5. Electromagnetism

5.1. Currents I_1 and I_2 flow in the same direction along two parallel conductors, with $I_1 > I_2$. In which of the three regions *I*, *II* or *III*, and at what distance from the

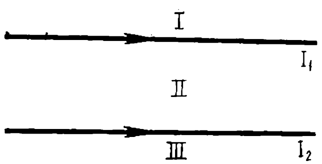

Fig. 5.1

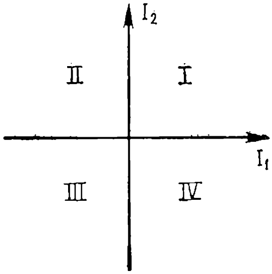

Fig. 5.2

conductor carrying current I_1 is the magnetic induction equal to zero?

5.2. Two mutually perpendicular conductors carrying currents I_1 and I_2 lie in one plane. Find the locus of points at which the magnetic induction is zero.

5.3. Equal currents are flowing along three conductors: a ring of radius R (Figure (a)), an infinitely long straight

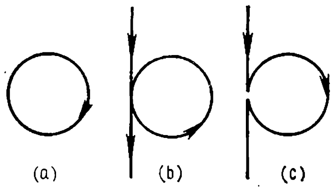

Fig. 5.3

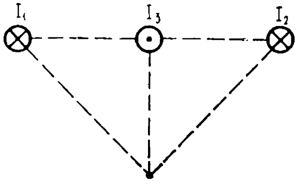

Fig. 5.4

conductor that forms a loop of the same radius R (Figure (b)), and an infinitely long straight conductor that also forms a loop of radius R but is broken at the point where the loop touches the conductor (Figure (c)). Find the relationships that link the magnetic induction vectors at the center of each circle.

5.4. Three conductors carrying currents are perpendicular to the plane of the drawing. They intersect the plane at three points that lie on a single straight line, with the distances from the middle conductor to the other two being equal. The currents in the outer conductors flow away from the reader, while the current in the middle conductor flows toward the reader. How is the magnetic field vector directed at the point on the straight line that is perpendicular to the straight line passing through the

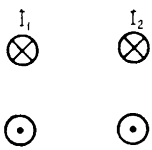

Fig. 5.5

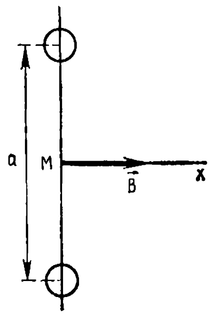

Fig. 5.6

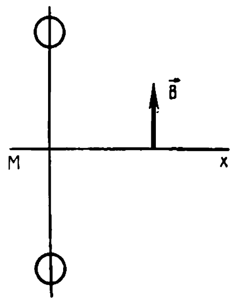

Fig. 5.7

three conductors in the plane of the drawing and is separated from the middle conductor by a distance equal to the distances between that conductor and the outer conductors? All three currents are equal in magnitude.

5.5. Along four parallel conductors whose sections with the plane of the drawing lie at the vertices of a square there flow four equal currents (the directions of these currents are as follows: those marked with an "x" point away from the reader, while those marked with a dot point to the reader. How is the vector of magnetic induction directed at the center of the square?

5.6. Two infinitely long parallel conductors carrying currents are directed at right angles to the plane of the drawing. The maximum of magnetic induction is at a point M that lies in the middle between the conductors. The direction of the magnetic induction vector **B** at this point coincides with the positive direction on the x axis. Determine the direction of the currents flowing in the conductors and the relationship that exists between these currents.

5.7. Two infinitely long parallel conductors carrying currents are directed at right angles to the plane of the drawing. The magnetic induction at a point M that lies in the middle between the conductors is zero. To the right of this point, the magnetic induction vector points upward, at right angles to the x axis. Find the direction of the currents flowing in the conductors, the direction of the magnetic induction vector to the left of point M, the relationship between the currents, and the point on the x axis at which the magnetic induction is maximal. The distance between the conductors is a.

5.8. Prove solely by reasoning (without performing any calculations) that the magnetic induction on the axis at an end face of a very long solenoid is half the value in the middle of the solenoid. A "very long solenoid" is one whose length is much greater than the diameter.

5.9. A current flows clockwise in a flat square loop. In

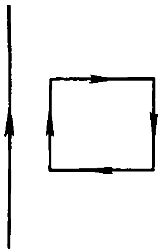

Fig. 5.9

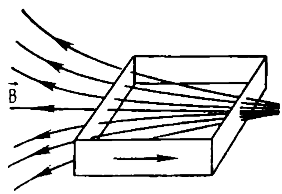

Fig. 5.10

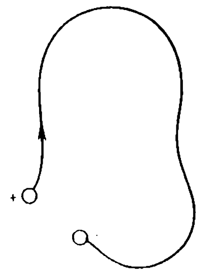

Fig. 5.11

the plane of the loop there lies an infinitely long straight conductor carrying a current whose direction is designated by the arrow in the figure. How will the loop move in the magnetic field created by the current flowing in the straight conductor and how will the shape of the loop change as a result of the action of this field?

5.10. A conducting loop carrying a current is placed in a nonuniform magnetic field. How will it move as a result of the action of this field?

5.11. A direct current (constant in magnitude and direction) flows in a contour made from soft wire. What shape does this contour tend to acquire as a result of the action of the magnetic field created by the current?

5.12. A small flat contour with a current flowing in it is placed successively at three points on the axis of a solenoid in which a current also flows in the same direction.

The points are at the middle of the solenoid (point *1*), at an end face (point *2*), and outside the solenoid at a distance from an end face equal to one-half the length of the solenoid (point *3*). The plane of the contour and the plane of the cross section of the solenoid are parallel. At which of these three points does the contour experience the greatest interaction with the solenoid and at which is the force minimal? Is the force attractive or repulsive at these points? The length of solenoid is considerably larger than the diameter.

5.13. At a small distance from a solenoid carrying a current there is placed a contour with a current in such a

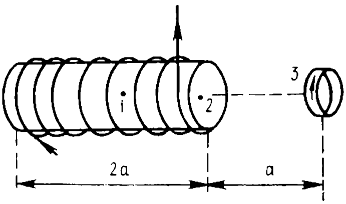

Fig. 5.12

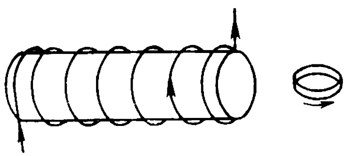

Fig. 5.13

manner that the solenoid's axis lies in the plane of the contour. The directions of the currents in solenoid and contour are shown by arrows. How does the contour move? How will it move if the current in it flows in the direction opposite to the one shown in the figure?

5.14. Between two fixed contours, *1* and *3*, carrying currents that flow in the same direction there is suspended

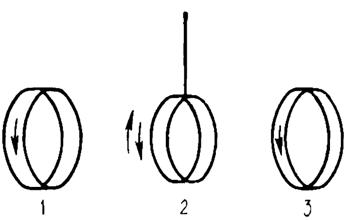

Fig. 5.14

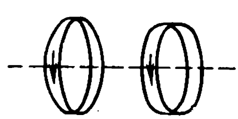

Fig. 5.15

another contour, *2*, that also carries a current. Contour *2* is oriented in such a manner that the forces caused by the currents in contours *1* and *3* are opposite in direction, equal in magnitude, and lie along a single straight line; thus, contour *2* is in equilibrium. Is this state of equilibrium stable or unstable? Consider the case where the current in contour *2* has the same direction as the currents in *1* and *3* and the case where the directions are opposite.

5.15. Two contours whose planes are parallel to each other and are separated by a certain distance carry currents that flow in the same direction. One contour is left fixed while the other is positioned in a different manner with respect to the first: in one case its plane is turned by 90°, in the other by 180°, while in the third case it is just moved parallel to itself over a certain distance. In which of these three cases one will have to perform the greatest work and in which, the smallest?

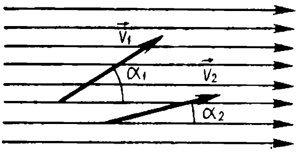

Fig. 5.16

5.16. In a uniform magnetic field there are two charged particles moving with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ and carrying equal charges, with $|\mathbf{v}_1| = |\mathbf{v}_2| = v$. The velocity of one particle forms an angle α_1 with the direction of the field, while the other velocity forms an angle α_2. In what parameters does the motion of one particle differ from that of the other? Determine which of the parameters is greater for which particle.

5.17. The device shown in Figure (a) is commonly used to measure the charge-to-mass ratio of the electron. The electrons that leave the cathode C are accelerated by an electric field that exists in the space between the cathode and the anode A. A fraction of electrons fly through the hole in the anode. These electrons, leaving region *I* of the device, fly into the region where there is no electric field. In this region the electrons are deflected from a straight line via a magnetic field directed perpendicularly to the plane of the drawing. This field is generated by two solenoids. The region *II* where the trajectory of the electrons is bent lies between these two solenoids. By increasing the current flowing through the two solenoids connected in series we can direct the electrons into a Faraday cylinder F, with a galvanometer G registering the resulting current. Any further increase in the solenoid current results in a drop in the current flowing through G, since the electrons begin to move along a circle of a smaller radius. The dependence of the galvanometer current on the solenoid current is illustrated by the curve in Figure (b). The following quantities are known in measurements: the potential difference U between anode and cathode, the curvature radius R of the axial line of region II (assuming that the majority of electrons deflected by the magnetic field travel along this line), the number of turns N_0 per unit length of solenoid, and the solenoid current I at which the galvanometer current is maximal. How to determine the charge-to-mass ratio of the electron knowing the values of these quantities?

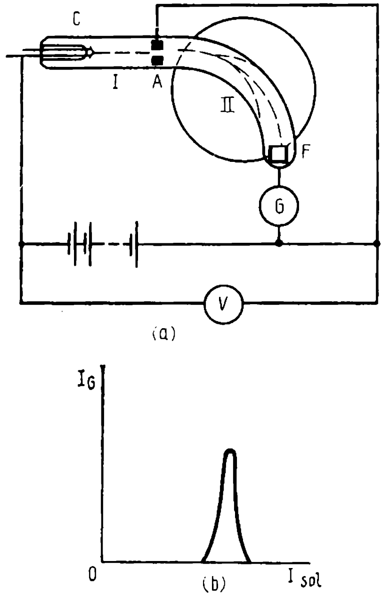

Fig. 5.17

5.18. A charged particle of mass m and charge Q has acquired a certain velocity by passing through a potential difference U_0. With this velocity it flies into the field of a parallel-plate capacitor, with the distance between the plates being l, the potential difference being U. The velocity of the particle is directed parallel to the plates. Where should the magnetic field that makes the particle move along a straight line in the capacitor be directed and what should its value be (the induction B)?

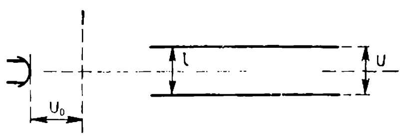

Fig. 5.18

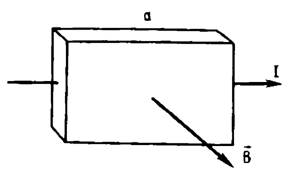

Fig. 5.19

5.19. A direct current I is flowing through a plane in the direction designated by an arrow. The plate is placed in a transverse magnetic field **B**. As a result of the Hall effect there appears a transverse potential difference. What is the sign of the potential at point a if the plate is made of metal and if the plate is an n-type or p-type semiconductor?

5.20. Two contours are positioned in such a manner that their planes are parallel to each other. Contour *1* carries a current whose direction is designated by an arrow. The

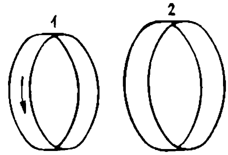

Fig. 5.20

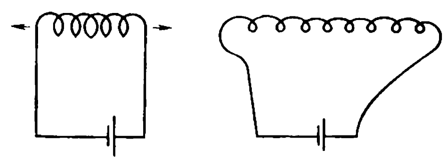

Fig. 5.21

contours move in relation to one another, but their planes remain parallel in the process. What is the direction of the current induced in contour *2* when the contours are moved toward each other or away from each other?

5.21. A spiral made from elastic wire is connected to a DC source. The spiral is stretched. Will the current flowing in the spiral become greater or smaller in the stretching process than the initial current or will it remain unchanged?

5.22. A solenoid carrying a current supplied by a DC source with a constant emf contains an iron core inside

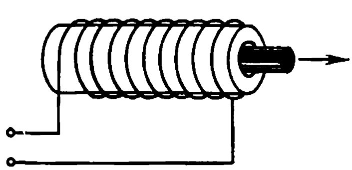

Fig. 5.22

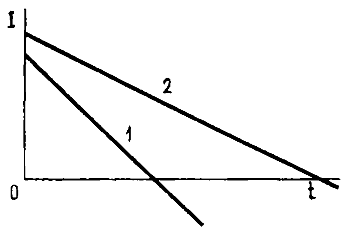

Fig. 5.23

it. How will the current change when the core is pulled out of the solenoid: will it increase, decrease, or remain the same?

5.23. Two identical inductances carry currents that vary with time according to linear laws. In which of the two inductances is the self-induction emf greater? Will the values or signs of the self-induction emf's change if the currents begin to increase in the opposite direction after they pass through zero (with the linear laws retained in the process)?

5.24. A current that varies with time according to a law depicted graphically in the figure passes through an induction coil. In which of the moments denoted in the figure

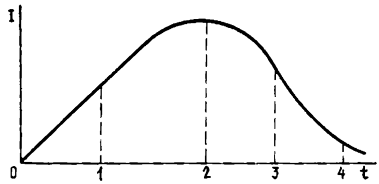

Fig. 5.24

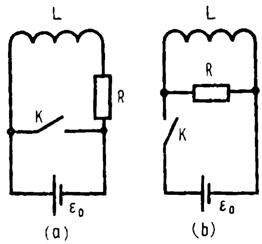

Fig. 5.25

will the self-induction emf be maximal (the inductance of the coil remains unchanged in the process)?

5.25. Various circuits are used to observe the phenomenon of self-induction. Among these are the circuits shown in Figures (a) and (b). In Figure (a), key K is initially opened and the current flows through the induction coil L and resistor R connected in series. In Figure (b), key K is initially closed and the current branches off to R and L. In both circuits the resistance of the coil L is much lower than R. Can an induction emf be generated in either one of these circuits that is higher than the emf of the DC source?

5.26. When a certain circuit consisting of a constant emf, an inductance, and a resistance is closed, the current in it increases with time according to curve *1* (see the figure accompanying the problem). After one parameter ($\mathscr{E}$, L, or R) is changed, the increase in current follows curve *2* when the circuit is closed a second time. Which parameter was changed and in what direction?

5.27. A current is flowing in a circular contour *1* whose radius is R. A second contour, *2*, whose radius is much smaller than that of the first, is moving with a constant velocity $\mathbf{v}$ along the r axis in such a manner that the planes of the contours remain parallel to each other in the course of the motion. At what distance from contour *1* will the emf induced in contour *2* be maximal?

5.28. A certain circuit consists of a DC source with emf $\mathscr{E}$, an induction coil *L1*, and a key *K1*. No resistance is present in the circuit. Another coil, *L2*, which is connected electrically to a resistor R through a key *K2*, is fastened to *L1*. At some moment in time key *K1* is closed. After a certain time interval *K2* is closed. How do

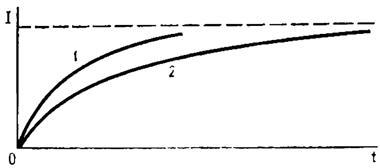

Fig. 5.26

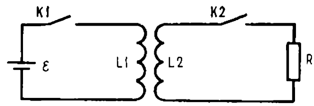

Fig. 5.28

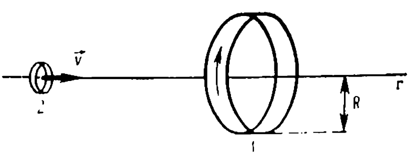

Fig. 5.27

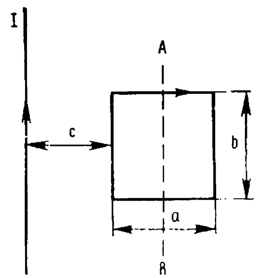

Fig. 5.29

the current in the primary circuit (the one containing $\mathscr{E}$), the induction emf in the secondary circuit (the one with *L2* and R), and the current in the secondary circuit vary with time?

5.29. An infinitely long straight conductor and a flat rectangular contour with sides a and b and with N turns lie in a single plane. The distance between the straight conductor and the side of the contour closest to the straight conductor is c. Determine the following quantities: (1) the mutual inductance of the conductor and the contour; (2) the quantity of electricity induced in the contour if the contour is rotated through 90° about the AB axis provided that a current I is flowing in the contour and the resistance of the contour is R; (3) the work that must be done to rotate the contour through 180° about the AB axis provided that there is current I both in the long conductor and in the contour and that the sense of the current in the contour is clockwise (in the plane of the drawing).

5.30. A common device used in electrical measurements is the so-called Rogowski loop. It constitutes a flexible solenoid that can be transformed into a torus if the two ends are brought together (Figure (a)). The leads can be connected to an AC ammeter, a ballistic galvanometer*, or an oscillograph. By circling a conductor with a Rogowski loop one can measure an alternating current flowing constantly in the conductor or even isolated changes in the current, such as those that occur when the current is switched on or off or when pulses pass through the circuit. Suppose the Rogowski loop forms a toroid that encircles a conductor carrying a direct current I (Figure (b)). The parameters of the loop are as follows: the cross-sectional area is S, the number of turns is N, the resistance of the winding is R, and the radius of the toroid is r. It is assumed that the width d of the loop proper is very small compared to r. At a certain moment the current is switched off; the current becomes zero in a very short interval. The ballistic galvanometer in the circuit of the loop measures the quantity of electricity Q that has passed through the loop (and the galvanometer). How can one find the current I that was flowing in the conductor prior to switch-off knowing the values of the above-mentioned parameters?

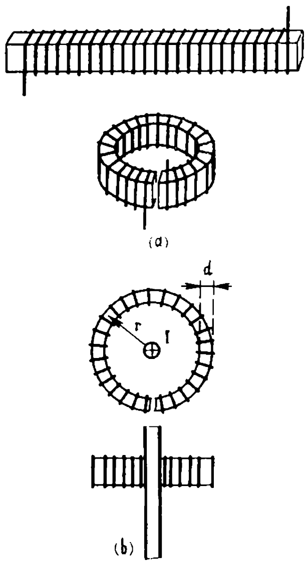

Fig. 5.30

* A ballistic galvanometer has a large period of oscillations. It is commonly used to measure the quantity of electricity that flows in a circuit in the form of a short pulse.

5.31. A flat coil with a cross-sectional area S and with N turns is placed in a magnetic field. The leads of the coil are connected to an oscillograph. When the coil is moved out of the field, an induction emf is generated in it, and the oscillogram of this emf is shown in the figure. How do the maximal value of the emf, $\mathscr{E}_{im}$, and the area under the curve depend on the rate with which the coil is moved out of the field?

5.32. Suppose that we have two solenoids of the same length. Their diameters differ only to the extent to which

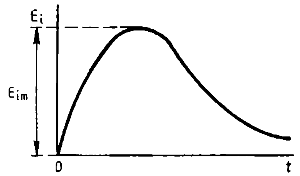

Fig. 5.31

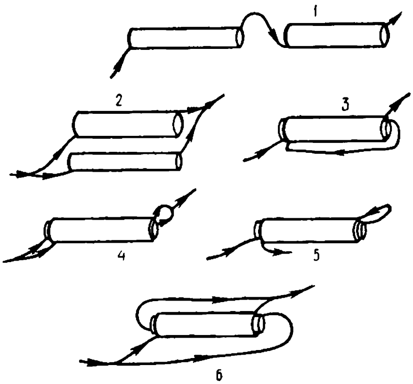

Fig. 5.32

one can be fitted onto the other. The inductances of the two solenoids can be considered the same and equal to L. Here are the ways in which the solenoids can be connected:

(1) the solenoids are connected in series and are separated by a large distance;

(2) the solenoids are connected in parallel and are separated by a large distance;

(3) the solenoids are connected in series, one is fitted onto the other, and the senses of the turns coincide;

(4) the solenoids are connected in parallel, one is fitted onto the other, and the senses of the turns coincide;

(5) the solenoids are connected in series, one is fitted onto the other, and the senses of the turns are opposite;

(6) the solenoids are connected in parallel, one is fitted onto the other, and the senses of the turns are opposite.

Determine the total inductance for each of the above cases.

5.33. The current flowing in a certain inductance coil varies in time according to the curve shown schematically in the figure. Draw the curve representing the induced emf as a function of time (also schematically).

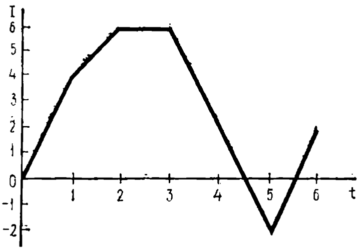

Fig. 5.33

5.34. Two similar parallel electron beams point in the same direction. The linear dimensions of the cross section of each beam are small compared to the distance between the beams. Suppose that v is the electron velocity and n is the electron concentration in either beam. In a coordinate system with respect to which the electrons are in motion there are two types of interactions, the electrostatic and the magnetic. Which of the two is greater in magnitude?

5.35. Electric charges do not generate magnetic field in a system of coordinates (better to say, frame of reference) where they are at rest. The magnetic field that surrounds a conductor carrying a current is generated by the charges that are moving in the conductor. Since the electron concentration in a conductor is of the order of 10^{22} cm^{-3}, the directional velocity of the electrons in the conductor is of the order of one millimeter per second (if the current density is estimated at 100 A/cm^2). We position the conductor carrying the current in such a manner that it follows the magnetic meridian at the point where the conductor is present. Just as in Oersted's experiment, a magnetic compass needle placed under the conductor will be deflected. If the needle is moved along the conductor with a speed equal to the directional velocity of the electrons in the conductor (i.e. of the order of several millimeters per second), the electron will be at rest in relation to the needle and, since the magnetic field in the system connected with the needle must be nil, the needle will not be deflected. More than that, if the needle is moved along the conductor with a speed greater than that of the electrons, the needle will be deflected in the opposite direction. Are these assertions correct?

5.36. How are the magnetic induction vector and the magnetic field vector directed inside and outside a bar magnet?

5.37. Two types of steel are characterized by the hysteresis loops shown in Figure (a) and (b). The loops are obtained in the processes of magnetization and demagnetization of the steels. Which of the two types is best suited

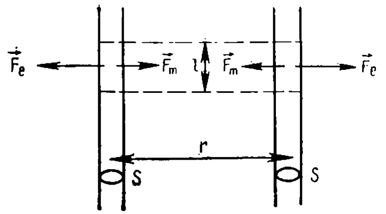

Fig. 5,34

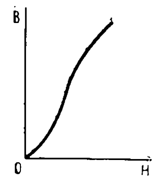

Fig. 5.38

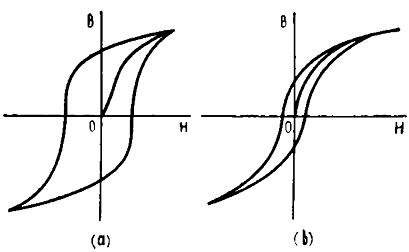

Fig. 5.37

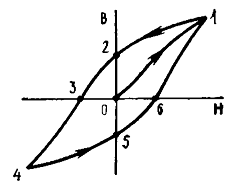

Fig. 5.39

for using as the core of a transformer and which, for using as a permanent magnet?

5.38. How can one use the B vs. H graph (the magnetization curve) to determine the work that a source of current must perform to magnetize a ferromagnetic core of a solenoid whose length is l and whose cross-sectional area is S? The magnetization curve is shown in the figure accompanying the problem.

5.39. Does a hysteresis loop possess sections in which we can formally assign to permeability a value that is zero or infinite or negative?

5.40. A straight conductor passes through a ferromagnetic toroid, as shown in the figure accompanying the problem. The conductor carries a current that first grows to a certain maximal value and then falls off to zero, as a result of this the toroid becomes magnetized. Indicate the directions of the lines of force for magnetic induction in the toroid and find the sections or points on the hysteresis loop corresponding to the state of the toroid after the current has ceased to flow (see the figure accompanying Problem 5.39).

5.41. Suppose we wish to calculate the circulation integrals of the magnetic field strength and magnetic induction along various contours, some of which lie entirely

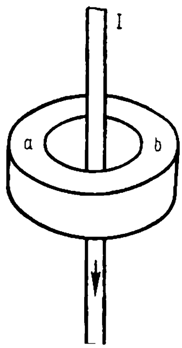

Fig. 5.40

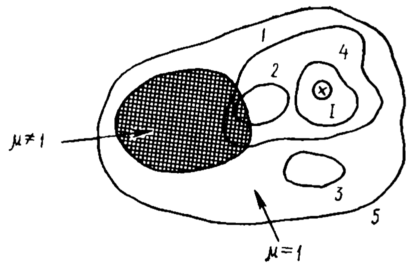

Fig. 5.41

in a vacuum while the other partially overlap a medium with a permeability μ. The "x" inside a small circle marks the section of a conductor carrying a current by the plane of the drawing. Are all the circulation integrals of the magnetic induction equal to each other? Is this also true of the circulation integrals of the magnetic field strength?

5. Electromagnetism

5.1. If we use the right-hand screw rule, we will find that both in region *I* and in region *III* the directions of the magnetic induction vectors coincide and the resultant induction may vanish only at infinity. The same rule shows that only in region *II* can the magnetic induction vectors point in opposite directions (i.e. the induction created by the two currents), with the resultant induction vanishing somewhere inside *II*. If a is the distance separating the conductors, then the distance x from a conductor carrying the current I_1 to the point where the induction is zero can be found from the equation

$$\frac{\mu_0\mu I_1}{2\pi x}-\frac{\mu_0\mu I_2}{2\pi (a-x)}=0.$$

Hence,

$$x=\frac{I_1}{I_1+I_2}a.$$

5.2. If we use the right-hand screw rule, we will establish that the magnetic induction can vanish only in sectors *I* and *III*. If y is the distance from a certain point on the conductor carrying the current I_1 to the point where the magnetic induction is zero, and x is the distance from this point to the conductor carrying the current I_2, then

$$\frac{\mu_0\mu I_1}{2\pi y}=\frac{\mu_0\mu I_2}{2\pi x}.$$

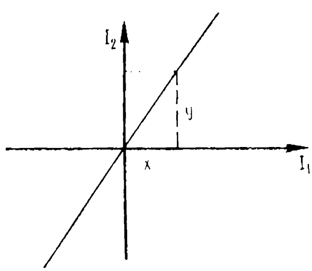

Fig. 5.2

Hence, the locus of points where the magnetic induction is zero is the straight line that passes through the point of intersection of the conductors and whose equation is $y=(I_1/I_2)\,x$.

5.3. The magnetic inductions generated by a straight conductor with a current and a circular conductor are, respectively,

$$B=\frac{\mu_0\mu I}{2\pi r}\quad\text{and}\quad B=\frac{\mu_0\mu I}{2r}.$$

In the case corresponding to Figure (b), the directions of the two induction vectors coincide, while in the case corresponding to Figure (c) they are opposite. Thus,

$$B_a = \frac{\mu_0\mu I}{2r}, \quad B_b = \frac{\mu_0\mu I}{2r}\left(1+\frac{1}{\pi}\right),$$

$$B_c = \frac{\mu_0\mu I}{2r}\left(1-\frac{1}{\pi}\right),$$

whence

$$B_b = \frac{\pi+1}{\pi} B_a = 1.32\, B_a, \quad B_c = \frac{\pi-1}{\pi} B_a = 0.68\, B_b.$$

5.4. If the distance from the middle conductor to each of the other two conductors and to the point where we wish to determine the field is a, the magnetic field generated by each outer conductor at this point is

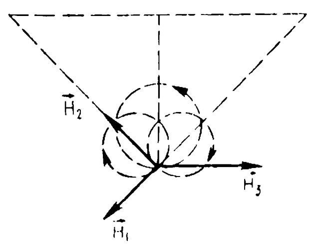

Fig. 5.4

$$H_1 = \frac{I}{2\pi a\sqrt{2}}.$$

Using the right-hand screw rule, we find that the vectors of the magnetic fields generated by the outer conductors are directed at an angle of 90°, so that the resultant magnetic field strength is

$$H_{1,2} = \sqrt{2}\, H_1 = \frac{I}{2\pi a},$$

with the vector representing this resultant directed parallel to the line passing through the conductors. Employing the same rule, we will find that the magnetic field H_3 generated by the middle conductor points in the direction opposite to the one of the resultant $H_{1,2}$, with $H_3 = I/2\pi a$, that is, $|H_3| = |H_{1,2}|$. Thus, the resultant of all three fields is zero.

5.5. A magnetic induction vector is always directed along a tangent to a line of force (for each of the four conductors the line of force is a circle in the plane of the drawing). As the figure accompanying the answer shows, the magnetic inductions generated by currents I_1 and I_4 are directed along the diagonal of the square from the conductor carrying I_2 to the conductor carrying I_3. Reasoning along the same line, we conclude that the magnetic inductions generated by currents I_2 and I_3 are directed along the diagonal of the square from the conductor carrying I_4 to the conductor carrying I_1. The resultant induction of the magnetic field generated by all four currents, or the geometric sum of the magnetic induction vectors of the four currents, lies in the plane of the drawing and points from right to left.

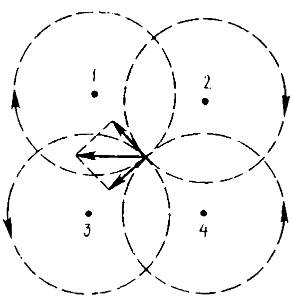

Fig. 5.5

5.6. The presence of a maximum in the middle between the conductors suggests that the currents in the conductors are flowing in opposite directions and that, the currents are equal in magnitude. Allowing for the direction of the induction vector at point M and employing the right-hand screw rule (see the figure accompanying the answer), we conclude that in the upper conductor the current is directed toward the reader and in the lower, away from the reader.

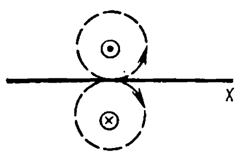

Fig. 5.6

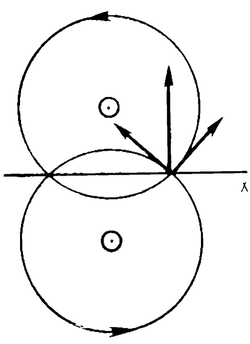

Fig. 5.7

5.7. At the point that lies in the middle between the conductors the induction is zero, which means that both currents flow in the same direction. Employing the right-hand screw rule, we can determine the direction of the magnetic induction vector in the region to the right of the conductors for both possible directions of current. As the figure accompanying the problem shows, the induction vector to the right of the conductors is directed upward. Hence, the currents are flowing toward the reader (see the figure accompanying the answer). At a distance x from point M the induction is

$$B = \frac{\mu_0 \mu I x}{\pi \left[x^2 + \left(\frac{a}{2} \right)^2 \right]} .$$

The derivative

$$\frac{dB}{dx} = \frac{\mu_0 \mu I \left[x^2 + \left(\frac{a}{2}\right)^2 - 2x^2 \right]}{\pi \left[x^2 + \left(\frac{a}{2}\right)^2 \right]^2}$$

vanishes at $x = a/2$. It is at this distance that B is maximal, with $B_{max} = \mu_0 \mu I / \pi a$.

5.8. The induction in the middle of a very long solenoid depends only on the number of ampere-turns per unit length of solenoid. Suppose that we have two very long,

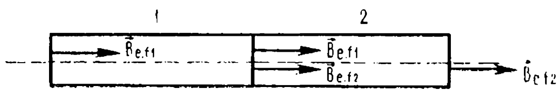

Fig. 5.8

similar solenoids with equal ampere-turns per unit length and that these solenoids are placed far apart. We denote the induction in the middle of a solenoid by B_m and that at an end face, by $B_{e.f}$. Let us bring these two solenoids together in such a manner that the directions of their magnetic inductions coincide and that the solenoids form a new long solenoid. At the point where the two solenoids touch (the right end face of the left solenoid touches the left end face of the right solenoid), the two induction vectors $B_{e.f}$ add up and form the total field with induction $2B_{e.f}$. But this point is simply the middle of the new solenoid, where the induction, as we already know, is B_m. Thus, $B_m = 2B_{e.f}$.

5.9. Employing the left-hand rule, we will find that the force acting on the side of the loop parallel to the conductor and closest to it is directed toward the conductor while the force acting on the opposite side of the loop parallel to the conductor and farthest from it is directed away from the conductor. Since the first force is greater in magnitude, the loop moves toward the conductor. Employing the same rule once more, we will see that the force acting on the upper side of the loop is directed upward while that acting on the lower side of the loop is directed downward. Thus, the forces tend to pull the loop apart, that is, to increase the area subtended by the loop.

This will actually happen if the material of the loop is elastic.

The answer to the question can be obtained from a more general reasoning. The work done in the process of displacing a loop carrying a current in a magnetic field is $A = I\Delta\Psi$, where $\Delta\Psi$ is the increment of the magnetic flux coupled with the loop. The loop tends to move or change its form in such a manner that the magnetic flux coupled with it acquires the greatest possible value. The flux is assumed to be positive if inside the loop it coincides in direction with the flux created by the current in the loop. In allowing for the various changes in the flux coupled with the loop one must take into account the changes that are due to the changes in the shape of the loop. In the case at hand the direction of the magnetic flux created by the current in the straight conductor and that of the magnetic flux created by the current in the loop coincide, and since the induction of the field created by the current in the straight conductor increases as we move closer to the conductor, this will lead to a certain displacement of the loop. The fact that the square loop transforms into a circle as the loop's area increases also leads to an increase in both the outer and inner magnetic fluxes.

5.10. Both a torque and a force act on the loop. The direction of the torque is determined by the fact that the positive normal to the plane of the loop must point in the direction of the induction of the external field. The right-hand screw rule is used to determine the positive direction of this normal, which therefore coincides with the direction of the magnetic field of the loop proper. In accord with the direction of the current in the loop, the positive normal points upward. In the external field the loop turns counterclockwise, with the magnetic field generated by the current flowing in the loop coinciding in direction with the external magnetic field. The direction of the force acting on the loop is determined by the nature of the inhomogeneity of the external field. Since a loop carrying a current and placed in an external magnetic field moves in such a manner that the magnetic flux coupled with it attains the maximal possible value (in the algebraic sense), when the directions of the external and the intrinsic flux coincide, the motion occurs in the direction of the field with the higher induction, which in the case at hand means from left to right.

5.11. A similar question has been considered in Problem 5.9. Although in this case no external magnetic flux is present, the contour may influence the magnitude of the flux coupled with it by changing its shape. Since the area and, hence, the flux through the contour are maximal when the contour is in the form of a circle, the magnetic forces acting on the contour tend to transform the contour in just this manner. We can arrive at the same conclusion by considering the interaction of two elements of the contour that are opposite to each other. The currents that flow in these elements tend to move the elements apart, since they flow in opposite directions. The collection of all such forces tends to stretch the contour.

5.12. The following force acts on a contour carrying a current and placed in a nonuniform magnetic field with

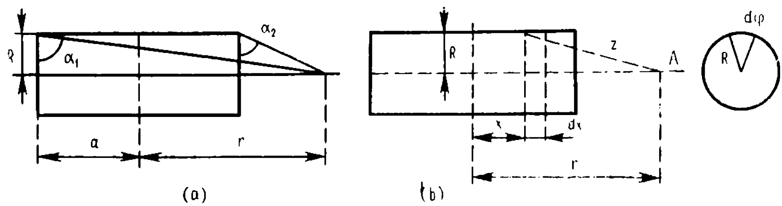

Fig. 5.12

the directions of the lines of force of this field coinciding with those of the field generated by the current in the contour: $F = p_m \dfrac{dB}{dr}$

In the case at hand, the force is determined by the values of the derivative dB/dr at different points of the field of the solenoid. The induction of the field of a solenoid of a finite length is given by the formula* (see Figure (a) accompanying the answer)

$$B = \frac{\mu_0 I N_0}{2} (\sin\alpha_1 - \sin\alpha_2). \qquad (5.12.1)$$

After simple transformations, the derivative dB/dr can be written as

$$\frac{dB}{dr} = \frac{\mu_0 I N_0}{2R} \left\{ \frac{1}{\left[1+\left(\frac{r+a}{R}\right)\right]^{3/2}} - \frac{1}{\left[1+\left(\frac{r-a}{R}\right)^2\right]^{3/2}} \right\}. \qquad (5.12.2)$$

Formula (5.12.2) shows that dB/dr is nonpositive for $r > 0$. This means that the force acting on the contour is attractive (in Figure (a) this force points from right to left).

At $r = 0$ we have $dB/dr = 0$, with the result that at point *1* the force is zero. This also follows from the fact that point *1* in the middle of the solenoid is the equilibrium point of the contour positioned inside the solenoid. At point *2* ($r = a$),

$$\frac{dB}{dr} = \frac{\mu_0 I N_0}{2R}\left[\frac{1}{(1+4a^2/R^2)^{3/2}} - 1\right], \qquad (5.12.3)$$

while at point *3* ($r = 2a$),

$$\frac{dB}{dr} = \frac{\mu_0 I N_0}{2r}\left[\frac{1}{(1+9a^2/R^2)^{3/2}} - \frac{1}{(1+a^2/R^2)^{3/2}}\right]. \qquad (5.12.4)$$

Comparison of (5.12.3) and (5.12.4) shows that the numerical value of the derivative is greater at point *2* than at point *3*. It can also be verified that at all points outside the solenoid the attractive force (if the direction of the current in the contour is opposite to that of the current in the solenoid, the force is repulsive) is smaller than at an end face of the solenoid, and decreases as the distance from the solenoid grows.

Formula (5.12.1) can be obtained in the following manner (see Figure (b) accompanying the answer). The element dx of the length of the solenoid contains $N_0\,dx$ turns (with N_0 the number of turns per unit length). The induction at point A generated by the current flowing in these turns is

$$dB = \frac{\mu_0}{4\pi}\,\frac{I N_0\,dx\,R\,d\varphi}{z^2},$$

where $R\,d\varphi$ is the element of length of the turn subtended by an angle $d\varphi$. The projection of $d\mathbf{B}$ on the solenoid axis is

$$dB_{\parallel} = \frac{\mu_0 I N_0\,dx\,R\,d\varphi}{z^2}\cos\alpha. \qquad (5.12.5)$$

The perpendicular component of $d\mathbf{B}$ is compensated by the induction generated by the symmetric elements of the same turn. Expressing all quantities in terms of angle α and the solenoid radius R, we get

$$z = \frac{R}{\cos\alpha}, \quad r - x = R\tan\alpha, \quad dx = -\frac{R}{\cos^2\alpha}\,d\alpha.$$

Substituting all this into (5.12.5), we find that

$$dB_{\parallel} = -\mu_0 I N_0\,d\varphi\cos\alpha\,d\alpha,$$

which yields formula (5.12.1) after we integrate from 0 to 2π with respect to φ and from α_1 to α_2 with respect to α.

5.13. Initially the external magnetic flux coupled with the contour is zero. In tending to increase this flux, the contour moves in such a manner that (a) the magnetic moment vector associated with the contour aligns with the induction vector of the external field and (b) contour moves into the region of higher induction after the alignment is completed. Under the given directions of the currents, the induction generated by the solenoid is directed from right to left and the magnetic moment vector of the contour is directed upward. Thus, the contour rotates counterclockwise and moves toward the solenoid. If the direction of current in the contour is opposite to the one shown in the figure accompanying the problem, the contour rotates clockwise and also moves toward the solenoid.

5.14. Contour *2* is in a nonuniform magnetic field. If the current in this contour flows in the same direction as the currents in contours *1* and *3*, contour *2* is attracted to the other two contours. If it is deflected from the state of equilibrium in some direction, then from this direction there acts on it an attractive force that is greater than the other attractive force (since contour *2* is in a nonuniform magnetic field), and this means that it will move in that direction and will be drawn closer to the corresponding contour. If the current in contour *2* flows in the direction opposite to that of the currents in contours *1* and *3*, then it might seem that contour *2* is in a state of stable equilibrium, since repulsive forces act from both directions. But there is another reason for instability. For an arbitrarily small rotation of contour *2* there appears a torque acting on this contour, and this torque tends to rotate the contour into such a position in which the direction of the current in contour *2* coincide with that of the currents in contours *1* and *3*. When this process is completed, we again have to deal with the instability considered in the first case.

Analyzing the behavior of contour *2*, we see that in both cases the instability manifests itself through a general rule, according to which a contour moves in an external field or changes its form in such a manner that the magnetic flux coupled with the contour acquires maximal value.

5.15. The work performed in moving a contour carrying a current is equal to $A = I\,(\Psi_2 - \Psi_1)$. If the flux coupled with the contour whose position is changed is initially Ψ_1, then upon rotating the plane of the contour by 180° this flux becomes $-\Psi_1$, upon rotating the plane by 90° it drops to zero, and upon moving the contour whose position is changed away from the fixed contour the flux decreases but does not become zero. Thus, in the first case $A_1 = -2I\Psi_1$, in the second $A_2 = -I\Psi_1$, and in the third $A_3 = -I\,(\Psi_1 - \Psi_2)$, where Ψ_2 is the flux coupled with the contour upon moving the mobile contour away from the fixed contour. The minus that is present in each formula shows that the work must be done against the interaction of the contours.

5.16. The velocity of each particle may be decomposed into two components, one pointing along the induction vector, and the other at right angles to the induction vector. The component directed along the field does not change since the Lorentz force affects only the component that is perpendicular to the field. If we denote this latter component by $v_\perp$ the Lorentz force is

$$F = ev_\perp B. \tag{5.16.1}$$

This force, which is perpendicular both to the velocity of a charged particle and to the induction vector, imparts a normal acceleration to the particle in question, with the equation of motion of the particle in the direction perpendicular to the field being

$$mv_\perp^2/R = ev_\perp B. \tag{5.16.2}$$

Combining (5.16.1) with (5.16.2), we can determine the radius of the circle along which the particle moves and the time it takes the particle to complete one circle (which does not depend on the velocity). In the course of the same time interval, T, the particle also moves along the field by a distance $h = v_\parallel T$, where $v_\parallel$ is the component of the velocity along the field. The result is the motion of the particle along a helical line with radius R and lead h. Since for an initial velocity v and an angle α the longitudinal component of the velocity is $v_\parallel = v \cos \alpha$ and the transverse component is $v_\perp = v \sin \alpha$, the trajectory of the particle with the larger angle α has a greater radius and a smaller lead of the helical line.

5.17. An electron accelerated by a potential difference U acquires kinetic energy

$$mv^2/2 = eU. \tag{5.17.1}$$

The force acting on the electron in a magnetic field is the Lorentz force

$$F = evB,$$

which makes the electron move along a circular arc whose radius is R, so that, according to Newton's second law,

$$mv^2/R = evB. \tag{5.17.2}$$

The induction of the magnetic field generated by the current in the solenoid is

$$B = \mu_0 I N_0. \tag{5.17.3}$$

Excluding velocity v from Eqs. (5.17.1) and (5.17.2) and substituting the value of B from (5.17.3), we find the sought for charge-to-mass ratio:

$$\frac{e}{m} = \frac{2U}{\mu_0^2 I^2 N_0^2 R^2}.$$

5.18. The electric field vector inside the capacitor is directed at right angles to the capacitor plates. The force $F_e = QE = QU/l$ with which the electric field acts on the particle is directed in the same manner. A force equal in magnitude to F_e but pointing in the opposite direction acts on the particle from the magnetic field. According to the Lorentz formula, this force is $F_m = QvB$ and is directed at right angles to the velocity of the particle and the magnetic induction vector. This means that the induction vector must be perpendicular to the electric field. As we have said earlier, the two forces must be equal: $QU/l = QvB$, or

$$B = U/(vl). \tag{5.18.1}$$

The velocity the particle acquired in an electric field can be found by employing the energy conservation law:

$$mv^2/2 = QU_0.$$

Solving this for v and substituting the result into (5.18.1), we finally obtain

$$B = \frac{U}{l}\sqrt{\frac{m}{2QU_0}}.$$

5.19. According to the Lorentz formula, charges moving in a magnetic field are subjected to a force whose direction is determined via the left-hand rule, where the positive direction of a current is defined in the "electrical-engineering" sense, that is, the direction in which the positive charges move in the conductor. Therefore, irrespective of the sign of the charge carriers, the forces acting on these carriers point in the same direction. In the case illustrated in the figure accompanying the problem, the charges move downward. In a metal or an n-type semiconductor, where electrons are the charge carriers, this will result in a depletion of charge carriers in the region about point a; the region will acquire a positive potential. In the case of a p-type semiconductor the sign of the charge is obviously minus.

5.20. According to Lenz's law, the induced current is in such a direction as to oppose the change in the magnetic field that produces it (that is, oppose the change in magnetic flux coupled with the contour). When the two contours approach each other, the flux coupled with the second contour increases, which means that the direction of the induction current in that contour is opposite to the current in the contour. On the other hand, when the contours are moved away from each other, the decrease in the flux in contour *2* leads to an induction current in that contour that is directed in same sense as in contour *1*.

5.21. The induction emf is

$$\mathscr{E}_i = -\frac{\mathrm{d}(LI)}{\mathrm{d}t} = -L\frac{\mathrm{d}I}{\mathrm{d}t} - I\frac{\mathrm{d}L}{\mathrm{d}t}.$$

In the case at hand the variable quantity is the inductance. When the spiral is stretched, the inductance falls, so that $\mathrm{d}L/\mathrm{d}t < 0$ and $\mathscr{E}_i > 0$. The generated induction emf leads to an increase in the current in the circuit. For an exact calculation one is forced to solve the equation

$$I = \left(\mathscr{E}_0 - L\frac{\mathrm{d}I}{\mathrm{d}t} - I\frac{\mathrm{d}L}{\mathrm{d}t}\right)\Big/R,$$

which requires knowing the time dependence of the inductance, $L = L(t)$.

5.22. Since removal of the iron core results in a decrease in the induction and the magnetic field flux in the solenoid, during removal there emerges a self-induction emf, which opposes the reduction in the flux and, hence, increases the current flowing in the solenoid (the direction of the external emf, which supplies DC power to the solenoid, and that of the self-induction emf are the same).

5.23. The self-induction emf defined by the formula $\mathscr{E}_{s.i} = -L(\mathrm{d}I/\mathrm{d}t)$ is proportional to the derivative $\mathrm{d}I/\mathrm{d}t$ (for equal inductances), which is the greater the steeper the straight line. Hence, the self-induction emf is higher for the inductance for which the time dependence of the current is depicted by straight line *1*. Since the slopes of the straight lines do not change when the currents pass through zero, both the numerical values and the directions of the self-induction emf's are retained.

5.24. The self-induction emf defined by the formula

$$\mathscr{E}_{s.i.} = -L\frac{\mathrm{d}I}{\mathrm{d}t}$$

has its maximal value, obviously, at the point where the rate of decrease of current is greatest, that is, at point *3*.

5.25. In Figure (a), after key K is closed, the current flowing through the circuit that consists of L and R connected in series is initially the same as the current that was flowing before K was closed. For this circuit we can write Kirchhoff's law in the form

$$-L\frac{\mathrm{d}I}{\mathrm{d}t} = RI.$$

Separation of variables and subsequent integration yield

$$I = I_0 \exp(-Rt/L).$$

The current falls off according to an exponential law, with the self-induction emf being initially

$$\mathscr{E}_{s.i.} = -L\frac{\mathrm{d}I}{\mathrm{d}t} = I_0 R,$$

which means that the self-induction emf is equal to the emf of the DC source.

In Figure (b), after key K is opened, the current initially is the same as the one that was flowing in the circuit before K was opened. In this case, however, ther esistor R closes the circuit. Since prior to opening the key the current flowing in the resistor was much weaker than that flowing in the induction coil, the voltage across the resistor after K is opened may initially become considerably higher than that prior to opening the key, which is possible only if R is considerably higher than the resistance of the DC source. One must bear in mind also that after opening the key the current in the resistor will reverse its direction.

5.26. The increase in current in the circuit with a resistance and an inductance occurs according to the law

$$I = \frac{\mathscr{E}}{R}\left[1 - e^{(R/L)t}\right]. \qquad (5.26.1)$$

Since by hypothesis only one parameter can vary, the parameter may be only the inductance because conservation of the limiting current is possible only when two parameters, $\mathscr{E}$ and R, are varied simultaneously. Formula (5.26.1) implies that the increase in current is the slower the higher the inductance in the circuit. Hence, curve *2* corresponds to a higher inductance.

5.27. The magnetic flux coupled with contour *2* is

$$\Psi = BS,$$

where S is the area of the contour, and B is the magnetic induction at the point where the contour has been placed. Accordingly, the induction emf generated in the contour is

$$\mathscr{E}_1 = -S = \frac{dB}{dt} - S\frac{dB}{dr}\frac{dr}{dt} = -Sv\frac{dB}{dr}.$$

The induction of the magnetic field generated by the current flowing in a circular contour and measured at a certain distance from the contour on the contour's axis is given by the formula

$$B = \frac{\mu_0 p_m}{2\pi (R^2 + r^2)^{3/2}},$$

where p_m is the magnetic moment of the contour. Therefore, the induction emf in contour *2* is

$$\mathscr{E}_1 = \frac{3\mu_0 p_m S v}{2\pi}\frac{r}{(R^2 + r^2)^{5/2}} = C\frac{r}{(R^2 + r^2)^{5/2}},$$

with $C = 3\mu_0 p_m Sv/2\pi$. The sign of r determines the sign of the induced emf; when contour *2* is moved closer to contour *1*, r is negative and so is the induced emf, so that the direction of the current in contour *2* is opposite to that of the current in contour *1*. When contour *2* passes through contour *1*, the induced emf changes sign. The maximal value of this emf can be obtained by nullifying the derivative,

$$\frac{d\mathscr{E}_1}{dt} = C\,\frac{(R^2+r^2)^{5/2}-(5/2)\,(R^2+r^2)^{3/2}\,2r^2}{(R^2+r^2)^5} = 0.$$

Thus the emf is maximal at $r = R/2$. An emf of equal magnitude but of opposite sign is generated at the same distance but on the other side when the contours are brought together.

5.28. When key *K1* is closed, a closed circuit consisting of a DC source and the induction coil *L1* is formed.

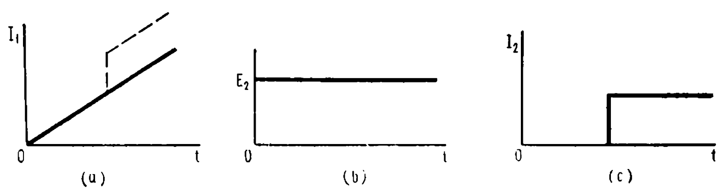

Fig. 5.28

Since there is no resistance in this circuit, the sum of the emf's is zero:

$$\mathscr{E} - L_1\frac{dI_1}{dt} = 0.$$

The fact that $\mathscr{E}$ and L_1 are constant requires that dI_1/dt be constant, too. Thus, a current linearly increasing with time will flow in the circuit (the solid line in Figure (a) accompanying the answer). The magnetic flux generated by this current is coupled with both coils and also linearly increases with time:

$$\Psi = L_1 I_1.$$

In the second coil there appears a constant emf (Figure (b) accompanying the answer) whose direction is opposite to that of the current in *L1*:

$$\mathscr{E}_2 = -\frac{d\Psi}{dt} = -L_1\frac{dI_1}{dt} = -L_2\frac{dI_1}{dt}.$$

The current in the first coil increases with a constant time-rate as long as $K2$ is open. When $K2$ is closed, a current flows in $L2$, with the magnetic field generated by this current opposing the field generated by the current in $L1$. The fact that the instantaneous value of the flux coupled with both coils must be preserved requires that there be a jump in the current in $L1$, after which the current will continue to grow according to the same linear law (the dashed broken line in Figure (a)). The current in $L2$ will remain constant during the entire process (Figure (c) accompanying the answer).

5.29. (1) To find the mutual inductance we determine the magnetic flux coupled with the contour and generated by current I flowing in the straight conductor. In this case the mutual inductance is determined from the equation $M = \Psi/I$. The induction at a distance x from the straight conductor is $B = \mu_0\mu I/2\pi x$. The fluxes that flow through a part of the contour dx wide and b high and through the entire contour are, respectively,

$$d\Phi = \frac{\mu_0\mu}{2\pi}\frac{I}{x}\, b\, dx,$$

$$\Phi = \frac{\mu_0\mu}{2\pi} Ib \int\limits_{c}^{c+a} \frac{dx}{x} = \frac{\mu_0\mu}{2\pi} Ib \ln\frac{c+a}{c}.$$

When there are N turns in the contour, the flux coupled with the contour is

$$\Psi = N\Phi = \frac{\mu_0\mu}{2\pi} INb \ln\frac{c+a}{c},$$

which implies that the mutual inductance is

$$M = \frac{\mu_0\mu}{2\pi} bN \ln\frac{c+a}{c}.$$

(2) Since the rotation of the contour through 90° makes the flux coupled with the contour vanish, the amount of electricity induced in the contour as a result of such a rotation is determined by the formula

$$Q = \frac{\Psi}{R} = \frac{\mu_0\mu b\, NI}{2\pi R} \ln\frac{c+a}{c}.$$

(3) The rotation of the contour through 180° requires the following work to be done:

$$A = -2\Psi I = -\frac{\mu_0\mu I^2 Nb}{\pi} \ln\frac{c+a}{c}$$

(since after the rotation the flux coupled with the contour will become $-\Psi$). The "minus" shows that the work is done against the forces induced by the magnetic field.

5.30. When a current flows in a conductor, the induction of the magnetic field generated by this current at a distance r from the conductor is

$$B = \mu_0 I/2\pi r.$$

The magnetic flux coupled with the contour formed by the winding of the loop is $\Psi = BSN$, or

$$\Psi = \mu_0 ISN/2\pi r.$$

When the current drops to zero, the flux follows it, and the amount of electricity flowing in the contour is determined by the formula $Q = \Delta\Psi/R$. Hence, the current that had been flowing in the conductor prior to switch-off is

$$I = \frac{2\pi rRQ}{\mu_0 SN}.$$

5.31. The following emf is induced in the coil:

$$\mathscr{E}_i = -\frac{d\Psi}{dt}. \qquad (5.31.1)$$

We see that the maximal possible value of $\mathscr{E}_i$ is the higher the greater the rate with which the coil is moved out of the field. The area under the curve is given by the integral

$$\int_0^t \mathscr{E}_i \, dt = -\int_{\Psi_1}^{\Psi_2} d\Psi = \Psi_1 - \Psi_2 = \Psi_1 = BSN$$

and, hence, is independent of the rate of coil removal from the region with the field.

5.32. (1) The system can be considered as being a new solenoid whose length is twice as large as that of one solenoid, with a density of turns the same as that in one solenoid and with the same cross-sectional area. Since the inductance of one solenoid is $L = \mu_0 N_0 V$, where N_0 is the number of turns per unit length, and V is the solenoid volume, which in this case is twice the volume of one solenoid, we have $L_1 = 2L_0$. The same result can be achieved by considering the self-induction emf that is generated in the two solenoids connected in series. The total emf is equal to the sum of the emf's generated in each solenoid; hence,

$$\mathscr{E}_1 = -2L_0 \frac{dI}{dt},$$

which yields $L_1 = 2L_0$.

(2) When the solenoids are connected in parallel, the self-induction emf in each solenoid is

$$\mathscr{E}_1 = -L_0 \frac{d(I/2)}{dt} = -\frac{1}{2} L_0 \frac{dI}{dt}.$$

Because the solenoids are connected in parallel, the total emf has the same value. Thus, with a current I in a circuit that is external with respect to the solenoid, the induced emf is one-half the value for the inductance L_0. Hence,

$$L_2 = \frac{1}{2} L_0.$$

(3) In this case, the number of turns per unit length is twice as large as that of one solenoid, and since the inductance is proportional to N_0^2, we have (provided that the current remains unchanged)

$$L_3 = 4L_0.$$

(4) If one solenoid is fitted onto the other and the senses of the turns coincide and the solenoids are connected in parallel, the current through each solenoid is $I/2$ if the current in the circuit is I, while the flux associated with current $I/2$ is $\Phi/2$. The total flux is Φ and the flux coupled with each solenoid is $\Psi = \Phi N_0$. In each solenoid there is generated a self-induction emf equal to the one induced in a separate solenoid when current I varies. Since the solenoids are connected in parallel, this emf is the common emf of both solenoids. Hence,

$$L_4 = L_0.$$

(5, 6) In both cases the induction flux in the solenoids is zero, so that $L_5 = L_6 = 0$.

5.33. The induced emf is

$$\mathscr{E}_1 = -L \frac{dI}{dt},$$

Hence, the value of the emf is determined by the rate with which the current decreases (the sign of this rate is opposite to dI/dt). The slope of the straight line on the *0-1* section is twice as large as that of the straight line on the *1-2* section and coincides (numerically) with the slopes of the straight lines on sections *3-4-5* and *5-6*. Hence, in the time interval between points *1* and *2* the

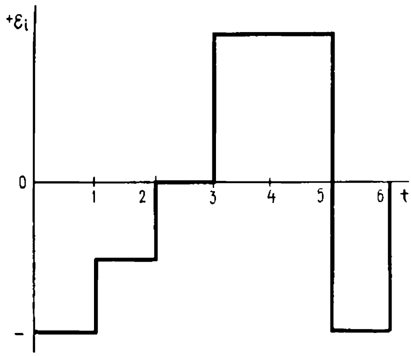

Fig. 5.33

induced emf is one-half of the emf's in the other intervals except the interval from point *2* to point *3* where $\mathscr{E}_i = 0$ (I = const).

5.34. In each beam we isolate an element of length l. On the one hand, the element can be thought of as a charge $Q = enSl$, or, on the other, as an element of current $I = envS$. An electrostatic repulsive force $F_e = EQ$ acts on each charge element, where E can be assumed to be the electric field generated by an infinitely long straight conductor carrying a charge whose linear density is $\tau = enS$. This field, which acts on the charges in the second beam, can be written in the form

$$E = \frac{enS}{2\pi\varepsilon_0 r},$$

so that

$$F_e = \frac{e^2n^2S^2l}{2\pi\varepsilon_0 r}.$$

The isolated element, if considered as an element of current, is under a force $F_m = BIl$, where B is the induction generated by the other current:

$$B = \mu_0 \frac{envS}{2\pi r}.$$

Thus

$$F_m = \mu_0 \frac{e^2 n^2 v^2 S^2 l}{2\pi r}.$$

The ratio of these two forces is $F_m/F_e = v^2 \varepsilon_0 \mu_0$. Since $\varepsilon_0 \mu_0 = 1/c^2$, where c is the speed of light in vacuo, we obtain

$$\frac{F_m}{F_e} = \frac{v^2}{c^2}.$$

5.35. The reasoning is all wrong. Even if the electrons in the conductors are at rest in relation to the needle, the positive ions that are moving in this case in the opposite direction create, obviously, a magnetic field equal to the one generated by the moving electrons when the needle was at rest. If the electrons are moving in a vacuum, then the electrodes and the electric field move in the opposite direction (when the needle is at rest in relation to the electrons).

5.36. The permeability of air is practically unity and at any point the magnetic field vector coincides in direction with the magnetic induction vector. In the emu system of units both vectors coincide in magnitude as well, while in the SI system they are related thus: $H = B/\mu_0$. Since the lines of force of induction are continuous, inside a bar magnet they are directed from the south pole to the north pole and are continued outside the magnet by lines directed from the north pole to the south pole. To determine the direction of the magnetic field inside the magnet, one must bear in mind that the circulation integral of the magnetic field vector along a closed contour must be equal to the algebraic sum of the currents encompassed by the contour. Since in the case at hand there are no currents, the circulation integral along any con-

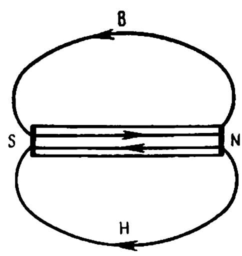

Fig. 5.36

tour lying inside the magnet must be zero. If the contour passes partially through the air surrounding the magnet and partially in the magnet, the circulation integral may be equal to zero only if inside the magnet the magnetic field vector is directed from the north pole to the south pole. Formally this means that inside the magnet the permeability is negative.

5.37. Alternating magnetization results in liberation of heat in the steels, with the amount of heat proportional to the area bounded by the hysteresis loop. Since a transformer operates on alternating currents, the amount of heat liberated in the core of a transformer will be the greater the bigger the area bounded by the loop. From this fact one can conclude that the steel whose hysteresis loop is depicted in Figure (b) accompanying the problem is more desirable. On the other hand, it is desirable that a permanent magnet have as high a residual magnetic induction and a coercive force as possible. This implies that the steel more suitable for manufacturing a permanent magnet is the one whose hysteresis loop is depicted in Figure (a).

5.38. The elementary work involved in changing the magnetic flux coupled with a contour carrying a current I is

$$\mathrm{d}A = I\,\mathrm{d}\Psi, \quad \text{or} \quad \mathrm{d}A = ISN\,\mathrm{d}B.$$

If we use the relationship that exists between the current in a solenoid and the magnetic field generated by this current, $H = IN/l$, we obtain

$$\mathrm{d}A = H\,Sl\,\mathrm{d}B = VH\mathrm{d}B,$$

where V is the volume of the core. The entire work is

$$A = V\int_0^B H\mathrm{d}B.$$

The integral on the right-hand side is the area bounded by the B vs. H curve, the ordinate, and the segment of a straight line parallel to the H axis (see the figure accompanying the answer).

5.39. As shown in the answer to Problem 5.36, the magnetic field inside a permanent magnet is directed from the north pole to the south pole, while the induction is directed from the south pole to the north pole, with the result that these quantities have opposite signs. On the hysteresis loop, this condition is met on sections *2-3* and *5-6* (see the figure accompanying the problem). Formally, to the permeability on these sections we can assign a negative value. Correspondingly, to point *0* we may assign a permeability equal to $\pm\infty$, while points *3* and *6* correspond to zero values of the permeability.

5.40. After the current in the conductor has ceased, the circulation integral of the magnetic field strength along any closed contour is zero (this is true even for a closed

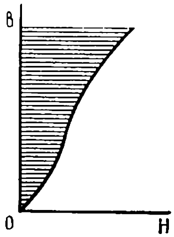

Fig. 5.38

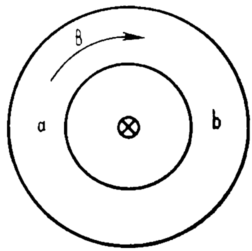

Fig. 5.40

contour that passes in the toroid). Since all points of a contour that is a circle concentric with the section of the conductor are identical, the magnetic field strength at all points inside the toroid is zero, too. At the same time, the toroid carries a residual magnetic induction whose lines of force are circles directed in the manner shown by the arrow in the figure accompanying the answer. This magnetic state of the toroid corresponds to point *2* or *5* on the hysteresis loop (the choice of the point depends on which of the two directions is assumed to be positive). If the positive direction of the magnetic field vector is the one the toroid acquires during magnetization, this magnetic state of the toroid is depicted by point *2*.

5.41. The circulation integral of the magnetic field is uniquely determined by the current flowing inside the contour. Because of this, the circulation integrals along contours *1*, *4* and *5* (see the figure accompanying the problem) are the same and equal to the current *I*, while the circulation integrals along contours *2* and *3* are zero. However, the situation with the circulation integrals of the magnetic induction along these contours is quite different. When the circulation integrals are evaluated along contours that pass through a homogeneous medium (in the case at hand, in a vacuum), they do not depend on the shape and size of the contours, with the result that the circulation integrals along contours *4* and *5* are equal. Reasoning in the same manner, we conclude that the circulation integral along contour *3* is zero. But in evaluating the circulation integrals along contours *1* and *2* that include sections of a medium with a permeability greater than unity, the circulation elements in this medium are μ times greater than in the vacuum (if μ is greater than unity). For this reason the circulation integral along contour *1* is greater than that along contours *4* and *5*, while the circulation integral along contour *2* is nonzero.