ALGEBRAIC EXPRESSIONS AND IDENTITIES

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CHAPTER

CONTENTS

➤ CONSTANT & VARIABLE

- **Constant :** A symbol having a fixed numerical value is called a constant.
- **Variable :** A symbol which takes various numerical values is called a variable.

  Eg. We know that the perimeter P of a square of side s is given by P = 4 × s. Here, 4 is a constant and P and s are variables.

  Eg. The perimeter P of a rectangle of sides *l* and b is given by P = 2 (*l* + b). Here, 2 is a constant and *l* and b are variables.

➤ ALGEBRAIC EXPRESSIONS

A combination of constants and variables connected by the signs of fundamental operation of addition, subtraction, multiplication and division is called an algebraic expression.

- **Terms :** Various parts of an algebraic expression which are separated by the signs of + or – are called the 'terms' of the expression.

  Eg. $2x^2 - 3xy + 5y^2$ is an algebraic expression consisting of three terms, namely, $2x^2$, $-3xy$ and $5y^2$.

  Eg. The expression $2x^3 - 3x^2 + 4x - 7$ is an algebraic expression consisting of four terms, namely, $2x^3$, $-3x^2$, $4x$ and -7.

- **Monomial :** An algebraic expression containing only one term is called a monomial.

  Eg. $-5, 3y, 7xy, \frac{2}{3}x^2yz, \frac{5}{3}a^2bc^3$ etc. are all monomials.

- **Binomial :** An algebraic expression containing two terms is called a binomial.

  Eg. The expression $2x - 3$, $3x + 2y$, $xyz - 5$ etc. are all binomials.

- **Trinomial :** An algebraic expression containing three terms is called a trinomial.

  Eg. The expressions $a - b + 2$, $x^2 + y^2 - xy$,

  $x^3 - 2y^3 - 3x^2y^2z$ etc. are trinomial.

- **Factors :** Each terms in an algebraic expression is a product of one or more number(s) and / or literal(s). These number(s) and liteal(s) are known as the factors of that terms.

  A constant factor is called a numerical factor, while a variable factor is known as a literal factor.

- **Coefficient :** In a term of an algebraic expression any of the factors with the sign of the term is called the coefficient of the other factors.

  Eg. In $-5xy$, the coefficient of x is $-5y$; the coefficient of y is $-5x$ and the coefficient of xy is -5.

  Eg. In $-x$, the coefficient of x is -1.

- **Constant Term :** A term of the expression having no literal factor is called a constant term.

  Eg. In the algebraic expression $x^2 - xy + yz - 4$, the constant term is -4.

- **Like and Unlike Terms :** The terms having the same literal factors are called like or similar terms, otherwise they are called unlike terms.

Eg. In the algebraic expression $2a^2b + 3ab^2 - 7ab - 4ba^2$, we have $2a^2b$ and $-4ba^2$ as like terms, whereas $3ab^2$ and $-7ab$ are unlike terms.

❖ EXAMPLES ❖

Ex.1 Add : $7x^2 - 4x + 5$, $-3x^2 + 2x - 1$ and $5x^2 - x + 9$.

Sol. We have,

Required sum

$= (7x^2 - 4x + 5) + (-3x^2 + 2x - 1) + (5x^2 - x + 9)$

$= 7x^2 - 3x^2 + 5x^2 - 4x + 2x - x + 5 - 1 + 9$ [Collecting like terms]

$= (7 - 3 + 5)x^2 + (-4 + 2 - 1)x + (5 - 1 + 9)$ [Adding like terms]

$= 9x^2 - 3x + 13$

Ex.2 Add : $5x^2 - \frac{1}{3}x + \frac{5}{2}$, $-\frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{3}$ and $-2x^2 + \frac{1}{5}x - \frac{1}{6}$.

Sol. Required sum

$$= \left(5x^2 - \frac{1}{3}x + \frac{5}{2}\right) + \left(-\frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{3}\right) + \left(-2x^2 + \frac{1}{5}x - \frac{1}{6}\right)$$

$$= 5x^2 - \frac{1}{2}x^2 - 2x^2 - \frac{1}{3}x + \frac{1}{2}x + \frac{1}{5}x + \frac{5}{2} - \frac{1}{3} - \frac{1}{6}$$ [Collecting like terms]

$$= \left(5 - \frac{1}{2} - 2\right)x^2 + \left(-\frac{1}{3} + \frac{1}{2} + \frac{1}{5}\right)x + \left(\frac{5}{2} - \frac{1}{3} - \frac{1}{6}\right)$$

[Adding like term]

$$= \left(\frac{10 - 1 - 4}{2}\right)x^2 + \left(\frac{-10 + 15 + 6}{30}\right)x + \left(\frac{15 - 2 - 1}{6}\right)$$

$$= \frac{5}{2}x^2 + \frac{11}{30}x + 2$$

➤ MULTIPLICATION OF ALGEBRAIC EXPRESSIONS

Multiplication Of Algebraic Expressions

(i) The product of two factors with like signs is positive and the product of two factors with unlike signs is negative

i.e., (a) $(+) \times (+) = +$

(b) $(+) \times (-) = -$

(c) $(-) \times (+) = -$

and, (d) $(-) \times (-) = +$

(ii) If a is any variable and m, n are positive integers, then

$a^m \times a^n = a^{m+n}$

For example , $a^3 \times a^5 = a^{3+5} = a^8$,

$y^4 \times y = y^{4+1} = y^5$ etc.

Ex.3 Find the product of the following pairs of polynomials :

(i) 4, 7x (ii) – 4a, 7a

(iii) – 4x, 7xy (iv) $4x^3$, – 3xy

(v) 4x, 0

Sol. We have,

(i) $4 \times 7x = (4 \times 7) \times x = 28 \times x = 28x$

(ii) $(-4a) \times (7a) = (-4 \times 7) \times (a \times a) = -28a^2$

(iii) $(-4x) \times (7xy) = (-4 \times 7) \times (x \times xy) = -28x^{1+1}y = -28x^2y$

(iv) $(4x^3) \times (-3xy) = (4 \times -3) \times (x^3 \times xy) = -12(x^{3+1}y) = -12x^4y$

(v) $4x \times 0 = (4 \times 0) \times x = 0 \times x = 0$

Ex.4 Find the areas of rectangles with the following pairs of monomials as their length and breadth respectively :

(i) (x, y) (ii) (10x, 5y)

(iii) $(2x^2, 5y^2)$ (iv) $(4a, 3a^2)$

(v) (3mn, 4np)

Sol. We know that the area of a rectangle is the product of its length and breadth.

	Length	Breadth	Length × Breadth = Area	
(i)	x	y	x × y	= xy
(ii)	10 x	5y	10x × 5y	= 50xy
(iii)	$2x^2$	$5y^2$	$2x^2 \times 5y^2$	$= (2 \times 5) \times (x^2 \times y^2) = 10x^2y^2$
(iv)	4a	$3a^2$	$4a \times 3a^2$	$= (4 \times 3) \times (a \times a^2) = 12\,a^3$
(v)	3mn	4np	3mn × 4np	$= (3 \times 4) \times (m \times n \times n \times p) = 12\,mn^2p$

Ex.5 Multiply :

(i) $3ab^2c^3$ by $5a^3b^2c$

(ii) $4x^2yz$ by $-\frac{3}{2}x^2yz^2$

(iii) $-\frac{8}{5}x^2yz^3$ by $-\frac{3}{4}xy^2z$

(iv) $\frac{3}{14}x^2y$ by $\frac{7}{2}x^4y$

(v) $2.1a^2bc$ by $4ab^2$

Sol. (i) We have,

$(3ab^2c^3) \times (5a^3b^2c)$

$= (3 \times 5) \times (a \times a^3 \times b^2 \times b^2 \times c^3 \times c)$

$= 15a^{1+3}b^{2+2}c^{3+1}$

$= 15a^4b^4c^4$

(ii) We have,

$(4x^2yz) \times \left(-\frac{3}{2}x^2yz^2\right)$

$= \left(4 \times -\frac{3}{2}\right) \times (x^2 \times x^2 \times y \times y \times z \times z^2)$

$= -6x^{2+2}y^{1+1}z^{1+2} = -6x^4y^2z^3$

(iii) We have,

$\left(-\frac{8}{5}x^2yz^3\right) \times \left(-\frac{3}{4}xy^2z\right)$

$= \left(-\frac{8}{5} \times -\frac{3}{4}\right) \times (x^2 \times x \times y \times y^2 \times z^3 \times z)$

$= \frac{6}{5}x^{2+1}y^{1+2}z^{3+1} = \frac{6}{5}x^3y^3z^4$

(iv) We have,

$\left(\frac{3}{14}x^2y\right) \times \left(\frac{7}{2}x^4y\right)$

$= \left(\frac{3}{14} \times \frac{7}{2}\right) \times (x^2 \times x^4 \times y \times y)$

$= \frac{3}{4}x^{2+4}y^{1+1} = \frac{3}{4}x^6y^2$

(v) We have, $(2.1a^2bc) \times (4ab^2)$

$= (2.1 \times 4) \times (a^2 \times a \times b \times b^2 \times c)$

$= 8.4a^{2+1}b^{1+2}c = 8.4a^3b^3c$

Ex.6 Multiply :

(i) $-6a^2bc$, $2a^2b$ and $-\frac{1}{4}$

(ii) $\frac{4}{9}a^5b^2$, $10a^3b$ and 6

(iii) 3.15x and $-23x^2y$

(iv) – x, x^2yz and $-\frac{3}{7}xyz^2$

Sol. (i) We have,

$(-6a^2bc) \times (2a^2b) \times \left(-\frac{1}{4}\right)$

$= \left(-6 \times 2 \times -\frac{1}{4}\right) \times (a^2 \times a^2 \times b \times b \times c)$

$= 3a^{2+2}b^{1+1}c = 3a^4b^2c$

(ii) We have,

$\left(\frac{4}{9}a^5b^2\right) \times (10a^3b) \times (6)$

$= \left(\frac{4}{9} \times 10 \times 6\right) \times (a^5 \times a^3 \times b^2 \times b)$

$= \frac{80}{3}a^{5+3}b^{2+1} = \frac{80}{3}a^8b^3$

(iii) We have, $(3) \times (15x) \times (-23x^2y)$

$= (3 \times 15 - 23) \times (x \times x^2 \times y)$

$= -1035x^{1+2}y = -1035x^3y.$

(iv) We have,

$(-x) \times (x^2yz) \times \left(\frac{-3}{7}xyz^2\right)$

$= \left(-1 \times \frac{-3}{7}\right) \times (x \times x^2 \times x \times y \times y \times z \times z^2)$

$= \frac{3}{7}x^{1+2+1}y^{1+1}z^{1+2} = \frac{3}{7}x^4y^2z^3$

Ex.7 Multiply each of the following monomials :

(i) $3xyz, 5x, 0$ (ii) $\frac{6}{5}ab, \frac{5}{6}bc, \frac{12}{9}abc$

(iii) $\frac{3}{4}x^2yz^2, 0.5xy^2z^2, 1.16x^2yz^3, 2xyz$

(vi) $20x^{10}y^{20}z^{30}, (10xyz)^2$

(v) $(-3x^2y), (4xy^2z), (-xy^2z^2)$ and $\left(\frac{4}{5}z\right)$

Sol. (i) We have,

$(3xyz) \times (5x) \times 0$

$= (3\times5\times0)\times (x\times x\times y\times z)$

$= 0 \times x^2yz = 0$

(ii) We have,

$\left(\frac{6}{5}ab\right) \times \left(\frac{5}{6}bc\right) \times \left(\frac{12}{9}abc\right)$

$\left(\frac{6}{5}\times\frac{5}{6}\times\frac{12}{9}\right) \times (a \times a \times b \times b \times b \times c \times c)$

$= \frac{12}{9}a^{1+1}b^{1+1+1}c^{1+1} = \frac{4}{3}a^2b^3c^2$

(iii) We have,

$\left(\frac{3}{4}x^2yz^2\right) \times (0.5xy^2z^2) \times (1.16x^2yz^3) \times (2xyz)$

$= \left(\frac{3}{4}\times 0.5 \times 1.16 \times 2\right) \times (x^2 \times x \times x^2 \times x \times y \times y^2 \times y \times y \times z^2 \times z^2 \times z^3 \times z)$

$= \left(\frac{3}{4}\times\frac{5}{10}\times\frac{116}{100}\times 2\right) \times (x^{2+1+2+1} \times y^{1+2+1+1} \times z^{2+2+3+1})$

$= \frac{87}{100}x^6y^5z^8$

(iv) We have,

$(20x^{10}y^{20}z^{30}) \times (10xyz)^2$

$= (20x^{10}y^{20}z^{30}) \times (10xyz) \times (10xyz)$

$= (20 \times 10 \times 10) \times (x^{10} \times x \times x \times y^{20} \times y \times y \times z^{30} \times z \times z)$

$= 2000x^{10+1+1}y^{20+1+1}z^{30+1+1}$

$= 2000x^{12}y^{22}z^{32}$

(v) We have,

$(-3x^2y) \times (4xy^2z) \times (-xy^2z^2) \times \left(\frac{4}{5}z\right)$

$= \left(-3 \times 4 \times -1 \times \frac{4}{5}\right) \times (x^2 \times x \times x \times y \times y^2 \times y^2 \times z \times z^2 \times z)$

$= \frac{48}{5}x^{2+1+1}y^{1+2+2}z^{1+2+1} = \frac{48}{5}x^4y^5z^4$

Ex.8 Express the following product as a monomial:

$(x^3) \times (7x^5) \times \left(\frac{1}{5}x^2\right) \times (-6x^4)$

Verify the product for x = 1

Sol. We have,

$(x^3) \times (7x^5) \times \left(\frac{1}{5}x^2\right) \times (-6x^4)$

$= \left(1 \times 7 \times \frac{1}{5} \times -6\right) \times (x^3 \times x^5 \times x^2 \times x^4)$

$= -\frac{42}{5}x^{3+5+2+4} = -\frac{42}{5}x^{14}$

Verification : For x = 1, we have

L.H.S. $= (x^3) \times (7x^5) \times \left(\frac{1}{5}x^2\right) \times (-6x^4)$

$= (1)^3 \times \{7 \times (1^5)\} \times \left\{\frac{1}{5}\times(1)^2\right\} \times \{-6 \times (1)^4\}$

$= 1 \times 7 \times \frac{1}{5} \times -6 = -\frac{42}{5}$

and, R.H.S. $= -\frac{42}{5} \times (1)^{14} = -\frac{42}{5}$

$\therefore$ L.H.S. = R.H.S.

Ex.9 Find the value of $(5a^6) \times (-10ab^2) \times (-2.1a^2b^3)$ for $a = 1$ and $b = \frac{1}{2}$

Sol. We have,

$(5a^6) \times (-10ab^2) \times (-2.1a^2b^3)$

$= (5 \times -10 \times -2.1) \times (a^6 \times a \times a^2 \times b^2 \times b^3)$

$= \left(5 \times -10 \times -\frac{21}{10}\right) \times (a^6 \times a \times a^2 \times b^2 \times b^3)$

$= 105\, a^{6+1+2}b^{2+3} = 105a^9b^5$

Putting $a = 1$ and $b = \frac{1}{2}$, we have

$105a^9b^5 = 105 \times (1)^9 \times \left(\frac{1}{2}\right)^5$

$= 105 \times 1 \times \frac{1}{32} = \frac{105}{32}$

Multiplication of a Monomial & a Binomial

Ex.10 Multiply : $2x$ by $(3x + 5y)$

Sol. We have,

$2x \times (3x + 5y) = 2x \times 3x + 2x \times 5y = 6x^2 + 10xy$

Ex.11 Multiply : $(7xy + 5y)$ by $3xy$

Sol. We have,

$(7xy + 5y) \times 3xy$

$= 7xy \times 3xy + 5y \times 3xy$

$= 21x^{1+1}y^{1+1} + 15xy^{1+1} = 21x^2y^2 + 15xy^2$

Ex.12 Multiply : $-\frac{3ab^2}{5}$ by $\left(\frac{2a}{3} - b\right)$

Sol. We have,

$\left(-\frac{3ab^2}{5}\right) \times \left(\frac{2a}{3} - b\right)$

$= \left(-\frac{3ab^2}{5}\right) \times \frac{2a}{3} - \left(-\frac{3ab^2}{5}\right) \times b$

$= -\frac{3}{5} \times \frac{2}{3} a^{1+1}b^2 + \frac{3}{5} ab^{2+1} = -\frac{2}{5} a^2b^2 + \frac{3}{5} ab^3$

Ex.13 Multiply : $\left(3x - \frac{4}{5}y^2x\right)$ by $\frac{1}{2}xy$.

Sol. Horizontal method

We have,

$\left(3x - \frac{4}{5}y^2x\right) \times \frac{1}{2}xy$

$= 3x \times \frac{1}{2}xy - \frac{4}{5}y^2x \times \frac{1}{2}xy$

$= \left(3 \times \frac{1}{2}\right) \times x \times x \times y -$

$\left(\frac{4}{5} \times \frac{1}{2}\right) \times y^2 \times y \times x \times x$

$= \frac{3}{2}x^2y - \frac{2}{5}y^3x^2 = \frac{3}{2}x^2y - \frac{2}{5}x^2y^3$

Column method

We have,

$3x - \frac{4}{5}y^2x$

$\times \frac{1}{2}xy$

$\frac{3}{2}x^2y - \frac{2}{5}x^2y^3$

Ex.14 Determine each of the following products and find the value of each for $x = 2$, $y = 1.15$, $z = 0.01$.

(i) $27x^2(1 - 3x)$ (ii) $xz(x^2 + y^2)$

(iii) $z^2(x - y)$ (iv) $(2z - 3x) \times (-4y)$

Sol. (i) We have,

$27x^2(1 - 3x)$

$= 27x^2 \times (1 - 3x)$

$= 27x^2 \times 1 - 27x^2 \times 3x$

[Expanding the bracket]

$= 27x^2 - 81x^3$

Putting $x = 2$, we have

$27x^2(1 - 3x)$

$= 27 \times (2)^2 \times (1 - 3 \times 2) = 27 \times 4 \times (1 - 6)$

$= 27 \times 4 \times -5 = -540$

(ii) We have, $xz(x^2 + y^2)$

$= xz \times (x^2 + y^2)$

$= xz \times x^2 + xz \times y^2 = x^3z + xy^2z$

Putting $x = 2$, $y = 1.15$ and $z = 0.01$, we get

$xz(x^2 + y^2)$

$= 2 \times 0.01 \times \{(2)^2 + (1.15)^2\}$

$= 0.02 \times (4 + 1.3225) = 0.02 \times 5.3225$

$= 0.106450$

(iii) We have,

$z^2(x - y)$

$= z^2 \times (x - y)$

$= z^2 \times x - z^2 \times y = z^2x - z^2y$

Putting x = 2 y = 1.15 and z = 0.01, we get

$z^2(x - y)$

$= (0.01)^2 \times (2 - 1.15)$

$= (0.0001) \times (0.85) = 0.000085$

(vi) We have,

$(2z - 3x) \times (-4y)$

$= (2z) \times (-4y) - 3x \times (-4y) = -8zy + 12xy$

Putting x = 2, y= 1.15 and z = 0.01, we have

$(2z - 3x) \times -4y$

$= [(2 \times 0.01) - (3 \times 2)] \times (-4 \times 1.15)$

$= (0.02 - 6) \times (-4.6) = -5.98 \times -4.6 = 27.508$

Ex.15 Simplify the expression and evaluate them as directed :

(i) $x(x - 3) + 2$ for $x = 1$

(ii) $3y(2y - 7) - 3(y - 4) - 63$ for $y = -2$

Sol. (i) We have,

$x(x - 3) + 2 = x^2 - 3x + 2$

For x = 1, we have

$x^2 - 3x + 2 = (1)^2 - 3 \times 1 + 2 = 1 - 3 + 2$

$= 3 - 3 = 0$

(ii) We have,

$3y(2y - 7) - 3(y - 4) - 63$

$= (6y^2 - 21y) - (3y - 12) - 63$

$= 6y^2 - 21y - 3y + 12 - 63$

$= 6y^2 - 24y - 51$

For y = –2, we have

$6y^2 - 24y - 51 = 6 \times (-2)^2 - 24(-2) - 51$

$= 6 \times 4 + 24 \times 2 - 51 = 24 + 48 - 51$

$= 72 - 51 = 21$

Ex.16 Subtract $3pq(p - q)$ from $2pq(p + q)$

Sol. (i) We have,

$3pq(p - q) = 3p^2q - 3pq^2$

and, $2pq(p + q) = 2p^2q + 2pq^2$

Subtraction :

$2p^2q + 2pq^2$

$3p^2q - 3pq^2$

$-\quad\quad +$

$-p^2q + 5pq^2$

Ex.17 Add : (i) $p(p - q)$, $q(q - r)$ and $r(r - p)$

(ii) $2x(z - x - y)$ and $2y(z - y - x)$

Sol. (i) We have,

$p(p - q) + q(q - r) + r(r - p)$

$= p^2 - pq + q^2 - qr + r^2 - rp$

$= p^2 + q^2 + r^2 - pq - qr - rp$

(ii) We have,

$2x(z - x - y) + 2y(z - y - x)$

$= 2xz - 2x^2 - 2xy + 2yz - 2y^2 - 2xy$

$= 2xz - 2x^2 - 4xy + 2yz - 2y^2$

Ex.18 Simplify each of the following expressions :

(i) $15a^2 - 6a(a - 2) + a(3 + 7a)$

(ii) $x^2(1 - 3y^2) + x(xy^2 - 2x) - 3y(y - 4x^2y)$

(iii) $4st(s - t) - 6s^2(t - t^2) - 3t^2(2s^2 - s) + 2st(s - t)$

Sol. (i) We have,

$15a^2 - 6a(a - 2) + a(3 + 7a)$

$= 15a^2 - 6a^2 + 12a + 3a + 7a^2$

$= 15a^2 - 6a^2 + 7a^2 + 12a + 3a = 16a^2 + 15a$

(ii) We have,

$x^2(1 - 3y^2) + x(xy^2 - 2x) - 3y(y - 4x^2y)$

$= x^2 \times 1 - 3y^2 \times x^2 + x \times xy^2 - x \times 2x - 3y \times y + 3y \times 4x^2y$

$= x^2 - 3x^2y^2 + x^2y^2 - 2x^2 - 3y^2 + 12x^2y^2$

$= (x^2 - 2x^2) + (-3x^2y^2 + x^2y^2 + 12x^2y^2) - 3y^2$

$= -x^2 + 10x^2y^2 - 3y^2$

(iii) $4st(s - t) - 6s^2(t - t^2) - 3t^2(2s^2 - s) + 2st(s - t)$

$= 4st \times s - 4st \times t - 6s^2 \times t + 6s^2 \times t^2 - 3t^2 \times 2s^2 + 3t^2 \times s + 2st \times s - 2st \times t$

$= 4s^2t - 4st^2 - 6s^2t + 6s^2t^2 - 6s^2t^2 + 3st^2 + 2s^2t - 2st^2$

$= (4s^2t - 6s^2t + 2s^2t) + (-4st^2 + 3st^2 - 2st^2) + (6s^2t^2 - 6s^2t^2)$

$= -3st^2$

Multiplication of Two Binomials

Ex.19 Multiply (3x + 2y) and (5x + 3y).

Sol. We have,

$(3x + 2y) \times (5x + 3y)$

$= 3x \times (5x + 3y) + 2y \times (5x + 3y)$

$= (3x \times 5x + 3x \times 3y) + (2y \times 5x + 2y \times 3y)$

$= (15x^2 + 9xy) + (10xy + 6y^2)$

$= 15x^2 + 9xy + 10xy + 6y^2$

$= 15x^2 + 19xy + 6y^2$

Ex.20 Multiply (2x + 3y) and (4x – 5y)

Sol. We have,

$(2x + 3y) \times (4x - 5y)$

$= 2x \times (4x - 5y) + 3y \times (4x - 5y)$

$= (2x \times 4x - 2x \times 5y) + (3y \times 4x - 3y \times 5y)$

$= (8x^2 - 10xy) + (12xy - 15y^2)$

$= 8x^2 - 10xy + 12xy - 15y^2$

$= 8x^2 + 2xy - 15y^2$

Ex.21 Multiply (7a + 3b) and (2a + 3b) by column method.

Sol. We have,

$7a + 3b$

$\times\ 2a + 3b$

$14a^2 + 6ab$ — Multiplying $7a + 3b$ by $2a$

$+ 21ab + 9b^2$ — Multiplying $7a + 3b$ by $3b$

$14a^2 + 27ab + 9b^2$ — Adding the like term

Ex.22 Multiply (7x – 3y) by (4x – 5y) by column method.

Sol. We have,

$7x - 3y$

$\times\ 4x - 5y$

$28x^2 - 12xy$ — Multiplying $7x - 3y$ by $4x$

$- 35xy + 15y^2$ — Multiplying $7x - 3y$ by $-5y$

$28x^2 - 47xy + 15y^2$ — Adding the like terms

Ex.23 Multiply (0.5x – y) by (0.5x + y)

Sol. Horizontal Method :

We have,

$(0.5x - y) \times (0.5x + y)$

$= 0.5x\,(0.5x + y) - y\,(0.5x + y)$

$= 0.5x \times 0.5x + 0.5x \times y - y \times 0.5x - y \times y$

$= 0.25x^2 + 0.5xy - 0.5xy - y^2$

$= 0.25x^2 - y^2$

Column method:

We have,

$0.5x - y$

$\times\ 0.5x + y$

$0.25x^2 - 0.5xy$ — Multiplying $0.5x - y$ by $0.5x$

$+ 0.5xy - y^2$ — Multiplying $0.5x - y$ by y

$0.25x^2 - y^2$ — Adding the like terms

Ex.24 Multiplying $\left(4x + \frac{3y}{5}\right)$ and $\left(3x - \frac{4y}{5}\right)$

Sol. Horizontal Method :

$\left(4x + \frac{3y}{5}\right) \times \left(3x - \frac{4y}{5}\right)$

$= 4x \times \left(3x - \frac{4y}{5}\right) + \frac{3y}{5} \times \left(3x - \frac{4y}{5}\right)$

$= 4x \times 3x - 4x \times \frac{4y}{5} + \frac{3y}{5} \times 3x - \frac{3y}{5} \times \frac{4y}{5}$

$= 12x^2 - \frac{16}{5}xy + \frac{9}{5}xy - \frac{12}{25}y^2$

$= 12x^2 - \frac{7}{5}xy - \frac{12}{25}y^2$

Column method:

We have,

$4x + \frac{3y}{5}$

$\times\ 3x - \frac{4y}{5}$

$12x^2 + \frac{9}{5}xy$ — Multiplying $4x + \frac{3y}{5}$ by $3x$.

$-\frac{16}{5}xy - \frac{12}{25}y^2$ — Multiplying $4x + \frac{3y}{5}$ by $-\frac{4y}{5}$.

$12x^2 - \frac{7}{5}xy - \frac{12}{25}y^2$ — Adding the like terms

Ex.25 Find the value of the following products:

(i) $(x + 2y)(x - 2y)$ at $x = 1, y = 0$

(ii) $(3m - 2n)(2m - 3n)$ at $m = 1, n = -1$

(iii) $(4a^2 + 3b)(4a^2 + 3b)$ at $a = 1, b = 2$

Sol. (i) We have,

$(x + 2y)(x - 2y)$

$= x(x - 2y) + 2y(x - 2y)$

$= x \times x - x \times 2y + 2y \times x - 2y \times 2y$

$= x^2 - 2xy + 2yx - 4y^2$

$= x^2 - 4y^2$

When x = 1, y = 0, we get

$(x + 2y)(x - 2y)$

$= x^2 - 4y^2 = (1)^2 - 4 \times (0)^2 = 1 - 0 = 1.$

(ii) We have,

$(3m - 2n)(2m - 3n)$

$= 3m(2m - 3n) - 2n(2m - 3n)$

$= 3m \times 2m - 3m \times 3n - 2n \times 2m + 2n \times 3n$

$= 6m^2 - 9mn - 4mn + 6n^2$

$= 6m^2 - 13mn + 6n^2$

When m = 1, n = –1, we get

$(3m - 2n)(2m - 3n)$

$= 6m^2 - 13mn + 6n^2$

$= 6 \times (1)^2 - 13 \times 1 \times (-1) + 6 \times (-1)^2$

$= 6 + 13 + 6 = 25$

(iii) We have

$(4a^2 + 3b)(4a^2 + 3b)$

$= 4a^2 \times (4a^2 + 3b) + 3b \times (4a^2 + 3b)$

$= 4a^2 \times 4a^2 + 4a^2 \times 3b + 3b \times 4a^2 + 3b \times 3b$

$= 16a^4 + 12a^2b + 12a^2b + 9b^2$

$= 16a^4 + 24a^2b + 9b^2$

When, a = 1, b = 2, we get

$(4a^2 + 3b)(4a^2 + 3b)$

$= 16a^4 + 24a^2b + 9b^2$

$= 16 \times (1)^4 + 24 \times (1)^2 \times 2 + 9 \times (2)^2$

$= 16 + 48 + 36 = 100$

Ex.26 Simplify the following :

(i) $(2x + 5)(3x - 2) + (x + 2)(2x - 3)$

(ii) $(3x + 2)(2x + 3) - (4x - 3)(2x - 1)$

(iii) $(2x + 3y)(3x + 4y) - (7x + 3y)(x + 2y)$

Sol. (i) We have,

$(2x + 5)(3x - 2) + (x + 2)(2x - 3)$

$= 2x(3x - 2) + 5(3x - 2) + x(2x - 3) + 2(2x - 3)$

$= 6x^2 - 4x + 15x - 10 + 2x^2 - 3x + 4x - 6$

$= (6x^2 + 2x^2) + (-4x + 15x - 3x + 4x) + (-10 - 6)$

$= 8x^2 + 12x - 16$

(ii) We have,

$(3x + 2)(2x + 3) - (4x - 3)(2x - 1)$

$= \{3x(2x+3) + 2(2x+3)\} - \{4x(2x-1) - 3(2x-1)\}$

$= (6x^2 + 9x + 4x + 6) - (8x^2 - 4x - 6x + 3)$

$= (6x^2 + 13x + 6) - (8x^2 - 10x + 3)$

$6x^2 + 13x + 6 - 8x^2 + 10x - 3$

$= -2x^2 + 23x + 3$

(iii) We have,

$(2x + 3y)(3x + 4y) - (7x + 3y)(x + 2y)$

$= \{2x(3x + 4y) + 3y(3x + 4y) - 7x(x + 2y) + 3y(x + 2y)\}$

$= (6x^2 + 8xy + 9xy + 12y^2) - (7x^2 + 14xy + 3xy + 6y^2)$

$= (6x^2 + 17xy + 12y^2) - (7x^2 + 17xy + 6y^2)$

$= 6x^2 + 17xy + 12y^2 - 7x^2 - 17xy - 6y^2$

$= 6x^2 - 7x^2 + 17xy - 17xy + 12y^2 - 6y^2$

$= -x^2 + 6y^2.$

Ex.27 Multiply : $(2x^2 - 3x + 5)$ by $(5x + 2)$.

Sol. Horizontal method:

We have,

$(2x^3 - 3x + 5) \times (5x + 2)$

$= (2x^2 - 3x + 5) \times 5x + (2x^2 - 3x + 5) \times 2$

$= (10x^3 - 15x^2 + 25x) + (4x^2 - 6x + 10)$

$= 10x^3 - 11x^2 + 19x + 10$

Column Method:

We have,

$2x^2 - 3x + 5$	
$\times \quad 5x + 2$	
$10x^3 - 15x^2 + 25x$	Multiplying $2x^2 - 3x + 5$ by 5x
$+ 4x^2 - 6x + 10$	Multiplying $2x^2 - 3x + 5$ by 2
$10x^3 - 11x^2 + 19x + 10$	Adding the like terms

Ex.28 Simplify :

(i) $(3x - 2)(x - 1)(3x + 5)$

(ii) $(5 - x)(3 - 2x)(4 - 3x)$

Sol. (i) We have,

$(3x - 2)(x - 1)(3x + 5)$

$= \{(3x - 2)(x - 1)\} \times (3x + 5)$

[By Associativity of Multiplication]

$= \{3x(x - 1) - 2(x - 1)\} \times (3x + 5)$

$= (3x^2 - 3x - 2x + 2) \times (3x + 5)$

$= (3x^2 - 5x + 2) \times (3x + 5)$

$= 3x^2 \times (3x + 5) - 5x(3x + 5) + 2 \times (3x + 5)$

$= (9x^3 + 15x^2) + (-15x^2 - 25x) + (6x + 10)$

$= 9x^3 + 15x^2 - 15x^2 - 25x + 6x + 10$

$= 9x^3 - 19x + 10$

(ii) We have,

$(5 - x)(3 - 2x)(4 - 3x)$

$= \{(5 - x)(3 - 2x)\} \times (4 - 3x)$

$= \{5(3 - 2x) - x(3 - 2x)\} \times (4 - 3x)$

$= (15 - 10x - 3x + 2x^2) \times (4 - 3x)$

$= (2x^2 - 13x + 15) + (4 - 3x)$

$= 2x^2 \times (4 - 3x) \quad - 13x \times (4 - 3x) + 15 \times (4 - 3x)$

$= 8x^2 - 6x^3 - 52x + 39x^2 + 60 - 45x$

$= -6x^3 + 47x^2 - 97x + 60$

➤ IDENTITIES

Identities

- **Identity** An indentity is an equality which is true for all values of the variables (s).
- **Standard Identities :**
  Identity 1. $(a + b)^2 = a^2 + 2ab + b^2$
  Identity 2. $(a - b)^2 = a^2 - 2ab + b^2$
  Identity 3. $(a + b)(a - b) = a^2 - b^2$
  Other identity
  $(x + a)(x + b) = x^2 + x(a + b) + ab$

Ex.29 Evaluate :

(i) $(2x + 3y)^2$

(ii) $(2x - 3y)^2$

(iii) $(2x + 3y)(2x - 3y)$

Sol. (i) We have,

$(2x + 3y)^2 = (2x)^2 + 2 \times (2x) \times (3y) + (3y)^2$

[Using: $(a + b)^2 = a^2 + 2ab + b^2$]

$= 4x^2 + 12xy + 9y^2$

(ii) We have,

$(2x - 3y)^2 = (2x)^2 - 2 \times (2x) \times (3y) + (3y)^2$

[Using: $(a - b)^2 = a^2 - 2ab + b^2$]

$= 4x^2 - 12xy + 9y^2$

(iii) We have,

$(2x + 3y)(2x - 3y) = (2x)^2 - (3y)^2$

[Using : $(a + b)(a - b) = a^2 - b^2$]

$= 4x^2 - 9y^2$.

Ex.30 Write down the squares of each of the following binomials :

(i) $\left(x + \frac{a}{2}\right)$ (ii) $\left(5b - \frac{1}{2}\right)$ (iii) $\left(y + \frac{y^2}{2}\right)$

Sol. (i) We have,

$\left(x + \frac{a}{2}\right)^2 = x^2 + 2 \times x \times \frac{a}{2} + \left(\frac{a}{2}\right)^2$

[Using : $(a + b)^2 = a^2 + 2ab + b^2$]

$= x^2 + xa + \frac{a^2}{4}$

(ii) We have,

$\left(5b - \frac{1}{2}\right)^2 = (5b)^2 - 2 \times 5b \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$

[Using : $(a - b)^2 = a^2 - 2ab + b^2$]

$= 25b^2 - 5b + \frac{1}{4}$

(iii) We have,

$\left(y + \frac{y^2}{2}\right)^2 = y^2 + 2 \times y \times \frac{y^2}{2} + \left(\frac{y^2}{2}\right)^2$

$= y^2 + y^3 + \frac{y^4}{4}$

Ex.31 Find the product of the following binomials :

(i) $\left(\frac{4}{3}x^2 + 3\right)\left(\frac{4}{3}x^2 + 3\right)$

(ii) $\left(\frac{2}{3}x^2 + 5y^2\right)\left(\frac{2}{3}x^2 + 5y^2\right)$

Sol. (i) We have,

$$\left(\frac{4}{3}x^2+3\right)\left(\frac{4}{3}x^2+3\right)$$

$$=\left(\frac{4}{3}x^2+3\right)^2 \qquad [\because a.a = a^2]$$

$$=\left(\frac{4}{3}x^2\right)^2 + 2\times\frac{4}{3}x^2\times 3 + (3)^2$$

[Using : $(a + b)^2 = a^2 + 2ab + b^2$]

$$=\frac{16}{9}x^4 + 8x^2 + 9$$

(ii) We have,

$$\left(\frac{2}{3}x^2+5y^2\right)\left(\frac{2}{3}x^2+5y^2\right)$$

$$=\left(\frac{2}{3}x^2+5y^2\right)^2 \qquad [\because a.a = a^2]$$

$$=\left(\frac{2}{3}x^2\right)^2 + 2\times\frac{2}{3}x^2\times 5y^2 + (5y^2)^2$$

[Using : $(a + b)^2 = a^2 + 2ab + b^2$]

$$=\frac{4}{9}x^4 + \frac{20}{3}x^2y^2 + 25y^4$$

Ex.32 If $x+\frac{1}{x}=4$, find the values of

(i) $x^2+\frac{1}{x^2}$ (ii) $x^4+\frac{1}{x^4}$

Sol. (i) We have,

$$x+\frac{1}{x}=4$$

On squaring both sides, we get

$$\left(x+\frac{1}{x}\right)^2=4^2$$

$$\Rightarrow x^2+2\times x\times\frac{1}{x}+\left(\frac{1}{x}\right)^2=16$$

$$\Rightarrow x^2+2+\frac{1}{x^2}=16$$

$$\Rightarrow x^2+\frac{1}{x^2}=16-2$$

[On transposing 2 on RHS]

$$\Rightarrow x^2+\frac{1}{x^2}=14$$

(ii) We have,

$$x^2+\frac{1}{x^2}=14$$

On squaring both sides, we get

$$\left(x^2+\frac{1}{x^2}\right)^2=14^2$$

$$\Rightarrow (x^2)^2+\left(\frac{1}{x^2}\right)^2+2\times x^2\times\frac{1}{x^2}=196$$

$$\Rightarrow x^4+\frac{1}{x^4}+2=196$$

$$\Rightarrow x^4+\frac{1}{x^4}=196-2$$

[On transposing 2 on RHS]

$$\Rightarrow x^4+\frac{1}{x^4}=194$$

Ex.33 If $x-\frac{1}{x}=9$, find the value of $x^2+\frac{1}{x^2}$.

Sol. We have,

$$x-\frac{1}{x}=9$$

On squaring both sides, we get

$$\left(x-\frac{1}{x}\right)^2=81$$

$$\Rightarrow x^2-2\times x\times\frac{1}{x}+\left(\frac{1}{x}\right)^2=81$$

$$\Rightarrow x^2-2+\frac{1}{x^2}=81$$

$$\Rightarrow x^2+\frac{1}{x^2}=81+2$$

[On transposing – 2 on RHS]

$$\Rightarrow x^2+\frac{1}{x^2}=83$$

Ex.34 If $x^2+\frac{1}{x^2}=27$, find the values of each of the following :

(i) $x+\frac{1}{x}$ (ii) $x-\frac{1}{x}$

Sol. (i) We have,

$$\left(x+\frac{1}{x}\right)^2 = x^2 + 2 \times x \times \frac{1}{x} + \frac{1}{x^2}$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = 27 + 2$$

$$\left[\Theta\ x^2 + \frac{1}{x^2} = 27(\text{given})\right]$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = 29$$

$$\Rightarrow x + \frac{1}{x} = \pm\sqrt{29}$$

[Taking square root of both sides]

(ii) We have,

$$\left(x-\frac{1}{x}\right)^2 = x^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2}$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2 = 27 - 2$$

$$\left[\Theta\ x^2 + \frac{1}{x^2} = 27(\text{given})\right]$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2 = 25$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2 = 5^2$$

$$\Rightarrow x - \frac{1}{x} = \pm 5$$

Ex.35 If $3x + 2y = 12$ and $xy = 6$, find the value of $9x^2 + 4y^2$.

Sol. We have,

$$(3x + 2y)^2 = (3x)^2 + (2y)^2 + 2 \times 3x \times 2y$$

$$\Rightarrow (3x + 2y)^2 = 9x^2 + 4y^2 + 12xy$$

$$\Rightarrow 12^2 = 9x^2 + 4y^2 + 12 \times 6$$

[Putting $3x + 2y = 12$ and $xy = 6$]

$$\Rightarrow 144 = 9x^2 + 4y^2 + 72$$

$$\Rightarrow 144 - 72 = 9x^2 + 4y^2$$

$$\Rightarrow 9x^2 + 4y^2 = 72$$

Ex.36 If $4x^2 + y^2 = 40$ and $xy = 6$, find the value of $2x + y$.

Sol. We have,

$$(2x + y)^2 = (2x)^2 + y^2 + 2 \times 2x \times y$$

$$\Rightarrow (2x + y)^2 = (4x^2 + y^2) + 4xy$$

$$\Rightarrow (2x + y)^2 = 40 + 4 \times 6$$

[Using $4x^2 + y^2 = 40$ and $xy = 6$]

$$\Rightarrow (2xy + y)^2 = 64 \Rightarrow 2x + y = \pm\sqrt{64}$$

$$\Rightarrow 2x + y = \pm 8$$

[Taking square root of both sides]

Ex.37 Find the continued product :

(i) $(x + 2)(x - 2)(x^2 + 4)$

(ii) $(2x + 3y)(2x - 3y)(4x^2 + 9y^2)$

(iii) $(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$

(iv) $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

(v) $\left(x-\frac{y}{5}-1\right)\left(x+\frac{y}{5}+1\right)$

Sol. (i) We have,

$(x + 2)(x - 2)(x^2 + 4)$

$= \{(x + 2)(x - 2)\}(x^2 + 4)$

[By associativity of multiplication]

$= (x^2 - 2^2)(x^2 + 4)$ [Θ $(a + b)(a - b) = a^2 - b^2$]

$= (x^2 - 4)(x^2 + 4)$

$= (x^2)^2 - 4^2$ [Θ $(a + b)(a - b) = a^2 - b^2$]

$= x^4 - 16$

(ii) We have,

$(2x + 3y)(2x - 3y)(4x^2 + 9y^2)$

$= \{(2x + 3y)(2x - 3y)\}(4x^2 + 9y^2)$

$= \{(2x + 3y)(2x - 3y)\}(4x^2 + 9y^2)$

$= \{(2x)^2 - (3y)^2\}(4x^2 + 9y^2)$

[Using : $(a + b)(a - b) = a^2 - b^2$]

$= (4x^2 - 9y^2)(4x^2 + 9y^2)$

$= (4x^2)^2 - (9y^2)^2$

[Using : $(a + b)(a - b) = a^2 - b^2$]

$= 16x^4 - 81y^4$.

(iii) We have,

$(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$

$= \{(x - 1)(x + 1)\}(x^2 + 1)(x^4 + 1)$

$= (x^2 - 1)(x^2 + 1)(x^4 + 1)$

$= \{(x^2 - 1)(x^2 + 1)\} + (x^4 + 1)$

$= \{(x^2)^2 - 1^2\}(x^4 + 1)$

$= (x^4 - 1)(x^4 + 1)$

$= \{(x^4)^2 - 1^2\}$

$= x^8 - 1$

(iv) We have

$\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right)$

$= \left\{\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\right\}\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right)$

$= \left(x^2 - \frac{1}{x^2}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right)$

$= \left\{(x^2)^2 - \left(\frac{1}{x^2}\right)^2\right\}\left(x^4 + \frac{1}{x^4}\right)$

$= \left(x^4 - \frac{1}{x^4}\right)\left(x^4 + \frac{1}{x^4}\right)$

$= (x^4)^2 - \left(\frac{1}{x^4}\right)^2$

$= x^8 - \frac{1}{x^8}$

(v) We have,

$\left(x - \frac{y}{5} - 1\right)\left(x + \frac{y}{5} + 1\right)$

$= \left\{x - \left(\frac{y}{5} + 1\right)\right\}\left\{x + \left(\frac{y}{5} + 1\right)\right\}$

$= x^2 - \left(\frac{y}{5} + 1\right)^2$

$= x^2 - \left(\frac{y^2}{25} + \frac{2y}{5} + 1\right)$

$= x^2 - \frac{y^2}{25} - \frac{2y}{5} - 1$

Ex.38 Using the formulae for squaring a binomial, evaluate the following :

(i) $(101)^2$ (ii) $(99)^2$ (iii) $(93)^2$

Sol. We have,

(i) $(101)^2 = (100 + 1)^2$

$= (100)^2 + 2 \times 100 \times 1 + (1)^2$

[Using : $(a + b)^2 = a^2 + 2ab + b^2$]

$= 10000 + 200 + 1$

$= 10201$

(ii) $(99)^2 = (100 - 1)^2$

$= (100)^2 - 2 \times 100 \times 1 + (1)^2$

[Using : $(a - b)^2 = a^2 - 2ab + b^2$]

$= 10000 - 200 + 1$

$= 9801$

(iii) $(93)^2 = (90 + 3)^2$

$= (90)^2 + 2 \times 90 \times 3 + (3)^2$

$= 8100 + 540 + 9 = 8649$

Ex.39 Find the value of x, if

(i) $6x = 23^2 - 17^2$ (ii) $4x = 98^2 - 88^2$

(iii) $25x = 536^2 - 136^2$

Sol. (i) We have,

$6x = 23^2 - 17^2$

$\Rightarrow 6x = (23 + 17) \times (23 - 17)$

[Using : $a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow 6x = 40 \times 6$

$\Rightarrow \frac{6x}{6} = \frac{40 \times 6}{6}$ [Dividing both sides by 6]

$\Rightarrow x = 40$

(ii) We have,

$4x = 98^2 - 88^2$

$\Rightarrow 4x = (98 + 88) \times (98 - 88)$

[Using : $a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow 4x = 186 \times 10$

$\Rightarrow \frac{4x}{4} = \frac{186 \times 10}{4}$ [Dividing both sides by 4]

$\Rightarrow x = \frac{1860}{4}$

$\Rightarrow x = 465$

(iii) We have,

$25x = 536^2 - 136^2$

$\Rightarrow 25x = (536 + 136) \times (536 - 136)$

[Using : $(a^2 - b^2) = (a + b)(a - b)$]

$\Rightarrow 25x = 672 \times 400$

$\Rightarrow \frac{25x}{25} = \frac{672 \times 400}{25}$ [Dividing both sides by 25]

$\Rightarrow x = 672 \times 16$

$\Rightarrow x = 10752$

Ex.40 What must be added to $9x^2 - 24x + 10$ to make it a whole square ?

Sol. We have,

$9x^2 - 24x + 10 = (3x)^2 - 2 \times 3x \times 4 + 10$

It is evident from the above expression that

First term = 3x and , Second term = 4

To make the given expression a whole square, we must have $(4)^2 = 16$ in place of 10.

Hence, we must add 6 to it to make a perfect square.

Adding and subtracting 6, we get

$9x^2 - 24x + 10 + 6 - 6 = 9x^2 - 24x + 16 - 6$

$= (3x - 4)^2 - 6$

Ex.41 Find the following products:

(i) $(x + 2)(x + 3)$

(ii) $(x + 7)(x - 2)$

(iii) $(y - 4)(y - 3)$

(iv) $(y - 7)(y + 3)$

(v) $(2x - 3)(2x + 5)$

(iv) $(3x + 4)(3x - 5)$

Sol. Using the identity :

$(x + a)(x + b) = x^2 + (a + b)x + ab$, we have

(i) $(x + 2)(x + 3) = x^2 + (2 + 3)x + 2 \times 3$

$= x^2 + 5x + 6$

(ii) $(x + 7)(x - 2) = (x + 7)\{x + (-2)\}$

$= x^2 + \{7 + (-2)\}x + 7 \times 2$

$= x^2 + 5x - 14$

(iii) $(y - 4)(y - 3) = \{y + (-4)\}\{y + (-3)\}$

$= y^2 + \{(-4) + (-3)\}y + (-4) \times (-3)\}$

$= y^2 - 7y + 12$

(iv) $(y - 7)(y + 3) = \{y + (-7)\}(y + 3)$

$= y^2 + \{(-7) + 3\} y + (-7) \times 3$

$= y^2 - 4y - 21$

(v) $(2x - 3)(2x + 5) = (y - 3)(y + 5)$,

where $y = 2x$

$= \{y + (-3)\}(y + 5)$

$= y^2 + \{(-3) + 5\}y + (-3) \times 5$

$= y^2 + 2y - 15$

Ex.42 Evaluate the following:

(i) 107×103 (ii) 56×48 (iii) 95×97

Sol. Using the identity :

$(x + a)(x + b) = x^2 + (a + b)x + ab$ we have

(i) $107 \times 103 = (100 + 7) \times (100 + 3)$

$= (100)^2 + (7 + 3) \times 100 + 7 \times 3$

$= 10000 + 10 \times 100 + 21$

$= 10000 + 1000 + 21$

$= 11021$

(ii) $56 \times 48 = (50 + 6) \times (50 - 2)$

$= (50 + 6) \times \{50 + (-2)\}$

$= (50)^2 + \{6 + (-2)\} \times 50 + 6 \times -2$

$= 2500 + 4 \times 50 - 12$

$= 2500 + 200 - 12$

$= 2700 - 12 = 2688$

(iii) $95 \times 97 = (100 - 5) \times (100 - 3)$

$= \{100 + (-5)\} \times \{100 + (-3)\}$

$= (100)^2 + \{(-5) + (-3)\} \times 100 + (-5) \times (-3)$

$= 10000 - 8 \times 100 + 15$

$= 10000 - 800 + 15 = 9215$

EXERCISE # 1

Q.1 Add the following algebraic expressions:

$2, \frac{2y}{3} - \frac{5y^2}{3} + \frac{5y^3}{2}, -\frac{4}{3} + \frac{2y^2}{3} - \frac{y}{2},$

$\frac{5y^3}{3} + 3y^2 + 3y + \frac{6}{5}$

Q.2 Subtract : $\left(-2y^2 + \frac{1}{2}y - 3\right)$ from $7y^2 - 2y + 10$.

Q.3 Subtract: $\frac{3}{2}x^2y + \frac{4}{5}y - \frac{1}{3}x^2yz$ from

$\frac{12}{5}x^2yz - \frac{3}{5}xyz + \frac{2}{3}x^2y$.

Q.4 Find the volume of the rectangular boxes with following length, breadth and height :

	Length	Breadth	Height
(i)	2ax	3by	5cz
(ii)	m^2n	n^2p	p^2m
(iii)	2q	$4q^2$	$8q^3$

Q.5 Find each of the following products:

(i) $(-2x^2) \times (7a^2x^7) \times (6a^5x^5)$

(ii) $(4s^2t) \times (3s^3t^3) \times (2st^4) \times (-2)$

Q.6 Multiply $-\frac{4}{3}xy^3$ by $\frac{6}{7}x^2y$ and verify your result for x = 2 and y = 1.

Q.7 Find the product of $-5x^2y$, $-\frac{2}{3}xy^2z$, $\frac{8}{15}xyz^2$ and $-\frac{1}{4}$. Verify the result when x = 1, y = 2 and y = q.

Q.8 Find the product of $\frac{7}{2}s^2t$ and s + t. Verify the result for $s = \frac{1}{2}$ and t = 5.

Q.9 Find the following products:

(i) $100x \times (0.01x^4 - 0.01x^2)$

(ii) $121.5ab \times \left(ac + \frac{b}{10}\right)$

(iii) $0.1a \times (0.01a \times 0.001b)$

Q.10 Add:

(i) $5m(3 - m)$ and $6m^2 - 13m$

(ii) $4y(3y^2 + 5y - 7)$ and $2(y^3 - 4y^2 + 5)$

Q.11 (i) Subtract: $3l\,(l - 4m + 5n)$ from $4l\,(10n - 3m + 2l)$

(ii) Subtract : $3a\,(a + b + c) - 2b\,(a + b + c)$ from $4c(-a + b + c)$

Q.12 Multiply $\left(\frac{1}{5}x - \frac{1}{4}y\right)$ and $(5x^2 - 4y^2)$

Q.13 Multiply $(3x^2 + y^2)$ by $(x^2 + 2y^2)$.

Q.14 Multiply: $\{2m + (-n)\}$ by $\{-3m + (-5)\}$

Q.15 Find the product of $\left(y + \frac{2}{7}y^2\right)$ and $(7y - y^2)$ and verify the result for y = 3.

Q.16 Simplify the following:

(i) $\frac{1}{3}(6x^2 + 15y^2)(6x^2 - 15y^2)$

(ii) $9x^4(2x^3 - 5x^4) \times 5x^6(x^4 - 3x^2)$

Q.17 Multiply: $(2x^2 - 4x + 5)$ by $(x^2 + 3x - 7)$

Q.18 Find the product of the following binomials:

(i) $(6x^2 - 7y^2)(6x^2 - 7y^2)$

(ii) $\left(\frac{1}{2}x - \frac{1}{5}y\right)\left(\frac{1}{2}x - \frac{1}{5}y\right)$

Q.19 Find the product of the following binomials:

(i) $\left(\frac{3}{4}x + \frac{5}{6}y\right)\left(\frac{3}{4}x - \frac{5}{6}y\right)$

(ii) $\left(2a + \frac{3}{b}\right)\left(2a - \frac{3}{b}\right)$

(iii) $(a^2 + b^2)(-a^2 + b^2)$

(iv) $(-a + c)(-a - c)$

Q.20 If $x + \frac{1}{x} = 9$ and $x^2 + \frac{1}{x^2} = 53$, find the value of $x - \frac{1}{x}$.

Q.21 If $x + y = 12$ and $xy = 14$, find the value of $x^2 + y^2$.

Q.22 Simplify the following products:

(i) $(x^2 + x + 1)(x^2 - x + 1)$

(ii) $(x^2 + 2x + 2)(x^2 - 2x + 2)$

Q.23 Simplify the following by using: $(a + b)(a - b) = a^2 - b^2$.

(i) 68×72 (ii) 101×99

(iii) 67×73 (iv) $128^2 - 77^2$

Q.24 Find the greatest common factor of the monomials $6x^3a^2b^2c$, $8x^2ab^3c^3$ and $12a^3b^2c^2$.

ANSWER KEY

EXERCISE # 1

1. $\frac{28}{15} + \frac{19}{6}y + 2y^2 + \frac{25}{6}y^3$

2. $9y^2 - \frac{5}{2}y + 13$

3. $\frac{41}{15}x^2yz - \frac{5}{6}x^2y - \frac{3}{5}xyz - \frac{4}{5}y$

4. (i) 30 abcxyz (ii) $m^3n^3p^3$ (iii) $64q^6$

5. (i) $-84x^{14}a^7$ (ii) $-48s^6t^8$ (iii) $1000x^{14}y^{11}$

6. $-\frac{8}{7}x^3y^4$

7. $-\frac{4}{9}x^4y^4z^4$

8. $\frac{7}{2}s^3t + \frac{7}{2}s^2t^2$

9. (i) $x^5 - x^3$
(ii) $121.5a^2bc + 12.15ab^2$
(iii) $0.001a^2 + 0.0001ab$

10. (i) $2m + m^2$

11. (i) $25ln + 5l^2$
(ii) $-7ac + 6bc + 4c^2 - 3a^2 - ab - 2b^2$

12. $x^3 - \frac{4}{5}xy^2 - \frac{5}{4}x^2y + y^3$

13. $3x^4 + 7x^2y^2 + 2y^4$

15. $7y^2 + y^3 - \frac{2}{7}y^4$

16. (i) $12x^4 - 75y^4$
(ii) $-225x^{18} + 90x^{17} + 675x^{16} - 270x^{15}$

17. $2x^4 + 2x^3 - 21x^2 + 43x - 35$

18. (i) $36x^4 - 84^2y^2 + 49y^4$ (ii) $\frac{1}{4}x^4 - \frac{xy}{5} + \frac{1}{25}y^2$

19. (i) $\frac{9}{16}x^2 - \frac{25}{36}y^2$ (ii) $4a^2 - \frac{9}{b^2}$
(iii) $b^4 - a^4$ (iv) $a^2 - c^2$

20. ± 5

21. 116

22. (i) $x^4 + x^2 + 1$, (ii) $x^4 - 2x^2 + 4$

23. (i) 4896 (ii) 9999 (iii) 4891 (iv) 10455

EXERCISE # 2

Q.1 If $\left(x+\frac{1}{x}\right)=3$, then find value of $\left(x^2+\frac{1}{x^2}\right)$.

Q.2 If $\left(x-\frac{1}{x}\right)=\frac{1}{2}$, then find value of $\left(4x^2+\frac{4}{x^2}\right)$.

Q.3 If $\left(x+\frac{1}{x}\right)=4$, then find value of $\left(x^4+\frac{1}{x^4}\right)$.

Q.4 If $\left(x+\frac{1}{x}\right)=\sqrt{3}$, then find the value of $\left(x^3+\frac{1}{x^3}\right)$

Q.5 If $\left(x+\frac{1}{x}\right)=2$, then find the value of $\left(x^3+\frac{1}{x^3}\right)$

Q.6 If $\left(x^2+\frac{1}{x^2}\right)=102$, then find the value of $\left(x-\frac{1}{x}\right)$

Q.7 If $\left(x^4+\frac{1}{x^4}\right)=322$, then find the value of $\left(x-\frac{1}{x}\right)$

Q.8 If $\left(x^3+\frac{1}{x^3}\right)=52$, then find the value of $\left(x+\frac{1}{x}\right)$

Q.9 If $\left(x^3-\frac{1}{x^3}\right)=14$, then find the value of $\left(x-\frac{1}{x}\right)$.

Q.10 If x is an integer such that $\left(x+\frac{1}{x}\right)=\left(\frac{17}{4}\right)$, then find the value of $\left(x-\frac{1}{x}\right)$

Q.11 If $\left(x^4+\frac{1}{x^4}\right)=727$, then find the value of $\left(x^3-\frac{1}{x^3}\right)$

Q.12 If $\left(2x-\frac{3}{x}\right)=5$, then find the value of $\left(4x^2-\frac{9}{x^2}\right)$

Q.13 If $x+y=7$ and $xy=12$, then find the value of (x^2+y^2)

Q.14 If $\frac{5^x}{125}=1$, then find the value of x

Q.15 Find the values of -

(i) 998^2	(ii) 5.2^2
(iii) 78×82	(iv) 1.05×9.5
(v) 51^2-49^2	(vi) $(1.02)^2-(0.98)^2$
(vii) $12.1^2-7.9^2$	(viii) 103×104
(ix) 5.1×5.2	(x) 9.7×9.8

Q.16 Show that

(i) $(3x+7)^2-84x=(3x-7)^2$

(ii) $(9p-5q)^2+180pq=(9p+5q)^2$

(iii) $\left(\frac{4}{3}m-\frac{3}{4}n\right)^2+2mn=\frac{16}{9}m^2+\frac{9}{16}n^2$

(iv) $(4pq+3q)^2-(4pq-3q)^2=48pq^2$

(v) $(a-b)(a+b)+(b-c)(b+c)+(c-a)(c+a)=0$

ANSWER KEY

EXERCISE # 2

1. 7

2. 9

3. 194

4. 0

5. 2

6. 10

7. 4

8. 4

9. 2

10. 15/4

11. 140

12. 35

13. 25

14. 3

15. (i) 996004 (ii) 27.04 (iii) 6396 (iv) 9.975 (v) 200 (vi) 0.08 (vii) 84
(viii) 10712 (ix) 26.52 (x) 95.06