# Class X Session 2023-24
# Subject - Mathematics (Basic)
# Sample Question Paper - 4

**Time Allowed: 3 hours** **Maximum Marks: 80**

**General Instructions:**

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

**Section A**

1. HCF of two numbers is 113, their LCM is 56952. If one number is 904, the second number is **[1]**

a) 7791 b) 7911

c) 7719 d) 7119

2. The LCM of two numbers is 1200. Which of the following cannot be their HCF? **[1]**

a) 500 b) 200

c) 600 d) 400

3. The values of k for which the quadratic equation $2x^2 – kx + k = 0$ has equal roots is **[1]**

a) 0 only b) 8 only

c) 0, 8 d) 4

4. Solve for x and y in the following question. **[1]**

$\frac{2}{x+2y} + \frac{1}{2x-y} + \frac{5}{9} = 0, \frac{9}{x+2y} + \frac{6}{2x-y} + 4 = 0$

a) x = 2, y = 1 b) x = 1, y = 2

c) x = 2, y = $\frac{1}{2}$ d) x =$\frac{1}{2}$, y = 2

5. If one root of the equation $3x^2 - 10x + 3 = 0$ is $\frac{1}{3}$ then the other root is **[1]**

a) $\frac{1}{3}$ b) 3

c) $\frac{-1}{3}$ d) -3

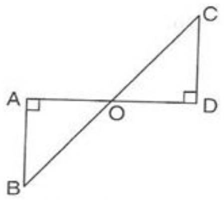
6. The points A(9, 0), B(9, 6), C(-9, 6) and D(-9, 0) are the vertices of a **[1]**

a) rhombus
b) trapezium
c) rectangle
d) square

7. In $\triangle$ ABC, a line XY parallel to BC cuts AB at X and AC at Y. If BY bisects $\angle XYC$, then **[1]**

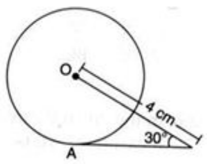
a) BC = CY
b) BC = BY
c) BC $\neq$ BY
d) BC $\neq$ CY

8. In the given figure $\Delta ABO \sim \Delta DCO$. If CD = 2cm, AB = 3 cm, OC = 3.2 cm, OD = 2.4 cm, then **[1]**




a) OA = 3 cm, OB = 4 cm.
b) OA = 4.3 cm, OB = 3.5 cm.
c) OA = 3.6 cm, OB = 4.8 cm.
d) OA = 3.2 cm, OB = 4.6 cm

9. AP is a tangent to the circle with centre O such that OP = 4 cm and $\angle OPA = 30^\circ$. Then, AP is equal to **[1]**




a) $3\sqrt{2}$ cm
b) $2\sqrt{3}$ cm.
c) 2 cm
d) $2\sqrt{2}$

10. If $\sqrt{3}\tan\theta = 3\sin\theta$, then the value of $\sin^2\theta - \cos^2\theta$ is **[1]**

a) 1
b) $\frac{1}{2}$
c) 0
d) $\frac{1}{3}$

11. If the length of a shadow of a tower is increasing, then the angle of elevation of the sun is **[1]**

a) neither increasing nor decreasing
b) zero
c) decreasing
d) increasing

12. 5 $\cot^2$ A - 5 $\operatorname{cosec}^2$ A = **[1]**

a) 0
b) 5
c) 1
d) -5

13. The length of an arc of a sector of angle $\theta°$ of a circle with radius R is **[1]**

a) $\frac{\pi R^2\theta}{180}$
b) $\frac{\pi R^2\theta}{360}$
c) $\frac{2\pi R\theta}{360}$
d) $\frac{2\pi R\theta}{180}$

14. The length of the minute hand of a clock is 21 cm. The area swept by the minute hand in 10 minutes is **[1]**

a) 252 $cm^2$
b) 126 $cm^2$
c) 231 $cm^2$
d) 210 $cm^2$

15. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is **[1]**

a) 21

b) 7

c) 28

d) 14

16. Median = ? **[1]**

a) $l+\left\{h\times\frac{\left(cf-\frac{N}{2}\right)}{f}\right\}$

b) $l-\left\{h\times\frac{\left(\frac{N}{2}-cf\right)}{f}\right\}$

c) $l+\left\{h\times\frac{\left(\frac{N}{2}-cf\right)}{f}\right\}$

d) none of these

17. If a sphere is inscribed in a cube, then the ratio of the volume of the cube to the volume of the sphere is **[1]**

a) $6 : \pi$

b) $\pi : 6$

c) $\pi : 4$

d) $4 : \pi$

18. The mean of the first 10 multiples of 6 is **[1]**

a) 3.3

b) 33

c) 35

d) 34

19. **Assertion (A):** Point A is on the y-axis at a distance of 4 units from the origin. If the coordinates of the point B are (-3, 0), then the length of AB is 5 units. **[1]**

**Reason (R):** Distance between points A($x_1$, $y_1$) and B($x_2$, $y_2$) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ .

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** 3 is a rational number. **[1]**

**Reason (R):** The square roots of all positive integers are irrationals.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

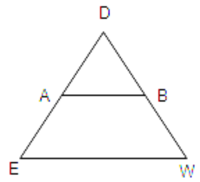
c) A is true but R is false.

d) A is false but R is true.

## Section B

21. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the pair of linear equation is consistent, or inconsistent: **[2]**
5x - 3y = 11; -10x + 6y = -22

22. In $\triangle DEW$, AB || EW. If, AD= 4 cm, DE = 12 cm and DW = 24 cm, find the value of DB. **[2]**




OR

If one diagonal of a trapezium divides the other diagonal in the ratio 1: 2, prove that one of the parallel sides is double the other.

23. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle is bisected at the point of contact. **[2]**

24. If 3 cot A = 4, find the value of $\frac{cosec^2A+1}{cosec^2A-1}$. **[2]**

25. In a circle with centre O and radius 5 cm, AB is a chord of length $5\sqrt{3}$ cm. Find the area of sector AOB. **[2]**

OR

The perimeter of a certain sector of a circle of radius 6.5 cm is 31 cm. Find the area of the sector.

**Section C**

26. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet again at the starting point? **[3]**

27. If $\alpha, \beta$ are the zeroes of the $x^2 + 7x + 7$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$. **[3]**

28. If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction? Solve the pair of the linear equation obtained by the elimination method. **[3]**

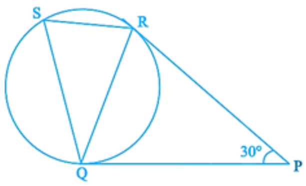
OR

Use elimination method to find all possible solutions of the following pair of linear equations

ax + by - a + b = 0 and bx - ay - a - b = 0

29. In the given figure, tangents PQ and PR are drawn to a circle such that $\angle$RPQ = $30^o$. A chord RS is drawn parallel to tangent PQ. Find the $\angle$RQS. **[3]**

Hint: Draw a line through Q and perpendicular to QP.]




30. If $\angle B$ and $\angle Q$ are acute angles such that sin B = sin Q, then prove that $\angle B = \angle Q$. **[3]**

OR

Prove: $\frac{1}{(\cot A)(\sec A) - \cot A} - \operatorname{cosec} A = \operatorname{cosec} A - \frac{1}{(\cot A)(\sec A) + \cot A}$

31. A group consists of 12 persons, of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is **[3]**

i. extremely patient,
ii. extremely kind or honest.

which of the above values you prefer more?

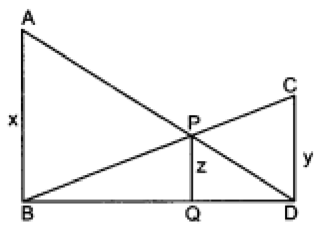
**Section D**

32. The sum of ages of a father and his son is 45 years. Five years ago, the product of their ages (in years) was 124. Determine their present ages. **[5]**

OR

The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

33. In figure AB || PQ || CD, AB = x units, CD = y units and PQ = z units, prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ **[5]**




34. A building is in the form of a cylinder surmounted by a hemispherical dome. The base diameter of the dome is **[5]**

equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains $67\frac{1}{21}\text{m}^3$ of air.

OR

A solid consisting of a right cone standing on a hemisphere is placed upright in a right circular cylinder full of water and touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm, the radius of the hemisphere is 60 cm and height of the cone is 120 cm, assuming that the hemisphere and the cone have common base.

35. The following table shows the ages of the patients admitted in a hospital during a year: **[5]**

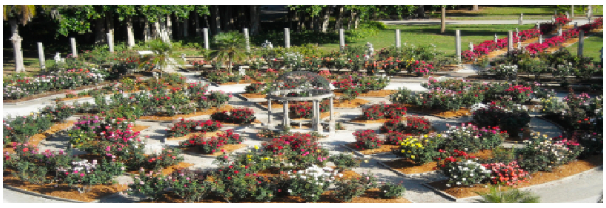
| **Age (in years)** | 5-15 | 15-25 | 25-35 | 35-45 | 45-55 | 55-65 |
|---|---|---|---|---|---|---|
| **Number of patients** | 6 | 11 | 21 | 23 | 14 | 5 |

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

**Section E**

36. **Read the text carefully and answer the questions:** **[4]**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.



In their garden bed, there are 23 rose plants in the first row, 21 are in the $2^{nd}$, 19 in $3^{rd}$ row and so on. There are 5 plants in the last row.

(i) How many rows are there of rose plants?

(ii) Also, find the total number of rose plants in the garden.

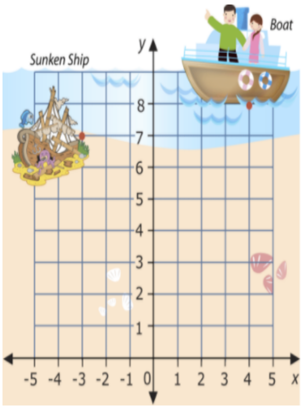
**OR**

If total number of plants are 80 in the garden, then find number of rows?

(iii) How many plants are there in 6th row.

37. **Read the text carefully and answer the questions:** **[4]**

Mary and John are very excited because they are going to go on a dive to see a sunken ship. The dive is quite shallow which is unusual because most sunken ship dives are found at depths that are too deep for two junior divers. However, this one is at 40 feet, so the two divers can go to see it.



They have the following map to chart their course. John wants to figure out exactly how far the boat will be from the sunken ship. Use the information in this lesson to help John figure out the following.

(i) What are the coordinates of the boat and the sunken ship respectively?

(ii) How much distance will Mary and John swim through the water from the boat to the sunken ship?

**OR**

If the distance between the points (x, -1) and (3, 2) is 5, then what is the value of x?

(iii) If each square represents 160 cubic feet of water, how many cubic feet of water will Mary and John swim through from the boat to the sunken ship?

38. **Read the text carefully and answer the questions:** **[4]**

Two trees are standing on flat ground. The angle of elevation of the top of Both the trees from a point X on the ground is $60^{o}$. If the horizontal distance between X and the smaller tree is 8 m and the distance of the top of the two trees is 20 m.


(i) Calculate the distance between the point X and the top of the smaller tree.

(ii) Calculate the horizontal distance between the two trees.

**OR**

Find the height of big tree.

(iii) Find the height of small tree.

# Solution

**Section A**

1.

**(d)** 7119

**Explanation:** LCM $\times$ HCF = Product of two numbers

56952 $\times$ 113 = 904 $\times$ second number

$\frac{56952\times113}{904}$ = second number

Therefore, second number = 7119

2. **(a)** 500

**Explanation:** It is given that the LCM of two numbers is 1200 .

We know that the HCF of two numbers is always the factor of LCM.

500 is not the factor of 1200.

So this cannot be the HCF.

3.

**(c)** 0, 8

**Explanation:** If a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has two equal roots, then its discriminant value will be equal to zero i.e., $D = b^2 - 4ac = 0$

Given, $2x^2 - kx + k = 0$

For equal roots,

$D = b^2 - 4ac = 0$

$\Rightarrow (-k)^2 - 4(2)(k) = 0$

$\Rightarrow k^2 - 8k = 0$

$\Rightarrow k(k - 8) = 0$

$\therefore k = 0,8$

4.

**(d)** $x = \frac{1}{2}$, $y = 2$

**Explanation:** We have, $\frac{2}{x+2y} + \frac{1}{2x-y} + \frac{5}{9} = 0$

and $\frac{9}{x+2y} + \frac{6}{2x-y} + 4 = 0$

Let $\frac{1}{x+2y}$ = a and $\frac{1}{2x-y}$ = b

Thus, equations would reduce to

$2a + b = -\frac{5}{9}$ ...(i)

and $9a + 6b = -4$ ...(ii)

Solving (i) and (ii), we get $a = \frac{2}{9}$ and $b = -1$

$\Rightarrow \frac{2}{9} = \frac{1}{x+2y}$ and $-1 = \frac{1}{2x-y}$

$\Rightarrow 2x + 4y = 9$ ...(iii)

and $2x - y = -1$ ...(iv)

Solving (iii) and (iv), we get $y = 2$ and $x = \frac{1}{2}$

5.

**(b)** 3

**Explanation:** Given:

$3x^2 - 10x + 3 = 0$

One root of the equation is $\frac{1}{3}$.

Let the other root be $\alpha$.

We know that:

Product of the roots $= \frac{c}{a}$

$\Rightarrow \frac{1}{3} \times \alpha = \frac{3}{3}$

$\Rightarrow \alpha$ = 3

6.

**(c)** rectangle

**Explanation:** A (9, 0), B(9, 6), C( - 9, 6) and D( - 9, 0) are the given vertices.

Then,

$AB^2 = (9 - 9)^2 + (6 - 0)^2$

$= (0)^2 + (6)^2$ = 0 + 36 = 36 units

$BC^2 = ( - 9 - 9)^2 + (6 - 6)^2$

$= (-18)^2 + (0)^2$ = 324 + 0 = 324 units

$CD^2 = (- 9 + 9)^2 + (0 - 6)^2 = (0)^2 + (-6)^2$ = 0 + 3 = 36 units

$DA^2 = ( - 9 - 9)^2 + (0 - 0)^2 = (- 18)^2 + (0)^2$ = 324 + 0 = 324 units

Therefore, we have:

$AB^2 = CD^2$ and $BC^2 = DA^2$

Now, the diagonals are:

$AC^2 = ( - 9 - 9)^2 + (6 - 0)^2 = (- 18)^2 + (6)^2$ = 324 + 36 = 360 units

$BD^2 = ( - 9 - 9)^2 + (0 - 6)^2 = ( - 18)^2 + ( - 6)^2$ = 324 + 36 = 360 units

Therefore,

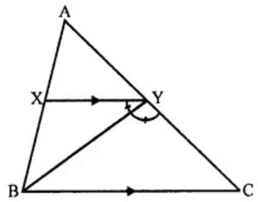
$AC^2 = BD^2$

Hence, *ABCD* is a rectangle.

7. **(a)** BC = CY

**Explanation:** In $\triangle$ ABC, XY || BC

Also BY is the bisector $\angle XYC$




$\angle XYB = \angle CYB$...... (i)

$XY \| BC$

$\angle XYB = \angle YBC$ ( Alternate angles are equal).......(ii)

$\angle CYB = \angle YBC$

BC = CY

8.

**(c)** OA = 3.6 cm, OB = 4.8 cm.

**Explanation:** Since $\Delta \mathrm{ABO} \sim \Delta \mathrm{DCO}$,

$\therefore \frac{\mathrm{OA}}{\mathrm{OD}} = \frac{\mathrm{AB}}{\mathrm{CD}}$

$\Rightarrow \frac{\mathrm{OA}}{2.4} = \frac{3}{2}$

$\Rightarrow$ OA = 3.6 cm

Again, Since $\Delta \mathrm{ABO} \sim \Delta \mathrm{DCO}$,

$\therefore \frac{\mathrm{OB}}{\mathrm{OC}} = \frac{\mathrm{AB}}{\mathrm{CD}}$

$\Rightarrow \frac{\mathrm{OB}}{3.2} = \frac{3}{2}$

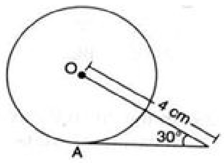
$\Rightarrow$ OB = 4.8 cm

Therefore, OA = 3.6 cm, OB = 4.8 cm

9.

**(b)** $2\sqrt{3}$ cm.

**Explanation:** Construction: Joined OA which is perpendicular to AP.




Since, OA $\perp$ AP, therefore, $\triangle$OAP is a right-angled triangle.

$\therefore \cos 30^\circ = \frac{\text{AP}}{\text{OP}}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\text{AP}}{4}$

$\Rightarrow \text{AP} = 2\sqrt{3}$ cm

10.

**(d)** $\frac{1}{3}$

**Explanation:** Given: $\sqrt{3} \tan \theta = 3 \sin \theta$

$\Rightarrow \sqrt{3} \frac{\sin \theta}{\cos \theta} = 3 \sin \theta$

$\Rightarrow \frac{\sqrt{3}}{3} = \cos \theta$

$\Rightarrow \cos \theta = \frac{1}{\sqrt{3}}$

And $\sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \frac{1}{3}} = \sqrt{\frac{2}{3}}$

$\therefore \sin^2 \theta - \cos^2 \theta = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

11.

**(c)** decreasing

**Explanation:**

If the elevation moves towards the tower, it is increasing and if its elevation moves away from the tower, it decreases. Hence if the shadow of a tower is increasing, then the angle of elevation of the sun is not increasing.

12.

**(d)** -5

**Explanation:** Given: 5cot$^2$A - 5 cosec$^2$A

= 5(cot$^2$A - cosec$^2$A)

= 5 $\times$ -1 = -5

[$\because$ cosec$^2\theta$ - cot$^2\theta$ = 1]

13.

**(c)** $\frac{2\pi R\theta}{360}$

**Explanation:** $\frac{2\pi R\theta}{360}$

14.

**(c)** 231 cm$^2$

**Explanation:** Area swept by minute hand in 60 minutes = $\pi R^2$

Area swept by it in 10 minutes

$= \left(\frac{\pi R^2}{60} \times 10\right) \text{cm}^2 = \left(\frac{22}{7} \times 21 \times 21 \times \frac{1}{6}\right) \text{cm}^2$

= 231 cm$^2$

15.

**(d)** 14

**Explanation:** Probability of getting bad eggs = $\frac{\text{No. of bad eggs}}{\text{Total no. of eggs}}$

$\Rightarrow 0.035 = \frac{\text{No. of bad eggs}}{400}$

$\Rightarrow$ No. of bad eggs = 0.035 $\times$ 400 = 14

16.

**(c)** $l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$

17. **(a)** $6 : \pi$

**Explanation:** Let side of cube be a

Here, side of cube= diameter of sphere

so, radius of sphere = $\frac{a}{2}$

The volume of cube : volume of sphere

$a^3 : \frac{4}{3}\pi r^3$

$a^3 : \frac{4}{3}\pi(\frac{a}{2})^3$

$3 \times 8 \times a^3 : 4\pi a^3$

$6 : \pi$

18.

**(b)** 33

**Explanation:** The first 10 multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60

$\therefore$ Mean = $\frac{\text{Sum of first 10 multiples of 6}}{10}$

= $\frac{6+12+18+24+30+36+42+48+54+60}{10}$

= $\frac{330}{10}$

= 33

19. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** Both A and R are true and R is the correct explanation of A.

20.

**(c)** A is true but R is false.

**Explanation:** Here, reason is not true.

$\sqrt{9} = \pm 3$, which is not an irrational number.

A is true but R is false.

**Section B**

21. From the given equations, We get,

$\frac{a_1}{a_2} = \frac{5}{-10} = -\frac{1}{2}$

$\frac{b_1}{b_2} = -\frac{3}{6} = -\frac{1}{2}$

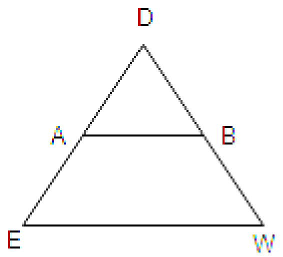
$\frac{c_1}{c_2} = \frac{11}{-22} = -\frac{1}{2}$

Hence,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore the given pair of line has an infinite number of solutions. So the given pair of linear equation is consistent.

22.




Given, AB || EW

$\therefore \frac{DA}{AE} = \frac{DB}{BW}$ [by basic proportionality theorem]

$\Rightarrow \frac{DA}{DE-DA} = \frac{DB}{DW-DB}$

$\Rightarrow \frac{4}{12-4} = \frac{DB}{24-DB}$

$\Rightarrow \frac{4}{8} = \frac{DB}{24-DB}$

$\Rightarrow 24 - DB = 2DB$

$\Rightarrow 24 = 3DB$

$\Rightarrow DB = \frac{24}{3} = 8cm$

OR

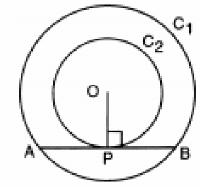
According to the question , the diagonal BD divides the AC in AO : OC with 2 : 1

To prove : AB = 2CD

$\triangle$AOC and $\triangle DOC$

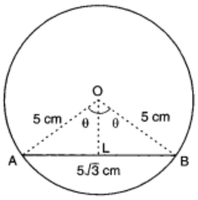
AOB = COD

OBA = ODC ( Because DC is parallel to AB, and DB is transversal. So these are alternates)
Therefore,by AA criteria of similar triangles,we have,
$\triangle \mathrm{AOB} \sim \triangle \mathrm{COD}$
Now, $\frac{\mathrm{AO}}{\mathrm{OC}} = \frac{\mathrm{AB}}{\mathrm{DC}}$ (Because in similar triangles, corresponding sides are proportional)
$\Rightarrow \frac{2}{1} = \frac{\mathrm{AB}}{\mathrm{DC}}$ (Given that AO : OC = 2 : 1)
So, AB = 2CD

23. 

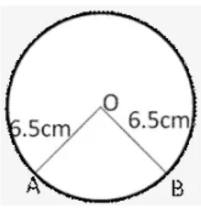

In larger circle C1 , AB is the chord and OP is the tangent.
Therefore, $\angle OPB = 90^\circ$
Hence, AP = PB ( perpendicular from center of the circle to the chord bisects the chord)

24. Given,
3 cot A = 4
$\Rightarrow$ cot A = $\frac{4}{3}$
We know that,
$\text{cosec}^2$A - $\cot^2$A = 1
$\text{cosec}^2$A - $\left(\frac{4}{3}\right)^2$ = 1
$\text{cosec}^2$A = 1 + $\frac{16}{9} = \frac{9+16}{9} = \frac{25}{9}$
Thus,
$\frac{cosec^2 A+1}{cosec^2 A-1} = \frac{25/9+1}{25/9-1} = \frac{34}{16}$

25. 


It is given that AB = 5$\sqrt{3}$ cm.
$\Rightarrow$ AL = BL = $\frac{5\sqrt{3}}{2}$ cm
Let $\angle AOB = 2\theta$ . Then, ∠AOL = ∠BOL =$\theta$
In △OLA, we have
$\sin \theta = \frac{AL}{OA} = \frac{\frac{5\sqrt{3}}{2}}{5} = \frac{\sqrt{3}}{2}$
$\Rightarrow \theta = 60^\circ$
$\Rightarrow \angle AOB = 120^\circ$
$\therefore$ Area of sector AOB = $\frac{120}{360} \times \pi \times 5^2 \mathrm{cm}^2 = \frac{25\pi}{3} \mathrm{cm}^2$

OR




Let sector of circle is OAB
Perimeter of a sector of circle = 31 cm
A O+ OB + length of arc AB = 31 cm
6.5 + 6.5 + arc AB = 31 cm
arc AB = 31 - 13
= 18 cm
Area of circle $= \frac{1}{2} \times r \times arc$

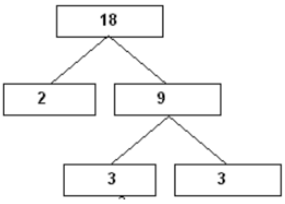
$= \frac{1}{2} \times 18 \times 6.5$

= 58.5 cm$^2$

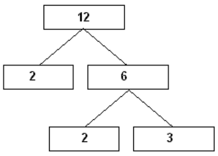
**Section C**

26. By taking LCM of time taken (in minutes) by Sonia and Ravi, We can get the actual number of minutes after which they meet again at the starting point after both start at the same point and at the same time, and go in the same direction.

$18 = 2 \times 3 \times 3 = 2 \times 3^2$




12 = 2 $\times$ 2 $\times$ 3 = $2^2 \times 3$




LCM $(18, 12) = 2^2 \times 3^2 = 36$

Therefore, both Sonia and Ravi will meet again at the starting point after 36 minutes.

27. Let the given polynomial is p(x) =$x^2$ + 7x + 7

Here,a = 1, b = 7, c = 7

$\therefore \alpha, \beta$ are both zeroes of p(x)

$\therefore \quad \alpha + \beta = \frac{-b}{a}$ = -7.........(i)

$\alpha\beta = \frac{c}{a} = 7$ ...........(ii)

Now,

$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta$

$= \frac{-7}{7} - 2 \times 7$

= -1 - 14

= -15

Hence the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ is - 15.

28. Let the fraction be $\frac{x}{y}$

Then, according to the question,

$\frac{x+1}{y-1} = 1$ ......(1)

$\frac{x}{y+1} = \frac{1}{2}$ .........(2)

$\Rightarrow$x + 1 = y - 1 ...........(3)

2x = y + 1................(4)

$\Rightarrow$x - y = - 2................(5)

2x - y = 1......................(^)

Substituting equation (5) from equation (6), we get x =3

Substituting this value of x in equation (5), we get

3 - y = -2

$\Rightarrow$ y = 3 + 2

$\Rightarrow$ y = 5

Hence, the required fraction is $\frac{3}{5}$

Verification: Substituting the value of x = 3 and y = 5,

we find that both the equations(1) and ( 2) are satisfied as shown below:

$\frac{x+1}{y-1} = \frac{3+1}{5-1} = \frac{4}{4} = 1$

$\frac{x}{y+1} = \frac{3}{5+1} = \frac{3}{6} = \frac{1}{2}$

Hence, the solution is correct.

OR

Given pair of linear equation is $ax + by - a + b = 0$ .....(i)

and $bx - ay - a - b = 0$ ........... (ii)

Multiplying $ax + by - a + b = 0$ by a and $bx - ay - a - b = 0$ by b, and adding them, we get

$a^2x + aby - a^2 + ab = 0$ and $b^2x - aby - ab - b^2 = 0$

$(a^2x + aby - a^2 + ab) + (b^2x - aby - ab - b^2) = 0$

$a^2x + aby - a^2 + ab + b^2x - aby - ab - b^2 = 0$

$a^2x + b^2x - a^2 - b^2 = 0$

$\Rightarrow (a^2 + b^2)x = (a^2 + b^2)$

$\Rightarrow x = \frac{(a^2+b^2)}{(a^2+b^2)} = 1$

On putting x =1 in first equation, we get

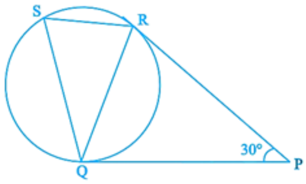
$ax + by - a + b = 0$

$a + by = a - b$

$\Rightarrow y = -\frac{b}{b} = -1$

Hence, x=1 and y=-1, which is the required unique solution.

29. In the given figure,we are given that, tangents PQ and PR are drawn to a circle such that $\angle RPQ = 30^o$. A chord RS is drawn parallel to tangent PQ. We have to find the $\angle RQS$.




In $\triangle PRQ$, PQ and PR are tangents from an external point P to circle.

Therefore, PR = PQ

$\Rightarrow \angle PRQ = \angle PQR$ [$\angle$s opp. to equal sides in $\triangle PRQ$ are equal]

$\angle PRQ + \angle PQR + \angle RPQ = 180^o$ [Int. $\angle$s of $\triangle$]

$\Rightarrow \angle PRQ + \angle PRQ + 30^o = 180^o$

$\Rightarrow 2\angle PRQ = 180^o - 30^o$

$\Rightarrow \angle PRQ = \frac{150^\circ}{2}$

Therefore, $\angle PRQ = \angle PQR = 75^o$

Tangent PQ $\parallel$ SR [Given]

Therefore, $\angle PQR = \angle SRQ = 75^o$ [Alternate segment of circle]

PQ is tangent at Q and QR is chord at Q.

Therefore, $\angle RSQ = \angle PQR = 75^o$ [$\angle$SQ in alternate segment of circle]

In $\triangle SRQ$,

$\angle RSQ + \angle SRQ + \angle SQR = 180^o$ [Angle sum property of a triangle]

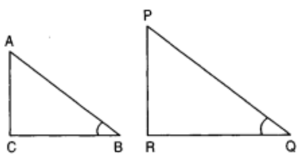
$\Rightarrow 75^o + 75^o + \angle SQR = 180^o$

$\Rightarrow \angle SQR = 180^o - 150^o$

$\Rightarrow \angle SQR = 30^o$

30. Consider two right triangles ABC and PQR in which $\angle B$ and $\angle Q$ are the right angles.

We have,




In $\triangle ABC$

$\sin B = \frac{AC}{AB}$

and, In $\triangle PQR$

$\sin Q = \frac{PR}{PQ}$

$\because \quad \sin B = \sin Q$

$\Rightarrow \quad \frac{AC}{AB} = \frac{PR}{PQ}$

$\Rightarrow \quad \frac{AC}{PR} = \frac{AB}{PQ} = k$ (say) ...... (i)

$\Rightarrow$ AC = kPR and AB = kPQ .....(ii)

Using Pythagoras theorem in triangles ABC and PQR, we obtain

$AB^2 = AC^2 + BC^2$ and $PQ^2 = PR^2 + QR^2$

$\Rightarrow \quad BC = \sqrt{AB^2 - AC^2}$ and $QR = \sqrt{PQ^2 - PR^2}$

$\Rightarrow \quad \frac{BC}{QR} = \frac{\sqrt{AB^2-AC^2}}{\sqrt{PQ^2-PR^2}} = \frac{\sqrt{k^2PQ^2-k^2PR^2}}{\sqrt{PQ^2-PR^2}}$ [ using (ii) ]

$\Rightarrow \quad \frac{BC}{QR} = \frac{k\sqrt{PQ^2-PR^2}}{\sqrt{PQ^2-PR^2}} = k$ ...(iii)

From (i) and (iii), we get

$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow \quad \Delta ACB - \Delta PRQ$ [By S.A.S similarity]

$\therefore \quad \angle B = \angle Q$

Hence proved.

OR

To prove-

$\frac{1}{(\cot A)(\sec A)-\cot A} - \operatorname{cosec} A = \operatorname{cosec} A - \frac{1}{(\cot A)(\sec A)+\cot A}$

Taking LHS

$= \frac{1}{(\cot A)(\sec A)-\cot A} - \operatorname{cosec} A$

$= \frac{1}{(\frac{\cos A}{\sin A})(\frac{1}{\cos A})-\left(\frac{\cos A}{\sin A}\right)} - \frac{1}{\sin A}$

$\frac{1}{(\frac{1}{\sin A})-\left(\frac{\cos A}{\sin A}\right)} - \frac{1}{\sin A} = \frac{1}{\frac{1-\cos A}{\sin A}} - \frac{1}{\sin A} = \frac{\sin A}{1-\cos A} - \frac{1}{\sin A} = \frac{\sin^2 A-1+\cos A}{(1-\cos A)\sin A}$

$= \frac{-\cos^2 A+\cos A}{(1-\cos A)\sin A} = \frac{\cos A(1-\cos A)}{(1-\cos A)\sin A}$ $\{\because \sin^2 A + \cos^2 A = 1\}$

$= \frac{\cos A}{\sin A} = cotA$

Now, taking RHS

$= \operatorname{cosec} A - \frac{1}{(\cot A)(\sec A)+\cot A}$

$= \frac{1}{\sin A} - \frac{1}{(\frac{\cos A}{\sin A})(\frac{1}{\cos A})+\frac{\cos A}{\sin A}}$

$= \frac{1}{\sin A} - \frac{1}{(\frac{1}{\sin A})+\frac{\cos A}{\sin A}} = \frac{1}{\sin A} - \frac{\sin A}{(1+\cos A)}$

$= \frac{1+\cos A-\sin^2 A}{(1+\cos A)\sin A} = \frac{\cos^2 A+\cos A}{(1+\cos A)\sin A}$

$= \frac{\cos A(\cos A+1)}{(1+\cos A)\sin A} = \frac{\cos A}{\sin A}$

= cotA = LHS

31. The total number of persons = 12.

The number of persons who are extremely patient = 3.

The number of persons who are extremely honest = 6.

Number of persons who are extremely kind = 12 – 3 – 6 = 3.

i. P(selecting a person who is extremely patient)=$\frac{\text{Number of favorable outcomes}}{\text{Number of all possible outcomes}}$

$= \frac{3}{12} = \frac{1}{4}$

Thus, the probability of selecting a person who is extremely patient is $\frac{1}{4}$.

ii. P(selecting a person who is extremely kind or honest) = $\frac{\text{Number of favorable outcomes}}{\text{Number of all possible outcomes}} = \frac{6+3}{12} = \frac{9}{12} = \frac{3}{4}$

Thus, the probability of selecting a person who is extremely kind or honest is $\frac{3}{4}$.

From the three given values, we prefer honesty more.

**Section D**

32. Let the present age of father be x years.

Son's present age = (45 - x) years.

Five years ago:

Father's age = (x - 5) years

Son's age = (45 - x - 5) years = (40 - x) years.

According to question,

$\therefore$ (x - 5) (40 - x) = 124

$\Rightarrow$ 40x - $x^2$ - 200 + 5x = 124

$\Rightarrow$ $x^2$ - 45x + 324 = 0

Spilting the middle term,

$\Rightarrow$ $x^2$ - 36x - 9 x + 324 = 0

$\Rightarrow$ x(x - 36) - 9(x - 36) = 0

$\Rightarrow$ (x - 9)(x - 36) = 0

$\Rightarrow$ x = 9, or 36

We can't take father age as 9 years

So, x = 36, we have

Father's present age = 36 years

Son's present age = 9 years

Hence, Father's present age = 36 years and Son's present age = 9 years.

OR

Let the first number be x

$\therefore$ Second number = x + 5

Now according to the question

$\frac{1}{x}-\frac{1}{x+5}=\frac{1}{10}$

$\Rightarrow \quad \frac{x+5-x}{x(x+5)}=\frac{1}{10}$

$\Rightarrow \quad 50=x^2+5x$

$\Rightarrow \quad x^2+5x-50=0$

$\Rightarrow \quad x^2+10x-5x-50=0$

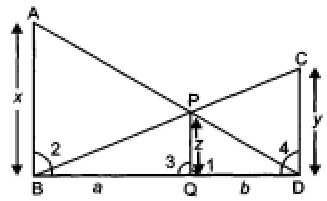
$\Rightarrow x(x+10)-5(x+10)=0$

$\Rightarrow \quad (x+10)(x-5)=0$

x = 5 ,- 10 rejected

The numbers = 5 and 10.

33. Let BQ = a units, DQ = b units




$\because \quad \mathrm{PQ} \parallel \mathrm{AB} \ \therefore \angle 1=\angle 2$ ,

and $\angle ADB=\angle PDQ$

$\therefore \quad \triangle \mathrm{ADB} \sim \triangle \mathrm{PDQ}$

Similarly $\triangle BCD \sim \triangle BPQ$

$\because \quad \triangle ADB \sim \triangle PDQ$

$\therefore \quad \frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{\mathrm{BD}}{\mathrm{DQ}}$

$\frac{x}{z}=\frac{a+b}{b}$

$\frac{x}{z}=\frac{a}{b}+1 \Rightarrow \frac{x}{z}-1=\frac{a}{b}$ ..(i)

Also, $\triangle BCD \sim \triangle BPQ$

$\therefore \quad \frac{\mathrm{BD}}{\mathrm{BQ}}=\frac{\mathrm{CD}}{\mathrm{PQ}} \Rightarrow \frac{a+b}{a}=\frac{y}{z}$

$1+\frac{b}{a}=\frac{y}{z} \Rightarrow \frac{b}{a}=\frac{y}{z}-1$

$\Rightarrow \quad \frac{b}{a}=\frac{y-z}{z} \Rightarrow \frac{a}{b}=\frac{z}{y-z}$ ..(ii)

From (i) and (ii)

$\frac{x}{z}-1=\frac{z}{y-z} \Rightarrow \frac{x}{z}=\frac{z}{y-z}+1$

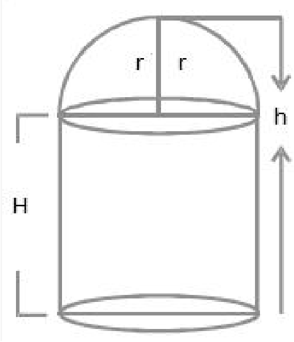
$\frac{x}{z}=\frac{z+y-z}{y-z}$

$\frac{x}{z}=\frac{y}{y-z} \Rightarrow \frac{z}{x}=\frac{y-z}{y}$

$\frac{z}{x}=1-\frac{z}{y}$

$z\left(\frac{1}{x}\right)=z\left(\frac{1}{z}-\frac{1}{y}\right) \Rightarrow \frac{1}{x}=\frac{1}{z}-\frac{1}{y}$

$\Rightarrow \quad \frac{1}{x}+\frac{1}{y}=\frac{1}{z}$ (Hence proved)

34. 


Let the radius of the hemispherical dome be r and the total height of the building be h.

Since, the base diameter of the dome is equal to $\frac{2}{3}$ of the total height

$2r = \frac{2}{3}h$

$\Rightarrow r = \frac{h}{3}$

Let H be the height of the cylindrical position.

$\Rightarrow H = h - r = h - \frac{h}{3} = \frac{2h}{3}$

Volume of air inside the building = Volume of air inside the dome + Volume of air inside the cylinder

$\Rightarrow 67\frac{1}{21} = \frac{2}{3}\pi r^3 + \pi r^2 H$

$\Rightarrow \frac{1408}{21} = \pi r^2\left(\frac{2}{3}r + H\right)$

$\Rightarrow \frac{1408}{21} = \frac{22}{7} \times \left(\frac{h}{3}\right)^2\left(\frac{2}{3} \times \frac{h}{3} + \frac{2h}{3}\right)$

$\Rightarrow \frac{1408 \times 7}{22 \times 21} = \frac{h^2}{9} \times \left(\frac{2h}{9} + \frac{2h}{3}\right)$

$\Rightarrow \frac{64}{3} = \frac{h^2}{9} \times \left(\frac{8h}{9}\right)$

$\Rightarrow \frac{64 \times 9 \times 9}{3 \times 8} = h^3$

$\Rightarrow h^3 = 8 \times 27$

$\Rightarrow h = 6$

Thus, the height of the building is 6 m.

OR

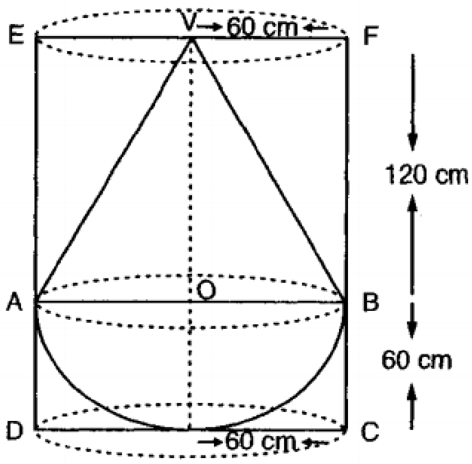
We have radius of cylinder = radius of cone = radius of hemisphere = 60 cm

Height of cone = 120 cm

$\therefore$ Height of cylindrical vessel = 120 + 60 =180 cm

$\therefore$ V = Volume of water that the cylinder contains = $\pi r^2 h = \left\{\pi \times (60)^2 \times 180\right\} \text{cm}^3$

Let $V_1$ be the volume of the conical part. Then,




$V_1 = \frac{1}{3}\pi r^2 h_1$

$\Rightarrow \quad V_1 = \frac{1}{3} \times \pi \times 60^2 \times 120\text{cm}^3 = \left\{\pi \times 60^2 \times 40\right\} \text{cm}^3$

For hemispherical part r = Radius = 60 cm

Let $V_2$ be the volume of the hemisphere. Then,

$V_2 = \left\{\frac{2}{3}\pi \times 60^3\right\} \text{cm}^3$

$\Rightarrow \quad V_2 = \left\{2\pi \times 20 \times 60^2\right\} \text{cm}^3 = \left\{40\pi \cdot 60^2\right\} \text{cm}^3$

Let $V_3$ the the volume of the water left-out in the cylinder. Then,

$V_3$= V - $V_1$ - $V_2$

$V_3 = \{\pi \times 60^2 \times 180 - \pi \times 60^2 \times 40 - 40\pi \times 60^2\}$ cm$^3$

$V_3 = \pi \times 60^2 \times \{180 - 40 - 40\}\text{cm}^3$

$V_3 = \frac{22}{7} \times 3600 \times 100\text{cm}^3$

$\Rightarrow \quad V_3 = \frac{22 \times 360000}{7}\text{cm}^3 = \frac{22 \times 360000}{7 \times (100)^3}\text{m}^3 = \frac{22 \times 36}{700}\text{m}^3 = 1.1314\text{m}^3.$

35. Mode:

Here, the maximum frequency is 23 and the class corresponding to this frequency is 35 - 45.

So, the modal class is 35 - 45.

Now, size (h) = 10

lower limit it (l) of modal class = 35

frequency ($f_1$) of the modal class = 23

frequency ($f_0$) of class previous the modal class = 21

frequency ($f_2$) of class succeeding the modal class = 14

$\therefore$ Mode = l + $\frac{f_1 - f_0}{2f_1 - f_0 - f_2}$ × h = 35 + $\frac{23-21}{2 \times 23 - 21 - 14}$ × 10

= 35 + $\frac{2}{11}$ × 10 = 35 + $\frac{20}{11}$

= 35 + 1.8 (approx.)

= 36.8 years (approx.)

Mean:-

Take a = 40, h = 10.

| **Age (in years)** | **Number of patients ($f_i$)** | **Class marks ($x_i$)** | **$d_i = x_i$ - 40** | $u_i = \frac{x_i - 40}{10}$ | **$f_iu_i$** |
|---|---|---|---|---|---|
| 5-15 | 6 | 10 | –30 | –3 | –18 |
| 15-25 | 11 | 20 | –20 | –2 | –22 |
| 25-35 | 21 | 30 | –10 | –1 | –21 |
| 35-45 | 23 | 40 | 0 | 0 | 0 |
| 45-55 | 14 | 50 | 10 | 1 | 14 |
| 55-65 | 5 | 60 | 20 | 2 | 10 |
| Total | $\sum f_i$ = 80 | | | | $\sum f_iu_i$ = - 37 |

Using the step deviation method,

$\overline{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right)$ × h = 40 + $\left(\frac{-37}{80}\right)$ × 10

= 40 - $\frac{37}{8}$ = 40 - 4.63

= 35.37 years

Interpretation:- Maximum number of patients admitted in the hospital are of the age 36.8 years (approx.), while on an average the age of a patient admitted to the hospital is 35.37 years.

**Section E**

36. **Read the text carefully and answer the questions:**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.

In their garden bed, there are 23 rose plants in the first row, 21 are in the 2nd, 19 in 3rd row and so on. There are 5 plants in the last row.

(i) The number of rose plants in the $1^{st}$, $2^{nd}$, .... are 23, 21, 19, ... 5

$a = 23$, $d = 21 - 23 = -2$, $a_n = 5$

$\therefore a_n = a + (n - 1)d$

or, $5 = 23 + (n - 1)(-2)$

or, $5 = 23 - 2n + 2$

or, $5 = 25 - 2n$

or, $2n = 20$

or, $n = 10$

(ii) Total number of rose plants in the flower bed,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(23) + (10-1)(-2)]$

$S_{10} = 5[46 - 20 + 2]$

$S_{10} = 5(46 - 18)$

$S_{10} = 5(28)$

$S_{10} = 140$

OR

$S_n = 80$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 80 = \frac{n}{2}[2 \times 23 + (n-1) \times -2]$

$\Rightarrow 80 = 23n - n^2 + n$

$\Rightarrow n^2 - 24n + 80 = 0$

$\Rightarrow (n - 4)(n - 20) = 0$

$\Rightarrow n = 4$ or $n = 20$

$n = 20$ not possible

$a_{20} = 23 + 19 \times (-2) = -15$

Number of plants cannot be negative.

$n = 4$

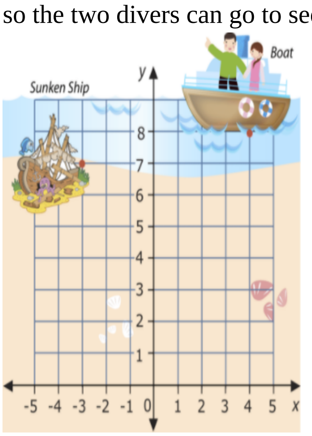
(iii) $a_n = a + (n - 1)d$

$\Rightarrow a_6 = 23 + 5 \times (-2)$

$\Rightarrow a_6 = 13$

37. **Read the text carefully and answer the questions:**

Mary and John are very excited because they are going to go on a dive to see a sunken ship. The dive is quite shallow which is unusual because most sunken ship dives are found at depths that are too deep for two junior divers. However, this one is at 40 feet, so the two divers can go to see it.




They have the following map to chart their course. John wants to figure out exactly how far the boat will be from the sunken ship. Use the information in this lesson to help John figure out the following.

(i) (4, 8) and (-3, 7)

(ii) 8 units

OR

7 or -1

(iii)1280 cubic feet

38. **Read the text carefully and answer the questions:**

Two trees are standing on flat ground. The angle of elevation of the top of Both the trees from a point X on the ground is $60^o$. If the horizontal distance between X and the smaller tree is 8 m and the distance of the top of the two trees is 20 m.


(i) In △DCX

tan $60^o$ = $\frac{DC}{CX}$

$\sqrt{3} = \frac{DC}{8}$

DC = $8\sqrt{3}$ m

DX = $\sqrt{DC^2 + CX^2}$

$= \sqrt{(8\sqrt{3})^2 + 8^2}$

$= \sqrt{192 + 64}$

$= \sqrt{256}$

= 16 m

Hence, distance between X and top of smaller tree is 16 m.

(ii) In △BAX

cos $60^o$ = $\frac{AX}{BX}$

$\frac{1}{2} = \frac{AC+8}{36}$

36 = 2AC + 16

20 = 2AC

$\frac{20}{2}$ = 10 AC

AC = 10

∴ horizontal distance between both trees is 10 m.

OR

Height of big tree = AB

∴ In △BAX

tan $60^o$ = $\frac{AB}{AX} = \frac{AB}{18}$

AB = $18\sqrt{3}$ m

(iii)Height of small tree = CD

In △ CDX

tan $60^o$ = $\frac{CD}{CX}$

$\sqrt{3} = \frac{CD}{8}$

CD = $8\sqrt{3}$ m