CHAPTER – 3

Playing with Numbers

EXERCISE – 3.7

Q. 1

Renu purchases two bags of fertiliser of weights 75 kg and 69 kg. Find the maximum value of weight which can measure the weight of the fertiliser exact number of times.

Answer:

Weight of the two bags = 75 kg and 69 kg

Maximum weight = HCF (75, 69)

```
3 | 75
5 | 25
5 | 5
  | 1
```

```
3  | 69
23 | 23
   | 1
```

$75 = 3 \times 5 \times 5$

$69 = 3 \times 23$

HCF = 3

Therefore, the maximum value of weight, which can measure the weight of the fertilizer exact number of time, is 3 kg.

Q. 2

Three boys step off together from the same spot. Their steps measure 63 cm, 70 cm and 77 cm respectively. What is the minimum distance each should cover so that all can cover the distance in complete steps?

Answer:

Step measure of first boy = 63 cm

Step measure of second boy = 70 cm

Step measure of third boy = 77cm

Now calculate the LCM of 63, 70 and 77

2	63, 70, 77
3	63, 35, 77
3	21, 35, 77
5	7, 35, 77
7	7, 7, 77
11	1, 1, 11
	1, 1, 1

LCM = 2 × 3 × 3 × 5 × 7 × 11 = 6930

Therefore,

The maximum distance each should cover so that all can cover the distance in complete steps is 6930 cm.

Q. 3

The length, breadth, and height of a room are 825 cm, 675 cm, and 450 cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.

Answer:

Length of the room = 825 cm

Breadth of the room = 675 cm

Height of the room = 450 cm

Longest tape = HCF of 825, 675 and 450

$825 = 3 \times 5 \times 5 \times 11$

$675 = 3 \times 3 \times 3 \times 5 \times 5$

$450 = 2 \times 3 \times 3 \times 5 \times 5$

HCF = Product of common factors = $3 \times 5 \times 5 = 75$ cm

Therefore, the longest tape = 75 cm

Q. 4

Determine the smallest 3-digit number which is exactly divisible by 6, 8 and 12.

Answer:

Smallest 3-digit number = LCM of 6, 8 and 12

2	6, 8, 12
2	3, 4, 6
2	3, 2, 3
3	3, 1, 3
	1, 1, 1

LCM = $2 \times 2 \times 2 \times 3 = 24$

LCM of these 3 numbers is 24 so to find out 3-digit number we have to calculate the multiples of 24,

24 × 1 = 24

24 × 2 = 48

24 × 3 = 72

24 × 4 = 96

24 × 5 = 120

24 × 6 = 144

It can be seen that, 120 is the smallest 3-digit multiple of 24 so, the smallest 3-digit number which is exactly divisible by 6, 8 and 12 is 120.

Q. 5

Determine the greatest 3-digit number exactly divisible by 8, 10 and 12.

Answer:

Let's take the LCM of 8, 10 and 12

2	8, 10, 12
2	4, 5, 6
2	2, 5, 3
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

LCM = 2 × 2 × 2 × 3 × 5 = 120

Now to get greatest 3-digit number we have to calculate the multiples of 120

120 × 1 = 120

120 × 2 = 240

120 × 3 = 360

120 × 4 = 480

120 × 5 = 600

120 × 6 = 720

120 × 7 = 840

120 × 8 = 960

120 × 9 = 1080

120 × 10 = 1200

Hence the greatest 3-digit number exactly divisible by 8, 10 and 12 is 960.

Q. 6

The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change simultaneously again?

Answer:

Time-period after which these lights will change = LCM of 48, 72 and 108

LCM is the lowest common multiple. LCM of two numbers It is the smallest number that is divisible by both of them.

```
2 | 48, 72, 108
2 | 24, 36, 54
2 | 12, 18, 27
2 | 6, 9, 27
3 | 3, 9, 27
3 | 1, 3, 9
3 | 1, 3, 3
  | 1, 1, 1
```

LCM = 2 × 2 × 2 × 2 × 3 × 3 × 3 = 432

The lights will change together after every 432 seconds.

60 seconds = 1 minute

$$60\overline{\smash{)}432}\;\;\;\;^{7}$$

432 seconds = 432/60 min

$$- \underline{420}$$

$$12$$

= 7 minutes and 12 seconds Total time will be 7 am + 7 min + 12 sec.

Hence these lights will change simultaneously again at 7:07:12 am.

Q. 7

Three tankers contain 403 litres, 434 litres and 465 litres of diesel respectively. Find the maximum capacity of a

container that can measure the diesel of the three containers exact number of times.

Answer:

Maximum capacity of the required tanker = HCF of 403, 434 and 465

```
13 | 403
31 | 31
   | 1
```

```
 2 | 434
 7 | 217
31 | 31
   | 1
```

```
 3 | 465
 5 | 155
31 | 31
   | 1
```

$403 = 13 \times 31$

$434 = 2 \times 7 \times 31$

$465 = 3 \times 5 \times 31$

HCF = 31

So, the container of capacity 31 litres can measure the diesel of 3 containers exact number of times.

Q. 8

Find the least number which when divided by 6, 15 and 18 leave remainder 5 in each case.

Answer:

To find the required number we have to calculate LCM of 6, 15 and 18 because LCM is the least number divided by all 6, 15 and 18

```
2 | 6, 15, 18
3 | 3, 15, 9
3 | 1, 5, 3
5 | 1, 5, 1
  | 1, 1, 1
```

LCM = 2 × 3 × 3 × 5 = 90
Since the number required always leaves remainder 5

Required number = 90 + 5 = 95

Q. 9

Find the smallest 4-digit number which is divisible by 18, 24 and 32.

Answer:

LCM 0f 18, 24 and 32

```
2 | 18, 24, 32
2 | 9, 12, 16
2 | 9, 6, 8
2 | 9, 3, 4
2 | 9, 3, 2
3 | 9, 3, 1
3 | 3, 1, 1
  | 1, 1, 1
```

LCM = 2 × 2 × 2 × 2 × 2 × 3 × 3 = 288

As, 288 is the smallest number divisible by 18, 24 and 32 and we have to find the smallest 4-digit multiple of 288 therefore we have to find the smallest 4-digit multiple of 288.

It can be observed that

288 × 3 = 864 and

288 × 4 = 1152

1152 is the smallest 4-digit multiple of 288.

Therefore, the smallest 4-digit number which is divisible by 18, 24 and 32 is 1152.

Q. 10

Find the LCM of the following numbers:
(a) 9 and 4 (b) 12 and 5
(c) 6 and 5 (d) 15 and 4
Observe a common property in the obtained LCMs. Is LCM the product of two numbers in each case?

Answer:

LCM – Least common multiple

The LCM of two numbers is the smallest number that is a multiple of both the numbers.

a. 9 and 4

```
2 | 9, 4
2 | 9, 2
3 | 9, 1
3 | 3, 1
  | 1, 1
```

LCM = 2 × 2 × 3 × 3 = 36

b. 12 and 5

```
2 | 12, 5
2 | 6, 5
3 | 3, 5
5 | 1, 5
  | 1, 1
```

LCM = 2 × 2 × 3 × 5 = 60

c. 6 and 5

```
2 | 6, 5
3 | 3, 5
5 | 1, 5
  | 1, 1
```

LCM = 2 × 3 × 5 = 30

d. 15 and 4

```
2 | 15, 4
2 | 15, 2
3 | 15, 1
5 | 5, 1
  | 1, 1
```

LCM = 2 × 2 × 3 × 5 = 60

Yes, it can be seen that in each case, LCM of given numbers is the product of these numbers. When two numbers are co-prime, their LCM is the product of those numbers. Also, in each case, LCM is a multiple of 3.

Q. 11

Find the LCM of the following numbers in which one number is the factor of the other:

(a) 5, 20   (b) 6, 18
(c) 12, 48   (d) 9, 45
What do you observe in the results obtained?

Answer:

a. 5, 20

```
2 | 5, 20
2 | 5, 10
5 | 5, 5
  | 1, 1
```

LCM = 2 × 2 × 5 = 20

b. 6, 18

2	6, 18
3	3, 9
3	1, 3
	1, 1

LCM = 2 × 3 × 3 = 18

c. 12, 48

2	12, 48
2	6, 24
2	3, 12
2	3, 6
3	3, 3
	1, 1

LCM = 2 × 2 × 2 × 2 × 3 = 48

d. 9, 45

3	9, 45
3	3, 15
5	1, 5
	1, 1

LCM = 3 × 3 × 5 = 45

Yes, it can be seen that in each case the LCM of the given number is the larger number. When one number is a factor of the other, their LCM will be the larger number.