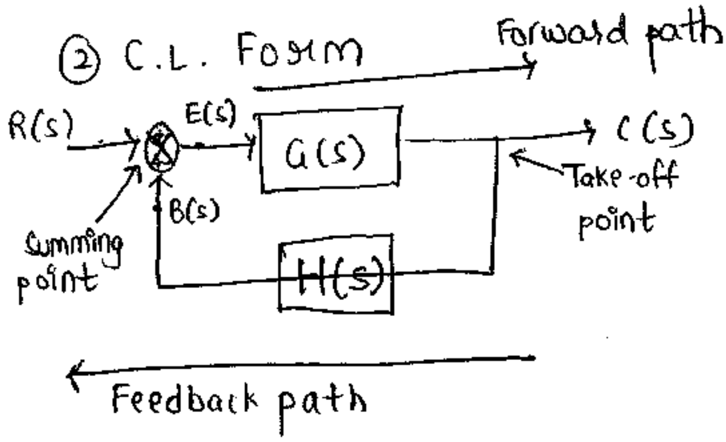
(2) Close loop form

① O.L. Form



$$\text{OL Gain} = \text{OLTF} = \frac{C[s]}{R[s]} = G[s]$$

② C.L. Form

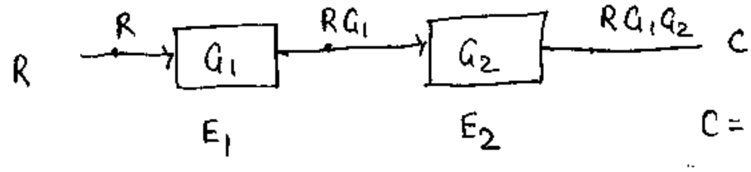


$$\text{CLTF} = \text{CL gain} = \frac{C[s]}{R[s]} = \frac{G[s]}{1 \pm G[s]H[s]}$$

-ve feedback　+ve feedback

* Block diagram reduction techniques

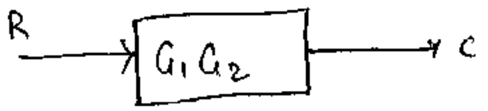
(1) Blocks are connected in series



$C = RG_1G_2$

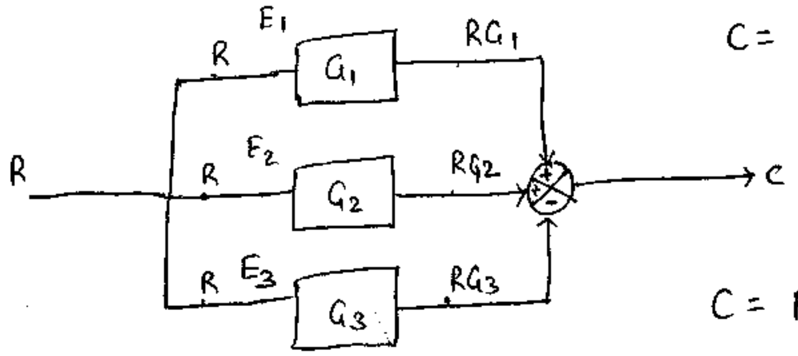
$E = E_1 + E_2$

|||



$C = G_1 \cdot G_2 \; R$

(2) Blocks are connected in parallel

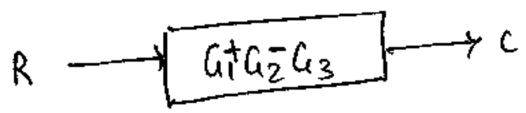


$C = RG_1 + RG_2 - RG_3$

$E = \frac{E_1 E_2}{E_3}$

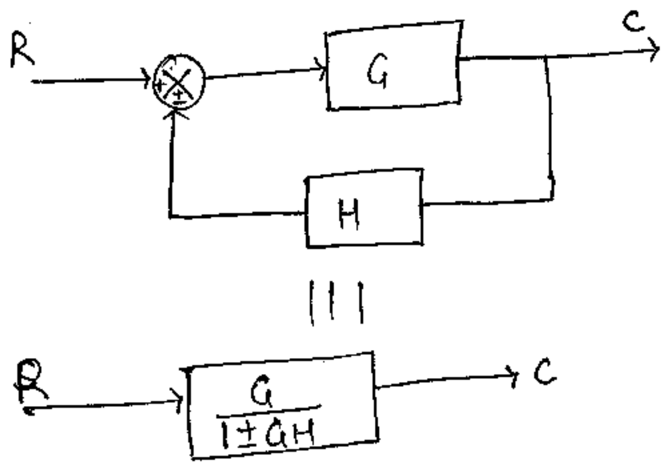
$C = R\,[G_1 + G_2 - G_3]$

|||

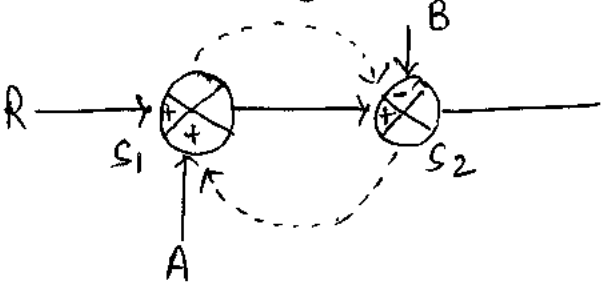


$\frac{C}{R} = G_1 G_2 G_3$

③ Closed loop

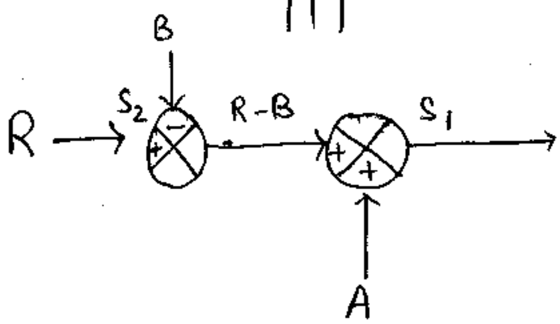


④ Interchanging of summing point



$C = A \mp B + R$

|||

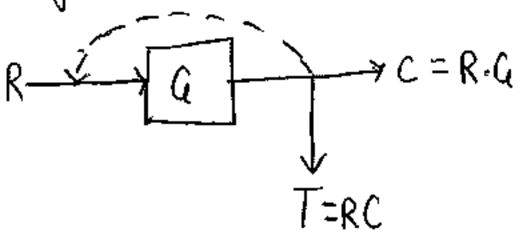


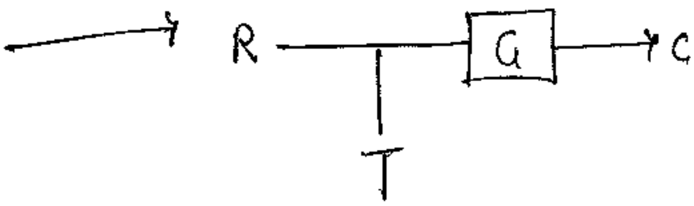
$C = R - B + A$

⑤ Shifting of Take-off point

(a) Shifting of take-off point before gain block

Before modification





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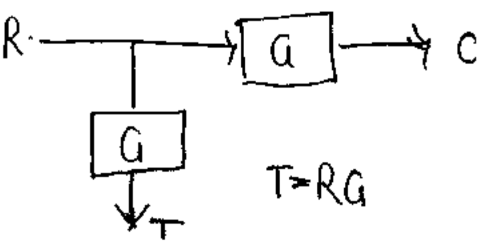
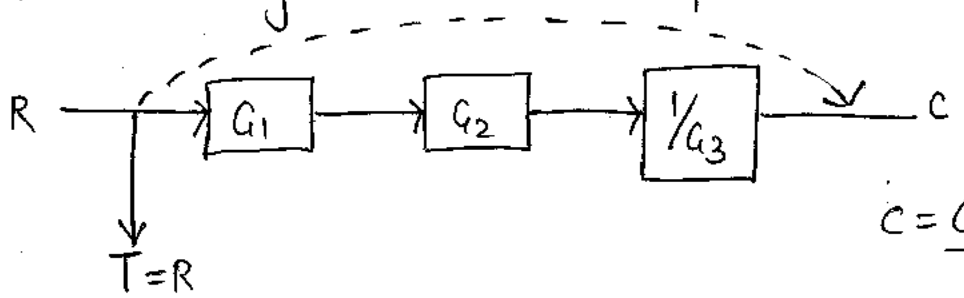


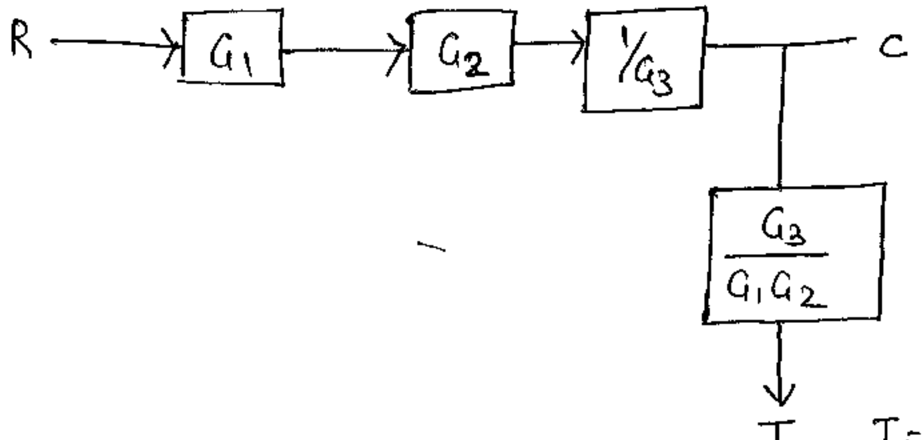
$C = RG$

After modification

(b) Shifting of take-off point after gain block



$$C = \frac{G_1 G_2}{G_3}$$



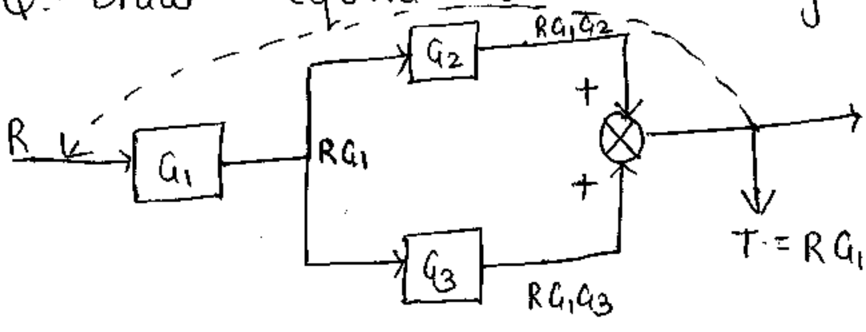
$$C = \frac{G_1 G_2}{G_3} \cdot R$$

T = R

NOTE:-

- While shifting of take-off point observes left hand side block changes. After shifting if there are extra gain blocks then divide (Gain) and if we are losing some gain block then multiply.

Q:- Draw equivalent block diagram?

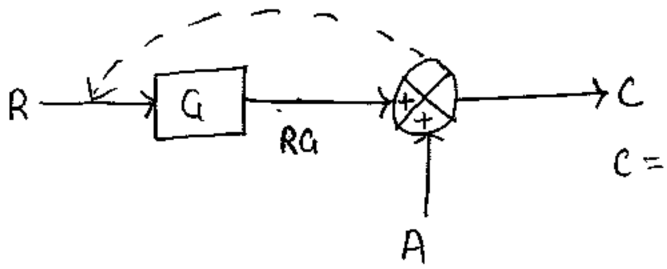


$C = RG_1[G_2+G_3]$

$T = RG_1[G_2+G_3]$

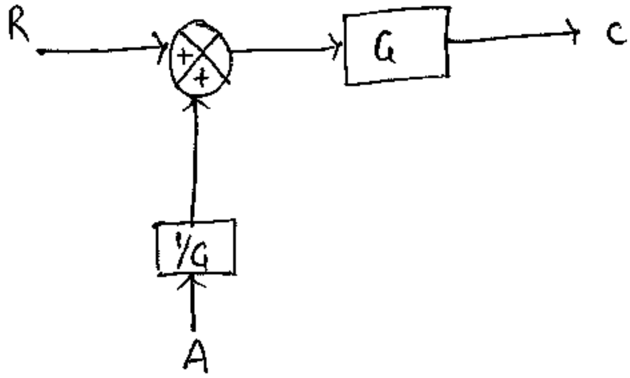
⑥ Shifting of summing point

(a) Shifting of summing point before gain block

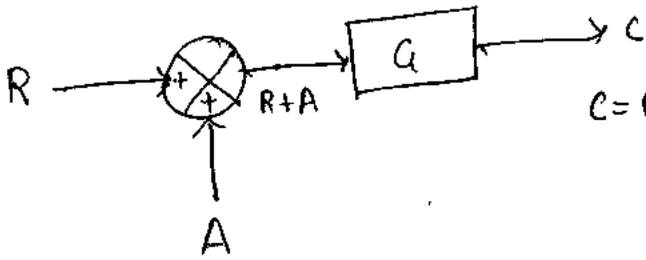


$C = RG + A$

$\equiv$

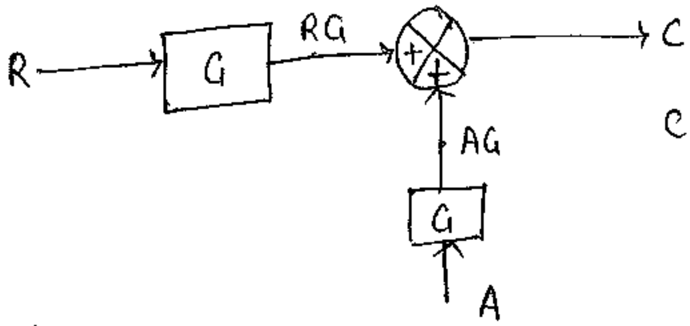


(b) Shifting of summing point after gain block



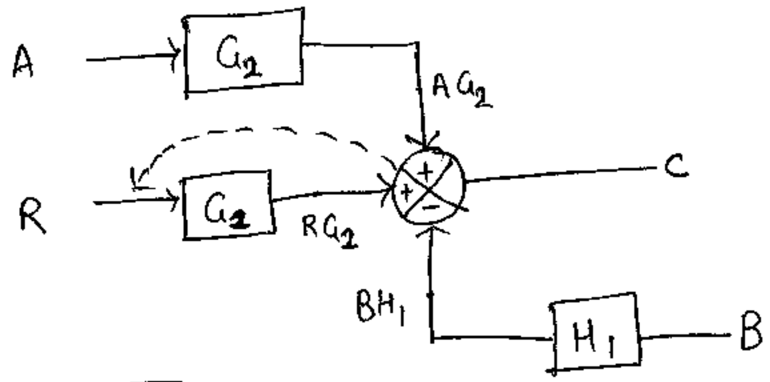
$C = G(R+A)$

$\equiv$

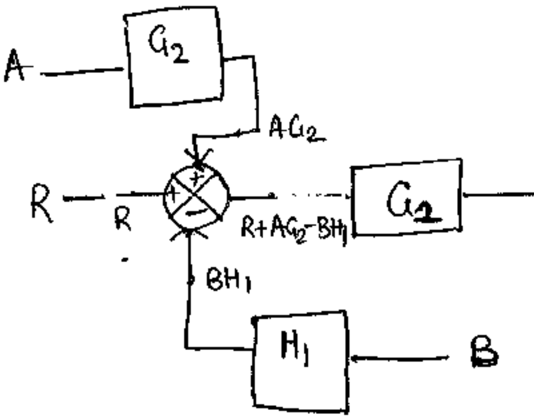


$C = RG + AG$

Q:- Draw eq^n block diagram:-

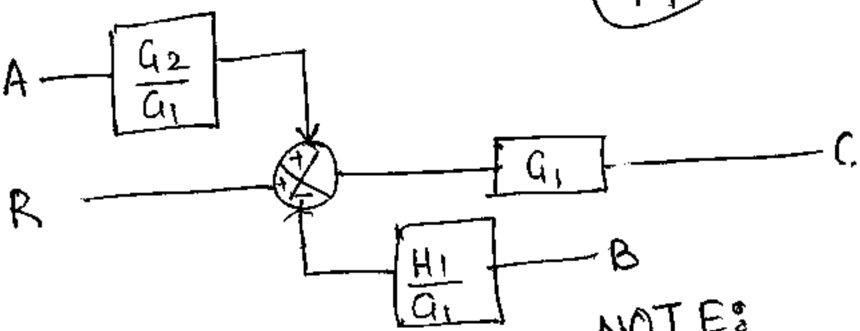


$C = AG_2 + RG_2 - BH_1$



$C = RG_1 + A\underline{G_1}G_2 - BH_1\underline{G_1}$

$|||$



$C = RG_1 + AG_2 - BH_1$

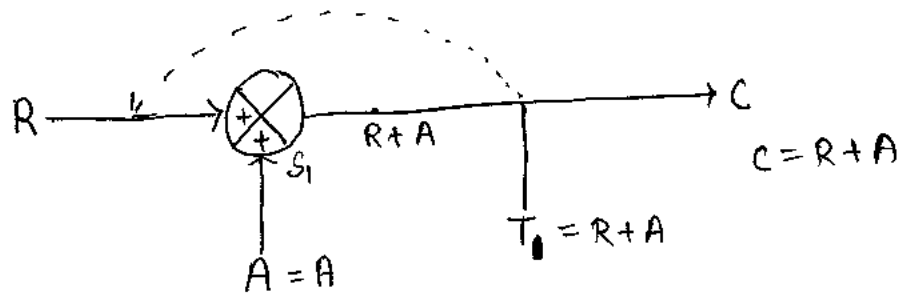
NOTE:

- While shifting operation changes are occured only in additional forward path & feedback path connected to that summing point only.
- Before shifting & after shifting forward path gain should be remain same we doen't want to loss and we don't want any extra gain so if it is extra gain after shifting then divide and if we miss any gain then we should multiply.

- For take-off point observe left hand side block change
- For summing point observe right hand side block change

⑦ Adjusting the take-off and summing point

(a) Shifting of take-off point before summing point



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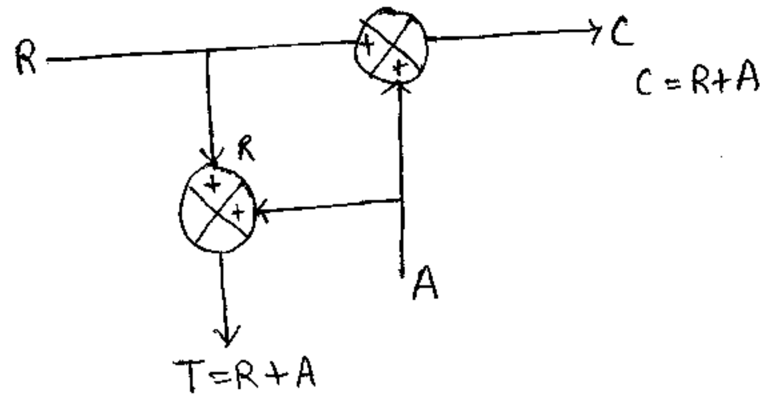
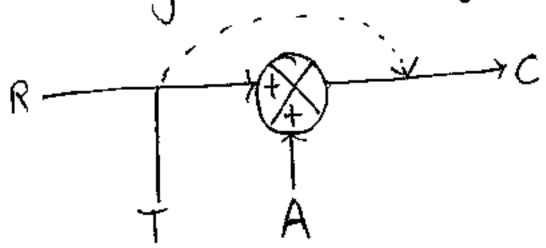


(b) Shifting of take off point after summing point



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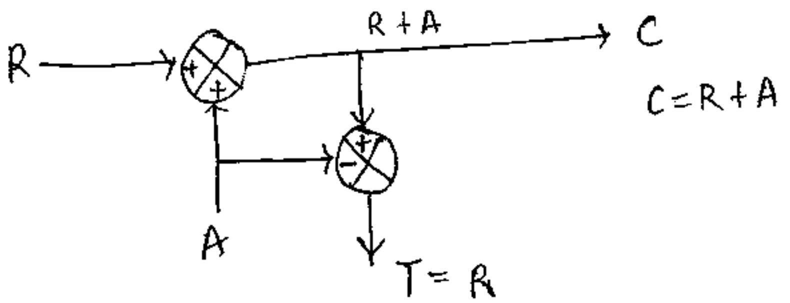


* Steps:-

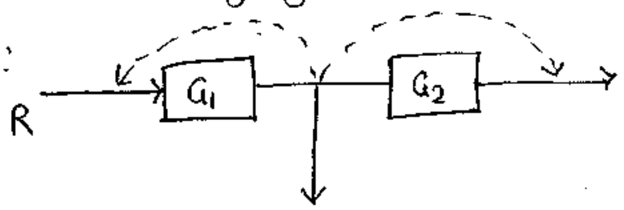
S_1 : series

S_2 : parallel

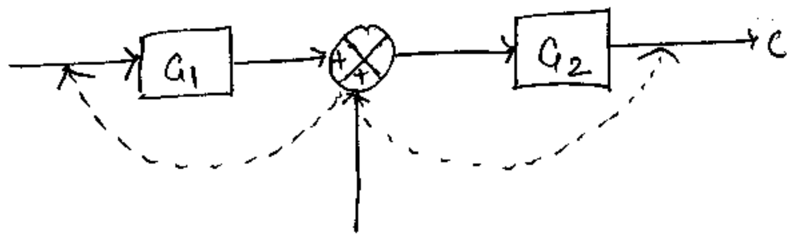
S_3 : closed loop

S_4 : interchanging of SP

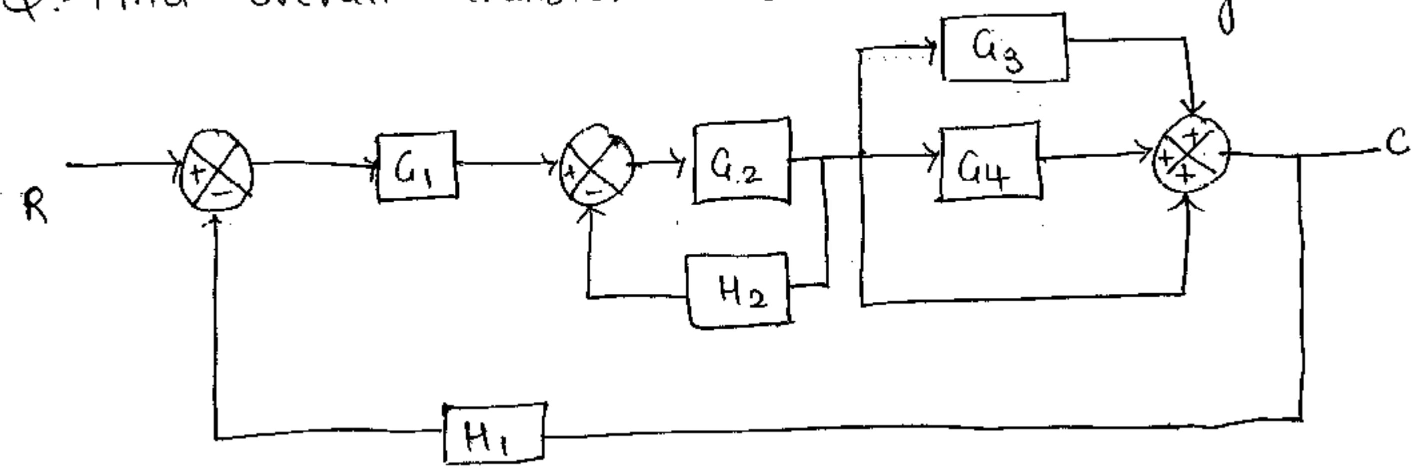
S_5 :



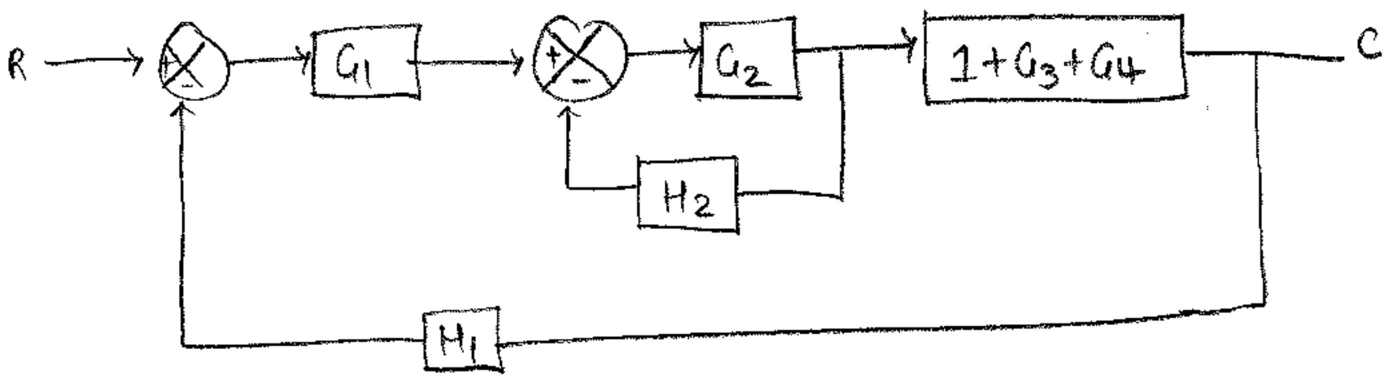
S_6 :



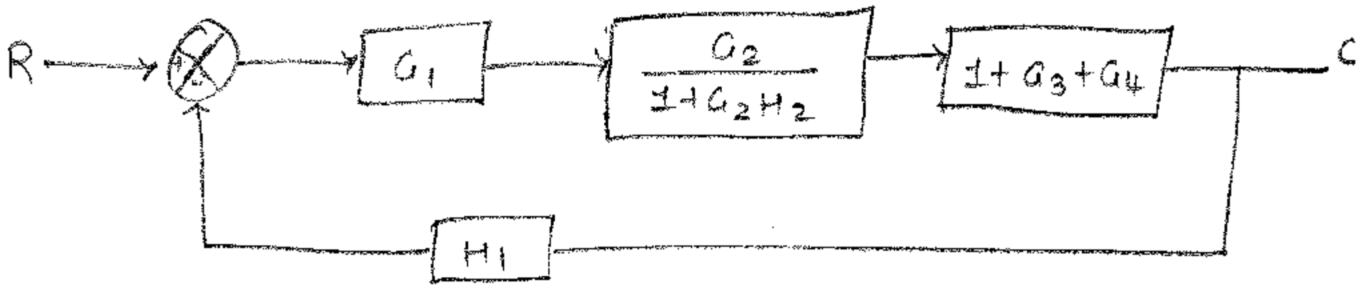
Q:- Find overall transfer function of block diagram:-

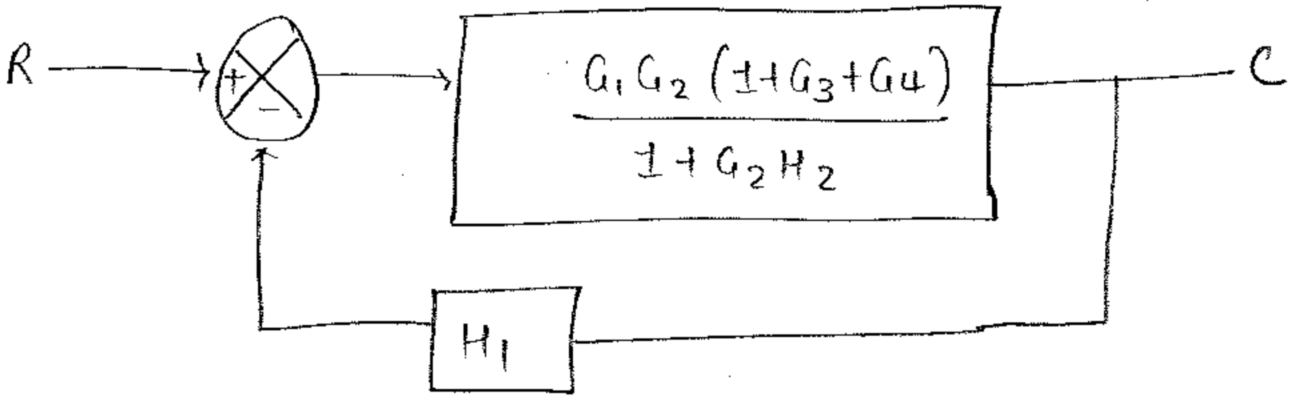


G_3, G_4 and 1 are in parallel



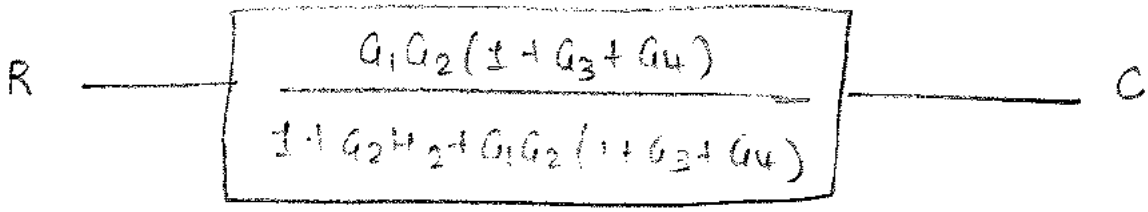
G_2 and H_2 are in feedback



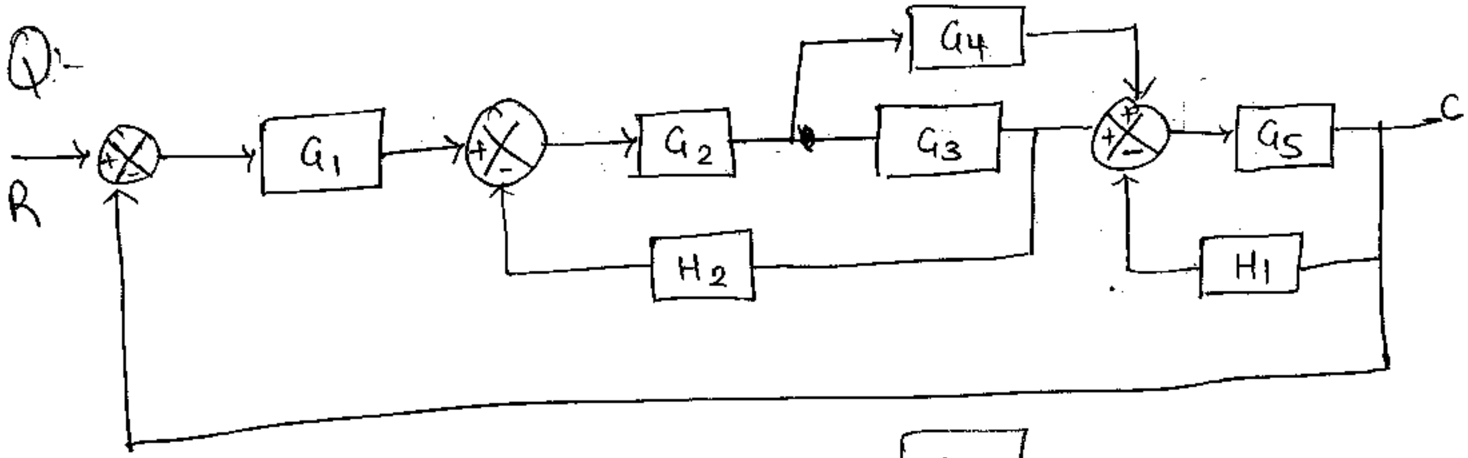


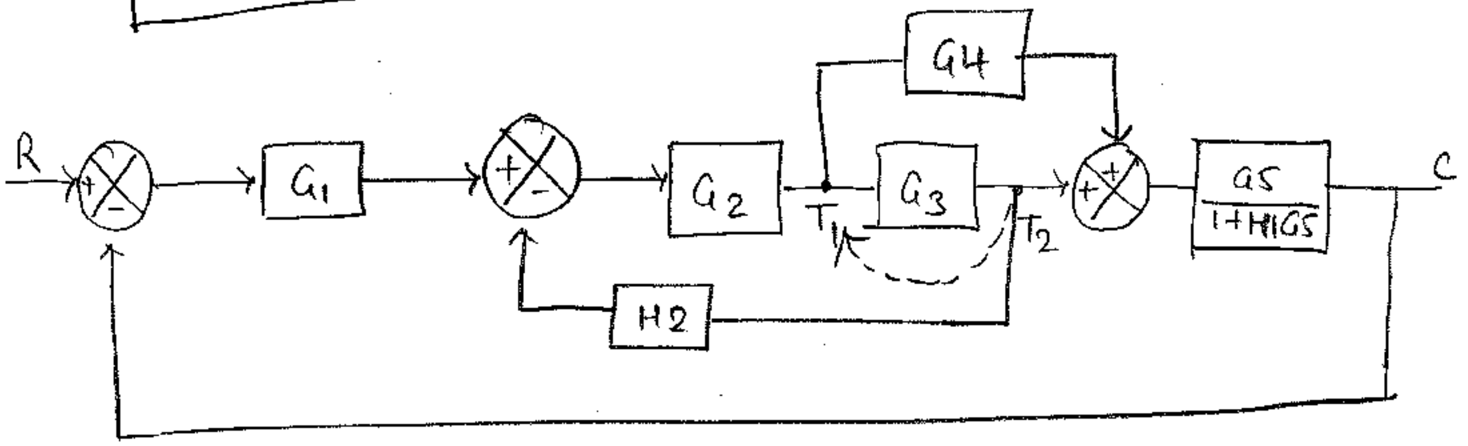
$$\frac{C}{R} = \frac{\dfrac{G_1G_2(1+G_3+G_4)}{1+G_2H_2}}{1 + \dfrac{G_1G_2(1+G_3+G_4)\cdot H_1}{1+G_2H_2}}$$

$$\frac{C}{R} = \frac{G_1G_2(1+G_3+G_4)}{1+G_2H_2+G_1G_2(1+G_3+G_4)}$$



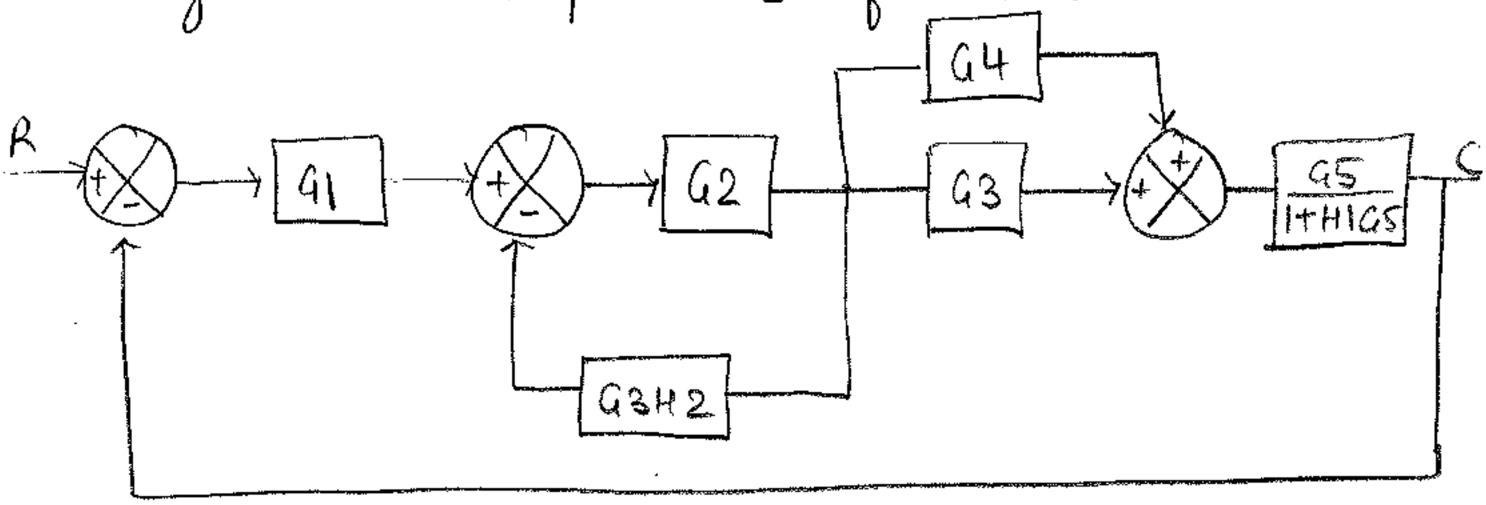
Q:-



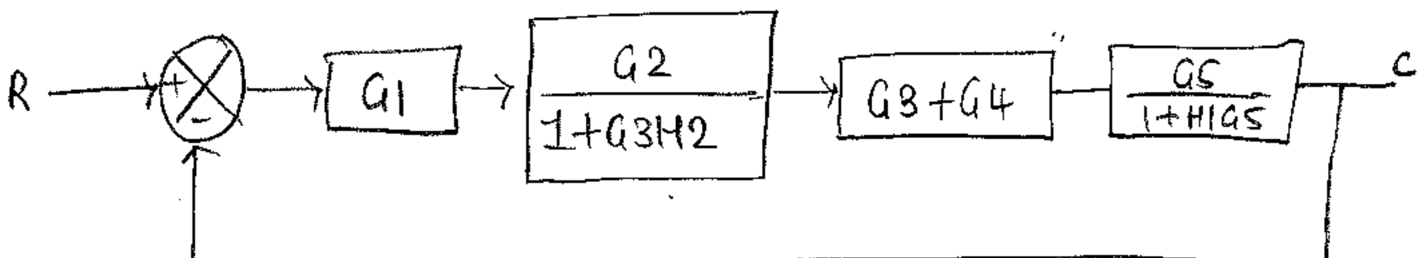


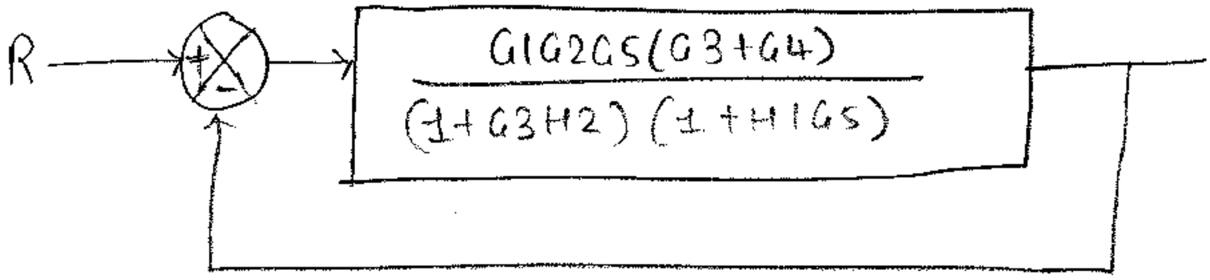
Method :- I

Shifting of take-off point T_2 before G_3

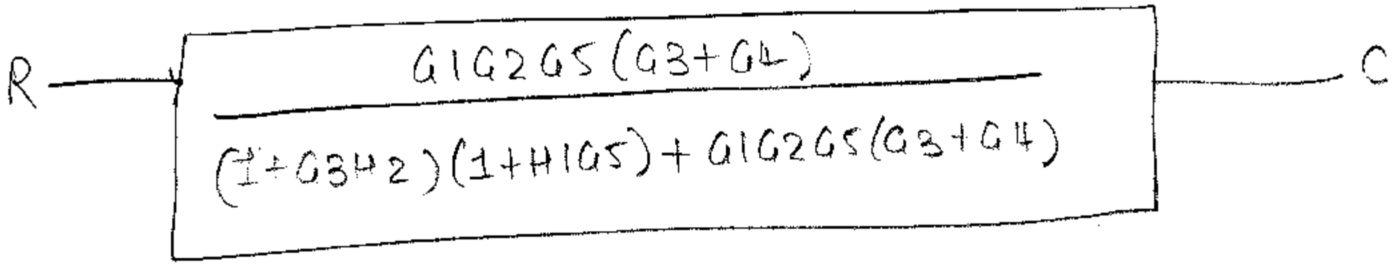


Now, G_3 & G_4 are in parallel, G_2 & G_3H_2 are in feedback form



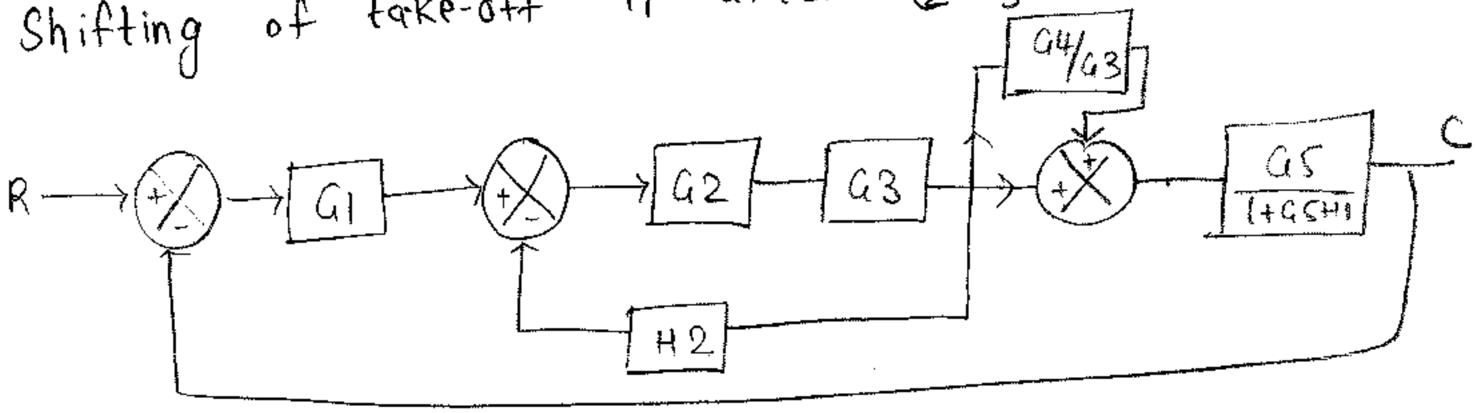


$$\frac{C}{R} = \frac{G1G2G5(G3+G4)}{(1+G3H2)(1+H_1G5)+G1G2G5(G_3+G4)}$$

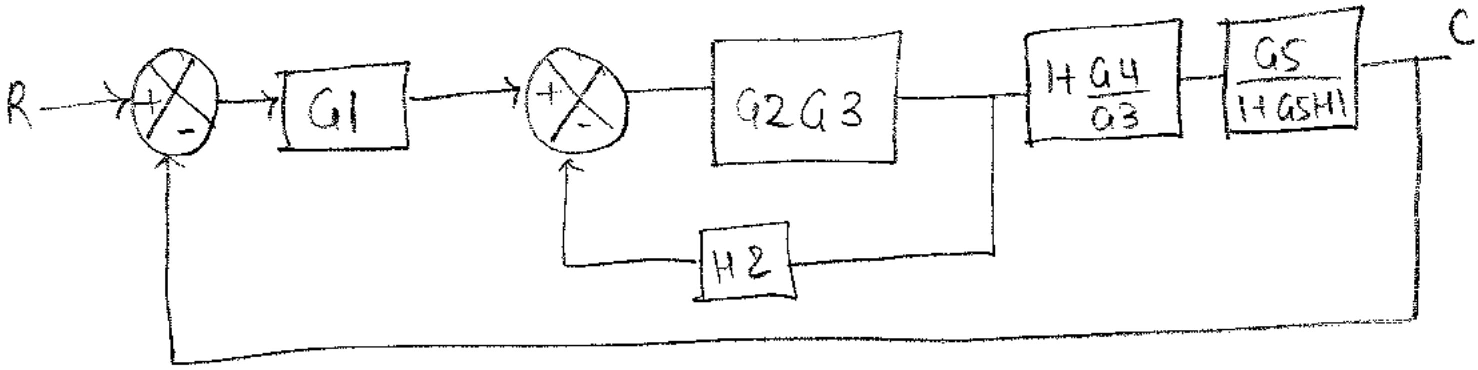


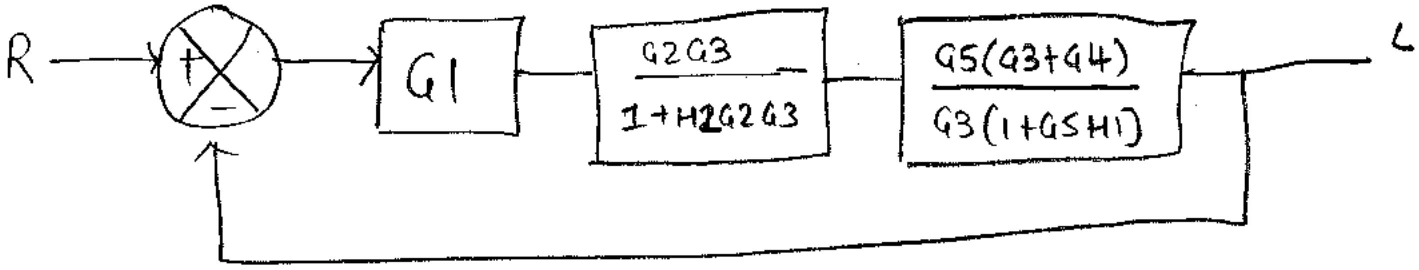
Method :- II

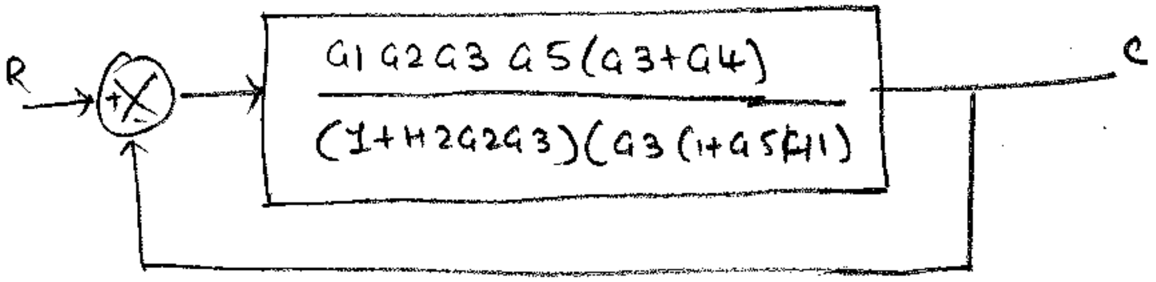
Shifting of take-off T_1 after ~~G2~~ G_3



Now, G2 & G3 are in series and whole with H2 are in feedback, Also $\frac{G4}{G3}$ & 1 are in parallel



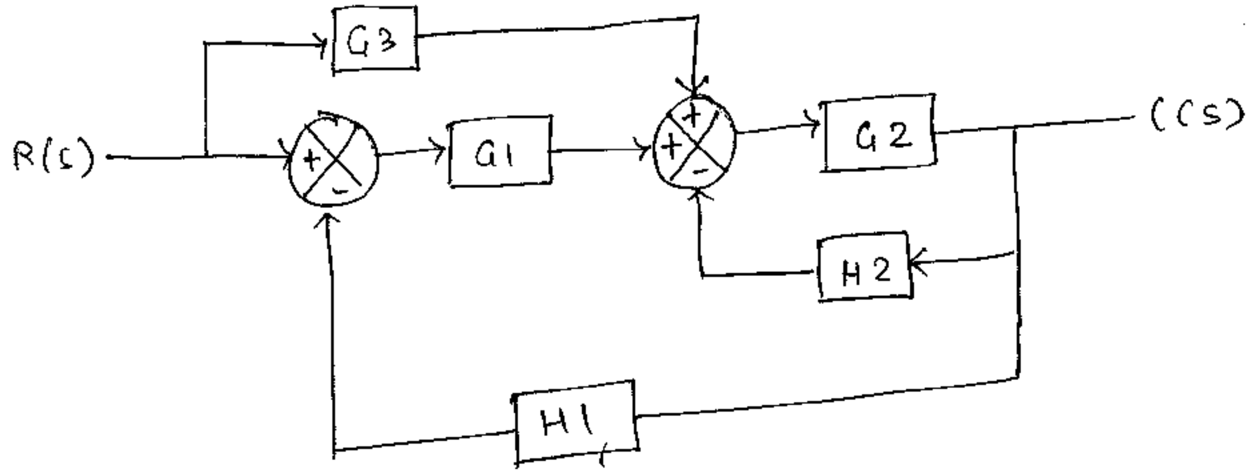




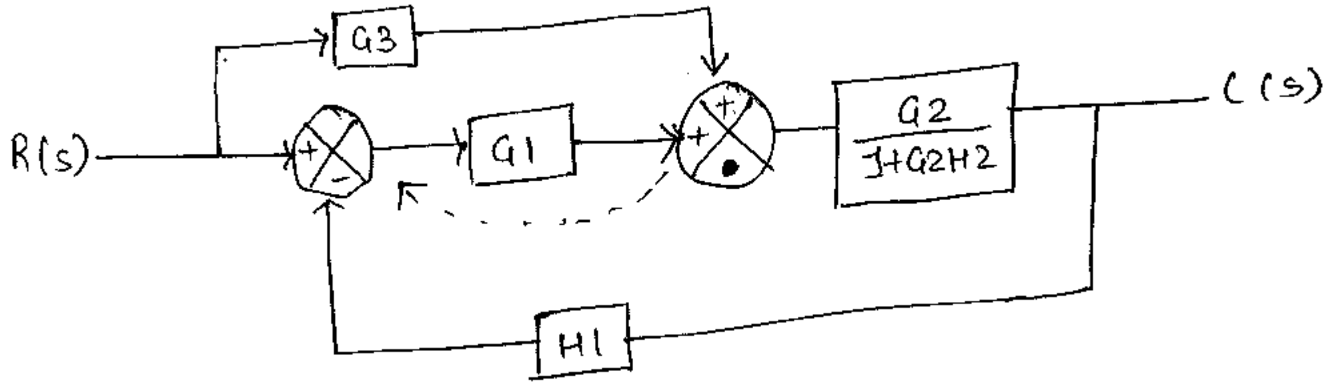
$$= \frac{\dfrac{G_1G_2G_3G_5(G_3+G_4)}{(G_3+G_5G_3H_1)(1+H_2G_2G_3)}}{1 + \dfrac{G_1G_2G_3G_5(G_3+G_4)}{(1+H_2G_2G_3)(G_3+G_3G_5H_1)}}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_5(G_3+G_4)}{G_3 + G_3G_5H_1 + H_2G_2G_3^2 + H_2G_2G_3^2G_5H_1 + G_1G_2G_3G_5(G_3+G_4)}$$

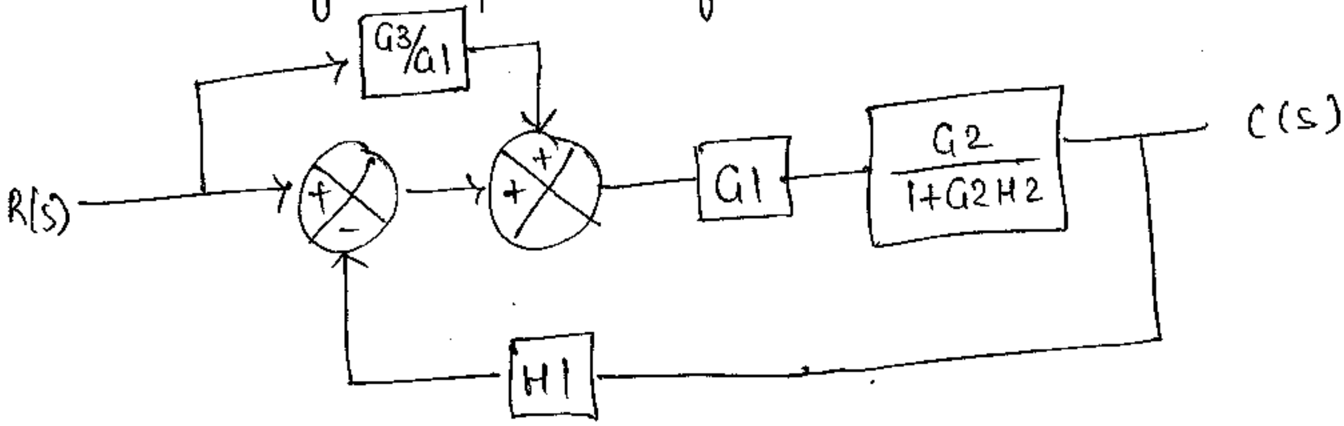
Q:- Find overall transfer function of block diagram



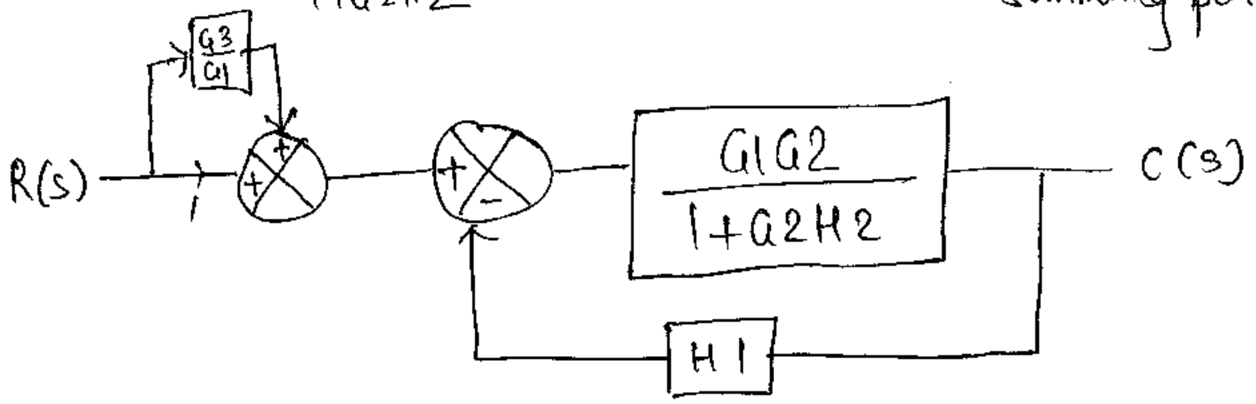
G2 and H_2 are in feedback form

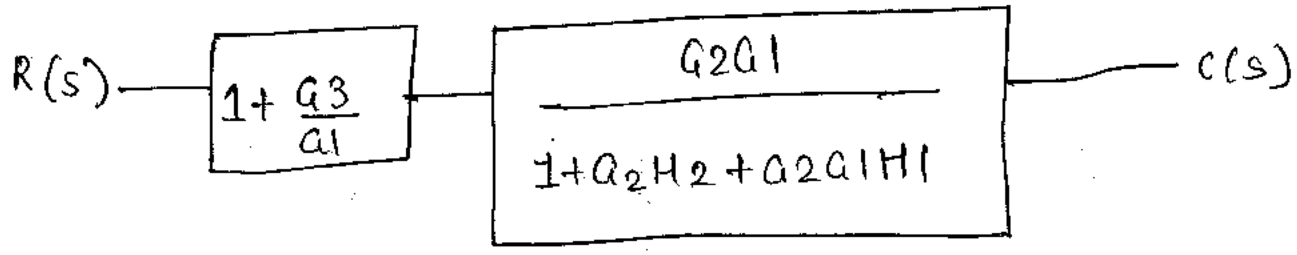


Now, shifting summing point before G1

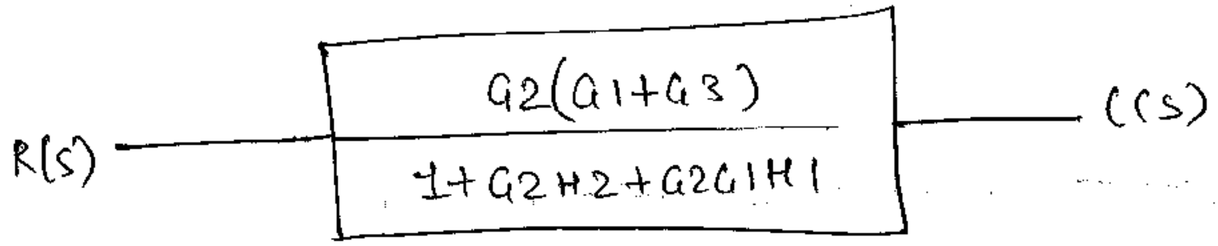


G1 and $\frac{G2}{1+G2H2}$ are in series & interchanging of summing point

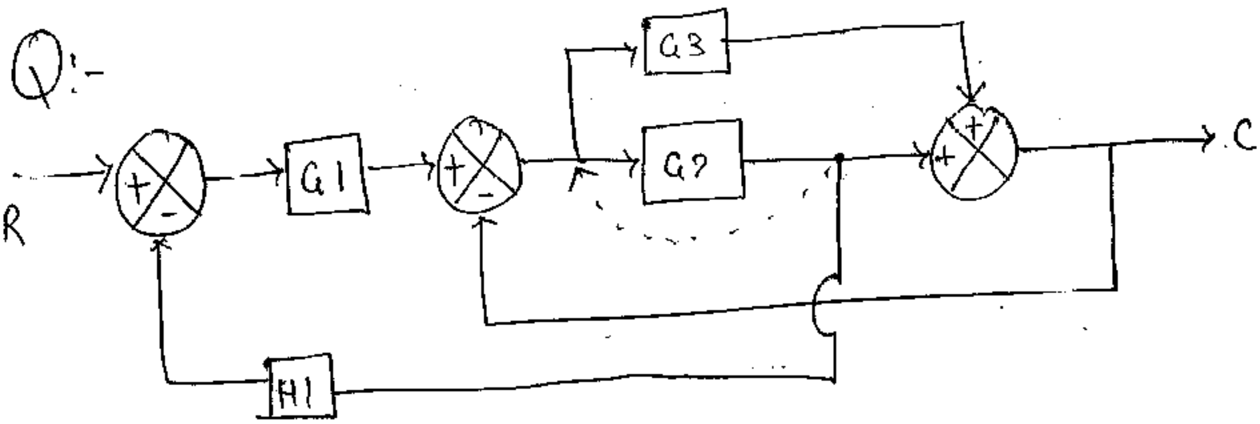




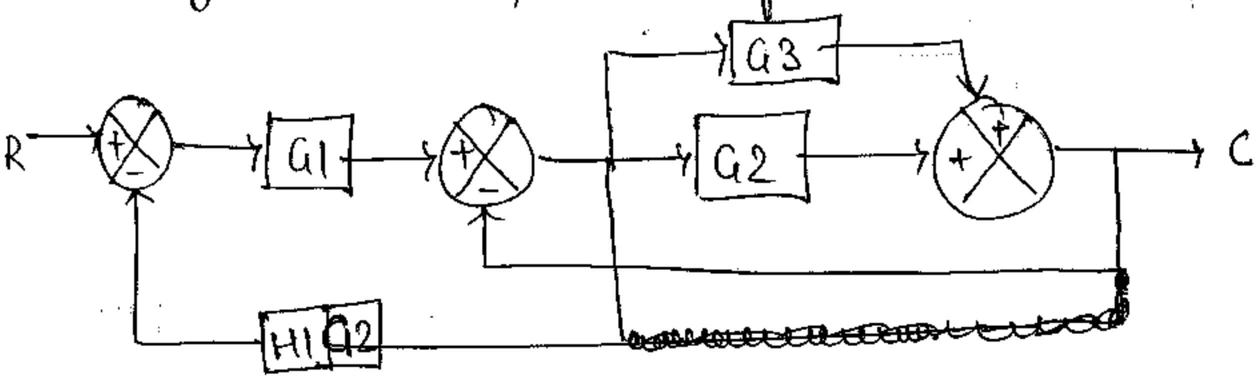
$$\frac{G1+G3}{\cancel{G1}} \cdot \frac{G2\cancel{G1}}{1+G2H2+G2G1H1}$$



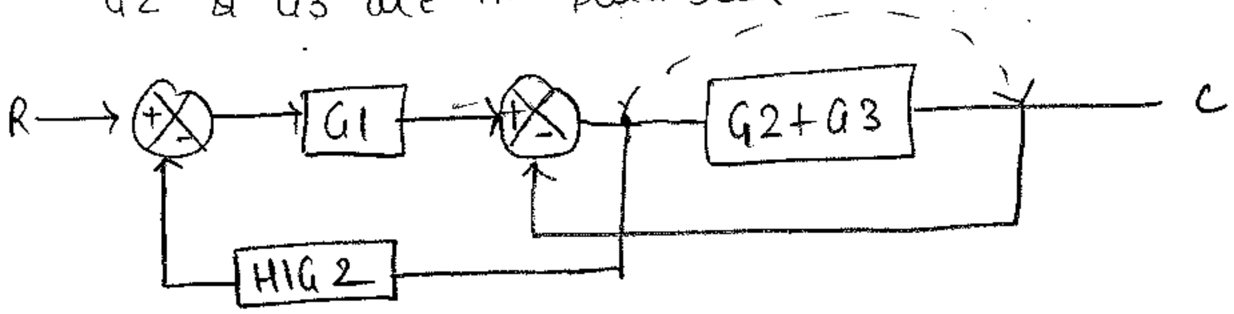
Q:-

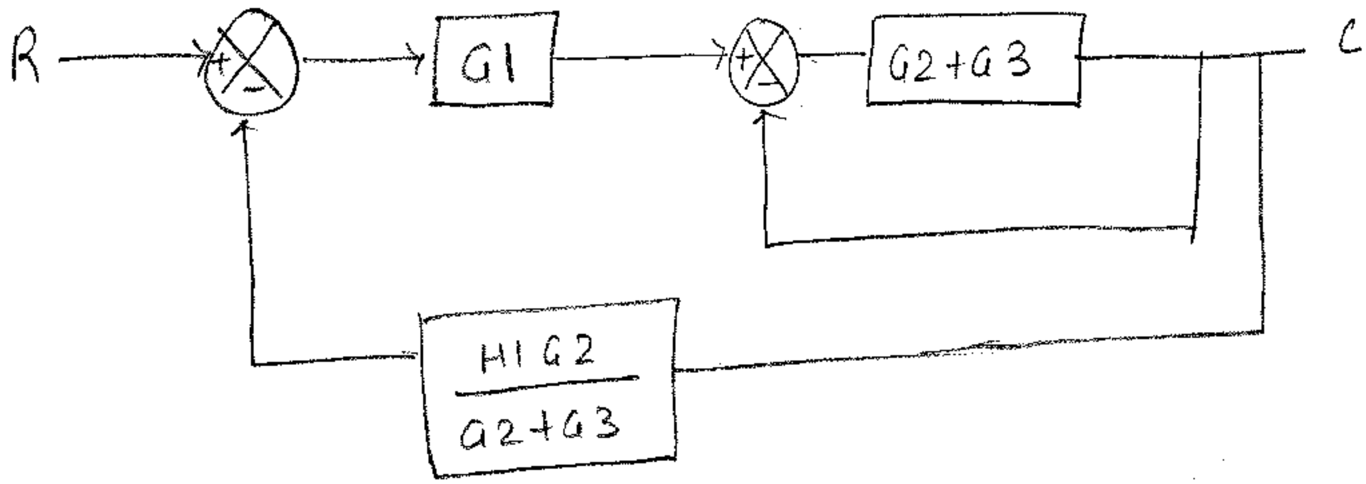


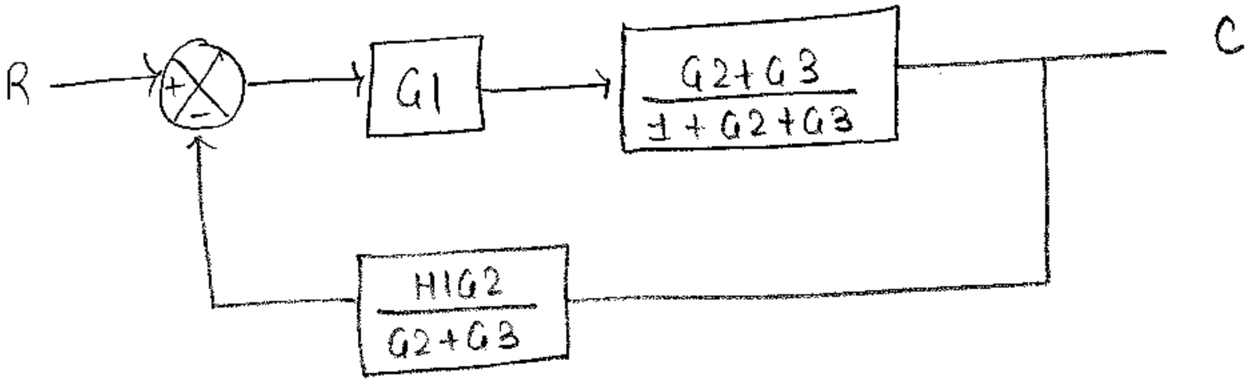
Taking take-off point before G2 block

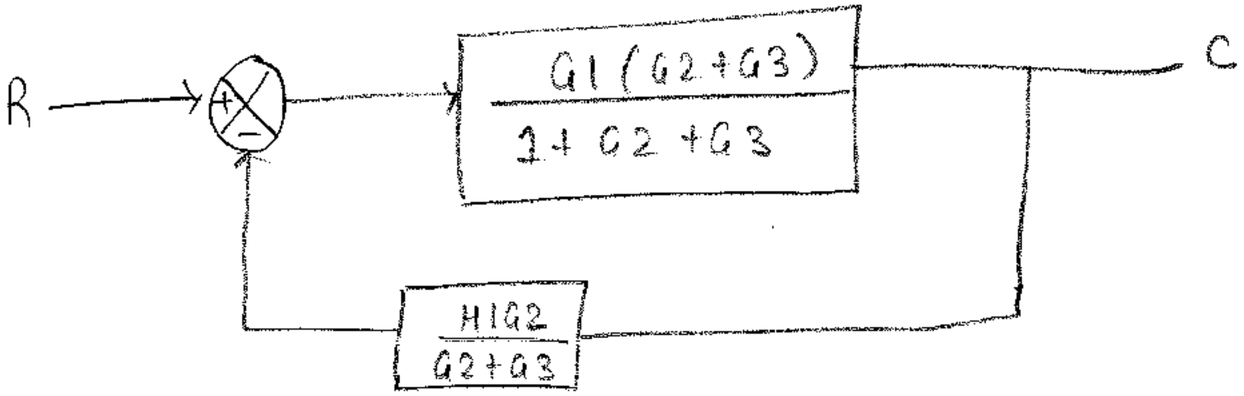


G2 & G3 are in parallel



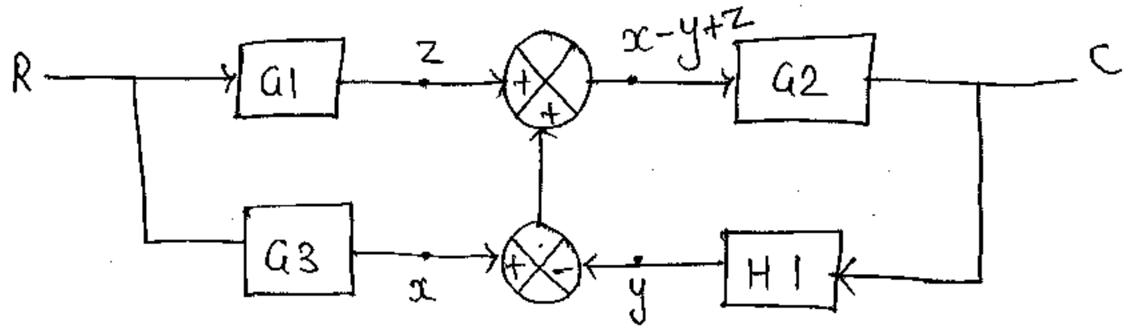


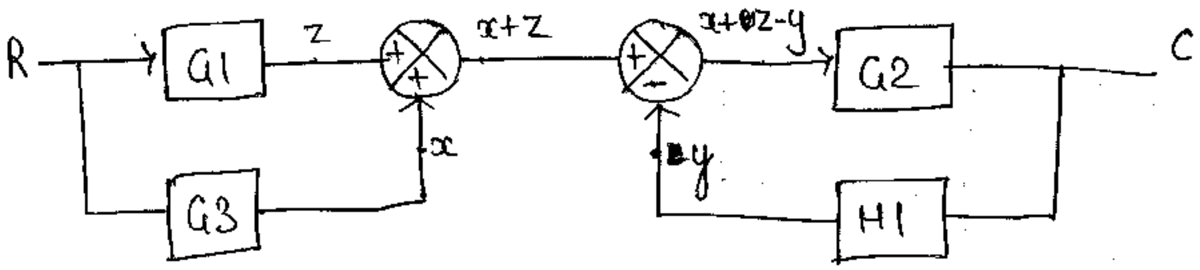




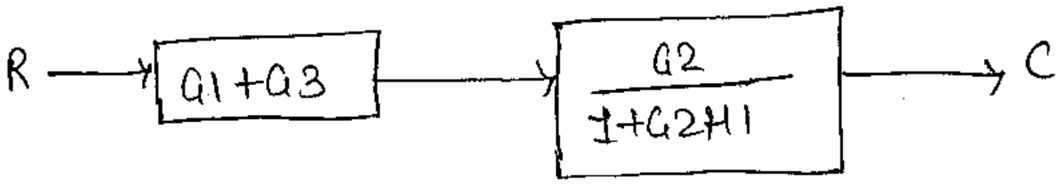
$$\frac{C}{R} = \frac{\dfrac{G1(G2+G3)}{1+G2+G3}}{1+\left(\dfrac{H1G2}{G2\cancel{+G3}}\right)\left(\dfrac{G1(G2\cancel{+G3})}{(1+G2+G3)}\right)} = \frac{G1(G2+G3)}{1+G2+G3+G1G2H1}$$

Q:-



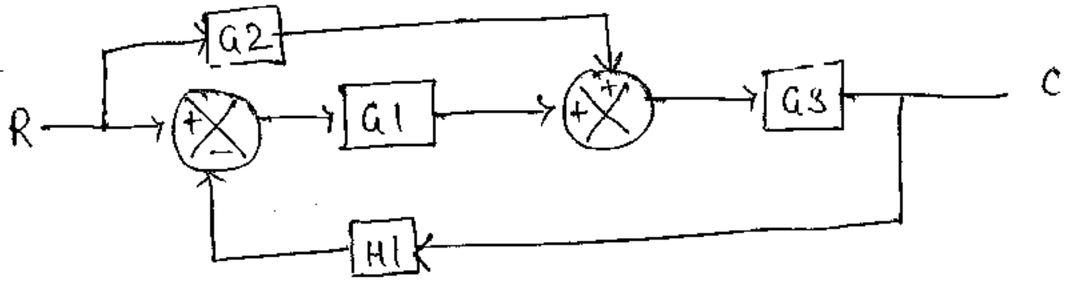


G1 & G3 are in parallel, G2 & G1 are in feedback form

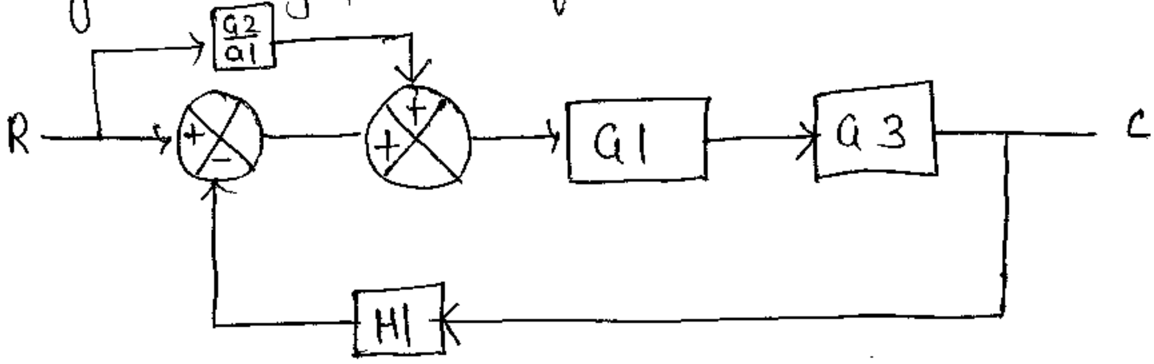


$$\frac{C}{R} = \frac{(G1+G3)G2}{1+G2H1}$$

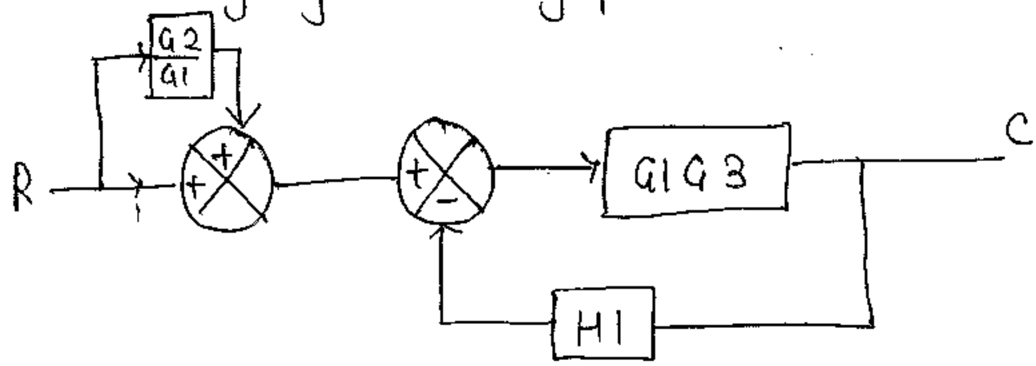
Q:-

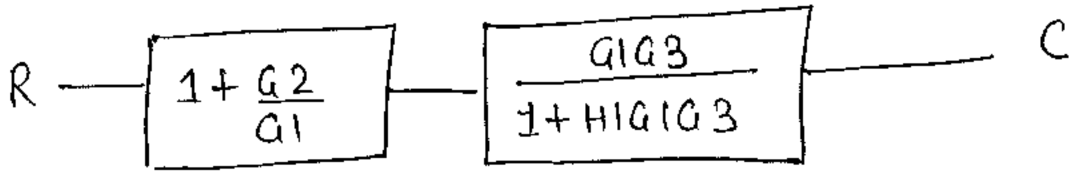


Taking somming point before G1

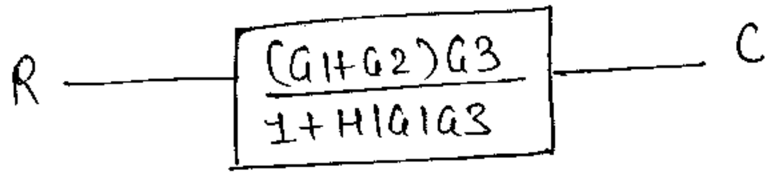


Interchanging summing point & G_1 & G_3 are in series

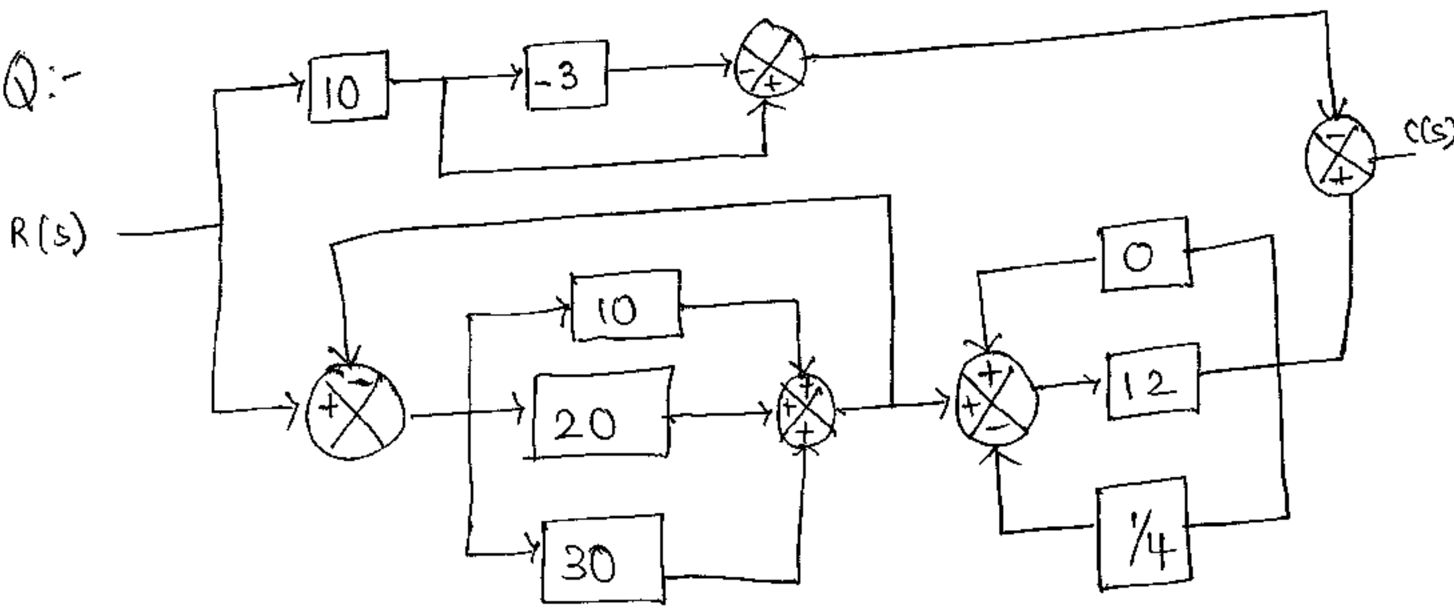


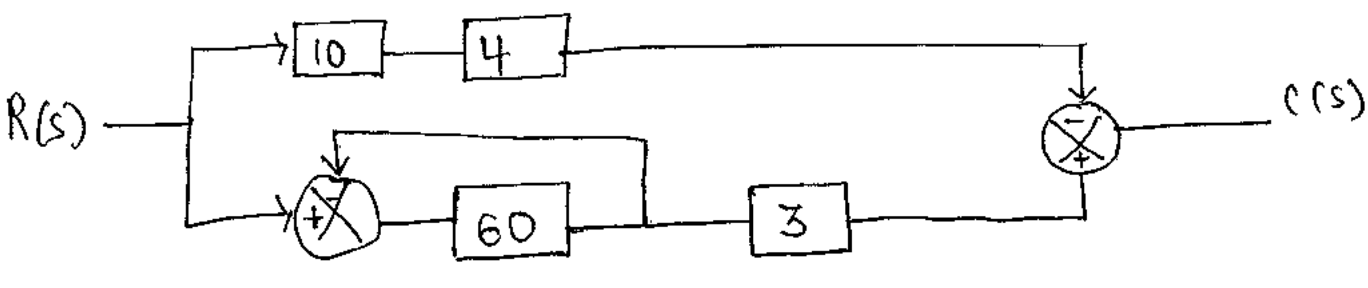


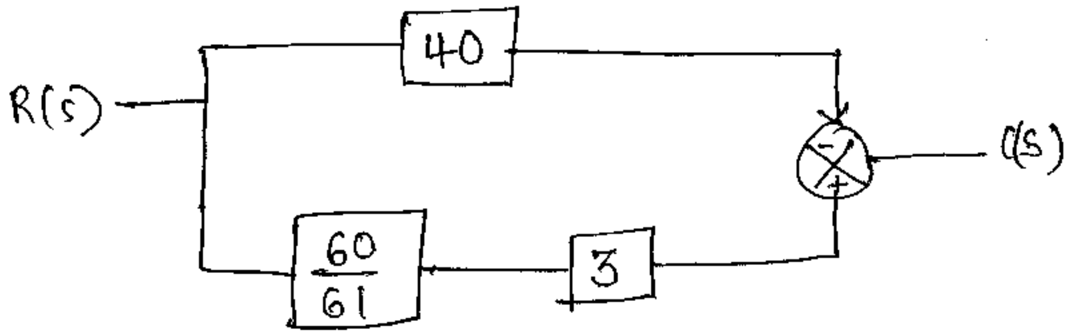
$$\frac{C}{R} = \frac{(G1+G2)G3}{1 + H1G1G3}$$

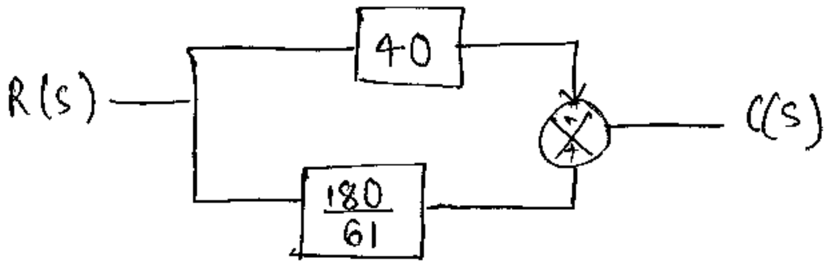


Q:-







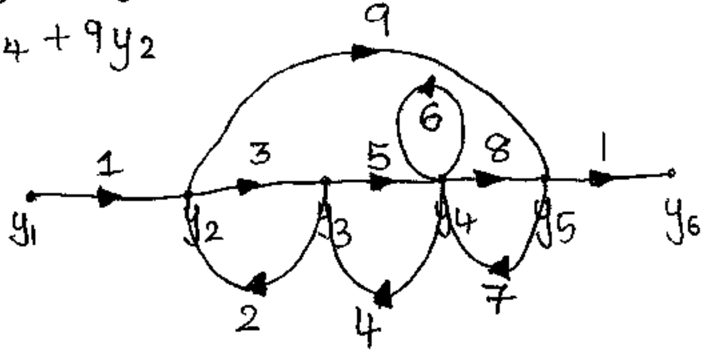


$$\frac{C(s)}{R(s)} = \frac{180}{61} - 40 = \frac{180 - 2440}{61} = \frac{-2260}{61}$$

Q:- construct

$$y_2 = y_1 + 2y_3$$
$$y_3 = 3y_2 + 4y_4$$
$$y_4 = 5y_3 + 6y_4 + 7y_5$$
$$y_5 = 8y_4 + 9y_2$$

Sol:-



Input node:- The node which has only ~~output~~ outgoing branches.

Output node: The node which has only incoming branch.

Chain Node/Link node : The node which has both incoming and outgoing branches is called chain node

In above SFG,

y_1 = input node

y_2, y_3, y_4, y_5 = chain node

y_6 dummy node

Forward path: It is the path to input to output

Loop: It is the path which terminates where it has started.

(1) No. of forward path:

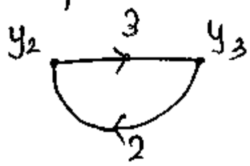
$P_1 = 1 \cdot 3 \cdot 5 \cdot 8 \cdot 1 = 120$

$P_2 = 1 \cdot 9 \cdot 1 = 9$

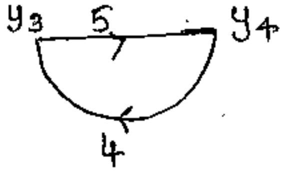
(2) No. of individual loops:

Node

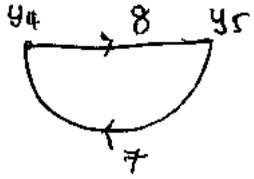
1) $L_1 = 3 \cdot 2$ (y_2, y_3)



2) $L_2 = 5 \cdot 4$ (y_3, y_4)

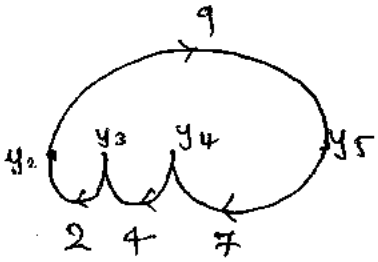


3) $L_3 = 8 \cdot 7$ (y_4, y_5)



4) $L_4 = 6$ (y_4)

5) $L_5 = 9 \cdot 7 \cdot 4 \cdot 2$ (y_2, y_3, y_4, y_5)



(3) No. of two non-touching loops:-

If there is no common node between two individual ~~nod~~ loop is called non-touching loops.

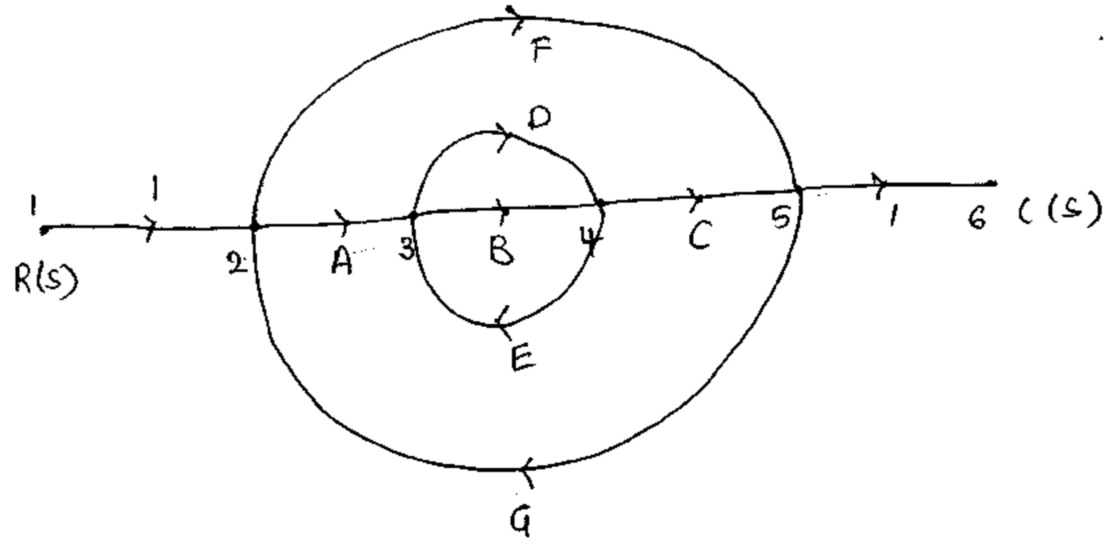
1) $L_1 L_3$

2) $L_1 L_4$

(4) No. of three non-touching loops:

There is no three non-touching loops.

Q:- For the given SFG find no. of forward path, individual loops, two non-touching loops & three non-touching loops



No. of forward path $= 1 \cdot A \cdot B \cdot C \cdot 1 = P_1$

$= 1 \cdot F \cdot 1 = P_2$

$= 1 \cdot A \cdot D \cdot C \cdot 1 = P_3$

Individual loops:- $L_1 = F \cdot G$

$L_2 = D \cdot E$

$L_3 = B \cdot E$

$L_4 = A \cdot B \cdot C \cdot G$

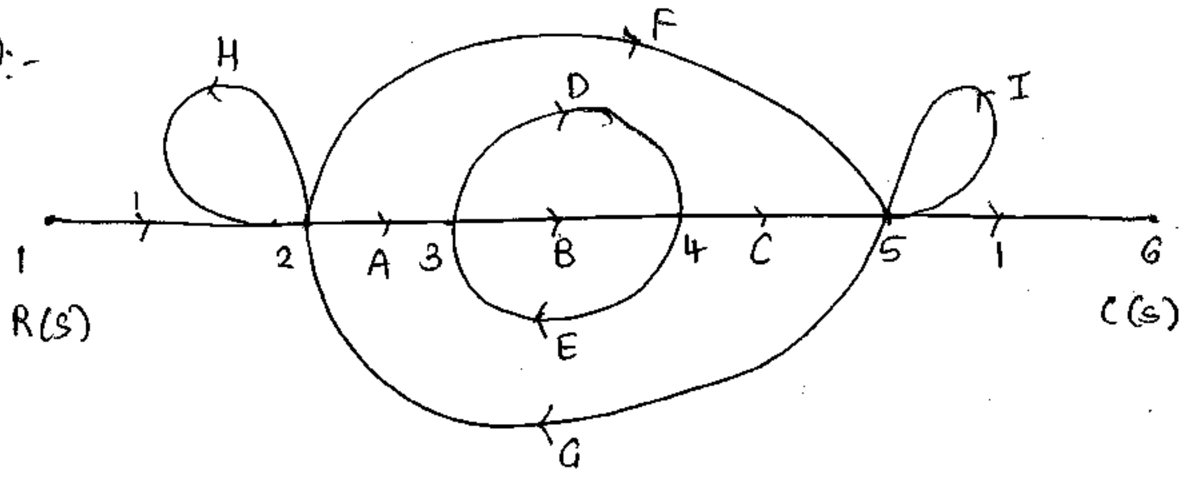
$L_5 = ADCG$

Two non-touching loops: $L_1 \cdot L_2 = FG \cdot DE$

$L_1 L_3 = FG \cdot BE$

Three non-touching loops: zero.

Q:-



(1) No. of forward path:

$P_1 = 1 \cdot A \cdot B \cdot C \cdot 1 = ABC$

$P_2 = 1 \cdot F \cdot 1 = F$

$P_3 = 1 \cdot ADC \cdot 1 = ADC$

(2) No. of loops:

$L_1 = BE$

$L_2 = DE$

$L_3 = ABCG$

$L_4 = FG$

$L_5 = ADCG$

$L_6 = H$

$L_7 = I$

(3) Two non-touching loops

$L_1 L_4 = BE \cdot FG$

$L6L7 = HI$

$L2L4 = DEFG$

$L6L1 = HBE$

$L6L2 = HDE$

$L7L1 = IBE$

$L7L2 = IDE$

Three non-touching loops:

L6L7L1 = HIBE

L6L7L2 = HIDE

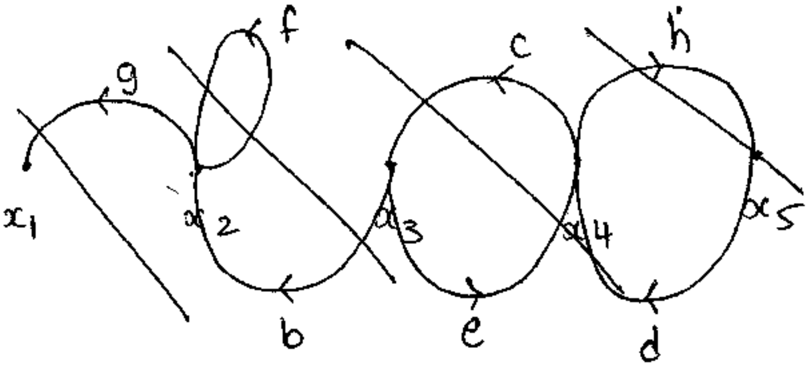
Q:- Represent by SFG and find forward path, two non-touching loops and three non-touching loops.

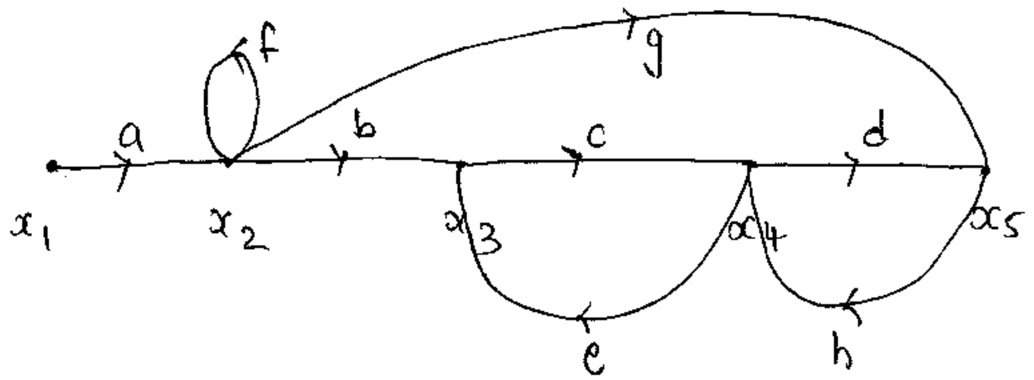
$x_2 = a x_1 + f x_2$

$x_3 = b x_2 + e x_4$

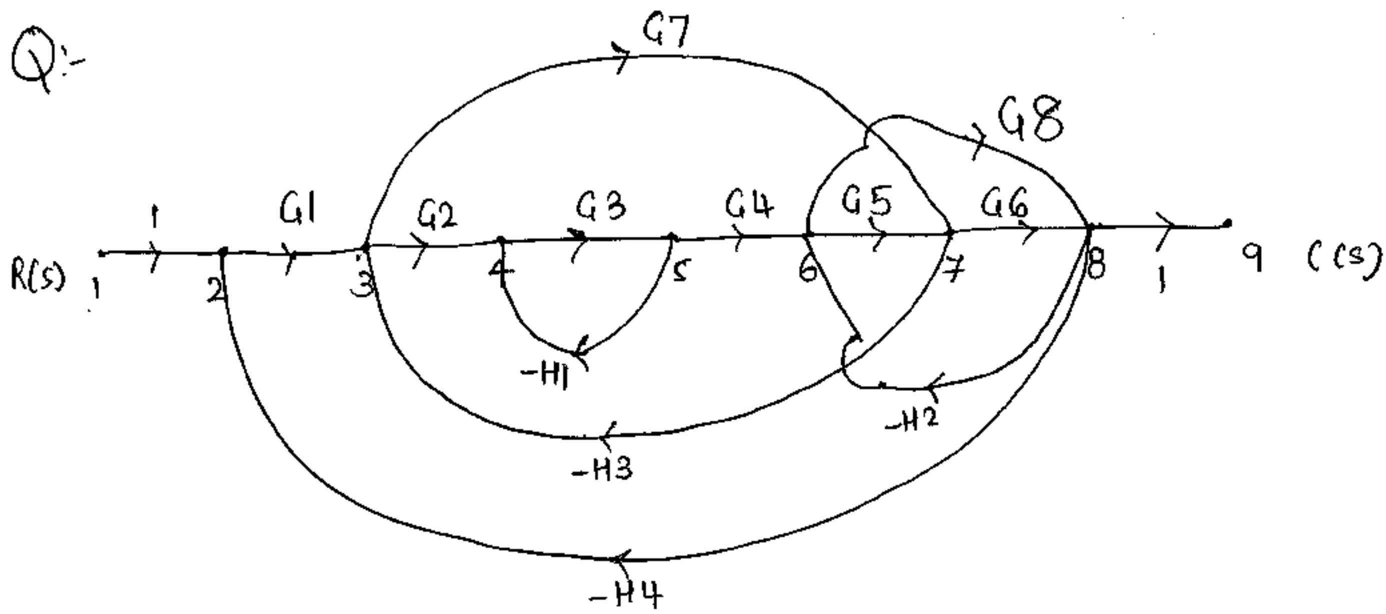
$x_4 = c x_3 + h x_5$

$x_5 = d x_4 + g x_2$





Q:-



NOTE:

If there is one feedback path between two nodes then no. of individual loops is equal to no. of forward path between that two nodes.

Sol:- Forward path:-

$P1 = G1G2G3G4G5G6$

$P2 = G1\,G7G6$

$P3 = G1G2\,G3G4G8$

No. of single loops:

$L1 = -H3G7$

$L2 = -G1G3G4G5H3$

$L3 = -H1G3$

$L4 = -H2G6G5$

$L5 = -H2G8$

$L6 = -H4G1G2G3G4G5G6$

$L7 = G1G2G3G4G8(-H4)$

$L8 = G1G7G6(-H4)$

No. of two non-touching loops:

$L3L1 = G3G7H1H3$ $\quad$ $L1L5 = G7H3G8H2$

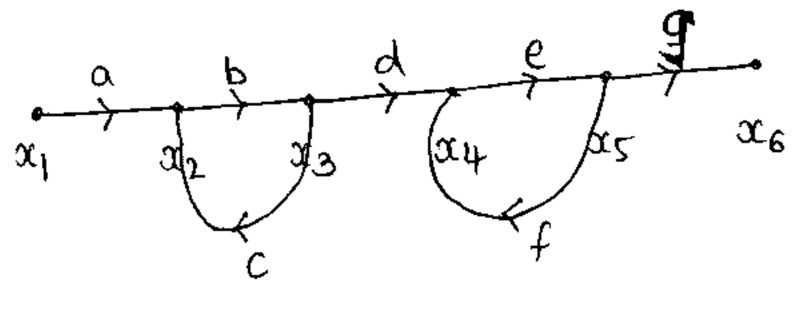
$L3L5 = G3H1G8H2$ $\quad$ $L1L4 = G3G7G5G6H2$

$L3L4 = G3H1G5G6H2$

No. of three non-touching

$L3L1L5 = H1G3G7H3G5G6(-H2)$

Q:- The SFG shown in figure has one forward path, two isolated loop determine overall transmitance OR overall gain relating x_1 and x_6



$x_2 = ax_1 + cx_2$

$x_3 = bx_2$

$x_4 = dx_3 + fx_5$

$x_5 = ex_4$

$x_6 = gx_5$

We have to find $\frac{x_6}{x_1} = ?$

Now,
$$x_3 = b(ax_1 + cx_3)$$

$$x_3(1-cb) = bax_1$$

$$\rightarrow x_3 = \frac{ab}{1-cb} \cdot x_1$$

$$\rightarrow x_4 = \frac{abd}{1-cb} x_1 + fx_5$$

$$\rightarrow x_4 = \frac{abd}{1-cb} x_1 + fex_4$$

$$x_4(1-fe) = \frac{abd}{1-cb} x_1$$

$$x_4 = \frac{abd}{(1-cb)(1-fe)} \cdot x_1$$

$$x_6 = gex_4$$

$$x_6 = \frac{abdeg}{(1-cb)(1-fe)} \cdot x_1$$

$$\frac{x_6}{x_1} = \frac{abdeg}{1-cb-fe+cbfe}$$

$$\frac{x_6}{x_1} = \frac{abdeg}{1-\underbrace{(cb+fe)}_{\text{loops}}+\underbrace{cbfe}_{\text{two-nontouching loop}}}$$

Forward path

$$\frac{x_6}{x_1} = \frac{P}{\Delta}$$

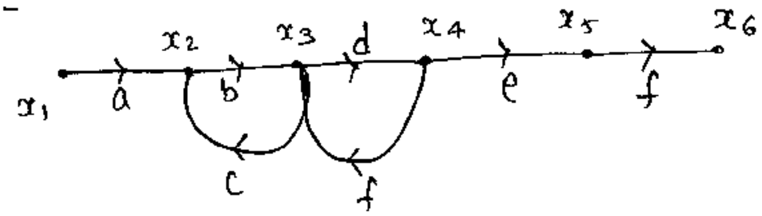
$$\Delta = 1-(cb+fe)+cb \cdot fe = 1-(L_1+L_2)+L_1L_2$$

$\Delta = 1 - $ [Sum of individual loop] + [Sum of gain product of two non-touching loop]

- [Sum of gain product of 3 non-touching loop]

$+ \ldots\ldots$

Q:-



$$\frac{x_6}{x_1} = ?$$

$x_2 = ax_1 + cx_3$

$x_3 = bx_2 + fx_4$

$x_4 = dx_3$

$x_5 = x_4 e$

$x_6 = fx_5$

$\rightarrow x_4 = d[bx_2 + fx_4]$

$x_4(1-df) = dbx_2 \quad \therefore x_4 = \frac{dbx_2}{1-df}$

$$x_5 = \frac{dbx_2}{1-df} \cdot e$$

$\cancel{x_4(1-df) = db[ax_1 + cx_3]}$

$\cancel{= db[ax_1 + c[bx_2 + fx_4]]}$

$\cancel{x_4(1-df) = db[ax_1 + cbx_2}$

$$x_6 = \frac{fedbx_2}{1-df}$$

$$x_6 = \frac{fedb}{1-df} \cdot (ax_1 + cx_3)$$

$$x_6 = \frac{fedb}{1-df}\left[ax_1 + \frac{cx_4}{d}\right]$$

$$= \frac{fedb}{1-df}\left[ax_1 + \frac{cbx_2}{d(1-df)}\right]$$

$$x_2 = ax_1 + cx_3$$

$$x_3 = bx_2 + fx_4$$

$$x_4 = dx_3$$

$$x_5 = ex_4$$

$$x_6 = gx_5$$

$$x_3 = b(ax_1 + cx_3) + fdx_3$$

$$x_3 = bax_1 + bcx_3 + fdx_3$$

$$x_3(1 - bc - fd) = bax_1$$

$$\frac{x_3}{x_1} = \frac{ab}{1 - bc - fd}$$

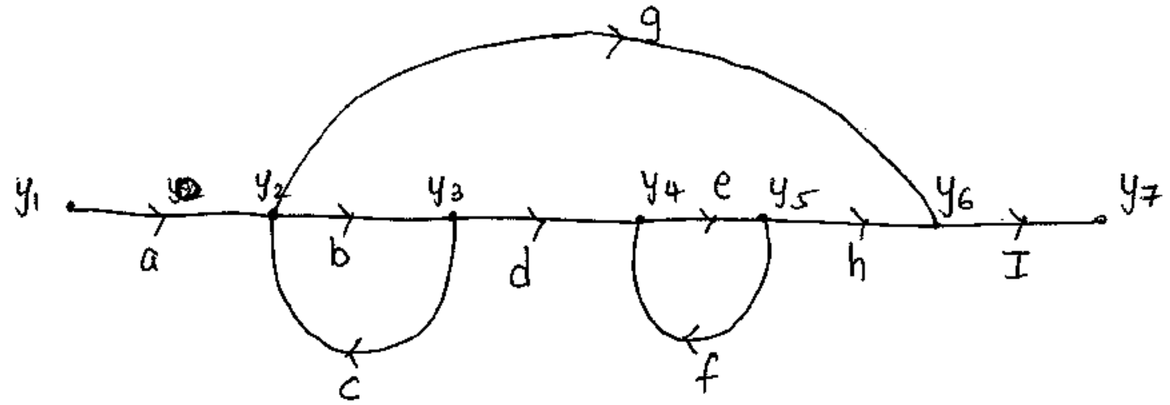
$$x_4 = \frac{abdx_1}{1 - bc - fd}$$

$$x_5 = \frac{abde\,x_1}{1 - bc - fd}$$

$$x_6 = \frac{abdegx_1}{1 - bc - fd}$$

$$\frac{x_6}{x_1} = \frac{abdeg}{1 - bc - fd}$$

Q:-



$$\frac{y_7}{y_1} = ?$$

$$y_2 = ay_1 + cy_3$$

$$y_3 = by_2$$

$$y_4 = dy_3 + fy_5$$

$$y_5 = ey_4$$

$$y_6 = hy_5 + gy_2$$

$$y_7 = y_6 i$$

$$\Rightarrow \quad y_7 = [hy_5 + gy_2]\, i \quad \text{——} \ (1)$$

$$y_3 = b\,[ay_1 + cy_3]$$

$$y_3(1-bc) = aby_1$$

$$y_3 = \frac{aby_1}{1-bc}$$

$$y_4 = \frac{abd}{1-bc} y_1 + fy_5$$

$$y_4 = \frac{abd}{1-bc} \cdot y_1 + fey_4$$

$$y_4[1-fe] = \frac{abd}{1-bc} \cdot y_1$$

$$y_4 = \frac{abd}{(1-bc)(1-fe)} \cdot y_1$$

$$y_5 = \frac{abde}{(1-bc)(1-fe)} \cdot y_1$$

Now,

$$y_2 = ay_1 + \frac{cab}{(1-cb)} y_1$$

$$y_2 = \left(a + \frac{abc}{1-cb}\right) y_1$$

$$= \left(\frac{a(1-cb) + abc}{1-cb}\right) y_1$$

$$= \left(\frac{a - \cancel{abc} + \cancel{abc}}{1-bc}\right) y_1$$

$$y_2 = \frac{a}{1-bc} y_1$$

$$y_7 = \left[\frac{abcde\,hi}{(1-bc)(1-fe)} + \frac{ag \cdot i}{(1-bc)}\right] y_1$$

$$\boxed{\frac{y_7}{y_1} = \frac{abcdehi(1) + agi(1-fe)}{1-(bc+fe)+bcfe}}$$

$$\frac{y_7}{y_1} = \frac{\overset{P_1}{abdehi}\,\overset{\Delta_1}{(1)} + \overset{P_2}{agi}\,\overset{\Delta_2}{(1-fe)}}{1-\underset{L_1+L_2}{(cb+fe)}+\underset{L_1L_2}{cbfe}}$$

* Mason's Gain Formula

$$\text{Overall T.F.} = \frac{\sum_{K=1}^{1} P_K \cdot \Delta_K}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2 + \cdots\cdots}{\Delta}$$

P_K = no. of forward path gain

Δ_K = It is obtained from Δ which can be obtained by removing all loops which are connected to that forward path

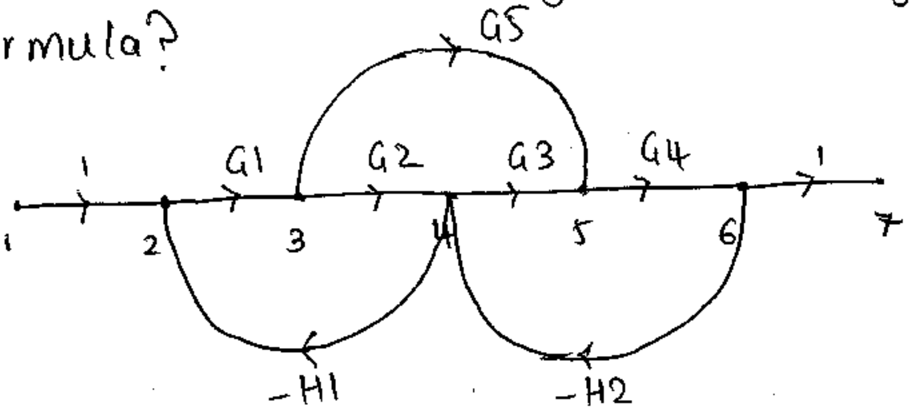
Means here for above example:-

$$\Delta_1 = 1-(0+0)+0 = 1$$

$$\Delta_2 = 1-(fe+0)+0 = 1-fe$$

$$\Delta = 1-\left[\sum \text{loop}\right] + \left[\sum \text{two non-touching loop}\right] - \left[\sum \text{3 non-touching loops}\right]$$

Q:- Find overall T.F. using SFG's using Mason's Gain Formula?



(1) No. of forward path

$P1 = G1G2G3G4$, $\Delta_1 = 1$

$P2 = G1G5G4$, $\Delta_2 = 1$

(2) No. of loops

$L1 = -G1G2H1$

$L2 = -G3G4H2$

$L3 = G1G5G4H1H2$

(3) Two non-touching loops

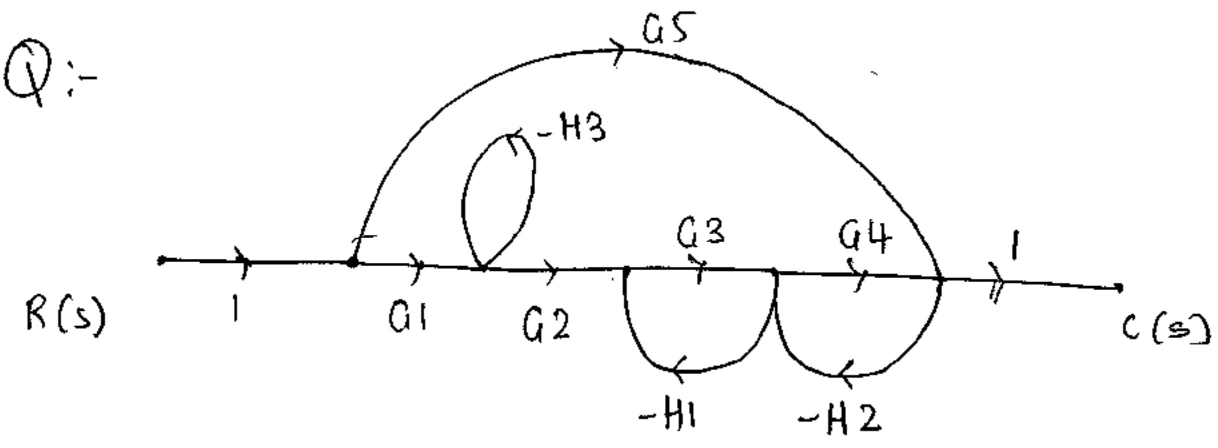
Zero

$$\Delta = 1 + G1G2H1 + G3G4H2 - G1G5G4H1H2$$

$$T.F. = \frac{G1G2G3G4(1) + G1G5G4(1)}{\Delta}$$

$$T.F. = \frac{G1G2G3G4 + G1G5G4}{1 + G1G2H1 + G3G4H2 - G1G5G4H1H2}$$

Q:-



(1) No. of forward path

$P1 = G1G2G3G4 \quad , \Delta_1 = 1$

$P2 = G5 \quad , \Delta_2 = 1 + H1G3 + H3 + G3H1H3$

(2) No. of single loops

$L1 = -H3$

$L2 = -H1G3$

$L3 = -H2G4$

(3) Two non-touching loops

$L1L2 = H1H3G3$

$L1L3 = H2H3G4$

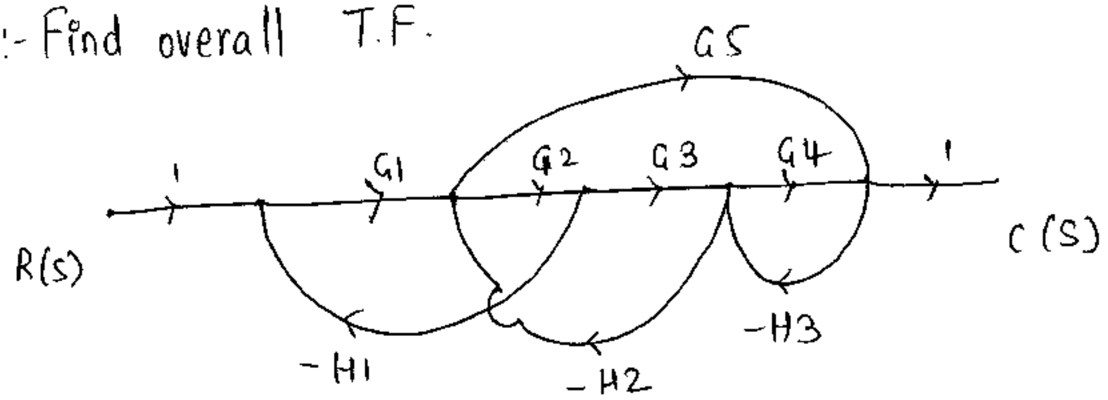
$\Delta = 1 + H3 + H1G3 + H2G4 + H1H3G3 + H2G3G4$

From Mason's gain formula,

$$T.F. = \frac{P1\Delta1 + P2\Delta2}{\Delta}$$

$$\frac{C(s)}{R(s)} = T.F. = \frac{G1G2G3G4 + G5(1 + H1G3 + H3 + G3H1H3)}{1 + H3 + H1G3 + H2G4 + H1H3G3 + H2G3G4}$$

Q:- Find overall T.F.



$P1 = G1G2G3G4 \quad , \Delta_1 = 1$

$P2 = G1G5 \quad , \Delta_2 = 1$

$\Delta = 1 + G1G2H1 + G2G3H2 + G4H3 + G1G2H1G4H3 - G5H2H3$

single loops:

$L1 = -G1G2H1$

$L2 = -G2G3H2$

$L3 = -G4H3$

$L4 = G5H2H3$

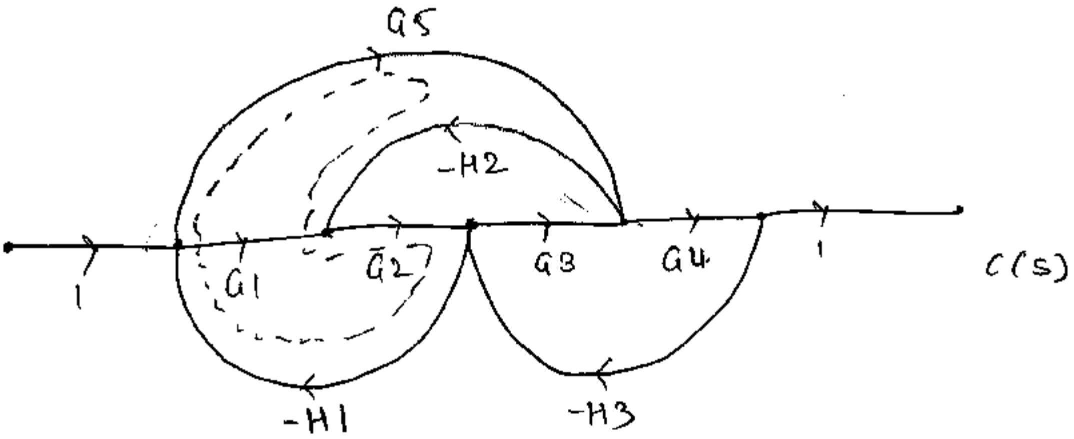
two non-touching loops

$L1L3 = G1G2H1G4H3$

$$T.F. = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G1G2G3G4 + G1G5}{1 + G1G2H1 + G2G3H2 + G4H3 - G5H2H3 + G1H1G2G4H3}$$

Q:-



No. of forward path

$P_1 = G1G2G3G4 \quad , \Delta_1 = 1$

$P2 = G5G4 \quad , \Delta_2 = 1.$

No. of individual loops

$L1 = -G1G2H1$

$L5 = G5H2G2H1$

$L2 = -G3G4H3$

$L3 = G5G4H1H3$

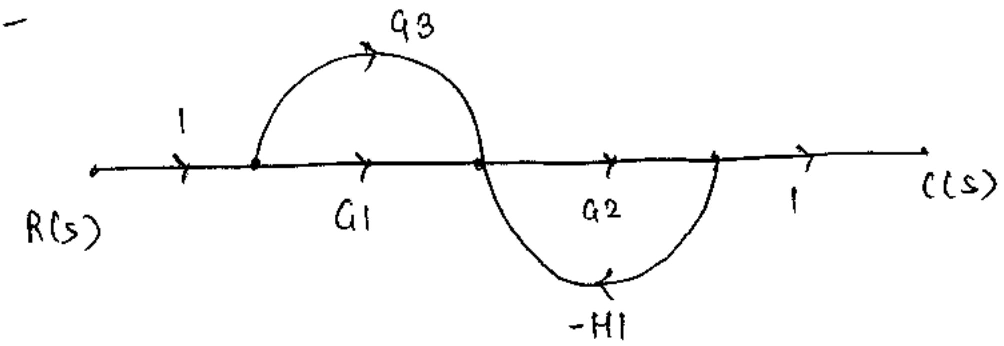
$L4 = -G2G3H2$

Two non-touching loops:-

zero

$$T.F. = \frac{G1G2G3G4 + G5G4}{1 + G1G2H1 + G3G4H3 - G5G4H1H3 + G2G3H2 - G5H2G2H1}$$

Q:-



Forward path:-

$P1 = G1G2 \quad , \Delta_1 = 1$

$P2 = G2G3 \quad , \Delta_2 = 1$

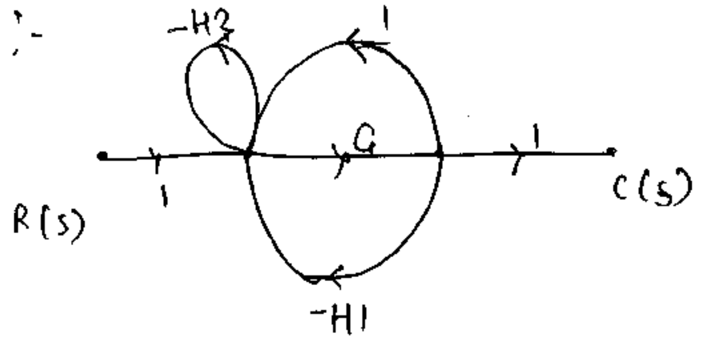
single loop:-

$L1 = -G2H1$

$\Delta = 1 + G2H1$

$$T.F. = \frac{G1G2 + G2G3}{1 + G2H1}$$

Q:-



Forward path

$P1 = G \quad , \Delta 1 = 1$

single loop:-

$L1 = -H2$

$L2 = G$

$L3 = -GH1$

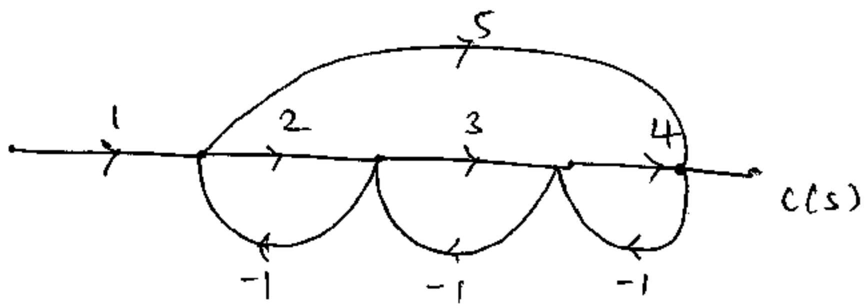
two non-touching loop:-

zero

$$\Delta = 1 + H2 - G + GH1$$

$$T.F. = \frac{G}{1 + H2 + GH1 - G}$$

Q:-



Forward path

$P1 = 24$

$P2 = 5$

single loop

$L1 = -2$

$L2 = -3$

$L3 = -4$

$L4 = -5$

two non-touching loops

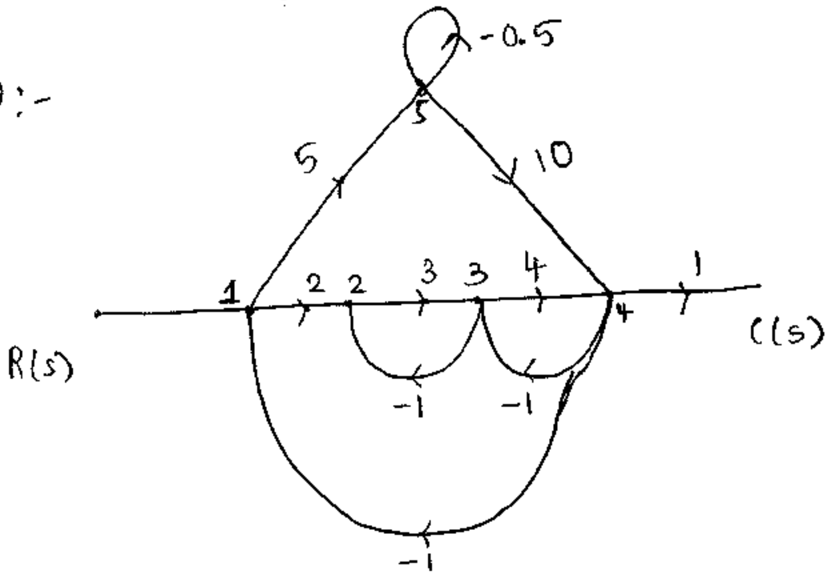
$L1L3 = 8$

$\Delta = 1 + 2 + 3 + 4 + 5 + 8 = 23$

$$T.F. = \frac{24\,(1) + 5\,(1+3)}{23}$$

$$T.F. = \frac{44}{23} = 1.913$$

Q:-



Forward path

$P_1 = 24$

$P_2 = 50$

Single loop

$L1 = -3$

$L2 = -4$

$L3 = -24$

$L4 = -0.5$

$L5 = -50$

Two-non touching loops

$L1 \cdot L4 = 1.5$

$L2 \cdot L4 = 2$

$L3 \cdot L4 = 12$

$L5 L1 = 150$

$\Delta = 1 + 3 + 4 + 24 + 0.5 + 50 + 1.5 + 2 + 12 + 150 = 248$

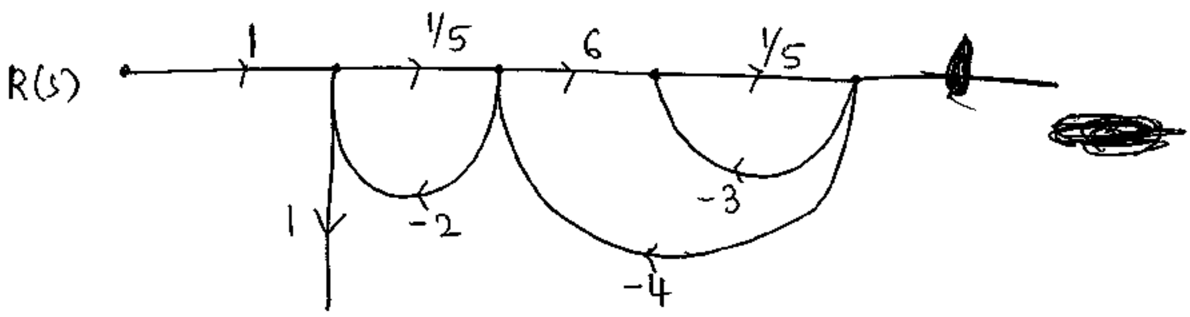
$\Delta_1 = 1 + 0.5 = 1.5$

$\Delta_2 = 1 + 3 = 4$

$$\frac{C(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{24(1.5) + 50(4)}{248} = \frac{236}{248}$$

Q:-



* Forward gain

$$P_1 = 1$$

* Single loop

$$L_1 = \frac{-2}{5}$$

$$L_2 = \frac{-3}{5}$$

$$L_3 = 6 \cdot \frac{1}{5} (-4) = \frac{-24}{5}$$

* Two non-touching loop

$$L_1 L_2 = \frac{6}{25}$$

$$\Delta = 1 + \frac{2}{5} + \frac{3}{5} + \frac{24}{5} + \frac{6}{25} = \frac{25+10+15+120+6}{25}$$

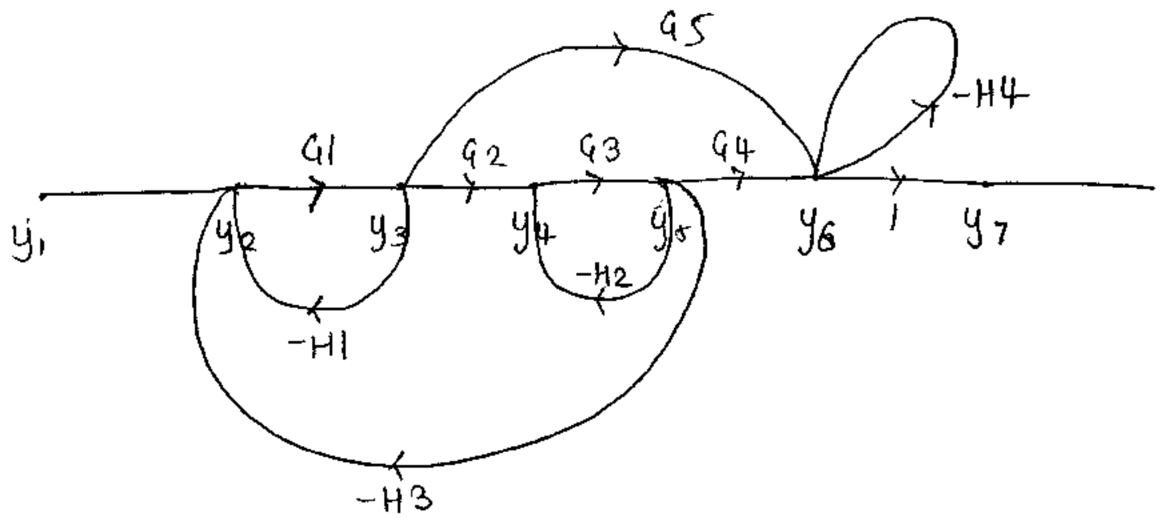
$$\Delta = \frac{176}{25}$$

$$\Delta_1 = 1 + \frac{3}{5} + \frac{24}{5} = \frac{5+27}{5} = \frac{32}{5}$$

$$T.F. = \frac{P_1 \Delta_1}{\Delta} = \frac{1 \cdot 32/\cancel{5}}{176/\cancel{25}5} = \frac{160}{176}$$

IMP ★ —

Q:- $\frac{y_7}{y_1}, \frac{y_5}{y_1}, \frac{y_2}{y_1}, \frac{y_7}{y_2}, \frac{y_5}{y_3}, \frac{y_5}{y_4}$ and so on ratio of any two node for SFG.



Forward Path

$P1 = G1G2G3G4$

$P2 = G1G5$

loop

$L1 = -G1H1$

$L2 = -G3H2$

$L3 = -H4$

$L4 = -G1G2G3H3$

Two non-touching loop

$L1L2 = G1G3H1H2$

$L1L3 = G1H1H4$

$L2L3 = G3H2H4$

$L4L3 = G1G2G3H3H4$

Three non-touching loop

$L1L2L3 = -G1G3H1H2H4$

$$\Delta = 1 + G1H1 + G3H2 + H4 + G1G2G3H3 + G1G3H1H2 + G1H1H4 + G3H2H4 + G1G2G3H3H4 + G1G3H1H2H4$$

$$\frac{y_6}{y_1} = \frac{G1G2G3G4(1) + G1G5(1+G3H2)}{\Delta}$$

$$\frac{y_7}{y_1} = \frac{G1G2G3G4(1) + G1G5(1+G3H2)}{\Delta}$$

$$\frac{y_5}{y_1} = \frac{G1G2G3(1+H4)}{\Delta}$$

$$\frac{y_2}{y_1} = \frac{1\cdot(1+G3H2+H4+G3H4H2)}{\Delta}$$

NOTE:-

$$\boxed{\frac{y_7}{y_2} = \frac{y_7/y_1}{y_2/y_1}}$$

$$\frac{y_5}{y_3} = \frac{G_2G_3(1)}{\Delta} \Rightarrow \text{wrong}$$

$$\frac{y_5}{y_3} = \frac{y_5/y_1}{y_3/y_1} = \frac{\cancel{G1}G2G3(1+H4)}{\cancel{G1}} = G2G3(1+H4)$$

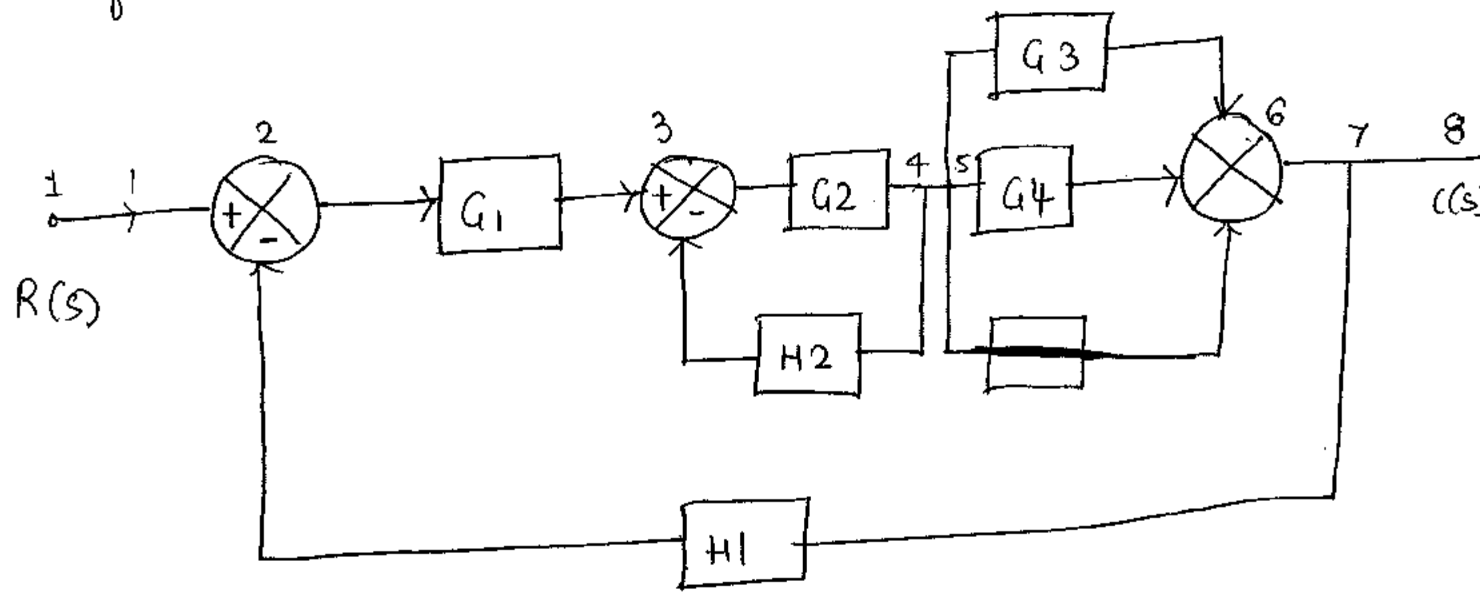
$$\frac{y_3}{y_1} = \frac{G1(1)}{\Delta}$$

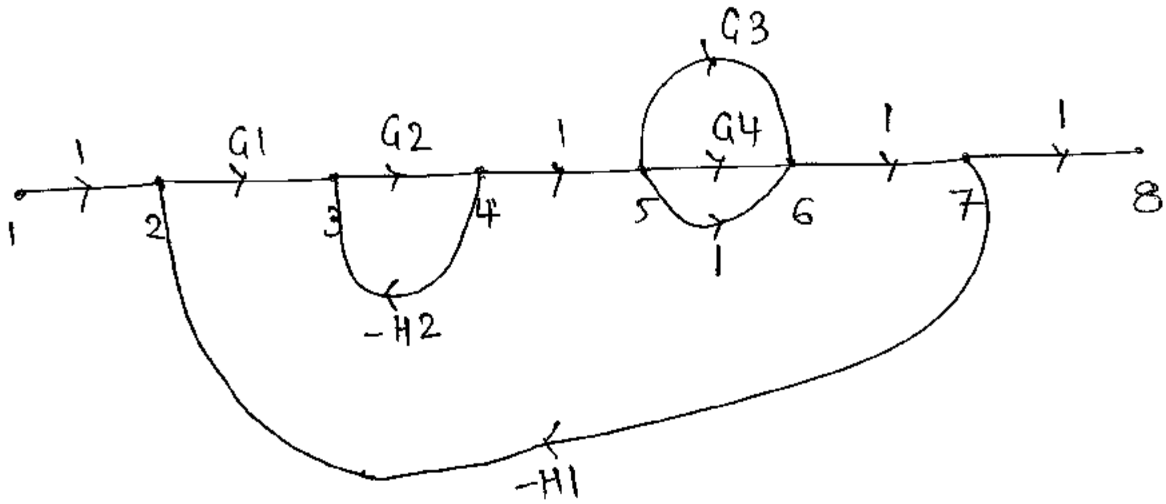
$$\frac{y_5}{y_4} = \frac{y_5/y_1}{y_4/y_1} = \frac{G1G2G3(1+H4)}{G1G2(1)} = G3(1+H4)$$

$$\frac{y_4}{y_1} = \frac{G1G2(1)}{\Delta}$$

* Construction of SFG from block diagram

Q:- From block diagram construct SFG and find overall transfer function from Meson's gain formula.





$P1 = G1G2G4$

$P2 = G1G2G3$

$P3 = G1G2$

single loop:

$L1 = -G2H2$

$L2 = -G1G2G4H1$

$L3 = -G1G2G3H1$

$L4 = -G1G2H1$

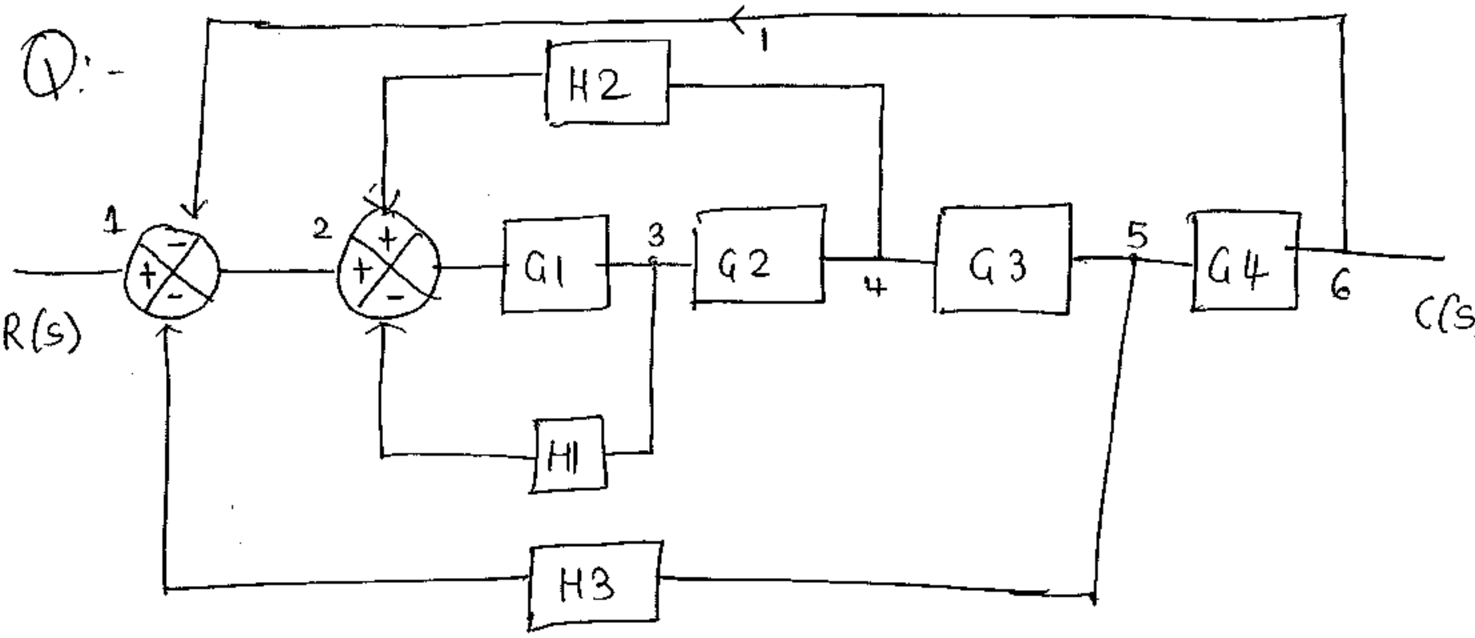
Two - non ~~tach~~ touching loop:-

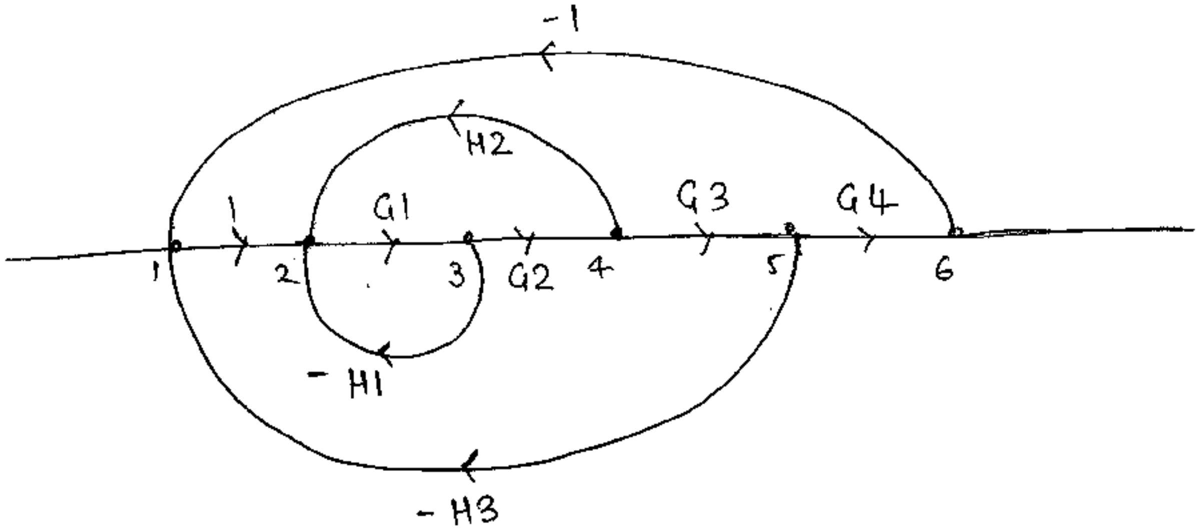
zero.

$\Delta = 1 + G2H2 + G1G2G4H1 + G1G2G3H1 + G1G2H1$

$$T.F. = \frac{G1G2G4 + G1G2G3 + G1G2}{1 + G2H2 + G1G2G4H1 + G1G2G3H1 + G1G2H1}$$

Q:-

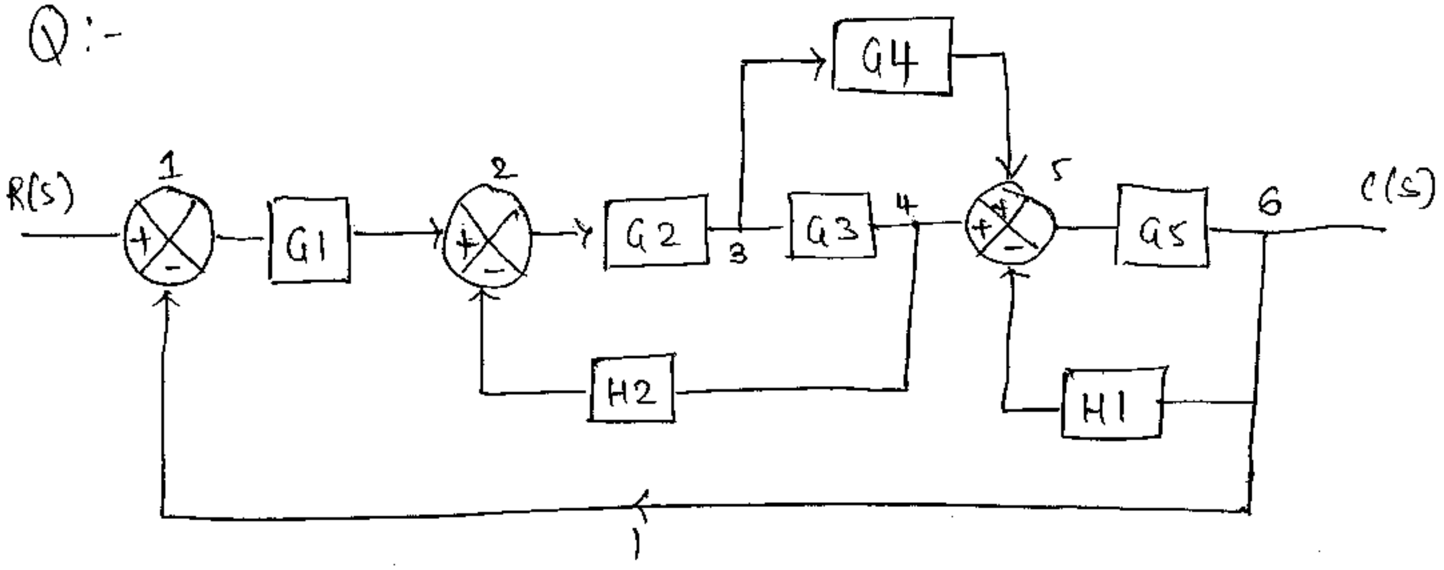


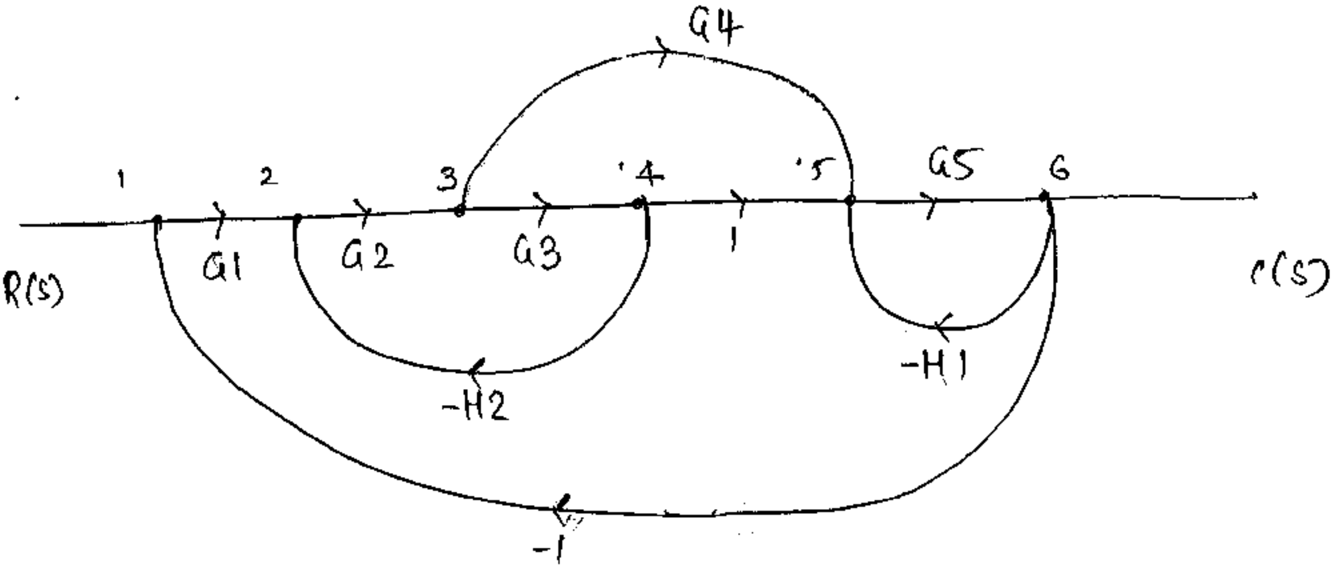


$$P1 = G1G2G3G4$$

$$T.F. = \frac{G1G2G3G4}{1 + G1H1 - G1G2H2 + G1G2G3H3 + G1G2G3G4}$$

Q:-





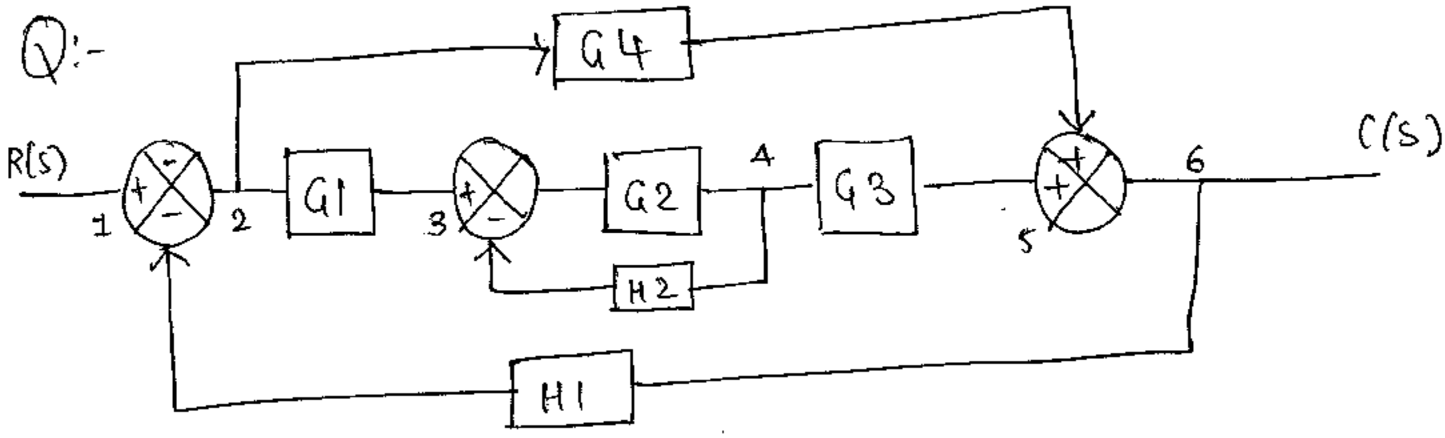
Forward path:-

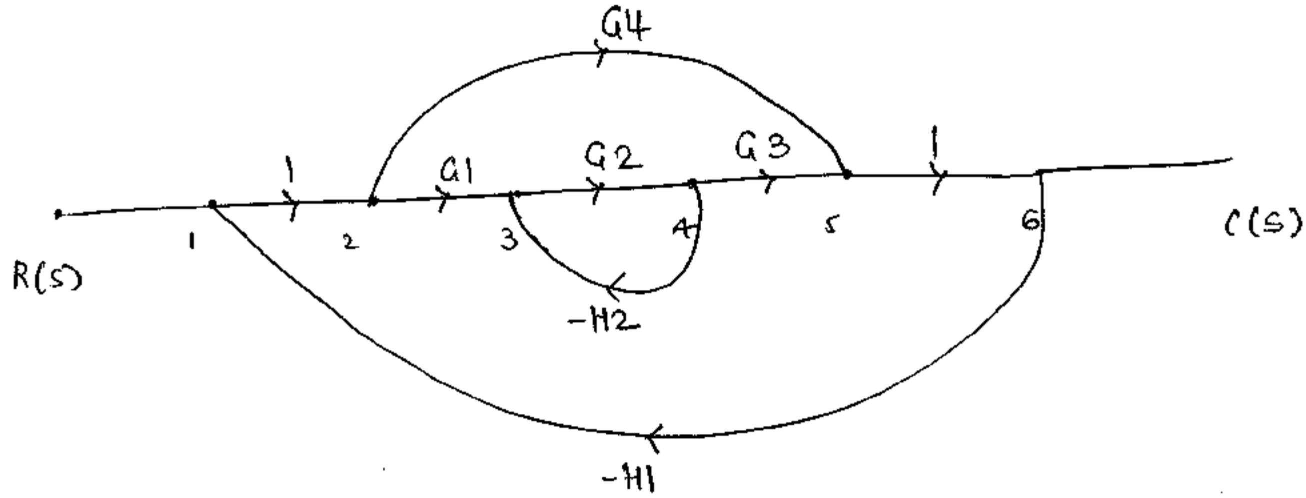
$P1 = G1G2G3G5$

$P2 = G1G2G4G5$

$$T.F. = \frac{G1G2G3G5 + G1G2G4G5}{1 + G2G3H2 + G5H1 + G1G2G3G5 + G1G2G4G5}$$

Q:-





Forward path

$P1 = G1G2G3$

$P2 = G4$

single loop

$L1 = -G2H2$

$L2 = -G4H1$

$L3 = -G1G2G3H1$

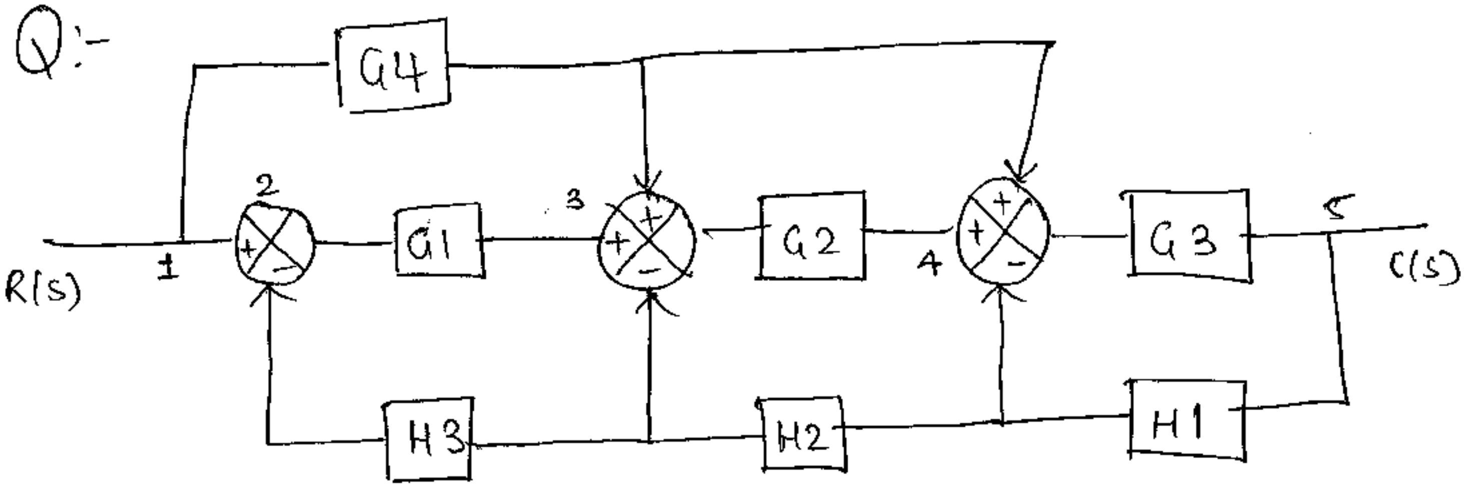
two non-touching loop

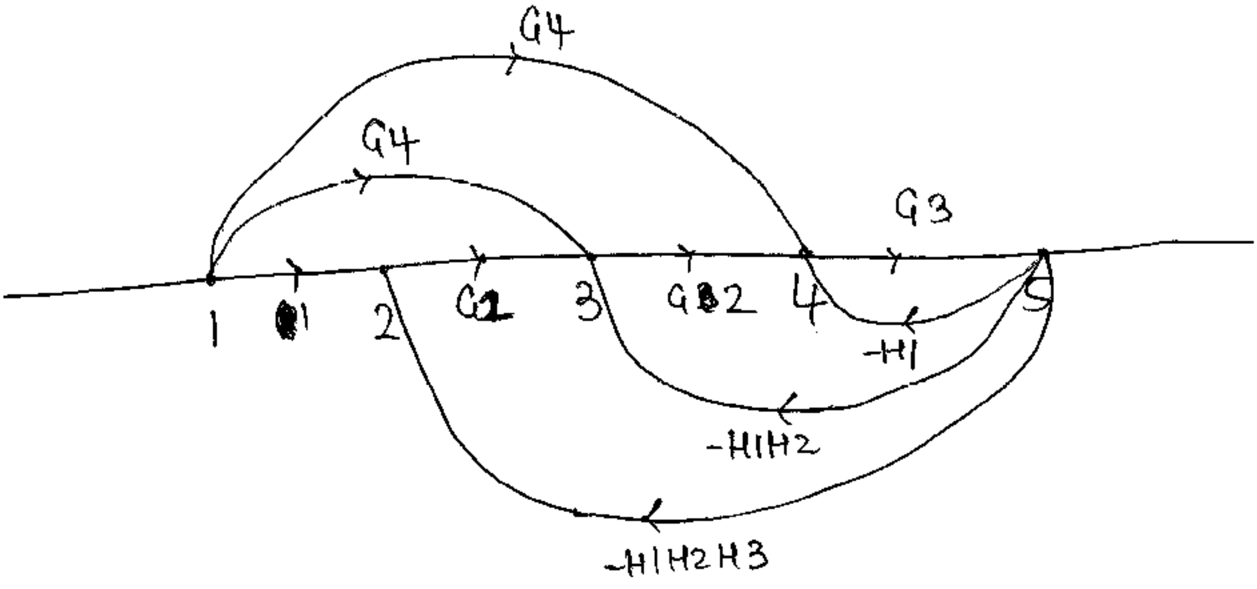
$L1L2 = G2G4H1H2$

$\Delta = 1 + G2H2 + G4H1 + G1G2G3H1 + G2G4H1H2$

$$T.F. = \frac{G1G2G3 + G4(1 + G2H2)}{1 + G2H2 + G4H1 + G1G2G3H1 + G2G4H1H2}$$

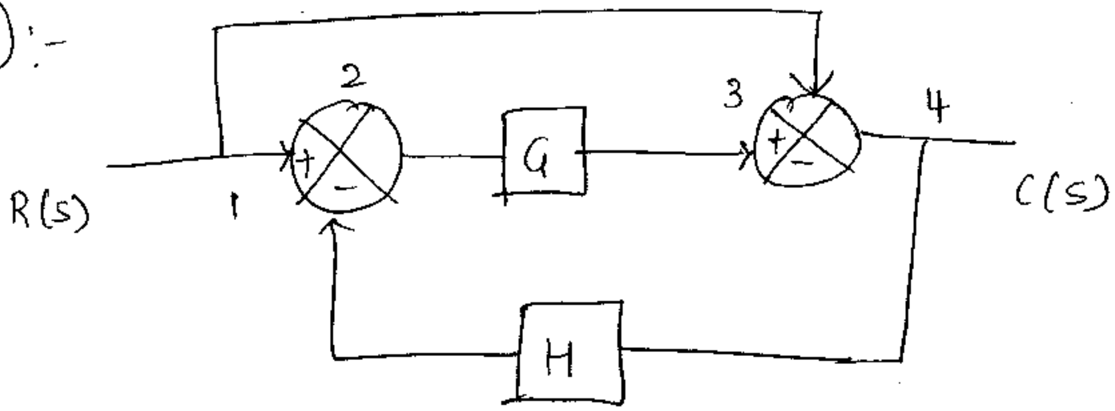
Q:-

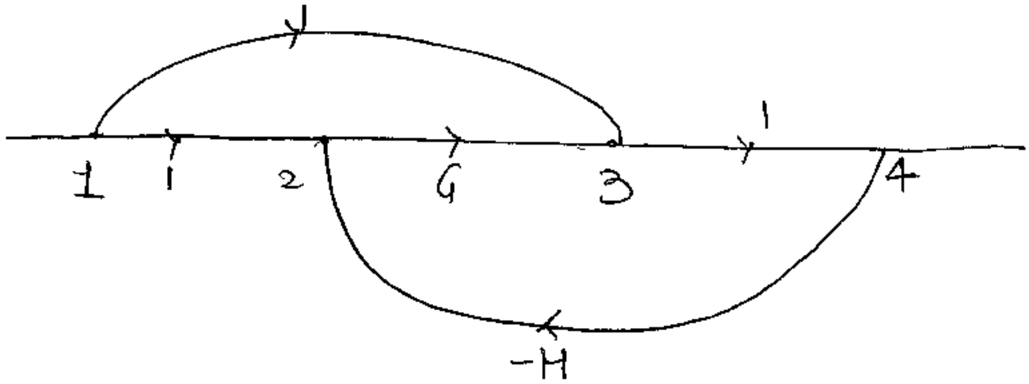




$$T.F. = \frac{G1G2G3 + G4G2G3 + G4G3}{1 + H1G3 - G3G2H1H2 + G1G2G3H1H2H3}$$

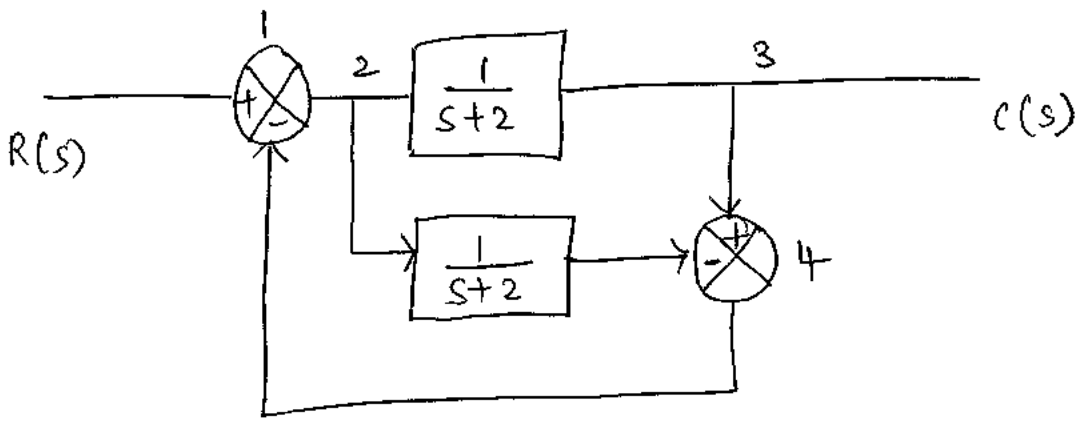
Q:-



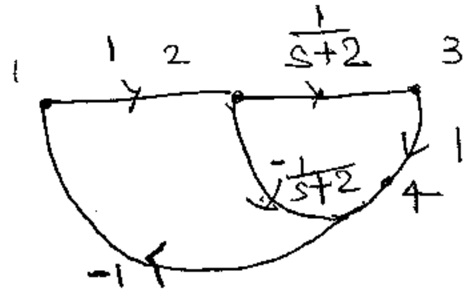


$$T.F. = \frac{1 + G}{1 + GH}$$

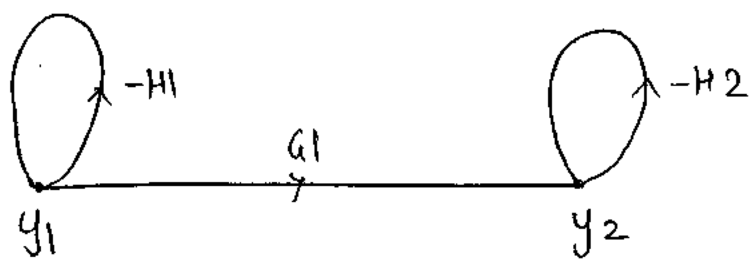
Q:-



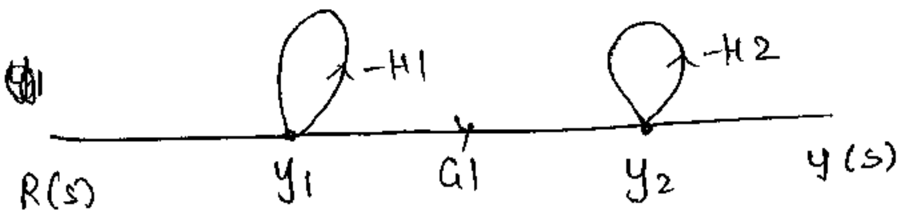
$$T.F. = \frac{\frac{1}{s+2}}{1 + \frac{1}{s+2} - \frac{1}{s+2}} = \frac{1}{s+2}$$



Q:-



$$\frac{y_2}{y_1} = ?$$



$$\frac{y_1}{R(s)} = \frac{1\,(1+H2)}{1+H1+H2+H1H2}$$

$$\frac{y_2}{R(s)} = \frac{G_1(1)}{1+H1+H2+H1H2}$$

$$\frac{y_2}{y_1} = \frac{y_2/R(s)}{y_1(R(s))} = \frac{G1}{1+H2}$$

$$\boxed{\frac{y_2}{y_1} = \frac{G1}{1+H2}}$$

Method: 2

$$y_2 = G1y_1 - H_2y_2$$

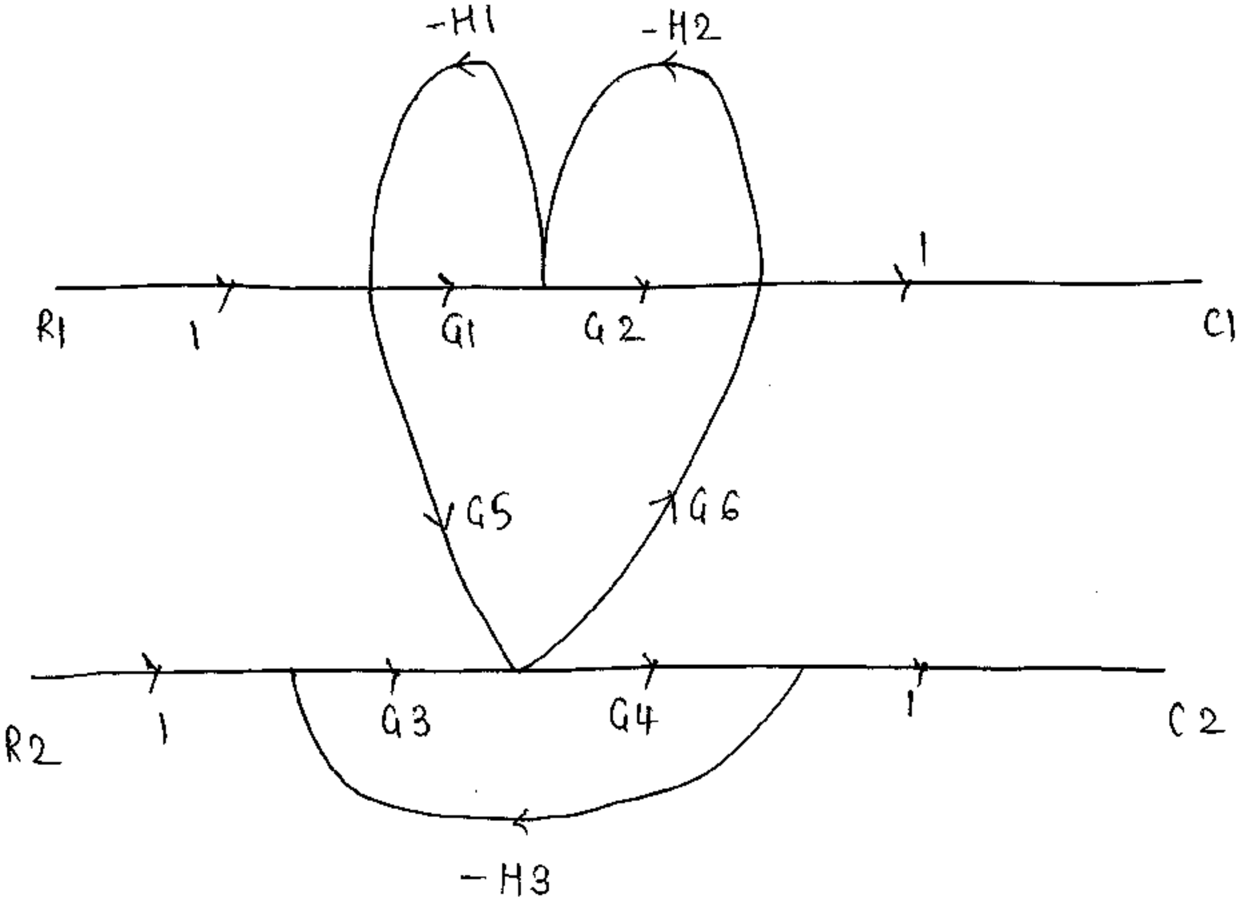
$$y_2(1+H2) = G1y_1$$

$$y_2 = \frac{G1}{1+H2}\,y_1$$

$$\frac{y_2}{y_1} = \frac{G1}{1+H2}$$ //

Method of Meson's gain formula gives ratio with respect to input node only it cannot give ratio with respect to middle node.

Q:- Find $\frac{C_1}{R_1}$, $\frac{C_1}{R_2}$, $\frac{C_2}{R_1}$, $\frac{C_2}{R_2}$ to given multiple input multiple output system.



Sol:-

$$\Delta = 1 + G1H1 + G2H2 - G5G6H1H2 + G3G4H3 + G3G4G1H3H1 + G3G2G4H2H3$$

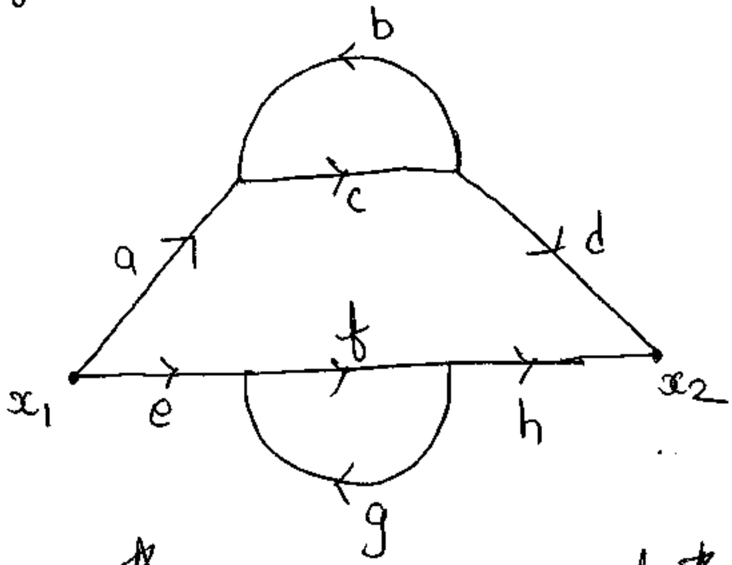
$$\frac{C1}{R1} = \frac{G1G2(1+G3G4H3) + G5G6(1)}{\Delta}$$

$$\frac{C1}{R2} = \frac{G3G6\,(1+G1H1)}{\Delta}$$

$$\frac{C2}{R2} = \frac{G3G4\,(1+G2H2+G1H1)}{\Delta}$$

$$\frac{C2}{R1} = \frac{G5G4\,(1+G2H2)}{\Delta}$$

Q:- For the SFG determine overall transmittance relating x_2, x_1



☆ No need to add dummy node ☆

☆ ☆

$$\frac{x_2}{x_1} = \frac{acd(1-fg) + efh\,(1-bc)}{1-fg-bc+bcfg}$$

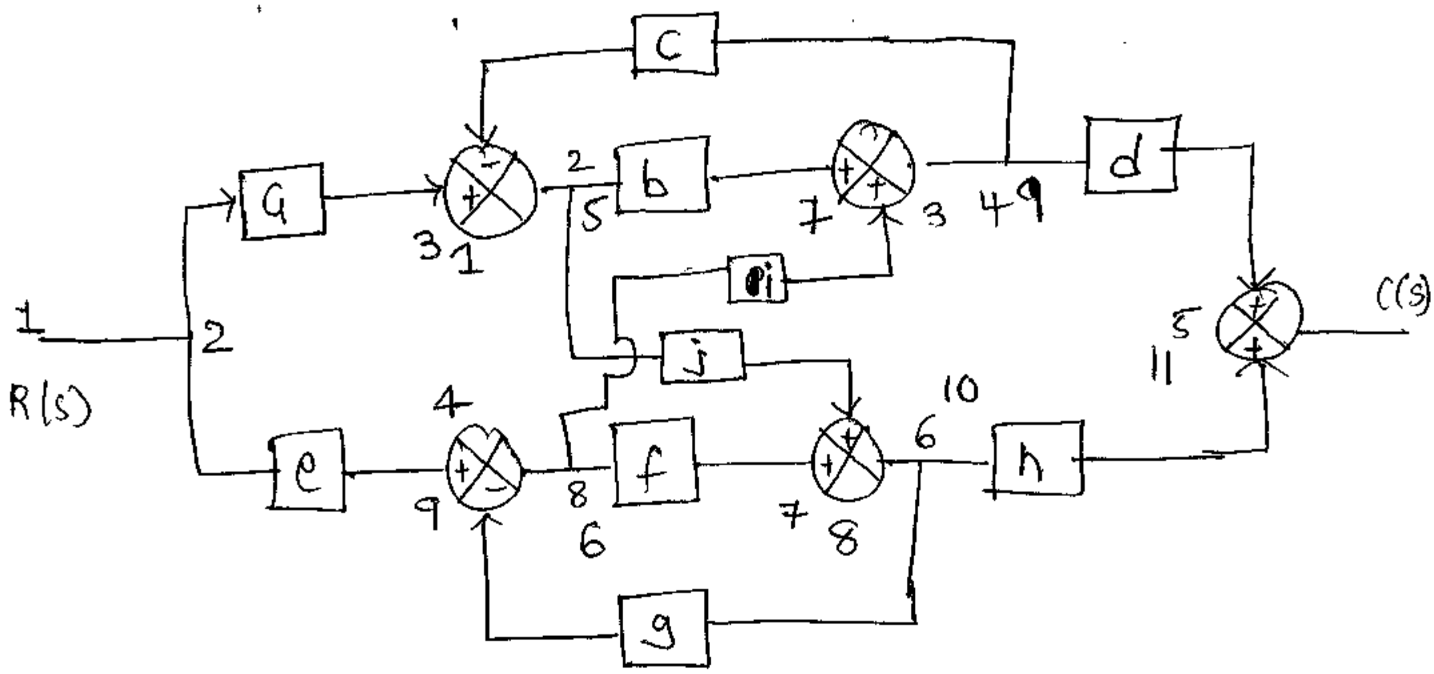
Obtain SFG representation of the system whose block diagram is given below

(1) Forward path

(2) Individual loop

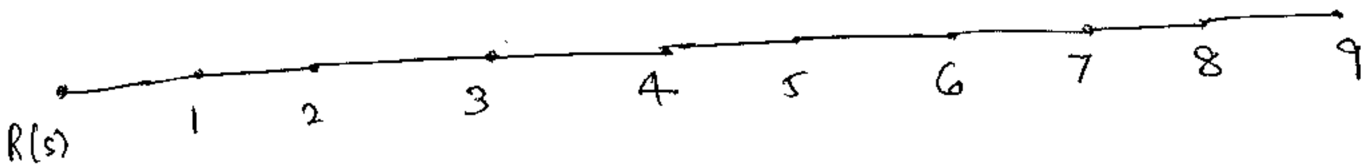
(3) Non-touching loops

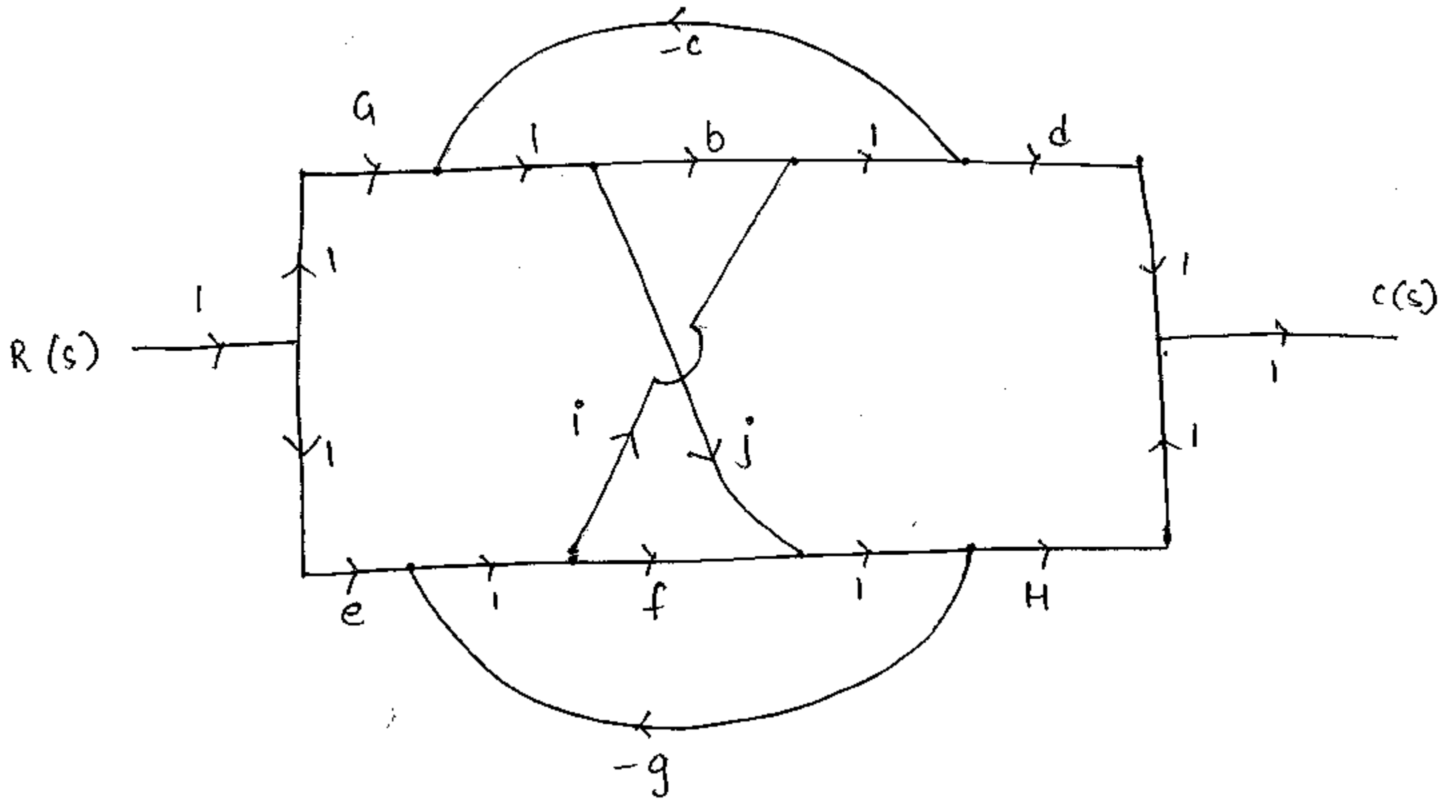
(4) Overall transfer function



| → wrong node

1 → right node





$P_1 = Gbd$, $P_2 = GjH$, $P_3 = efH$, $P_4 = eid$

$P_5 = -Gjgid$, $P_6 = -eicjH$

single loops:

$L1 = -fg$

$L2 = -bc$

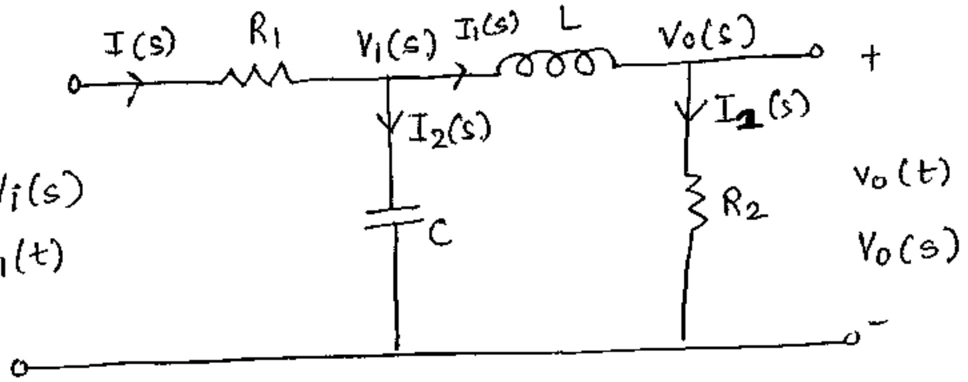
$L3 = jgic$

Two non-touching loops

$L1L2 = fgbc$

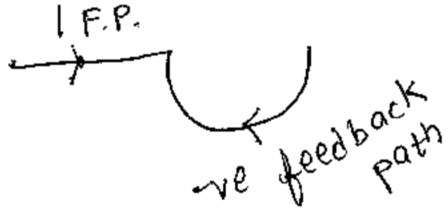
$$T.F. = \frac{Gbd(1+fg) + GjH + efH(1+bc) + eid - Gjgid - eicjH}{1 + fg + bc - jgic + fgbc}$$

* Construction of SFG from electrical network:



$$\Rightarrow I(s) = \frac{V_i(s) - V_1(s)}{R_1} = \frac{1}{R_1}\left[V_i(s) - V_1(s)\right] \quad \text{——— (1)}$$

$$V_1(s) = \frac{1}{sC} \cdot I_2(s)$$

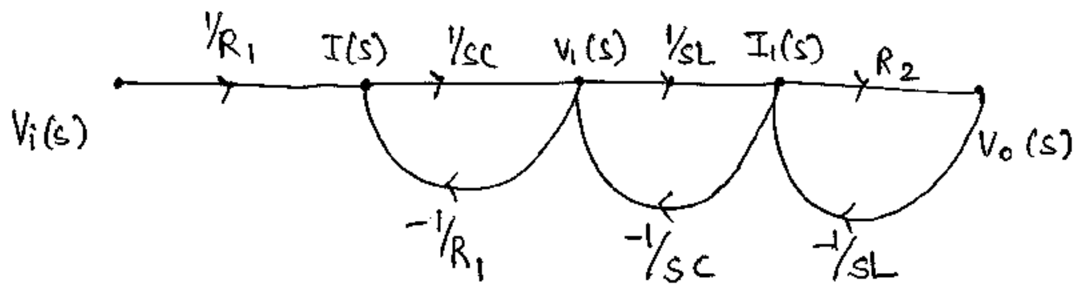


$$I_2(s) = I(s) - I_1(s)$$

$$\Rightarrow V_1(s) = \frac{1}{sC}\left[I(s) - I_1(s)\right] \quad \text{——— (2)}$$

$$\Rightarrow I_1(s) = \frac{V_1(s) - V_0(s)}{R} \quad \text{——— (3)}$$

$$\Rightarrow V_0(s) = R_2 I_1(s) \quad \text{——— (4)}$$



$$\frac{C(s)}{R(s)} = \frac{\dfrac{R_2}{s^2 LCR_1}}{1 + \dfrac{1}{sCR_1} + \dfrac{1}{s^2 LC} + \dfrac{R_2}{sL} + \dfrac{R_2}{s^2 LCR_1}}$$

Also, from above result

$$\frac{V_0(s)}{V_i(s)} = \frac{Z_2 \cdot Z_4}{Z_1(Z_2+Z_3+Z_4)+Z_2(Z_3+Z_4)}$$

* Procedure for drawing SFG from electrical network.

- Mark voltage & current along the series path element.

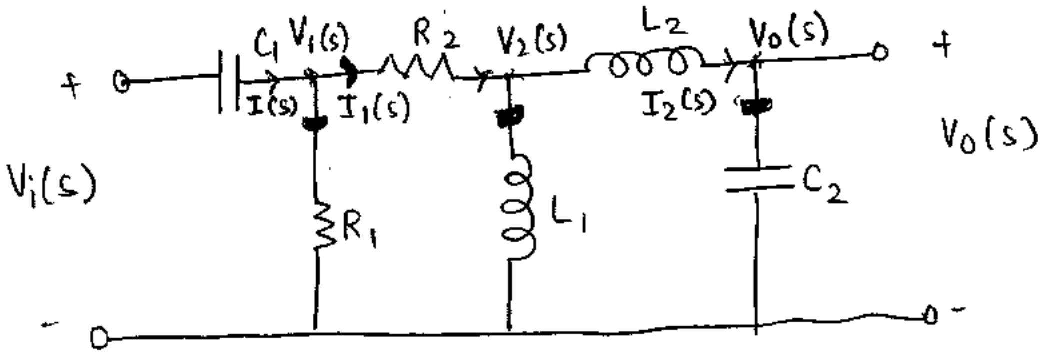
- No. of nodes in SFG are variables along the series path

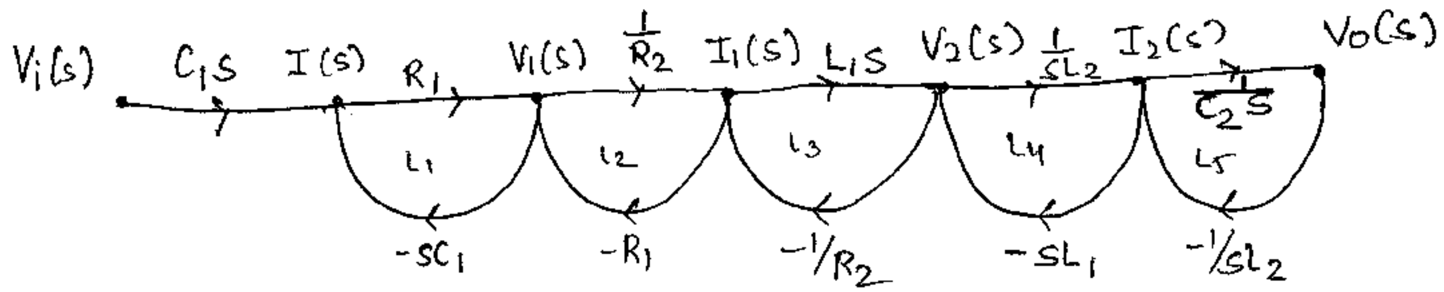
- Each and every element of electrical network gives one forward path and one negative feedback path except last element gives only one forward path

- Take inverse of impedance as a path gain for series element and take same impedance as a path gain for shunt element.

In parallel R, L, C write as it is.
In series write $( )^{-1}$ of it.

Q:- Find T.F. and draw SFG.





$$\frac{V_o(s)}{V_i(s)} = \frac{\dfrac{R_1L_1C_1}{R_2L_2C_2}}{1 + sR_1C_1 + \dfrac{R_1}{R_2} + \dfrac{sL_1}{R_2} + \dfrac{L_1}{L_2} + \dfrac{1}{s^2L_2C_2} + \dfrac{s^2R_1L_1C_1}{R_2} + \dfrac{sR_1C_1L_1}{L_2} + \cancel{\dfrac{R_1C_1L_1}{R_2}} + \dfrac{R_1C_1}{sL_2C_2} + \dfrac{L_1}{sR_2C_2L_2} + \dfrac{R_1L_1}{R_2L_2} + \dfrac{R_1}{s^2R_2C_2L_2} + \dfrac{R_1C_1L_1}{R_2C_2L_2}}$$

If $R = L = C$ then

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1+s+1+s+1+\dfrac{1}{s^2}+s^2+s+\dfrac{1}{s}+\dfrac{1}{s}+1+\dfrac{1}{s^2}+1}$$

$$= \frac{1}{\dfrac{2}{s^2} + s^2 + 3s + \dfrac{2}{s} + 5}$$

$$= \frac{s^2}{2 + s^4 + 3s^3 + 2s + 5s^2}$$

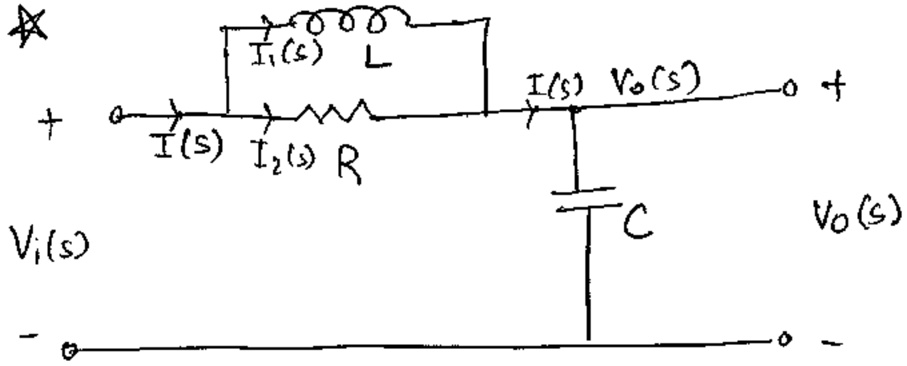
$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{s^2}{s^4 + 3s^3 + 5s^2 + 2s + 2}}$$

Total order $= 4$

storage element $= 4$

DC gain $= 0$

Q:-

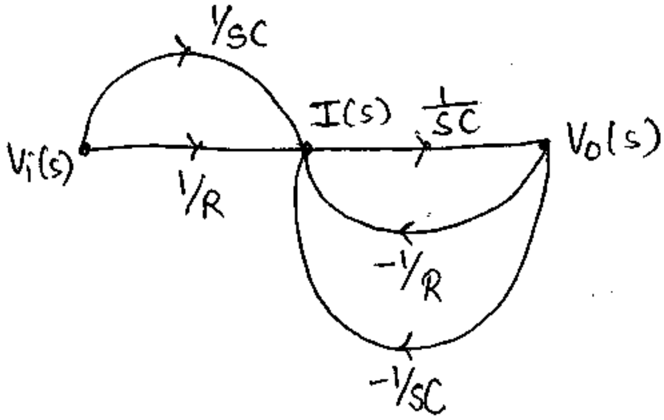


$$I(s) = I_1(s) + I_2(s)$$

$$I(s) = \frac{V_i(s) - V_o(s)}{SL} + \frac{V_i(s) - V_o(s)}{R}$$

$$I(s) = V_i(s)\left[\frac{1}{R} + \frac{1}{SL}\right] - V_o(s)\left[\frac{1}{R} + \frac{1}{SL}\right]$$

$$V_o(s) = \frac{1}{SC} I(s)$$



$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{SCR} + \frac{1}{S^2C^2}}{1 + \frac{1}{SCR} + \frac{1}{S^2C^2}}$$