CHAPTER 20 LINES AND ANGLES

DEFINITION

Point

A point is like a dot marked by a very sharp pencil on a plane paper. A point is named by a capital letter like *P*. In the figure *P* is a point. Length, breadth and height of a point are negligible and hence cannot be measured.

. *P*

Line

A line is defined as a group of points. Which are straight one after another. Each line is extended infinitely in two directions. Examples:

(i) (ii)

A line is named by either any two points on it or by a single small letter. In figure (i), *AB* is a line. In figure (ii), *l* is a line.

Arrows on both sides of a line indicate that the line is extended both sides infinitely. A line has only length. It does not have any width or height.

Line Segment

If a part of the line is cut out, then this cut out piece of the line is called a line segment. A line segment has no arrow at its any end.

This means that no line segment is extended infinitely in any direction.

Ray

A ray is a part of a line extended infinitely in any one direction only. Example:

A ray is named by two points, one of which is the end point on the ray called initial point and other point is any point on the ray.

In the figure, *AB* is a ray. The point *A* is called the initial point. Arrow of the ray indicates that the ray is extended infinitely towards arrow head.

Plane

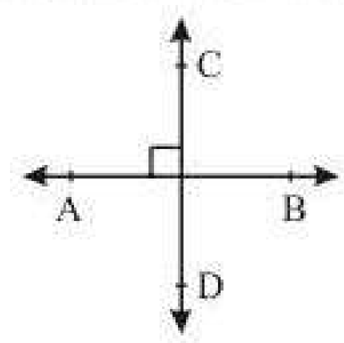
It is a flat surface extended infinitely. It has only length and breadth but no thickness. Surface of a black board, surface of a wall, surface of a table are some examples of parts of planes because they are flat surfaces but not extended infinitely.

Intersecting Lines

If two or more lines intersect each other, then they are called intersecting lines. In the figure *AB* and *CD* are intersecting lines.

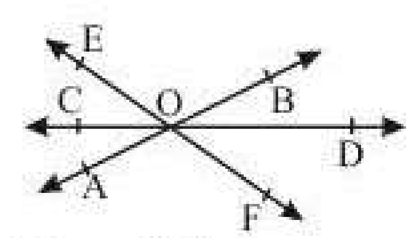
If two lines intersect at right angles, then two lines are called perpendicular lines

In the following figure *AB* and *CD* are perpendicular lines.



Symbolically it is represented as $AB \perp CD$ or $CD \perp AB$.

Concurrent Lines: If three or more lines pass through a point, then they are called concurrent lines and the point through which these all lines pass is called point of concurrent.



In the figure, *AB*, *CD* and *EF* are concurrent lines and point *O* is the point of concurrent.

Parallel Lines

Two straight lines are parallel if they lie in the same plane and do not intersect even if they produced.

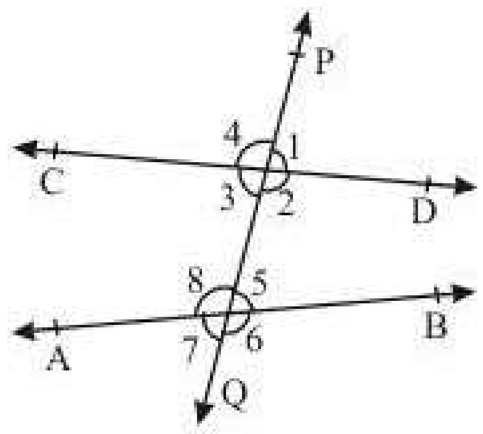
Perpendicular distances between two parallel lines are the same at all places.

In the figure *AB* and *CD* are parallel lines.

Symbol for parallel lines is ||. Hence parallel lines *AB* and *CD* represented symbolically as $AB \parallel CD$.

Transversal Line

A line which intersects two or more given lines at distinct points is called a transversal of the given lines.



In figure straight lines *AB* and *CD* are intersected by a transversal *PQ*.

(i) Corresponding angles: In the figure ∠1 and ∠5, ∠4 and ∠8, ∠2 and ∠6, ∠3 and ∠7 are four pairs of corresponding angles.

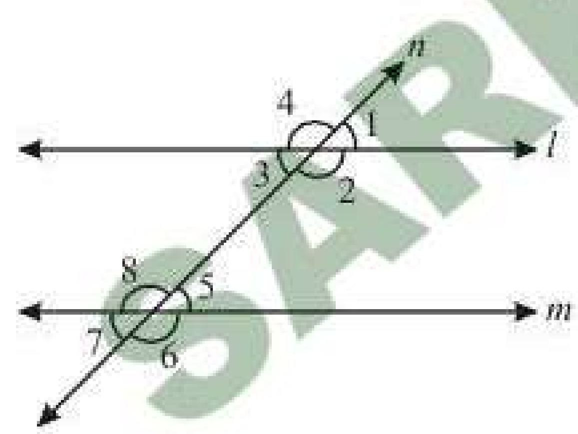
(ii) Alternate interior angles: ∠3 and ∠5, ∠2 and ∠8, are two pairs of alternate interior angles.

(iii) Alternate exterior angles: ∠1 and ∠7, ∠4 and ∠6 are two pairs of alternate exterior angles.

(iv) Consecutive interior angles: In the figure, ∠2 and ∠5, ∠5 and ∠8, ∠8 and ∠3, ∠3 and ∠2 are four pairs of consecutive interior angles.

Interior angles on the same side of a transversal are called **cointerior angles**. In the fig. ∠2 and ∠5, ∠3 and ∠8 are two pairs of **cointerior angles.**

When a transversal intersects two parallel lines:



In the figure two parallel lines *l* and *m* are intersected by a transversal line *n*, then

(a) Two angles of each pair of corresponding angles are equal i.e. ∠1 = ∠5 ; ∠2 = ∠6 ; ∠4 = ∠8 ; ∠3 = ∠7

(b) Two angles of each pair of alternate interior angles are equal i.e.
∠2 = ∠8; ∠3 = ∠5

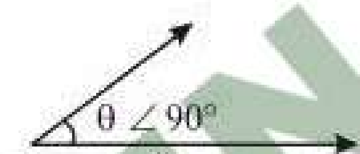
(c) Two angles of each pair of alternate exterior angles are equal i.e.
∠1 = ∠7; ∠4 = ∠6

(d) Any two consecutive interior angles are supplementary. i.e. their sum is 180°. Hence
∠2 + ∠5 = 180°; ∠5 + ∠8 = 180°; ∠8 + ∠3 = 180°; ∠3 + ∠2 = 180°

Remember

- ✧ If two angles of any pair of corresponding angles are equal, then the two lines are parallel.
- ✧ If two angles of any pair of alternate interior angles are equal, then the two lines are parallel.
- ✧ If two angles of any pair of alternate exterior angles are equal, then the two lines are parallel.
- ✧ If any two consecutive interior angles are supplementary (i.e. their sum is 180°), then the two lines are parallel.

Acute angle: An angle is said to be acute angle if it is less than 90°.



Here 0° ∠ θ ∠ 90°, hence θ is acute angle.

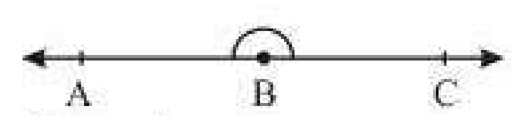
Right angle: An angle is said to be right angle if it is of 90°.

Here θ is right angle.

Obtuse angle: An angle is said to be obtuse angle if it is of more than 90°.

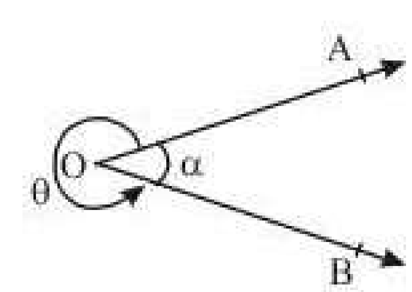
Here θ is obtuse angle.

Straight angle: An angle is said to be straight angle if it is of 180°.



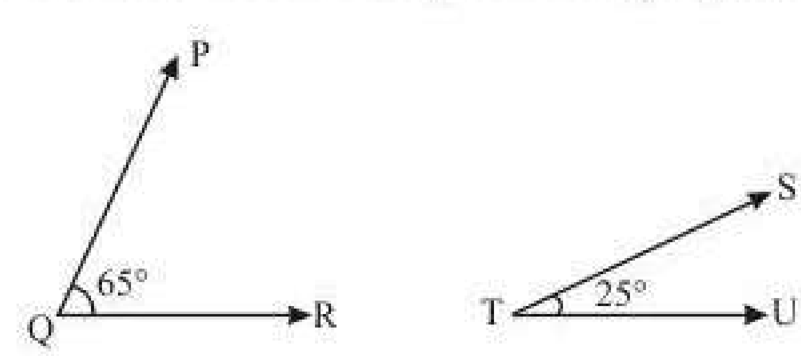
Here θ is a straight angle.

Reflex angle: An angle is said to be reflex angle if it is of greater than 180°.

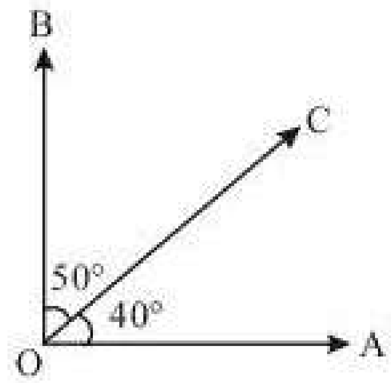


Here θ is the reflex angle.
Reflex angle θ is written as
θ = 360° – ∠*AOB* (or 360° – α)
Here ∠*AOB* or α is less than 180°

Complementary angles: Two angles, the sum of whose measures is 90°, are called the complementary angles.

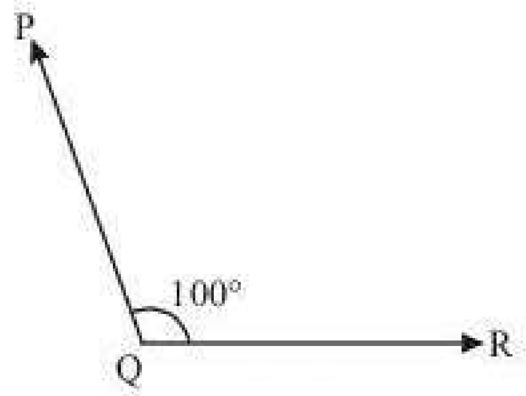


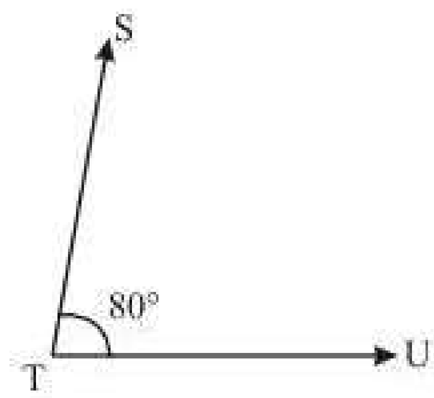
∠*PQR* and ∠*STU* are complementary angles.



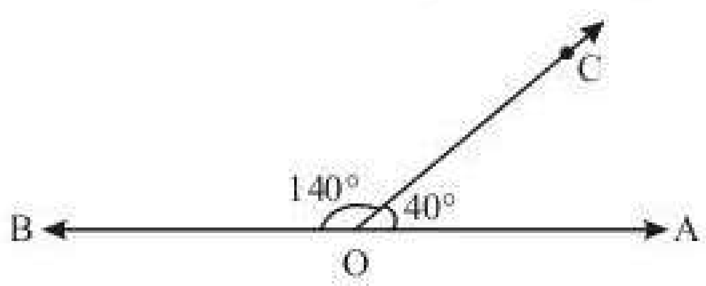
In figure $\angle AOC$ and $\angle BOC$ are also complementary angles.

Supplementary angles: Two angles, the sum of whose measures is 180°, are called the supplementary angles.





In figure, $\angle PQR$ and $\angle STU$ are supplementary angles.



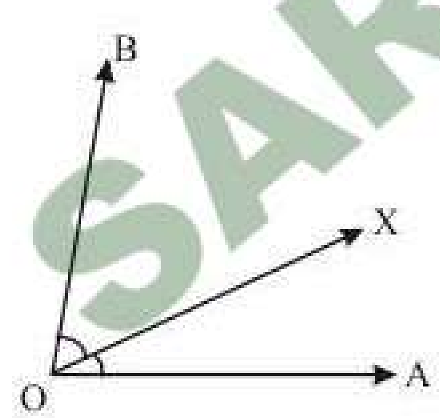
In figure, $\angle$AOC and $\angle$BOC are also supplementary angles.

Adjacent angles: Two angles are called adjacent angles, if

(i) they have the same vertex

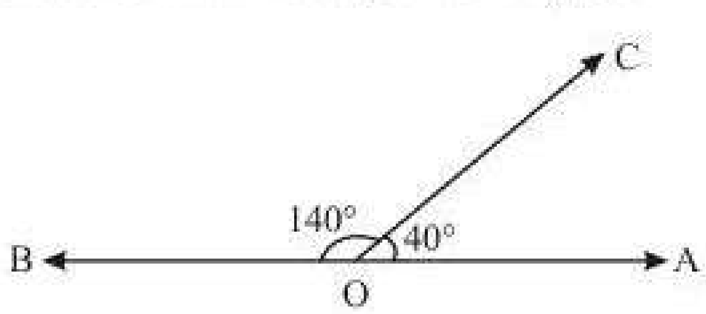
(ii) they have a common arm and

(iii) non-common arms are on either side of the common arm.



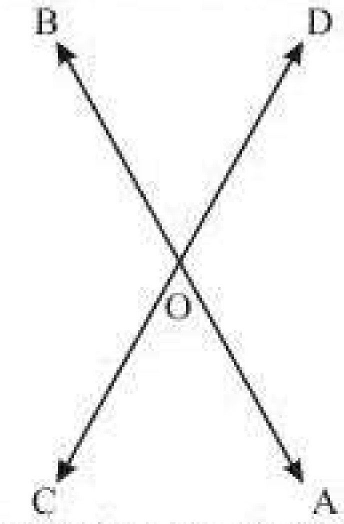
In figure, $\angle AOX$ and $\angle BOX$ are adjacent angles because O is the common vertex, OX is common arm, non-common arm OA and OB are on either side of OX.

Linear pair of angles: Two adjacent angles are said to form a linear pair of angles, if their non common arms are two opposite rays. In other words if the sum of two adjacent angles is 180°, then they are said to form a linear pair of angles.



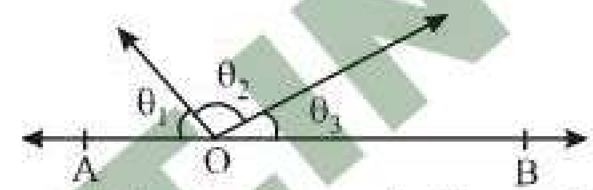
In figure, $\angle AOC$ and $\angle BOC$ are linear pair angles.

Vertically opposite angles: Two angles are called a pair of vertically opposite angles, if their arms form two intersecting lines.



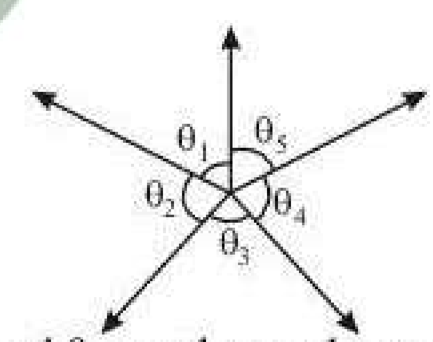
In figure, $\angle AOC$ and $\angle BOD$ form a pair of vertically opposite angles. Also $\angle AOD$ and $\angle BOC$ from a pair of vertically opposite angles.

Angles on one side of a line at a point on the line: Sum of all the angles on any one side of a line at a point on the line is always 180°.



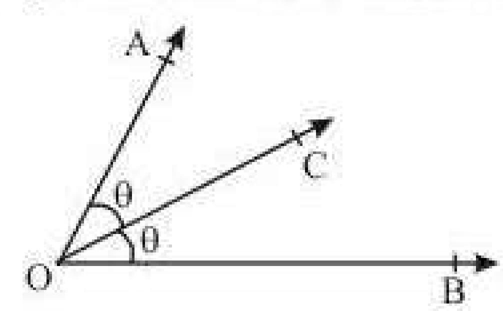
Here AOB is a straight line, hence in figure, $\theta_1 + \theta_2 + \theta_3 = 180°$.

Angle around a point: Sum of all the angles around a point is always 360°.



Here θ_1, θ_2, θ_3, θ_4 and θ_5 are the angles around a point. Hence $\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 = 360°$

Angle bisector: An angle bisector is a ray which bisects the angle whose initial point be the vertex of the angle.



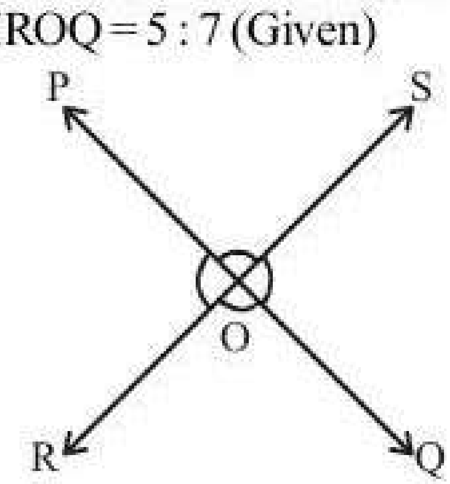
Since $\angle AOC = \angle BOC = \theta$

Hence ray OC is the bisector of $\angle AOB$.

Example 1: In figure given below, lines PQ and RS intersect each other at point O. If ∠POR : ∠ROQ = 5 : 7, find all the angles.

Solution : $\angle POR + \angle ROQ = 180°$ (Linear pair of angles)

But $\angle POR : \angle ROQ = 5 : 7$ (Given)



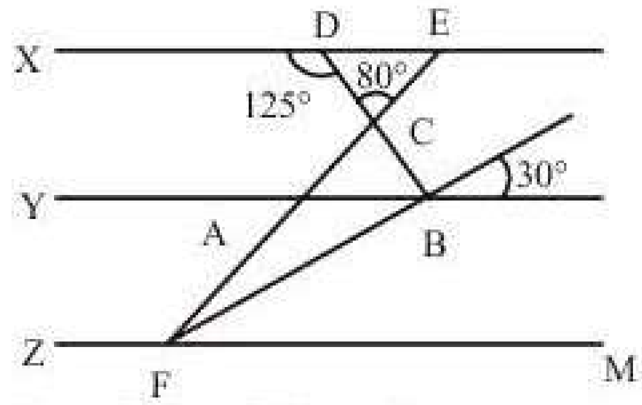
$$\therefore \angle POR = \frac{5}{12} \times 180° = 75°$$

Similarly, $\angle ROQ = \frac{7}{12} \times 180° = 105°$

Now, $\angle POS = \angle ROQ = 105°$ (Vertically opposite angles)

and $\angle SOQ = \angle POR = 75°$ (Vertically opposite angles)

Example 2: **Three straight lines, *X, Y* and *Z* are parallel and the angles are as shown in the figure above. What is $\angle AFB$ equal to?**



***Solution*:** $\angle CDE = 180° - 125° = 55°$

In ΔDCE,

$\angle CED = 180° - 55° - 80° = 45°$

and $\angle ABF = 30°$ (vertically opposite)

Also, $\angle ABF = \angle BFM = 30°$ (alternate angle)

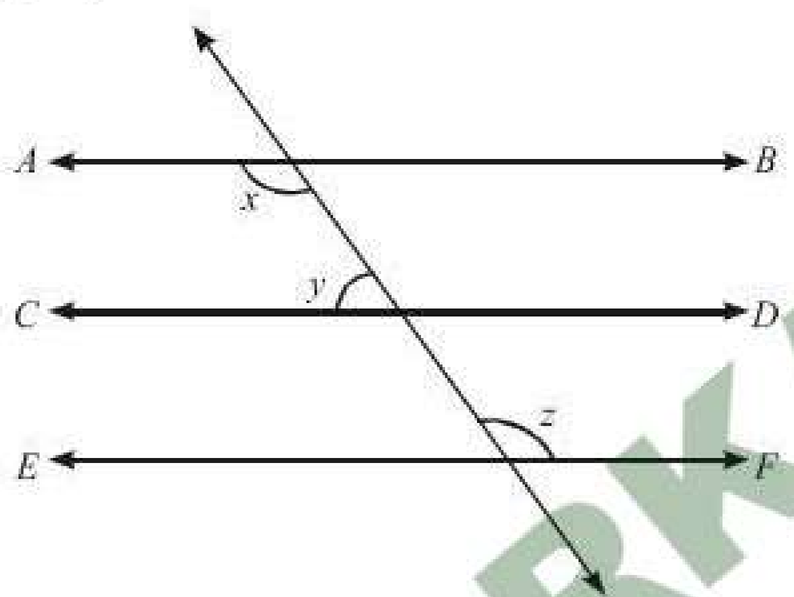
and, $\angle DEF = \angle EFM$ (alternate angle)

$\angle EFM = 45°$

$\Rightarrow \angle EFB + \angle BFM = 45° \Rightarrow \angle EFB = 45° - 30°$

$\Rightarrow \angle AFB = 15°$

Example 3: **In figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, $x = ?$**

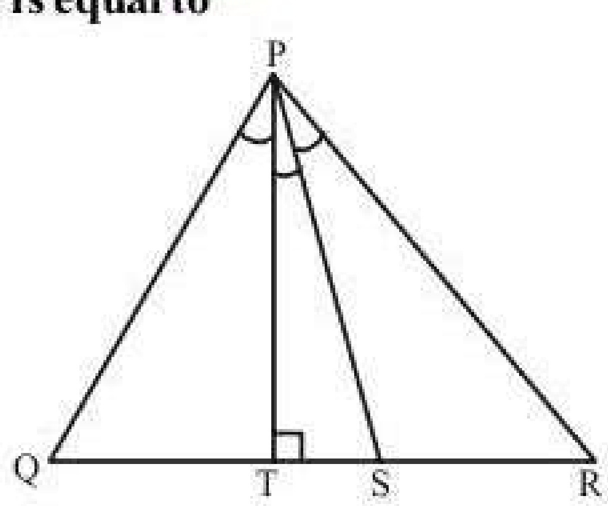


***Solution*:** As $y + z = 180°$, $\therefore y = 54°$

$x + y = 180°$

$x = 180 - 54 = 126°$

Example 4: **In the ΔPQR, PS is the bisector of $\angle P$ and PT ? QR, then $\angle TPS$ is equal to**



(a) $\angle Q + \angle R$ (b) $90° + \angle Q$
(c) $90° - \angle R$ (d) $(\angle Q - \angle R)$

***Solution*:** **(d)** PS is the bisector of $\angle QPR$

$\therefore \angle 1 + \angle 2 = \angle 3$ ...(1)

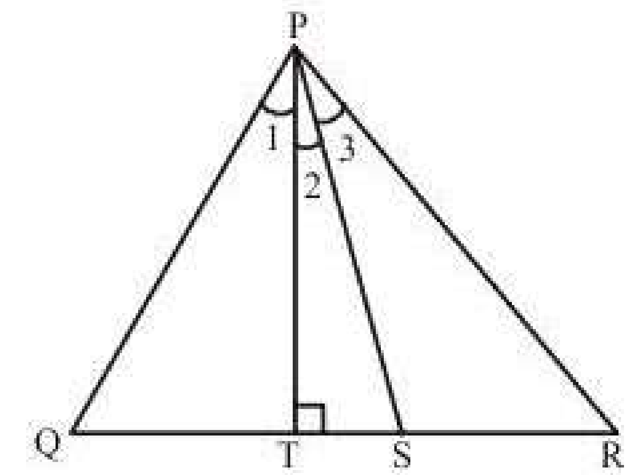
$\Rightarrow \angle Q = 90° - \angle 1$

$\angle R = 90° - \angle 2 - \angle 3$

So, $\angle Q - \angle R = [90° - \angle 1] - [90° - \angle 2 - \angle 3]$

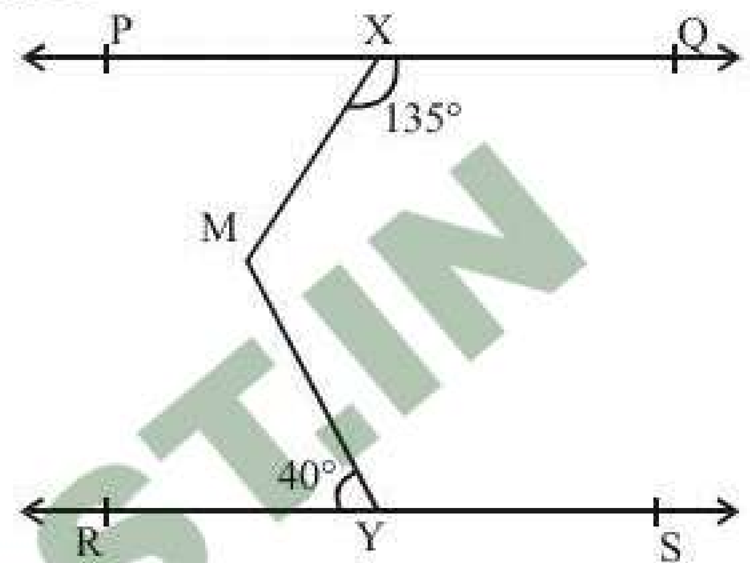
$\Rightarrow \angle Q - \angle R = \angle 2 + \angle 3 - \angle 1$

$= \angle 2 + (\angle 1 + \angle 2) - \angle 1$ [From Eq. (1)]



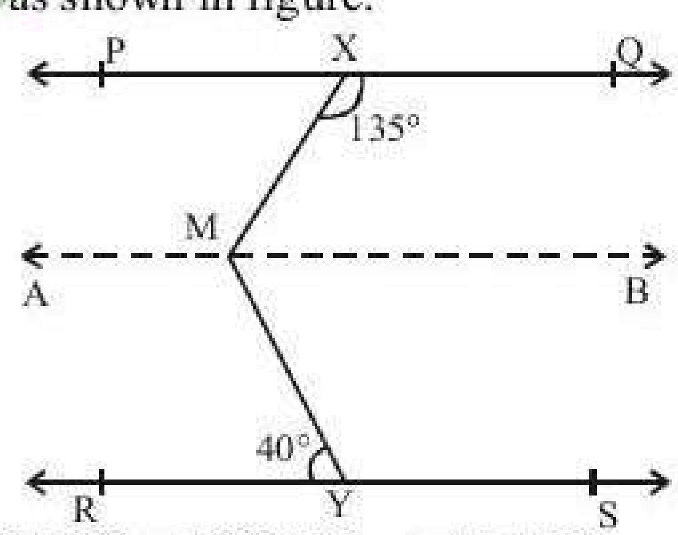
$\Rightarrow \angle Q - \angle R = 2\angle 2 \Rightarrow (\angle Q - \angle R) = \angle TPS$

Example 5: **In fig. if PQ || RS, ∠MXQ = 135° and ∠MYR = 40°, find ∠XMY.**



Solution :

Here, we need to draw a line AB parallel to line PQ, through point M as shown in figure.



Now, AB || PQ and PQ || RS $\Rightarrow$ AB || RS

Now, $\angle QXM + \angle XMB = 180°$

($\because$ AB || PQ, interior angles on the same side of the transversal)

But $\angle QXM = 135° \Rightarrow 135° + \angle XMB = 180°$

$\therefore \angle XMB = 45°$ ....(i)

Now, $\angle BMY = \angle MYR$ ($\because$ AB || RS, alternate angles)

$\therefore \angle BMY = 40°$ ....(ii)

Adding (i) and (ii), we get

$\angle XMB + \angle BMY = 45° + 40°$

i.e. $\angle XMY = 85°$

Example 6: **The supplement of an angle is one-fifth of itself. Determine the angle and its supplement.**

Solution :

Let the measure of the angle be x°. Then the measure of its supplementary angle is 180° – x°.

It is given that $180 - x = \frac{1}{5}x$

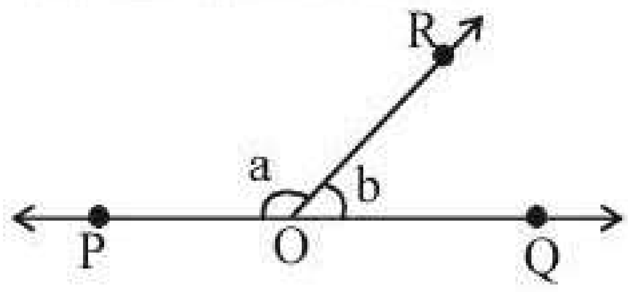
$\Rightarrow 5(180° - x) = x$

$\Rightarrow 900 - 5x = x \Rightarrow 900 = 5x + x$

$\Rightarrow 900 = 6x \Rightarrow 6x = 900 \Rightarrow x = \frac{900}{6} = 150$

Supplementary angle is $180° - 150° = 30°$

Example 7: In figure, $\angle POR$ and $\angle QOP$ form a linear pair. If a – b = 80°, find the values of a and b.



Solution :

$\because$ $\angle POR$ and $\angle QOR$ for a linear pair

$\therefore$ $\angle POR + \angle QOR = 180°$ (Linear pair axiom)

or $a + b = 180°$ .... (i)

But $a - b = 80°$ .... (ii) [Given]

Adding eqs. (i) and (ii), we get

$2a = 260°$ $\therefore$ $a = \frac{260}{2} = 130°$

Substituting the value of a in (1), we get

$130° + b = 180°$

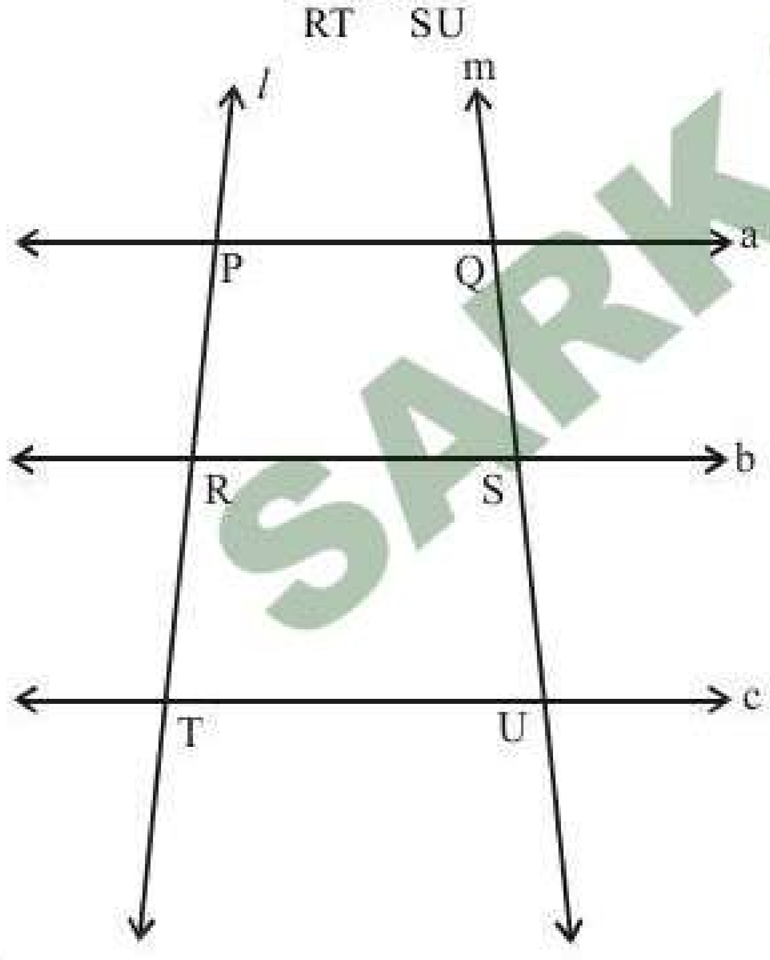
$b = 180° - 130° = 50°$

PROPORTIONALITY THEOREM

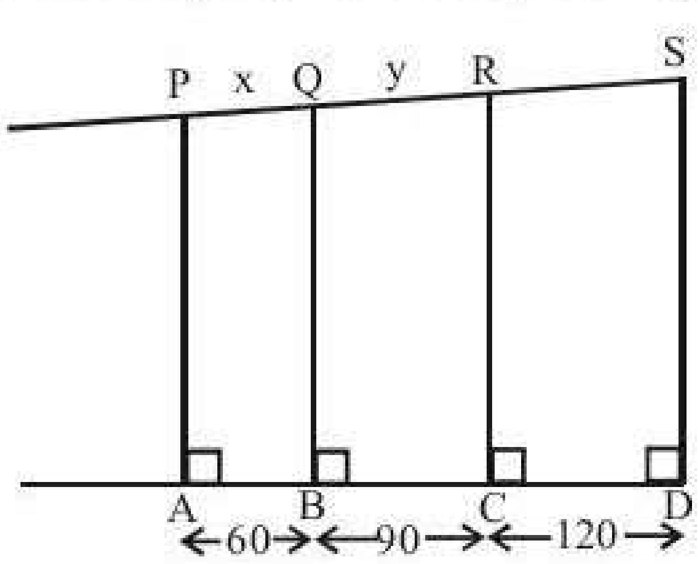
The ratio of intercepts made by three parallel lines on a transversal is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines.

If line a || b || c, and lines *l* and *m* are two transversals, then

$$\frac{PR}{RT} = \frac{QS}{SU}$$



Example 8: In the figure, if PS = 360, find PQ, QR and RS.



Solution :

PA, QB, RC and SD are perpendicular to AD. Hence, they are parallel. So the intercepts are proportional.

$\therefore$ $\frac{AB}{BD} = \frac{PQ}{QS}$ $\Rightarrow$ $\frac{60}{210} = \frac{x}{360 - x}$

$\Rightarrow$ $\frac{2}{7} = \frac{x}{360 - x}$ $\Rightarrow$ $x = \frac{720}{9} = 80$

$\therefore$ PQ = 80

So, QS = 360 – 80 = 280

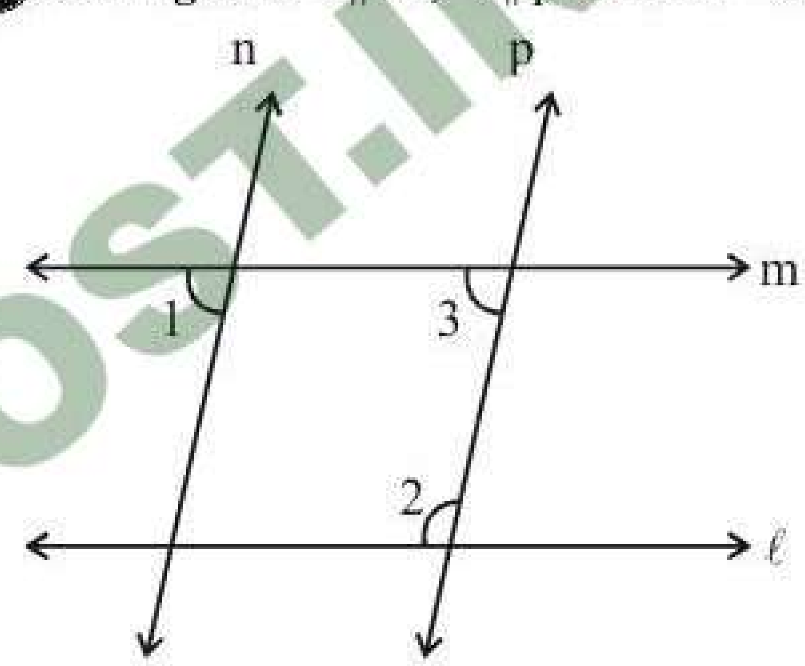
Again, $\frac{BC}{CD} = \frac{QR}{RS}$

$\therefore$ $\frac{90}{120} = \frac{y}{280 - y}$ $\Rightarrow$ $\frac{3}{4} = \frac{y}{280 - y}$

$\Rightarrow$ $y = 120$

$\therefore$ QR = 120 and SR = 280 – 120 = 160

Example 9: In figure if $\ell \parallel m$, $n \parallel p$ and $\angle 1 = 85°$ find $\angle 2$.



Solution :

$\because$ $n \parallel p$ and m is transversal

$\therefore$ $\angle 1 = \angle 3 = 85°$ (Corresponding angles)

Also, $m \parallel \ell$ & p is transversal

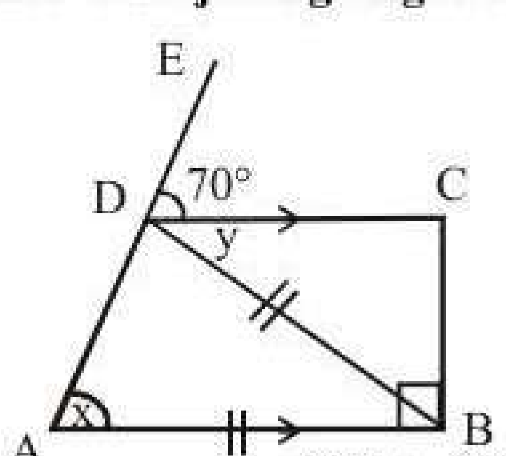
$\therefore$ $\angle 2 + \angle 3 = 180°$ ($\therefore$ Consecutive interior angles)

$\Rightarrow$ $\angle 2 + 85° = 180°$

$\Rightarrow$ $\angle 2 = 180° - 85°$

$\Rightarrow$ $\angle 2 = 95°$

Example 10: From the adjoining diagram Find (i) $\angle x$ (ii) $\angle y$



$\angle CBA = 90°$

Solution :

$\angle x = \angle EDC = 70°$ (corresponding angles)

Now, $\angle ADB = x = 70°$ [AB = DB]

In ΔABD,

$\angle ABD = 180 - \angle x - \angle x$

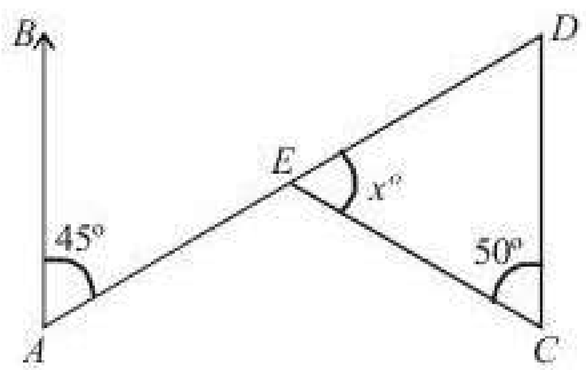
$= 180 - 70 - 70 = 40°$

$\Rightarrow \angle BDC = \angle ABD = 40°$ (alternate angles)

$\Rightarrow \angle y = 40°$

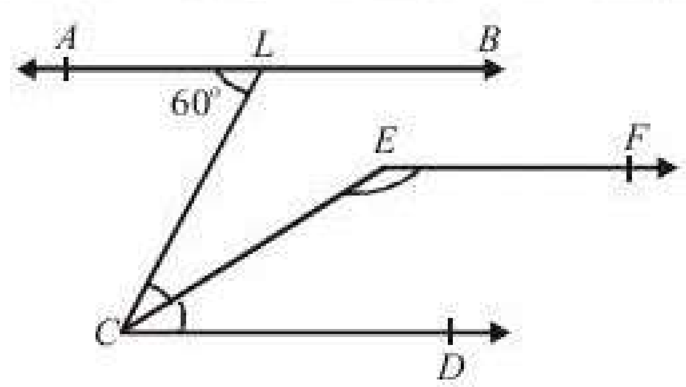
EXERCISE

1. In the given figure, $AB \parallel CD$, $\angle BAE = 45°$, $\angle DCE = 50°$ and $\angle CED = x$, then find the value of x.



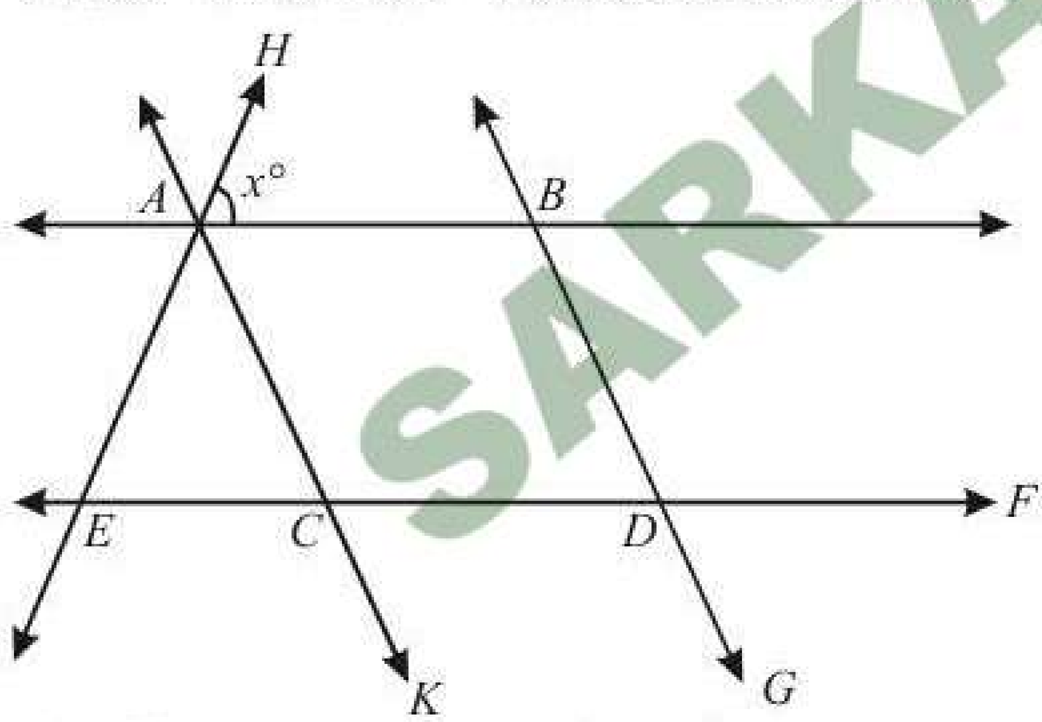
(a) 85° (b) 95°
(c) 60° (d) 20°

2. In the given figure, $AB \parallel CD$, $\angle ALC = 60°$, EC is the bisector of $\angle LCD$ and $EF \parallel AB$. Then, find the measure of $\angle CEF$.



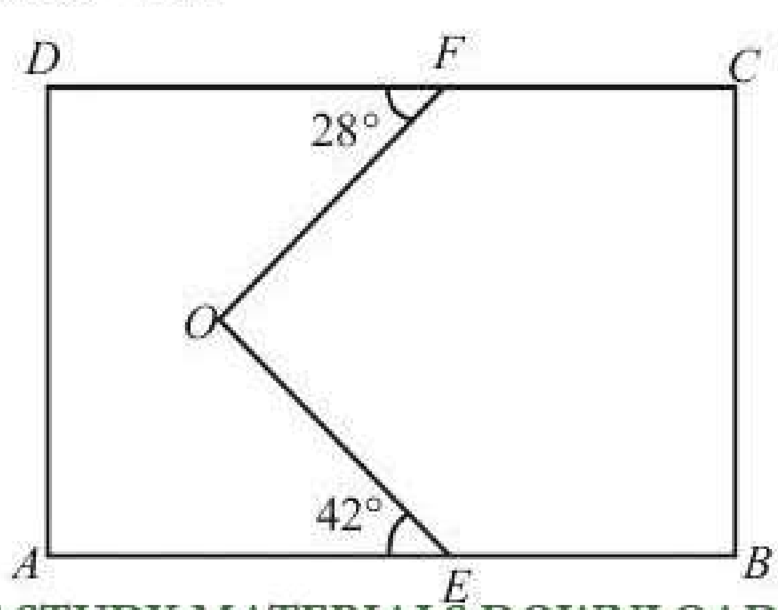
(a) 80° (b) 130°
(c) 120° (d) 150°

3. In the given figure $AB \parallel CD$ and $AC \parallel BD$. If $\angle EAC = 40°$, $\angle FDG = 55°$, $\angle HAB = x°$, then find the value of x.



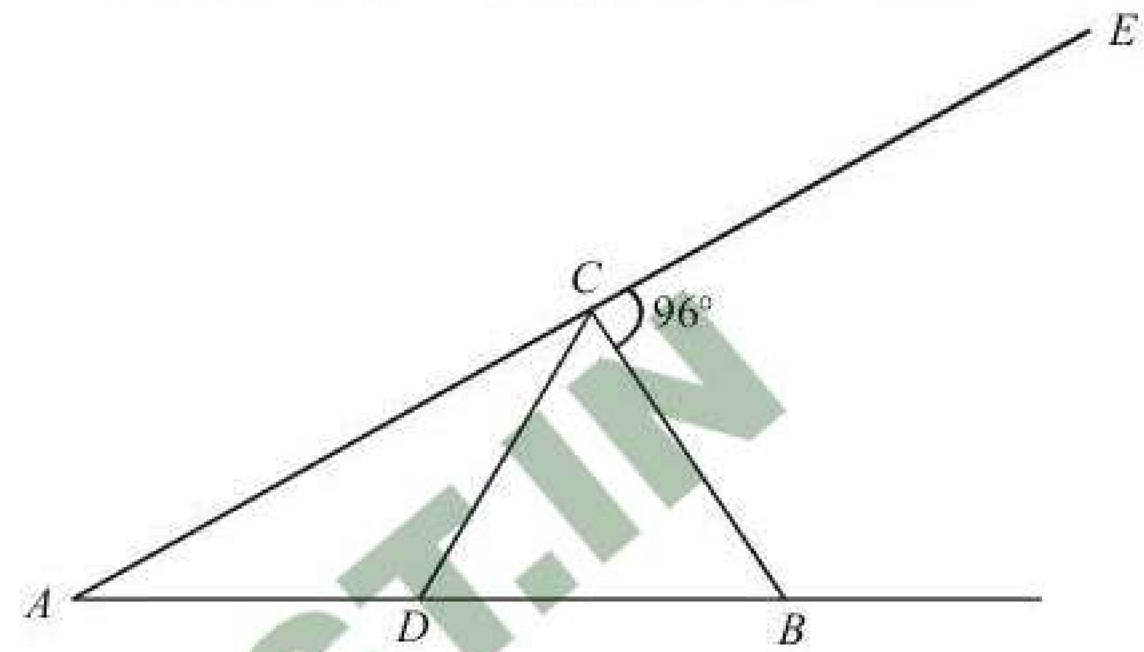
(a) 85° (b) 75°
(c) 65° (d) 55°

4. In the adjoining figure $ABCD$ is a rectangle and $DF = CF$ also, $AE = 3BE$. What is the value of $\angle EOF$, if $\angle DFO = 28°$ and $\angle AEO = 42°$?



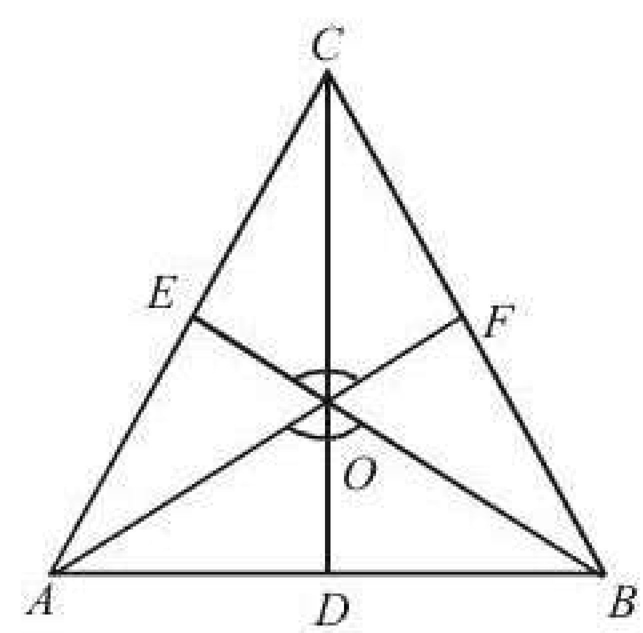
(a) 14° (b) 42°
(c) 70° (d) 90°

5. In the figure (not drawn to scale) given below, if $AD = CD = BC$, and $\angle BCE = 96°$, how much is $\angle DBC$?



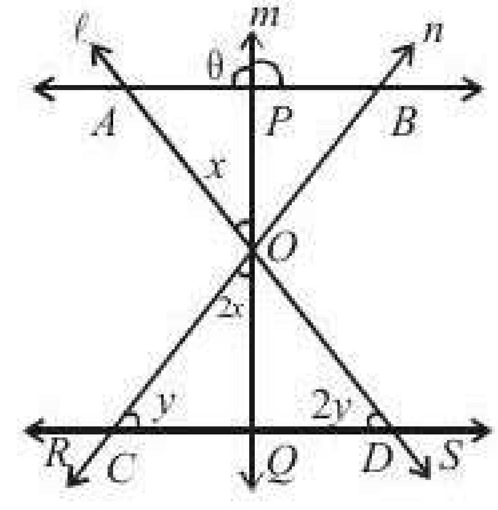
(a) 32° (b) 84°
(c) 64° (d) Cannot be determined

6. ABC is a triangle in which $\angle CAB = 80°$ and $\angle ABC = 50°$, AE, BF and CD are the altitudes and O is the orthocentre. What is the value of $\angle AOB$?



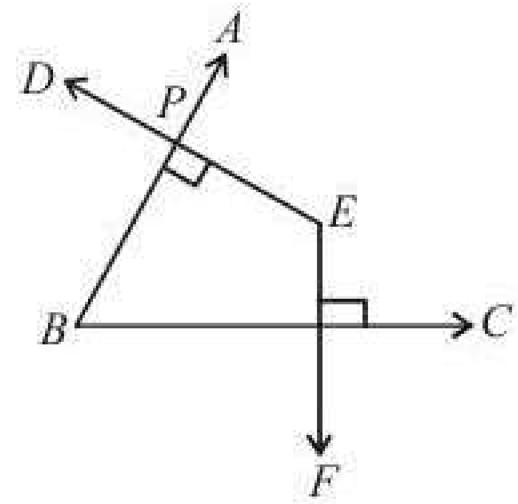
(a) 65° (b) 70°
(c) 50° (d) 130°

7. Give that segment AB and CD are parallel, if lines ℓ, m and n intersect at point O. Find the ratio of θ to $\angle ODS$



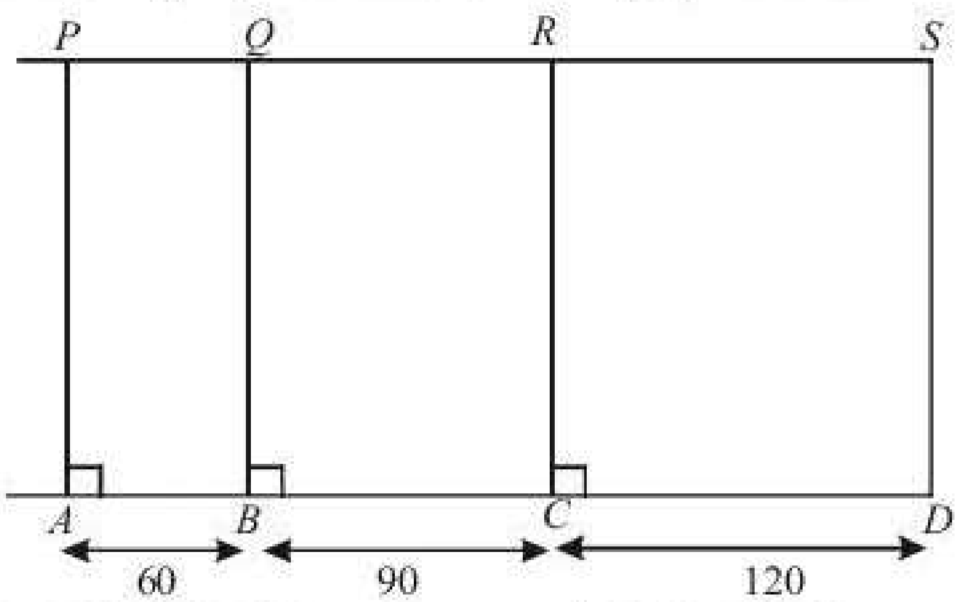
(a) 2 : 3 (b) 3 : 2
(c) 3 : 4 (d) Data insufficient

8. In the given figure, $\angle ABC$ and $\angle DEF$ are two angles such that $BA \perp ED$ and $EF \perp BC$, then find value of $\angle ABC + \angle DEF$.



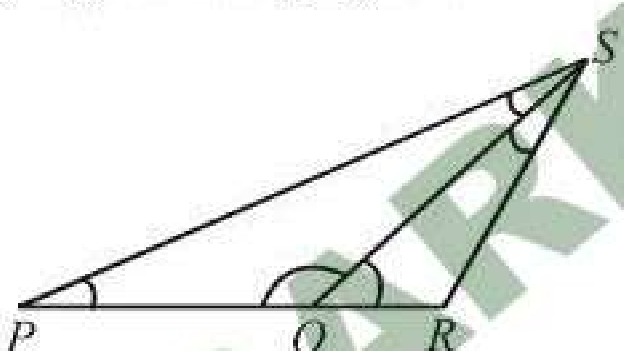
(a) 120° (b) 180°
(c) 150° (d) 210°

9. In the figure, if $PS = 360$, find PQ, QR and RS.



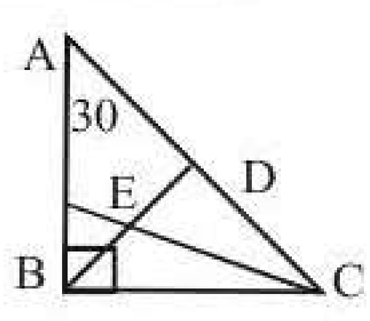
(a) 150, 120, 130 (b) 160, 120, 80
(c) 180, 150, 60 (d) 190, 70, 140

10. In the figure below, $PQ = QS$, $QR = RS$ and angle $SRQ = 100°$. How many degrees is angle QPS?



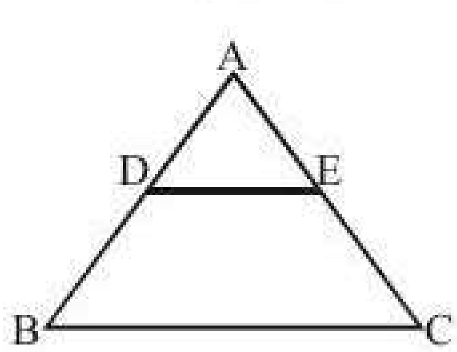
(a) 20° (b) 40°
(c) 15° (d) 30°

11. $AB \perp BC$ and $BD \perp AC$. And CE bisects the angle C. $\angle A = 30°$. The, what is $\angle CED$.



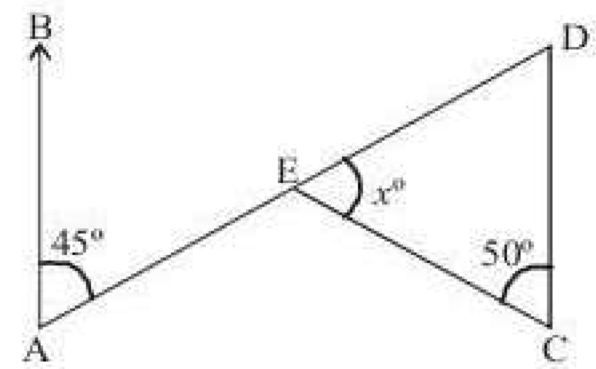
(a) 30° (b) 60°
(c) 45° (d) 65°

12. In ΔABC, $DE \parallel BC$ and $\frac{AD}{DB} = \frac{3}{5}$. If $AC = 5.6$ cm, find AE.



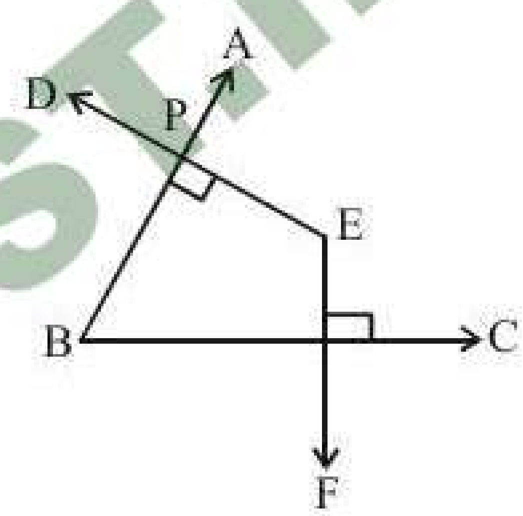
(a) 2.1 cm (b) 3.1 cm
(c) 1.2 cm (d) 2.3 cm

13. In the given figure, $AB \parallel CD$, $\angle BAE = 45°$, $\angle DCE = 50°$ and $\angle CED = x$, then find the value of x.



(a) 85° (b) 95°
(c) 60° (d) 20°

14. In the given figure, $\angle ABC$ and $\angle DEF$ are two angles such that $BA \perp ED$ and $EF \perp BC$, then find value of $\angle ABC + \angle DEF$.

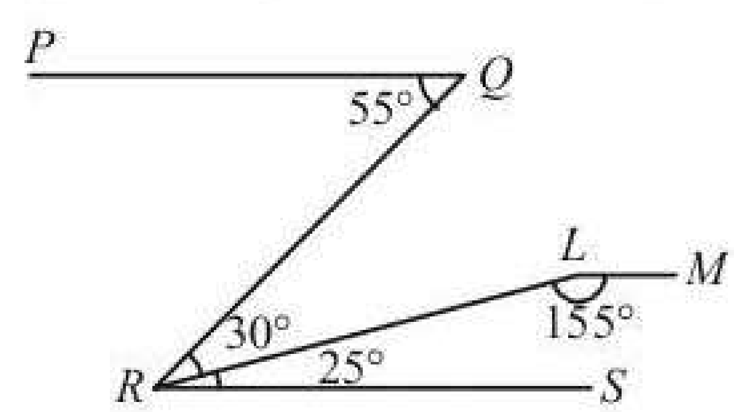


(a) 120° (b) 180°
(c) 150° (d) 210°

15. D and E are the mid-points of AB and AC of ΔABC. If $\angle A = 80°$, $\angle C = 35°$, then $\angle EDB$ is equal to
(a) 100° (b) 115°
(c) 120° (d) 125°

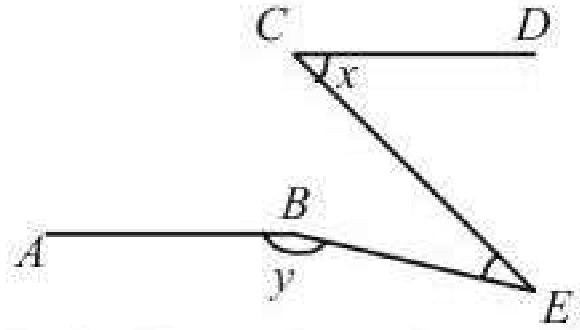
16. In ΔABC, $\angle A < \angle B$. The altitude to the base divides vertex angle C into two parts C_1 and C_2, with C_2 adjacent to BC. Then
(a) $C_1 + C_2 = A + B$ (b) $C_1 - C_2 = A - B$
(c) $C_1 - C_2 = B - A$ (d) $C_1 + C_2 = B - A$

17.

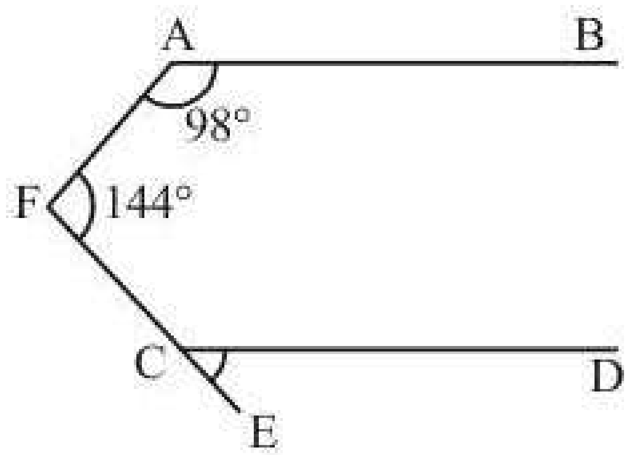


In the figure given above, PQ is parallel to RS, What is the angle between the lines PQ and LM?
(a) 175° (b) 177°
(c) 179° (d) 180°

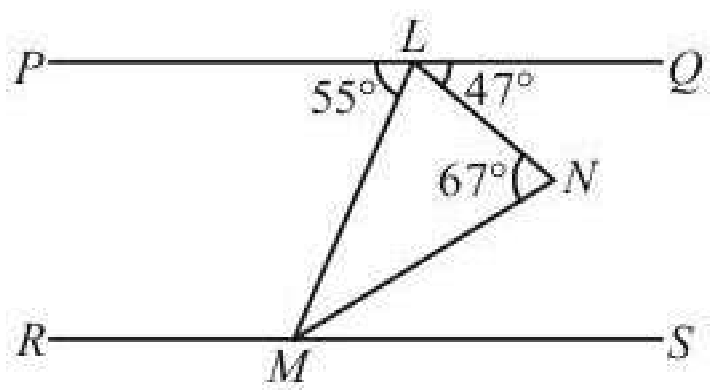
18. In a ΔABC, side AB is extended beyond B, side BC beyond C and side CA beyond A, What is the sum of the three exterior angles?
(a) 270° (b) 305°
(c) 360° (d) 540°

19. 

In the figure given above, AB is parallel to CD. If $\angle DCE = x$ and $\angle ABE = y$, then what is $\angle CEB$ equal to?
(a) $y - x$ (b) $(x + y)/2$
(c) $x + y - (\pi/2)$ (d) $x + y - \pi$

20. 

In the figure given above, AB is parallel to CD. If $\angle BAF = 98^o$ and $\angle AFC = 144^o$, then what is $\angle ECD$ equal to?
(a) 62º (b) 64º
(c) 82º (d) 84º

21. 

In the figure given above, PQ is parallel to RS. What is $\angle NMS$ equal to?
(a) 20º (b) 23º
(c) 27º (d) 47º

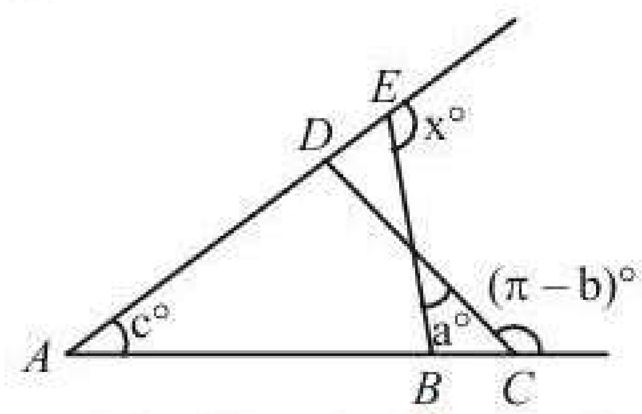
22. Consider the following statements
If two straight lines intersect, then
I. Vertically opposite angles are equal.
II. Vertically opposite angles are supplementary.
III. adjacent angles are complementary.
Which of the statements given above is/are correct?
(a) Only III (b) Only I
(c) I and III (d) II and III

23. Two transversals S and T cut a set of distinct parallel lines. S cuts the parallel lines in points A, B, C, D, and T cuts the parallel lines in points E, F, G and H, respectively. If $AB = 4$, $CD = 3$ and $EF = 12$, then what is the length of GH?
(a) 4 (b) 6
(c) 8 (d) 9

24. The ratio of two complementary angle is 1 : 5. What is the difference between the two angles?
(a) 60º
(b) 90º
(c) 120º
(d) Cannot be determined with the given data

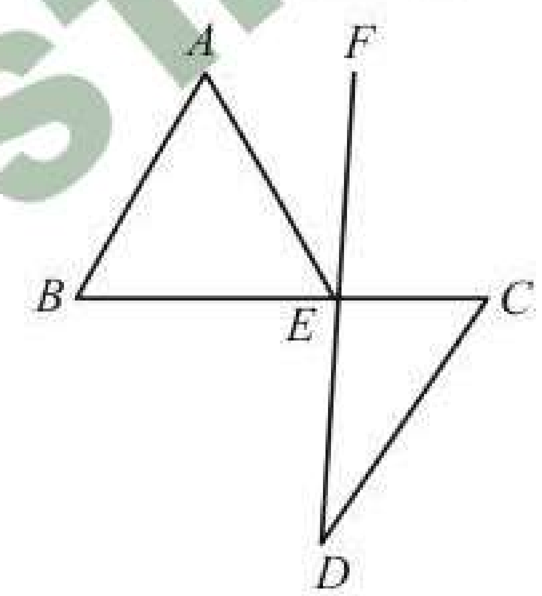
25. In a ΔABC, $\frac{1}{2}\angle A + \frac{1}{3}\angle C + \frac{1}{2}\angle B = 80^o$, then what is the value of $\angle C$?
(a) 35º (b) 40º
(c) 60º (d) 70º

26. The angles x^o, a^o, c^o and $(\pi - b)^o$ are indicated in the figure given below



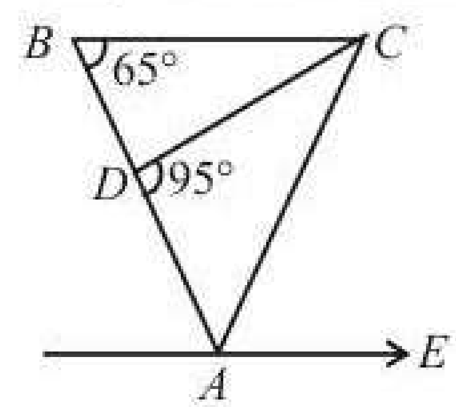
Which one of the following is correct?
(a) $x = a + c - b$ (b) $x = b - a - c$
(c) $x = a + b + c$ (d) $x = a - b + c$

27. In the figure given below, AB is parallel to CD. $\angle ABC = 65^o$, $\angle CDE = 15^o$ and $AB = AE$,



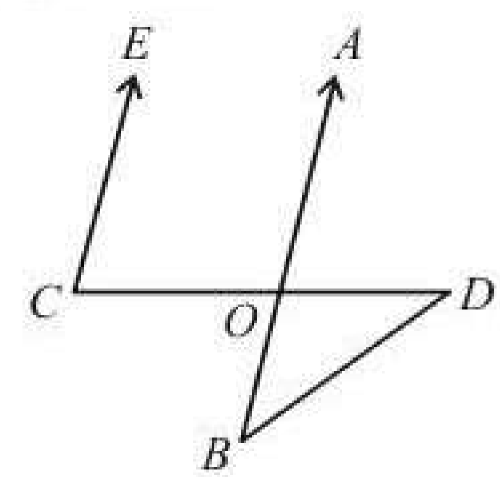
What is the value of $\angle AEF$?
(a) 30º (b) 35º
(c) 40º (d) 45º

28. In the figure given below, ABC is a triangle. BC is parallel to AE. If $BC = AC$, then what is the value of $\angle CAE$?



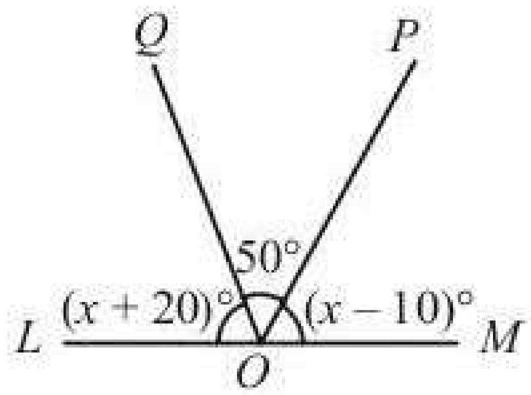
(a) 20º (b) 30º
(c) 40º (d) 50º

29. In the figure given below, EC is parallel to AB, $\angle ECD = 70^o$ and $\angle BDO = 20^o$.



What is the value of $\angle OBD$?
(a) 20º (b) 30º
(c) 40º (d) 50º

30. In the given figure below LOM is a straight line.



What is the value of x^{o} ?

(a) 45° (b) 60°

(c) 70° (d) 80°

31. If the arms of one angle are respectively parallel to the arms of another angle, then the two angles are

(a) neither equal nor supplementary

(b) not equal but supplementary

(c) equal but not supplementary

(d) either equal or supplementary

32. Let OA, OB, OC and OD are rays in the anticlockwise direction such that $\angle AOB = \angle COD = 100°$, $\angle BOC = 82°$ and $\angle AOD = 78°$. Consider the following statements : **(CDS)**

1. AOC and BOD are lines.
2. $\angle BOC$ and $\angle AOD$ are supplementary.

Which of the above statements is /are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

33. There are five lines in a plane, no two of which are parallel. The maximum number of points in which they can intersect is **(CDS)**

(a) 4 (b) 6

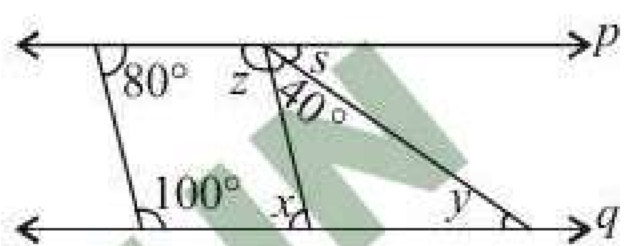
(c) 10 (d) None of the above

34. If a transversal intersects four parallel straight lines, then the number of distinct values of the angles formed will be **(CDS)**

(a) 2 (b) 4

(c) 8 (d) 16

35.



In the figure given above, P and q are parallel lines. What are the values of the angles x, y and z? **(CDS)**

(a) $x = 80°, y = 40°, z = 100°$

(b) $x = 80°, y = 50°, z = 105°$

(c) $x = 70°, y = 40°, z = 110°$

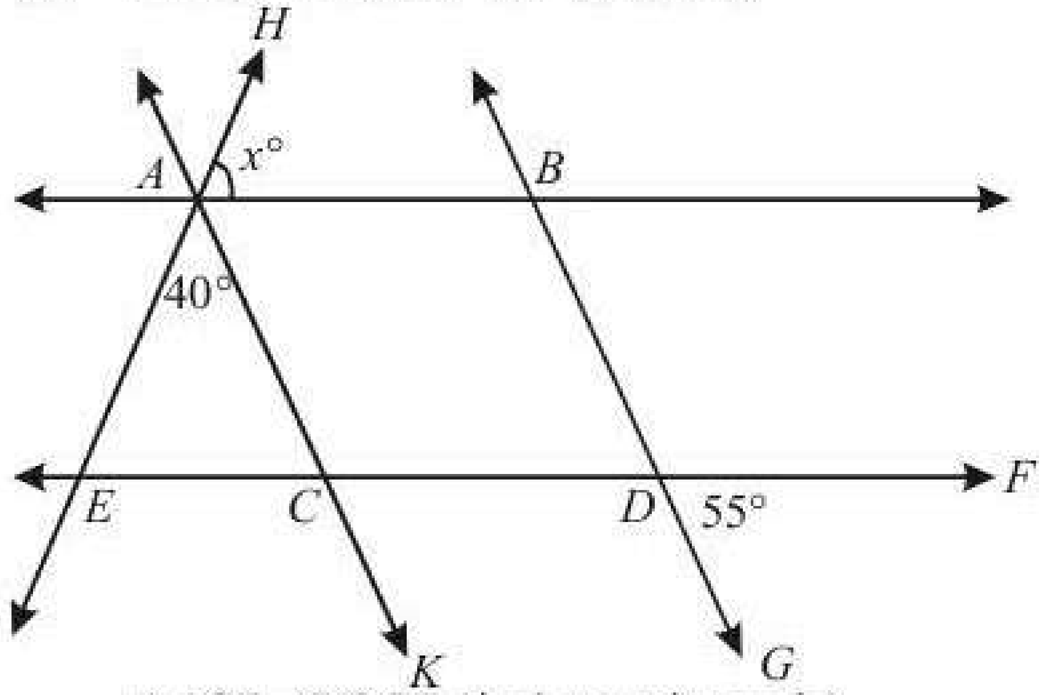
(d) $x = 60°, y = 20°, z = 120°$

HINTS & SOLUTIONS

1. (a) $\angle EDC = \angle BAD = 45^o$ (alternate angles)
$\therefore\ x = DEC = 180^o - (50^o + 45^o) = 85^o$.

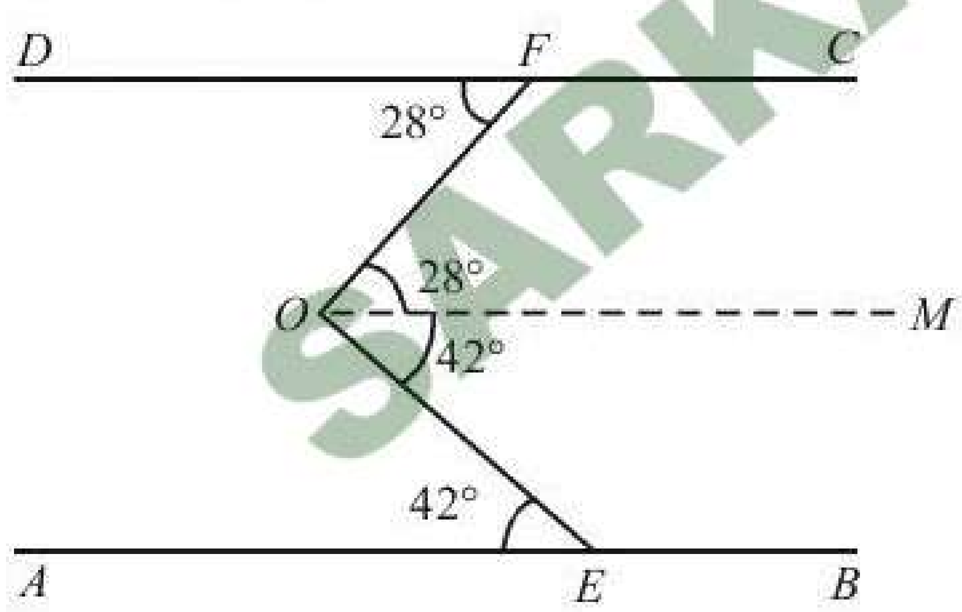
2. (d) $\angle LCD = \angle ALC = 60°$ (alternate angles)
$\angle DCE = \frac{1}{2}\angle LCD = 30°$. ($EC$ is the angle bisector)
$\therefore\ \angle FEC = (180° - 30°) = 150°$.

3. (a) $\angle DCK = \angle FDG = 55°$ (corr. $\angle s$)



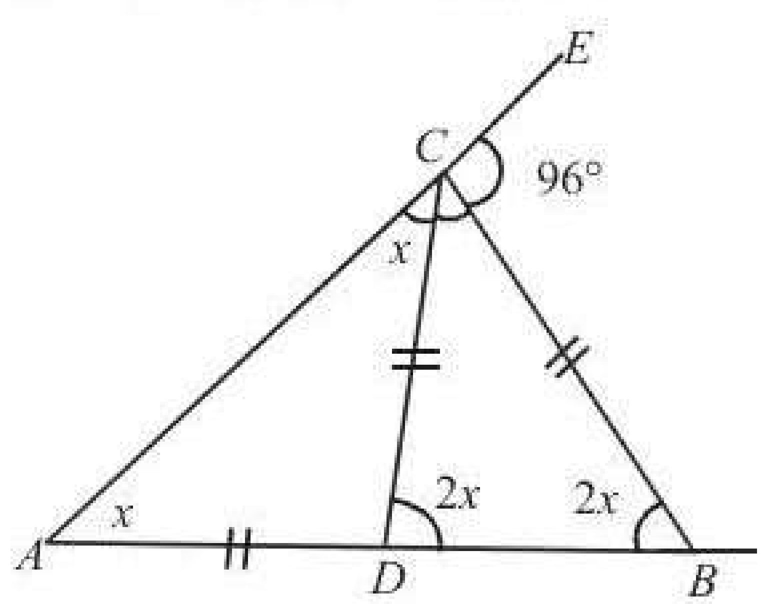
$\angle ACE = 55°$ (Vertical opposite angle)
$\therefore \angle ACE = 180° - (\angle EAC + \angle ACE)$
$\therefore \angle HAB = \angle AEC = 85^o$ (corr. $\angle s$)
Hence, $x = 85°$

4. (c) $\angle DFO = \angle FOM$
and $\angle AEO = \angle EOM$
(since $CD \parallel AB$)



$\therefore\ \angle FOE = (28° + 42°) = 70°$

5. (c)



Let $\angle CAD = \angle ACD = x$
At point C,
$x + (180° - 4x) + 96° = 180°$
$\Rightarrow 180° - 3x + 96° = 180°$
$\therefore\ x = 32°$
Hence, $\angle DBC = 2 \times 32 = 64^o$

6. (d) $\angle ACB = 50°$
$\angle CFO = \angle CEO = 90°$
$\therefore\ \angle FOE = 360° - (90° + 90° + 50°) = 130°$
but $\angle AOB = \angle FOE = 130°$

7. (c) Let the line m cut AB and CD at point P and Q respectively
$\angle DOQ = x$ (exterior angle)
Hence, $Y + 2x$ (corresponding angle)
$\therefore y = x$ ...(i)
Also . $\angle DOQ = x$ (vertically opposite angles)
In ΔOCD, sum of the angles = 180°
$\therefore y + 2y + 2x + x = 180°$
$3x + 3y = 180°$
$x + y = 60$ ...(ii)
From (i) and (ii) $x = y = 30 = 2y = 60$
$\therefore \angle ODS = 180 - 60 = 120°$
$\therefore \theta = 180 - 3x = 180 - 3(30) = 180 - 90 = 90°$.
$\therefore$ The required ratio = 90 : 120 = 3 : 4.

8. (b) Since the sum of all the angle of a quadrilateral is 360^o
We have $\angle ABC + \angle BQE + \angle DEF + \angle EPB = 360^o$
$\therefore\ \angle ABC + \angle DEF = 180^o$
$[\because BPE = EQB = 90^o]$

9. (b) PA, AB, RC and SD are perpendicular to AD. Hence they are a parallel. So, the intercepts are proportional.
$\therefore\ \frac{AB}{BD} = \frac{PQ}{QS}$ $\quad \therefore\ \frac{60}{210} = \frac{x}{360 - x}$
$\therefore\ \frac{2}{7} = \frac{x}{360 - x}$ $\quad \therefore\ x = \frac{720}{9} = 80$
$\therefore PQ = 80$ $\quad \therefore QS = 360 - 80 = 280$
Again, $\frac{BC}{CD} = \frac{QR}{RS}$ $\quad \therefore\ \frac{90}{120} = \frac{y}{280 - y}$
$\therefore\ \frac{3}{4} = \frac{y}{280 - y}$ $\quad \therefore 7y = 280 \times 3\ \therefore y = 120$
$\therefore QR = 120$
$\therefore SR = 280 - 120 = 160$
Another method: $60 : 90 : 120 = 2 : 3 : 4$
$\therefore$ Divide 360 in the ratio 2 : 3 : 4
$\Rightarrow PQ = 80, QR = 120$ and $RS = 160$

10. (a) In ΔQRS; $QR = RS$, therefore $\angle RQS = \angle RSQ$
(because angles opposite to equal sides are equal).
Thus $\angle RQS + \angle RSQ = 180° - 100° = 80°$
$\therefore \angle RQS = \angle RSQ = 40°$
$\angle PQS = 180° - 40° = 140°$
(sum of angles on a line = 180°)
Then again $\angle QRS = \angle QSP$
($\therefore$ angles opposite to equal sides are equal)
Thus $\angle QPS + \angle QSP = 180° - 140° = 40°$
And $\angle QPS = \angle QSP = 20°$

11. (b) In $\Delta ABC, \angle C = 180 - 90 - 30 = 60°$

$\therefore \Delta DCE = \frac{60}{2} = 30°$

Again in ΔDEC, $\angle CED = 180 - 90 - 30 = 60°$

12. (a) In ΔABC, $DE \parallel BC$

By applying basic Proportionality theorem,

$\frac{AD}{DB} = \frac{AE}{EC}$

But $\frac{AD}{DB} = \frac{3}{5}$ (Given)

$\therefore \frac{AE}{EC} = \frac{3}{5}$ or $\frac{AE}{EC + AE} = \frac{3}{5+3}$ or $\frac{AE}{AC} = \frac{3}{8}$

or $\frac{AE}{5.6} = \frac{3}{8} \Rightarrow 8AE = 3 \times 5.6 \Rightarrow AE = 3 \times 5.6/8$

$\therefore AE = 2.1$ cm.

13. (a) $\angle EDC = \angle BAD = 45°$ (alternate angles)

$\therefore x = DEC = 180° - (50° + 45°) = 85°$.

14. (b) Since the sum of all the angle of a quadrilateral is 360°

We have $\angle ABC + \angle BQE + \angle DEF + \angle EPB = 360°$

$\therefore \angle ABC + \angle DEF = 180°$ [$\because$ BPE = EQB = 90°]

15. (b) DE is parallel to BC

So $\angle AED = \angle C = 35°$

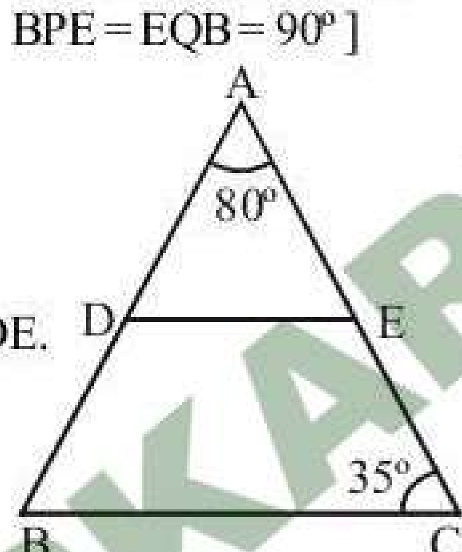
Since $\angle A = 80°$

Then $\angle ADE = 65°$

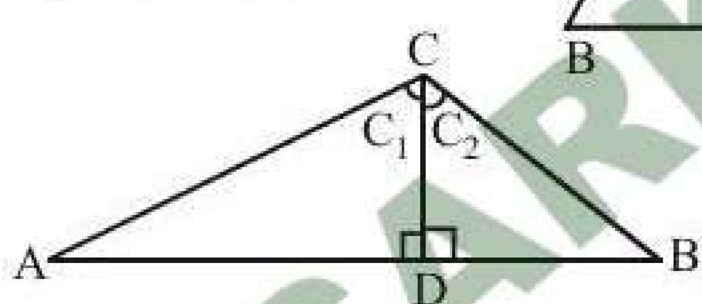
$\angle EDB$ is supplement to $\angle ADE$.

So, $\angle EDB = 180° - \angle ADE$

$= 180° - 65° = 115°$



16. (c)



In ΔADC,

$A + D + C_1 = 180°$; $A + C_1 = 180° - 90° = 90°$

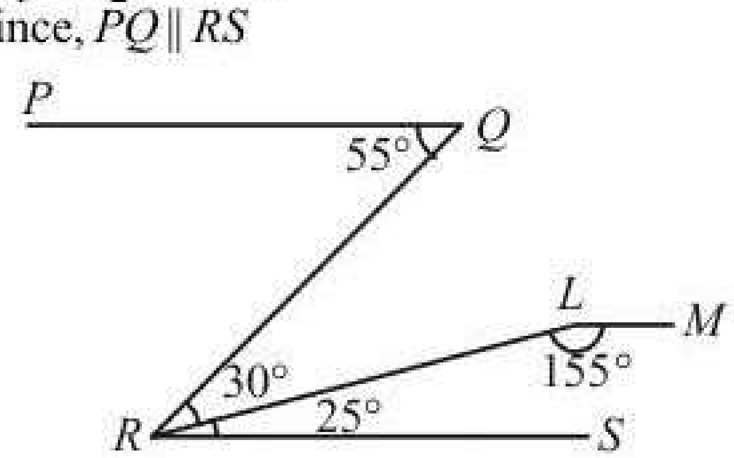
In ΔBDC,

$B + D + C_2 = 180°$; $B + C_2 = 180° - 90° = 90°$

$A + C_1 = B + C_2$

$C_1 - C_2 = B - A$

17. (d) Since, $PQ \parallel RS$



$\therefore \angle PQR = \angle QRS$ (alternate angle)

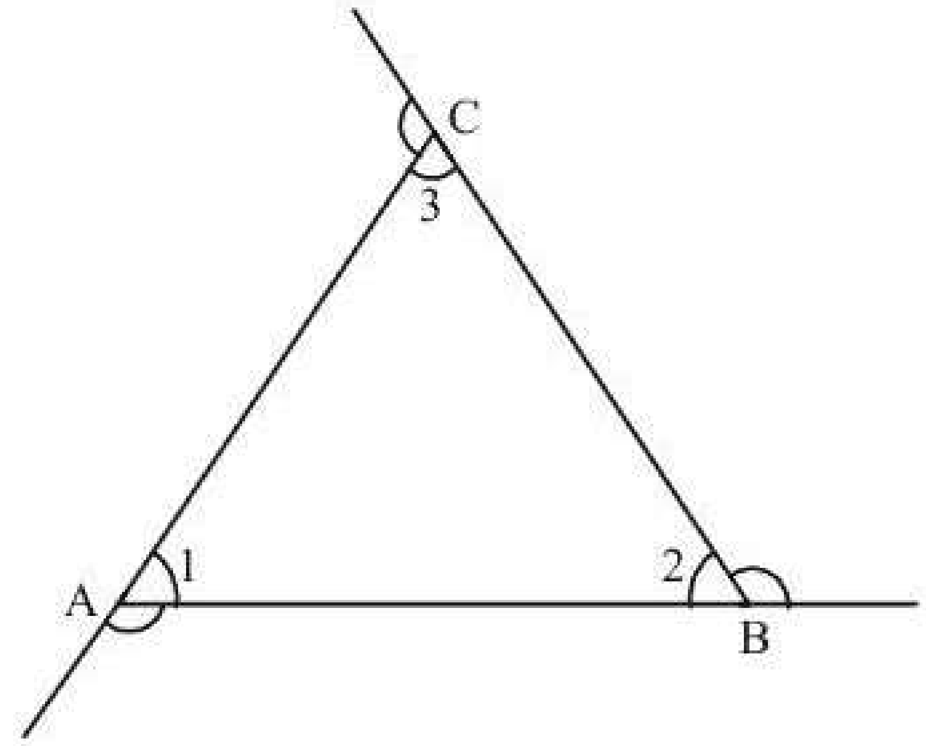
and $\angle SRL + \angle RLM = 180° \Rightarrow RS \parallel LM$ ...(ii)

From Eqs.(i) and (ii),

$PQ \parallel LM$

Therefore, angle between the lines PQ and LM is 180°

18. (c)



Sum of the three exterior angles

$= (\angle 1 + \angle 2) + (\angle 2 + \angle 3) + (\angle 3 + \angle 1)$

$= 2(\angle 1 + \angle 2 + \angle 3)$

(Sum of the interior angles are 180°)

$= 2 \times 180° = 360°$

Sum of exterior angles of any polygon = 360°

Alternate Method

Sum of the Exterior angles

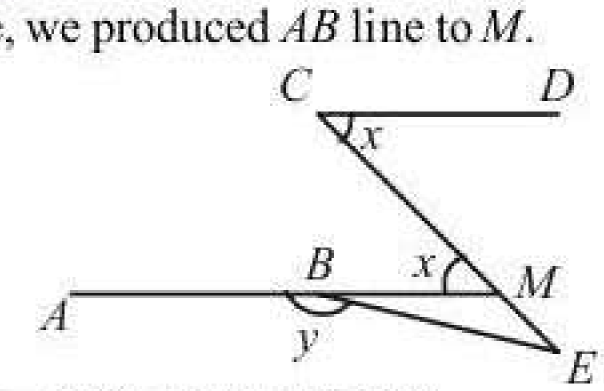
$= (180° - \angle A) + (180° - \angle B) + (180° - \angle C)$

$= 540° - (\angle A + \angle B + \angle C)$

$= 540° - 180°$ (Sum of angles of a triangle is 180°)

$= 360°$

19. (d) Here, we produced AB line to M.



Since, AM is parallel to CD.

$\angle DCM = \angle BMC = x$ (alternate angle)

Also, ABM is a straight line.

$\angle EBM = \pi - y$

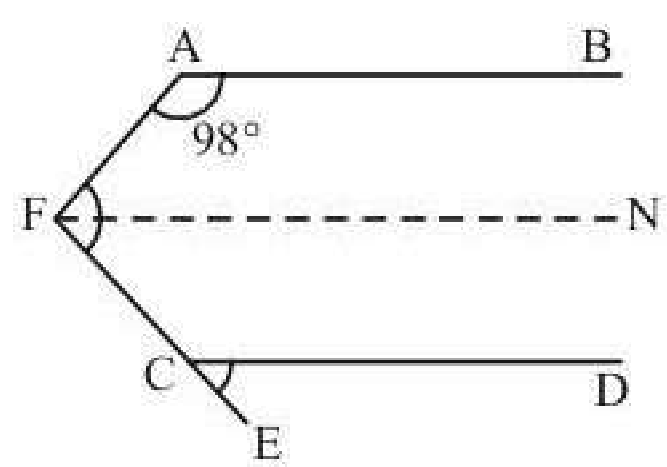
Now in ΔBEM.

$\angle B + \angle M + \angle E = \pi$

$\Rightarrow \pi - y + \pi - x + \angle E = \pi$

$\Rightarrow \angle E = x + y - \pi$

20. (a)



parallel to AB and CD.

$\therefore \angle AFN = 180° - 98° = 82°$

$\Rightarrow \angle CFN = 144° - 82° = 62°$

$\therefore \angle FCD = 180° - 62° = 118°$

$\Rightarrow \angle ECD = 180° - 118° = 62°$

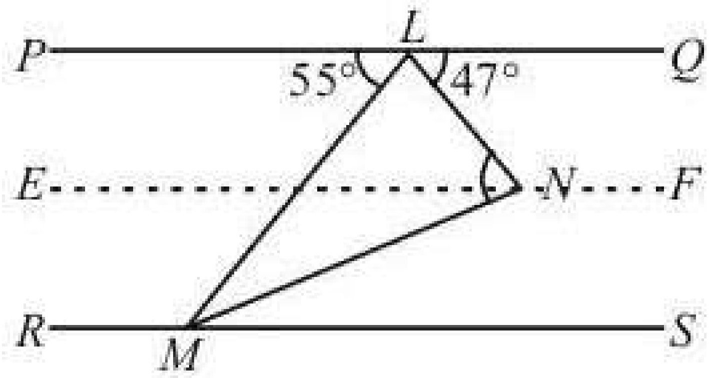
21. (a) $\angle PLM = \angle LMS$ (alternate angle)

$= 55°$

Let draw EF line which is parallel to PQ and bisects by LN

Then, $\angle QLN = \angle LNE = 47^o$

$\because \angle ENL + \angle MNE = 67^o$



Then, $\angle LNE = 47^o$

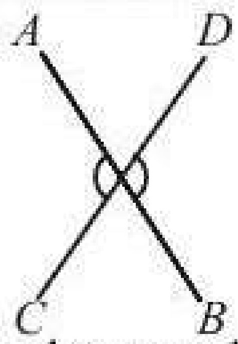
$\Rightarrow 47^\circ + \angle MNE = 67^o$

$\angle MNE = 67^o - 47^o$

$\angle MNE = 20^o$

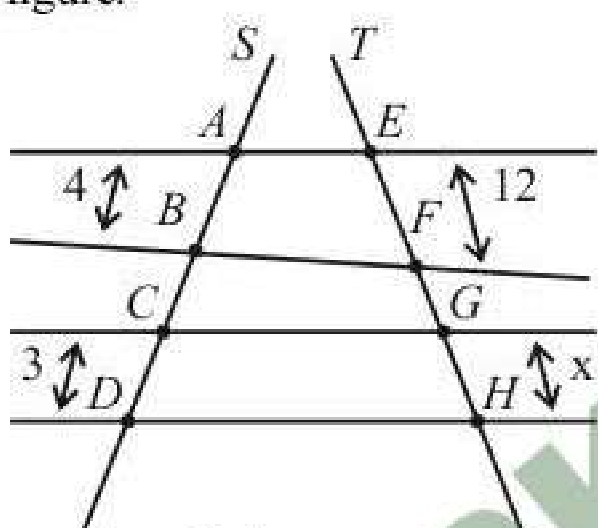
Similarly, $EF \| RS$, then $\angle ENM = \angle NMS = 20^o$

22. (b) AB and CD are two lines.



We know that when two lines intersect each other then opposite vertically angles are equal

23. (d) From figure.



By proportionality law,

$$\frac{AB}{CD} = \frac{EF}{GH} \Rightarrow \frac{4}{3} = \frac{12}{x}$$

$\therefore \quad x = 3 \times 3 = 9$

24. (a) Given that, $\frac{\alpha}{\beta} = \frac{1}{5} \Rightarrow \alpha = k$ and $\beta = 5k$ (say)

or complementary angles,

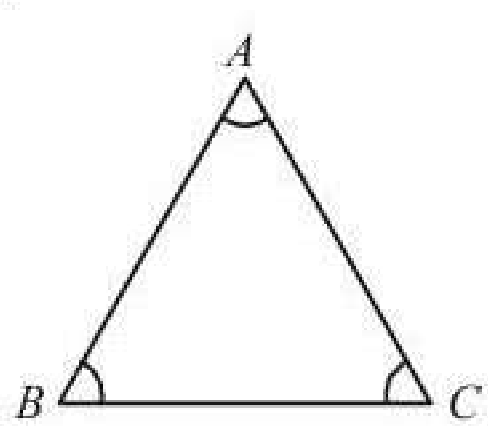
$\alpha = 90^o - \beta \Rightarrow k = 90^o - 5k$

$\Rightarrow \quad k = 15^o$

$\therefore \quad \alpha = 15^o$ and $\beta = 75^o$

$\therefore$ Difference between angles $= 75^o - 15^o = 60^o$

25. (c) We know



$\angle A + \angle B + \angle C = 180^o$

$\angle A + \angle B = 180^o - \angle C$ ...(i)

$$\frac{\angle A}{2} + \frac{\angle B}{2} + \frac{\angle C}{3} = 80^o \text{ (Given)}$$

$$\left(\frac{\angle A + \angle B}{2}\right) + \frac{\angle C}{3} = 80^o$$

from equation (i)

$$\Rightarrow \quad \frac{180^o - \angle C}{2} + \frac{\angle C}{3} = 80^o$$

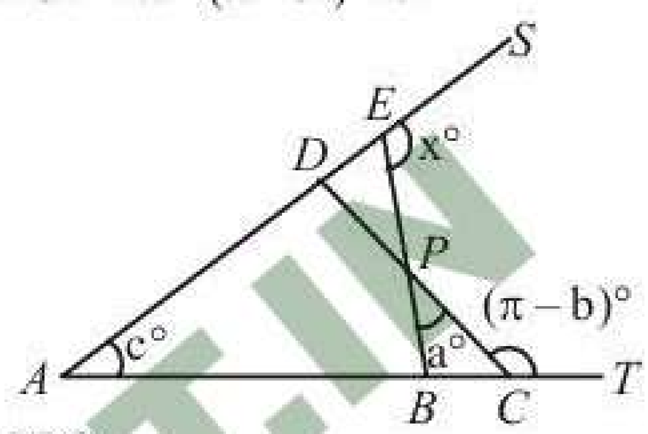
$$\Rightarrow \quad \frac{180^o \times 3 - 3\angle C + 2\angle C}{6} = 80^o$$

$\Rightarrow \quad \angle C = (540^o - 480^o)$

$\Rightarrow \quad \angle C = 60^o$

26. (c) $\because \angle PCT + \angle PCB = \pi$

$\angle PCB = \pi - (\pi - b^o) = b^o$



In ΔBPC,

$\angle PCB + \angle BPC + \angle PBC = \pi$

$\Rightarrow \angle PBC = \pi - \angle PCB - \angle BPC$

$= \pi - b^o - a^o$

$\because \angle ABE + \angle EBC = \pi$ (linear pair)

$\Rightarrow \angle ABE = \pi - \angle PBC = \pi - (\pi - b^o - a^o)$

$= a^o + b^o$ $(\because \angle PBC = \angle EBC)$

Now,

Sum of two interior angles = Exterior angle

$\therefore$ in $\Delta ABE, \angle EAB + \angle ABE = \angle BES \Rightarrow c^o + b^o + a^o = x^o$

$\therefore x^o = a^o + b^o + c^o$

27. (b) $AB = AE$ (Given)

$\therefore \quad \angle ABC = 65^o = \angle AEB$

$AB \| CD$, then

$\angle ABE = \angle DCE = 65^o$ (Alternate angles)

In ΔDCE,

$\angle D + \angle C + \angle E = 180^o$

$\angle E = 180^o - 65^o - 15^o = 100^o$

BC and FD intersect each other at E, then

$\angle BEF = \angle DEC = 100^o$

$\angle AEF = 100^o - 65^o = 35^o$

(Because $\angle BEF = \angle BEA + \angle AEF$)

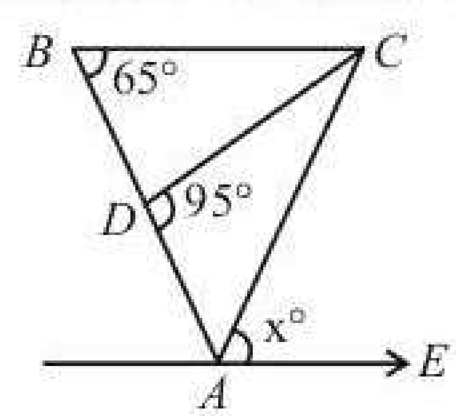
28. (d) Given that, $BC \| AE$

$\angle CBA + \angle EAB = 180^o$

$\Rightarrow \quad \angle EAB = 180^o - 65^o = 115^o$

$\because \quad BC = AC$

Hence, ΔABC is an isosceles triangle



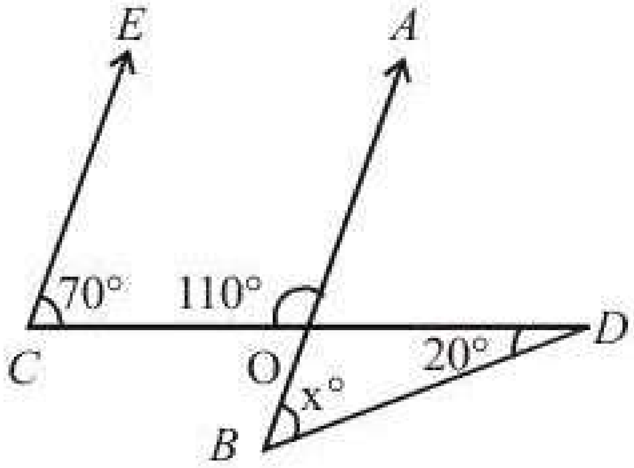
$\Rightarrow \quad \angle CBA = \angle CAB = 65^o$

Now, $\angle EAB = \angle EAC + \angle CAB$

$\Rightarrow \quad 115^o = x + 65^o \Rightarrow x = 50^o$

29. (d) Given that, $EC \| AB$

$\angle ECO + \angle AOC = 180^o$



$\Rightarrow \angle AOC = 180^\circ - 70^\circ = 110^\circ$

$\therefore \angle BOD = \angle AOC = 110^\circ$ (alternate angle)

Now, in ΔOBD

$\angle BOD + \angle ODB + \angle DBO = 180^\circ$

$\therefore \quad 110^\circ + 20^\circ + x^\circ = 180^\circ \Rightarrow x^\circ = 50^\circ$

30. (b) From the given figure. We know that sum of all angles is 180° because of straight lines.

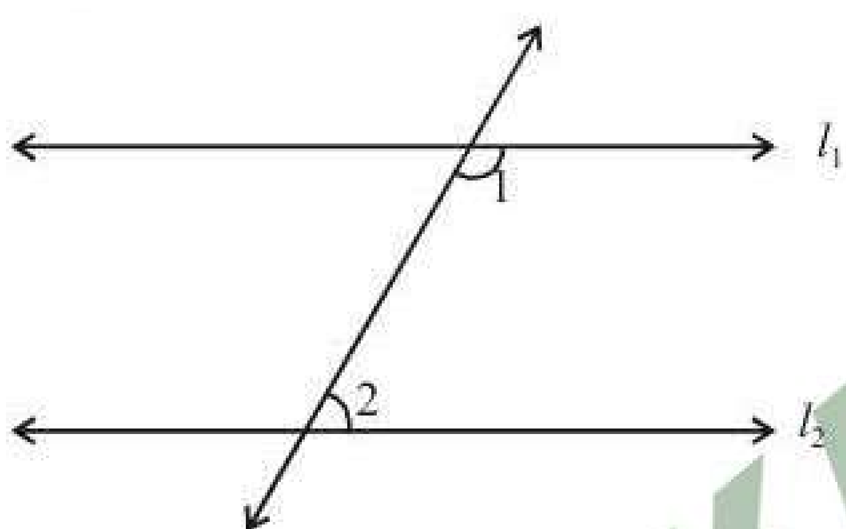
$\angle LOQ + \angle QOP + \angle POM = 180^\circ$

$\therefore \quad (x^\circ + 20^\circ) + 50^\circ + (x^\circ - 10^\circ) = 180^\circ$

$\Rightarrow \quad 2x^\circ + 60^\circ = 180^\circ \Rightarrow 2x^\circ = 120^\circ$

$x^\circ = 60^\circ$

31. (b) l_1 and l_2 are two parallel lines and $\angle 1$ and $\angle 2$ are interior angles on the same side of the transversal.

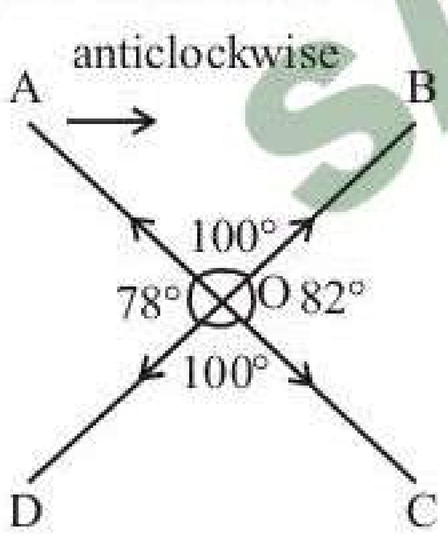


$\angle 1 \neq \angle 2$

$\angle 1 + \angle 2 = 180^\circ$.

Therefore, these are supplementary angles or consecutive interior angles.

32. (d)



Statement 1

AOC and BOD are not lines because $\angle AOD + \angle COD \neq 180^\circ$ and $\angle AOB + \angle COB \neq 180^\circ$

So, it is not correct.

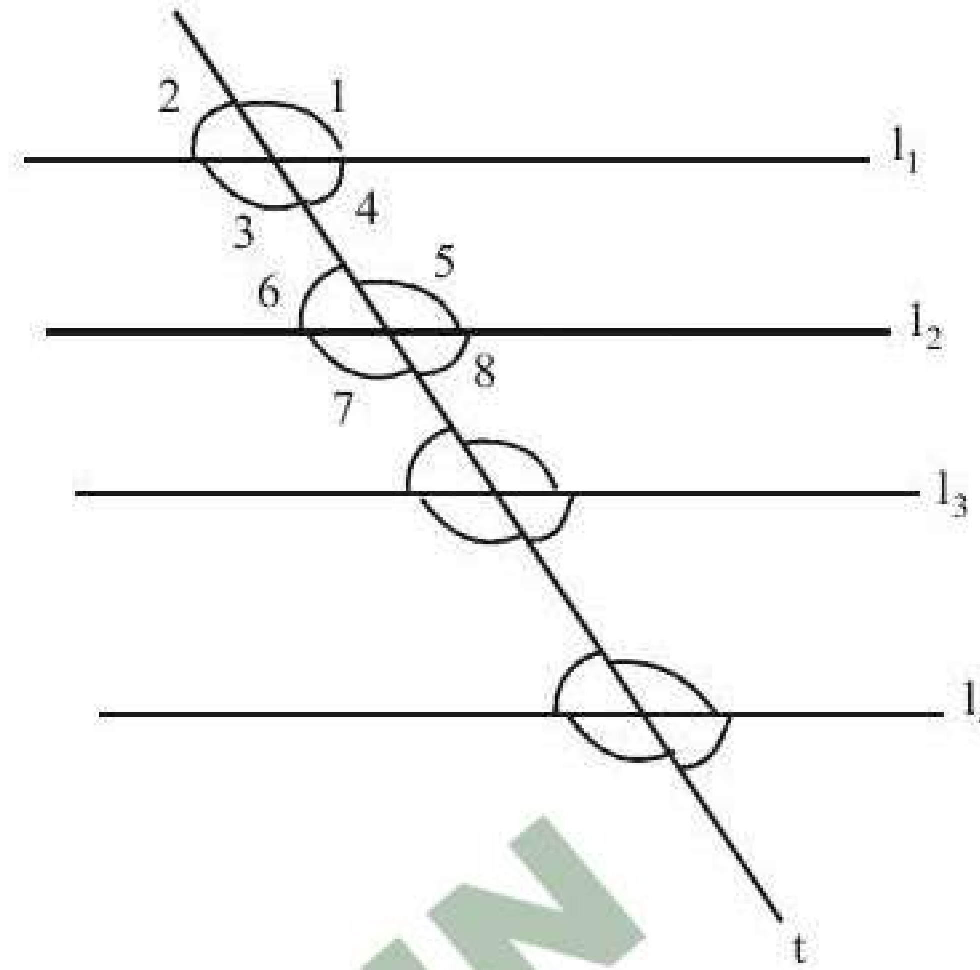
Statement 2

$\angle BOC$ and $\angle AOD$ are 82° and 78° respectively, so it is not supplementry angle.

So, neither statement 1 nor 2 are correct.

33. (d)

34. (a) Let l_1, l_2, l_3 and l_4 be your straight lines and t be a transversal.



$\angle 1 = \angle 3$

$\angle 2 = \angle 4$

[alternate angles]

Similarly angles formed by

l_2, l_3 and l_4 are also equal in same way as l_1

corresponding angles

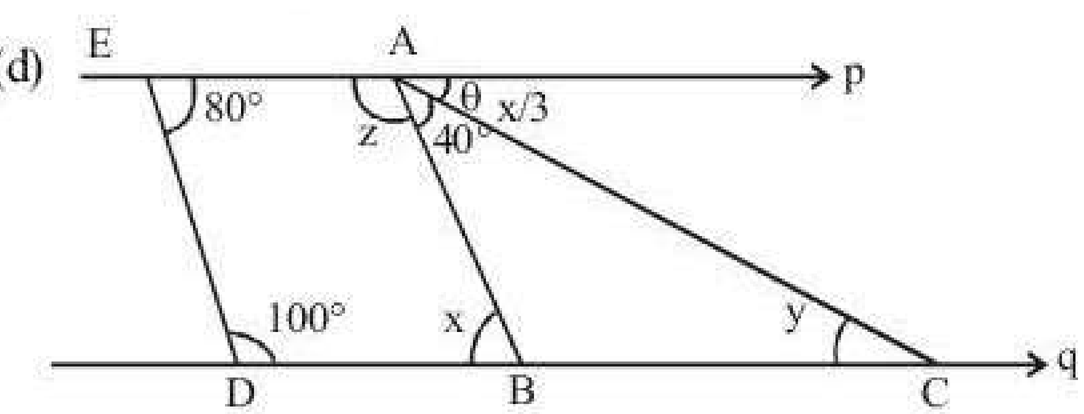
$\angle 1 = \angle 5$ & $\angle 5 = \angle 7$

$\angle 6 = \angle 2$ & $\angle 6 = \angle 8$ and so on

Only two distinct angles are formed

$\therefore$ Option (a) is correct.

35. (d)



Now ABDE is a quadrilateral

AB works as a transversal on parallel lines.

$\Rightarrow \quad \angle x = 40^\circ + \frac{x}{3}$ (alternative angles)

$\Rightarrow \quad x - \frac{x}{3} = 40$

$\Rightarrow \quad x = \frac{40 \times 3}{2}$

$\Rightarrow \quad x = 60^\circ$

$\Rightarrow \quad \frac{x}{3} = 20^\circ = \angle y$ (alternate angles)

$\Rightarrow \quad x = 60^\circ$

$y = 20^\circ$

Since ABDE is a quadrilateral.

$\Rightarrow \quad \angle z + \angle x = 180^\circ$

$\Rightarrow \quad \angle z = 180 - \angle x$

$= 180^\circ - 60^\circ$

$= 120^\circ$