Chapter 8

DESCRIPTIVE STATISTICS AND PROBABILITY

Learning Objectives

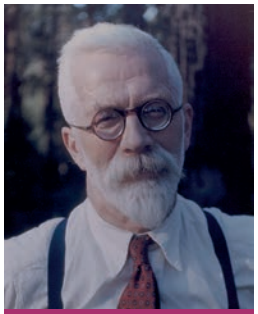
After studying this chapter students will be able to understand the following concepts.

- Measures of Central Tendency like A.M, G.M & H.M
- Relationship among the averages.
- Related positional measures like Quartiles, Deciles and percentiles etc.,
- Measures of Dispersion like Quartile Deviation, mean Deviation
- Relative measures like co-efficient of Quartile Deviation, co-efficient of Mean Deviation
- Concept of conditional probability and multiplication theorem.
- Baye's theorem and its applications.

8.1 Measures of Central Tendency

Introduction:

One of the most important objectives of Statistical analysis is to get one single value that describes the characteristic of the entire value for data. Such a value represent the measure of central tendency for the complete data. The word average is very commonly used in day-to-day conversation. For example, we often talk of average boy in a class, average height or average life of an Indian, average income, etc,. Sir Ronald Fisher who is known to be a father of statistics and he made his pioneering contributions in the applications of statistics in various disciplines.


Sir Ronald Fisher

IKFN76

8.1.1 Average

- Recall

There are several measures of central tendency for the data. They are

- Arithmetic Mean
- Median
- Mode
- Geometric Mean
- Harmonic Mean

Arithmetic Mean (discrete case)

Arithmetic mean of a set of observations is their sum divided by the number of observations. The observation are classified into a) Ungrouped data and b) Grouped data.

a) Ungrouped data

(i) Direct Method:

$$\overline{X} = \frac{X_1 + X_2 + X_3 + ...X_n}{n} = \frac{\sum X}{n}$$

where $\overline{X}$ is Arithmetic Mean, $\sum X$ is sum of all the values of the variable X and n is number of observations.

(ii) Short-cut method

The arithmetic mean can be calculated by using any arbitrary value A as origin and write d as the deviation of the variable X then,

$$\overline{X} = A + \frac{\Sigma d}{n} \text{ where } d = X - A.$$

b) Grouped data

(i) Direct method

The formula for computing the mean is $\overline{X} = \frac{\Sigma fx}{N}$

where f is frequency, X is the variable, $N = \Sigma f$ i.e. total frequency.

(ii) Short-cut method

The arithmetic mean is computed by applying the following formula:

$\overline{X} = A + \frac{\Sigma fd}{N}$, where A is assumed mean, $d = X - A$, $N = \Sigma f$

Arithmetic mean for Continuous case

The arithmetic mean may be computed by applying any of the following methods:

(i) Direct method

(ii) Short-cut method

(iii) Step deviation method

(i) Direct method

When direct method is used arithmetic mean is defined as

$$\overline{X} = \frac{\Sigma fm}{N}$$

Where m = midpoint of each of the class interval,

f = the frequency of each class interval

$N = \Sigma f =$ total frequency

(ii) Short-cut method

The arithmetic mean is computed by applying the following formula.

$$\overline{X} = A + \frac{\Sigma fd}{N}$$

where A is assumed mean (or) arbitrary value, $d = m - A$ is deviations of mid-point from assumed mean and $N = \Sigma f$

(iii) Step Deviation Method

In case of grouped (or) continuous frequency distribution, the arithmetic mean is

$\overline{X} = A + \left(\frac{\Sigma fd}{N} \times c\right)$, where $d = \frac{(m - A)}{c}$

A is any arbitrary value (or) assumed mean and c is the magnitude of class interval.

DO YOU KNOW?

All the above three methods of finding arithmetic mean in continuous case gives us the same answer.

Mode:

Mode is the value which repeats maximum number of times among the given observations.

Median:

Median is exactly a middle value and it exceeds and exceeded by the same number of observations. Median is one of the positional measure. Some other related positional measures are also described below.

NOTE

It is believed that the students might be familiar with the above concepts and our present syllabus continues from the following.

8.1.2 Related Positional Measures - Quartiles, Deciles and Percentiles :

Besides median there are other measures which divide a series into equal parts. Important amongst these are quartiles, deciles and percentiles.

(i) Quartiles:

A measure which divides an array into four equal parts is known as quartiles.

Each portion contains equal number of items. The first, second and third points are termed as first quartile (Q_1), second quartile (Q_2) (better named as median) and the third quartile(Q_3). The first quartile (Q_1) or lower quartile, has 25% of the items of the distribution below it and 75% of the items are greater than it. Q_2(median) the second quartile or median has 50 % of the observations above it and 50% of the observations below it. The upper quartile or third quartile(Q_3) has 75% of the items of the distribution below it and 25% of the items are above it. Similarly the other two positional measures can be defined.

(ii) Deciles:

A measure which divides an array into ten equal parts is known as deciles.

That is Deciles are the values which divides the series into ten equal parts. We get nine dividing positions namely $D_1, D_2, \ldots, D_9$ which are called as deciles. Therefore there are nine deciles. It is to be noted that D_5 is equal to median.

(iii) Percentiles:

A measure which divides an array into hundred equal parts is known as Percentiles.

That is percentiles are the values which divides the series into hundred equal parts. We get 99 dividing positions $P_1, P_2, \ldots, P_{99}$ which are called as percentiles. Therefore there are 99 percentiles. It is to be noted that P_{50} is equal to median.

8.1.3 Computations for Related positional measure

The procedure for computing quartiles, deciles and percentiles are the same as the median.

(i) Ungrouped data:

Steps:

1. Arrange the data either in ascending or descending order of magnitude.

2. Apply the formula

$$Q_1 = \text{Size of } \left(\frac{n+1}{4}\right)^{th} \text{ value}$$

$$Q_3 = \text{Size of } \left(\frac{3(n+1)}{4}\right)^{th} \text{ value}$$

$$D_1 = \text{Size of } \left(\frac{n+1}{10}\right)^{th} \text{ value}$$

$$D_2 = \text{Size of } 2\left(\frac{n+1}{10}\right)^{th} \text{ value}$$

$$P_{60} = \text{Size of } 60\left(\frac{n+1}{100}\right)^{th} \text{ value}$$

$$P_{99} = \text{Size of } 99\left(\frac{n+1}{100}\right)^{th} \text{ value}$$

DO YOU KNOW?

$Q_2 = D_5 = P_{50} =$ **Median**

Example 8.1

Find D_2 and D_6 for the following series 22, 4, 2, 12, 16, 6, 10, 18, 14, 20, 8

Solution:

Here n = 11 observations are arranged into ascending order

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22

$$D_2 = \text{size of } 2\left(\frac{n+1}{10}\right)^{th} \text{ value}$$

$$D_6 = \text{size of } 6\left(\frac{n+1}{10}\right)^{th} \text{ value}$$

D_2 = size of 2.4th value ≈size of 2nd value = 4

D_6 = size of 7.2th value ≈size of 7th value =14

Example 8.2

Calculate the value of Q_1, Q_3, D_6 and P_{50} from the following data

Roll No	1	2	3	4	5	6	7
Marks	20	28	40	12	30	15	50

Solution:

Marks are arranged in ascending order

12 15 20 28 30 40 50

n = number of observations =7

$$Q_1 = \text{Size of } \left(\frac{n+1}{4}\right)^{th} \text{ value}$$

$$= \text{Size of } \left(\frac{7+1}{4}\right)^{th} \text{ value}$$

$$= \text{Size of } 2^{nd} \text{ value } = 15$$

$$Q_3 = \text{Size of } \left(\frac{3(n+1)}{4}\right)^{th} \text{ value}$$

$$= \text{Size of} \left(\frac{3\times 8}{4}\right)^{th} \text{ value}$$

$$= \text{Size of } 6^{th} \text{ value} = 40$$

$$D_6 = \text{Size of } \left(\frac{6(n+1)}{10}\right)^{th} \text{ value}$$

$$= \text{Size of } \left(\frac{6\times 8}{10}\right)^{th} \text{ value}$$

$$= \text{Size of } 4.8^{th} \text{ value}$$

$$= \text{Size of } 5^{th} \text{ value} = 30$$

$$P_{50} = \text{Size of } \left(\frac{50(n+1)}{100}\right)^{th} \text{ value}$$

$$= \text{Size of } 4^{th} \text{ value} = 28$$

Hence Q_1=15, Q_3=40, D_6 =30 and P_{50}=28

(ii) Grouped data (discrete case):

Steps:

1. Arrange the data in ascending or descending order of magnitude.
2. Find out cumulative frequencies.
3. Apply the formula:

$$Q_1 = \text{Size of } \left(\frac{N+1}{4}\right)^{th} \text{ value}$$

$$Q_3 = \text{Size of } \left(\frac{3(N+1)}{4}\right)^{th} \text{ value}$$

Now look at the cumulative frequency (*cf*) column and find that total which is either equal to $\left(\frac{N+1}{4}\right)$ or next higher than that and determine the value of the variable corresponding to this. That gives the value of Q_1. Similarly Q_3 is determined with reference of $\frac{3(N+1)}{4}$ value of the variable.

Example 8.3

Compute Q_1, D_2 and P_{90} from the following data

Marks	10	20	30	40	50	60
No. of Students	4	7	15	8	7	2

Solution:

Marks X	Frequency *f*	Cumulative Frequency cf
10	4	4
20	7	11
30	15	26
40	8	34
50	7	41
60	2	*N*= 43

Table : 8.1

$$Q_1 = \text{Size of } \left(\frac{N+1}{4}\right)^{th} \text{ value}$$

$$= \text{Size of } \frac{43+1}{4} = 11^{th} \text{ value} = 20.$$

$$D_2 = \text{Size of } \left(\frac{2(N+1)}{10}\right)^{th} \text{ value}$$

$$= \text{Size of } \frac{88}{10} = 8.8^{th} \text{ value} = 20.$$

$$P_{90} = \text{Size of } \left(\frac{90(N+1)}{100}\right)^{th} \text{ value}$$

$$= \text{Size of } \frac{3960}{100} = 39.6^{th} \text{ value} = 50.$$

(iii) Grouped data (Continuous case):

In the case of continuous frequency distribution, the classes are arranged either in ascending or descending order and the class corresponding to the cumulative frequency (cf) just equal or greater than (N/4) is called Q_1 class and the value of Q_1 is obtained by the following formula:

$$Q_1 = L + \left(\frac{\frac{N}{4} - pcf}{f}\right) \times c$$

where L is the lower limit of the Q_1 class,

f is the frequency of the Q_1 class,

c is the magnitude of the Q_1 class,

pcf is the cumulative frequency of the pre Q_1 class.

Similarly third quartile value can be obtained by the same procedure with Q_3 class by the following formula:

$$Q_3 = L + \left(\frac{\frac{3N}{4} - pcf}{f}\right) \times c$$

where L is the lower limit of the third quartile class,

f is the frequency of the third quartile class,

c is the magnitude of the third quartile class,

pcf is the cumulative frequency of the pre Q_3 class.

Similarly the same procedure is to be followed for other positional measures such as deciles and percentiles,

$$D_4 = L + \left(\frac{\frac{4N}{10} - pcf}{f}\right) \times c$$

$$P_{60} = L + \left(\frac{\frac{60N}{100} - pcf}{f}\right) \times c$$

Example 8.4

Compute upper Quartiles, lower Quartiles, D_4 and P_{60}, P_{75} from the following data.

CI	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	12	19	5	10	9	6	6

Solution:

CI	f	cf
10 – 20	12	12
20 – 30	19	31
30 – 40	5	36
40 – 50	10	46
50 – 60	9	55
60 – 70	6	61
70 – 80	6	N= 67
	N = 67	

Table : 8.2

Q_1 = Size of $\left(\frac{N}{4}\right)^{th}$ value = $\frac{67}{4}$ = 16.75^{th} value.

Thus Q_1 lies in the class (20 – 30) and its corresponding values are $L = 20$;

$\frac{N}{4} = 16.75$; $pcf = 12$; $f = 19$; $c = 10$

$$Q_1 = L + \left(\frac{\frac{N}{4} - pcf}{f}\right) \times c$$

$$Q_1 = 20 + \left(\frac{16.75 - 12}{19}\right) \times 10 = 20 + 2.5 = 22.5$$

Q_3 = Size of $\left(\frac{3N}{4}\right)^{th}$ value = 50.25^{th} value

So Q_3 lies in the class (50-60) corresponding values are $L = 50$, $\left(\frac{3N}{4}\right) = 50.25$; $pcf = 46$, $f = 9$, $c = 10$

$$Q_3 = L + \left(\frac{\frac{3N}{4} - pcf}{f}\right) \times c$$

$$Q_3 = 50 + \left(\frac{50.25 - 46}{9}\right) \times 10 = 54.72$$

$$D_4 = L + \left(\frac{\frac{4N}{10} - pcf}{f}\right) \times c$$

D_4 = Size of $\left(\frac{4N}{10}\right)^{th}$ value = 26.8^{th} value. Thus D_4 lies in the class (20 – 30) and its corresponding values are

$L = 20$, $\frac{4N}{10} = 26.8$; $pcf = 12$, $f = 19$, $c = 10$.

$$D_4 = 20 + \left(\frac{26.8 - 12}{19}\right) \times 10 = 27.79$$

P_{75} = Size of $\left(\frac{75N}{100}\right)^{th}$ value = 50.25th value. Thus P_{75} lies in the class (50 – 60) and its corresponding values are

$$L = 50;\ \frac{75N}{100} = 50.25;\ pcf = 46, f = 9, c = 10.$$

$$P_{75} = L + \left(\frac{\frac{75N}{100} - pcf}{f}\right) \times c$$

$$= 50 + \left(\frac{50.25 - 46}{9}\right) \times 10 = 54.72$$

8.1.4 Geometric mean

Geometric mean is defined as the nth root of the product of *n* observations or values.

If there are two observations, we take the square root; if there are three observations we have to take the cube root and so on

$$GM = \sqrt[n]{X_1 . X_2 . X_3 ..... X_n} = (X_1, X_2, X3, ...X_n)^{1/n}$$

where $X_1, X_2, X_3, ..., X_n$ refer to the various items of the series which are all greater than zero and n refers number of observations.

Thus the geometric mean of 3 values 2,3,4 would be

$$GM = \sqrt[3]{(2)(3)(4)} = 2.885$$

When the number of items is three or more the task of multiplying the numbers and of extracting the root becomes excessively difficult. To simplify calculations, logarithms are used. Geometric mean is calculated as follows:

$$\log GM = \frac{\log X_1 + \log X_2 + ... + \log X_n}{n}$$

(or) $$\log GM = \left(\frac{\Sigma \log X}{n}\right)$$

$$GM = Anti\log\left(\frac{\Sigma \log X}{n}\right),$$

where n is number of observation.

(i) In discrete observation

$$GM = Anti\log\left(\frac{\Sigma f \log X}{N}\right);\ \text{where } N = \Sigma f$$

(ii) In Continuous observation

GM = $Anti\log\left[\frac{\Sigma f \log m}{N}\right]$; where m is midpoint and $N = \Sigma f$

Example 8.5

Daily income (in Rs) of ten families of a particular place is given below. Find out GM

85, 70, 15, 75, 500, 8, 45, 250, 40, 36.

Solution:

X	log X
85	1.9294
70	1.8451
15	1.1761
75	1.8751
500	2.6990
8	0.9031
45	1.6532
250	2.3979
40	1.6021
36	1.5563
	$\Sigma \log X = 17.6373$

Table : 8.3

$$GM = Anti\log\left(\frac{\Sigma \log X}{n}\right);\ \text{where } n = 10$$

$$GM = Anti\log\left(\frac{17.6373}{10}\right)$$

$$= Anti\log(1.7637)$$

$$GM = 58.03$$

Example 8.6

Calculate the geometric mean of the data given below giving the number of families and the income per head of different classes of people in a village of Kancheepuram District.

Class of people	No. of Families	Income per head in 1990 (Rs)
Landlords	1	1000
Cultivators	50	80
Landless labourers	25	40
Money- lenders	2	750
School teachers	3	100
Shop-keepers	4	150
Carpenters	3	120
Weavers	5	60

Solution:

Calculation of Geometric Mean

Class of people	Income per head in 1990 (₹) X	No of Families f	$\log X$	$f \log X$
Landlords	1000	1	3.0000	3.0000
Cultivators	80	50	1.9031	95.1550
Landless labourers	40	25	1.6021	40.0525
Money- lenders	750	2	2.8751	5.7502
School teachers	100	3	2.0000	6.0000
Shopkeepers	150	4	2.1761	8.7044
Carpenters	120	3	2.0792	6.2376
Weavers	60	5	1.7782	8.8910
		N = 93		Σ=173.7907

Table : 8.4

$$GM = Anti\log\left(\frac{\Sigma f \log X}{N}\right)$$

$$= Anti\log\left(\frac{173.7907}{93}\right)$$

$$= Anti\log(1.8687)$$

GM = 73.91

Example 8.7

Compute the Geometric mean from the data given below:

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	8	12	18	8	6

Solution:

Marks	m	f	$\log m$	$f \log m$
0-10	5	8	0.6990	5.5920
10-20	15	12	1.1761	14.1132
20-30	25	18	1.3979	25.1622
30-40	35	8	1.5441	12.3528
40-50	45	6	1.6532	9.9192
		N = 52		$\Sigma f \log m = 67.1394$

Table : 8.5

$$GM = Anti\log\left(\frac{\Sigma f \log m}{N}\right)$$

$$= Anti\log\left(\frac{67.1394}{52}\right)$$

$$= Anti\log(1.2911)$$

GM = 19.54

Specific uses of Geometric mean

The most useful application of geometric mean is to average the rate of changes. For example, from 2006 to 2008 prices increased by 5%, 10% and 18% respectively. The average annual increase is not 11% $\left(\frac{5+10+18}{3} = 11\right)$ as given by the arithmetic average but 10.9% as obtained by the geometric mean. This average is also useful in measuring the growth of population, because population increases in geometric progression.

Example 8.8

Compared to the previous year the overhead expenses went up by 32% in 1995, they increased by 40% in the next year and by 50% in the following year. Calculate the average rate of increase in overhead expenses over the three years.

Solution:

In averaging ratios and percentages, geometric mean is more appropriate. Let us consider X represents Expenses at the end of the year.

% Rise	X	log X
32	132	2.1206
40	140	2.1461
50	150	2.1761
		$\Sigma \log X = 6.4428$

Table : 8.6

$$GM = Anti\log\left(\frac{\Sigma \log X}{n}\right)$$

$$= Anti\log\left(\frac{6.4428}{3}\right)$$

$$= Anti\log(2.1476)$$

$$GM = 140.5$$

Average rate of increase in overhead expenses

$$140.5 - 100 = 40.5\ \%$$

DO YOU KNOW?

Geometric mean cannot be calculated if one of the observations is zero.

8.1.5 Harmonic mean

Harmonic mean is defined as the reciprocal of the arithmetic mean of the reciprocal of the individual observations. It is denoted by HM.

$$\text{Thus, HM} = \frac{n}{\left(\frac{1}{X_1}+\frac{1}{X_2}+\ldots..+\frac{1}{X_n}\right)}$$

When the number of items is large the computation of harmonic mean in the above manner becomes tedious. To simplify calculations we obtain reciprocals of the various items from the tables and apply the following formulae:

(i) In individual observations

$$HM = \frac{n}{\left(\frac{1}{X_1}+\frac{1}{X_2}+\ldots..+\frac{1}{X_n}\right)} \quad \text{(or)}$$

$$HM = \frac{n}{\Sigma\left(\frac{1}{X}\right)}$$

where n is number of observations or items or values.

(ii) In discrete frequency distribution

$$HM = \frac{N}{\Sigma\left(\frac{f}{X}\right)} \quad \text{where}$$

N = total frequency= Σf

(iii) In continuous frequency distribution

$$HM = \frac{N}{\Sigma\left(\frac{f}{m}\right)}$$

Where m is midpoint and N is total frequency

Example 8.9

Calculate the Harmonic Mean of the following values:

1, 0.5, 10, 45.0, 175.0, 0.01, 4.0, 11.2.

Solution:

X	$\frac{1}{X}$
1	1.0000
0.5	2.0000
10	0.1000
45	0.0222
175	0.0057
0.01	100.0000
4.0	0.2500
11.2	0.0893
	$\Sigma\left(\frac{1}{X}\right)=103.4672$

Table : 8.7

$n = 8$

$$HM = \frac{n}{\Sigma\left(\frac{1}{X}\right)} = \frac{8}{103.467} = 0.077$$

Example 8.10

From the following data compute the value of Harmonic Mean

Marks	10	20	25	40	50
No. of students	20	30	50	15	5

Solution:

Calculation of Harmonic Mean

Marks X	No. of Students f	$\frac{f}{x}$
10	20	2.000
20	30	1.500
25	50	2.000
40	15	0.375
50	5	0.100
	$N = 120$	$\Sigma\left(\frac{f}{X}\right) = 5.975$

Table : 8.8

$$HM = \frac{N}{\Sigma\left(\frac{f}{X}\right)} = \frac{120}{5.975} = 20.08$$

Example 8.11

Calculate Harmonic Mean for the following data given below:

Value	0-10	10-20	20-30	30-40	40-50
Frequency	8	12	20	6	4

Solution:

Calculation of Harmonic Mean

Value	Mid-Point m	f	$\left(\frac{f}{m}\right)$
0-10	5	8	1.60
10-20	15	12	0.80
20-30	25	20	0.80
30-40	35	6	0.17
40-50	45	4	0.09
		N = 50	$\Sigma\left(\frac{f}{m}\right)$ =3.46

Table : 8.9

$$HM = \frac{N}{\Sigma\left(\frac{f}{m}\right)} = \frac{50}{3.46} = 14.45$$

Special applications of Harmonic Mean

The Harmonic Mean is restricted in its field of usefulness. It is useful for computing the average rate of increase of profits of a concern or average speed at which a journey has been performed or the average price at which an article has been sold. The rate usually indicates the relation between two different types of measuring units that can be expressed reciprocally.

For example, if a man walked 20km in 5 hours, the rate of his walking speed can be expressed

$$\frac{20\,km}{5\,hours} = 4\,km\ per\ hour$$

where the units of the first term is a km and the unit of the second term is an hour or reciprocally,

$$\frac{5\,hours}{20\,km} = \frac{1}{4}\ hour\ per\ km\ .$$

where the unit of the first term is an hour and the unit of the second term is a km.

Example 8.12

An automobile driver travels from plain to hill station 100km distance at an average speed of 30km per hour. He then makes the return trip at average speed of 20km per hour what is his average speed over the entire distance (200km)?

Solution:

If the problem is given to a layman he is most likely to compute the arithmetic mean of two speeds.

i.e., $\overline{X} = \dfrac{30\,km + 20\,km}{2} = 25\ kmph$

But this is not the correct average.

So harmonic mean would be mean suitable in this situation.

Harmonic Mean of 30 and 20 is

$$HM = \frac{2}{\left(\frac{1}{20}\right)+\left(\frac{1}{30}\right)} = \frac{2}{\left(\frac{5}{60}\right)} = \frac{2(60)}{5} = 24\,kmph$$

8.1.6 Relationship among the averages

In any distribution when the original items differ in size, the values of AM, GM and HM would also differ and will be in the following order

$AM \geq GM \geq HM$

DO YOU KNOW?

If all the numbers $X_1, X_2, \ldots, X_n$ are identical then, AM = GM = HM.

Example 8.13

Verify the relationship among AM, GM and HM for the following data

X	7	10	13	16	19	22	25	28
f	10	22	24	28	19	9	12	16

Solution:

X	f	Xf	logX	f logX	f / X
7	10	70	0.8451	8.4510	1.4286
10	22	220	1	22.0000	2.2000
13	24	312	1.1139	26.7346	1.8462
16	28	448	1.2041	33.7154	1.7500
19	19	361	1.2788	24.2963	1.0000
22	9	198	1.3424	12.0818	0.4091
25	12	300	1.3979	16.7753	0.4800
28	16	448	1.4472	23.1545	0.5714
	$\sum f = N = 140$	$\sum fX = 2357$		$\sum f \log x = 167.209$	$\sum \frac{f}{x} = 9.6852$

Table : 8.10

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$$AM = \frac{\sum fX}{N} = \frac{2357}{140} = 16.84$$

$$GM = Anti\log\left(\frac{\sum f\log X}{N}\right)$$

$$= Anti\log\left(\frac{167.209}{140}\right) = Anti\log(1.1944) = 15.65$$

$$HM = \frac{N}{\Sigma\left(\frac{f}{X}\right)} = \frac{140}{9.6852} = 14.46$$

i.e. $16.84 > 15.65 > 14.46$

$\therefore\ AM > GM > HM$

NOTE

The Harmonic Mean is a measure of central tendency for data expressed as rates such as kilometres per hour, kilometres per litre, periods per semester, tonnes per month etc.

Example 8.14

A's scooter gives an average of 40km a litre while B's scooter gives an average of 30km a litre . Find out the mean, if

(i) each one of them travels 120 km.

(ii) the petrol consumed by both of them is 2 litres per head.

Solution:

(i) Here the distance is constant. Hence harmonic mean is appropriate.

$$HM = \frac{n}{\frac{1}{a}+\frac{1}{b}}$$

$$= \frac{2}{\frac{1}{40}+\frac{1}{30}} = \frac{2}{\frac{7}{120}}$$

$$= \frac{2\times 120}{7} = 34.3 \text{ km per litre}$$

(ii) Here the quantity of petrol consumed is fixed i.e 2 litres. Here the arithmetic mean will give the correct answer.

$$\overline{X} = \frac{\textit{Total distance Covered}}{\textit{Total Petrol Consumed}}$$

$$\overline{X} = \frac{40\times 2+30\times 2}{4} = 35$$

$\therefore$ Average speed = 35 km per litre.

Example 8.15

A person purchases tomatoes from each of the 4 places at the rate of 1kg., 2kg., 3kg., and 4kg. per rupee respectively .On the average, how many kilograms has he purchased per rupee?

Solution:

Since we are given rate per rupee, harmonic mean will give the correct answer.

$$HM = \frac{n}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$$

$$= \frac{4}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}$$

$$= \frac{4\times 12}{25}$$

$= 1.92$ kg per rupee.

8.2 Measures of Dispersion

Average gives us an idea on the point of the concentration of the observations about the central part of the distribution. If we know the average alone we cannot form a complete idea about the distribution as will be clear from the following example.

Consider the series:

(i) 7,8,9,10,11 (ii) 3,6,9,12,15 and (iii) 1,5,9, 13,17. In all these cases we see that n, the number of observations is 5 and the mean is 9. If we are given that the mean of 5 observations is 9, we cannot form an idea as to whether it is the average of first series or second series or third series or of any other series of 5 observations whose sum is 45. Thus we see that the measures of central tendency are inadequate to give us a complete idea of the distribution. So they must be supported and supplemented by some other measures. One such measure is Dispersion, which provides the nature of spreadness of the data.

Literal meaning of dispersion is "scatteredness" we study dispersion to have an idea about the homogeneity or heterogeneity of the distribution. In the above case we say that series (i) is more homogeneous (less dispersed) than the series (ii) or (iii) or we say that series (iii) is more heterogeneous (more scattered) than the series (i) or (ii).

Various measures of dispersion can be classified into two broad categories.

(a) The measures which express the spread of observations in terms of distance between the values of selected observations. These are also termed as distance measures.

Example: Range and interquartile range (or) quartile deviation.

(b) The measures which express the spread of observations in terms of the average of deviations of observation from some central value.

Example: Mean deviation and Standard deviation.

8.2.1 Quartile Deviation

Quartile Deviation is defined as

$QD = \frac{1}{2}(Q_3 - Q_1)$. It may also be called as semi-inter quartile. Where Q_1 and Q_3 are the first and third quartiles of the distribution respectively and $Q_3 - Q_1$ is called as inter quartile range.

(i) Relative measures for QD

Quartile deviation is an absolute measure of dispersion. The relative measure corresponding to this measure, called the coefficient of quartile deviation is calculated as follows:

$$\text{Coefficient of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

Coefficient of quartile deviation can be used to compare the degree of variation in different distributions.

(ii) Computation of Quartile Deviation

The process of computing quartile deviation is very simple since we just have to compute the values of the upper and lower quartiles that is Q_3 and Q_1 respectively.

Example 8.16

Calculate the value of quartile deviation and its coefficient from the following data

Roll No.	1	2	3	4	5	6	7
Marks	20	28	40	12	30	15	50

Solution:

Marks are arranged in ascending order

12 15 20 28 30 40 50

n = number of observations =7

Q_1= Size of $\left(\frac{(n+1)}{4}\right)^{th}$ value = Size of $\left(\frac{7+1}{4}\right)^{th}$ value = Size of 2nd value = 15

Hence $Q_1 = 15$

Q_3 = Size of $\left(\frac{3(n+1)}{4}\right)^{th}$ value = Size of $\left(\frac{3\times 8}{4}\right)^{th}$ value = Size of 6th value = 40

Hence $Q_3 = 40$

$$QD = \frac{1}{2}(Q_3 - Q_1) = \frac{40-15}{2} = 12.5$$

Coefficient of QD

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{40-15}{40+15} = \frac{25}{55} = 0.455$$

Hence coefficient of QD = 0.455

Example 8.17

Compute coefficient of quartile deviation from the following data

Marks	10	20	30	40	50	60
No. of Students	4	7	15	8	7	2

Solution:

Marks	Frequency	Cumulative Frequency
X	f	cf
10	4	4
20	7	11
30	15	26
40	8	34
50	7	41
60	2	43
		$N = \sum f = 43$

Table : 8.11

$$Q_1 = \text{Size of} \left(\frac{N+1}{4}\right)^{th} \text{value} = \text{Size } 11^{th} \text{ value} = 20$$

$$Q_3 = \text{Size of} \left(\frac{3(N+1)}{4}\right)^{th} \text{value}$$

$$= \text{Size of } 33^{rd} \text{ value} = 40.$$

$$QD = \frac{1}{2}(Q_3 - Q_1) = \frac{40-20}{2} = 10$$

$$\text{Coefficient of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{40-20}{40+20}$$

$$= \frac{20}{60} = 0.333$$

Example 8.18

Compute Quartile deviation from the following data

CI	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
f	12	19	5	10	9	6	6

Solution:

CI	f	cf
10 – 20	12	12
20 – 30	19	31
30 – 40	5	36
40 – 50	10	46
50 – 60	9	55
60 – 70	6	61
70 – 80	6	67
	N = 67	

Table : 8.12

$$Q_1 = \text{Size of} \left(\frac{N}{4}\right)^{th} \text{value} = \left(\frac{67}{4}\right)^{th} = 16.75^{th} \text{ value}$$

Thus Q_1 lies in the class 20 – 30; and the corresponding values are

$$L = 20, \frac{N}{4} = 16.75, pcf = 12, f = 19, c = 10$$

$$Q_1 = L + \left(\frac{\frac{N}{4} - pcf}{f}\right) \times c$$

$$Q_1 = 20 + \left(\frac{16.75 - 12}{19}\right) \times 10 = 20 + 2.5 = 22.5$$

$$Q_3 = \text{Size of} \left(\frac{3N}{4}\right)^{th} \text{value} = 50.25^{th} \text{ value}$$

Thus Q_3 lies in the class 50 – 60 and the corresponding values are

$$L = 50, \frac{3N}{4} = 50.25, pcf = 46, f = 9, c = 10$$

$$Q_3 = L + \left(\frac{\frac{3N}{4} - pcf}{f}\right) \times c$$

$$Q_3 = 50 + \left[\frac{50.25 - 46}{9}\right] \times 10 = 54.72$$

$$QD = \frac{1}{2}(Q_3 - Q_1)$$

$$= \frac{54.72 - 22.5}{2} = 16.11$$

$\therefore$ QD = 16.11

8.2.2 Mean deviation

Mean Deviation (MD) is defined as the average of the absolute difference between the items in a distribution and the mean or median of that series.

(i) Computation of Mean Deviation - Individual observations

If $X_1, X_2, X_3, ... X_n$ are n given observation then the mean deviation about mean or median is as follows

$$\text{MD about Mean} = \frac{\sum|X - \bar{X}|}{n} = \frac{\sum|D|}{n}$$

where $|D| = |X - \bar{X}|$ and n is the number of observations.

$$\text{MD about Median} = \frac{\Sigma|x - median|}{n} = \frac{\Sigma|D|}{n}$$

where $|D| = |X - median|$ and n is the number of observations.

NOTE

If the Mean deviation is computed from Median then in that case $|D|$ shall denote deviations of the items from Median, ignoring signs.

(ii) Computation of Mean Deviation - Discrete series

In discrete series the formula for calculating mean deviation is

$$\text{MD about Mean} = \frac{\sum f|X - \bar{X}|}{N} = \frac{\sum f|D|}{N}$$

where $|D| = |X - \bar{X}|$ by ignoring negative signs and N is total frequencies.

$$\text{MD about Median} = \frac{\sum f|X - Median|}{N}$$

$$= \frac{\sum f|D|}{N}$$

where $|D|$ = $|X$ – Median$|$ by ignoring negative sign and N is total frequencies.

(iii) Calculation of Mean Deviation- Continuous Series

For calculating Mean deviation in continuous series we have to obtain the midpoints of the various classes and take the deviations of these mid points from mean or median.

$$\text{MD about Mean} = \frac{\sum f|M - \bar{X}|}{N} \text{ (or)}$$

$$\text{MD about Mean} = \frac{\sum f|D|}{N}$$

where M is a mid value, $|D| = |M - \bar{X}|$ and N is the total frequencies.

$$\text{MD about Median} = \frac{\sum f|M - Median|}{N} \text{ (or)}$$

$$\text{MD about Mean} = \frac{\sum f|D|}{N}$$

where M is a mid value, $|D|$ = $|M$–Median$|$ (by ignoring negative sign) and N is the total frequencies.

(iv) Relative Measure for Mean Deviation

The relative measure corresponding to the mean deviation is called the coefficient of mean deviation and it is obtained as follows

Coefficient of MD about mean

$$= \frac{\text{Mean Deviation about Mean}}{\text{Mean}}$$

Coefficient of MD about median

$$= \frac{\text{Mean Deviation about Median}}{\text{Median}}$$

NOTE

However, in practice the arithmetic mean is more frequently used in calculating the mean deviation. If specifically stated to calculate mean deviation about median, median can be used.

Example 8.19

Calculate the Mean Deviation about mean and its coefficient of the income groups of five, given below.

Income Rs.	4000	4200	4400	4600	4800

Solution:

Calculation of Mean Deviation about Mean

$$\text{Mean} = \frac{\sum X}{n} = \frac{22000}{5} = 4400$$

Income (Rs) X	$\lvert D\rvert=(X-4400)$
4000	400
4200	200
4400	0
4600	200
4800	400
$\sum X = 22000$	$\sum \lvert D\rvert = 1200$

Table : 8.13

$$\text{Median Deviation about Mean } MD = \frac{\Sigma\lvert D\rvert}{n}$$

$$\text{MD} = \frac{1200}{5} = 240$$

$$\text{Coefficient of } MD = \frac{240}{4400} = 0.055$$

Example 8.20

Calculate the mean deviation about median and its relative measure for seven numbers given below: 55, 45, 40, 20, 60, 80, and 30.

Solution:

Arrange the values in ascending order 20, 30, 40, 45, 55, 60, 80.

$$\text{Median} = \text{size of } \left(\frac{(n+1)}{2}\right)^{th} \text{ value when n is odd}$$

$$= \text{size of } \left(\frac{(7+1)}{2}\right)^{th} \text{ value}$$

$$= \text{size of } 4^{th} \text{ item} = 45$$

X	$\lvert X - \text{Median}\rvert = \lvert X - 45\rvert$
20	25
30	15
40	5
45	0
55	10
60	15
80	35
	$\Sigma\ \lvert X - \text{Median}\rvert = 105$

Table : 8.14

$$\text{MD about Median} = \frac{\sum\lvert X - Median\rvert}{n}$$

$$= \frac{105}{7} = 15.0$$

Coefficient of MD about median

$$= \frac{15}{45} = 0.33$$

Example 8.21

Calculate the Mean deviation about mean for the following data.

Size	2	4	6	8	10	12	14	16
frequency	2	2	4	5	3	2	1	1

Solution:

Calculation of Mean Deviation about Mean

From the Table 8.15, we get

$$\overline{X} = \frac{\Sigma fX}{N} = \frac{160}{20} = 8$$

Mean Deviation about Mean

$$= \frac{\Sigma f\lvert D\rvert}{N} = \frac{56}{20} = 2.8$$

X	f	fX	$\|D\| = X - 8$	f|D|
2	2	4	6	12
4	2	8	4	8
6	4	24	2	8
8	5	40	0	0
10	3	30	2	6
12	2	24	4	8
14	1	14	6	6
16	1	16	8	8
	$N = 20$	$\Sigma fX = 160$		$\Sigma f\|D\| = 56$

Table : 8.15

Example 8.22

Calculate the Mean deviation about median and its relative measure for the following data.

X	15	25	35	45	55	65	75	85
frequency	12	11	10	15	22	13	18	19

Solution:

Already the values are arranged in ascending order then Median is obtained by the following:

X	f	cf
15	12	12
25	11	23
35	10	33
45	15	48
55	22	70
65	13	83
75	18	101
85	19	120
	N = 120	

Table : 8.16

$$\text{Median} = \text{size of} \left(\frac{(n+1)}{2}\right)^{th} \text{value}$$

$$= \text{size of} \left(\frac{(120+1)}{2}\right)^{th} \text{value}$$

$$= \text{size of } 60.5^{th} \text{ item} = 55$$

MD about Median

$$= \frac{\sum f|X - Median|}{N} = \frac{\sum f|D|}{N}$$

Mean deviation about Median

X	f	$\|D\| = \|X - 55\|$	f|D|
15	12	40	480
25	11	30	330
35	10	20	200
45	15	10	150
55	22	0	0
65	13	10	130
75	18	20	360
85	19	30	570
	N = 120		$\Sigma f\|D\| = 2220$

Table : 8.17

$$\text{MD about Median} = \frac{2220}{120} = 18.5$$

Coefficient of mean deviation about median

$$= \frac{MD\ about\ Median}{Median}$$

$$= \frac{18.5}{55} = 0.34$$

Example 8.23

Find out the coefficient of mean deviation about median in the following series

Age in years	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of persons	20	25	32	40	42	35	10	8

Calculations have to be made correct to two places of decimals.

Solution:

Calculation for median follows by the following table

X	f	cf
0-10	20	20
10-20	25	45
20-30	32	77
30-40	40	117
40-50	42	159
50-60	35	194
60-70	10	204
70-80	8	N=212

Table : 8.18

$\frac{N}{2}=\frac{212}{2}$ =106. Class interval corresponding to cumulative frequency 106 is (30 – 40). So, the corresponding values from the median class are $L = 30$, $pcf = 77$, $f = 40$ and $c = 10$.

$$\text{Median} = L + \left(\frac{\left(\frac{N}{2}\right) - pcf}{f}\right) \times c$$

$$\text{Median} = 30 + \left(\frac{106-77}{40}\right) \times 10$$

$\therefore$ Median =37.25 (corrected to two places of decimals)

Calculations proceeded for mean deviation about the median.

X	f	M	$\lvert D\rvert = \lvert X-37.25\rvert$	$f\lvert D\rvert$
0-10	20	5	32.25	645
10-20	25	15	22.25	556.25
20-30	32	25	12.25	392
30-40	40	35	2.25	90
40-50	42	45	7.75	325.5
50-60	35	55	17.75	621.25
60-70	10	65	27.75	277.5
70-80	8	75	37.75	302
	N=212			$\Sigma f\lvert D\rvert$=3209.5

Table : 8.19

Then the mean deviation about median is to be computed by the following

$$\text{MD about Median} = \frac{\Sigma f\lvert D\rvert}{N} = \frac{3209.5}{212} = 15.14$$

Coefficient of MD about Median

$$= \frac{\text{M.D. about median}}{\text{Median}} = \frac{15.14}{37.25} = 0.4064 = 0.41$$

(corrected to two decimal places).

NOTE

The above problem can also be solved for mean deviation about mean instead of median.

Exercise 8.1

1. Find the first quartile and third quartile for the given observations.
   2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22

2. Find Q_1, Q_3, D_8 and P_{67} of the following data:

Size of Shares	4	4.5	5	5.5	6	6.5	7	7.5	8
Frequency	10	18	22	25	40	15	10	8	7

3. Find lower quartile, upper quartile, 7th decile, 5th decile and 60th percentile for the following frequency distribution.

Wages	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	1	3	11	21	43	32	9

4. Calculate GM for the following table gives the weight of 31 persons in sample survey.

Weight (lbs):	130	135	140	145	146	148	149	150	157
Frequency	3	4	6	6	3	5	2	1	1

5. The price of a commodity increased by 5% from 2004 to 2005 , 8% from 2005 to 2006 and 77% from 2006 to 2007. Calculate the average increase from 2004 to 2007?

6. An aeroplane flies, along the four sides of a square at speeds of 100,200,300 and 400 kilometres per hour respectively. What is the average speed of the plane in its flight around the square.

7. A man travelled by car for 3 days. He covered 480 km each day. On the first day he drove for 10 hours at 48 km. an hour. On the second day, he drove for 12 hours at 40 km an hour and for the last day he drove for 15 hours at 32 km. What is his average speed?

8. The monthly incomes of 8 families in rupees in a certain locality are given below. Calculate the mean, the geometric mean and the harmonic mean and confirm that the relations among them holds true. Verify their relationships among averages.

Family:	A	B	C	D	E	F	G	H
Income (Rs.):	70	10	50	75	8	25	8	42

9. Calculate AM, GM and HM and also verify their relations among them for the following data

X	5	15	10	30	25	20	35	40
f	18	16	20	21	22	13	12	16

10. Calculate AM, GM and HM from the following data and also find its relationship:

Marks:	0-10	10-20	20-30	30-40	40-50	50-60
No. of students:	5	10	25	30	20	10

11. Calculate the quartile deviation and its coefficient from the following data:

Age in Years:	20	30	40	50	60	70	80
No. of Members:	13	61	47	15	10	18	36

12. Calculate quartile deviation and its relative measure from the following data:

X	0-10	10-20	20-30	30-40	40-50	50-60
f	5	10	13	18	14	8

13. Compute mean deviation about median from the following data:

Height in inches	No. of students
58	15
59	20
60	32
61	35
62	35
63	22
64	20
65	10
66	8

14. Compute the mean deviation about mean from the following data:

Class Interval:	0-5	5-10	10-15	15-20	20-25
Frequency f	3	5	12	6	4

15. Find out the coefficient of mean deviation about median in the following series

Age in years	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of persons	8	12	16	20	37	25	19	13

8.3 Probability

The word 'probability' or 'chance' is very commonly used in day-to-day conversation and generally people have a rough idea about its meaning. For example, we come across statements like

"Probably it may rain tomorrow";

"The chances of teams A and B winning a certain match are equal";

All these terms – possible, probable, etc., convey the same sense, i.e., the event is not certain to take place or, in other words, there is uncertainty about happening of the event in question. In Layman's terminology the word 'Probability' thus can notes that there is uncertainty about what has happened. However, in mathematics and statistics we try to present conditions under which we can make sensible numerical statements about uncertainty and apply certain methods of calculating numerical values of probabilities.

Galileo (1564-1642), an Italian mathematician, was the first man to attempt quantitative measure of probability while dealing with some problems related to the theory of dice in gambling. The figure (8.1) given below represents the basic concepts of probability.

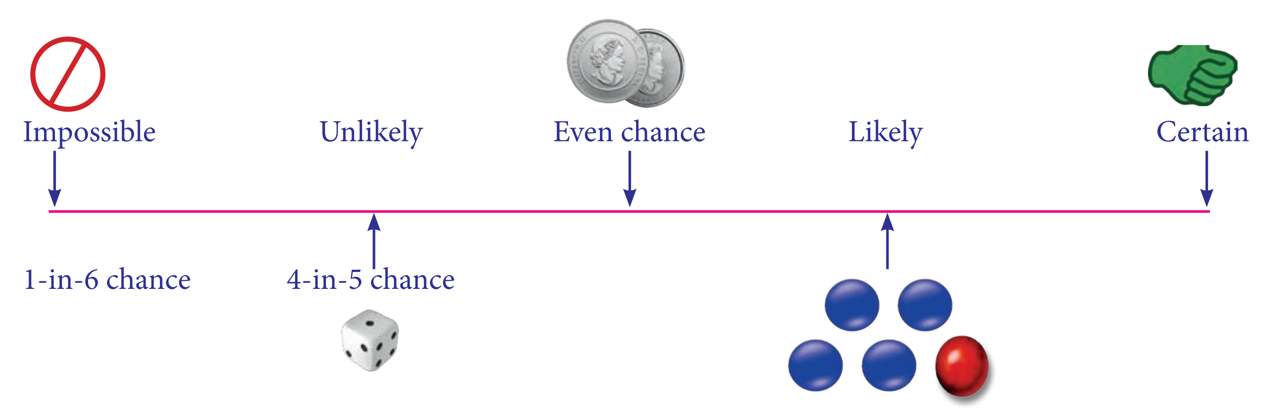


Fig. 8.1

8.3.1 Basic concepts of Probability

Recall

(i) Random Experiment

If an experiment or trial can be repeated under the same conditions, any number of times and it is possible to count the total number of outcomes, but individual result ie., individual outcome is not predictable, then the experiment is known as random experiment.

Example: Tossing a coin, throwing a die, selecting a card from a pack of playing cards, etc.

(ii) Outcome:

The result of a random experiment will be called an outcome.

(iii) Trial and Event:

Any particular performance of a random experiment is called a trial and outcome or combinations of outcomes are termed as events.

(iv) Exhaustive Events:

The total number of possible outcomes of a random experiment is known as the exhaustive events.

(v) Favourable Events:

The number of cases favourable to an event in a trial is the number of outcomes which entail the happening of the event.

(vi) Mutually Exclusive events:

Events are said to be mutually exclusive if the happening of any one of them precludes the happening of all the others, ie., if no two or more of them can happen simultaneously in the same trial. Symbolically the event A and B are mutually exclusive if $A \cap B = \varnothing$.

(vii) Equally Likely Events:

Events (two or more) of an experiment are said to be equally likely, if any one of them cannot be expected to occur in preference to the others.

(viii) Classical definition of Probability

If a random experiment or trial results in 'n' exhaustive, mutually exclusive and equally likely outcomes (or cases), out of which m are favourable to the occurrence of an event E, then the probability 'p' of occurrence (or happening) of E, usually denoted by $P(E)$, is given by

$$P = P(E) = \frac{Number\ of\ \ favourable\ cases}{Total\ number\ of\ \ exhastive\ cases} = \frac{m}{n}$$

DO YOU KNOW?

James Bernoulli who was the first person to obtain a quantitative measure of uncertainty.

(ix) Properties

(i) $0 \leq P(E) \leq 1$

(ii) Sum of all the probability equal to 1.

(iii) If $P(E) = 0$ then E is an impossible event.

For example : A coin is tossed. Find the probability of getting a head.

Solution: The total possible outcomes of an experiment {H,T}

Therefore $n = 2$

The favourable outcome for getting a head {H}. Therefore m =1. Thus the required probability is

$$P(\text{getting a } H\}= \frac{m}{n} = \frac{1}{2}$$

(x) Modern Definition of Probability

The modern approach to probability is purely axiomatic and it is based on the set theory concepts. In order to study, the theory of probability with an axiomatic approach it is necessary to define certain basic concepts. They are

(i) Sample Space: Each possible outcome of an experiment that can be repeated under similar or identical conditions is called a sample point and the collection of sample points is called the sample space, denoted by S.

(ii) Event: Any subset of a sample space is called an event.

(iii) Mutually Exclusive events: Two events A and B are said to be mutually exclusive events if $A \cap B = \phi$ i.e., if A and B are disjoint sets.

Example: Consider S = { 1,2,3,4,5}

Let A = the set of odd numbers = {1,3,5}

and B = the set of even numbers = {2,4}

Then $A \cap B = \phi$

(iv) Therefore the events A and B are mutually exclusive.

(xi) Observation:

Statement meaning in terms of Set theory approach.

(i) $A \cup B \Rightarrow$ at least one of the events A or B occurs

(ii) $A \cap B \Rightarrow$ both events A and B occurs

(iii) $\overline{A} \cap \overline{B} \Rightarrow$ Neither A nor B occur

(iv) $A \cap \overline{B} \Rightarrow$ Event A occurs and B does not occur

(xii) Definition of Probability (Axiomatic approach)

Let E be an experiment. Let S be a sample space associated with E. With every event in S we associate a real number denoted by $P(A)$ called the probability of the event A satisfying the following axioms.

Axiom 1 : $P(A) \geq 0$

Axiom 2 : $P(S) = 1$

Axiom 3 : If $A_1, A_2, \supset, A_n$ be a sequence of n mutually exclusive events in S then

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$$

(xiii) Basic Theorems on probability

Theorem 1:

$P(\varnothing) = 0$ i.e., probability of an impossible event is zero.

Theorem 2:

Let S be the sample space and A be an event in S, then $P(\overline{A}) = 1 - P(A)$.

Theorem 3: Addition Theorem

If A and B are any two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(xiv) Observation:

(i) If the two events A and B are mutually exclusive, then $A \cap B = \varnothing$

$\therefore P(A \cap B) = 0$

$\Rightarrow P(A \cup B) = P(A) + P(B)$

(ii) The addition theorem may be extended to any three events A, B, C and we have

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

It is believed that the students might be familiar with the above concepts and our present syllabus continues from the following.

8.3.2 Independent and Dependent events

(i) Independent Events

Two or more events are said to be **independent** when the outcome of one does not affect and is not affected by, the other. For example, if a coin is tossed twice, the result of the second throw would in no way be affected by the result of the first throw.

(ii) Dependent events

Are those in which the occurrence or non-occurrence of one event in any one trial affects the other events in other trials.

For example the probability of drawing a queen from a pack of 52 cards is $\frac{4}{52}$ or $\frac{1}{13}$. But if the card drawn (queen) is not replaced in the pack, the probability of drawing again a queen is $\frac{3}{51}$.

8.3.3 Conditional Probability

If two events A and B are dependent, then the conditional probability of B given that A as occurred already is

$$P(B/A) = \frac{P(A \cap B)}{P(A)};\ P(A) \neq 0$$

Similarly $P(A/B) = \frac{P(A \cap B)}{P(B)};\ P(B) \neq 0$

(i) Multiplication Theorem:

The probability of the simultaneous happening of two events A and B is given by

$P(A \cap B) = P(A).P(B/A)$ (or)

$P(A \cap B) = P(B).P(A/B)$

NOTE

If A and B are two independent events then $P(A \text{ and } B) = P(A \cap B) = P(A)\ P(B)$

The theorem can be extended to three or more independent events. Thus for three events the theorem states that

$$P(A \text{ and } B \text{ and } C) = P(A \cap B \cap C) = P(A)\ P(B) P(C)$$

Example 8.24

An unbiased die is thrown. If A is the event 'the number appearing is a multiple of 3' and B be the event 'the number appearing is even' number then find whether A and B are independent?

Solution:

We know that the sample space is

$S = \{1,2,3,4,5,6\}$

Now, $A = \{3,6\}$; $B = \{2,4,6\}$ then $(A \cap B) = \{6\}$

$$P(A) = \frac{2}{6} = \frac{1}{3}$$

$$P(B) = \frac{3}{6} = \frac{1}{2} \text{ and } P(A \cap B) = \frac{1}{6}$$

Clearly $P(A \cap B) = P(A)\ P(B)$

Hence A and B are independent events.

Example 8.25

Let $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$. Find $P(A \cap B)$ if A and B are independent events.

Solution:

Since A and B are independent events then $P(A \cap B) = P(A)\ P(B)$

Given that $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$,

then $P(A \cap B) = \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$

Example 8.26

Three coins are tossed simultaneously. Consider the events A 'three heads or three tails', B 'atleast two heads' and C 'at most two heads' of the pairs (A,B), (A,C) and (B,C), which are independent? Which are dependent?

Solution:

Here the sample space of the experiment is

S = {HHH, HHT, HTH, HTT, THH, TTH, THT, TTT}

A = {Three heads or Three tails}
= {HHH, TTT}

B = {at least two heads}
= {HHH, HHT, HTH, THH} and

C = {at most two heads} = {HHT, HTH, HTT, THH, TTH, THT, TTT}

Also (A∩B)= {HHH}; (A∩C) = {TTT} and (B∩C) ={HHT, HTH, THH}

$\therefore P(A) = \frac{2}{8} = \frac{1}{4}$; $P(B) = \frac{1}{2}$; $P(C) = \frac{7}{8}$ and

$P(A \cap B) = \frac{1}{8}$, $P(A \cap C) = \frac{1}{8}$, $P(B \cap C) = \frac{3}{8}$

Also $P(A).\ P(B) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$

$P(A).\ P(C) = \frac{1}{4} \cdot \frac{7}{8} = \frac{7}{32}$

and $P(B).\ P(C) = \frac{1}{2} \cdot \frac{7}{8} = \frac{7}{16}$

Thus, $P(A \cap B) = P(A).\ P(B)$

$P(A \cap C) \neq P(A).P(C)$ and

$P(B \cap C) \neq P(B).\ P(C)$

Hence, the events (A and B) are independent, and the events (A and C) and (B and C) are dependent.

Example 8.27

A can solve 90 per cent of the problems given in a book and B can solve 70 per cent. What is the probability that at least one of them will solve a problem selected at random?

Solution:

Given the probability that A will be able to solve the problem $= \frac{90}{100} = \frac{9}{10}$ and the probability that B will be able to solve the problem $= \frac{70}{100} = \frac{7}{10}$

i.e., $P(A) = \frac{9}{10}$ and $P(B) = \frac{7}{10}$

$P(\bar{A}) = 1 - P(A) = \frac{9}{10} = \frac{1}{10}$

$P(\bar{B}) = 1 - P(B) = \frac{7}{10} = \frac{3}{10}$

P(at least one solve the problem) $= P(A \cup B)$

$= 1 - P(\overline{A \cup B}) = 1 - P(\bar{A} \cap \bar{B})$

$= 1 - P(\bar{A}) \cdot P(\bar{B})$

$= 1 - \frac{3}{100} = \frac{97}{100}$

Hence the probability that at least one of them will solve the problem $= \frac{97}{100}$.

Example 8.28

A bag contains 5 white and 3 black balls. Two balls are drawn at random one after the other without replacement. Find the probability that both balls drawn are black.

Solution:

Let A, B be the events of getting a black ball in the first and second draw.

Probability of drawing a black ball in the first attempt is

$P(A) = \frac{3}{5+3} = \frac{3}{8}$

Probability of drawing the second black ball given that the first ball drawn is black

$P(B/A) = \frac{2}{5+2} = \frac{2}{7}$

$\therefore$ The probability that both balls drawn are black is given by

$P(A \cap B) = P(A)\ P(B/A) = \frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$

Example 8.29

In a shooting test the probability of hitting the target are $\frac{3}{4}$ for A, $\frac{1}{2}$ for B and $\frac{2}{3}$ for C. If all of them fire at the same target, calculate the probabilities that

(i) All the three hit the target

(ii) Only one of them hits the target

(iii) At least one of them hits the target

Solution:

Given $P(A) = \frac{3}{4}$, $P(B) = \frac{1}{2}$, $P(C) = \frac{2}{3}$

Then $P(\bar{A}) = 1 - \frac{3}{4} = \frac{1}{4}$; $P(\bar{B}) = 1 - \frac{1}{2} = \frac{1}{2}$ and

$P(\bar{C}) = 1 - \frac{2}{3} = \frac{1}{3}$

(i) P(A)=(all the three hit the targets)

$= P(A \cap B \cap C) = P(A)P(B)P(C)$

(since A, B, C hits independently)

$= \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{4}$

(ii) P (only one of them hits the target)

$= P\{(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)\}$

$= P\{(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C})$

$+ P(\bar{A} \cap \bar{B} \cap C)\}$

$= \left(\frac{3}{4} \cdot \frac{1}{2} \cdot \frac{1}{3}\right) + \left(\frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{3}\right) + \left(\frac{1}{4} \cdot \frac{1}{2} \cdot \frac{2}{3}\right) = \frac{1}{4}$

(iii) P (at least one of them hit the target)

$= 1 -$ P(none of them hit the target)

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 - P(\bar{A}) P(\bar{B}) P(\bar{C})$

$= 1 - \frac{1}{24} = \frac{23}{24}$

Example 8.30

Find the probability of drawing a queen, a king and a knave (Jack) in that order from a pack of cards in three consecutive draws, the card drawn not being replaced.

Solution:

Let A : the card drawn is a queen

B: the card drawn is a king

C: the card is drawn is a knave(jack)

P (drawing a queen card) = $P(A) = \frac{4}{52}$

P (drawing a king card given that a queen card has been already drawn)

$= P(B/A) = \frac{4}{51}$

P (drawing a knave card given that a queen and a king cards have been drawn)

$= P(C/AB) = \frac{4}{50}$

Since they are dependent events, the required probability of the compound event is

$$P(ABC) = P(A)\,P(B/A)\,P(C/AB)$$

$$= \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50} = \frac{64}{132600}$$

$$= 0.00048$$

8.3.4 Baye's Theorem

If $E_1, E_2, E_3 ..., E_n$ are a set of n mutually exclusive and collectively exhaustive events with $P(E_i) \neq 0$ ($i = 1,2,3\ldots, n$), then for any arbitrary event A which is associated with sample space $S = \bigcup_{i=1}^{n} E_i$ such that $P(A) > 0$, we have

$$P(E_i / A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}; i = 1,2,3,\ldots, n$$

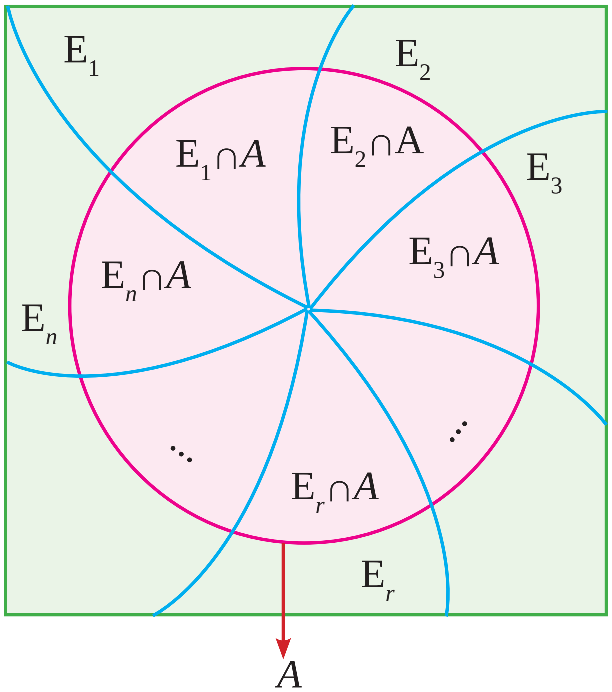


Fig. 8.2

where $P(A) = \sum_{i=1}^{n} P(E_i)P(A/E_i)$

Example 8.31

Bag I contains 3 red and 4 blue balls while another Bag II contains 5 red and 6 blue balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from second Bag.

Solution:

Let E_1 be the event of choosing the first bag, E_2 the event of choosing the second bag and A be the events of drawing a red ball. Then $P(E_1) = P(E_2) = \frac{1}{2}$.

Also $P(A/E_1) = P$ (drawing a red ball from Bag I) $= \frac{3}{7}$

and $P(A/E_2) = P$ (drawing a red ball from Bag II) $= \frac{5}{11}$.

Now, the probability of drawing a ball from Bag II, being given that it is red, is $P(E_2/A)$.

By using Baye's theorem, we have

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{\sum_{i=1}^{2} P(E_i)P(A/E_i)}$$

$$= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{5}{11}}{\left(\frac{1}{2}\times\frac{3}{7}\right)+\left(\frac{1}{2}\times\frac{5}{11}\right)} = \frac{35}{68}$$

Example 8.32

X speaks truth 4 out of 5 times. A die is thrown. He reports that there is a six. What is the chance that actually there was a six?

Solution:

Let us define the following events.

E_1 : X speaks truth

E_2 : X tells a lie

E : X reports a six

From the data given in the problem, we have

$P(E_1) = \frac{4}{5}$; $P(E_2) = \frac{1}{5}$

$P(E/E_1) = \frac{1}{6}$; $P(E/E_2) = \frac{5}{6}$

The required probability that actually there was six (by Bayes theorem) is

$$P(E_1/E) = \frac{P(E_1)P(E/E_1)}{P(E_1)P(E/E_1)+P(E_2)P(E/E_2)}$$

$$= \frac{\frac{4}{5}\times\frac{1}{6}}{\left(\frac{4}{5}\times\frac{1}{6}\right)+\left(\frac{1}{5}\times\frac{5}{6}\right)} = \frac{4}{9}$$

Example 8.33

A factory has 3 machines A_1, A_2, A_3 producing 1000, 2000, 3000 screws per day respectively. A_1 produces 1% defectives, A_2 produces 1.5% and A_3 produces 2% defectives. A screw is chosen at random at the end of a day and found defective. What is the probability that it comes from machines A_1?

Solution:

$P(A_1)$ = P(that the machine A_1 produces screws) $= \frac{1000}{6000} = \frac{1}{6}$

$P(A_2)$ = P(that the machine A_2 produces screws) $= \frac{2000}{6000} = \frac{1}{3}$

$P(A_3)$ = P(that the machine A_3 produces screws) $= \frac{3000}{6000} = \frac{1}{2}$

Let B be the event that the chosen screw is defective

$\therefore P(B/A_1)$ = P(that defective screw from the machine A_1) = 0.01

$P(B/A_2)$ = P(that defective screw from the machine A_2) = 0.015 and

$P(B/A_3)$ = P(that defective screw from the machine A_3) = 0.02

We have to find $P(A_1/B)$

Hence by Baye's theorem, we get

$P(A_1/B)$

$$= \frac{P(A_1)P(B/A_1)}{P(A_1)P(B/A_1)+P(A_2)P(B/A_2)+P(A_3)P(B/A_3)}$$

$$= \frac{\left(\frac{1}{6}\right)(0.01)}{\left(\frac{1}{6}\right)(0.01)+\left(\frac{1}{3}\right)(0.015)+\left(\frac{1}{2}\right)(0.02)}$$

$$= \frac{0.01}{0.01+0.03+0.06} = \frac{0.01}{0.1} = \frac{1}{10}$$

Exercise 8.2

1. A family has two children. What is the probability that both the children are girls given that at least one of them is a girl?

2. A die is thrown twice and the sum of the number appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?

3. An unbiased die is thrown twice. Let the event A be odd number on the first throw and B the event odd number on the second throw. Check whether A and B events are independent.

4. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that the problem is

   (i) solved

   (ii) exactly one of them solves the problem

5. Suppose one person is selected at random from a group of 100 persons are given in the following

Title	Psychologist	Socialist	Democrate	Total
Men	15	25	10	50
Women	20	15	15	50
Total	35	40	25	100

   What is the probability that the man selected is a Psychologist?

6. Two urns contains the set of balls as given in the following table

Title	White	Red	Black
Urn 1	10	6	9
Urn 2	3	7	15

   One ball is drawn from each urn and find the probability that

   (i) both balls are red

   (ii) both balls are of the same colour.

7. Bag I contains 3 Red and 4 Black balls while another Bag II contains 5 Red and 6 Black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag I.

8. The first of three urns contains 7 White and 10 Black balls, the second contains 5 White and 12 Black balls and third contains 17 White balls and no Black ball. A person chooses an urn at random and draws a ball from it. And the ball is found to be White. Find the probabilities that the ball comes from

   (i) the first urn

   (ii) the second urn

   (iii) the third urn

9. Three boxes B_1, B_2, B_3 contain lamp bulbs some of which are defective. The defective proportions in box B_1, box B_2 and box B_3 are respectively $\frac{1}{2}$, $\frac{1}{8}$ and $\frac{3}{4}$. A box is selected at random and a bulb drawn from it. If the selected bulb is found to be defective, what is the probability that box B_1 was selected?

10. Three horses A, B, C are in race. A is twice as likely to win as B and B is twice as likely to win as C. What are their respective probabilities of winning?

11. A die is thrown. Find the probability of getting

    (i) a prime number

    (ii) a number greater than or equal to 3

12. Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is

more than 4. What is the probability that it is an even number?

13. There are 1000 students in a school out of which 450 are girls. It is known that out of 450, 20% of the girls studying in class XI. A student is randomly selected from 1000 students. What is the probability that the selected student is from class XI given that the selected student is a girl?rom a pack of 52 cards, two cards are drawn at random. Find the probability that one is a king and the other is a queen.

14. From a pack of 52 cards, two cards are drawn at random. Find the probability that one is a king and the other is a queen.

15. A card is drawn from a pack of playing cards and then another card is drawn without the first being replaced. What is the probability of drawing
    (i) two aces
    (ii) two spades

16. A company has three machines A, B, C which produces 20%, 30% and 50% of the product respectively. Their respective defective percentages are 7, 3 and 5. From these products one is chosen and inspected. If it is defective what is the probability that it has been made by machine C?10.

Exercise 8.3

Choose the correct answer:

1. Which of the following is positional measure?
   (a) Range (b) Mode
   (c) Mean deviation (d) Percentiles

2. When calculating the average growth of economy, the correct mean to use is?
   (a) Weighted mean
   (b) Arithmetic mean
   (c) Geometric mean
   (d) Harmonic mean

3. When an observation in the data is zero, then its geometric mean is
   (a) Negative (b) Positive
   (c) Zero (d) Cannot be calculated

4. The best measure of central tendency is
   (a) Arithmetic mean
   (b) Harmonic mean
   (c) Geometric mean
   (d) Median

5. The harmonic mean of the numbers 2,3,4 is
   (a) $\frac{12}{13}$ (b) 12 (c) $\frac{36}{13}$ (d) $\frac{13}{36}$

6. The geometric mean of two numbers 8 and 18 shall be
   (a) 12 (b) 13 (c) 15 (d) 11.08

7. The correct relationship among A.M.,G.M.and H.M.is
   (a) $A.M.<G.M.<H.M.$
   (b) $G.M.\geq A.M.\geq H.M.$
   (c) H.M.≥G.M.≥A.M.
   (d) $A.M.\geq G.M.\geq H.M.$

8. Harmonic mean is the reciprocal of
   (a) Median of the values.
   (b) Geometric mean of the values.
   (c) Arithmetic mean of the reciprocal of the values.
   (d) Quartiles of the values.

9. Median is same as
   (a) Q_1 (b) Q_2 (c) Q_3 (d) D_2

10. The median of 10,14,11,9,8,12,6 is
(a) 10 (b) 12 (c) 14 (d) 9

11. The mean of the values 11,12,13,14 and 15 is
(a) 15 (b) 11 (c) 12.5 (d) 13

12. If the mean of 1,2,3, …, n is $\frac{6n}{11}$, then the value of n is
(a) 10 (b) 12 (c) 11 (d) 13

13. Harmonic mean is better than other means if the data are for
(a) Speed or rates.
(b) Heights or lengths.
(c) Binary values like 0 and 1.
(d) Ratios or proportions.

14. The first quartile is also known as
(a) median. (b) lower quartile.
(c) mode. (d) third decile

15. If $Q_1 = 30$ and $Q_3 = 50$, the coefficient of quartile deviation is
(a) 20 (b) 40 (c) 10 (d) 0.25

16. If median = 45 and its coefficient is 0.25, then the mean deviation about median is
(a) 11.25 (b) 180 (c) 0.0056 (d) 45

17. The two events A and B are mutually exclusive if
(a) $P(A\cap B) = 0$ (b) $P(A\cap B) = 1$
(c) $P(A\cup B) = 0$ (d) $P(A\cup B) = 1$

18. The events A and B are independent if
(a) $P(A\cap B) = 0$
(b) $P(A\cap B) = P(A)\times P(B)$
(c) $P(A\cap B) = P(A)+P(B)$
(d) $P(A\cup B) = P(A)\times P(B)$

19. If two events A and B are dependent then the conditional probability of $P(B/A)$ is
(a) $P(A)\ P(B/A)$ (b) $\frac{P(A\cap B)}{P(B)}$
(c) $\frac{P(A\cap B)}{P(A)}$ (d) $P(A)\ P(A/B)$

20. The probability of drawing a spade from a pack of card is
(a) 1/52 (b) 1/13 (c) 4/13 (d) 1/4

21. If the outcome of one event does not influence another event then the two events are
(a) Mutually exclusive
(b) Dependent
(c) Not disjoint
(d) Independent

22. Let a sample space of an experiment be $S = \{E_1, E_2, \ldots, E_n\}$, then $\sum_{i=1}^{n} P(E_i)$ is equal to
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

23. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is
(a) 1/36 (b) 0 (c) 1/3 (d) 1/6

24. Probability of an impossible event is
(a) 1 (b) 0 (c) 0.2 (d) 0.5

25. Probability that at least one of the events A, B occur is
(a) $P(A\cup B)$ (b) $P(A\cap B)$
(c) $P(A/B)$ (d) $(A\cup B)$

Miscellaneous Problems

1. Find out the GM for the following

Yield of Rice (tones)	No. of farms
7.5-10.5	5
10.5-13.5	9
13.5-16.5	19
16.5-19.5	23
19.5-22.5	7
22.5-25.5	4
25.5-28.5	1

2. An investor buys ₹ 1,500 worth of shares in a company each month. During the first four months he bought the shares at a price of ₹ 10, ₹ 15, ₹ 20 and ₹ 30 per share. What is the average price paid for the shares bought during these four months? Verify your result.

3. Calculate Mean deviation about median of the following data.

Class interval :	0-10	10-20	20-30	30-40	40-50	50-60
Frequency :	6	7	15	16	4	2

4. Calculate Mean deviation about Mean of the following data.

X	2	5	6	8	10	12
f	2	8	10	7	8	5

5. Calculate Quartile deviation and Coefficient of Quartile deviation of the following data.

Marks:	0	10	20	30	40	50	60	70
No. of students:	150	142	130	120	72	30	12	4

6. In a screw factory machines A, B, C manufacture respectively 30%, 40% and 30% of the total output of these 2% , 4% and 6% percent are defective screws. A screws is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by Machine C?

7. A committee of two persons is formed from 3 men and 2 women. What is the probability that the committee will have
(i) No woman
(ii) One man
(iii) No man

8. A, B and C was 50% , 30% and 20% of the cars in a service station respectively. They fail to clean the glass in 5% , 7% and 3% of the cars respectively. The glass of a washed car is checked. What is the probability that the glass has been cleaned?

9. Data on readership of a magazine indicates that the proportion of male readers over 30 years old is 0.30 and the proportion of male reader under 30 is 0.20. If the proportion of readers under 30 is 0.80. What is the probability that a randomly selected male subscriber is under 30?

10. Gun 1 and Gun 2 are shooting at the same target. Gun 1 shoots on the average nine shots during the same time Gun 2 shoots 10 shots. The precision of these two guns is not the same. On the average, out of 10 shots from Gun 2 seven hit the target. In the course of shooting the target has been hit by a bullet, but it is not known which Gunshot this bullet. Find the chance that the target was hit by Gun 2?

Summary

- A measure which divides an array into four equal parts in known as **quartiles**.
- A measure which divides an array into ten equal parts is known as **deciles**.
- A measure which divides an array into hundred equal parts is known as **percentiles**
- $Q_2 = D_5 = P_{50} =$ Median
- Inter quartile range = $Q_3 - Q_1$
- QD = $\dfrac{Q_3 - Q_1}{2}$
- Harmonic mean = $\dfrac{n}{\dfrac{1}{X_1} + \dfrac{1}{X_2} + \dfrac{1}{X_3} + \cdots \dfrac{1}{X_n}} = \dfrac{n}{\sum \dfrac{1}{X}}$
- Mean Deviation for Individual series MD = $\dfrac{\sum |X - \bar{X}|}{n} = \dfrac{\sum |D|}{n}$
- The conditional probability of an event A given the occurrence of the event B is given by $P(A/B) = \dfrac{P(A \cap B)}{P(B)}, P(B) \neq 0$
- Baye's Theorem:

  If $E_1, E_2, E_3, \ldots, E_n$ are a set of n mutually exclusive and collectively exhaustive events with $P(E_i) \neq 0$ (i =1,2,3…,n), then for any arbitrary event A which is associated with sample space $S = \bigcup_{i=1}^{n} E_i$ such that $P(A) > 0$, we have

  $$P(E_i / A) = \frac{P(E_i) P(A / E_i)}{\sum_{i=1}^{n} P(E_i) P(A / E_i)} \quad ; \quad i = 1,2,3\ldots, n$$

  where $P(A) = P(A) = \sum_{i=1}^{n} P(E_i) P(A / E_i)$

GLOSSARY (கலைச்சொற்கள்)	
Conditional probability	நிபந்தனைக்குட்பட்ட நிகழ்தகவு
Continuous series	தொடர்ச்சியான தொடர்
Decile	பதின்மானம்
Dependent events	சார்பு நிகழ்வுகள்
Discrete series	தனித்த தொடர்
Equally likely events	சம வாய்ப்புள்ள நிகழ்வுகள்
Exhaustive events	முழுமையான நிகழ்வுகள்
Frequency	அலைவெண் / நிகழ்வெண்
Grouped data	தொகுக்கப்பட்ட விவரங்கள்
Independent events	சார்பில்லா நிகழ்வுகள்
Mean deviation	சராசரி விலக்கம்
Mode	முகடு
Mutually exclusive events/ disjoint events	ஒன்றை ஒன்று விலக்கும் நிகழ்வுகள்
Percentile	நூற்றுமானம்/ சதமானம்
Probability	நிகழ்தகவு
Quartile	கால்மானம்
Quartile deviation	கால்மான விலக்கம்
Random experiment	சமவாய்ப்பு சோதனை
Range	வீச்சு
Sample space	கூறுவெளி

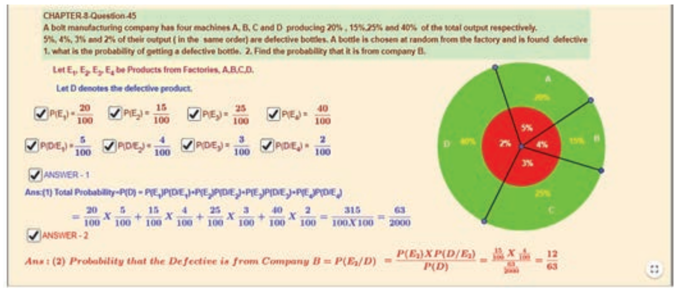
ICT Corner

Descriptive statistics and probability

Step – 1

Open the Browser type the URL Link given below (or) Scan the QR Code.

GeoGebra Workbook called "**11th BUSINESS MATHEMATICS and STATISTICS**" will appear. In that there are several worksheets related to your Text Book.

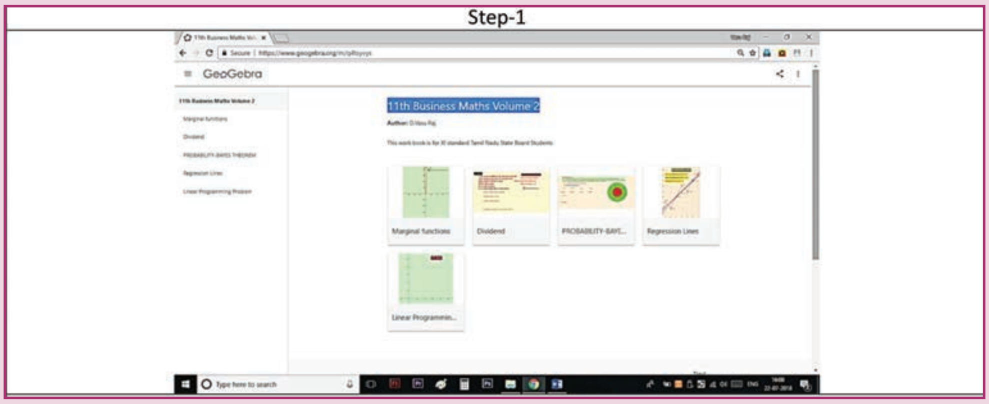


Expected Outcome ⇒

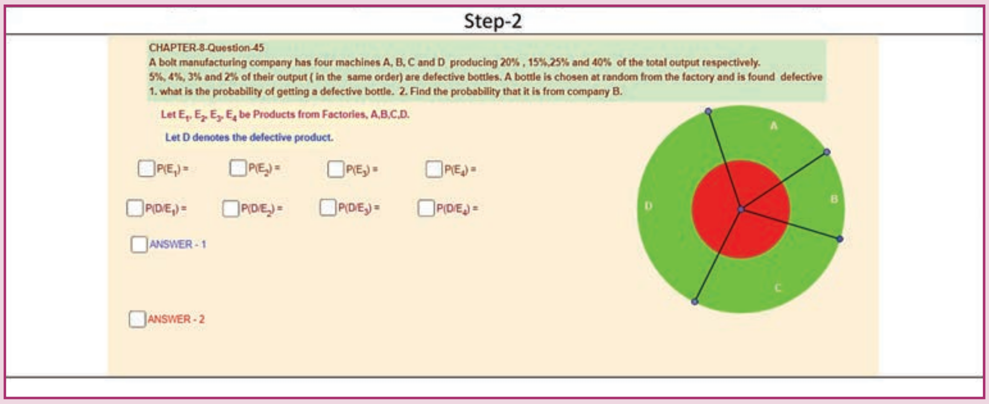
Step-2

Select the work sheet "Probability-Bayes theorem" Find each probabilities step by step as shown and Click on the respective boxes to see the answers.

Step 1



Step 2



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