Chapter 2

Inverse Trigonometric Functions

Exercise 2.1

Q. 1 Find the principal values of the following:

$\text{Sin}^{-1}\left(-\frac{1}{2}\right)$

Answer:

Let us take $\sin^{-1}\left(-\frac{1}{2}\right) = x$

Therefore,

$\text{Sinx} = -\frac{1}{2} = -\sin\frac{\pi}{6} = \sin\left(-\frac{\pi}{6}\right)$

We know that principle value range of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

And,

$\text{Sin}\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

Therefore principle value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Q. 2 Find the principal values of the following:

$\text{Cos}^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Answer:

Let us take $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

Then, $\cos x = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right)$

We know that principle value range of $\cos^{-1}$ is $[0, \pi]$

And cos $\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Therefore, principle value of cos^{-1} $\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$

Q. 3 Find the principal values of the following:

cosec^{-1} (2)

Answer:

Let cosec^{-1} 2 = x

Therefore, cosec x = 2 = cosec $\left(\frac{\pi}{6}\right)$

And cosec $\left(\frac{\pi}{6}\right)$ = 2

We know that principle value range of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ – {0}

Therefore, principle value of cosec^{-1}(2) is $\frac{\pi}{6}$.

Q. 4 Find the principal values of the following:

tan^{-1} $(-\sqrt{3})$

Answer:

Let us take tan^{-1} $\left(-\sqrt{3}\right)$ = x

Then we get,

tanx = - $\sqrt{3}$ = - tan $\frac{\pi}{3}$ = tan $\left(-\frac{\pi}{3}\right)$

And tan $\left(-\frac{\pi}{3}\right)$ = - $\sqrt{3}$

We know that principle value range of $\tan - 1$ is $\left[-\frac{\pi}{2}, \frac{\pi}{3}\right]$

Therefore, principle value of tan^{-1}$\left(-\sqrt{3}\right)$ is $\left(-\frac{\pi}{3}\right)$.

Q. 5 Find the principal values of the following:

Cos$^{-1}\left(-\frac{1}{2}\right)$

Answer:

Let us take cos$^{-1}\left(-\frac{1}{2}\right)$ = x

Then we will get,

Cosx = - $\frac{1}{2}$ = - cos $\left(\frac{\pi}{3}\right)$ = cos $\left(\pi - \frac{\pi}{3}\right)$ = cos $\left(\frac{2\pi}{3}\right)$

And cos $\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

We know that principle value range of cos^{-1} is [0, π]

Therefore, principle value of cos$^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

Q. 6 Find the principal values of the following:

tan^{-1} (–1)

Answer:

Let us take tan^{-1}(-1) = x then we get,

= tanx = - 1 = - tan $\left(\frac{\pi}{4}\right)$

= tan $\left(-\frac{\pi}{4}\right)$

And tan $\left(-\frac{\pi}{4}\right)$ = - 1

We know that principle value range of tan^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, principle value of tan^{-1}(-1) is $-\frac{\pi}{4}$

Q. 7 Find the principal values of the following:

Sec-1 $\left(\frac{2}{\sqrt{3}}\right)$

Answer:

Let sec-1 $\left(\frac{2}{\sqrt{3}}\right)$ = x

Then,

Secx = $\frac{2}{\sqrt{3}}$ = sec $\left(\frac{\pi}{6}\right)$

We know that range of the principle value branch of sec-1 is [0, π]$-\left\{\frac{\pi}{2}\right\}$

And sec$\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$

Therefore principle value of sec-1 $\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$

Q. 8 Find the principal values of the following:

Cot-1 $\left(-\sqrt{3}\right)$ = x

Answer:

Let us consider cot-1 $\left(-\sqrt{3}\right)$ = x

Then we get,

Cotx = $-\sqrt{3}$ = $-$ cot $\left(\frac{\pi}{6}\right)$ = cot $\left(\pi-\frac{\pi}{6}\right)$ = cot $\frac{5\pi}{6}$

We know that the range of the principal value branch of cot-1 is [0, π].

And, cot $\left(\frac{5\pi}{6}\right) = -\sqrt{3}$

Therefore, the principle value of cot-1$\left(-\sqrt{3}\right)$ is $\frac{5\pi}{6}$.

Q. 9 Find the principal values of the following:

Cos-1 $\left(-\frac{1}{\sqrt{2}}\right)$

Answer:

Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = x$

Therefore,

$\text{Cos}x = -\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$

We know that range of the principle value branch of $\cos^{-1}$ is $[0, \pi]$

And $\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

Therefore principle value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$

Q. 10 Find the principal values of the following:

$\text{Cosec}^{-1}\left(-\sqrt{2}\right)$

Answer:

Let us take the values of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$,= x

Then,

$\text{Cosec}x = -\sqrt{2} = \text{cosec}\left(-\frac{\pi}{4}\right)$

We know that range of the principle value branch of $\text{cosec}-1$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

And $\text{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

Therefore principle value of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$.

Q. 11 Find the values of the following:

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

Answer:

Let us consider $\tan^{-1}(1) = x$ then we get

$\tan x = 1 = \tan \frac{\pi}{4}$

We know that range of the principle value branch of $\tan^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, $\tan^{-1}(1) = \frac{\pi}{4}$

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$

$\cos y = -\frac{1}{2} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$

We know that range of the principle value branch of $\cos^{-1}$ is $[0, \pi]$

Therefore, $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = z$

$\sin z = -\sin\frac{\pi}{6} = \sin\left(-\frac{\pi}{6}\right)$

We know that range of the principle value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Now,

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin\left(-\frac{1}{2}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6} = \frac{3\pi+8\pi-2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$

Q. 12 Find the values of the following:

$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

Answer:

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$

Then, we get,

$\cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

We know that range of the principle value branch of $\cos^{-1}$ is $[0, \pi]$

Therefore $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$ then $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

We know that range of the principle value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

Now,

$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2 \times \frac{\pi}{6} = \frac{2\pi}{3}$

Q. 13 Find the values of the following:

$\sin^{-1} x = y$, then

A. $0 \le y \le \pi$

B. $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

C. $0 < y < \pi$

D. $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Answer:

$\sin^{-1} x = y$

We know that range of the principle value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

Hence, the option (B) is correct.

Q. 14 Find the values of the following: $\tan^{-1} \sqrt{3} - \sec^{-1}$ (-2) is equal to

A. π

B. $-\frac{\pi}{3}$

C. $\frac{\pi}{3}$

D. $\frac{2\pi}{3}$

Answer:

Let us take

$\tan^{-1}\left(\sqrt{3}\right) = x$ Then we get,

$\tan x = \sqrt{3} = \tan \frac{\pi}{3}$

We know that range of the principle value branch of $\tan^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, $\tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$

Let $\sec^{-1}(-2) = y$ Then we get,

$\text{Sec } y = -2 = -\sec \frac{\pi}{3} = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\left(\frac{2\pi}{3}\right)$

We know that range of the principle value branch of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

Therefore, $\sec^{-1}(-2) = \frac{2\pi}{3}$

Now,

$\tan^{-1} \sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$

Hence, the option (B) is correct.

Exercise 2.2

Q. 1

Prove the following:

3 $\sin^{-1} x = \sin^{-1} (3x - 4x3)3$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Answer:

Let $x = \sin\theta$ then $\sin^{-1}x = \theta$

We have,

R.H.S $= \sin^{-1} (3x - 4x^3)^3 = \sin^{-1} (3 \sin\theta - 4 \sin^3\theta)$

Now, we know that,

$\sin 3x = 3\sin x - 4 \sin 3x$

Therefore,

$= \sin^{-1}(\sin (3\theta))$

$= 3\theta$

$= 3\sin^{-1}x$

= L.H.S

Hence Proved

Q. 2

Prove the following:

$3 \cos^{-1} x = \cos^{-1} (4x^3 - 3x)$, $x \in \left[\frac{1}{2}, 1\right]$

Answer:

Let $x = \cos\theta$

Then, $\text{Cos}^{-1} = \theta$

Now, R.H.S. = $\cos^{-1}(4x^3 - 3x)$

$= \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$= \cos^{-1}(\cos^3\theta)$

$= 3\theta$

$= 3\cos^{-1}x$

= L.H.S.

Hence Proved

Q. 3

Prove the following:

$\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Answer:

L.H.S. $\tan{-1}\frac{2}{11} + \tan{-1}\frac{7}{24}$

$= \tan^{-1}\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}.\frac{7}{24}}$ $\left[\therefore \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$

$= \tan^{-1}\frac{\frac{48+77}{11\times24}}{\frac{11\times24-14}{11\times24}}$

$= \tan^{-1}\frac{48+77}{264-14}$

$= \tan^{-1}\frac{125}{250}$

$= \tan^{-1}\frac{1}{2}$

= R.H.S.

Hence Proved.

Q. 4

Prove the following:

2 $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Answer:

L.H.S. $= 2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\frac{2.\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\frac{1}{\frac{3}{4}} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}.\frac{1}{7}}$ $\left[since \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$

$= \tan^{-1}\frac{\frac{28+3}{21}}{\frac{21-4}{21}}$

$= \tan^{-1}\frac{31}{17}$

= R.H.S.

Hence Proved.

Q. 5

Write the following functions in the simplest form:

$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$, $x \neq 0$

Answer:

$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$, $x \neq 0$

Now, Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

Therefore, $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x} = \tan^{-1} \left(\frac{\sqrt{1+\tan\theta^2}-1}{\tan\theta}\right)$

$= \tan^{-1} \left(\frac{\sec\theta - 1}{\tan\theta}\right)$

$= \tan^{-1} \left(\frac{1-\cos\theta}{\sin\theta}\right)$

$= \tan^{-1} \left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$

$= \tan^{-1} \left(\tan\frac{\theta}{2}\right)$

$=\frac{\theta}{2} = \frac{1}{2} \tan^{-1}x$

Therefore, $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x} = \frac{1}{2} \tan^{-1}x$

Q. 6

Write the following functions in the simplest form:

$\tan^{-1} \frac{1}{\sqrt{x^2-1}}$, $|x| > 1$

Answer:

$\tan^{-1} \frac{1}{\sqrt{x^2-1}}$, $|x| > 1$

Let us take,

x = cosec θ = θ = cosec^{-1} x

[We have done this substitution on the bases of identity sec2θ – 1 = tan2θ]

Therefore, $\tan^{-1} \frac{1}{\sqrt{x^2-1}} = \tan^{-1} \frac{1}{\sqrt{cosec^2\theta-1}}$

Now we know that, cosec2θ – 1 = cot2θ

Therefore,

$= \tan^{-1} \frac{1}{cot\theta} = \tan^{-1} (\tan \theta) = \theta = cosec^{-1} x$

Q. 7

Write the following functions in the simplest form:

$\tan^{-1} = \left(\sqrt{\frac{1-cos x}{1+cos x}}\right) x < \pi$

Answer:

$\tan^{-1} = \left(\sqrt{\frac{1-cos x}{1+cos x}}\right) x < \pi$

$= \tan^{-1} \left(\sqrt{\frac{2sin^2\frac{x}{2}}{2cos^2\frac{x}{2}}}\right)$

$= \tan^{-1} \left(\frac{sin\frac{x}{2}}{cos\frac{x}{2}}\right)$

$= \tan^{-1} \left(tan\frac{x}{2}\right)$

$= \frac{x}{2}$

Hence, $\tan^{-1} \left(\sqrt{\frac{1-cos x}{1+cos x}}\right) = \frac{x}{2}$

Q. 8

Write the following functions in the simplest form:

$\tan^{-1} \left(\frac{cos x - sin x}{cos x + sin x}\right), 0 < x < \pi$

Answer:

$\tan^{-1} \left(\frac{cos x - sin x}{cos x + sin x}\right)$

Dividing by cos x,

$= \tan^{-1}\left(\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\right)$

$= \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right), \left[\therefore \frac{\sin x}{\cos x} = \tan x\right]$

$= \tan^{-1} - \tan^{-1} (\tan x) \left[\therefore \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$

As we know $\tan(\pi/4) = 1$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}\right)\right) + \tan^{-1} (\tan x)$

$= \frac{\pi}{4} - x$

Hence, $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \frac{\pi}{4} - x.$

Q. 9

Write the following functions in the simplest form:

$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}, |x| < a$

Answer:

$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$

We will solve this problem on the bases of the identity 1 - sin2θ = cos2θ

So, for $a^2 - x^2$, we can substitute x = a sin θ or x = a cos θ

Now, let us put x = a sin θ

$= \frac{x}{a} = \sin\theta$

$= \theta = \sin^{-1}\left(\frac{x}{a}\right)$

Therefore,

$\tan^{-1} \frac{x}{\sqrt{a^2-x^2}} = \tan^{-1} \left(\frac{a\, sin\theta}{\sqrt{a^2-a^2 sin^2\theta}}\right) = \tan^{-1} \left(\frac{a\, sin\theta}{a \cos\theta}\right)$

$= \tan^{-1} (\tan\theta) = \theta = \sin^{-1} \left(\frac{x}{a}\right)$

Hence, $\tan^{-1} \frac{x}{\sqrt{a^2-x^2}} = \sin^{-1} \left(\frac{x}{a}\right)$

Q. 10

Write the following functions in the simplest form:

$\tan^{-1} \left\{\frac{3a^3x-x^3}{a^3-3ax^2}\right\}$, $a > 0; \frac{-a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}$

Answer:

$\tan^{-1} \left\{\frac{3a^3x-x^3}{a^3-3ax^2}\right\}$

Put $x = a \tan\theta \Rightarrow \frac{x}{a} = \tan\theta = \tan^{-1}\frac{x}{a}$

Now,

$\tan^{-1} \left(\frac{3a^3x-x^3}{a^3-3ax^2}\right) = \tan^{-1} \left(\frac{3a^3.a\, tan\theta - a^3 tan^3\theta}{a^3-3a.a^2 tan^2\theta}\right)$

$= \tan^{-1} \left(\frac{3a^3 tan\theta - a^3 tan^3\theta}{a^3-3a^3 tan^2\theta}\right)$

$= \tan^{-1} (\tan 3\theta) \left[\therefore \frac{3tan\theta - tan^3\theta}{1-3tan^2\theta} = tan3\theta\right]$

$= 3\theta$

$= 3 \tan^{-1} \frac{x}{a}$

Q. 11

Find the values of each of the following:

$\tan^{-1} \left[2cos\left(2sin^{-1} \frac{1}{2}\right)\right]$

Answer:

$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

We will solve the inner bracket first.

So, we will first find the principal value of $\sin^{-1}\frac{1}{2}$

We know that, $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

Therefore,

$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] = \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\cos\left(\frac{\pi}{3}\right)\right]$

$= \tan^{-1}\left[2\times\frac{1}{2}\right]\left[\text{since} \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}\right]$

$= \tan^{-1}1$

$= \pi/4$

Hence,

The value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] = \frac{\pi}{4}$

Q. 12

Find the values of each of the following:

$\cot(\tan^{-1}a + \cot^{-1}a)$

Answer:

$\cot(\tan^{-1}a + \cot^{-1}a)$

$= \cot\left(\frac{\pi}{2}\right), \left[\because \tan^{-1}x + \cot^{-1}y = \frac{\pi}{2}\right]$

$= 0$

Hence, the value of $\cot(\tan^{-1}a + \cot^{-1}a) = 0$

Q. 13

Find the values of each of the following:

$\tan \frac{1}{2}\left[s i n^{-1} \frac{2x}{1+x^2} + c os^{-1} \frac{1-y^2}{1+y^2}\right]$, |x| < 1, y = 0

Answer:

$\tan \frac{1}{2}\left[s i n^{-1} \frac{2x}{1+x^2} + c os^{-1} \frac{1-y^2}{1+y^2}\right]$

We will solve this problem by expressing sin2θ and cos2θ in terms of tan θ

Now let us put, x = tan θ. Then we will have,

$\theta = \tan^{-1}x$

$\therefore \sin^{-1} \frac{2x}{1+x^2} = \sin^{-1} \frac{2\tan\theta}{1+tan^2\theta} = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1}x$

Now again, Let's put, y = tan∅. Then we will have,

∅ = tan-1y

$\therefore \cos^{-1} \frac{1-y^2}{1+y^2} = \sin^{-1}\left(\frac{1-tan^2\emptyset}{1+tan^2\emptyset}\right) = \cos^{-1}(\cos 2\emptyset) = 2\emptyset = 2\tan^{-1} y$

Now,

$\tan \frac{1}{2}\left[s i n^{-1} \frac{2x}{1+x^2} + c os^{-1} \frac{1-y^2}{1+y^2}\right]$

$= \tan \frac{1}{2}[2\tan^{-1} x + 2\tan^{-1} y]$

$= \tan[\tan^{-1} x + \tan^{-1} y]$

$= \tan\left[tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$= \frac{x+y}{1-xy}$

Hence, the value of

$\tan \frac{1}{2}\left[\sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-y^2}{1+y^2}\right] = \frac{x+y}{1-xy}$

Q. 14

Find the values of each of the following:

If sin $\left(\sin^{-1} \frac{1}{5} + \cos^{-1} x\right)$ = 1, then find the value of x

Answer:

Sin $\left(\sin^{-1} \frac{1}{5} + \cos^{-1} x\right)$ = 1

= $\sin^{-1} \frac{1}{5} + \cos^{-1}$ x = sin-1 1

= $\sin^{-1} \frac{1}{5} + \cos^{-1}$ x = $\frac{\pi}{2}$, $\left[since, \sin^{-1} 1 = \frac{\pi}{2}\right]$

= $\sin^{-1} \frac{1}{5} = \frac{\pi}{2} - \cos^{-1}$ x

= $\sin^{-1} \frac{1}{5} = \sin^{-1}$ x, $\left[since, \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right]$

On comparing the co-efficient on both sides we get,

= x = $\frac{1}{5}$

Q. 15

Find the values of each of the expression

$\text{Sin}^{-1} \left(\sin \frac{2\pi}{3}\right)$

Answer:

$\text{Sin}^{-1} \left(\sin \frac{2\pi}{3}\right)$

(For $\sin^{-1}$ (sin x) type of problem we have to always check whether the angle is in the principal range or not. This angle must be in the principal range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

So here, $\frac{2\pi}{2} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Now, sin^{-1} $\left(s\,in\frac{2\pi}{3}\right)$ can be written as,

Sin^{-1} $\left(s\,i\,n\frac{2\pi}{3}\right)$

= sin^{-1} $\left(\sin \pi - \frac{\pi}{3}\right)$

= sin^{-1} $\left(\sin \frac{\pi}{3}\right)$ where $\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

= $\frac{\pi}{3}$

Hence, sin^{-1} $\left(s\,i\,n\frac{2\pi}{3}\right) = \frac{\pi}{3}$.

Q. 16

Find the values of each of the expression

tan^{-1} $\left(\tan \frac{3\pi}{4}\right)$

Answer:

tan^{-1}$\left(\tan \frac{3\pi}{4}\right)$

(For tan^{-1} (tan x) type of problem we have to always check whether the angle is in the principal range or not. This angle must be in the principal range.$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$)

So here, $\frac{3\pi}{4} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Now, tan^{-1} $\left(\tan \frac{3\pi}{4}\right)$can be written as,

tan^{-1} $\left(\tan \frac{3\pi}{4}\right)$

= tan^{-1} $\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$

= - tan-1 $\left(\tan\frac{\pi}{4}\right)$ where $-\frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, [since, tan ($\pi$ – x) = - tanx

$= -\frac{\pi}{4}$

Hence, tan-1 $\left(\tan\frac{3\pi}{4}\right) = -\frac{\pi}{4}$

Q. 17

Find the values of each of the expression

tan $\left(s\,i\,\bar{n}^{1}\frac{3}{5} + c\,ot^{-1}\frac{3}{2}\right)$

Answer:

Let $\sin^{-1}\left(\frac{3}{5}\right)$ = y so sin y = $\frac{3}{5}$ and y $\in \left(0, \frac{\pi}{2}\right)$, so all ratio of y are positive and

Hence, cos y = $\frac{4}{5}$ and tan y = $\frac{3}{4}$, so $\tan^{-1}\left(\frac{3}{4}\right)$ = y

Also,

$\text{Cot}^{-1}\left(\frac{3}{2}\right) = \tan^{-1}\frac{2}{3}$ as $\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$

So, tan $\left(s\,i\,\bar{n}^{1}\frac{3}{5} + c\,ot^{-1}\frac{3}{2}\right)$

= tan $\left(tan^{-1}\ \frac{3}{4} + tan^{-1}\frac{2}{3}\right)$

= tan $\left(tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right)$

= tan $\left(tan^{-1}\ \frac{17}{6}\right) = \frac{17}{6}$

Hence, tan $\left(s\,i\,\bar{n}^{1}\frac{3}{5} + c\,ot^{\,-1}\frac{3}{2}\right) = \frac{17}{6}$

Q. 18

Find the values of each of the expression

$\text{Cos}^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to

A. $\frac{7\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

Answer:

$\text{Cos}^{-1}\left(\cos\frac{7\pi}{6}\right)$

(For $\cos^{-1}(\cos x)$ type of problem we have to always check whether the angle is in the principal range or not. This angle must be in the principal range. $\left[-\frac{(\pi}{2}, \frac{(\pi}{2}\right]$)

So here,

$\frac{7\pi}{6} \notin [0, \pi]$

Now, $\cos^{-1}\left(\cos\frac{7\pi}{67}\right)$ can be written as,

$\text{Cos}^{-1}\left(\cos\frac{7\pi}{6}\right)$

$= \cos^{-1}\left[\cos\left(\pi + \frac{\pi}{6}\right)\right]$

$= -\cos^{-1}\left(\cos\frac{\pi}{6}\right)$ where $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, [since, $\cos(\pi + x) = -\cos x$]

$= \pi - \cos^{-1}\left(\cos\frac{\pi}{6}\right)$ as $\cos\text{-}1(-x) = \pi - \cos\text{-}1$

$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Hence, $\cos\left(\cos\frac{7\pi}{6}\right) = \frac{5\pi}{6}$.

Q. 19

Find the values of each of the expression

Sin $\left(\frac{\pi}{3} - s\,i\,\bar{n}^{1}\left(-\frac{1}{2}\right)\right)$ is equal to

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. 1

Answer:

$\text{Sin}^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(\frac{1}{2}\right)$ as $\sin^{-1}(-x) = \sin^{-1} x$

$= -\frac{\pi}{6}$ as $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

We all know that the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore s\,i\,\bar{n}^{1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Therefore, $\sin\left(\frac{\pi}{3} - s\,i\,\bar{n}^{1}\left(-\frac{1}{2}\right)\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

Hence, the value of $\sin\left(\frac{\pi}{3} - s\,i\,\bar{n}^{1}\left(-\frac{1}{2}\right)\right)$

Q. 20

Find the values of each of the expression

$\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right)$

is equal to

A. π

B. $-\frac{\pi}{2}$

C. 0

D. $2\sqrt{3}$

Answer:

$\tan^{-1}\sqrt{3-}\cot^{-1}\left(-\sqrt{3}\right)$

$= \tan^{-1}\sqrt{3}-\left(\pi-c\,ot^{-1}\sqrt{3}\right)$

$= \tan^{-1}\sqrt{3}+\cot^{-1}\sqrt{3}-\pi$

$= \frac{\pi}{2}-\pi$

$= -\frac{\pi}{2}$

Miscellaneous Exercise

Q. 1

Find the value of the following:

Cos^{-1} $\left(\cos\frac{13\pi}{6}\right)$

Answer:

Cos^{-1} $\left(\cos\frac{13\pi}{6}\right)$

(Forcos-1(cos x) type of problem we have to always check whether the angle is in the principal range or not. This angle must be in the principal range. [0, π])

So here, $\frac{13\pi}{6} \notin [0, \pi]$

Now, cos^{-1} $\left(\cos\frac{13\pi}{6}\right)$can be written as,

Cos^{-1} $\left(\cos\frac{13\pi}{6}\right)$

= cos^{-1} $\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right]$

= cos^{-1} $\left(\cos\frac{\pi}{6}\right)$ where $\frac{\pi}{6} \in [0, \pi]$

$= \frac{\pi}{6}$

Hence, cos^{-1} $\left(\cos\frac{13\pi}{6}\right) = \frac{\pi}{6}$

Q. 2

Find the value of the following:

tan^{-1} $\left(\tan\frac{7\pi}{6}\right)$

Answer:

$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

(For $\tan^{-1}(\tan x)$ type of problem we have to always check whether the angle is in the principal range or not. This angle must be in the principal range. $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$)

So here, $\frac{7\pi}{6} \notin \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Now, $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ can be written as,

$= \tan^{-1}\left[\tan\left(\pi+\frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left(\tan\frac{\pi}{6}\right)$ where, $\frac{\pi}{6} \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ [since, $\tan(\pi+x) = \tan x$]

$= \frac{\pi}{6}$

Hence, $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \frac{\pi}{6}$

Q. 3

Prove that

$2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

Answer:

Taking LHS

Let

$\mathrm{Sin}^{-1}\frac{3}{5} = x$

Then,

$\mathrm{Sin}\ x = \frac{3}{5}$

Therefore,

tan x = $\frac{3}{\sqrt{5^2-3^2}} = \frac{3}{\sqrt{25-9}}$

∴ tan x = $\frac{3}{4}$ = x = $\tan^{-1}\frac{3}{4}$

= $\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$ ……. (1)

and

$2\sin^{-1}\frac{3}{5} = 2\tan^{-1}\frac{3}{4}$

Now, we know

$2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= 2\tan^{-1}\frac{3}{4} = \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right)$

$= \tan^{-1}\left(\frac{\frac{3}{2}}{1-\frac{9}{16}}\right)$

$= \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right)$

$= \tan^{-1}\frac{24}{7}$

As, LHS = RHS

Hence Proved!

Q. 4

Prove that

$\text{Sin}^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

Answer:

Sin^{-1} $\frac{8}{17}$ + sin^{-1} $\frac{3}{5}$ = tan^{-1} $\frac{77}{36}$

Sin^{-1} $\left(\frac{8}{17}\right)$ = x

Sin x = $\frac{8}{17}$

Cos x = $\sqrt{1-\left(\frac{8}{17}\right)^2}$

= $\sqrt{\frac{225}{289}}$

= $\frac{15}{17}$

∴ tan x = $\frac{8}{15}$ = x = tan^{-1} $\frac{8}{15}$

= sin^{-1} $\frac{8}{17}$ = tan^{-1} $\frac{8}{15}$ … (1)

Let sin-1 $\frac{3}{5}$ = y

Then, sin y = $\frac{3}{5}$

= cos y = $\sqrt{1-\left(\frac{3}{5}\right)^2}$ = $\frac{4}{5}$

= tan y = $\frac{3}{4}$ = y = tan^{-1} $\frac{3}{4}$

= sin^{-1} $\frac{3}{5}$ = tan^{-1} $\frac{3}{4}$ … (2)

Now,

L.H.S. = sin^{-1} $\frac{8}{17}$ + sin^{-1} $\frac{3}{5}$

= tan^{-1} $\frac{8}{15}$ + tan^{-1} $\frac{3}{4}$ putting the value from equation (1) and (2)

$= \tan^{-1} \frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}$ $\left[since\, tan^{-1}x - tan^{-1}y = tan^{-1}\frac{x-y}{1-xy}\right]$

$= \tan^{-1} \frac{32+45}{60-24}$

$= \tan^{-1} \frac{77}{36}$

= R.H.S

Hence proved.

Q. 5

Prove that

$\text{Cos}^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$

Answer:

$\text{Cos}^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$

Let $\cos^{-1} \frac{4}{5} = x$

Then, $\cos x = \frac{4}{5}$

$= \sin x = \sqrt{1-\left(\frac{4}{5}\right)^2} = \frac{3}{5}$

$= \tan x = \frac{3}{4} = x = \tan^{-1} \frac{3}{4}$

$= \cos^{-1} \frac{4}{3} = \tan^{-1} \frac{3}{4} \ldots (1)$

Let, $\cos^{-1} \frac{12}{13} = y$

Then $\cos y = \frac{12}{13}$

$= \sin y = \sqrt{1-\left(\frac{12}{13}\right)^2} = \frac{5}{13}$

$= \tan y = \frac{5}{12} = y = \tan^{-1} \frac{5}{12}$

$= \cos^{-1} \frac{12}{13} = \tan^{-1} \frac{5}{12}$ … (2)

Let, $\cos^{-1} \frac{33}{65} = z$

Then, $\cos z = \frac{33}{65}$

$= \sin z = \sqrt{1 - \left(\frac{33}{65}\right)^2} = \frac{56}{65}$

$\tan z = \frac{56}{33} = y = \tan-1 \frac{56}{33}$

$= \cos\text{-}1 \frac{33}{65} = \tan-1 \frac{56}{33}$ … (3)

Now,

L.H.S. $= \cos-1 \frac{4}{5} + \cos^{-1} \frac{12}{13}$

$= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{5}{12}$ putting the value from the equation (1) and (2)

$= \tan^{-1} \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}} \left[since\, tan^{-1}x - tan^{-1}y = tan^{-1}\frac{x-y}{1-xy}\right]$

$= \tan^{-1} \frac{36+20}{48-15}$

$= \tan^{-1} \frac{56}{33}$ … by equation (3)

= R.H.S.

Hence, proved.

Q. 6

Prove that

$\cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{56}{65}$

Answer:

$\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

We can also solve this problem by using the identity Sin (A + B) = sin A Cos B + cos A Sin B

Let, sin-1 $\frac{3}{5}$ = A and cos-1 $\frac{12}{13}$ = B

So,

sin A = 3/5 and cos B = 12/13Therefore, cos A = 4/5 and sin B = 5/13

As R.H.S. is sin-1 we will use sin (A+B)

= sin (A + B) = sin A Cos B + cos A Sin B

$= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13} = \frac{36}{65} + \frac{20}{65}$

$= \frac{56}{65}$

Thus, A + B = $\sin^{-1}\frac{56}{65}$

= R.H.S.

Hence Proved.

Q. 7

Prove that

$\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

Answer:

$\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

let, $\sin^{-1}\frac{5}{13}$ = x then, sin x = $\frac{5}{13}$

$= \cos x = \sqrt{1 - \left(\frac{5}{13}\right)^2} = \frac{12}{13}$

$= \tan x = \frac{5}{12} = x = \tan^{-1} \frac{5}{12}$

$= \sin^{-1} \frac{5}{13} = \tan^{-1} \frac{5}{12}$ … (1)

Let, $\cos^{-1} \frac{3}{5} = y$. then, $\cos y = \frac{3}{5}$

$= \sin y = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$

$= \tan y = \frac{4}{3} = y = \tan^{-1} \frac{4}{3}$

$= \cos^{-1} \frac{3}{5} = \tan^{-1} \frac{4}{3}$ … (2)

Now,

R.H.S. $= \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}$

$= \tan^{-1} \frac{5}{12} + \tan^{-1} \frac{4}{3}$ putting the value from equation (1) and (2)

$= \tan^{-1} \frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}} \left(since\, tan^{-1}x - tan^{-1}y = tan^{-1}\frac{x-y}{1-xy}\right)$

$= \tan^{-1} \frac{15+48}{36-20}$

$= \tan^{-1} \frac{63}{16}$

= L.H.S.

Hence, proved.

Q. 8

Prove that

$\tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$

Answer

L.H.S. $= \tan{-1}\ \frac{1}{5} + \tan{-1}\ \frac{1}{7} + \tan{-1}\ \frac{1}{3} + \tan{-1}\ \frac{1}{8}$

$= \tan{-1}\ \frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}} + \tan{-1}\ \frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{38}} \left[since\ tan^{-1}x - tan^{-1}y = tan^{-1}\frac{x-y}{1-xy}\right]$

$= \tan{-1}\ \frac{7+5}{35-1} + \tan{-1}\ \frac{8+3}{24-1}$

$= \tan^{-1} \frac{12}{34} + \tan{-1}\ \frac{11}{23}$

$= \tan^{-1} \frac{6}{17} + \tan{-1}\ \frac{11}{23}$

$= \tan^{-1} \frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\times\frac{11}{23}} \left[since\ tan^{-1}x - tan^{-1}y = tan^{-1}\frac{x-y}{1-xy}\right]$

$= \tan^{-1} \frac{138+187}{391-66}$

$= \tan^{-1} \frac{325}{325} = \tan^{-1} = \frac{\pi}{4}$

= R.H.S.

Hence, proved.

Q. 9

Prove that

$\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \left(\frac{1-x}{1+x}\right)$, $x \in [0, 1]$

Answer:

$\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \left(\frac{1-x}{1+x}\right)$

Let $x = \tan 2\theta$.Then, $\sqrt{x} = \tan\theta$

$= \theta = \tan^{-1} \sqrt{x}$

$\therefore \frac{1-x}{1+x} = \frac{1-tan^2}{1+tan^2} = \cos 2\theta$

So now putting the value, we get,

R.H.S. $= \frac{1}{2} \cos^{-1} \left(\frac{1-x}{1+x}\right)$

$= \frac{1}{2} \cos^{-1} (\cos 2\theta) = \frac{1}{2} \times 2\theta = \tan^{-1} \sqrt{x}$

= L.H.S.

Q. 10

Prove that

$\text{Cot}^{-1} \left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$

Answer:

$\text{Cot}^{-1} \left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

Consider, $\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

On Rationalizing, we get,

$= \frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}\right)^2-\left(\sqrt{1+\sin x}\right)^2}$

$= \frac{(1+\sin x)+(1-\sin x)+2\sqrt{(1+\sin x)(1-\sin x)}}{1+\sin x-1+\sin x}$

$= \frac{2\left(1+\sqrt{1-sin^2 x}\right)}{2\sin x} = \frac{1+\cos x}{\sin x} = \frac{2cos^2\frac{x}{2}}{2sin\frac{x}{2}cos\frac{x}{2}} = \cot \frac{x}{2}$

Now,

L.H.S. $= \cot^{-1} \left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

$= \cot^{-1}\left(cot\frac{x}{2}\right) = \frac{x}{2}$

= R.H.S.

Hence Proved

Q. 11

Prove that

$\tan{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos{-1}\, x$

Answer:

$\tan\text{-}1\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos{-1}\, x$

Let $x = \cos 2\theta$ so that $\theta = \frac{1}{2}\cos{-1}\, x$

Now,

L.H.S. $= \tan{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

$= \tan{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{2cos^2\theta}-\sqrt{2sin^2\theta}}{\sqrt{2cos^2\theta}+\sqrt{2sin^2\theta}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$

$= \tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$

$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

$= \tan^{-1} 1 - \tan^{-1}(\tan\theta) = \frac{\pi}{4} - \theta$

$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$

= R.H.S.

Hence Proved.

Q. 12

Prove that

$\frac{9\pi}{8} - \frac{9}{4}$ sin^{-1} $\frac{1}{3} = \frac{9}{4}$ sin^{-1} $\frac{2\sqrt{2}}{3}$π

Answer:

$\frac{9\pi}{8} - \frac{9}{4}$ sin^{-1} $\frac{1}{3} = \frac{9}{4}$ sin^{-1} $\frac{2\sqrt{2}}{3}$

Now, L.H.S. $\frac{9\pi}{8} - \frac{9}{4}$ sin^{-1} $\frac{1}{3}$

$= \frac{9}{4}\left(\frac{\pi}{2} - sin^{-1}\frac{1}{3}\right)$

$= \frac{9}{4} \times$ cos^{-1} $\frac{1}{3}$ … eq. (1)

Now, Let

Cos^{-1} $\frac{1}{3}$ = x. then cos x = $\frac{1}{3}$ = sin x = $\sqrt{1 - \left(\frac{1}{3}\right)^3} = \frac{2\sqrt{2}}{3}$

∴ x = sin^{-1} $\frac{2\sqrt{2}}{3}$ = cos^{-1} $\frac{1}{3}$ = sin^{-1} $\frac{2\sqrt{2}}{3}$

L.H.S. $= \frac{9}{4}$ sin^{-1} $\frac{2\sqrt{2}}{3}$

= R.H.S.

Hence Proved

Q. 13

Solve the following equations:

2 tan^{-1} (cos x) = tan^{-1} (2 cosec x)

Answer:

$2\tan^{-1}(\cos x) = \tan^{-1}(2\text{ cosec } x)$

$= \tan^{-1}\left(\frac{2\cos x}{1-cos^2 x}\right) = \tan^{-1}(2\text{ cosec } x)$

$= \frac{2\cos x}{1-cos^2 x} = 2\text{ cosec } x$

$= \frac{2\cos x}{sin^2 x} = \frac{2}{\sin x} = \cos x = \sin x$

$= \tan x = 1$

Hence, $x = \frac{\pi}{4}$

Q. 14

Solve the following equations:

$\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1} x, (x > 0)$

Answer:

$\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1} x$

As we know, $\tan^{-1}(x) - \tan^{-1}(y) = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

$= \tan^{-1} 1 - \tan^{-1} x = \frac{1}{2}\tan^{-1} x$

We know, $\tan\frac{\pi}{4} = 1$

So, $\tan^{-1}(1) = \frac{\pi}{4}$

$= \frac{\pi}{4} - \tan^{-1}(x) = \frac{1}{2}\tan^{-1}(x)$

$= \frac{\pi}{4} = \frac{3}{2}\tan^{-1} x$

$= \tan^{-1} x = \frac{\pi}{6}$

$= x = \tan\frac{\pi}{6}$

Hence,

x = $\frac{1}{\sqrt{3}}$

Q. 15

Solve the following equations:

Sin (tan^{-1} x), |x| < 1 is equal to

A. $\frac{x}{\sqrt{1+x^2}}$

B. $\frac{1}{\sqrt{1-x^2}}$

C. $\frac{1}{\sqrt{1+x^2}}$

D. $\frac{x}{\sqrt{1+x^2}}$

Answer:

Let tan-1 x = y, then tan y = x = sin y = $\frac{x}{\sqrt{1+x^2}}$

∴ y = sin^{-1} $\left(\frac{x}{\sqrt{1+x^2}}\right)$

= tan^{-1} x = sin^{-1} $\left(\frac{x}{\sqrt{1+x^2}}\right)$

= sin (tan^{-1} x) = sin $\left(si\ n^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)$

= $\frac{x}{\sqrt{1+x^2}}$

Q. 16

Solve the following equations:

Sin^{-1} (1 – x) – 2 sin^{-1} x = $\frac{\pi}{2}$, then x is equal to

A. 0, $\frac{1}{2}$

B. 1, $\frac{1}{2}$

C. 0

D. $\frac{1}{2}$

Answer:

$\text{Sin}^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$

Now we will put Now, we will put x= sin y in the given equation, and we get,

$\text{Sin}^{-1}(1 - \sin y) - 2\sin^{-1}\sin y = \frac{\pi}{2}$

$= \sin^{-1}(1 - \sin y) - 2y = \frac{\pi}{2}\pi$

$= \sin^{-1}(1 - \sin y) = \frac{\pi}{2} + 2y$

$= 1 - \sin y = \sin\left(\frac{\pi}{2} + 2y\right)$

$\Rightarrow 1 - \sin y = \cos 2y$ as $\sin\left(\frac{\pi}{2} + x\right) = \cos x$

$\Rightarrow 1 - \cos 2y = \sin y$

$\Rightarrow 2\sin 2y = \sin y$

$\Rightarrow \sin y\,(2\sin y - 1) = 0$

$\Rightarrow \sin y = 0$ or $1/2$

$\therefore x = 0$ or $1/2$

Now, if we put then we will see that,

L.H.S. $= \sin^{-1}\left(\frac{1}{2}\right) = -2\sin^{-1}\frac{1}{2}$

$= -\sin^{-1}\frac{1}{2}$

$= -\frac{\pi}{6} \neq \frac{\pi}{2} \neq$ R.H.S

Hence, x = $\frac{1}{2}$is not the solution of the given equation.

Thus, x = 0

Q. 17

Solve the following equations:

$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$ is equal to

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{-3\pi}{4}$

Answer:

$\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$

$= \tan^{-1}\left[\frac{\frac{x}{y}-\frac{x-y}{x+y}}{1-\frac{x}{y}.\frac{x-y}{x+y}}\right]$

$= \tan^{-1}\left[\frac{\frac{x(x+y)-y(x-y)}{y(x+y)}}{\frac{y(x+y)+x(x-y)}{y(x+y)}}\right]$

$= \tan^{-1}\left(\frac{x^2+xy-xy+y^2}{xy+y^2+x^2-xy}\right)$

$= \tan^{-1}\left(\frac{x^2+y^2}{x^2+y^2}\right) = \tan^{-1} 1 = \frac{\pi}{4}$