Chapter

8

Determinants and Matrices

Contents

G. Cramer

The theory of determinants may be said to have begun with G.W. Leibnitz (1646-1716) *who gave a rule for the solution of simultaneous linear equations equivalent to that of the Chinese. This rule was simplified by* G. Cramer (1704-1752), *a Swiss mathematician, in* 1750. E. Bezout (1730-1783), *a French mathematician simplified it further in* 1764. *However, A.T. Vandermonde (1735-1796) gave the first systematic account of determinants in* 1771. P.S. Laplace (1749-1827), *in* 1772, *gave the general method of expanding a determinant in terms of minors. In* 1773, J.L,. Lagrange (1736-1813) *treated determinants fo order 2 and 3 and used them for purposes other than the solution of equations.* Carl F. Gauss (1777-1855) *used determinants in the theory of numbers.*

J.J. Sylvester, *who was very fond of assigning imaginative names to his creations and inventions, wrote down in 1850, certain terms and expressions in the form of a rectangular arrangement, Sylvester gave this rectangular arrangement the name 'matrix'. In particular, the names of* Sir William Rowan Hamilton, (1805-1865) *and Arthur Cayley deserve special mention. Sir Hamilton, in* 1853*, and Arthur Cayley, in 1858, made significant contributions to the theory of matrices.*

8.1 Determinants

8.1.1 Definition

(1) Consider two equations, $a_1x+b_1y=0$ .....(i) and $a_2x+b_2y=0$ .....(ii)

Multiplying (i) by b_2 and (ii) by b_1 and subtracting, dividing by *x*, we get, $a_1b_2-a_2b_1=0$

The result $a_1b_2-a_2b_1$ is represented by $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

Which is known as determinant of order two and $a_1b_2-a_2b_1$ is the expansion of this determinant. The horizontal lines are called rows and vertical lines are called columns.

Now let us consider three homogeneous linear equations

$$a_1x+b_1y+c_1z=0,\ a_2x+b_2y+c_2z=0 \text{ and } a_3x+b_3y+c_3z=0$$

Eliminated *x, y, z* from above three equations we obtain

$$a_1(b_2c_3-b_3c_2)-b_1(a_2c_3-a_3c_2)+c_1(a_2b_3-a_3b_2)=0 \quad .....(iii)$$

The L.H.S. of (iii) is represented by $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Its contains three rows and three columns, it is called a determinant of third order.

Note : ❑ The number of elements in a second order is $2^2=4$ and the number of elements in a third order determinant is $3^2=9$.

(2) **Rows and columns of a determinant :** In a determinant horizontal lines counting from top 1st, 2nd, 3rd,..... respectively known as rows and denoted by $R_1, R_2, R_3, ...$ and vertical lines counting left to right, 1st, 2nd, 3rd,..... respectively known as columns and denoted by $C_1, C_2, C_3, ...$

(3) **Shape and constituents of a determinant :** Shape of every determinant is square. If a determinant of *n* order then it contains *n* rows and *n* columns.

i.e., Number of constituents in determinants = n^2

(4) **Sign system for expansion of determinant :** Sign system for order 2, order 3, order 4,..... are given by $\begin{vmatrix} + & - \\ - & + \end{vmatrix}$, $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$, $\begin{vmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{vmatrix}$,.....

8.1.2 Expansion of Determinants

Unlike a matrix, determinant is not just a table of numerical data but (quite differently) a short hand way of writing algebraic expression, whose value can be computed when the values of terms or elements are known.

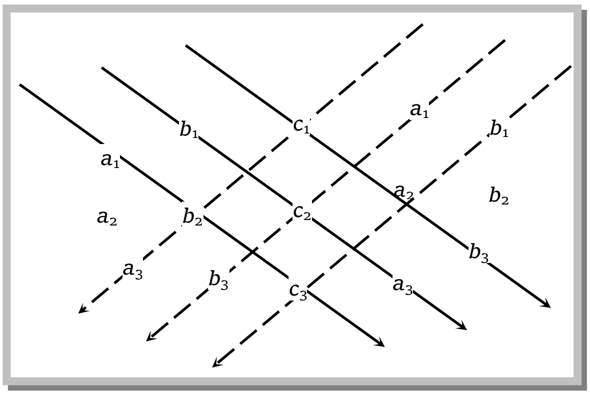
(1) The 4 numbers a_1, b_1, a_2, b_2 arranged as $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is a determinant of second order. These numbers are called elements of the determinant. The value of the determinant is defined as $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$.

The expanded form of determinant has 2! terms.

(2) The 9 numbers $a_r, b_r, c_r\ (r = 1, 2, 3)$ arranged as $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is a determinant of third order. Take any row (or column); the value of the determinant is the sum of products of the elements of the row (or column) and the corresponding determinant obtained by omitting the row and the column of the element with a proper sign, given by the rule $(-1)^{i+j}$, where i and j are the number of rows and the number of columns respectively of the element of the row (or the column) chosen. Thus $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

The diagonal through the left-hand top corner which contains the element a_1, b_2, c_3 is called the leading diagonal or principal diagonal and the terms are called the leading terms. The expanded form of determinant has 3! terms.

Short cut method or Sarrus diagram method : To find the value of third order determinant, following method is also useful



Taking product of R.H.S. diagonal elements positive and L.H.S. diagonal elements negative and adding them. We get the value of determinant as $= a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - c_1b_2a_3 - a_1c_2b_3 - b_1a_2c_3$

Note : ❑ This method does not work for determinants of order greater than three.

8.1.3 Evaluation of Determinants

If A is a square matrix of order 2, then its determinant can be easily found. But to evaluate determinants of square matrices of higher orders, we should always try to introduce zeros at maximum number of places in a particular row (column) by using the properties and then we should expand the determinant along that row (column).

We shall be using the following notations to evaluate a determinant :

(1) R_i to denote i^{th} row.

(2) $R_i \leftrightarrow R_j$ to denote the interchange of i^{th} and j^{th} rows.

(3) $R_i \to R_i + \lambda R_j$ to denote the addition of λ times the elements of j^{th} row to the corresponding elements of i^{th} row.

(4) $R_i(\lambda)$ to denote the multiplication of all element of i^{th} row by λ.

Similar notations are used to denote column operations if R is replaced by C.

8.1.4 Properties of Determinants

P-1 : The value of determinant remains unchanged, if the rows and the columns are interchanged.

If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$. Then $D' = D$, D and D' are transpose of each other.

Note : ❑ Since the determinant remains unchanged when rows and columns are interchanged, it is obvious that any theorem which is true for 'rows' must also be true for 'columns'.

P-2 : If any two rows (or columns) of a determinant be interchanged, the determinant is unaltered in numerical value but is changed in sign only.

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$. Then $D' = -D$

P-3 : If a determinant has two rows (or columns) identical, then its value is zero.

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$. Then, $D = 0$

P-4 : If all the elements of any row (or column) be multiplied by the same number, then the value of determinant is multiplied by that number.

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$. Then $D' = kD$

P-5 : If each element of any row (or column) can be expressed as a sum of two terms, then the determinant can be expressed as the sum of the determinants.

e.g., $\begin{vmatrix} a_1+x & b_1+y & c_1+z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

P-6 : The value of a determinant is not altered by adding to the elements of any row (or column) the same multiples of the corresponding elements of any other row (or column)

e.g., $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} a_1+ma_2 & b_1+mb_2 & c_1+mc_2 \\ a_2 & b_2 & c_2 \\ a_3-na_1 & b_3-nb_1 & c_3-nc_1 \end{vmatrix}$. Then $D' = D$

Note : ❑ It should be noted that while applying **P-6** at least one row (or column) must remain unchanged.

P-7 : If all elements below leading diagonal or above leading diagonal or except leading diagonal elements are zero then the value of the determinant equal to multiplied of all leading diagonal elements.

e.g., $\begin{vmatrix} a_1 & b_1 & c_1 \\ 0 & b_2 & c_2 \\ 0 & 0 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{vmatrix} = a_1b_2c_3$

P-8 : If a determinant D becomes zero on putting $x=\alpha$, then we say that $(x-\alpha)$ is factor of determinant.

e.g., if $D = \begin{vmatrix} x & 5 & 2 \\ x^2 & 9 & 4 \\ x^3 & 16 & 8 \end{vmatrix}$. At $x=2$, $D=0$ (because C_1 and C_2 are identical at $x=2$)

Hence $(x-2)$ is a factor of D.

Note : ❑ It should be noted that while applying operations on determinants then at least one row (or column) must remain unchanged.or, Maximum number of operations = order or determinant –1

❑ It should be noted that if the row (or column) which is changed by multiplied a non zero number, then the determinant will be divided by that number.

Example: 1 If $n \neq 3k$ and 1, ω, ω^2 are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$ has the value

[Pb. CET 1991; Rajasthan PET 2001; AIEEE 2003]

(a) 0 (b) ω (c) ω^2 (d) 1

Solution: (a) Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} 1+\omega^n+\omega^{2n} & \omega^n & \omega^{2n} \\ 1+\omega^n+\omega^{2n} & 1 & \omega^n \\ 1+\omega^n+\omega^{2n} & \omega^{2n} & 1 \end{vmatrix} = \begin{vmatrix} 0 & \omega^n & \omega^{2n} \\ 0 & 1 & \omega^n \\ 0 & \omega^{2n} & 1 \end{vmatrix} = 0 \qquad (\because 1+\omega^n+\omega^{2n} = 0 \text{ if } n \text{ is not multiple of } 3)$$

Example: 2 $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} =$ **[Rajasthan PET 1992; Kerala (Engg.) 2002]**

(a) $xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ (b) xyz (c) $1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ (d) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$

Solution: (a) $\Delta = xyz\begin{vmatrix} 1+\frac{1}{x} & \frac{1}{x} & \frac{1}{x} \\ \frac{1}{y} & 1+\frac{1}{y} & \frac{1}{y} \\ \frac{1}{z} & \frac{1}{z} & 1+\frac{1}{z} \end{vmatrix} = xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{y} & 1+\frac{1}{y} & \frac{1}{y} \\ \frac{1}{z} & \frac{1}{z} & 1+\frac{1}{z} \end{vmatrix}$ $(by\ R_1 \to R_1 + R_2 + R_3)$

$= xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{y} & 1 & 0 \\ \frac{1}{z} & 0 & 1 \end{vmatrix}$ $(by\ C_2 \to C_2 - C_1 \text{ and } C_3 \to C_3 - C_1)$

$= xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

Trick : Put $x = 1,\ y = 2$ and $z = 3$, then $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{vmatrix} = 2(11) - 1(3) + 1(1-3) = 17$.

option (a) gives $1 \times 2 \times 3\left(1+\frac{1}{1}+\frac{1}{2}+\frac{1}{3}\right) = 17$

Example: 3 The value of $\Delta = \begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix}$ is equal to zero, where m is

(a) 6 (b) 4 (c) 5 (d) None of these

Solution: (c) $\Delta = \begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$

Applying $C_2 \to C_1 + C_2$

$\Delta = \begin{vmatrix} {}^{10}C_4 & {}^{10}C_4 + {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_6 + {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_8 + {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0 \Rightarrow \Delta = \begin{vmatrix} {}^{10}C_4 & {}^{11}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{12}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{13}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$

Clearly $m = 5$ satisfies the above result $[\because C_2, C_3 \text{ will be identical}]$

Example: 4 If $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots\ldots$ are in $G.P.$ then the value of the determinant $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is

[AIEEE 2004; IIT 1993]

(a) –2 (b) 1 (c) 2 (d) 0

Solution: (d) $\because a_1, a_2, a_3, \ldots\ldots, a_n, \ldots\ldots$ are in $G.P.$

$\therefore a_{n+1}^2 = a_n . a_{n+2} \Rightarrow 2\log a_{n+1} = \log a_n + \log a_{n+2}$

$a_{n+4}^2 = a_{n+3} . a_{n+5} \Rightarrow 2\log a_{n+4} = \log a_{n+3} + \log a_{n+5}$

$a_{n+7}^2 = a_{n+6} . a_{n+8} \Rightarrow 2\log a_{n+7} = \log a_{n+6} + \log a_{n+8}$

Putting these values in the second column of the given determinant, we get

$\Delta = \frac{1}{2}\begin{vmatrix} \log a_n & \log a_n + \log a_{n+2} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+3} + \log a_{n+5} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+6} + \log a_{n+8} & \log a_{n+8} \end{vmatrix}$

$= \frac{1}{2}(0) = 0$ [$\because C_2$ is the sum of two elements, first identical with C_1 and second with C_3]

Example: 5 The value of $\begin{vmatrix} 1 & 1 & 1 \\ (2^x+2^{-x})^2 & (3^x+3^{-x})^2 & (5^x+5^{-x})^2 \\ (2^x-2^{-x})^2 & (3^x-3^{-x})^2 & (5^x-5^{-x})^2 \end{vmatrix}$

(a) 0 (b) 30^x (c) 30^{-x} (d) None of these

Solution: (a) Applying $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2.2^x.2.2^{-x} & 2.3^x.2.3^{-x} & 2.5^x.2.5^{-x} \\ (2^x-2^{-x})^2 & (3^x-3^{-x})^2 & (5^x-5^{-x})^2 \end{vmatrix} = 4\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ (2^x-2^{-x})^2 & (3^x-3^{-x})^2 & (5^x-5^{-x})^2 \end{vmatrix} = 0$$ [$\because R_1$ and R_2 are identical]

Trick : Putting $x = 0$, we get option (*a*) is correct

Example: 6 If *x, y, z* are integers in *A.P.* lying between 1 and 9 and *x*51, *y*41 and *z*31 are three digit numbers then the value of $\begin{vmatrix} 5 & 4 & 3 \\ x51 & y41 & z31 \\ x & y & z \end{vmatrix}$ is

(a) $x+y+z$ (b) $x-y+z$ (c) 0 (d) None of these

Solution: (c) $\because$ $x51 = 100x + 50 + 1$,

$y41 = 100y + 40 + 1$

$z31 = 100z + 30 + 1$

$$\therefore \quad \Delta = \begin{vmatrix} 5 & 4 & 3 \\ 100x+50+1 & 100y+40+1 & 100z+30+1 \\ x & y & z \end{vmatrix}$$

Applying $R_2 \to R_2 - 100R_3 - 10R_1$

$$\Delta = \begin{vmatrix} 5 & 4 & 3 \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = x - 2y + z$$

$\because$ *x, y, z* are in A.P. , $\therefore$ $x - 2y + z = 0$, $\therefore$ $\Delta = 0$

Example: 7 If $a \neq b \neq c$, the value of *x* which satisfies the equation $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$ is

[EAMCET 1988; Karnataka CET 1991; MNR 1980; MP PET 1988, 99, 2001; DCE 2001]

(a) $x = 0$ (b) $x = a$ (c) $x = b$ (d) $x = c$

Solution: (a) Expanding determinant, we get, $\Delta = -(x-a)[-(x+b)(x-c)] + (x+b)[(x+a)(x+c)] = 0 \Rightarrow 2x^3 - (2\Sigma ab)x = 0$

$\Rightarrow$ Either $x = 0$ or $x^2 = \Sigma ab$. Since $x = 0$ satisfies the given equation.

Trick : On putting $x = 0$, we observe that the determinant becomes $\Delta_{x=0} = \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$

$\therefore$ $x = 0$ is a root of the given equation.

Example: 8 The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is

(a) 0 (b) 2 (c) 1 (d) 3

Solution: (c) $(2\cos x+\sin x)\begin{vmatrix}1 & \cos x & \cos x\\ 1 & \sin x & \cos x\\ 1 & \cos x & \sin x\end{vmatrix}=0$

Applying, $R_2\to R_2-R_1$ and $R_3\to R_3-R_1$

$$(2\cos x+\sin x)\begin{vmatrix}1 & \cos x & \cos x\\ 0 & \sin x-\cos x & 0\\ 0 & 0 & \sin x-\cos x\end{vmatrix}=0 \Rightarrow (2\cos x+\sin x)(\sin x-\cos x)^2=0$$

$\therefore \tan x=-2,1$ But $\tan x\neq -2$ in $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$. Hence $\tan x=1\Rightarrow x=\frac{\pi}{4}$

Example: 9 If $p\lambda^4+q\lambda^3+r\lambda^2+s\lambda+t=\begin{vmatrix}\lambda^2+3\lambda & \lambda-1 & \lambda+3\\ \lambda+1 & 2-\lambda & \lambda-4\\ \lambda-3 & \lambda+4 & 3\lambda\end{vmatrix}$, then value of t is **[IIT 1981]**

(a) 16 (b) 18 (c) 17 (d) 19

Solution: (b) Since it is an identity in λ so satisfied by every value of λ. Now put $\lambda=0$ in the given equation, we have

$$t=\begin{vmatrix}0 & -1 & 3\\ 1 & 2 & -4\\ -3 & 4 & 0\end{vmatrix}=-12+30=18$$

Example: 10 If $f(x)=\begin{vmatrix}1 & x & (x+1)\\ 2x & x(x-1) & (x+1)x\\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1)\end{vmatrix}$, then $f(100)$ is equal to **[IIT 1999, MP PET 2000]**

(a) 0 (b) 1 (c) 100 (d) –100

Solution: (a) $f(x)=\begin{vmatrix}1 & x & (x+1)\\ 2x & x(x-1) & (x+1)x\\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1)\end{vmatrix}$

Applying $C_3\to C_3-C_2$, we get

$$f(x)=\begin{vmatrix}1 & x & 1\\ 2x & x(x-1) & 2x\\ 3x(x-1) & x(x-1)(x-2) & 3x(x-1)\end{vmatrix}=0$$. Hence $f(100)=0$

Example: 11 The value of $\begin{vmatrix}1 & 0 & 0 & 0 & 0\\ 2 & 2 & 0 & 0 & 0\\ 4 & 4 & 3 & 0 & 0\\ 5 & 5 & 5 & 4 & 0\\ 6 & 6 & 6 & 6 & 5\end{vmatrix}$ is

(a) 6! (b) 5! (c) $1.2^2.3.4^3.5^4.6^4$ (d) None of these

Solution: (b) The elements in the leading diagonal are 1, 2, 3, 4, 5. On one side of the leading diagonal all the elements are zero.

$\therefore$ The value of the determinant

= The product of the elements in the leading diagonal = 1.2.3.4.5 = 5!

Example: 12 The determinant $\begin{vmatrix}a & b & a\alpha+b\\ b & c & b\alpha+c\\ a\alpha+b & b\alpha+c & 0\end{vmatrix}=0$ if **[DCE 2000, 2001]**

(a) *a, b, c* are in A.P.

(b) *a, b, c* are in G.P. or $(x-\alpha)$ is a factor of $ax^2+2bx+c=0$

(c) *a, b, c* are in H.P.

(d) α is a root of the equation

Solution: (b) Applying $R_3 \to R_3 - \alpha R_1 - R_2$, we get $\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ 0 & 0 & -a\alpha^2 - b\alpha - b\alpha - c \end{vmatrix} = 0$

$\Rightarrow -(a\alpha^2 + 2b\alpha + c)(ac - b^2) = 0 \;\Rightarrow a\alpha^2 + 2b\alpha + c = 0$ or $b^2 = ac$

$\Rightarrow$ $x = \alpha$ is a root of $ax^2 + 2bx + c = 0$ or *a, b, c* are in G.P.

$\Rightarrow$ $(x - \alpha)$ is a factor of $ax^2 + 2bx + c = 0$ or *a, b, c* are in G.P.

8.1.5 Minors and Cofactors

(1) **Minor of an element :** If we take the element of the determinant and delete (remove) the row and column containing that element, the determinant left is called the minor of that element. It is denoted by M_{ij}

Consider the determinant $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, then determinant of minors $M = \begin{vmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{vmatrix}$,

where M_{11} = minor of $a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$, M_{12} = minor of $a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

M_{13} = minor of $a_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Similarly, we can find the minors of other elements . Using this concept the value of determinant can be

$\Delta = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$

or, $\Delta = -a_{21}M_{21} + a_{22}M_{22} - a_{23}M_{23}$ or, $\Delta = a_{31}M_{31} - a_{32}M_{32} + a_{33}M_{33}$.

(2) **Cofactor of an element :** The cofactor of an element a_{ij} (*i.e.* the element in the i^{th} row and j^{th} column) is defined as $(-1)^{i+j}$ times the minor of that element. It is denoted by C_{ij} or A_{ij} or F_{ij}.

$C_{ij} = (-1)^{i+j} M_{ij}$

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, then determinant of cofactors is $C = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$, where

$C_{11} = (-1)^{1+1} M_{11} = +M_{11}$, $C_{12} = (-1)^{1+2} M_{12} = -M_{12}$ and $C_{13} = (-1)^{1+3} M_{13} = +M_{13}$

Similarly, we can find the cofactors of other elements.

Note : ❑ The sum of products of the element of any row with their corresponding cofactor is equal to the value of determinant *i.e.* $\Delta = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$ $= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$

where the capital letters C_{11}, C_{12}, C_{13} *etc.* denote the cofactors of a_{11}, a_{12}, a_{13} *etc.*

- In general, it should be noted

  $a_{i1}C_{j1} + a_{i2}C_{j2} + a_{i3}C_{j3} = 0,\ \text{if } i \neq j$ or $a_{1i}C_{1j} + a_{2i}C_{2j} + a_{3i}C_{3j} = 0,\ \text{if } i \neq j$

- If Δ' is the determinant formed by replacing the elements of a determinant Δ by their corresponding cofactors, then if $\Delta = 0$, then $\Delta^C = 0$, $\Delta' = \Delta^{n-1}$, where n is the order of the determinant.

Example: 13 The cofactor of the element 4 in the determinant $\begin{vmatrix} 1 & 3 & 5 & 1 \\ 2 & 3 & 4 & 2 \\ 8 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \end{vmatrix}$ is **[MP PET 1987]**

(a) 4 (b) 10 (c) –10 (d) –4

Solution: (b) The cofactor of element 4, in the 2nd row and 3rd column is $(-1)^{2+3}\begin{vmatrix} 1 & 3 & 1 \\ 8 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$ = $-\{1(-2) - 3(8-0) + 1.16\} = 10$

Example: 14 If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and A_1, B_1, C_1 denote the cofactors of a_1, b_1, c_1 respectively, then the value of the determinant $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$ is

(a) Δ (b) Δ^2 (c) Δ^3 (d) 0

Solution: (b) We know that $\Delta.\Delta' = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} . \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = \begin{vmatrix} \Sigma a_1A_1 & 0 & 0 \\ 0 & \Sigma a_2A_2 & 0 \\ 0 & 0 & \Sigma a_3A_3 \end{vmatrix} = \begin{vmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{vmatrix} = \Delta^3$

$\Rightarrow \Delta' = \Delta^2$

Trick : According to property of cofactors $\Delta' = \Delta^{n-1} = \Delta^2$ $(\because \text{Hence } n = 3)$

Example: 15 If the value of a third order determinant is 11, then the value of the square of the determinant formed by the cofactors will be

(a) 11 (b) 121 (c) 1331 (d) 14641

Solution: (d) $\Delta' = \Delta^{n-1} = \Delta^{3-1} = \Delta^2 = (11)^2 = 121$. But we have to find the value of the square of the determinant, so required value is $(121)^2 = 14641$.

8.1.6 Product of two Determinants

Let the two determinants of third order be,

$D_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D_2 = \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}$. Let D be their product.

(1) **Method of multiplying (Row by row) :** Take the first row of D_1 and the first row of D_2 *i.e.* a_1, b_1, c_1 and $\alpha_1, \beta_1, \gamma_1$ multiplying the corresponding elements and add. The result is $a_1\alpha_1 + b_1\beta_1 + c_1\gamma_1$ is the first element of first row of *D*.

Now similar product first row of D_1 and second row of D_2 gives $a_1\alpha_2 + b_1\beta_2 + c_1\gamma_2$ is the second element of first row of *D*, and the product of first row D_1 and third row of D_2 gives $a_1\alpha_3 + b_1\beta_3 + c_1\gamma_3$ is the third element of first row of *D*. The second row and third row of *D* is obtained by multiplying second row and third row of D_1 with 1st , 2nd , 3rd row of D_2, in the above manner.

Hence,

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix} = \begin{vmatrix} a_1\alpha_1 + b_1\beta_1 + c_1\gamma_1 & a_1\alpha_2 + b_1\beta_2 + c_1\gamma_2 & a_1\alpha_3 + b_1\beta_3 + c_1\gamma_3 \\ a_2\alpha_1 + b_2\beta_1 + c_2\gamma_1 & a_2\alpha_2 + b_2\beta_2 + c_2\gamma_2 & a_2\alpha_3 + b_2\beta_3 + c_2\gamma_3 \\ a_3\alpha_1 + b_3\beta_1 + c_3\gamma_1 & a_3\alpha_2 + b_3\beta_2 + c_3\gamma_2 & a_3\alpha_3 + b_3\beta_3 + c_3\gamma_3 \end{vmatrix}$$

Note : ❑ We can also multiply rows by columns or columns by rows or columns by columns.

Example: 16 For all values of *A, B, C* and *P, Q, R* the value of $\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix}$ is

(a) 0 (b) $\cos A \cos B \cos C$ (c) $\sin A \sin B \sin C$ (d) $\cos P \cos Q \cos R$

Solution: (a) The determinant can be expanded as

$$\begin{vmatrix} \cos A\cos P + \sin A \sin P & \cos A \cos Q + \sin A \sin Q & \cos A \cos R + \sin A \sin R \\ \cos B \cos P + \sin B \sin P & \cos B \cos Q + \sin B \sin Q & \cos B \cos R + \sin B \sin R \\ \cos C \cos P + \sin C \sin P & \cos C \cos Q + \sin C \sin Q & \cos C \cos R + \sin C \sin R \end{vmatrix} = \begin{vmatrix} \cos A & \sin A & 0 \\ \cos B & \sin B & 0 \\ \cos C & \sin C & 0 \end{vmatrix} \times \begin{vmatrix} \cos P & \sin P & 0 \\ \cos Q & \sin Q & 0 \\ \cos R & \sin R & 0 \end{vmatrix} = 0$$

Example: 17 $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix} =$ **[Tamilnadu (Engg.) 2002]**

(a) 7 (b) 10 (c) 13 (d) 17

Solution: (b) $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix} = \left(\frac{\log 512}{\log 3} \times \frac{\log 9}{\log 4} - \frac{\log 3}{\log 4} \times \frac{\log 8}{\log 3}\right) \times \left(\frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} - \frac{\log 3}{\log 8} \times \frac{\log 4}{\log 3}\right)$

$$= \left(\frac{\log 2^9}{\log 3} \times \frac{\log 3^2}{\log 2^2} - \frac{\log 2^3}{\log 2^2}\right) \times \left(\frac{\log 2^2}{\log 2} - \frac{\log 2^2}{\log 2^3}\right) = \left(\frac{9 \times 2}{2} - \frac{3}{2}\right)\left(2 - \frac{2}{3}\right) = 10$$

8.1.7 Summation of Determinants

Let $\Delta_r = \begin{vmatrix} f(r) & a & l \\ g(r) & b & m \\ h(r) & c & n \end{vmatrix}$, where *a, b, c, l, m* and *n* are constants, independent of *r*.

Then, $\sum_{r=1}^{n}\Delta_r = \begin{vmatrix} \sum_{r=1}^{n} f(r) & a & l \\ \sum_{r=1}^{n} g(r) & b & m \\ \sum_{r=1}^{n} h(r) & c & n \end{vmatrix}$. Here function of r can be the elements of only one row or one column.

Example: 18 If $D_p = \begin{vmatrix} p & 15 & 8 \\ p^2 & 35 & 9 \\ p^3 & 25 & 10 \end{vmatrix}$, then $D_1 + D_2 + D_3 + D_4 + D_5 =$

(a) 0 (b) 25 (c) 625 (d) None of these

Solution: (d) $\because D_1 = \begin{vmatrix} 1 & 15 & 8 \\ 1 & 35 & 9 \\ 1 & 25 & 10 \end{vmatrix}$, $D_2 = \begin{vmatrix} 2 & 15 & 8 \\ 4 & 35 & 9 \\ 8 & 25 & 10 \end{vmatrix}$, $D_3 = \begin{vmatrix} 3 & 15 & 8 \\ 9 & 35 & 9 \\ 27 & 25 & 10 \end{vmatrix}$, $D_4 = \begin{vmatrix} 4 & 15 & 8 \\ 16 & 35 & 9 \\ 64 & 25 & 10 \end{vmatrix}$, $D_5 = \begin{vmatrix} 5 & 15 & 8 \\ 25 & 35 & 9 \\ 125 & 25 & 10 \end{vmatrix}$

$\Rightarrow D_1 + D_2 + D_3 + D_4 + D_5 = \begin{vmatrix} 15 & 75 & 40 \\ 55 & 175 & 45 \\ 225 & 125 & 50 \end{vmatrix}$

$= 15(3125) - 75(-7375) + 40(-32500) = 46875 + 553125 - 1300000 = -700000$

Example: 19 The value of $\sum_{n=1}^{N} U_n$, if $U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N \end{vmatrix}$ is

(a) 0 (b) 1 (c) –1 (d) None of these

Solution: (a) $\sum_{n=1}^{N} U_n = \begin{vmatrix} \frac{N(N+1)}{2} & 1 & 5 \\ \frac{N(N+1)(2N+1)}{6} & 2N+1 & 2N+1 \\ \left\{\frac{N(N+1)}{2}\right\}^2 & 3N^2 & 3N \end{vmatrix} = \frac{N(N+1)}{12}\begin{vmatrix} 6 & 1 & 5 \\ 4N+2 & 2N+1 & 2N+1 \\ 3N(N+1) & 3N^2 & 3N \end{vmatrix}$

Applying $C_3 \to C_3 + C_2$ $= \frac{N(N+1)}{12}\begin{vmatrix} 6 & 1 & 6 \\ 4N+2 & 2N+1 & 4N+2 \\ 3N(N+1) & 3N^2 & 3N(N+1) \end{vmatrix} = 0$ [$\because$ C_1 and C_3 are identical]

8.1.8 Differentiation and Integration of Determinants

(1) **Differentiation of a determinant :** (i) Let $\Delta(x)$ be a determinant of order two. If we write $\Delta(x) = |C_1 \quad C_2|$, where C_1 and C_2 denote the 1st and 2nd columns, then

$$\Delta'(x) = |C'_1 \quad C_2| + |C_1 \quad C'_2|$$

where C'_i denotes the column which contains the derivative of all the functions in the i^{th} column C_i.

In a similar fashion, if we write $\Delta(x) = \begin{vmatrix} R_1 \\ R_2 \end{vmatrix}$, then $\Delta'(x) = \begin{vmatrix} R'_1 \\ R_2 \end{vmatrix} + \begin{vmatrix} R_1 \\ R'_2 \end{vmatrix}$

(ii) Let $\Delta(x)$ be a determinant of order three. If we write $\Delta(x) = |C_1 \quad C_2 \quad C_3|$, then

$\Delta'(x) = \left| C'_1 \quad C_2 \quad C_3 \right| + \left| C_1 \quad C'_2 \quad C_3 \right| + \left| C_1 \quad C_2 \quad C'_3 \right|$

and similarly if we consider $\Delta(x) = \begin{vmatrix} R_1 \\ R_2 \\ R_3 \end{vmatrix}$, then $\Delta'(x) = \begin{vmatrix} R'_1 \\ R_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R'_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2 \\ R'_3 \end{vmatrix}$

(iii) If only one row (or column) consists functions of x and other rows (or columns) are constant, *viz.*

Let $\Delta(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$,

then $\Delta'(x) = \begin{vmatrix} f'_1(x) & f'_2(x) & f'_3(x) \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ and in general $\Delta^n(x) = \begin{vmatrix} f_1^n(x) & f_2^n(x) & f_3^n(x) \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

where n is any positive integer and $f^n(x)$ denotes the n^{th} derivative of $f(x)$.

Example: 20 If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinants, then

[MNR 1986; Kurukshetra CEE 1998; UPSEAT 2000]

(a) $\Delta_1 = 3(\Delta_2)^2$ (b) $\frac{d}{dx}(\Delta_1) = 3\Delta_2$ (c) $\frac{d}{dx}(\Delta_1) = 2(\Delta_2)^2$ (d) $\Delta_1 = 3\Delta_2^{3/2}$

Solution: (b) $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix} = x^3 - 3abx \Rightarrow \frac{d}{dx}(\Delta_1) = 3(x^2 - ab)$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix} = x^2 - ab$

$\therefore \frac{d}{dx}(\Delta_1) = 3(x^2 - ab) = 3\Delta_2$

Example: 21 If $y = \sin mx$, then the value of the determinant $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$, where $y_n = \frac{d^n y}{dx^n}$ is

(a) m^9 (b) m^2 (c) m^3 (d) None of these

Solution: (d) $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix} = \begin{vmatrix} \sin mx & m\cos mx & -m^2 \sin mx \\ -m^3 \cos mx & m^4 \sin mx & m^5 \cos mx \\ -m^6 \sin mx & -m^7 \cos mx & m^8 \sin mx \end{vmatrix}$

Taking $-m^6$ common from R_3, R_1 and R_3 becomes identical. Hence the value of determinant is zero.

(2) **Integration of a determinant :** Let $\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ l & m & n \end{vmatrix}$, where *a, b, c, l, m* and *n* are constants.

$$\Rightarrow \int_a^b \Delta(x)dx = \begin{vmatrix} \int_a^b f(x)dx & \int_a^b g(x)dx & \int_a^b h(x)dx \\ a & b & c \\ l & m & n \end{vmatrix}$$

Note : ❑ If the elements of more than one column or rows are functions of x then the integration can be done only after evaluation/expansion of the determinant.

Example: 22 If $\Delta(x)=\begin{vmatrix}1 & \cos x & 1-\cos x\\ 1+\sin x & \cos x & 1+\sin x-\cos x\\ \sin x & \sin x & 1\end{vmatrix}$, then $\int_0^{\pi/2}\Delta(x)\,dx$ is equal to

(a) 1/4 (b) 1/2 (c) 0 (d) –1/2

Solution: (d) Applying $C_3\to C_3+C_2-C_1$

$$\Delta(x)=\begin{vmatrix}1 & \cos x & 0\\ 1+\sin x & \cos x & 0\\ \sin x & \sin x & 1\end{vmatrix}=\cos x-\cos x(1+\sin x)=-\sin x\cos x$$

$$\therefore \int_0^{\pi/2}\Delta(x)dx=-\frac{1}{2}\int_0^{\pi/2}\sin 2x dx=-\frac{1}{2}\left[-\frac{\cos 2x}{2}\right]_0^{\pi/2}=\frac{1}{4}(\cos\pi-\cos 0)=-\frac{1}{2}$$

8.1.9 Application of Determinants in solving a system of Linear Equations

Consider a system of simultaneous linear equations is given by $\left.\begin{array}{l}a_1x+b_1y+c_1z=d_1\\ a_2x+b_2y+c_2z=d_2\\ a_3x+b_3y+c_3z=d_3\end{array}\right\}$

.....(i)

A set of values of the variables *x, y, z* which simultaneously satisfy these three equations is called a solution. A system of linear equations may have a unique solution or many solutions, or no solution at all, if it has a solution (whether unique or not) the system is said to be consistent. If it has no solution, it is called an inconsistent system.

If $d_1=d_2=d_3=0$ in (*i*) then the system of equations is said to be a homogeneous system. Otherwise it is called a non-homogeneous system of equations.

Theorem 1 : (Cramer's rule) The solution of the system of simultaneous linear equations

$a_1x+b_1y=c_1$ .....(i) and $a_2x+b_2y=c_2$ .....(ii)

is given by $x=\frac{D_1}{D}$, $y=\frac{D_2}{D}$, where $D=\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}$, $D_1=\begin{vmatrix}c_1 & b_1\\ c_2 & b_2\end{vmatrix}$ and $D_2=\begin{vmatrix}a_1 & c_1\\ a_2 & c_2\end{vmatrix}$, provided that $D\neq 0$

Note : ❑ Here $D=\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}$ is the determinant of the coefficient matrix $\begin{bmatrix}a_1 & b_1\\ a_2 & b_2\end{bmatrix}$.

The determinant D_1 is obtained by replacing first column in D by the column of the right hand side of the given equations. The determinant D_2 is obtained by replacing the second column in D by the right most column in the given system of equations.

(1) **Solution of system of linear equations in three variables by Cramer's rule :**

Theorem 2 : (Cramer's Rule) The solution of the system of linear equations

$a_1x+b_1y+c_1z=d_1$ .....(i)

$a_2x+b_2y+c_2z=d_2$ .....(ii)

$a_3x+b_3y+c_3z=d_3$ .....(iii)

is given by $x = \frac{D_1}{D}$, $y = \frac{D_2}{D}$ and $z = \frac{D_3}{D}$, where $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$,

$D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$, $D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$, and $D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$, Provided that $D \neq 0$

Note : ❑ Here D is the determinant of the coefficient matrix. The determinant D_1 is obtained by replacing the elements in first column of D by d_1, d_2, d_3. D_2 is obtained by replacing the element in the second column of D by d_1, d_2, d_3 and to obtain D_3, replace elements in the third column of D by d_1, d_2, d_3.

Theorem 3 : (Cramer's Rule) Let there be a system of n_simultaneous linear equation n unknown given by

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n = b_n$$

Let $D = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{vmatrix}$ and let D_j, be the determinant obtained from D after replacing the j^{th} column by $\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$. Then, $x_1 = \frac{D_1}{D}$, $x_2 = \frac{D_2}{D}$,....., $x_n = \frac{D_n}{D}$, Provided that $D \neq 0$

(2) **Conditions for consistency**

Case 1 : For a system of 2 simultaneous linear equations with 2 unknowns

(i) If $D \neq 0$, then the given system of equations is consistent and has a unique solution given by $x = \frac{D_1}{D}, y = \frac{D_2}{D}$.

(ii) If $D = 0$ and $D_1 = D_2 = 0$, then the system is consistent and has infinitely many solutions.

(iii) If $D = 0$ and one of D_1 and D_2 is non-zero, then the system is inconsistent.

Case 2 : For a system of 3 simultaneous linear equations in three unknowns

(i) If $D \neq 0$, then the given system of equations is consistent and has a unique solution given by $x = \frac{D_1}{D}$, $y = \frac{D_2}{D}$ and $z = \frac{D_3}{D}$

(ii) If $D = 0$ and $D_1 = D_2 = D_3 = 0$, then the given system of equations is consistent with infinitely many solutions.

(iii) If $D = 0$ and at least one of the determinants D_1, D_2, D_3 is non-zero, then given of equations is inconsistent.

(3) **Algorithm for solving a system of simultaneous linear equations by Cramer's rule (Determinant method)**

Step 1 : Obtain D, D_1, D_2 and D_3

Step 2 : Find the value of D. If $D \neq 0$, then the system of the equations is consistent has a unique solution. To find the solution, obtain the values of D_1, D_2 and D_3. The solutions is given by $x = \frac{D_1}{D}, y = \frac{D_2}{D}$ and $z = \frac{D_3}{D}$. If $D = 0$ go to step 3.

Step 3 : Find the values of D_1, D_2, D_3. If at least one of these determinants is non-zero, then the system is inconsistent. If $D_1 = D_2 = D_3 = 0$, then go to step 4

Step 4 : Take any two equations out of three given equations and shift one of the variables, say *z* on the right hand side to obtain two equations in *x, y*. Solve these two equations by Cramer's rule to obtain *x, y,* in terms of *z*.

Note: ❑ The system of following homogeneous equations $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$, $a_3x + b_3y + c_3z = 0$ is always consistent.

If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$, then this system has the unique solution $x = y = z = 0$ known as **trivial solution.** But if $\Delta = 0$, then this system has an infinite number of solutions. Hence for non-trivial solution $\Delta = 0$.

Example: 23 If the system of linear equations $x + 2ay + az = 0$, $x + 3by + bz = 0$, $x + 4cy + cz = 0$ has a non-zero solution, then *a, b, c*

[AIEEE 2003]

(a) Are in A.P. (b) Are in G.P. (c) Are in H.P. (d) Satisfy $a + 2b + 3c = 0$

Solution: (c) System of linear equations has a non-zero solution, then $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$

Applying $C_2 \to C_2 - 2C_3$; $\begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix} = 0$

Applying $R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$

$\begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c-b & c-b \end{vmatrix} = 0 \Rightarrow b(c-b) - (b-a)(2c-b) = 0$. On simplification $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$; $\therefore$ *a, b, c* are in H.P.

Example: 24 If the system of equations $x + ay = 0$, $az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of *a* is

[IIT Screening 2003]

(a) –1 (b) 1 (c) 0 (d) No real values

Solution: (a) $\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0 \Rightarrow 1 + a(a)^2 = 0 \Rightarrow a^3 = -1 \Rightarrow a = -1$

Example: 25 If the system of equations $ax + y + z = 0,\ x + by + z = 0$ and $x + y + cz = 0$, where $a, b, c \neq 1$ has a non-trivial solution, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is

(a) –1 (b) 0 (c) 1 (d) None of these

Solution: (c) As the system of the equations has a non-trivial solution $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix} = 0 \Rightarrow a(b-1)(c-1) - 1(1-a)(c-1) - 1(1-a)(b-1) = 0$$

$$\Rightarrow \frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0 \Rightarrow \frac{1}{1-a} - 1 + \frac{1}{1-b} + \frac{1}{1-c} = 0 \Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

Example: 26 If the system of equations $x + 2y - 3z = 1,\ (k+3)z = 3,\ (2k+1)x + z = 0$ is inconsistent, then the value of k is

[Roorkee 2000]

(a) –3 (b) $\frac{1}{2}$ (c) 0 (d) 2

Solution: (a) For the equations to be inconsistent $D = 0$

$\therefore\ D = \begin{vmatrix} 1 & 2 & -3 \\ 0 & 0 & k+3 \\ 2k+1 & 0 & 1 \end{vmatrix} = 0 \Rightarrow k = -3$ and $D_1 = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \neq 0$, Hence system is inconsistent for $k = -3$.

Example: 27. The equations $x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + mz = n$ give infinite number of values of the triplet (x, y, z) if

[]

(a) $m = 3,\ n \in R$ (b) $m = 3,\ n \neq 10$ (c) $m = 3,\ n = 10$ (d) None of these

Solution: (c) Each of the first three options contains $m = 3$. When $m = 3$, the last two equations become $x + 2y + 3z = 10$ and $x + 2y + 3z = n$.

Obviously, when $n = 10$ these equations become the same. So we are left with only two independent equations to find the values of the three unknowns.

Consequently, there will be infinite solutions.

Example: 28 The value of λ for which the system of equations $2x - y - z = 12$, $x - 2y + z = -4,\ x + y + \lambda z = 4$ has no solution is

[IIT Screening 2004]

(a) 3 (b) –3 (c) 2 (d) –2

Solution: (d) $D = \begin{vmatrix} 2 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = -3\lambda - 6$. For no solution the necessary condition is $-3\lambda - 6 = 0 \Rightarrow \lambda = -2$. It can be see that for $\lambda = -2$, there is no solution for the given system of equations.

8.1.10 Application of Determinants in Co-ordinate Geometry

(1) Area of triangle whose vertices are $(x_r, y_r);\ r = 1, 2, 3$ is

$$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

(2) If $a_r x + b_r y + c_r = 0$, $(r = 1, 2, 3)$ are the sides of a triangle, then the area of the triangle is given by

$$\Delta = \frac{1}{2C_1C_2C_3}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2$$, where $C_1 = a_2b_3 - a_3b_2$, $C_2 = a_3b_1 - a_1b_3$, $C_3 = a_1b_2 - a_2b_1$ are the cofactors of the elements c_1, c_2, c_3 respectively in the determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$.

(3) The equation of a straight line passing through two points (x_1, y_1) and (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

(4) If three lines $a_r x + b_r y + c_r = 0$; $(r = 1, 2, 3)$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

(5) If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines then

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

(6) The equation of circle through three non-collinear points $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ is

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ x_1^2 + y_1^2 & x_1 & y_1 & 1 \\ x_2^2 + y_2^2 & x_2 & y_2 & 1 \\ x_3^2 + y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

Example: 29 The three lines $ax + by + c = 0,\ bx + cy + a = 0,\ cx + ay + b = 0$ are concurrent only when

(a) $a + b + c = 0$ (b) $a^2 + b^2 + c^2 = ab + bc + ca$

(c) $a^3 + b^3 + c^3 = ab + bc + ca$ (d) None of these

Solution: (a, b)Three lines are concurrent if $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ or, $3abc - a^3 - b^3 - c^3 = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

Also, $a^3 + b^3 + c^3 - 3abc = 0 \Rightarrow (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$\Rightarrow (a + b + c) = 0$ or $a^2 + b^2 + c^2 = ab + bc + ca$.

8.1.11 Some Special Determinants

(1) **Symmetric determinant :** A determinant is called symmetric determinant if for its every element $a_{ij} = a_{ji} \forall i, j$ *e.g.,* $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$

(2) **Skew-symmetric determinant :** A determinant is called skew symmetric determinant if for its every element $a_{ij} = -a_{ji} \forall i, j$ *e.g.,* $\begin{vmatrix} 0 & 3 & -1 \\ -3 & 0 & 5 \\ 1 & -5 & 0 \end{vmatrix}$

Note : ❑ Every diagonal element of a skew symmetric determinant is always zero.

❑ The value of a skew symmetric determinant of even order is always a perfect square and that of odd order is always zero.

For example (i) $\begin{vmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{vmatrix} = -c(0-ab) - b(ac-0) = abc - abc = 0$

(ii) $\begin{vmatrix} 0 & a \\ -a & 0 \end{vmatrix} = 0 + a^2 = a^2$ (Perfect square) (iii) $\begin{vmatrix} 0 & a-b & e-f \\ b-a & 0 & l-m \\ f-e & m-l & 0 \end{vmatrix} = 0$

(3) **Cyclic order :** If elements of the rows (or columns) are in cyclic order.

i.e. (i) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(ii) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

(iii) $\begin{vmatrix} a & bc & abc \\ b & ca & abc \\ c & ab & abc \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$

(iv) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$ (v) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc)$

Note : ❑ These results direct applicable in lengthy questions (As behavior of standard results)

Example: 30 $\Delta = \begin{vmatrix} 0 & i-100 & i-500 \\ 100-i & 0 & 1000-i \\ 500-i & i-1000 & 0 \end{vmatrix}$ is equal to

(a) 100 (b) 500 (c) 1000 (d) 0

Solution: (d) This determinant of skew symmetric of odd order, hence is equal to 0.

Example: 31 $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} =$ **[Pb. CET 1997; DCE 2002]**

(a) $a^2+b^2+c^2$ (b) $(a+b)(b+c)(c+a)$ **(c)** $(a-b)(b-c)(c-a)$ (d) None of these

Solution: (c) $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix};$$

Applying $R_1 \to R_1 - R_2$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 0 & (a-c) \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(a-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(a-c)(-1) = (a-b)(b-c)(c-a)$$

Example: 32 If $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$ Where *a, b, c* are all different, then the determinant $\begin{vmatrix} 1 & 1 & 1 \\ (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (x-b)(x-c) & (x-c)(x-a) & (x-a)(x-b) \end{vmatrix}$ vanishes when

(a) $a+b+c=0$ (b) $x=\frac{1}{3}(a+b+c)$ (c) $x=\frac{1}{2}(a+b+c)$ (d) $x=a+b+c$

Solution: (b) $\because \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$ ...... (i)

Now, $\begin{vmatrix} 1 & 1 & 1 \\ (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (x-b)(x-c) & (x-c)(x-a) & (x-a)(x-b) \end{vmatrix} = 0$

$$\Rightarrow \frac{1}{(x-a)(x-b)(x-c)}\begin{vmatrix} (x-a) & (x-b) & (x-c) \\ (x-a)^3 & (x-b)^3 & (x-c)^3 \\ (x-a)(x-b)(x-c) & (x-a)(x-b)(x-c) & (x-a)(x-b)(x-c) \end{vmatrix} = 0$$

Applying $C_1 \to C_1(x-a),\ C_2 \to C_2(x-b),\ C_3 \to C_3(x-c)$

$$\begin{vmatrix} (x-a) & (x-b) & (x-c) \\ (x-a)^3 & (x-b)^3 & (x-c)^3 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$[(x-a)-(x-b)][(x-b)-(x-c)][(x-c)-(x-a)](x-a+x-b+x-c)=0$

$(b-a)(c-b)(a-c)[3x-(a+b+c)]=0$ or $x=\frac{1}{3}(a+b+c)$ $[\because a \neq b \neq c]$

Example: 33 If α, β and γ are the roots of the equations $x^3+px+q=0$ then value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is **[AMU 1990]**

(a) p (b) q (c) p^2-2q (d) 0

Solution: (d) Since α, β, γ are the roots of $x^3+px+q=0$, $\therefore \alpha+\beta+\gamma=0$

$$\Delta = \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, We get, $\begin{vmatrix} \alpha+\beta+\gamma & \alpha+\beta+\gamma & \alpha+\beta+\gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$

Example: 34 If $\Delta = \begin{vmatrix} x+1 & x^2+2 & x(x+1) \\ x(x+1) & x+1 & x(x^2+2) \\ x^2+2 & x(x+1) & x+1 \end{vmatrix} = p_0x^6 + p_1x^5 + p_2x^4 + p_3x^3 + p_4x^2 + p_5x + p_6$, then $(p_5,\ p_6) =$

(a) (−3, −9) (b) (−5, −9) (c) (−3, −5) (d) (3, −9)

Solution: (b) Putting $x = 0$ in both sides, we get, $\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{vmatrix} = p_6 \Rightarrow p_6 = 9$ by expansion.

p_5 is the coefficient of x or constant term in the differentiation of determinant.

Differentiate both sides,

$$\begin{vmatrix} 1 & 2x & 2x+1 \\ x(x+1) & x+1 & x(x^2+2) \\ x^2+2 & x(x+1) & x+1 \end{vmatrix} + \begin{vmatrix} x+1 & x^2+2 & x(x+1) \\ 2x+1 & 1 & 3x^2+2 \\ x^2+2 & x(x+1) & x+1 \end{vmatrix} + \begin{vmatrix} x+1 & x^2+2 & x(x+1) \\ x(x+1) & x+1 & x(x^2+2) \\ 2x & 2x+1 & 1 \end{vmatrix}$$

$$= 6p_0x^5 + 5p_1x^4 + 4p_2x^3 + 3p_3x^2 + 2p_4x + p_5$$

Putting $x = 0$ both sides, we get $p_5 = -5$; $\therefore (p_5,\ p_6) = (-5,\ 9)$.

***

Assignment

Expansion of Determinants

Basic Level

1. If a, b and c are non -zero real numbers, then $\Delta = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ is equal to **[AMU 1992; Karnataka CET 2000, 2003]**

(a) abc (b) $a^2b^2c^2$ (c) $ab+bc+ca$ (d) None of these

2. If ω is a cube root of unity and $\Delta = \begin{vmatrix} 1 & 2\omega \\ \omega & \omega^2 \end{vmatrix}$, then Δ^2 is equal to **[Rajas**

(a) $-\omega$ (b) ω (c) 1 (d) ω^2

3. The determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix}$ is not equal to **[MP PET 1988]**

(a) $\begin{vmatrix} 2 & 1 & 1 \\ 2 & 2 & 3 \\ 2 & 3 & 6 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 4 & 3 & 6 \end{vmatrix}$ (c) $\begin{vmatrix} 1 & 2 & 1 \\ 1 & 5 & 3 \\ 1 & 9 & 6 \end{vmatrix}$ (d) $\begin{vmatrix} 3 & 1 & 1 \\ 6 & 2 & 3 \\ 10 & 3 & 6 \end{vmatrix}$

4. If ω is the cube root of unity, then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} =$

[MP PET 1990, 2002; Karnataka CET 1992, 93, 2002; Rajasthan PET 1985, 93, 94]

(a) 1 (b) 0 (c) ω (d) ω^2

5. If $a+b+c=0$, then the solution of the equation $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ is **[UPSEAT 2001]**

(a) 0 (b) $\pm\frac{3}{2}(a^2+b^2+c^2)$ (c) 0, $\pm\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$ (d) 0, $\pm\sqrt{a^2+b^2+c^2}$

6. $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} =$

(a) $\begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$ (b) $\begin{vmatrix} y & q & b \\ x & p & a \\ z & r & c \end{vmatrix}$ (c) $\begin{vmatrix} b & y & q \\ a & p & x \\ c & z & r \end{vmatrix}$ (d) $\begin{vmatrix} y & b & q \\ z & c & r \\ x & a & p \end{vmatrix}$

7. $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix} =$ **[Rajasthan PET 1990, 99]**

(a) abc (b) $1/abc$ (c) $ab+bc+ca$ (d) 0

8. $\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} =$ **[IIT 1980]**

(a) abc (b) $4abc$ (c) $4a^2b^2c^2$ (d) $a^2b^2c^2$

9. If $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -1 \end{vmatrix} = 0$, then the value of k is **[IIT 1979]**

(a) –1 (b) 0 (c) 1 (d) None of these

10. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b+c-a & c+a-b & a+b-c \end{vmatrix}$ is **[Rajas**

(a) abc (b) $a+b+c$ (c) $ab+bc+ca$ (d) None of these

11. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} =$ **[Rajasthan PET 1996]**

(a) 1 (b) 0 (c) x (d) xy

12. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} =$ **[Rajasthan PET 1990, 95]**

(a) $(a+b+c)^2$ (b) $(a+b+c)^3$ (c) $(a+b+c)(ab+bc+ca)$ (d) None of these

13. The value of the determinant $\begin{vmatrix} 4 & -6 & 1 \\ -1 & -1 & 1 \\ -4 & 11 & -1 \end{vmatrix}$ is **[Rajasthan PET 1992, 96]**

(a) –75 (b) 25 (c) 0 (d) –25

14. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is **[MP PET 1993; Karnataka CET 1994; Rajasthan PET 1985]**

(a) $(a+b+c)$ (b) $(a+b+c)^2$ (c) 0 (d) $1+a+b+c$

15. $\begin{vmatrix} 1 & \log_b a \\ \log_a b & 1 \end{vmatrix} =$ **[MP PET 1995]**

(a) 1 (b) 0 (c) $\log_a b$ (d) $\log_b a$

16. $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ is equal to **[Rajasthan PET 1995]**

(a) $3abc-a^3-b^3-c^3$ (b) $(a+b)(b+c)(c+a)$ (c) $(a-b)(b-c)(c-a)$ (d) $(a-b)(b-c)(a-c)$

17. $\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & q-r & r-p \end{vmatrix} =$ **[MNR 1987]**

(a) $a(x+y+z)+b(p+q+r)+c$ (b) 0

(c) $abc + xyz + pqr$ (d) None of these

18. $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix} =$ **[IIT 1988; MP PET 1990, 91; Rajasthan PET 2002]**

(a) 0 (b) $a^3+b^3+c^3-3abc$ (c) 3 abc (d) $(a+b+c)^3$

19. $\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix} =$ **[MP PET 1992]**

(a) $-2abc$ (b) abc (c) 0 (d) $a^2+b^2+c^2$

20. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} =$ **[AMU 1979; Rajasthan PET 1990; DCE 1999]**

(a) $a^3+b^3+c^3-3abc$ (b) $a^3+b^3+c^3+3abc$ (c) $(a+b+c)(a-b)(b-c)(c-a)$ (d) None of these

21. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} =$ **[MP PET 1991]**

(a) $3abc+a^3+b^3+c^3$ (b) $3abc-a^3-b^3-c^3$ (c) $abc-a^3+b^3+c^3$ (d) $abc+a^3-b^3-c^3$

22. If ω is a cube root of unity, then $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} =$ **[MNR 1990; MP PET 1999]**

(a) x^3+1 (b) $x^3+\omega$ (c) $x^3+\omega^2$ (d) x^3

23. $\begin{vmatrix} a-1 & a & bc \\ b-1 & b & ca \\ c-1 & c & ab \end{vmatrix} =$ **[Rajasthan PET 1988]**

(a) 0 (b) $(a-b)(b-c)(c-a)$ (c) $a^3+b^3+c^3-3abc$ (d) None of these

24. The value of the determinant $\begin{vmatrix} 31 & 37 & 92 \\ 31 & 58 & 71 \\ 31 & 105 & 24 \end{vmatrix}$ is **[MP PET 1992]**

(a) –2 (b) 0 (c) 81 (d) None of these

25. The value of the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 8 & 14 & 20 \end{vmatrix}$ is **[MNR 1991]**

(a) 20 (b) 10 (c) 0 (d) 250

26. The value of the determinant $\begin{vmatrix} 7 & 9 & 79 \\ 4 & 1 & 41 \\ 5 & 5 & 55 \end{vmatrix}$ is **[MP PET 1992]**

(a) –7 (b) 0 (c) 15 (d) 27

27. $\begin{vmatrix} 19 & 17 & 15 \\ 9 & 8 & 7 \\ 1 & 1 & 1 \end{vmatrix} =$ **[MP PET 1990]**

(a) 0 (b) 187 (c) 354 (d) 54

28. $\begin{vmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{vmatrix} =$ **[MP PET 1991]**

(a) $1+a^2+b^2+c^2$ (b) $1-a^2+b^2+c^2$ (c) $1+a^2+b^2-c^2$ (d) $1+a^2-b^2+c^2$

29. The value of the determinant $\begin{vmatrix} 2 & 8 & 4 \\ -5 & 6 & -10 \\ 1 & 7 & 2 \end{vmatrix}$ is **[MP PET 1994]**

(a) - 440 (b) 0 (c) 328 (d) 488

30. $\begin{vmatrix} 19 & 6 & 7 \\ 21 & 3 & 15 \\ 28 & 11 & 6 \end{vmatrix}$ is equal to **[Rajasthan PET 1995]**

(a) 150 (b) –110 (c) 0 (d) None of these

31. $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} =$ **[MP PET 1990]**

(a) $a^3+b^3+c^3-3abc$ (b) $3abc-a^3-b^3-c^3$

(c) $a^3+b^3+c^3-a^2b-b^2c-c^2a$ (d) $(a+b+c)(a^2+b^2+c^2+ab+bc+ca)$

32. For non-zero *a, b, c* if $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$, then the value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} =$ **[Kerala (Engg.) 2002]**

(a) abc (b) $\frac{1}{abc}$ (c) $-(a+b+c)$ (d) None of these

33. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ e & \pi & \sqrt{2} \\ 2 & 2 & 2 \end{vmatrix}$ is equal to **[AMU 1982]**

(a) 0 (b) e (c) π (d) $2(e-\pi+\sqrt{2})$

34. If $D = \begin{vmatrix} 1 & a & b \\ 1 & b & c \\ 1 & c & a \end{vmatrix}$, then $\begin{vmatrix} a & b & c \\ b & c & a \\ 1 & 1 & 1 \end{vmatrix}$ equals **[AMU 1988, 90, 92]**

(a) *O* (b) *D* (c) –*D* (d) None of these

35. The value of the determinant $\begin{vmatrix} a_1 & la_1+mb_1 & b_1 \\ a_2 & la_2+mb_2 & b_2 \\ a_3 & la_3+mb_3 & b_3 \end{vmatrix}$ is **[AMU 1987]**

(a) 0 (b) *l* (c) *m* (d) *lm*

36. If ω is a complex cube root of unity, then the value of the determinant $\begin{vmatrix} 1 & \omega & \omega+1 \\ \omega+1 & 1 & \omega \\ \omega & \omega+1 & 1 \end{vmatrix}$ is **[AMU 1989]**

(a) 0 (b) ω (c) 2 (d) 4

37. If $\Delta = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 1 \\ 0 & 4 & 2 \end{vmatrix}$, the value of $\begin{vmatrix} 4 & 12 & 4 \\ 8 & -4 & 4 \\ 0 & 16 & 8 \end{vmatrix}$ is **[T.S. Rajendra 1990]**

(a) 12Δ (b) 64Δ (c) 4Δ (d) $4^2\Delta$

38. If T_p, T_q, T_r are pth, qth and rth terms of an $A.P.$, then $\begin{vmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ is equal to

(a) 1 (b) –1 (c) 0 (d) $p+q+r$

39. $\Delta = \begin{vmatrix} 1 & 1+ac & 1+bc \\ 1 & 1+ad & 1+bd \\ 1 & 1+ae & 1+be \end{vmatrix} =$ **[MP PET 1996]**

(a) 1 (b) 0 (c) 3 (d) $a+b+c$

40. The value of the determinant $\begin{vmatrix} 1 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 1 & 4 \end{vmatrix}$ is **[Rajasthan PET 1985]**

(a) 2 (b) –2 (c) 0 (d) 5

41. If $A = \begin{vmatrix} 0 & 3 & 4 \\ 5 & 7 & 8 \\ 0 & 6 & 8 \end{vmatrix}$, then the value of A is **[Rajasthan PET 1984]**

(a) 0 (b) 1 (c) 2 (d) 3

42. The value of $\begin{vmatrix} 1 & 2 & 4 \\ -3 & 1 & -2 \\ 2 & 2 & 4 \end{vmatrix}$ is **[Rajasthan PET 1984]**

(a) 88 (b) – 8 (c) – 40 (d) 56

43. The value of $\begin{vmatrix} 43 & 1 & 6 \\ 35 & 7 & 4 \\ 17 & 3 & 2 \end{vmatrix}$ is **[MNR 1991, 95; Rajasthan PET 1996]**

(a) 1 (b) – 1 (c) 0 (d) None of these

44. $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 8 & 14 & 20 \end{vmatrix} =$ **[MNR 1991]**

(a) 20 (b) 10 (c) 0 (d) 5

45. The value of $\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix}$ is **[Rajasthan PET 1989]**

(a) –100 (b) 0 (c) 100 (d) 1000

46. The value of $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$ is **[Rajasthan PET 1986]**

(a) 4 (b) 6 (c) 8 (d) 10

47. The value of the determinant $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$ is **[Rajasthan PET 1988]**

(a) $abc(a-b)(b-c)(c-a)$ (b) $(a-b)(b-c)(c-a)(a+b+c)$

(c) $(a-b)(b-c)(c-a)(ab+bc+ca)$ (d) None of these

48. The value of the determinant $\begin{vmatrix} a_1 & ma_1 & b_1 \\ a_2 & ma_2 & b_2 \\ a_3 & ma_3 & b_3 \end{vmatrix}$ is **[Rajasthan PET 1989]**

(a) 0 (b) $ma_1a_2a_3$ (c) $ma_1b_2a_2$ (d) $mb_1b_2b_3$

49. $\begin{vmatrix} 1/a & a & bc \\ 1/b & b & ca \\ 1/c & c & ab \end{vmatrix}$ equals to **[Rajasthan PET 1989]**

(a) $(a-b)(b-c)(c-a)$ (b) $a^3+b^3-c^3+3abc$ (c) 0 (d) None of these

50. $\begin{vmatrix} 2ac-b^2 & a^2 & c^2 \\ a^2 & 2ab-c^2 & b^2 \\ c^2 & b^2 & 2bc-a^2 \end{vmatrix}$ equals **[Rajasthan PET 1998]**

(a) $4abc$ (b) $-4abc$ (c) 0 (d) $(a^3+b^3+c^3-3abc)^2$

51. The value of the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 13 & 14 & 15 \end{vmatrix}$ is **[Rajasthan PET 1991]**

(a) 0 (b) 10 (c) 46 (d) 50

52. The value of the determinant $\begin{vmatrix} x & -y & z \\ -x & y & z \\ -x & -y & z \end{vmatrix}$ is **[Rajasthan PET 1991]**

(a) 0 (b) xyz (c) 2 xyz (d) 4 xyz

53. If a, b, c are all different and $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix}$ = 0, then correct statement is **[Rajasthan PET 1991]**

(a) $a+b+c=0$ (b) $ab+bc+ca=0$ (c) $a^2+b^2+c^2=bc+ca+ab$ (d) None of these

54. If $a\neq b\neq c$ and $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}=0$, then the value of $(a+b+c)$ is **[Rajasthan PET 1990]**

(a) 1 (b) 0 (c) 2 (d) $-a$

55. The value of $\begin{vmatrix} 13 & 16 & 19 \\ 14 & 17 & 20 \\ 15 & 18 & 21 \end{vmatrix}$ is **[Rajasthan PET 1992; MP 1996]**

(a) 652 (b) 576 (c) 0 (d) None of these

56. If ω is a cube root of unity, then one root of the equation $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}=0$ is **[MNR 1990; DCE 1998]**

(a) 1 (b) ω (c) ω^2 (d) 0

57. If $\begin{vmatrix} a & -b & -c \\ -a & b & -c \\ -a & -b & c \end{vmatrix}+\lambda abc=0$ then λ equals **[Rajasthan PET 1998]**

(a) –4 (b) 4 (c) 2 (d) None of these

58. If $a\neq b\neq c$ and $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix}=0$ then **[EAMCET 1989]**

(a) $a+b+c=0$ (b) abc = 1 (c) $a+b+c=1$ (d) $ab+bc+ca=0$

59. $\begin{vmatrix} \sin\theta & 1 & 0 \\ 0 & \cos\phi & -\cos\theta \\ \sin\phi & 0 & 1 \end{vmatrix}$ is equal to **[Rajasthan PET 1996]**

(a) $\cos(\theta+\phi)$ (b) $\sin(\theta+\phi)$ (c) $\cos(\theta-\phi)$ (d) $\sin(\theta-\phi)$

60. $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ = (where *x, y, z* being positive) **[IIT 1993; UPSEAT 2002]**

(a) $\log_y x$ (b) $\log_z y$ (c) $\log_x z$ (d) 0

61. If $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = kxyz$, then k = **[Roorkee 1980; Rajasthan PET 1999]**

(a) 1 (b) 2 (c) 3 (d) 4

62. Let $\Delta = \begin{vmatrix} 1 & \sin\alpha & 1 \\ -\sin\alpha & 1 & \sin\alpha \\ -1 & -\sin\alpha & 1 \end{vmatrix}$, then Δ lies in the interval

(a) [2, 3] (b) [3, 4] (c) [1, 4] (d) (2, 4)

63. If $a \ne p, b \ne q, c \ne r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, then the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ is

(a) 0 (b) 1 (c) –1 (d) 2

64. The value of the determinant $\Delta = \begin{vmatrix} \log x & \log y & \log z \\ \log 2x & \log 2y & \log 2z \\ \log 3x & \log 3y & \log 3z \end{vmatrix}$ is **[AMU 1993]**

(a) 0 (b) $\log(xyz)$ (c) $\log(6xyz)$ (d) $6\log(xyz)$

65. If *a, b, c* are negative distinct real numbers, then the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

(a) <0 (b) ≤ 0 (c) >0 (d) ≥ 0

66. If *A, B, C* are the angles of a triangle, then the value of $\Delta = \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$ is **[Karnataka CET 2002]**

(a) $\cos A \cos B \cos C$ (b) $\sin A \sin B \sin C$ (c) 0 (d) None of these

67. $\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix} =$ **[Karnataka CET 1994]**

(a) $(x+p)(x+q)(x-p-q)$ (b) $(x-p)(x-q)(x+p+q)$ (c) $(x-p)(x-q)(x-p-q)$ (d) $(x+p)(x+q)(x+p+q)$

68. $\begin{vmatrix} 11 & 12 & 13 \\ 12 & 13 & 14 \\ 13 & 14 & 15 \end{vmatrix} =$ **[Karnataka CET 1991]**

(a) 1 (b) 0 (c) –1 (d) 67

69. $\begin{vmatrix} x & 4 & y+z \\ y & 4 & z+x \\ z & 4 & x+y \end{vmatrix} =$ **[Karnataka CET 1991]**

(a) 4 (b) $x+y+z$ (c) xyz (d) 0

70. The value of the determinant $\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$ is equal to **[Roorkee 1992]**

(a) –4 (b) 0 (c) 1 (d) 4

71. $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} =$ **[EAMCET 1994]**

(a) 0 (b) $12\cos^2 x - 10\sin^2 x$ (c) $12\sin^2 x - 10\cos^2 x - 2$ (d) $10\sin 2x$

72. If *a, b, c* are in *A.P.*, then the value of $\begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$ is **[Rajasthan PET 1999]**

(a) $x-(a+b+c)$ (b) $9x^2+a+b+c$ (c) $a+b+c$ (d) 0

73. The value of the determinant given below $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 8 & 14 & 20 \end{vmatrix}$ is **[UPSEAT 2000]**

(a) 20 (b) 10 (c) 0 (d) 5

74. $\begin{vmatrix} 1 & 1 & 1 \\ \cos(nx) & \cos(n+1)x & \cos(n+2)x \\ \sin(nx) & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$ is not depend **[Rajasthan PET 2000]**

(a) On x (b) On n (c) Both on x and n (d) None of these

75. The value of $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$ is **[Kerala (Engg.) 2001]**

(a) 8 (b) –8 (c) 400 (d) 1

76. The value of $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$ is equal to **[Kerala (Engg.) 2001]**

(a) $9a^2(a+b)$ (b) $9b^2(a+b)$ (c) $a^2(a+b)$ (d) $b^2(a+b)$

77. The value of $\begin{vmatrix} 41 & 42 & 43 \\ 44 & 45 & 46 \\ 47 & 48 & 49 \end{vmatrix} =$ **[Karnataka CET 2001]**

(a) 2 (b) 4 (c) 0 (d) 1

78. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is **[IIT Screening 2002]**

(a) 3ω (b) $3\omega(\omega-1)$ (c) $3\omega^2$ (d) $3\omega(1-\omega)$

79. If $C = 2\cos\theta$, then the value of the determinant $\Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 6 & 1 & C \end{vmatrix}$ is **[Orissa JEE 2002]**

(a) $\frac{\sin 4\theta}{\sin\theta}$ (b) $\frac{2\sin^2 2\theta}{\sin\theta}$ (c) $4\cos^2\theta(2\cos\theta-1)$ (d) None of these

80. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{vmatrix} =$ **[Rajasthan PET 2002]**

(a) $3\sqrt{3}i$ (b) $-3\sqrt{3}i$ (c) $i\sqrt{3}$ (d) 3

81. $\begin{vmatrix} 1/a & 1 & bc \\ 1/b & 1 & ca \\ 1/c & 1 & ab \end{vmatrix} =$ **[Rajasthan PET 2002]**

(a) 0 (b) abc (c) $1/abc$ (d) None of these

82. $\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (b^x+b^{-x})^2 & (b^x-b^{-x})^2 & 1 \\ (c^x+c^{-x})^2 & (c^x-c^{-x})^2 & 1 \end{vmatrix} =$ **[UPSEAT 2002]**

(a) 0 (b) $2abc$ (c) $a^2b^2c^2$ (d) None of these

83. The determinant $\begin{vmatrix} a & b & a-b \\ b & c & b-c \\ 2 & 1 & 0 \end{vmatrix}$ is equal to zero if a, b, c are in **[UPSEAT 2002]**

(a) G.P. (b) A.P. (c) H.P. (d) None of these

84. If $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = k(a+b+c)(a^2+b^2+c^2-bc-ca-ab)$, then k = **[Rajasthan PET 2003]**

(a) 1 (b) 2 (c) –1 (d) –2

85. The value of the determinant $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$ is **[Orissa JEE 2003]**

(a) 2(10! 11!) (b) 2(10! 13!) (c) 2(10! 11! 12!) (d) 2(11! 12! 13!)

86. The value of $\begin{vmatrix} a^2 & -ab & -ac \\ -ab & b^2 & -bc \\ ca & bc & -c^2 \end{vmatrix}$ is **[Tamilnadu (Engg.) 2002]**

(a) $4a^2b^2$ (b) $4b^2c^2$ (c) $4c^2a^2$ (d) $4a^2b^2c^2$

87. $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix}$ is equal to

(a) $x^2(x+3)$ (b) $3x^3$ (c) 0 (d) x^3

88. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x+iy$, then

(a) $x=3, y=1$ (b) $x=1, y=3$ (c) $x=0, y=3$ (d) $x=0, y=0$

89. The determinant $\begin{vmatrix} a & a+d & a+2d \\ a^2 & (a+d)^2 & (a+2d)^2 \\ 2a+3d & 2(a+d) & 2a+d \end{vmatrix} = 0$, then

(a) $d=0$ (b) $a+d=0$ (c) $d=0$ or $a+d=0$ (d) None of these

90. The value of the determinant $\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix}$ is

(a) 0 (b) $2\sin\theta$ (c) $\sin 2\theta$ (d) None of these

Advance Level

91. If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax-12$, then the value of A is **[IIT 1982]**

(a) 12 (b) 24 (c) –12 (d) –24

92. $\begin{vmatrix} a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \end{vmatrix} =$ **[MNR 1985; IIT 1986; MP PET 1998]**

(a) $a^2+b^2+c^2-3abc$ (b) $3ab$ (c) $3a+5b$ (d) 0

93. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} =$ **[Roorkee 1980; Rajasthan PET 1997, 99, Karnataka CET 1999, MP PET 2001]**

(a) abc (b) $2abc$ (c) $3abc$ (d) $4abc$

94. If *a, b, c* are unequal what is the condition that the value of the following determinant is zero $\Delta = \begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix}$ **[IIT 1985; DCE 1999]**

(a) $1+abc=0$ (b) $(a-b)(b-c)(c-a)=0$ (c) $a+b+c+1=0$ (d) None of these

95. If $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$, then $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ is equal to **[Karnataka CET 1991; Rajasthan PET 2000]**

(a) 0 (b) abc (c) $-abc$ (d) None of these

96. The value of the determinant $\Delta = \begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix}$ is equal to **[AMU 1994]**

(a) 1 (b) 0 (c) 2 (d) 3

97. If $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = \lambda a^2b^2c^2$ then the value of λ is **[MP PET 1999; Kurukshetra CEE 1990, 2002]**

(a) 1 (b) 2 (c) 4 (d) 3

98. The parameter, on which the value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend upon is

[IIT 1997]

(a) a (b) p (c) d (d) x

99. The value of the determinant $\Delta = \begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}$ is **[Karnataka CET 1991]**

(a) 2! (b) 3! (c) 4! (d) 5!

100. If $0 < \theta < \frac{\pi}{2}$ and $\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$ than θ is equal to **[MNR 1992]**

(a) $\frac{\pi}{24}, \frac{5\pi}{24}$ (b) $\frac{5\pi}{24}, \frac{7\pi}{24}$ (c) $\frac{7\pi}{24}, \frac{11\pi}{24}$ (d) None of these

101. The value of $\begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos(\alpha-\gamma) \\ \cos(\alpha-\beta) & 1 & \cos(\beta-\gamma) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix}$ is **[Rajasthan PET 2000; Pb. CET 1992]**

(a) $\begin{vmatrix} \cos\alpha & \sin\alpha & 1 \\ \cos\beta & \sin\beta & 1 \\ \cos\gamma & \sin\gamma & 1 \end{vmatrix}^2$ (b) $\begin{vmatrix} \sin\alpha & \cos\alpha & 0 \\ \sin\beta & \cos\beta & 0 \\ \sin\gamma & \cos\gamma & 0 \end{vmatrix}^2$ (c) $\begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ \sin\beta & 0 & \cos\beta \\ 0 & \cos\gamma & \sin\gamma \end{vmatrix}^2$ (d) None of these

102. If a, b, c are all different and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$, then the value of $abc(ab+bc+ca)$ is **[Kurukshetra CEE 2002]**

(a) $a+b+c$ (b) 0 (c) $a^2+b^2+c^2$ (d) $a^2-b^2+c^2$

103. $\begin{vmatrix} a^2+x^2 & ab & ca \\ ab & b^2+x^2 & bc \\ ca & bc & c^2+x^2 \end{vmatrix}$ is divisor of **[Rajasthan PET 2000]**

(a) a^2 (b) b^2 (c) c^2 (d) x^2

104. If $\begin{vmatrix} a & b & a+b \\ b & c & b+c \\ a+b & b+c & 0 \end{vmatrix} = 0$, then a, b, c are in **[AMU 2000]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

105. If $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = k\,abc(a+b+c)^3$, then the value of k is **[Tamilnadu (Engg.) 2001]**

(a) –1 (b) 1 (c) 2 (d) –2

106. If $a > 0$ and discriminant of $ax^2+2bx+c$ is negative, then $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is **[AIEEE 2002]**

(a) Positive (b) $(ac-b^2)(ax^2+2bx+c)$ (c) Negative (d) 0

107. The determinant $\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix} = 0$, if a, b, c are in **[UPSEAT 2002]**

(a) *A.P.* (b) *G.P.* (c) *H.P.* (d) None of these

108. The value of the determinant $\begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$ is **[UPSEAT 2003]**

(a) $\alpha^2+\beta^2$ (b) $\alpha^2-\beta^2$ (c) 1 (d) 0

109. In a ΔABC, if $\begin{vmatrix}1 & a & b\\ 1 & c & a\\ 1 & b & c\end{vmatrix}=0$, then $\sin^2 A+\sin^2 B+\sin^2 C=$ **[Karnataka CET 2003]**

(a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) 1 (d) $3\sqrt{3}$

110. If $a\ne p, b\ne q, c\ne r$ and $\begin{vmatrix}p & b & c\\ p+a & q+b & 2c\\ a & b & r\end{vmatrix}=0$, then $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=$ **[EAMCET 2003]**

(a) 3 (b) 2 (c) 1 (d) 0

111. If $A=\begin{vmatrix}\sin(\theta+\alpha) & \cos(\theta+\alpha) & 1\\ \sin(\theta+\beta) & \cos(\theta+\beta) & 1\\ \sin(\theta+\gamma) & \cos(\theta+\gamma) & 1\end{vmatrix}$, then **[Orissa JEE 2003]**

(a) $A=0$ for all θ (b) A is an odd function of θ (c) $A=0$ for $\theta=\alpha+\beta+\gamma$ (d)A is in

112. *l, m, n* are the *pth, qth* and *rth* term of a G.P., all positive, then $\begin{vmatrix}\log l & p & 1\\ \log m & q & 1\\ \log n & r & 1\end{vmatrix}$ equals **[AIEEE 2002]**

(a) –1 (b) 2 (c) 1 (d) 0

113. If *a, b, c* are respectively the p^{th}, q^{th}, r^{th} terms of an A.P., then $\begin{vmatrix}a & p & 1\\ b & q & 1\\ c & r & 1\end{vmatrix}=$ **[Kerala (Engg.) 2002]**

(a) 1 (b) –1 (c) 0 (d) *pqr*

114. The value of the determinant $\begin{vmatrix}bc & ca & ab\\ p & q & r\\ 1 & 1 & 1\end{vmatrix}$, where *a, b, c* are the p^{th}, q^{th}, r^{th} terms of a H.P. is

(a) $ap+bq+cr$ (b) $(a+b+c)(p+q+r)$ (c) 0 (d) None of these

115. The value of $\begin{vmatrix}a_1x+b_1y & a_2x+b_2y & a_3x+b_3y\\ b_1x+a_1y & b_2x+a_2y & b_3x+a_3y\\ b_1x+a_1 & b_2x+a_2 & b_3x+a_3\end{vmatrix}$ is equal to

(a) x^2+y^2 (b) 0 (c) $a_1a_2a_3x^2+b_1b_2b_3y^2$ (d) None of these

116. If α, β are non-real numbers satisfying $x^3-1=0$ then the value of $\begin{vmatrix}\lambda+1 & \alpha & \beta\\ \alpha & \lambda+\beta & 1\\ \beta & 1 & \lambda+\alpha\end{vmatrix}$ is equal to

(a) 0 (b) λ^3 (c) λ^3+1 (d) None of these

117. The value of the determinant $\begin{vmatrix}{}^5C_0 & {}^5C_3 & 14\\ {}^5C_1 & {}^5C_4 & 1\\ {}^5C_2 & {}^5C_5 & 1\end{vmatrix}$ is

(a) 0 (b) – (6 !) (c) 80 (d) None of these

118. $\begin{vmatrix}\cos C & \tan A & 0\\ \sin B & 0 & -\tan A\\ 0 & \sin B & \cos C\end{vmatrix}$ has the value

(a) 0 (b) 1 (c) $\sin A\sin B\cos C$ (d) None of these

119. The value of $\begin{vmatrix}x & x^2-yz & 1\\ y & y^2-zx & 1\\ z & z^2-xy & 1\end{vmatrix}$ is

(a) 1 (b) –1 (c) 0 (d) $-xyz$

120. If $\sqrt{-1}=i$ and ω is non real cube root of unity then the value of $\begin{vmatrix} 1 & \omega^2 & 1+i+\omega^2 \\ -i & -1 & -1-i+\omega \\ 1-i & \omega^2-1 & -1 \end{vmatrix}$ is equal to

(a) 1 (b) i (c) ω (d) 0

121. The value of $\begin{vmatrix} i^m & i^{m+1} & i^{m+2} \\ i^{m+5} & i^{m+4} & i^{m+3} \\ i^{m+6} & i^{m+7} & i^{m+8} \end{vmatrix}$, where $i=\sqrt{-1}$, is

(a) 1 if m is a multiple of 4 (b) 0 for all real m (c) $-i$ if m is a multiple of 3 (d) None of these

122. If the determinant $\begin{vmatrix} \cos 2x & \sin^2 x & \cos 4x \\ \sin^2 x & \cos 2x & \cos^2 x \\ \cos 4x & \cos^2 x & \cos 2x \end{vmatrix}$ is expanded in powers of $\sin x$ then the constant term in the expansion is

(a) 1 (b) 2 (c) –1 (d) None of these

123. Let $f(n)=\begin{vmatrix} n & n+1 & n+2 \\ {}^nP_n & {}^{n+1}P_{n+1} & {}^{n+2}P_{n+2} \\ {}^nC_n & {}^{n+1}C_{n+1} & {}^{n+2}C_{n+2} \end{vmatrix}$ where the symbols have their usual meanings. The $f(n)$ is divisible by

(a) n^2+n+1 (b) $(n+1)!$ (c) $n!$ (d) None of these

124. $\begin{vmatrix} {}^xC_1 & {}^xC_2 & {}^xC_3 \\ {}^yC_1 & {}^yC_2 & {}^yC_3 \\ {}^zC_1 & {}^zC_2 & {}^zC_3 \end{vmatrix}$ is equal to

(a) $xyz(x-y)(y-z)(z-x)$ (b) $\frac{xyz}{6}(x-y)(y-z)(z-x)$ (c) $\frac{xyz}{12}(x-y)(y-z)(z-x)$ (d) None of these

125. $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}=$ **[MNR 1985]**

(a) 2 (b) –2 (c) x^2-2 (d) None of these

Solution of Equations in the from of Determinant

Basic Level

126. The roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix}=0$ are **[IIT 1987; MP PET 2002]**

(a) –1, –2 (b) –1, 2 (c) 1, –2 (d) 1, 2

127. The roots of the equation $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix}=0$ are **[MP PET 1989; Roorkee 1998]**

(a) 0, –3 (b) 0, 0, –3 (c) 0, 0, 0, –3 (d) None of these

128. If –9 is a root of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}=0$, then the other two roots are **[IIT 1983; MNR 1992; MP PET 1995]**

(a) 2, 7 (b) –2, 7 (c) 2, –7 (d) –2, –7

129. If $a+b+c=0$, then one root of $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix}=0$ is **[UPSEAT 2001; Karnataka CET 1993]**

(a) $x=1$ (b) $x=2$ (c) $x=a^2+b^2+c^2$ (d) $x=0$

130. If $\begin{vmatrix} 4 & 3 & 3 \\ 3 & x & 3 \\ 3 & 3 & 4 \end{vmatrix}=0$, then x equals **[Rajasthan PET 1988]**

(a) 2 (b) 3 (c) 4 (d) None of these

131. The number of roots of the equation $\begin{vmatrix} 1 & x & x+1 \\ 1 & x & x+2 \\ 1 & 0 & x+3 \end{vmatrix}=0$ is **[AMU 1998]**

(a) 1 (b) 2 (c) 3 (d) 4

132. The roots of the equation $\begin{vmatrix} 1 & 1 & 1 \\ a & x & a \\ b & b & x \end{vmatrix}=0$ are **[Rajasthan PET 1992]**

(a) 1, 1 (b) 1, *a* (c) 1, b (d) *a, b*

133. If $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix}=0$, then the values of x are **[IIT 1991]**

(a) 1, 2 (b) –1, 2 (c) –1, –2 (d) 1, –2

134. If $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix}=0$, then $x=$ **[Kurukshetra CEE 1991]**

(a) 8/3 (b) 2/3 (c) 1/3 (d) None of these

135. The factors of $\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix}$ are **[Karnataka CET 1993]**

(a) $x-a, x-b$ and $x+a+b$ (b) $x+a, x+b$ and $x+a+b$ (c) $x+a, x+b$ and $x-a-b$ (d) $x-a,\ x-b$ and $x-a-b$

136. The roots of the equation $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix}=0$ are **[Karnataka CET 1992]**

(a) 1, 2 (b) –1, 2 (c) 1, –2 (d) –1, –2

137. A root of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix}=0$ is **[Roorkee 1991; Rajasthan PET 2001]**

(a) 6 (b) 3 (c) 0 (d) None of these

138. One of the root of the given equation $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix}=0$ is **[Tamilnadu Engg. 2002; MP PET 1988, 2002]**

(a) $-(a+b)$ (b) $-(b+c)$ (c) $-a$ (d) $-(a+b+c)$

139. If $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$, then **[Karnataka CET 1999]**

(a) a is one of the cube root of unity
(b) b is one of the cube root of unity
(c) $\left(\frac{a}{b}\right)$ is one of the cube root of unity
(d) $\left(\frac{a}{b}\right)$ is one of the cube root of –1

140. If $a \neq 6, b, c$ satisfy $\begin{vmatrix} a & 2b & 2c \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$, then $abc =$ **[EAMCET 2000]**

(a) $a+b+c$
(b) 0
(c) b^3
(d) $ab+bc$

141. At what value of x, will $\begin{vmatrix} x+\omega^2 & \omega & 1 \\ \omega & \omega^2 & 1+x \\ 1 & x+\omega & \omega^2 \end{vmatrix} = 0$ **[DCE 2000, 01]**

(a) $x=0$
(b) $x=1$
(c) $x=-1$
(d) None of these

142. $\begin{vmatrix} x+1 & \omega & \omega^2 \\ x+\omega & \omega^2 & 1 \\ x+\omega^2 & 1 & \omega \end{vmatrix} = 3$ is an equation of x, where ω, ω^2 are the complex cube roots of unity, what is the value of x **[DCE 2001]**

(a) 0
(b) 1
(c) –1
(d) 2

143. If 5 is one root of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & -2 \\ 7 & 8 & x \end{vmatrix} = 0$, then other two roots of the equation are **[Karnataka CET 2002]**

(a) –2 and 7
(b) –2 and –7
(c) 2 and 7
(d) 2 and –7

144. Solutions of the equation $\begin{vmatrix} 1 & 1 & x \\ p+1 & p+1 & p+x \\ 3 & x+1 & x+2 \end{vmatrix} = 0$ are **[AMU 2002]**

(a) $x=1,2$
(b) $x=2,3$
(c) $x=1,p,2$
(d) $x=1,2,-p$

145. If $\begin{vmatrix} 1+ax & 1+bx & 1+cx \\ 1+a_1x & 1+b_1x & 1+c_1x \\ 1+a_2x & 1+b_2x & 1+c_2x \end{vmatrix} = A_0 + A_1x + A_2x^2 + A_3x^3$, then A_1 is equal to **[AMU 2002]**

(a) abc
(b) 0
(c) 1
(d) None of these

146. If $\begin{vmatrix} x+1 & 1 & 1 \\ 2 & x+2 & 2 \\ 3 & 3 & x+3 \end{vmatrix} = 0$, then x is **[Kerala (Engg.) 2002]**

(a) 0, –6
(b) 0, 6
(c) 6
(d) –6

147. The values of x in the following determinant equation $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ **[MP PET 2003]**

(a) $x=0, x=4a$
(b) $x=0, x=a$
(c) $x=0, x=2a$
(d) $x=0, x=3a$

148. If $\begin{vmatrix} x-1 & 3 & 0 \\ 2 & x-3 & 4 \\ 3 & 5 & 6 \end{vmatrix} = 0$, then x = **[Rajasthan PET 2003]**

(a) 0 (b) 2 (c) 3 (d) 1

149. If $i = \sqrt{-1}$ and $\sqrt[4]{1} = \alpha, \beta, \gamma, \delta$ then $\begin{vmatrix} \alpha & \beta & \gamma & \delta \\ \beta & \gamma & \delta & \alpha \\ \gamma & \delta & \alpha & \beta \\ \delta & \alpha & \beta & \gamma \end{vmatrix}$ is equal to

(a) i (b) $-i$ (c) 1 (d) 0

150. If $\begin{vmatrix} a+x & a & x \\ a-x & a & x \\ a-x & a & -x \end{vmatrix} = 0$, then x is

(a) 0 (b) a (c) 3 (d) $2a$

151. The sum of two non-integral roots of $\begin{vmatrix} x & 2 & 5 \\ 3 & x & 3 \\ 5 & 4 & x \end{vmatrix} = 0$ is

(a) 5 (b) –5 (c) – 18 (d) None of these

Advance Level

152. The solution set of the equation $\begin{vmatrix} 2 & 3 & m \\ 2 & 1 & m^2 \\ 6 & 7 & 3 \end{vmatrix} = 0$ is **[AMU 1997]**

(a) (1, 2) (b) (1, –2) (c) (1, –3) (d) (0, 1)

153. If a, b, c are in A.P., then the value of $\begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$ is **[Rajasthan PET 1999]**

(a) $x-(a+b+c)$ (b) $9x^2+a+b+c$ (c) $(a+b+c)$ (d) 0

154. The value of x obtained from the equation $\begin{vmatrix} x+\alpha & \beta & \gamma \\ \gamma & x+\beta & \alpha \\ \alpha & \beta & x+y \end{vmatrix} = 0$ will be **[UPSEAT 1999]**

(a) 0 and $-(\alpha+\beta+\gamma)$ (b) 0 and $(\alpha+\beta+\gamma)$ (c) 1 and $(\alpha-\beta-\gamma)$ (d) 0 and $(\alpha^2+\beta^2+\gamma^2)$

155. If $ab+bc+ca=0$ and $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$, then one of the value of x is **[AMU 2000]**

(a) $(a^2+b^2+c^2)^{\frac{1}{2}}$ (b) $\left[\frac{3}{2}(a^2+b^2+c^2)\right]^{\frac{1}{2}}$ (c) $\left[\frac{1}{2}(a^2+b^2+c^2)\right]^{\frac{1}{2}}$ (d) None of these

156. If $\Delta_1 = \begin{vmatrix} 7 & x & 2 \\ -5 & x+1 & 3 \\ 4 & x & 7 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} x & 2 & 7 \\ x+1 & 3 & -5 \\ x & 7 & 4 \end{vmatrix}$ then $\Delta_1 - \Delta_2 = 0$ for

(a) $x=2$ (b) All real x (c) $x=0$ (d) None of these

157. If $\Delta_1 = \begin{vmatrix} 10 & 4 & 3 \\ 17 & 7 & 4 \\ 4 & -5 & 7 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 4 & x+5 & 3 \\ 7 & x+12 & 4 \\ -5 & x-1 & 7 \end{vmatrix}$ such that $\Delta_1 + \Delta_2 = 0$ then

(a) $x = 5$ (b) x has no real value (c) $x = 0$ (d) None of these

158. Let $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = ax^5 + bx^4 + cx^3 + dx^2 + \lambda x + \mu$ be an identity in x, where a, b, c, d, λ, μ are independent of x. Then the value of λ is

(a) 3 (b) 2 (c) 4 (d) None of these

159. Using the factor theorem it is found that $b+c, c+a$ and $a+b$ are three factors of determinant $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$. The other factor in the value of the determinant is

(a) 4 (b) 2 (c) $a+b+c$ (d) None of these

160. The roots of $\begin{vmatrix} x & a & b & 1 \\ \lambda & x & b & 1 \\ \lambda & \mu & x & 1 \\ \lambda & \mu & v & 1 \end{vmatrix} = 0$ are independent of

(a) λ, μ, v (b) a, b (c) λ, μ, v, a, b (d) None of these

Properties of Determinants

Basic Level

161. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 5$, then $\begin{vmatrix} 3a & 3b \\ 3c & 3d \end{vmatrix} =$

(a) 15 (b) 45 (c) 405 (d) None of these

162. If $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 9 & 13 \end{vmatrix}$ and $\Delta' = \begin{vmatrix} 7 & 20 & 29 \\ 2 & 5 & 7 \\ 3 & 9 & 13 \end{vmatrix}$, then

(a) $\Delta' = 3\Delta$ (b) $\Delta' = \frac{3}{\Delta}$ (c) $\Delta' = \Delta$ (d) $\Delta' = 2\Delta$

163. If $A = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$, $B = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$, $C = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$, then which relation is correct

(a) $A = B$ (b) $A = C$ (c) $B = C$ (d) None of these

164. If $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$, then $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix}$ is equal to **[Rajasthan PET 1986]**

(a) Δ (b) $k\Delta$ (c) $3k\Delta$ (d) $k^3\Delta$

165. If the entries in a 3×3 determinant are either 0 or 1, then the greatest value of this determinant is **[AMU 1988]**

(a) 1 (b) 2 (c) 3 (d) 9

166. If the elements of the first row of the product of $\begin{vmatrix} a & b \\ -b & a \end{vmatrix}$ and $\begin{vmatrix} c & d \\ -d & c \end{vmatrix}$ are respectively A and B then the elements of the second row of the product are **[Rajasthan PET 1987]**

(a) *A, B* (b) *B, A* (c) –*B, A* (d) –*B*, –*A*

167. If for every element of any determinant of odd order $a_{ji} = -a_{ij}$, then the value of determinant is

(a) 0 (b) 1 (c) –1 (d) Δ^2

168. The sum of the products of the elements of any row of a determinant *A* with the corresponding cofactors of the same row is always equal to **[Karnataka CET 2000]**

(a) $|A|$ (b) $|A|/2$ (c) 1 (d) 0

169. If $\Delta = \begin{vmatrix} a & c & b \\ b & b & a \\ c & a & c \end{vmatrix}$ and $\Delta' = \begin{vmatrix} a^2 & c^2 & b^2 \\ b^2 & b^2 & a^2 \\ c^2 & a^2 & c^2 \end{vmatrix}$, then **[Kurukshetra CEE 1997]**

(a) $\Delta = a^2b^2c^2\Delta'$ (b) $\Delta' = a^2b^2c^2\Delta$ (c) $\Delta = abc\Delta'$ (d) None of these

170. If $\Delta = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$, then $\begin{vmatrix} 2x+4p & p+6a & a \\ 2y+4q & q+6b & b \\ 2z+4r & r+6c & c \end{vmatrix}$ is equal to **[Rajasthan PET 1999]**

(a) 2Δ (b) 4Δ (c) 6Δ (d) Δ

171. If every element of a third order determinant of value Δ is multiplied by 5, then the value of new determinant is **[Roorkee 1993]**

(a) Δ (b) 5Δ (c) 25Δ (d) 125Δ

172. If $\Delta = \begin{vmatrix} x^n & x^{n+2} & x^{2n} \\ 1 & x^p & p \\ x^{n+5} & x^{p+6} & x^{2n+5} \end{vmatrix} = 0$, then p is given by

(a) x^n (b) $(n+1)$ (c) Either *A* or *B* (d) Both *A* and *B*

173. If $\Delta = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$, then $\begin{vmatrix} x & 2y & z \\ 2p & 4q & 2r \\ a & 2b & c \end{vmatrix}$ equals **[Rajasthan PET 1999]**

(a) Δ^2 (b) 4Δ (c) 3Δ (d) None of these

174. If $\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} = k$, then $\begin{vmatrix} 6a & 2b & 2c \\ 3m & n & p \\ 3x & y & z \end{vmatrix} =$ **[Tamilnadu (Engg.) 2002]**

(a) $k/6$ (b) $2k$ (c) $3k$ (d) $6k$

175. If $A = \begin{vmatrix} -1 & 2 & 4 \\ 3 & 1 & 0 \\ -2 & 4 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} -2 & 4 & 2 \\ 6 & 2 & 0 \\ -2 & 4 & 8 \end{vmatrix}$, then B is given by **[Tamilnadu (Engg.) 2002]**

(a) $B = 4A$ (b) $B = -4A$ (c) $B = -A$ (d) $B = 6A$

176. Let $\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}$, then $\Delta_1 \times \Delta_2$ can be expressed as the sum of how many determinants

[Tamilnadu (Engg.) 2001]

(a) 9 (b) 3 (c) 27 (d) 2

177. If $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$ then

(a) $\Delta_1 + \Delta_2 = 0$ (b) $\Delta_1 + 2\Delta_2 = 0$ (c) $\Delta_1 = \Delta_2$ (d) None of these

Advance Level

178. In a third order determinant, each element of the first column consists of sum of two terms, each element of the second column consists of sum of three terms and each element of the third column consists of sum of four terms. Then it can be decomposed into *n* determinants, where *n* has the value **[Roorkee 1993]**

(a) 1 (b) 9 (c) 16 (d) 24

179. If $\Delta_1 = \begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$, then

(a) $\Delta_1 = 2\Delta_2$ (b) $\Delta_2 = 2\Delta_1$ (c) $\Delta_1 = \Delta_2$ (d) None of these

180. Consider the following statements with reference to determinants

(I) The value of determinant is unchanged if the rows and columns are interchanged

(II) If any two rows or columns of a determinant are interchanged, the sign of the determinant is changed.

(III) If any two rows or columns are identical, the value of determinant is zero

(a) I and III are correct (b) II and III are correct (c) Only I is correct (d) I, II and III are correct

181. Let a_{ij} denote the element of the i^{th} row and j^{th} column in a 3×3 determinant ($1 \le i \le 3, 1 \le j \le 3$) and let $a_{ij} = -a_{ji}$ for every i and j. Then the determinant has all the principal diagonal elements as **[AMU 1992]**

(a) 1 (b) –1 (c) 0 (d) None of these

182. If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 5$, then the value of $\begin{vmatrix} b_2c_3 - b_3c_2 & c_2a_3 - c_3a_2 & a_2b_3 - a_3b_2 \\ b_3c_1 - b_1c_3 & c_3a_1 - c_1a_3 & a_3b_1 - a_1b_3 \\ b_1c_2 - b_2c_1 & c_1a_2 - c_2a_1 & a_1b_2 - a_2b_1 \end{vmatrix}$ is

[Tamilnadu (Engg.) 2002]

(a) 5 (b) 25 (c) 125 (d) 0

183. Two non-zero distinct numbers *a, b* are used as elements to make determinants of the third order. The number of determinants whose value is zero for all *a, b* is

(a) 24 (b) 32 (c) $a+b$ (d) None of these

Minors and Cofactors

Basic Level

184. If in the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, A_1, B_1, C_1 etc. be the co-factors of a_1, b_1, c_1 etc., then which of the following relations is incorrect

(a) $a_1A_1 + b_1B_1 + c_1C_1 = \Delta$ (b) $b_1B_1 + b_2B_2 + c_2C_2 = \Delta$ (c) $a_3A_3 + b_3B_3 + c_3C_3 = \Delta$ (d) $a_1A_2 + b_1B_2 + c_1C_2 = \Delta$

185. If $A_1, B_1, C_1, ...$ are respectively the co-factors of the elements $a_1, b_1, c_1, ...$ of the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, then $\begin{vmatrix} B_2 & C_2 \\ B_3 & C_3 \end{vmatrix} =$

(a) $a_1\Delta$ (b) $a_1a_3\Delta$ (c) $(a_1 + b_1)\Delta$ (d) None of these

186. The cofactors of 1, −2, −3 and 4 in $\begin{vmatrix} 1 & -2 \\ -3 & 4 \end{vmatrix}$ are

(a) 4, 3, 2, 1 (b) −4, 3, 2, −1 (c) 4, −3, −2, 1 (d) −4, −3, −2, −1

187. The cofactor of 2 in $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 9 \end{vmatrix}$ is

(a) 1 (b) −5 (c) 8 (d) −8

188. If cofactor of $2x$ in the determinant $\begin{vmatrix} x & 1 & -2 \\ 1 & 2x & x-1 \\ x-1 & x & 0 \end{vmatrix}$ is zero, then x equals to

(a) 0 (b) 2 (c) 1 (d) −1

189. The cofactor of element 0 in determinant $\begin{vmatrix} -1 & 0 \\ 2 & 2 \end{vmatrix}$ is

(a) −1 (b) 0 (c) 2 (d) −2

190. The cofactor of element 0 in determinant $\begin{vmatrix} -1 & 2 & 1 \\ -2 & 3 & -3 \\ 4 & 0 & -4 \end{vmatrix}$ is

(a) 2 (b) 5 (c) −5 (d) 9

191. The minors of the element of the first row in the determinant $\begin{vmatrix} 2 & -1 & 4 \\ 4 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix}$ are

(a) 2, 7, 11 (b) 7, 11, 2 (c) 11, 2, 7 (d) 7, 2, 11

Advance Level

192. The value of the determinant Δ of 3rd order is 9 then the value of Δ'^2 where Δ' is a determinant formed by cofactors of the element of Δ is

(a) 9 (b) 81 (c) 729 (d) 6561

Product and Summation of Determinants

Basic Level

193. If $\Delta_1 = \begin{vmatrix} 1 & 0 \\ a & b \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 1 & 0 \\ c & d \end{vmatrix}$, then $\Delta_2\Delta_1$ is equal to **[Rajasthan PET 1984]**

(a) ac (b) bd (c) $(b-a)(d-c)$ (d) None of these

194. $\begin{vmatrix} 2 & 3 \\ -1 & 5 \end{vmatrix} \times \begin{vmatrix} 1 & 0 \\ -1 & 3 \end{vmatrix}$ equals

(a) $\begin{vmatrix} 2 & 0 \\ 1 & 15 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & -3 \\ 0 & 15 \end{vmatrix}$ (c) $\begin{vmatrix} -1 & 16 \\ 2 & 7 \end{vmatrix}$ (d) $\begin{vmatrix} 2 & 7 \\ -1 & 16 \end{vmatrix}$

195. The value of $\begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 4 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 3 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$ is

(a) $\begin{vmatrix} 10 & 10 \\ 4 & 12 \end{vmatrix}$ (b) $\begin{vmatrix} 10 & 10 \\ 1 & 3 \end{vmatrix}$

(c) $\begin{vmatrix} 24 & 24 \\ 1 & 3 \end{vmatrix}$ (d) $\begin{vmatrix} 24 & 24 \\ 1 & 81 \end{vmatrix}$

196. If $D_r = \begin{vmatrix} 1 & n & n \\ 2r & n^2+n+1 & n^2+n \\ 2r-1 & n^2 & n^2+n+1 \end{vmatrix}$ and $\sum_{r=1}^{n} D_r = 56$, then n equals

(a) 4 (b) 6 (c) 7 (d) 8

Advance Level

197. $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix} =$ **[Tamilnadu (Engg.) 2002]**

(a) 7 (b) 10 (c) 13 (d) 17

198. If $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$ = square of determinant Δ of the third order then Δ is equal to

(a) $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}$ (b) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ (c) $\begin{vmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$ (d) None of these

199. If $D_r = \begin{vmatrix} 2^{r-1} & 2.3^{r-1} & 4.5^{r-1} \\ x & y & z \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$, then the value of $\sum_{r=1}^{n} D_r =$

(a) 1 (b) –1 (c) 0 (d) None of these

200. Let $D_r = \begin{vmatrix} a & 2^r & 2^{16}-1 \\ b & 3(4^r) & 2(4^{16}-1) \\ c & 7(8^r) & 4(8^{16}-1) \end{vmatrix}$, then the value of $\sum_{r=1}^{16} D_r$ is

(a) 0 (b) $a+b+c$ (c) $ab+bc+ca$ (d) None of these

201. Let m be a positive integer and $\Delta_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$, $(0 \le r \le m)$, then the value of $\sum_{r=0}^{m} \Delta_r$ is given by

(a) 0 (b) m^2-1 (c) 2^m (d) $2^m \sin^2(2^m)$

202. If $U_n = \begin{vmatrix} n & 15 & 8 \\ n^2 & 35 & 9 \\ n^3 & 25 & 10 \end{vmatrix}$, then $\sum_{n=1}^{5} U_n =$

(a) 0 (b) 25 (c) 625 (d) None of these

203. If $\Delta_a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$, then $\sum_{a=1}^{n} \Delta_a$ is equal to

(a) 0 (b) 1 (c) $\left(\frac{n(n+1)}{2}\right)\left(\frac{a(a+1)}{2}\right)$ (d) None of these

204. If $\Delta_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2-1 & y & n^2(2n+3) \\ 4r^3-2nr & z & n^3(n+1) \end{vmatrix}$, then the value of $\sum_{r=1}^{n} \Delta_r$ is independent of

(a) x (b) y (c) z (d) n

205. If $D_r = \begin{vmatrix} r & x & \frac{n(n+1)}{2} \\ 2r-1 & y & n^2 \\ 3r-2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$, then $\sum_{r=1}^{n} D_r$ is equal to

(a) $\frac{1}{6}n(n+1)(2n+1)$ (b) $\frac{1}{4}n^2(n+1)^2$ (c) 0 (d) None of these

Differentiation and Integration of Determinants

Basic Level

206. Let f, g, h and k be differentiable in (a, b). If F is defined as $F(x) = \begin{vmatrix} f(x) & g(x) \\ h(x) & k(x) \end{vmatrix}$, then $F'(x)$ is given by **[SCRA 1999]**

(a) $\begin{vmatrix} f & g \\ h & k \end{vmatrix} + \begin{vmatrix} f & g \\ h' & k' \end{vmatrix}$ (b) $\begin{vmatrix} f' & g' \\ h & k \end{vmatrix} + \begin{vmatrix} f & g \\ h' & k' \end{vmatrix}$ (c) $\begin{vmatrix} f & g' \\ h & k' \end{vmatrix} + \begin{vmatrix} f' & g \\ h & k' \end{vmatrix}$ (d) $\begin{vmatrix} f & g \\ h & k' \end{vmatrix} + \begin{vmatrix} f' & g \\ h & k \end{vmatrix}$

207. If $f(x) = \begin{vmatrix} \cos x & 1 & 0 \\ 1 & \cos x & 1 \\ 0 & 1 & \cos x \end{vmatrix}$, then $f'(\pi/3)$ equals

(a) $\frac{11\sqrt{3}}{8}$ (b) $\frac{5\sqrt{3}}{8}$ (c) $\frac{-5\sqrt{3}}{8}$ (d) None of these

208. If $\Delta = \begin{vmatrix} x^2 & \sin x \\ x & e^x \end{vmatrix}$, then $\left(\frac{d\Delta}{dx}\right)_{x=0}$ is equal to

(a) 0 (b) –1 (c) 1 (d) 2

209. Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$, then $\lim_{x \to 0} \frac{f'(x)}{x}$ equals

(a) 0 (b) 1 (c) –2 (d) None of these

210. If $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec}^2 x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \operatorname{cosec}^2 x \end{vmatrix}$, then $\int_0^{\pi/2} f(x)dx =$

(a) $\frac{\pi}{4} + \frac{8}{15}$ (b) $\left(-\frac{\pi}{4} + \frac{8}{15}\right)$ (c) $-\left(\frac{\pi}{4} + \frac{8}{15}\right)$ (d) None of these

Advance Level

211. If $f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 2x\\ \sin^2 x & 1+\cos^2 x & 4\sin 2x\\ \sin^2 x & \cos^2 x & 1+4\sin 2x\end{vmatrix}$, then maximum value of $f(x)$ is

(a) 0 (b) 2 (c) 4 (d) 6

212. If $\begin{vmatrix}\alpha & x & x & x\\ x & \beta & x & x\\ x & x & \gamma & x\\ x & x & x & \delta\end{vmatrix}=f(x)-xf'(x)$, then $f(x)$ is equal to

(a) $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$ (b) $(x+\alpha)(x+\beta)(x+\gamma)(x+\delta)$

(c) $2(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$ (d) None of these

213. If $F(x), G(x)$ and $H(x)$ are three polynomials of degree 2, then $\phi(x)=\begin{vmatrix}F(x) & G(x) & H(x)\\ F'(x) & G'(x) & H'(x)\\ F''(x) & G''(x) & H''(x)\end{vmatrix}$ is polynomial of degree

(a) 2 (b) 3 (c) 4 (d) 0

214. Let $f(x)=\begin{vmatrix}x^3 & \sin x & \cos x\\ 6 & -1 & 0\\ p & p^2 & p^3\end{vmatrix}$, where p is a constant, then $\frac{d^3}{dx^3}[f(x)]$ at $x=0$ is **[IIT 1997]**

(a) p (b) $p+p^2$ (c) $p+p^3$ (d) Independent of p

215. If $f(x)=\begin{vmatrix}x^n & \sin x & \cos x\\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2}\\ p & p^2 & p^3\end{vmatrix}$, then $\frac{d^n}{dx^n}(f(x))$ at $x = 0$ is

(a) 0 (b) p (c) p^3 (d) Independent of p

216. If $f(x)=\begin{vmatrix}\cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2)\\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2)\\ \sin 2x & 0 & \sin 2x^2\end{vmatrix}$, then $f'(0)=$

(a) 4 (b) 2 (c) 3 (d) 0

System of Linear Equations

Basic Level

217. If $x+y-z=0,\ 3x-\alpha y-3z=0,\ x-3y+z=0$ has non-zero solution, then $\alpha=$ **[MP PET 1990]**

(a) –1 (b) 0 (c) 1 (d) –3

218. The number of solutions of the equations $x+4y-z=0, 3x-4y-z=0,\ x-3y+z=0$ is **[MP PET 1992]**

(a) 0 (b) 1 (c) 2 (d) Infinite

219. The following system of equations $3x-2y+z=0, \lambda x-14y+15z=0,\ x+2y-3z=0$ has a solution other than $x=y=z=0$ for λ equal to **[MP PET 1990]**

(a) 1 (b) 2 (c) 3 (d) 5

220. The number of solutions of equations $x+y-z=0,\ 3x-y-z=0,\ x-3y+z=0$ is **[MP PET 1992]**

(a) 0 (b) 1 (c) 2 (d) Infinite

221. If the system of following equations $2x+3y+5=0,\ x+ky+5=0,\ kx-12y-14=0$ be consistent, then k =

(a) $-2, \frac{12}{5}$ (b) $-1, \frac{1}{5}$ (c) $-6, \frac{17}{5}$ (d) $6, -\frac{12}{5}$

222. If the equation $x = ay + z,\ y = z + ax,\ z = x + y$ have non-zero solutions, then

(a) $a^2 + 1 = 0$ (b) $a^3 + 1 = 0$ (c) $a + 1 = 0$ (d) $a - 1 = 0$

223. If the system of equations $3x - y + 4z - 3 = 0$, $x + 2y - 3z + 2 = 0$, $6x + 5y + \lambda z + 3 = 0$ has infinite number of solutions, then $\lambda =$

(a) 7 (b) −7 (c) 5 (d) −5

224. The system of equations $4x - 5y - 2z = 2,\ 5x - 4y + 2z = 3$ and $2x + 2y + 8z = 1$ is

(a) Consistent (unique solution) (b) Inconsistent

(c) Consistent (infinite solutions) (d) None of these

225. The system of equations $\lambda x + y + z = 0$, $-x + \lambda y + z = 0$, $-x - y + \lambda z = 0$, will have a non-zero solution if real values of λ are given by **[IIT 1984]**

(a) 0 (b) 1 (c) 3 (d) $\sqrt{3}$

226. The equations $x + 2y + 3z = 1, x - y + 4z = 0$ and $2x + y + 7z = 1$ have **[Karnataka CET 1992]**

(a) Only one solution (b) Only two solutions (c) No solution (d) Infinitely many solutions

227. The number of solutions of $2x + y = 4,\ x - 2y = 2,\ 3x + 5y = 6$ is **[Karnataka CET 1991]**

(a) 0 (b) 1 (c) 2 (d) Infinitely many

228. The existence of unique solution of the system $x + y + z = b,\ 2x + 3y - z = 6,\ 5x - y + az = 10$ depends on **[Kurukshetra CEE 2**

(a) *b* only (b) *a* only (c) *a* and *b* (d) Neither *a* nor *b*

229. The value of *k* for which the set of equations $3x + ky - 2z = 0$, $x + ky + 3z = 0$ and $2x + 3y - 4z = 0$ has a non-trivial solution is

[Kurukshetra CEE 1996]

(a) 15 (b) 16 (c) 31/2 (d) 33/2

230. $x + y + z = 6, x - y + z = 2$ and $2x + y - z = 1$ then *x, y, z* are respectively

(a) 3, 2, 1 (b) 1, 2, 3 (c) 2, 1, 3 (d) None of these

231. Consider the system of equations $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$, $a_3x + b_3y + c_3z = 0$ if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, then the system has **[Roorkee 1990]**

(a) More than two solutions (b) One trivial and one non-trivial solutions

(c) No solution (d) Only trivial solution (0, 0, 0)

232. If $2x + 3y - 5z = 7$, $x + y + z = 6$, $3x - 4y + 2z = 1$, then $x =$ **[MP PET 1987]**

(a) $\begin{vmatrix} 2 & -5 & 7 \\ 1 & 1 & 6 \\ 3 & 2 & 1 \end{vmatrix} \div \begin{vmatrix} 7 & 3 & -5 \\ 6 & 1 & 1 \\ 1 & -4 & 2 \end{vmatrix}$ (b) $\begin{vmatrix} -7 & 3 & -5 \\ -6 & 1 & 1 \\ -1 & -4 & 2 \end{vmatrix} \div \begin{vmatrix} 2 & 3 & -5 \\ 1 & 1 & 1 \\ 3 & -4 & 2 \end{vmatrix}$

(c) $\begin{vmatrix} 7 & 3 & -5 \\ 6 & 1 & 1 \\ 1 & -4 & 2 \end{vmatrix} \div \begin{vmatrix} 2 & 3 & -5 \\ 1 & 1 & 1 \\ 3 & -4 & 2 \end{vmatrix}$ (d) None of these

233. The system of equations $x + y + z = 2$, $3x - y + 2z = 6$ and $3x + y + z = -18$ has **[Kurukshetra CEET 2002]**

(a) A unique solution (b) No solution

(c) An infinite number of solutions (d) Zero solution as the only solution

234. If $2x+3y+4z=9,\ 4x+9y+3z=10,\ 5x+10y+5z=11$ then find the value of x **[UPSEAT 2002]**

(a) $\begin{vmatrix} 9 & 3 & 4 \\ 10 & 9 & 3 \\ 11 & 10 & 5 \end{vmatrix} \div \begin{vmatrix} 2 & 3 & 4 \\ 4 & 9 & 3 \\ 5 & 10 & 5 \end{vmatrix}$ (b) $\begin{vmatrix} 9 & 4 & 3 \\ 10 & 3 & 9 \\ 11 & 5 & 10 \end{vmatrix} \div \begin{vmatrix} 2 & 3 & 4 \\ 4 & 9 & 3 \\ 5 & 10 & 5 \end{vmatrix}$

(c) $\begin{vmatrix} 9 & 4 & 9 \\ 10 & 3 & 3 \\ 11 & 5 & 10 \end{vmatrix} \div \begin{vmatrix} 3 & 2 & 4 \\ 9 & 4 & 3 \\ 10 & 5 & 5 \end{vmatrix}$ (d) None of these

235. Equations $x+y=2,\ 2x+2y=3$ will have **[UPSEAT 1999]**

(a) Only one solution (b) Many finite solutions (c) No solution (d) None of these

236. If the system of equations $x-ky-z=0,\ kx-y-z=0$ and $x+y-z=0$ has a non zero solution, then the possible value of k are **[IIT Screening 2000]**

(a) –1, 2 (b) 1, 2 (c) 0, 1 (d) –1, 1

237. The system of equations $x_1-x_2+x_3=2$, $3x_1-x_2+2x_3=-6$ and $3x_1+x_2+x_3=-18$ has **[AMU 2001]**

(a) No solution (b) Exactly one solution (c) Infinite solutions (d) None of these

238. The existence of the unique solution of the system $x+y+z=\lambda,\ \ 5x-y+\mu z=10,\ \ 2x+3y-z=6$ depends on **[Kurukshetra CEE 2002]**

(a) μ only (b) λ only (c) λ and μ both (d) Neither λ nor μ

239. The number of solutions of the following equations $x_2-x_3=1$, $-x_1+2x_3=-2$, $x_1-2x_2=3$ is **[MP PET 2000]**

(a) Zero (b) One (c) Two (d) Infinite

240. The number of values of k for which the system of equations $(k+1)x+8y=4k,\ \ kx+(k+3)y=3k-1$ has infinitely many solutions, is **[IIT Screening 2002]**

(a) 0 (b) 1 (c) 2 (d) Infinite

241. For what value of λ, the system of equations $x+y+z=6,\ \ x+2y+3z=10,\ \ x+2y+\lambda z=12$ is inconsistent **[AIEEE 2002]**

(a) $\lambda=1$ (b) $\lambda=2$ (c) $\lambda=-2$ (d) $\lambda=3$

242. The values of the *x, y, z* in order, of the system of equations $3x+y+2z=3,\ \ 2x-3y-z=-3,\ \ x+2y+z=4$ are **[MP PET**

(a) 2, 1, 5 (b) 1, 1, 1 (c) 1, –2, –1 (d) 1, 2, –1

243. The number of solutions of the system of equations $2x+y-z=7,\ \ x-3y+2z=1,\ \ x+4y-3z=5$ is **[EAMCET 2003]**

(a) 3 (b) 2 (c) 1 (d) 0

244. The system of equations $x+y+z=6,\ \ x+2y+3z=10,\ \ x+2y+\lambda z=\mu$ has no solution for **[Orissa JEE 2003]**

(a) $\lambda\neq 3,\ \mu=10$ (b) $\lambda=3,\ \mu\neq 10$ (c) $\lambda\neq 3,\ \mu\neq 10$ (d) None of these

245. The system of equations $ax+4y+z=0,\ bx+3y+z=0,\ \ cx+2y+z=0$ has non-trivial solution if *a, b, c* are in

(a) AP (b) GP (c) HP (d) None of these

246. The system of equations $2x-y+z=0, x-2y+z=0, \lambda x-y+2z=0$ [has infinite number of nontrivial solutions for]

(a) $\lambda=1$ (b) $\lambda=5$ (c) $\lambda=-5$ (d) No real value of λ

247. The system of equations $2x+3y=8,\ \ 7x-5y+3=0,\ \ 4x-6y+\lambda=0$ is solvable if λ is

(a) 6 (b) 8 (c) –8 (d) –6

248. The system of the equations $x+2y+3z=4,\ \ 2x+3y+4z=5,\ \ 3x+4y+5z=6$ has

(a) Infinitely many solutions (b) No solution (c) Unique solution (d)None

249. If the equations $x=ay+z,\ \ y=az+x$ and $z=ax+y$ are the consistent having non-trivial solution, then

(a) $a^3=1$ (b) $a^3+1=0$ (c) $a+1=0$ (d) None of these

Advance Level

250. The value of a for which the system of equations $a^3x+(a+1)^3y+(a+2)^3z=0$, $ax+(a+1)y+(a+2)z=0$ and $x+y+z=0$ has a non-zero solution is **[Pb. CET 2000]**

(a) –1 (b) 0 (c) 1 (d) None of these

251. The values of λ which make the system of linear equations $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ to have the so **[Kurukshetra CEE 1991]**

(a) $\lambda=3$ or 4 (b) $\lambda=0$ or ∞

(c) $\lambda=1$ or 2 (d) $\lambda=K$, where K is an arbitrary constant

252. If the equations $x=cy+bz$, $y=az+cx$ and $z=bx+ay$ have a solution other than $x=0, y=0$ and $z=0$, then *a, b* and *c* are connected by the relation **[MP PET 1998]**

(a) $a^3+b^3+c^3+3abc=0$ (b) $a^2+b^2+c^2+2abc=1$ (c) $a^2+b^2+c^2-bc-ca-ab=0$ (d) None of these

253. If $a=\frac{x}{y-z}$, $b=\frac{y}{z-x}$ and $c=\frac{z}{x-y}$ where *x, y, z* are not all zero, then $ab+bc+ca=$

(a) 0 (b) 1 (c) –1 (d) 2

254. Let *a, b, c* be positive real numbers. the following system of equations in *x, y,* and *z*

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, $-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ has **[IIT 1995]**

(a) No solution (b) Unique solution (c) Infinitely many solutions (d)Finitely many solutions

255. The system of the linear equations $x+y+z=2$, $2x+y-z=3$, $3x+2y+kz=4$ has a unique solution if **[EAMCET 1994; DCE 2000]**

(a) $k\neq 0$ (b) $-1<k<1$ (c) $-2<k<2$ (d) $k=0$

256. If $a>b>c$ and the system of equations $ax+by+cz=0$, $bx+cy+az=0$, $cx+ay+bz=0$ has a non-trivial solution then both the roots of the quadratic equation at^2+bt+c are

(a) Real (b) Of opposite sign (c) Positive (d) Complex

257. If $x^ay^b=e^m$, $x^cy^d=e^n$, $\Delta_1=\begin{vmatrix} m & b \\ n & d\end{vmatrix}$, $\Delta_2=\begin{vmatrix} a & m \\ c & n\end{vmatrix}$, $\Delta_3=\begin{vmatrix} a & b \\ c & d\end{vmatrix}$, then the values of x and y are respectively **[Tamilnadu (Engg.) 2002]**

(a) Δ_1/Δ_3 and Δ_2/Δ_3 (b) Δ_2/Δ_1 and Δ_3/Δ_1 (c) $\log(\Delta_1/\Delta_3)$ and $\log(\Delta_2/\Delta_3)$ (d) e^{Δ_1/Δ_3} and e^{Δ_2/Δ_3}

258. If the equations $a(y+z)=x$, $b(z+x)=y$, $c(x+y)=z$, where $a\neq -1$, $b\neq -1$, $c\neq -1$, admit of nontrivial solutions then $(1+a)^{-1}+(1+b)^{-1}+(1+c)^{-1}$ is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) None of these

259. If the system of equations $ax+by+c=0$, $bx+cy+a=0$ and $cx+ay+b=0$ has a solution then the system of equations

$(b+c)x+(c+a)y+(a+b)z=0$

$(c+a)x+(a+b)y+(b+c)z=0$

$(a+b)x+(b+c)y+(c+a)z=0$, has

(a) Only one solution (b) No solution (c) Infinite number of solutions (d) None of these

Some Special Determinants

Basic Level

260. The value of $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ is

(a) $abc + 2fgh - af^2 - bg^2 - ch^2$ (b) $abc + fgh - af^2 - bg^2 - ch^2$

(c) $2abc + fgh - af^2 - bg^2 - ch^2$ (d) None of these

261. The value of $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$ is

(a) 0 (b) abc (c) $(a-b)(b-c)(c-a)$ (d) None of these

262. The value of the determinant $\begin{vmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{vmatrix}$ is

(a) fgh (b) $-fgh$ (c) 0 (d) $1/fgh$

263. The value of an even order skew symmetric determinant is

(a) 0 (b) Perfect square (c) ± 1 (d) None of these

264. The value of an odd order skew symmetric determinant is

(a) Perfect square (b) Negative

(c) ± 1 (d) 0

265. In a skew-symmetric matrix, the diagonal elements are all **[MP PET 1987]**

(a) Different from each other (b) Zero (c) One (d)None

Advance Level

266. The value of $\begin{vmatrix} 0 & 2+3i & \frac{3}{2}-5i \\ -2+3i & 0 & 7-4i \\ -\frac{3}{2}-5i & -7-4i & 0 \end{vmatrix}$ is

(a) Purely real (b) Purely imaginary (c) Non real complex (d) None of these

Miscellaneous Problems

Basic Level

267. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$, then the two triangles whose vertices are (x_1, y_1), (x_2, y_2), (x_3, y_3) and (a_1, b_1), (a_2, b_2), (a_3, b_3), are

(a) Congruent (b) Similar (c) Equal in area (d) None of these

Advance Level

268. If *A, B* and *C* are the angles of a triangle and

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix}=0$$, then the triangle must be

(a) Equilateral (b) Isosceles (c) Any triangle (d) Right angled.

269. If $f(x)=ax^6+bx^5+cx^4+dx^3+ex^2+fx+g=\begin{vmatrix} x^2-2x+3 & 7x+2 & x+4 \\ 2x+7 & x^2-x+2 & 3x \\ 3 & 2x-1 & x^2-4x+7 \end{vmatrix}$, then $g=$

(a) –200 (b) 100 (c) 112 (d) –108

270. If *a, b, c* are the sides of a ΔABC and *A, B, C* are respectively the angles opposite to them, then $\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos(B-C) \\ c\sin A & \cos(B-C) & 1 \end{vmatrix}$ is equal to

(a) $\sin A-\sin B\sin C$ (b) *abc* (c) 1 (d) 0

271. If $\Delta=\begin{vmatrix} \frac{1}{z} & \frac{1}{z} & -\frac{(x+y)}{z^2} \\ -\frac{(y+z)}{x^2} & \frac{1}{x} & \frac{1}{x} \\ -\frac{y(y+z)}{x^2z} & \frac{x+2y+z}{xz} & -\frac{y(x+y)}{xz^2} \end{vmatrix}$, then Δ is independent of

(a) *x* (b) *y* (c) *z* (d) $\Delta=0$

272. If $f(x)=\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x)-f(x)$ is equal to

(a) *ax* (b) $ax(2a+3x)$ (c) $ax(2+3x)$ (d) None of these

273. If *f(x)* is a polynomial satisfying $f(x)=\frac{1}{2}\begin{vmatrix} f(x) & f\left(\frac{1}{x}\right)-f(x) \\ 1 & f\left(\frac{1}{x}\right) \end{vmatrix}$ and $f(2)=17$, then the value of $f(5)$ is

(a) 126 (b) 626 (c) –124 (d) 624

274. If $\Delta=\begin{vmatrix} \tan x & \tan(x+h) & \tan(x+2h) \\ \tan(x+2h) & \tan x & \tan(x+h) \\ \tan(x+h) & \tan(x+2h) & \tan x \end{vmatrix}$, then $\lim_{h\to 0}\frac{\Delta}{h^2}$ is equal to

(a) $\tan x\sec^4 x$ (b) $9\tan x\sec^2 x$ (c) $\tan x\sec^4 x$ (d) $9\tan x\sec x$

275. If $x^2+y^2+z^2=1$, then $\Delta=\begin{vmatrix} x^2+(y^2+z^2)\cos\theta & xy(1-\cos\theta) & xz(1-\cos\theta) \\ xy(1-\cos\theta) & y^2+(z^2+x^2)\cos\theta & yz(1-\cos\theta) \\ zx(1-\cos\theta) & yz(1-\cos\theta) & z^2+(x^2+y^2)\cos\theta \end{vmatrix}$ is independent of

(a) *x, y, z* (b) *y* only (c) *z* only (d) θ only

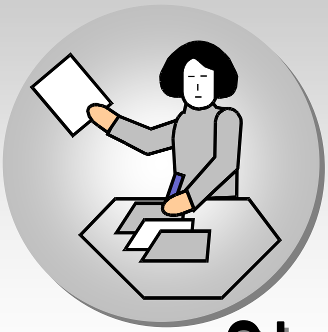
276. If $x+y+z=\pi$, then the value of the determinant $\begin{vmatrix} \sin(x+y+z) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$ is **[Pb. CET 1999]**

(a) 0 (b) $2\sin B\tan A\cos C$ (c) 1 (d) None of these

277. If $[a]$ denotes the greatest integer less than or equal to a and $-1\le x<0,\ 0\le y<1,\ 1\le z<2$, then $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is equal to

(a) $[x]$ (b) $[y]$ (c) $[z]$ (d) None of these

***



Answer Sheet

Determinants — ***Assignment (Basic and Advance Level)***

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
d	b	a	b	c	a	d	c	d	d	d	b	d	c	b	c	b	a	c	c
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
b	d	d	b	c	b	a	a	b	c	b	d	a	b	a	d	b	c	b	b
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
a	b	c	c	b	a	c	a	c	d	a	d	a	b	c	d	b	b	d	d
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
d	d	d	a	c,d	c	b	b	d	d	a	d	c	b	b	b	c	b	d	a
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
a	a	a	c	c	d	a	d	c	a	b	d	d	a	b	b	c	b	c	c
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
b	a	d	b	c	c	b	d	a	b	d	d	c	c	b	b	d	a	c	d
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
b	c	c	c	b	b	b	a	d	d	a	d	b	b	a	b	c	d	d	c
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
a	b	d	a	b	a	d	d	d	a	b	c	d	a	a	b	a	a	a	b
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
b	c	d	d	b	c	a	a	d	a	d	d	b	d	b	c	a	d	b	d
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
c	b	b	d	a	a	c	c	d	c	b	d	b	d	b	c	b	a	c	a
201	**202**	**203**	**204**	**205**	**206**	**207**	**208**	**209**	**210**	**211**	**212**	**213**	**214**	**215**	**216**	**217**	**218**	**219**	**220**
a	d	a	a,b,c,d	c	b	b	a	c	d	d	a	d	d	a,d	b	d	b	d	d
221	**222**	**223**	**224**	**225**	**226**	**227**	**228**	**229**	**230**	**231**	**232**	**233**	**234**	**235**	**236**	**237**	**238**	**239**	**240**
c	c	d	b	a	d	b	b	d	b	a	c	a	a	c	d	c	a	a	b
241	**242**	**243**	**244**	**245**	**246**	**247**	**248**	**249**	**250**	**251**	**252**	**253**	**254**	**255**	**256**	**257**	**258**	**259**	**260**
d	d	d	b	a	c	b	a	d	a	c	b	c	b	a	b	d	a	b	a
261	**262**	**263**	**264**	**265**	**266**	**267**	**268**	**269**	**270**	**271**	**272**	**273**	**274**	**275**	**276**	**277**			
a	c	b	d	b	b	c	b	d	d	a,b,c,d	b	b	b	a	a	c			

8.2 Matrices

8.2.1 Definition

A rectangular arrangement of numbers (which may be real or complex numbers) in rows and columns, is called a matrix. This arrangement is enclosed by small ( ) or big [ ] brackets. The numbers are called the elements of the matrix or entries in the matrix. A matrix is represented by capital letters *A, B, C etc.* and its elements by small letters *a,b,c,x,y etc.* The following are some examples of matrices:

$$A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix},\ B=\begin{bmatrix}2+i & -3 & 2\\ 1 & -3+i & -5\end{bmatrix},\ C=[1,\ 4,\ 9],\ \ D=\begin{bmatrix}a\\ g\\ h\end{bmatrix},\ \ E=[l]$$

8.2.2 Order of a Matrix

A matrix having m rows and n columns is called a matrix of order $m\times n$ or simply $m\times n$ matrix (read as 'an m by n matrix). A matrix A of order $m\times n$ is usually written in the following manner

$$A=\begin{bmatrix}a_{11} & a_{12} & a_{13} & ...a_{1j} & ...a_{1n}\\ a_{21} & a_{22} & a_{23} & ...a_{2j} & ...a_{2n}\\ ..... & ..... & ..... & ..... & .....\\ a_{i1} & a_{i2} & a_{i3} & ...a_{ij} & ...a_{in}\\ ..... & ..... & ..... & ..... & .....\\ a_{m1} & a_{m2} & a_{m3} & ...a_{mj} & ...a_{mn}\end{bmatrix} \text{ or } A=[a_{ij}]_{m\times n}, \text{ where } \begin{matrix}i=1,\ 2,.....m\\ j=1,\ 2,.....n\end{matrix}$$

Here a_{ij} denotes the element of ith row and jth column. *Example* : order of matrix $\begin{bmatrix}3 & -1 & 5\\ 6 & 2 & -7\end{bmatrix}$ is 2×3

Note : ❑ A matrix of order $m\times n$ contains mn elements. Every row of such a matrix contains n elements and every column contains m elements.

8.2.3 Equality of Matrices

Two matrix A and B are said to be equal matrix if they are of same order and their corresponding elements are equal *Example*: If $A=\begin{bmatrix}1 & 6 & 3\\ 5 & 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{bmatrix}$ are equal matrices.

Then $a_1=1, a_2=6, a_3=3, b_1=5, b_2=2, b_3=1$

8.2.4 Types of Matrices

(1) **Row matrix** : A matrix is said to be a row matrix or row vector if it has only one row and any number of columns. *Example* : $[5\ \ 0\ \ 3]$ is a row matrix of order 1×3 and $[2]$ is a row matrix of order 1×1.

(2) **Column matrix** : A matrix is said to be a column matrix or column vector if it has only one column and any number of rows. *Example* : $\begin{bmatrix} 2 \\ 3 \\ -6 \end{bmatrix}$ is a column matrix of order 3×1 and [2] is a column matrix of order 1×1. Observe that [2] is both a row matrix as well as a column matrix.

(3) **Singleton matrix** : If in a matrix there is only one element then it is called singleton matrix.

Thus, $A = [aij]_{m\times n}$ is a singleton matrix if $m = n = 1$ *Example* : [2], [3], [a], [–3] are singleton matrices.

(4) **Null or zero matrix** : If in a matrix all the elements are zero then it is called a zero matrix and it is generally denoted by *O*. Thus $A = [a_{ij}]_{m\times n}$ is a zero matrix if $a_{ij} = 0$ for all *i* and *j*.

Example : $[0], \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, [0\ 0]$ are all zero matrices, but of different orders.

(5) **Square matrix** : If number of rows and number of columns in a matrix are equal, then it is called a square matrix. Thus $A = [a_{ij}]_{m\times n}$ is a square matrix if $m = n$. *Example* : $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is a square matrix of order 3×3

(i) If $m \neq n$ then matrix is called a rectangular matrix.

(ii) The elements of a square matrix A for which $i = j, i.e.\ a_{11}, a_{22}, a_{33}, \ldots. a_{nn}$ are called diagonal elements and the line joining these elements is called the principal diagonal or leading diagonal of matrix *A*.

(iii) **Trace of a matrix :** The sum of diagonal elements of a square matrix. *A* is called the trace of matrix A , which is denoted by tr A. $tr\ A = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \ldots a_{nn}$

Properties of trace of a matrix : Let $A = [a_{ii}]_{n\times n}$ and $B = [b_{ij}]_{n\times n}$ and λ be a scalar

(i) $tr(\lambda A) = \lambda\, tr(A)$ (ii) $tr(A - B) = tr(A) - tr(B)$ (iii) $tr(AB) = tr(BA)$

(iv) $tr(A) = tr(A')$ or $tr\ (A^T)$ (v) $tr(I_n) = n$ (vi) tr (0)= 0

(vii) $tr(AB) \neq tr\,A . tr\,B$

(6) **Diagonal matrix** : If all elements except the principal diagonal in a square matrix are zero, it is called a diagonal matrix. Thus a square matrix $A = [a_{ij}]$ is a diagonal matrix if $a_{ij} = 0,$ when $i \neq j$.

Example : $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a diagonal matrix of order 3×3, which can be denoted by diag [2, 3, 4]

Note : ❑ No element of principal diagonal in a diagonal matrix is zero.

❑ Number of zeros in a diagonal matrix is given by $n^2 - n$ where n is the order of the matrix.

❑ A diagonal matrix of order $n \times n$ having $d_1, d_2, \ldots\ldots, d_n$ as diagonal elements is denoted by $\text{diag}\ [d_1, d_2, \ldots, d_n]$.

(7) **Identity matrix** : A square matrix in which elements in the main diagonal are all '1' and rest are all zero is called an identity matrix or unit matrix. Thus, the square matrix $A = [a_{ij}]$ is an identity matrix, if $a_{ij} = \begin{cases} 1, \text{ if } i = j \\ 0, \text{ if } i \neq j \end{cases}$

We denote the identity matrix of order n by I_n.

Example : [1], $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are identity matrices of order 1, 2 and 3 respectively.

(8) **Scalar matrix** : A square matrix whose all non diagonal elements are zero and diagonal elements are equal is called a scalar matrix. Thus, if $A = [a_{ij}]$ is a square matrix and $a_{ij} = \begin{cases} \alpha, \text{if } i = j \\ 0, \text{ if } i \neq j \end{cases}$, then A is a scalar matrix.

Example : [2], $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ are scalar matrices of order 1, 2 and 3 respectively.

Note : ❑ Unit matrix and null square matrices are also scalar matrices.

(9) **Triangular Matrix** : A square matrix $[a_{ij}]$ is said to be triangular matrix if each element above or below the principal diagonal is zero. It is of two types

(i) **Upper Triangular matrix :** A square matrix $[a_{ij}]$ is called the upper triangular matrix, if $a_{ij} = 0$ when $i > j$.

Example : $\begin{bmatrix} 3 & 1 & 2 \\ 0 & 4 & 3 \\ 0 & 0 & 6 \end{bmatrix}$ is an upper triangular matrix of order 3×3.

(ii) **Lower Triangular matrix :** A square matrix $[a_{ij}]$ is called the lower triangular matrix, if $a_{ij} = 0$ when $i < j$.

Example : $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 2 \end{bmatrix}$ is a lower triangular matrix of order 3×3.

Note : ❑ Minimum number of zeros in a triangular matrix is given by $\frac{n(n-1)}{2}$ where n is order of matrix.

❑ Diagonal matrix is both upper and lower triangular.

- A triangular matrix $a = [a_{ij}]_{n\times n}$ is called strictly triangular if $a_{ij} = 0$ for $1 \le i \le n$

Example: 1 A square matrix $A = [a_{ij}]$ in which $a_{ij} = 0$ for $i \ne j$ and $a_{ij} = k$ (constant) for $i = j$ is called a

(a) Unit matrix (b) Scalar matrix (c) Null matrix (d) Diagonal matrix

Solution: (b) When $a_{ij} = 0$ for $i \ne j$ and a_{ij} is constant for $i = j$ then the matrix $[a_{ij}]_{n\times n}$ is called a scalar matrix

Example: 2 If *A, B* are square matrix of order 3, A is non singular and $AB = 0$, then *B* is a

(a) Null matrix (b) Singular matrix (c) Unit matrix (d) Non singular matrix

Solution: (a) *AB* = 0 when *B* is null matrix.

Example: 3 The matrix $\begin{bmatrix} 2 & 5 & -7 \\ 0 & 3 & 11 \\ 0 & 0 & 9 \end{bmatrix}$ is known as

(a) Symmetric matrix (b) Diagonal matrix (c) Upper triangular matrix (d) Skew symmetric matrix

Solution: (c) We know that if all the elements below the diagonal in a matrix are zero, then it is an upper triangular matrix.

Example: 4 In an upper triangular matrix *n*×*n*, minimum number of zeros is **[Rajasthan PET 1999]**

(a) $\frac{n(n-1)}{2}$ (b) $\frac{n(n+1)}{2}$ (c) $\frac{2n(n-1)}{2}$ (d) None of these

Solution: (a) As we know a square matrix $A = [a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ for all *i>j*

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \dots & a_{1(n-2)} & a_{1(n-1)} & a_{1n} \\ 0 & a_{22} & a_{23} & a_{24} & \dots & a_{2(n-2)} & a_{2(n-1)} & a_{2n} \\ 0 & 0 & a_{33} & a_{34} & \dots & a_{3(n-2)} & a_{3(n-1)} & a_{3n} \\ 0 & 0 & 0 & a_{44} & \dots & a_{4(n-2)} & a_{4(n-1)} & a_{4n} \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & \dots & 0 & a_{(n-1)(n-1)} & a_{(n-1)n} \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & a_{nn} \end{bmatrix}$$

. Number of zeros

$= (n-1) + (n-2) + \dots + 2 + 1 = \frac{(n-1)n}{2}$

Example: 5 If $A = [a_{ij}]$ is a scalar matrix then trace of *A* is

(a) $\sum_i \sum_j a_{ij}$ (b) $\sum_i a_{ij}$ (c) $\sum_j a_{ij}$ (d) $\sum_i a_{ii}$

Solution: (d) The trace of $A = \sum_{i=1}^{n} a_{ii}$ = Sum of diagonal elements.

8.2.5 Addition and Subtraction of Matrices

If $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ are two matrices of the same order then their sum *A*+*B* is a matrix whose each element is the sum of corresponding elements. *i.e.* $A + B = [a_{ij} + b_{ij}]_{m\times n}$

Example : If $A = \begin{bmatrix} 5 & 2 \\ 1 & 3 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ 2 & 2 \\ 3 & 3 \end{bmatrix}$, then $A + B = \begin{bmatrix} 5+1 & 2+5 \\ 1+2 & 3+2 \\ 4+3 & 1+3 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 3 & 5 \\ 7 & 4 \end{bmatrix}$

Similarly, their subtraction $A - B$ is defined as $A - B = [a_{ij} - b_{ij}]_{m\times n}$

i.e. in above example $A-B=\begin{bmatrix}5-1 & 2-5\\ 1-2 & 3-2\\ 4-3 & 1-3\end{bmatrix}=\begin{bmatrix}4 & -3\\ -1 & 1\\ 1 & -2\end{bmatrix}$

Note : ❑ Matrix addition and subtraction can be possible only when matrices are of the same order.

Properties of matrix addition : If *A, B* and *C* are matrices of same order, then

(i) $A+B=B+A$ (Commutative law)

(ii) $(A+B)+C=A+(B+C)$ (Associative law)

(iii) $A+O=O+A=A,$ where O is zero matrix which is additive identity of the matrix.

(iv) $A+(-A)=0=(-A)+A$, where $(-A)$ is obtained by changing the sign of every element of *A,* which is additive inverse of the matrix.

(v) $\left.\begin{matrix}A+B=A+C\\ B+A=C+A\end{matrix}\right\}\Rightarrow B=C$ (Cancellation law)

8.2.6 Scalar Multiplication of Matrices

Let $A=[a_{ij}]_{m\times n}$ be a matrix and k be a number, then the matrix which is obtained by multiplying every element of A by k is called scalar multiplication of A by k and it is denoted by kA.

Thus, if $A=[a_{ij}]_{m\times n}$, then $kA=Ak=[ka_{ij}]_{m\times n}$. *Example* : If $A=\begin{bmatrix}2 & 4\\ 3 & 1\\ 4 & 6\end{bmatrix}$, then $5A=\begin{bmatrix}10 & 20\\ 15 & 5\\ 20 & 30\end{bmatrix}$

Properties of scalar multiplication:

If A, B are matrices of the same order and λ, μ are any two scalars then

(i) $\lambda(A+B)=\lambda A+\lambda B$ (ii) $(\lambda+\mu)A=\lambda A+\mu A$

(iii) $\lambda(\mu A)=(\lambda\mu A)=\mu(\lambda A)$ (iv) $(-\lambda A)=-(\lambda A)=\lambda(-A)$

Note : ❑ All the laws of ordinary algebra hold for the addition or subtraction of matrices and their multiplication by scalars.

8.2.7 Multiplication of Matrices

Two matrices A and B are conformable for the product AB if the number of columns in A (pre-multiplier) is same as the number of rows in *B* (post multiplier).Thus, if $A=[a_{ij}]_{m\times n}$ and $B=[b_{ij}]_{n\times p}$ are two matrices of order $m\times n$ and $n\times p$ respectively, then their product *AB* is of order $m\times p$ and is defined as $(AB)_{ij}=\sum_{r=1}^{n}a_{ir}b_{rj}$

$=[a_{i1}a_{i2}...a_{in}]\begin{bmatrix}b_{1j}\\ b_{2j}\\ \vdots\\ b_{nj}\end{bmatrix}$ = (i^{th} row of A)(j^{th} column of *B*) .....(i), where i=1, 2, ..., m and j=1, 2, ...p

Now we define the product of a row matrix and a column matrix.

Let $A = [a_1 a_2 \ldots. a_n]$ be a row matrix and $B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$ be a column matrix.

Then $AB = [a_1 b_1 + a_2 b_2 + \ldots. + a_n b_n]$ ...(ii). Thus, from (i),

$(AB)_{ij} =$ Sum of the product of elements of i^{th} row of A with the corresponding elements of j^{th} column of B.

Properties of matrix multiplication

If A,B and C are three matrices such that their product is defined, then

(i) $AB \neq BA$ (Generally not commutative)

(ii) $(AB)C = A(BC)$ (Associative Law)

(iii) $IA = A = AI$, where I is identity matrix for matrix multiplication

(iv) $A(B + C) = AB + AC$ (Distributive law)

(v) If $AB = AC \not\Rightarrow B = C$ (Cancellation law is not applicable)

(vi) If AB= 0 It does not mean that A= 0 or B = 0, again product of two non zero matrix may be a zero matrix.

Note : ❑ If A and B are two matrices such that AB exists, then BA may or may not exist.

❑ The multiplication of two triangular matrices is a triangular matrix.

❑ The multiplication of two diagonal matrices is also a diagonal matrix and

$$\text{diag } (a_1, a_2, \ldots. a_n) \times \text{diag } (b_1, b_2, \ldots. b_n) = \text{diag } (a_1 b_1, a_2 b_2, \ldots. a_n b_n)$$

❑ The multiplication of two scalar matrices is also a scalar matrix.

❑ If A and B are two matrices of the same order, then

(i) $(A + B)^2 = A^2 + B^2 + AB + BA$

(ii) $(A - B^2) = A^2 + B^2 - AB - BA$

(iii) $(A - B)(A + B) = A^2 - B^2 + AB - BA$

(iv) $(A + B)(A - B) = A^2 - B^2 - AB + BA$

(v) $A(-B) = (-A)B = -(AB)$

8.2.8 Positive Integral Powers of A Matrix

The positive integral powers of a matrix A are defined only when A is a square matrix. Also then $A^2 = A.A$, $A^3 = A.A.A = A^2 A$. Also for any positive integers m ,n.

(i) $A^m A^n = A^{m+n}$

(ii) $(A^m)^n = A^{mn} = (A^n)^m$

(iii) $I^n = I, I^m = I$

(iv) $A^0 = I_n$ where A is a square matrix of order n.

8.2.9 Matrix Polynomial

Let $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1} x + a_n$ be a polynomial and let A be a square matrix of order n. Then $f(A) = a_0 A^n + a_1 A^{n-1} + a_2 A^{n-2} + \ldots + a_{n-1} A + a_n I_n$ is called a matrix polynomial.

Example : If $f(x) = x^2 - 3x + 2$ is a polynomial and A is a square matrix, then $A^2 - 3A + 2I$ is a matrix polynomial.

Example: 6 If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then $A^2 =$ **[Rajasthan PET 2001]**

(a) $\begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$ (b) $\begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$ (c) $\begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$ (d) $\begin{bmatrix} -\cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & -\cos 2\alpha \end{bmatrix}$

Solution: (c) Since $A^2 = A.A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

Example: 7 If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$ then **[AIEEE 2003]**

(a) $\alpha = a^2 + b^2, \beta = ab$ (b) $\alpha = a^2 + b^2, \beta = 2ab$ (c) $\alpha = a^2 + b^2, \beta = a^2 - b^2$ (d) $\alpha = 2ab, \beta = a^2 + b^2$

Solution: (b) $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{bmatrix}$. On comparing, we get, $\alpha = a^2 + b^2, \beta = 2ab$

Example: 8 If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}, n \in N$, then A^{4n} equals **[AMU 1992]**

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ (c) $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Solution: (a) $A^2 = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $A^4 = A^2.A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$; $(A^4)^n = I^n = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Example: 9 If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$ then value of a and b are **[Kurukshetra CEE 2002]**

(a) $a = 4, b = 1$ (b) $a = 1, b = 4$ (c) $a = 0, b = 4$ (d) $a = 2, b = 4$

Solution: (b) We have $(A+B)^2 = A^2 + B^2 + A.B + BA$

$\because AB + BA = 0$ $\therefore \begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix} + \begin{bmatrix} a+2 & -a-1 \\ b-2 & -b+1 \end{bmatrix} = 0$

$\begin{bmatrix} 2a+2-b & -a+1 \\ 2a-2 & 4-b \end{bmatrix} = 0$. On comparing, we get, $-a+1 = 0 \Rightarrow a = 1$ and $4 - b = 0 \Rightarrow b = 4$

Example: 10 The order of $[x\ y\ z]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is **[EAMCET 1994]**

(a) 3×1 (b) 1×1 (c) 1×3 (d) 3×3

Solution: (b) Order will be $(1\times 3)(3\times 3)(3\times 1) = (1\times 1)$

Example: 11 Let $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then $F(\alpha).F(\alpha')$ is equal to

(a) $F(\alpha\alpha')$ (b) $F(\alpha/\alpha')$ (c) $F(\alpha+\alpha')$ (d) $F(\alpha-\alpha')$

Solution: (c) We have $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $F(\alpha') = \begin{bmatrix} \cos\alpha' & -\sin\alpha' & 0 \\ \sin\alpha' & \cos\alpha' & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$F(\alpha).F(\alpha') = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha' & -\sin\alpha' & 0 \\ \sin\alpha' & \cos\alpha' & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\alpha+\alpha') & -\sin(\alpha+\alpha') & 0 \\ \sin(\alpha+\alpha') & \cos(\alpha+\alpha') & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(\alpha+\alpha')$$

Example: 12 For the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$, which of the following is correct

(a) $A^3 + 3A^2 - I = 0$ (b) $A^3 - 3A^2 - I = 0$ (c) $A^3 + 2A^2 - I = 0$ (d) $A^3 - A^2 + I = 0$

Solution: (b) $A^2 = A.A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 6 & 2 \\ 3 & 4 & 1 \end{bmatrix}$, $A^3 = A.^2 A = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 6 & 2 \\ 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 9 & 3 \\ 15 & 19 & 6 \\ 9 & 12 & 4 \end{bmatrix}$

$$A^3 - 3.A^2 = \begin{bmatrix} 7 & 9 & 3 \\ 15 & 19 & 6 \\ 9 & 12 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 9 & 3 \\ 15 & 18 & 6 \\ 9 & 12 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \Rightarrow A^3 - 3A^2 - I = 0$$

Example: 13 If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then the value of α for which $A^2 = B$ is **[IIT Screening 2003]**

(a) 1 (b) –1 (c) 4 (d) No real values

Solution: (d) $A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$ $\because A^2 = B$ (given)

Then $\begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \Rightarrow \alpha^2 = 1$ and $\alpha + 1 = 5$. Clearly no real value of α

8.2.10 Transpose of a Matrix

The matrix obtained from a given matrix A by changing its rows into columns or columns into rows is called transpose of Matrix A and is denoted by A^T or A'.

From the definition it is obvious that if order of A is $m \times n$, then order of A^T is $n \times m$

Example : Transpose of matrix $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}_{2\times 3}$ is $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}_{3\times 2}$

Properties of transpose : Let A and B be two matrices then

(i) $(A^T)^T = A$

(ii) $(A+B)^T = A^T + B^T$, A and B being of the same order

(iii) $(kA)^T = kA^T, k$ be any scalar (real or complex)

(iv) $(AB)^T = B^T A^T$, A and B being conformable for the product AB

(v) $(A_1 A_2 A_3 .....A_{n-1} A_n)^T = A_n{}^T A_{n-1}{}^T ....A_3{}^T A_2{}^T A_1{}^T$

(vi) $I^T = I$

8.2.11 Determinant of a Matrix

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a square matrix, then its determinant, denoted by $|A|$ or Det (A) is defined as

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Properties of determinant of a matrix

(i) $|A|$ exists $\Leftrightarrow A$ is square matrix (ii) $|AB| = |A||B|$

(iii) $|A^T| = |A|$ (iv) $|kA| = k^n |A|$, if A is a square matrix of order n

(v) If A and B are square matrices of same order then $|AB| = |BA|$

(vi) If A is a skew symmetric matrix of odd order then $|A|=0$

(vii) If $A=\text{diag}\,(a_1, a_2, \ldots\ldots a_n)$ then $|A|=a_1a_2\ldots a_n$ (viii) $|A|^n \neq |A^n|,\ n\in N$.

Example: 14 If A and B are square matrices of same order then **[Pb. CET 1992; Roorkee 1995; MP PET 1990; Rajasthan PET 1992, 94]**

(a) $(AB)'=A'B'$ (b) $(AB)'=B'A'$

(c) $AB=0$, if $|A|=0$ or $|B|=0$ (d) $AB=0$, if $|A|=I$ or $B=I$

Solution: (b) $A=[a_{ij}]_{n\times n}, B=[b_{jk}]_{n\times n}$, $AB=[a_{ij}]_{n\times n}[b_{jk}]_{n\times n}=[c_{ik}]_{n\times n}$, where $c_{ik}=a_{ij}b_{jk}$

$(AB)'=[c_{ik}]'_{n\times n}=[c_{ki}]_{n\times n}=[b_{kj}]_{n\times n}[a_{ji}]_{n\times n}=B'A'$

Alternatively, Let $A=\begin{bmatrix}1&2\\3&4\end{bmatrix}_{2\times 2}, B=\begin{bmatrix}1&3\\0&4\end{bmatrix}_{2\times 2}$; $AB=\begin{bmatrix}1&11\\3&25\end{bmatrix}$

$(AB)'=\begin{bmatrix}1&3\\11&25\end{bmatrix}$ .....(i) and $B'A'=\begin{bmatrix}1&0\\3&4\end{bmatrix}\begin{bmatrix}1&3\\2&4\end{bmatrix}=\begin{bmatrix}1&3\\11&25\end{bmatrix}$ .....(ii)

From (i) and (ii), $(AB)=B'A'$

Example: 15 If A,B are 3×2 order matrices and C is a 2×3 order matrix, then which of the following matrices not defined **[Rajasthan PET 1998]**

(a) A^t+B (b) $B+C^t$ (c) A^t+C (d) A^t+B^t

Solution: (a) Order of A is 3 × 2 and order of B is 3 × 2 and order of A^t is 2 × 3 then $=A^t+B=[A^t]_{2\times 3}+[B]_{3\times 2}$ is not possible because order are not same.

8.2.12 Special Types of Matrices

(1) Symmetric and skew-symmetric matrix

(i) **Symmetric matrix** : A square matrix $A=[a_{ij}]$ is called symmetric matrix if $a_{ij}=a_{ji}$ for all *i, j* or $A^T=A$

Example : $\begin{bmatrix}a&h&g\\h&b&f\\g&f&c\end{bmatrix}$

Note : ❑ Every unit matrix and square zero matrix are symmetric matrices.

❑ Maximum number of different elements in a symmetric matrix is $\frac{n(n+1)}{2}$

(ii) **Skew-symmetric matrix** : A square matrix $A=[a_{ij}]$ is called skew- symmetric matrix if $a_{ij}=-a_{ji}$ for all *i, j*

or $A^T=-A$. *Example* : $\begin{bmatrix}0&h&g\\-h&0&f\\-g&-f&0\end{bmatrix}$

Note : ❑ All principal diagonal elements of a skew- symmetric matrix are always zero because for any diagonal element. $a_{ij}=-a_{ij}\Rightarrow a_{ij}=0$

❑ Trace of a skew symmetric matrix is always 0.

Properties of symmetric and skew-symmetric matrices:

(i) If A is a square matrix, then $A+A^T, AA^T, A^TA$ are symmetric matrices, while $A-A^T$ is skew-symmetric matrix.

(ii) If A is a symmetric matrix, then $-A, KA, A^T, A^n, A^{-1}, B^TAB$ are also symmetric matrices, where $n\in N$, $K\in R$ and B is a square matrix of order that of A

(iii) If A is a skew-symmetric matrix, then

(a) A^{2n} is a symmetric matrix for $n \in N$,

(b) A^{2n+1} is a skew-symmetric matrix for $n \in N$,

(c) kA is also skew-symmetric matrix, where $k \in R$,

(d) $B^T AB$ is also skew- symmetric matrix where B is a square matrix of order that of A.

(iv) If A, B are two symmetric matrices, then

(a) $A \pm B,\ AB + BA$ are also symmetric matrices,

(b) $AB - BA$ is a skew- symmetric matrix,

(c) AB is a symmetric matrix, when $AB = BA$.

(v) If A,B are two skew-symmetric matrices, then

(a) $A \pm B,\ AB - BA$ are skew-symmetric matrices,

(b) $AB + BA$ is a symmetric matrix.

(vi) If A a skew-symmetric matrix and C is a column matrix, then $C^T AC$ is a zero matrix.

(vii) Every square matrix A can uniquelly be expressed as sum of a symmetric and skew-symmetric matrix *i.e.*

$$A = \left[\frac{1}{2}(A + A^T)\right] + \left[\frac{1}{2}(A - A^T)\right].$$

(2) **Singular and Non-singular matrix :** Any square matrix A is said to be non-singular if $|A| \neq 0$, and a square matrix A is said to be singular if $|A| = 0$. Here $|A|$ (or det(A) or simply det $|A|$ means corresponding determinant of square matrix A.

Example : $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ then $|A| = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 10 - 12 = -2 \Rightarrow A$ is a non singular matrix.

(3) **Hermitian and skew-Hermitian matrix :** A square matrix $A = [a_{ij}]$ is said to be hermitian matrix if $a_{ij} = \bar{a}_{ji}\ \ \forall i.j\ i.e.\ A = A^\theta$. *Example* : $\begin{bmatrix} a & b+ic \\ b-ic & d \end{bmatrix}, \begin{bmatrix} 3 & 3-4i & 5+2i \\ 3+4i & 5 & -2+i \\ 5-2i & -2-i & 2 \end{bmatrix}$ are Hermitian matrices.

Note : ❑ If A is a Hermitian matrix then $a_{ii} = \bar{a}_{ii} \Rightarrow a_{ii}$ is real $\forall i,$ thus every diagonal element of a Hermitian matrix must be real.

❑ A Hermitian matrix over the set of real numbers is actually a real symmetric matrix and a square matrix, $A=|a_{ij}|$ is said to be a skew-Hermitian if $a_{ij} = -\bar{a}_{ji}.\forall i,j\, i.e.\, A^\theta = -A$.

Example : $\begin{bmatrix} 0 & -2+i \\ 2-i & 0 \end{bmatrix}, \begin{bmatrix} 3i & -3+2i & -1-i \\ 3+2i & -2i & -2-4i \\ 1-i & 2-4i & 0 \end{bmatrix}$ are skew-Hermitian matrices.

❑ If A is a skew-Hermitian matrix, then $a_{ii} = -\bar{a}_{ii} \Rightarrow a_{ii} + \bar{a}_{ii} = 0$ i.e. a_{ii} must be purely imaginary or zero.

❑ A skew-Hermitian matrix over the set of real numbers is actually a real skew-symmetric matrix.

(4) **Orthogonal matrix :** A square matrix A is called orthogonal if $AA^T = I = A^T A$ *i.e.* if $A^{-1} = A^T$

Example : $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ is orthogonal because $A^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = A^T$

In fact every unit matrix is orthogonal.

(5) **Idempotent matrix** : A square matrix A is called an idempotent matrix if $A^2 = A$.

Example : $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ is an idempotent matrix, because $A^2 = \begin{bmatrix} 1/4+1/4 & 1/4+1/4 \\ 1/4+1/4 & 1/4+1/4 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} = A$.

Also, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are idempotent matrices because $A^2 = A$ and $B^2 = B$.

In fact every unit matrix is indempotent.

(6) **Involutory matrix :** A square matrix A is called an involutory matrix if $A^2 = I$ or $A^{-1} = A$

Example : $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an involutory matrix because $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

In fact every unit matrix is involutory.

(7) **Nilpotent matrix :** A square matrix A is called a nilpotent matrix if there exists a $p \in N$ such that $A^p = 0$

Example : $A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ is a nilpotent matrix because $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ (Here $P = 2$)

(8) **Unitary matrix :** A square matrix is said to be unitary, if $\overline{A}' A = I$ since $|\overline{A}'| = |A|$ and $|\overline{A}' A| = |\overline{A}'||A|$ therefore if $\overline{A}' A$=I, we have $|\overline{A}'||A| = 1$

Thus the determinant of unitary matrix is of unit modulus. For a matrix to be unitary it must be non-singular.

Hence $\overline{A}' A = I \Rightarrow A\overline{A}' = I$

(9) **Periodic matrix** : A matrix A will be called a periodic matrix if $A^{k+1} = A$ where k is a positive integer. If, however k is the least positive integer for which $A^{k+1} = A$, then k is said to be the period of A.

(10) **Differentiation of a matrix :** If $A = \begin{bmatrix} f(x) & g(x) \\ h(x) & l(x) \end{bmatrix}$ then $\frac{dA}{dx} = \begin{bmatrix} f'(x) & g'(x) \\ h'(x) & l'(x) \end{bmatrix}$ is a differentiation of matrix A.

Example : If $A = \begin{bmatrix} x^2 & \sin x \\ 2x & 2 \end{bmatrix}$ then $\frac{dA}{dx} = \begin{bmatrix} 2x & \cos x \\ 2 & 0 \end{bmatrix}$

(11) **Submatrix :** Let A be $m \times n$ matrix, then a matrix obtained by leaving some rows or columns or both, of A is called a sub matrix of A. *Example* : If $A' = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 2 \\ 2 & 5 & 3 \end{bmatrix}$ and $\begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix}$ are sub matrices of matrix $A = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 3 & 2 & 2 & 4 \\ 2 & 5 & 3 & 1 \end{bmatrix}$

(12) **Conjugate of a matrix :** The matrix obtained from any given matrix A containing complex number as its elements, on replacing its elements by the corresponding conjugate complex

numbers is called conjugate of A and is denoted by $\overline{A}$. *Example* : $A=\begin{bmatrix}1+2i & 2-3i & 3+4i\\ 4-5i & 5+6i & 6-7i\\ 8 & 7+8i & 7\end{bmatrix}$ then

$\overline{A}=\begin{bmatrix}1-2i & 2+3i & 3-4i\\ 4+5i & 5-6i & 6+7i\\ 8 & 7-8i & 7\end{bmatrix}$

Properties of conjugates :

(i) $\overline{(\overline{A})}=A$ (ii) $\overline{(A+B)}=\overline{A}+\overline{B}$

(iii) $\overline{(\alpha A)}=\overline{\alpha}\overline{A}, \alpha$ being any number (iv) $\overline{(AB)}=\overline{A}\,\overline{B}, A$ and B being conformable for multiplication.

(13) **Transpose conjugate of a matrix :** The transpose of the conjugate of a matrix A is called transposed conjugate of A and is denoted by A^{θ}. The conjugate of the transpose of A is the same as the transpose of the conjugate of A *i.e.* $\overline{(A')}=(\overline{A})'=A^{\theta}$.

If $A=[a_{ij}]_{m\times n}$ then $A^{\theta}=[b_{ji}]_{n\times m}$ where $b_{ji}=\overline{a}_{ij}$ *i.e.* the $(j,i)^{th}$ element of $A^{\theta}=$ the conjugate of $(i,j)^{th}$ element of A.

Example : If $A=\begin{bmatrix}1+2i & 2-3i & 3+4i\\ 4-5i & 5+6i & 6-7i\\ 8 & 7+8i & 7\end{bmatrix}$, then $A^{\theta}=\begin{bmatrix}1-2i & 4+5i & 8\\ 2+3i & 5-6i & 7-8i\\ 3-4i & 6+7i & 7\end{bmatrix}$

Properties of transpose conjugate

(i) $(A^{\theta})^{\theta}=A$ (ii) $(A+B)^{\theta}=A^{\theta}+B^{\theta}$ (iii) $(kA)^{\theta}=\overline{K}A^{\theta}, K$ being any number (iv) $(AB)^{\theta}=B^{\theta}A^{\theta}$

Example: 16 The matrix $\begin{bmatrix}0 & 5 & -7\\ -5 & 0 & 11\\ 7 & -11 & 0\end{bmatrix}$ is known as **[Karnataka CET 2000]**

(a) Upper triangular matrix (b) Skew-symmetric matrix (c) Symmetric matrix (d)

Solution: (b) In a skew-symmetric matrix, $a_{ij}=-a_{ji}\ \forall\ i,j=1,2,3$ and $j=i$, $a_{ii}=-a_{ii}$ $\Rightarrow$ each $a_{ii}=0$

Hence the given matrix is skew-symmetric matrix $[\because A^T=-A]$.

Example: 17 The matrix $\begin{bmatrix}2 & \lambda & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$ is non singular if **[Kurukshetra CEE 2002]**

(a) $\lambda\neq-2$ (b) $\lambda\neq2$ (c) $\lambda\neq3$ (d) $\lambda\neq-3$

Solution: (a) The given matrix $A=\begin{bmatrix}2 & \lambda & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$ is non singular If $|A|\neq0$

$\Rightarrow |A|=\begin{vmatrix}2 & \lambda & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{vmatrix}\neq0 \Rightarrow \begin{vmatrix}1 & \lambda+3 & 0\\ -1 & 3 & 4\\ 1 & -2 & -3\end{vmatrix}\neq0\ [R_1\to R_1+R_2]$

$\Rightarrow |A|=\begin{bmatrix}1 & \lambda+3 & 0\\ 0 & 1 & 1\\ 0 & -\lambda-5 & -3\end{bmatrix}\neq0$, $\begin{bmatrix}R_2\to & R_2+R_3\\ R_3\to & R_3-R_1\end{bmatrix}$

$\Rightarrow 1(-3+\lambda+5)\neq0 \Rightarrow \lambda+2\neq0 \Rightarrow \lambda\neq-2$

Example: 18 The matrix $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is **[Kurukshetra CEE 2002]**

(a) Orthogonal (b) Involutary (c) Idempotent (d) Nilpotent

Solution: (a) Since for given $A = \frac{1}{3}\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{vmatrix}$. For orthogonal matrix $AA^T = A^T A = I_{(3\times 3)}$

$\Rightarrow AA^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = 3I$. Similarly $A^T A = 3I$. Hence A is orthogonal

Example: 19 If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric, then $x =$ **[Karnataka CET 1994]**

(a) 3 (b) 5 (c) 2 (d) 4

Solution: (b) For symmetric matrix, $A = A^T \Rightarrow \begin{bmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{bmatrix} = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix} \Rightarrow 2x-3 = x+2 \Rightarrow x = 5$

Example: 20 If A and B are square matrices of order $n\times n$, then $(A-B)^2$ is equal to **[Karnataka CET 1999; Kerala (Engg.) 2002]**

(a) $A^2 - B^2$ (b) $A^2 - 2AB + B^2$ (c) $A^2 + 2AB + B^2$ (d) $A^2 - AB - BA + B^2$

Solution: (d) Given A and B are square matrices of order $n\times n$ we know that $(A-B)^2 = (A-B)(A-B) = A^2 - AB - BA + B^2$

Example: 21 If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then which of the following statement is not correct **[DCE 2001]**

(a) A is orthogonal matrix (b) A^T is orthogonal matrix (c) Determinant $A = 1$ (d)

Solution: (d) $|A| = 1 \neq 0$, therefore A is invertible. Thus (d) is not correct

Example: 22 Matrix A is such that $A^2 = 2A - I$ where I is the identity matrix. Then for $n \geq 2$, $A^n =$

(a) $nA - (n-1)I$ (b) $nA - I$ (c) $2^{n-1}A - (n-1)I$ (d) $2^{n-1}A - I$

Solution: (a) We have, $A^2 = 2A - I \Rightarrow A^2.A = (2A - I)A$; $A^3 = 2A^2 - IA = 2[2A - I] - IA \Rightarrow A^3 = 3A - 2I$

Similarly $A^4 = 4A - 3I$ and hence $A^n = nA - (n-1)I$

Example: 23 Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$, the only correct statement about the matrix A is **[AIEEE 2004]**

(a) $A^2 = I$ (b) $A = (-1)I$, where I is unit matrix

(c) A^{-1} does not exist (d) A is zero matrix

Solution: (a) $A^2 = A.A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$. Also, A^{-1} exists as $|A| = 1$

8.2.13 Adjoint of a Square Matrix

Let $A = [a_{ij}]$ be a square matrix of order n and let C_{ij} be cofactor of a_{ij} in A. Then the transpose of the matrix of cofactors of elements of A is called the adjoint of A and is denoted by *adj* A

Thus, $adj\ A = [C_{ij}]^T \Rightarrow (adj A)_{ij} = C_{ji} =$ cofactor of a_{ji} in A.

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then $adj A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$;

Where C_{ij} denotes the cofactor of a_{ij} in A.

Example : $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}, C_{11} = s, C_{12} = -r, C_{21} = -q, C_{22} = p$

$$\therefore adj A = \begin{bmatrix} s & -r \\ -q & p \end{bmatrix}^T = \begin{bmatrix} s & -q \\ -r & p \end{bmatrix}$$

Note : ❑ The adjoint of a square matrix of order 2 can be easily obtained by interchanging the diagonal elements and changing signs of off diagonal elements.

Properties of adjoint matrix : If *A*, *B* are square matrices of order *n* and I_n is corresponding unit matrix, then (i) $A(adj\ A) = |A|\ I_n = (adj\ A)A$ (Thus *A* (*adj A*) is always a scalar matrix)

(ii) $|adj A| = |A|^{n-1}$

(iii) $adj(adj A) = |A|^{n-2}\ A$

(iv) $|adj\ (adj\ A)| = |A|^{(n-1)^2}$

(v) $adj(A^T) = (adj A)^T$

(vi) $adj\ (AB) = (adj\ B)(adj\ A)$

(vii) $adj(A^m) = (adj A)^m, m \in N$

(viii) $adj(kA) = k^{n-1}(adj A), k \in R$

(ix) $adj\ (I_n) = I_n$

(x) $adj\ (O) = O$

(xi) *A* is symmetric ⇒ *adj A* is also symmetric.

(xii) *A* is diagonal ⇒ *adj A* is also diagonal.

(xiii) *A* is triangular ⇒ *adj A* is also triangular.

(xiv) *A* is singular ⇒ |*adj A*|= 0

8.2.14 Inverse of a Matrix

A non-singular square matrix of order *n* is invertible if there exists a square matrix *B* of the same order such that $AB = I_n = BA$.

In such a case, we say that the inverse of *A* is *B* and we write $A^{-1} = B$

The inverse of A is given by $A^{-1} = \frac{1}{|A|}.adj A$

The necessary and sufficient condition for the existence of the inverse of a square matrix A is that $|A| \neq 0$

Properties of inverse matrix:

If *A* and *B* are invertible matrices of the same order, then

(i) $(A^{-1})^{-1} = A$

(ii) $(A^T)^{-1} = (A^{-1})^T$

(iii) $(AB)^{-1} = B^{-1}A^{-1}$

(iv) $(A^k)^{-1} = (A^{-1})^k, k \in N$ [In particular $(A^2)^{-1} = (A^{-1})^2$]

(v) $adj(A^{-1}) = (adj A)^{-1}$

(vi) $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$

(vii) *A* = diag $(a_1 a_2 ... a_n) \Rightarrow A^{-1} = diag(a_1^{-1} a_2^{-1} ... a_n^{-1})$

(viii) *A* is symmetric ⇒ A^{-1} is also symmetric.

(ix) *A* is diagonal, $|A| \neq 0 \Rightarrow A^{-1}$ is also diagonal.

(x) *A* is scalar matrix ⇒ A^{-1} is also scalar matrix.

(xi) *A* is triangular, $|A| \neq 0 \Rightarrow A^{-1}$ is also triangular.

(xii) Every invertible matrix possesses a unique inverse.

Note : ❑ (Cancellation law with respect to multiplication)

If *A* is a non singular matrix i.e., if $|A| \neq 0$,then A^{-1} exists and $AB = AC \Rightarrow A^{-1}(AB) = A^{-1}(AC)$

$\Rightarrow (A^{-1}A)B = (A^{-1}A)C \Rightarrow IB = IC \Rightarrow B = C\ \therefore AB = AC \Rightarrow B = C \Leftrightarrow |A| \neq 0$

Example: 24 If $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$, then $|adj A|$ is equal to **[UPSEAT 2003]**

(a) 16 (b) 10 (c) 6 (d) None of these

Solution: (b) $adj\ A = \begin{bmatrix} 4 & -2 \\ -3 & 4 \end{bmatrix}$

$|\ adj\ A| = \begin{vmatrix} 4 & -2 \\ -3 & 4 \end{vmatrix} = 16 - 6 = 10$

Example: 25 If 3, – 2 are the Eigen values of non-singular matrix *A* and |*A*| = 4 . Then Eigen values of *adj* (*A*) are **[Kurukshetra CEE 2002]**

(a) 3/4, –1/2 (b) 4/3, –2 (c) 12, –8 (d) –12, 8

Solution: (b) Since $A^{-1} = \frac{adj\ A}{|\ A|}$ and if λ is Eigen value of *A* then λ^{-1} is Eigen value of A^{-1}, thus for $adj\,(A)\,x = (A^{-1}x)|\ A|$

$= |\ A|\ .\lambda^{-1} I$

$adj(A)$ corresponding to Eigen value

$\lambda = 3$ is = 4/3 and for $\lambda = -2$ is = $4/-2$ = –2

Example: 26 If matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{K} adj\,(A)$, then *K* is **[UPSEAT 2002]**

(a) 7 (b) –7 (c) 1/7 (d) 11

Solution: (d) We know that $A^{-1} = \frac{adj(A)}{|\ A|}$. We have $A^{-1} = \frac{1}{K} adj(A)$ *i.e.* $K = |\ A|$

and $K = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 3(3) - 2(1) + 4(1) = 9 - 2 + 4 = 11$

Example: 27 The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

[AMU 2001]

(a) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (b) $\frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (c) $\frac{1}{|\ A|}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} b & -a \\ d & -c \end{bmatrix}$

Solution: (b) Here $|\ A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$, $adj\,(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. Hence $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Example: 28 Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10.B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If *B* is the inverse of matrix *A*, then α is **[AIEEE 2004]**

(a) 5 (b) –1 (c) 2 (d) –2

Solution: (a) We have, $A = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $\therefore |\ A| = 1(4) + 1(5) + 1(1) = 10$ and $adj\,(A) = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

Then $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

According to question, *B* is the inverse of matrix *A*. Hence $\alpha = 5$

Example: 29 Matrix $A = \begin{bmatrix} 1 & 0 & -K \\ 2 & 1 & 3 \\ K & 0 & 1 \end{bmatrix}$ is invertible for **[UPSEAT 2002]**

(a) $K = 1$ (b) $K = -1$ (c) $K = 0$ (d) All real *K*

Solution: (d) For invertible, $|\ A| \neq 0$ *i.e.,* $\begin{vmatrix} 1 & 0 & -K \\ 2 & 1 & 3 \\ K & 0 & 1 \end{vmatrix} \neq 0$

$\Rightarrow 1(1)-K(-K)\neq 0 \Rightarrow |A| = K^2+1\neq 0$, which is true for all real K.

Example: 30 Let p be a non-singular matrix, **[Orissa JEE 2002]**

$1+p+p^2+.......+p^n=0$ **(**0 denotes the null matrix**),** then $p^{-1}=$

(a) p^n (b) $-p^n$ (c) $-(1+p+.....+p^n)$ (d) None of these

Solution: (a) We have, $1+p+p^2+---+p^n=0$

Multiplying both sides by p^{-1}, $p^{-1}+I+Ip+.....+p^{n-1}I=0.p^{-1}$

$p^{-1}+I(1+p+....+p^{n-1})=0 \Rightarrow p^{-1}=-I(1+p+p^2+....+p^{n-1})=-(-p^n)=p^n$.

Example: 31 Let $f(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$, where $\alpha\in R$,then $[f(\alpha)]^{-1}$ is equal to **[AMU 2000]**

(a) $f(-\alpha)$ (b) $f(\alpha^{-1})$ (c) $f(2\alpha)$ (d) None

Solution: (a) $|f(\alpha)|=\begin{vmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{vmatrix}=1$, *adj* of $f(\alpha)=\begin{vmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{vmatrix}$

$[f(\alpha)]^{-1}=\begin{vmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{vmatrix}$ ......(i) and $f(-\alpha)=\begin{vmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{vmatrix}$ ......(ii)

From (i) and (ii), $[f(\alpha)]^{-1}=f[-\alpha]$

Example: 32 If *I* is a unit matrix of order 10, then the determinant of *I* is equal to **[Kerala (Engg.) 2002]**

(a) 10 (b) 1 (c) 1/10 (d) 9

Solution: (b) Determinant of unit matrix of any order =1.

Example: 33 If $A=\begin{bmatrix}\alpha & 2\\ 2 & \alpha\end{bmatrix}$ and $|A^3|=125$ then $\alpha=$ **[IIT Screening 2004]**

(a) ± 3 (b) ± 2 (c) ± 5 (d) 0

Solution: (a) $125=|A^3|=|A|^3 \Rightarrow |A|=5$ and $|A|=\alpha^2-4=5 \Rightarrow \alpha^2=9 \Rightarrow \alpha=\pm 3$

Example: 34 If |*A*| denotes the value of the determinant of the square matrix *A* of order 3, then $|-2A|=$ **[MP PET 1987, 89, 92, 2000]**

(a) $-8|A|$ (b) $8|A|$ (c) $-2|A|$ (d) None of these

Solution: (a) We know that, det. $(-A)=(-1)^n \det A$, where *n* is order of square matrix

If *A* is square matrix of order 3, Then $n=3$. Hence $|-2A|=(-2)^3|A|=-8|A|$.

Example: 35 For how many value (*s*) of *x* in the closed interval [–4, –1] is the matrix $\begin{bmatrix}3 & -1+x & 2\\ 3 & -1 & x+2\\ x+3 & -1 & 2\end{bmatrix}$ singular **[Karnataka CET 2002]**

(a) 2 (b) 0 (c) 3 (d) 1

Solution: (d) $\begin{vmatrix}3 & x-1 & 2\\ 3 & -1 & x+2\\ x+3 & -1 & 2\end{vmatrix}=0$

$\begin{vmatrix}0 & x & -x\\ 3 & -1 & x+2\\ x+3 & -1 & 2\end{vmatrix}=0 \quad [R_1\to R_1-R_2]$, $\begin{vmatrix}0 & x & -x\\ -x & 0 & x\\ x+3 & -1 & 2\end{vmatrix}=0 \quad [R_2\to R_2-R_3]$

$\begin{vmatrix}0 & 0 & -x\\ -x & x & x\\ x+3 & 1 & 2\end{vmatrix}=0 \quad [C_2\to C_2+C_3]$

$-x[(-x)-x(x+3)]=0 \Rightarrow x(x^2+4x)=0 \Rightarrow x=0,-4$

Hence only one value of x in closed interval [–4,–1] i.e. $x=-4$

Example: 36 Inverse of diagonal matrix (if it exists) is a

(a) Skew-symmetric matrix (b) Diagonal matrix (c) Non invertible matrix (d) None of these

Solution: (b) Let $A=\text{diag}(d_1,d_2,d_3......,d_n)$

As A is invertible, therefore $\det(A)\neq 0 \Rightarrow d_1,d_2,d_3.......,d_n\neq 0 \Rightarrow d_i\neq 0$ for i = 1, 2, 3…..n

Here, cofactor of each non diagonal entry is 0 and cofactor of a_{ii}

$$=(-1)^{i+1}\det[diag(d_1,d_2,d_3.....,d_{i-1},d_{i+1}.....,d_n)]=d_1,d_2,d_3.....d_{i-1}.d_{i+1},......,d_n=\frac{1}{d_i}[d_1,d_2,d_3.....,d_{i-1},d_i,d_{i+1}.....,d_n]=\frac{|A|}{d_i}$$

$$A^{-1}=\frac{1}{|A|}(adj\,A)=diag\left(\frac{1}{d_1},\frac{1}{d_2}.......,\frac{1}{d_n}\right),$$ which is a diagonal matrix

8.2.15 Elementary Transformations or Elementary Operations of a Matrix

The following three operations applied on the rows (columns) of a matrix are called elementary row (column) transformations

(1) Interchange of any two rows (columns)

If i^{th} row (column) of a matrix is interchanged with the j^{th} row (column), it will be denoted by $R_i\leftrightarrow R_j\,(C_i\leftrightarrow C_j)$

Example : $A=\begin{bmatrix}2&1&3\\-1&2&1\\3&2&4\end{bmatrix}$, then by applying $R_2\leftrightarrow R_3$, we get $B=\begin{bmatrix}2&1&3\\3&2&4\\-1&2&1\end{bmatrix}$

(2) Multiplying all elements of a row (column) of a matrix by a non-zero scalar

If the elements of i^{th} row (column) are multiplied by a non-zero scalar k, it will be denoted by $R_i\to R_i(k),[C_i\to C_i(k)]$ or $R_i\to kR_i,[C_i\to kC_i]$

If $A=\begin{bmatrix}3&2&-1\\0&1&2\\-1&2&-3\end{bmatrix}$, then by applying $R_2\to 3R_2$ we obtain $B=\begin{bmatrix}3&2&-1\\0&3&6\\-1&2&-3\end{bmatrix}$

(3) Adding to the elements of a row (column), the corresponding elements of any other row (column) multiplied by any scalar k. If k times the elements of j^{th} row (column) are added to the corresponding elements of the i^{th} row (column), it will be denoted by $R_i\to R_i+kR_j\left(C_i\to C_i+kC_j\right)$

If $A=\begin{bmatrix}2&1&3&1\\-1&-1&0&2\\0&1&3&1\end{bmatrix}$, then the application of elementary operation $R_3\to R_3+2R_1$ gives the matrix

$B=\begin{bmatrix}2&1&3&1\\-1&-1&0&2\\4&3&9&3\end{bmatrix}$, If a matrix B is obtained from a matrix A by one or more elementary transformations, then A and B are equivalent matrices and we write $A\sim B$

Let $A=\begin{bmatrix}1&2&3&4\\2&1&4&3\\3&1&2&4\end{bmatrix}$, then $A\sim\begin{bmatrix}1&2&3&4\\1&-1&1&-1\\3&1&2&4\end{bmatrix}$, applying $R_2\to R_2+(-1)R_1$

$\sim\begin{bmatrix}1&2&3&1\\1&-1&1&-2\\1&1&2&2\end{bmatrix}$, applying $C_4\to C_4+(-1)C_3$

An elementary transformation is called a row transformation or a column transformation according as it is applied to rows or columns.

8.2.16 Elementary Martix

A matrix obtained from an identity matrix by a single elementary operation (transformation) is called an elementary matrix. *Example :* $\begin{bmatrix}1&3&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix},\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$ are elementary matrices obtained from I_3 by subjecting it to the elementary transformations $R_1 \to R_1 + 3\,R_2,\ C_1 \leftrightarrow C_3$ and $R_2 \leftrightarrow R_3$ respectively.

Theorem 1 : Every elementary row (column) transformation of an *m*×*n* matrix (not identity matrix) can be obtained by pre-multiplication (post- multiplication) with the corresponding elementary matrix obtained from the identity matrix $I_m\ (I_n)$ by subjecting it to the same elementary row (column) transformation.

Theorem 2 : Let $C = AB$ be a product of two matrices. Any elementary row (column) transformation of *AB* can be obtained by subjecting the pre-factor *A* (post factor B) to the same elementary row (column) transformation.

Method of finding the inverse of a matrix by elementary transformations : Let A be a non singular matrix of order *n*. Then *A* can be reduced to the identity matrix I_n by a finite sequence of elementary transformation only. As we have discussed every elementary row transformation of a matrix is equivalent to pre-multiplication by the corresponding elementary matrix. Therefore there exist elementary matrices $E_1, E_2 .....E_k$ such that

$$(E_k E_{k-1...} E_2 E_1)A = I_n$$

$$\Rightarrow (E_k E_{k-1}...E_2 E_1)AA^{-1} = I_n A^{-1} \quad \text{(post multiplying by } A^{-1})$$

$$\Rightarrow (E_k E_{k-1}...E_2 E_1)I_n = A^{-1} \qquad (\because I_n A^{-1} = A^{-1} \text{and } AA^{-1} = I_n) \Rightarrow A^{-1} = (E_k E_{k-1}...E_2 E_1)I_n$$

Algorithm for finding the inverse of a non singular matrix by elementary row transformations

Let *A* be non- singular matrix of order *n*

Step I : Write $A = I_n A$

Step II : Perform a sequence of elementary row operations successively on A on the LHS and the pre factor I_n on the RHS till we obtain the result $I_n = BA$

Step III : Write $A^{-1} = B$

Note : ❑ The following steps will be helpful to find the inverse of a square matrix of order 3 by using elementary row transformations.

Step I : Introduce unity at the intersection of first row and first column either by interchanging two rows or by adding a constant multiple of elements of some other row to first row.

Step II : After introducing unity at (1,1) place introduce zeros at all other places in first column.

Step III Introduce unity at the intersection of 2nd row and 2nd column with the help of 2nd and 3rd row.

Step IV : Introduce zeros at all other places in the second column except at the intersection of 2nd row and 2nd column.

Step V : Introduce unity at the intersection of 3rd row and third column.

Step VI : Finally introduce zeros at all other places in the third column except at the intersection of third row and third column.

Example: 37 Using elementary row transformation find the inverse of the matrix $A = \begin{bmatrix}3&-1&-2\\2&0&-1\\3&-5&0\end{bmatrix}$

(a) $\begin{bmatrix} -5/8 & 5/4 & 1/8 \\ -3/8 & 3/4 & -1/8 \\ -5/4 & 3/2 & 1/4 \end{bmatrix}$ (b) $\frac{1}{8}\begin{bmatrix} 5 & -5 & 1 \\ 3 & -3 & 1 \\ 0 & 3 & 1 \end{bmatrix}$ (c) $\frac{1}{8}\begin{bmatrix} 5 & 5 & 1 \\ 3 & 6 & -1 \\ 10 & -12 & 2 \end{bmatrix}$ (d) None of these

Solution: (a) We have $A=IA \Rightarrow \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $(R_1 \to R_1 - R_2)$ $\begin{bmatrix} 1 & -1 & -1 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$, $\begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & -1 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ -3 & 3 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2/2$, $\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 1/2 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3/2 & 0 \\ -3 & 3 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 + R_2$ and $R_3 \to R_3 + 2R_2$, $\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 & 0 \\ -1 & 3/2 & 0 \\ -5 & 6 & 1 \end{bmatrix} A$

Applying $R_3 \to R_3/4$, $\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 & 0 \\ -1 & 3/2 & 0 \\ 5/4 & 6/4 & 1/4 \end{bmatrix} A$

Applying $R_1 \to R_1 + \frac{1}{2}R_3$ and $R_2 \to R_2 - \frac{1}{2}R_3$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5/8 & 5/4 & 1/8 \\ -3/8 & 3/4 & -1/8 \\ -5/4 & 3/2 & 1/4 \end{bmatrix} A$

$$A^{-1} = \begin{bmatrix} -5/8 & 5/4 & 1/8 \\ -3/8 & 3/4 & -1/8 \\ -5/4 & 3/2 & 1/4 \end{bmatrix}$$

8.2.17 Rank of Matrix

Definition : Let A be a $m \times n$ matrix. If we retain any r rows and r columns of A we shall have a square sub-matrix of order r. The determinant of the square sub-matrix of order r is called a minor of A order r.

Consider any matrix A which is of the order of 3×4 say, $A = \begin{vmatrix} 1 & 3 & 4 & 5 \\ 1 & 2 & 6 & 7 \\ 1 & 5 & 0 & 1 \end{vmatrix}$. It is 3×4 matrix so we can have minors of order 3, 2 or 1. Taking any three rows and three columns minor of order three. Hence minor of order $3 = \begin{vmatrix} 1 & 3 & 4 \\ 1 & 2 & 6 \\ 1 & 5 & 0 \end{vmatrix} = 0$

Making two zeros and expanding above minor is zero. Similarly we can consider any other minor of order 3 and it can be shown to be zero. Minor of order 2 is obtained by taking any two rows and any two columns.

Minor of order $2 = \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = 2 - 3 = -1 \neq 0$. Minor of order 1 is every element of the matrix.

Rank of a matrix: The rank of a given matrix A is said to be r if

(1) Every minor of A of order r+1 is zero

(2) There is at least one minor of A of order r which does not vanish

Note : ❑ If a minor of A is zero the corresponding submatrix is singular and if a minor of A is not zero then corresponding submatrix is non-singular.

Here we can also say that the rank of a matrix A is said to be r if

(i) Every square submatrix of order r+1 is singular.

(ii) There is at least one square submatrix of order r which is non-singular.

The rank r of matrix A is written as $\rho(A) = r$

Working rule : Calculate the minors of highest possible order of a given matrix A. If it is not zero, then the order of the minor is the rank. If it is zero and all other minors of the same order be also zero, then calculate minor of next lower order and if at least one of them is not zero then this next lower order will be the rank. If, however, all the minors of next lower orders are zero, then calculate minors of still next lower order and so on.

Note : ❑ The rank of the null matrix is not defined and the rank of every non-null matrix is greater than or equal to 1.

❑ The rank of a singular square matrix of order n cannot be n.

8.2.18 Echelon form of a Matrix

A matrix A is said to be in Echelon form if either A is the null matrix or A satisfies the following conditions:

(1) Every non- zero row in A precedes every zero row.

(2) The number of zeros before the first non-zero element in a row is less than the number of such zeros in the next row.

It can be easily proved that the rank of a matrix in Echelon form is equal to the number of non-zero row of the matrix. *Example* : The rank of the matrix $A = \begin{bmatrix} 0 & 3 & 2 & 1 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is 2 because it is in Echelon form and it has two non-zero rows. The matrix $A = \begin{bmatrix} 0 & 2 & 5 \\ 0 & 0 & 1 \\ 0 & 0 & -4 \end{bmatrix}$ is not in Echelon form, because the number of zeros in second row is not less than the number of zeros in the third row. To reduce A in the echelon form, we apply some elementary row transformations on it. Applying $R_3 \to R_3 + 4R_2$, we obtain $A \sim \begin{bmatrix} 0 & 2 & 5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, which is in Echelon form and contains 2 non zero rows. Hence, $r(A) = 2$

Rank of a matrix in Echelon form : The rank of a matrix in Echelon form is equal to. the number of non-zero rows in that matrix.

Algorithm for finding the rank of a matrix : Let $A = [a_{ij}]$ be an m×n matrix.

Step I : Using elementary row transformations make $a_{11} = 1$

Step II : Make $a_{21}, a_{31}, ..., a_{m1}$ all zeros by using elementary transformations,

$R_2 \to R_2 - a_{21}R_1, R_3 \to R_3 - a_{31}R_1, ...R_m \to R_m - a_{m1}R_1$

Step III : Make $a_{22} = 1$ by using elementary row transformations.

Step IV : Make $a_{32}, a_{42}, ..., a_{m2}$ all zeros by using $R_3 \to R_3 - a_{32}R_2, R_4 \to R_4 - a_{42}R_2, ...R_m \to R_m - a_{m2}R_2$

The process used in steps III and IV is repeated upto $(m-1)th$ row. Finally we obtain a matrix in Echelon form, which is equivalent to the matrix *A.* The rank of *A* will be equal to the number of non-zeros rows in it.

Example: 38 The rank of the matrix $A = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -4 & 2 \end{bmatrix}$ is **[Kurukshetra CEE 2002]**

(a) 2 (b) 3 (c) 1 (d) Indeterminate

Solution: (a) We have $A = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -4 & 2 \end{bmatrix}_{3\times 4}$, Considering 3×3 minor $\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 0 & -2 & -4 \end{bmatrix}_{3\times 3}$ its determinant is 0.

Similarly considering , $\begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & -1 \\ 0 & -2 & 2 \end{bmatrix}$, $\begin{bmatrix} 2 & 1 & 4 \\ 0 & 2 & -1 \\ 0 & -4 & 2 \end{bmatrix}$ and $\begin{bmatrix} 3 & 1 & 4 \\ 1 & 2 & -1 \\ -2 & -4 & 2 \end{bmatrix}$, their determinant is 0 each rank can not be 3

Then again considering a 2×2 minor, $\begin{bmatrix} 2 & 3 \\ 0 & -2 \end{bmatrix}$, which is non zero. Thus, rank =2

Example: 39 The rank of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$ is **[Roorkee 1988]**

(a) 1 if *a* = 6 (b) 2 if *a* =1 (c) 3 if *a* = 2 (d) 1 if *a* = – 6

Solution: (b,d) Let $A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix} = \begin{vmatrix} 0 & 0 & a+6 \\ 0 & 0 & -a-6 \\ 1 & -2 & a+1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -a-6 \\ 1 & -2 & a+1 \end{vmatrix}$

When $a = -6$, $A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -2 & -5 \end{vmatrix}$, $\therefore r(A) = 1$

When $a = 1$, $A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -7 \\ 1 & -2 & 2 \end{vmatrix}$ $\therefore r(A) = 2$, When $a = 6$, $A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -12 \\ 1 & -2 & 7 \end{vmatrix}$,$\therefore r(A) = 2$

When $a = 2$, $A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -8 \\ 1 & -2 & 3 \end{vmatrix}$, $\therefore r(A) = 2$

Example: 40 The value of x so that the matrix $\begin{bmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{bmatrix}$ has rank 3 is

(a) $x \neq 0$ (b) $x = a+b+c$
(c) $x \neq 0$ and $x \neq -(a+b+c)$ (d) $x = 0,\ x = a+b+c$

Solution: (c) Since rank is 3, $|A|_{3\times 3} \neq 0$, $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}_{3\times 3} \neq 0$

$\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix} \neq 0$, Applying $(C_1 \to C_1 + C_2 + C_3)$

$$(x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} \neq 0 \Rightarrow x+a+b+c \neq 0, \begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} \neq 0$$

$$x \neq -(a+b+c), \begin{vmatrix} 0 & -x & c \\ 0 & x & -x \\ 1 & b & x+c \end{vmatrix} \neq 0 \Rightarrow x \neq 0$$

8.2.19 System of Simultaneous Linear Equations

Consider the following system of *m* linear equations in *n* unknowns

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2$$

... ...

... ...

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m$$

The system of equations can be written in matrix form as $\begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & ... & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$ or $AX = B$,

Where $A = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & ... & a_{mn} \end{bmatrix}_{m\times n}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$ and $B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m\times 1}$

The *m*×*n* matrix A is called the coefficient matrix of the system of linear equations.

(1) **Solution :** A set of values of the variables $x_1, x_2 ......x_n$ which simultaneously satisfy all the equations is called a solution of the system of equations. *Example* : $x = 2, y = -3$ is a solution of the system of linear equations $3x + y = 3,\quad 2x + y = 1$, because 3(2) +(–3)=3 and 2(2)+(–3)=1

(2) **Consistent system :** If the system of equations has one or more solutions, then it is said to be a consistent system of equations, otherwise it is an inconsistent system of equations. *Example* : the system of linear equation $2x + 3y = 5,\ 4x + 6y = 10$ is consistent, because *x*=1, *y* =1 and $x = 2, y = 1/3$ are solutions of it.

However, the system of linear equations $2x + 3y = 5, 4x + 6y = 10$ is inconsistent, because there is no set of values of *x, y* which satisfy the two equations simultaneously.

(3) **Homogeneous and non-homogeneous system of linear equations:** A system of equations *AX*=*B* is called a homogeneous system if $B = 0$. Otherwise, it is called a non-homogeneous system of equations.

Example : The system of equations, $2x + 3y = 0,\ 3x - y = 5$ is a homogeneous system of linear equations whereas the system of equations given by $2x + 3y = 1,\ 3x - y = 5$ is a non homogeneous system of linear equations.

8.2.20 Solution of a Non Homogeneous System of Linear Equations

There are three methods of solving a non homogeneous system of simultaneous linear equations.

(1) Determinant Method (Cramer's Rule) (2) Matrix method (3) Rank method

We have already discussed the determinant method (Cramer's rule) in chapter determinants.

(1) **Matrix method :** Let $AX = B$ be a system of *n* linear equations with *n* unknowns. If *A* is non-singular, then A^{-1} exists. $\therefore AX = B \Rightarrow A^{-1}(AX) = A^{-1}B$, [pre-multiplying by A^{-1}]

$\Rightarrow (A^{-1}A)X = A^{-1}B$, [by associativity]

$\Rightarrow I_n X = A^{-1}B \Rightarrow X = A^{-1}B$. Thus, the system of equations $AX = B$ has a solution given by $X = A^{-1}B$.

Now, let X_1 and X_2 be two solutions of $AX = B$. then, $AX_1 = B$ and $AX_2 = B$

$\Rightarrow AX_1 = AX_2 \Rightarrow A^{-1}(AX_1) = A^{-1}(AX_2) \Rightarrow (A^{-1}A)X_1 = (A^{-1}A)X_2 \Rightarrow I_n X_1 = I_n X_2 \Rightarrow X_1 = X_2$.

Hence, the given system has a unique solution.

Thus, if *A* is a non-singular matrix, then the system of equations given by $AX = B$ has a unique solution given by $X = A^{-1}B$.

If *A* is a singular matrix, then the system of equations given by *AX=B* may be consistent with infinitely many solutions or it may be inconsistent also.

Criterion of consistency : Let $AX = B$ be a system of *n*-linear equations in *n* unknowns.

(i) If $|A| \neq 0$, then the system is consistent and has a unique solution given by $X = A^{-1}B$

(ii) If $|A| = 0$ and (*adj A*) $B = 0$, then the system is consistent and has infinitely many solutions.

(iii) If $|A| = 0$ and $(adj\ A)\ B \neq 0$, then the system is inconsistent

Algorithm for solving a non-homogeneous system of linear equations : We shall give the algorithm for three equations in three unknowns. But it can be generalized to any number of equations.

Let $AX = B$ be a non-homogenous system of 3 linear equations in 3 unknowns. To solve this system of equations we proceed as follows

Step I : Write the given system of equations in matrix form, $AX = B$ and obtain A, B.

Step II : Find $|A|$

Step III : If $|A| \neq 0$, then write "the system is consistent with unique solution". obtain the unique solution by the following procedure. Find A^{-1} by using $A^{-1} = \frac{1}{|A|} adj A$ obtain the unique solution given by $X = A^{-1}B$

Step IV : If $|A| = 0$, then write "the system is either consistent with infinitely many solutions or it is inconsistent. To distinguish these two, proceed as follows: Find (*adj A*) *B.*

If $(adj\ A)B \neq 0$, then write "the system is inconsistent”.

If $(adj\ A)B = 0$, then the system is consistent with infinitely many solution. To find these solutions proceed as follows. Put $z = k$ (any real number) and take any two equations out of three equations. Solve these equations for *x* and *y.* Let the values of *x* and *y* be λ and μ respectively. Then $x = \lambda$, $y = \mu$ $z = k$ is the required solution, where any two of λ, μ, k are functions of the third.

(2) **Rank method :** Consider a system of *m* simultaneous linear equations in *n* unknowns $x_1, x_2 ...., x_n$, given by $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + ... + a_{1n}x = b_1$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + ... + a_{2n}x_n = b_2$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + ... + a_{mn}x_n = b_m$$

This system of equations can be written in matrix form as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

or $AX = B$,where $\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}_{m\times n}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$ and $B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ a_m \end{bmatrix}_{m\times 1}$

The matrix *A* is called the coefficient matrix and the matrix

$[A : B] = \left[\begin{array}{ccccc:c} a_{11} & a_{12} & a_{13} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} & b_m \end{array}\right]$ is called the augmented matrix of the given system of equations. This matrix is obtained by adding $(n+1)$ column to *A.* The elements of this column are $b_1, b_2 ...., b_m$

For example, the augmented matrix of the system of equation

$$2x - y + 3z = 1$$
$$x + y - 2z = 5$$
$$x + y + z = -1 \text{ is}$$

$$\left[\begin{array}{ccc:c} 2 & -1 & 3 & 1 \\ 1 & 1 & -2 & 5 \\ 1 & 1 & 1 & -1 \end{array}\right]$$

A non-homogeneous system of linear equations may have a unique solution, or many solutions or no solution at all. If it has a solution (whether unique or not) the system is said to be consistent. Otherwise it is called an inconsistent system. The following theorems tell us about the condition for consistency of a system of linear equations

Theorem 1 : The system of linear equations $AX = B$ is consistent iff the rank of the augmented matrix $[A : B]$ is equal to the rank of the coefficient matrix *A.*

Theorem 2 : Let $AX = B$ be a system of *m* simultaneous linear equations in *n* unknowns.

Case I : If $m > n$, then

(i) if $r(A) = r(A : B) = n,$ then system of linear equations has a unique solution.

(ii) if $r(A) = r(A : B) = r < n,$ then system of linear equations is consistent and has infinite number of solutions. In fact, in this case $(n - r)$ variables can be assigned arbitrary values.

(iii) if $r(A) \neq r(A : B),$ then the system of linear equations is inconsistent i.e. it has no solution.

Case II : If $m < n$ and $r(A) = r(A : B) = r,$ then $r \leq m < n$ and so from (ii) in case I, there are infinite number of solutions.

Thus, when the number of equations is less than the number of unknowns and the system is consistent, then the system of equations will always have an infinite number of solutions.

Algorithm for solving a non-homogeneous system *AX=B* of linear equations by rank method

Step I: Obtain *A*, B.

Step II : Write the Augmented matrix [A : B].

Step III : Reduce the augmented matrix to Echelon form by applying a sequence of elementary row-operations.

Step IV : Determine the number of non-zero rows in *A* and [*A* : *B*] to determine the ranks of A and [*A* :*B*] respectively.

Step V: If $r(A) \neq r(A:B)$ then write "the system is inconsistent" STOP else write "the system is consistent", go to Step VI

Step VI : If $r(A) = r(A:B)$ = number of unknowns, then the system has a unique solution which can be obtained by back substitution.

If $r(A) = r(A:B)$ < number of unknowns, then the system has an infinite number of solutions which can also be obtained by back substitution.

Example: 41 If $\begin{bmatrix} x+y & 2x+z \\ x-y & 2z+w \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$ then values of x, y, z, w are

(a) 2, 2, 3, 4 (b) 2, 3, 1,2 (c) 3, 3, 0, 1 (d) None of these

Solution: (a) We have $\begin{bmatrix} x+y & 2x+z \\ x-y & 2z+w \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$

$x+y=4,\ 2x+z=7$, $x-y=0$ and $2z+\omega=10 \Rightarrow x=2$ and $y=2,\ z=3,\ w=4$

Example: 42 The system of linear equation $x+y+z=2$, $2x+y-z=3,\ 3x+2y+kz=4$ has unique solution if **[EAMCET 1994]**

(a) $K \neq 0$ (b) $-1 < K < 1$ (c) $-2 < K < 2$ (d) $K = 0$

Solution: (a) The given system of equation has a unique solution if $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0 \Rightarrow K \neq 0$

8.2.21 Cayley-Hamilton Theorem

Every matrix satisfies its characteristic equation e.g. let *A* be a square matrix then $|A - xI| = 0$ is the characteristics equation of A. If $x^3 - 4x^2 - 5x - 7 = 0$ is the characteristic equation for *A*, then $A^3 - 4A^2 + 5A - 7I = 0$

Roots of characteristic equation for *A* are called Eigen values of A or characteristic roots of *A* or latent roots of *A.*

If λ is characteristic root of *A*, then λ^{-1} is characteristic root of A^{-1}.

8.2.22 Geometrical Transformations

(1) **Reflexion in the *x*-axis:** If $P'(x', y')$ is the reflexion of the point $P(x, y)$ on the x-axis, then the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ describes the reflexion of a point $P(x, y)$ in the *x*-axis.

(2) **Reflexion in the *y*-axis :** Here the matrix is $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(3) **Reflexion through the origin :** Here the matrix is $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(4) **Reflexion in the line *y = x* :** Here the matrix is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(5) **Reflexion in the line *y* = – *x* :** Here the matrix is $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(6) **Reflexion in y = *x* tan θ :** Here matrix is $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

(7) **Rotation through an angle** θ **:** Here matrix is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

8.2.23 Matrices of Rotation of Axes

We know that if *x* and y axis are rotated through an angle θ about the origin the new coordinates are given by $x = X\cos\theta - Y\sin\theta$ and $y = X\sin\theta + Y\cos\theta$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} \Rightarrow \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is the matrix of rotation through an angle θ .

Example: 43 Characteristic equation of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

(a) $A^3 - 20A + 8I$ (b) $A^3 + 20A + 8I$ (c) $A^3 - 80A + 20I$ (d) None of these

Solution: (a) The characteristic equation is $|A - \lambda I| = 0$.

So, $\begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 3-\lambda & -3 \\ -2 & -4 & -4-\lambda \end{vmatrix} = 0$ i.e. $\lambda^3 - 20\lambda + 8 = 0$

By cayley-Hamilton theorem , $A^3 - 20A + 8I = 0$

Example: 44 The transformation due to the reflection of (x, y) through the origin is described by the matrix

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution: (b) If (x', y') is the new position

$x' = (-1)x + 0.y,\ y' = 0.x + (-1)yd$

$\therefore \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

$\therefore$ Transformation matrix is $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Example: 45 The rotation through 180^0 is identical to

(a) The reflection in *x* -axis (b) The reflection in *y*-axis (c) A point reflection (d) Identity transformation

Solution: (c) Rotation through 180^0 gives $x' = -x$

$y' = -y$. Hence this a point reflection.

***

Assignment

Types of Matrices

Basic Level

1. If $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & b & 1 \end{bmatrix}$ be a diagonal matrix, then $b =$

(a) 2 (b) 0 (c) 1 (d) 3

2. Which of the following is a diagonal matrix

(a) $\begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (d) None of these

3. If I is a unit matrix, then $3I$ will be

(a) A unit matrix (b) A triangular matrix (c) A scalar matrix (d) None of these

4. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 7 \\ 5 & 1 & 6 \end{bmatrix}$, then the value of X where $A+X$ is a unit matrix, is

(a) $\begin{bmatrix} 0 & -2 & 1 \\ -3 & -3 & -7 \\ -5 & -1 & -6 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -3 & 5 \\ -2 & -3 & 1 \\ -1 & -7 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & -1 & -2 \\ 3 & 3 & 7 \\ 5 & 1 & 6 \end{bmatrix}$ (d) None of these

5. If A is diagonal matrix of order 2×2, then wrong statement is

(a) $AB = BA$, where B is a diagonal matrix of order 2×2 (b) AB is a diagonal matrix

(c) $A^T = A$ (d) A is a scalar matrix

Algebra of Matrices

Basic Level

6. If $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - \lambda M - I_2 = O$, then $\lambda =$ **[MP PET 1990, 2001]**

(a) –2 (b) 2 (c) –4 (d) 4

7. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $B = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$, then the correct relation is

(a) $A^2 = B^2$ (b) $A + B = B - A$ (c) $AB = BA$ (d) None of these

8. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, then

(a) $AB = BA$ (b) $AB = BA = O$ (c) $AB = O,\ BA \neq O$ (d) $AB \neq BA = O$

9. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then $A^n =$ [**Rajasthan PET 1995**]

(a) $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} n & n \\ 0 & n \end{bmatrix}$ (c) $\begin{bmatrix} n & 1 \\ 0 & n \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 0 & n \end{bmatrix}$

10. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then $A^2 =$

(a) A (b) $2A$ (c) $-A$ (d) $-2A$

11. If $2A + \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 7 \\ 4 & 3 \end{bmatrix}$, then $A =$

(a) $\begin{bmatrix} -5 & 8 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -5/2 & 4 \\ 1 & 3/2 \end{bmatrix}$ (c) $\begin{bmatrix} -5 & 6 \\ 2 & 3 \end{bmatrix}$ (d) None of these

12. If $[m\ \ n]\begin{bmatrix} m \\ n \end{bmatrix} = [25]$ and $m < n$, then $(m,n) =$

(a) (2, 3) (b) (3, 4) (c) (4, 3) (d) None of these

13. If $A = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$, $B = \begin{bmatrix} \cos^2\phi & \sin\phi\cos\phi \\ \sin\phi\cos\phi & \sin^2\phi \end{bmatrix}$ and θ and ϕ differs by $\frac{\pi}{2}$, then $AB =$

(a) I (b) O (c) $-I$ (d) None of these

14. If $A = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix}$ and $B = \begin{bmatrix} p^2 & pq & pr \\ pq & q^2 & qr \\ pr & qr & r^2 \end{bmatrix}$, then $AB =$

(a) $\begin{bmatrix} p & 0 & 0 \\ 0 & q & 0 \\ 0 & 0 & r \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

15. If $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $A^2 - 6A =$ [**MP PET 1987**]

(a) $3I$ (b) $5I$ (c) $-5I$ (d) None of these

16. If $A = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$,then for what value of λ, $A^2 = O$ [**MP PET 1992**]

(a) 0 (b) ± 1 (c) –1 (d) 1

17. If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ and $A^n = O$, then the minimum value of n is

(a) 2 (b) 3 (c) 4 (d) 5

18. If $A = \begin{bmatrix} 1/3 & 2 \\ 0 & 2x-3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$ and $AB = I$, then $x =$ [**MP PET 1987**]

(a) –1 (b) 1 (c) 0 (d) 2

19. If $2A + B = \begin{bmatrix} 6 & 4 \\ 6 & -11 \end{bmatrix}$ and $A - B = \begin{bmatrix} 0 & 2 \\ 6 & 2 \end{bmatrix}$, then $A =$

(a) $\begin{bmatrix} 2 & 2 \\ 4 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 \\ 4 & -3 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 2 \\ 4 & 3 \end{bmatrix}$ (d) None of these

20. If A=[1 2 3] and $B=\begin{bmatrix} -5 & 4 & 0 \\ 0 & 2 & -1 \\ 1 & -3 & 2 \end{bmatrix}$, then $AB=$ **[MP PET 1988]**

(a) $\begin{bmatrix} -5 & 4 & 0 \\ 0 & 4 & -2 \\ 3 & -9 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & -1 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & 8 & 0 \\ 0 & 4 & -3 \\ 1 & -6 & 6 \end{bmatrix}$

21. If $A=\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$, then AB is

(a) Diagonal matrix (b) Null matrix (c) Unit matrix (d) None of these

22. If $A=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $A^5=$

(a) $5A$ (b) $10A$ (c) $16A$ (d) $32A$

23. If $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $AB=O$, then $B=$ **[MP PET 1989]**

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$

24. If $R(t)=\begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$,then $R(s)\ R(t)=$ **[Roorkee 1981]**

(a) $R(s)+R(t)$ (b) $R(st)$ (c) $R(s+t)$ (d) None of these

25. If $A=\begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{bmatrix}$ and $3A-4B=\begin{bmatrix} -4 & 3 & 6 \\ 6 & 5 & 12 \\ 12 & 15 & 14 \end{bmatrix}$,then $B=$

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ (d) None of these

26. If $A=\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$,then A^4 is equal to **[MP PET 1993]**

(a) $\begin{bmatrix} 1 & a^4 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 4a \\ 0 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & a^4 \\ 0 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4a \\ 0 & 1 \end{bmatrix}$

27. If $\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}X=\begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$, then $X=$

(a) $\begin{bmatrix} -3 & 4 \\ 14 & -13 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -4 \\ -14 & 13 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 4 \\ 14 & 13 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 4 \\ -14 & 13 \end{bmatrix}$

28. If $A=\begin{bmatrix} 5 & -3 \\ 2 & 4 \end{bmatrix}$ and $B=\begin{bmatrix} 6 & -4 \\ 3 & 6 \end{bmatrix}$, then $A-B=$ **[Rajasthan PET 1995]**

(a) $\begin{bmatrix} 11 & -7 \\ 5 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 1 \\ -1 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 11 & 7 \\ 5 & -10 \end{bmatrix}$ (d) $\begin{bmatrix} 12 & -7 \\ 5 & -10 \end{bmatrix}$

29. If $3X+2Y=I$ and $2X-Y=O$, where I and O are unit and null matrices of order 3 respectively, then[**MP PET 1995**]

(a) $X=\frac{1}{7}, Y=\frac{2}{7}$ (b) $X=\frac{2}{7}, Y=\frac{1}{7}$ (c) $X=\frac{1}{7}I, Y=\frac{2}{7}I$ (d) $X=\frac{2}{7}I, Y=\frac{1}{7}I$

30. If $A=\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$ and I is the identity matrix of order 2, then $(A-2I)(A-3I)=$ [**Rajasthan PET 2002**]

(a) I (b) O (c) $\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ (d) $\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$

31. If $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, then $A^4=$ [**EAMCET 1994**]

(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$ (c) $\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

32. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, then $A^2=$ [**Karnataka CET 1994**]

(a) $\begin{bmatrix}8 & -5\\ -5 & 3\end{bmatrix}$ (b) $\begin{bmatrix}8 & -5\\ 5 & 3\end{bmatrix}$ (c) $\begin{bmatrix}8 & -5\\ -5 & -3\end{bmatrix}$ (d) $\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$

33. If $X=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, then the value of X^n is [**EAMCET 1991**]

(a) $\begin{bmatrix}3n & -4n\\ n & -n\end{bmatrix}$ (b) $\begin{bmatrix}2+n & 5-n\\ n & -n\end{bmatrix}$ (c) $\begin{bmatrix}3^n & (-4)^n\\ 1^n & (-1)^n\end{bmatrix}$ (d) None of these

34. If $A=\begin{bmatrix}i & 0\\ 0 & i\end{bmatrix}$, then $A^2=$ [**EAMCET 1983**]

(a) $\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ (b) $\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$ (c) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$

35. If $A+B=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ and $A-2B=\begin{bmatrix}-1 & 1\\ 0 & -1\end{bmatrix}$, then $A=$ [**Karnataka CET 1994**]

(a) $\begin{bmatrix}1 & 1\\ 2 & 1\end{bmatrix}$ (b) $\begin{bmatrix}2/3 & 1/3\\ 1/3 & 2/3\end{bmatrix}$ (c) $\begin{bmatrix}1/3 & 1/3\\ 2/3 & 1/3\end{bmatrix}$ (d) None of these

36. If $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, then $AB=$ [**EAMCET 1984**]

(a) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ (c) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

37. If $A=\begin{bmatrix}1 & 3 & 0\\ -1 & 2 & 1\\ 0 & 0 & 2\end{bmatrix}$, $B=\begin{bmatrix}2 & 3 & 4\\ 1 & 2 & 3\\ -1 & 1 & 2\end{bmatrix}$, then $AB=$ [**EAMCET1987**]

(a) $\begin{bmatrix}5 & 9 & 13\\ -1 & 2 & 4\\ -1 & 2 & 4\end{bmatrix}$ (b) $\begin{bmatrix}5 & 9 & 13\\ -1 & 2 & 4\\ -2 & 2 & 4\end{bmatrix}$ (c) $\begin{bmatrix}1 & 2 & 4\\ -1 & 2 & 4\\ -2 & 2 & 4\end{bmatrix}$ (d) None of these

38. If $\begin{bmatrix}x & 0\\ 1 & y\end{bmatrix}+\begin{bmatrix}-2 & 1\\ 3 & 4\end{bmatrix}=\begin{bmatrix}3 & 5\\ 6 & 3\end{bmatrix}-\begin{bmatrix}2 & 4\\ 2 & 1\end{bmatrix}$, then [**Rajasthan PET 1994**]

(a) $x=-3, y=-2$ (b) $x=3, y=-2$ (c) $x=3, y=2$ (d) $x=-3, y=2$

39. If $A=\begin{bmatrix}1 & -2\\3 & 0\end{bmatrix}$, $B=\begin{bmatrix}-1 & 4\\2 & 3\end{bmatrix}$, $C=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$, then $5A-3B-2C=$

(a) $\begin{bmatrix}8 & 20\\7 & 9\end{bmatrix}$ (b) $\begin{bmatrix}8 & -20\\7 & -9\end{bmatrix}$ (c) $\begin{bmatrix}-8 & 20\\-7 & 9\end{bmatrix}$ (d) $\begin{bmatrix}8 & 7\\-20 & -9\end{bmatrix}$

40. If $A=\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$, I is the unit matrix of order 2 and a, b are arbitrary constants, then $(aI+bA)^2$ is equal to

(a) a^2I+abA (b) $a^2I+2abA$ (c) a^2I+b^2A (d) None of these

41. If $U=[2\ \ -3\ \ 4]$, $X=[0\ \ 2\ \ 3]$, $V=\begin{bmatrix}3\\2\\1\end{bmatrix}$ and $Y=\begin{bmatrix}2\\2\\4\end{bmatrix}$, then $UV+XY=$ [**MP PET 1997**]

(a) 20 (b) [–20] (c) –20 (d) [20]

42. Which one of the following is not true [**Kurukshetra CEE 1998**]

(a) Matrix addition is commutative (b) Matrix addition is associative

(c) Matrix multiplication is commutative (d) Matrix multiplication is associative

43. If $A=[1\ \ 2\ \ 3]$, $B=\begin{bmatrix}2\\3\\4\end{bmatrix}$ and $C=\begin{bmatrix}1 & 5\\0 & 2\end{bmatrix}$, then which of the following is defined [**Rajasthan PET 1996**]

(a) AB (b) BA (c) $(AB).C$ (d) $(AC).B$

44. If $A=\begin{bmatrix}1 & 2 & 3\\-2 & 3 & -1\\3 & 1 & 2\end{bmatrix}$ and I is a unit matrix of 3rd order, then (A^2+9I) equals [**Rajasthan PET 1999**]

(a) $2A$ (b) $4A$ (c) $6A$ (d) None of these

45. If $A=\begin{pmatrix}i & 1\\0 & i\end{pmatrix}$, then A^4 equals [**AMU 1999**]

(a) $\begin{pmatrix}1 & -4i\\0 & 1\end{pmatrix}$ (b) $\begin{pmatrix}-1 & 4i\\0 & -1\end{pmatrix}$ (c) $\begin{pmatrix}-i & 4\\0 & i\end{pmatrix}$ (d) $\begin{pmatrix}1 & 4\\0 & 1\end{pmatrix}$

46. $\begin{bmatrix}1\\-1\\2\end{bmatrix}[2\ 1\ -1]=$ [**MP PET 2000**]

(a) [–1] (b) $\begin{bmatrix}2\\-1\\-2\end{bmatrix}$ (c) $\begin{bmatrix}2 & 1 & -1\\-2 & -1 & 1\\4 & 2 & -2\end{bmatrix}$ (d) Not defined

47. If $2X-\begin{bmatrix}1 & 2\\7 & 4\end{bmatrix}=\begin{bmatrix}3 & 2\\0 & -2\end{bmatrix}$, then X is equal to [**Rajasthan PET 2001**]

(a) $\begin{bmatrix}2 & 2\\7 & 4\end{bmatrix}$ (b) $\begin{bmatrix}1 & 2\\7/2 & 2\end{bmatrix}$ (c) $\begin{bmatrix}2 & 2\\7/2 & 1\end{bmatrix}$ (d) None of these

48. If $A=\begin{bmatrix}0 & 2\\3 & -4\end{bmatrix}$ and $kA=\begin{bmatrix}0 & 3a\\2b & 24\end{bmatrix}$, then the values of k, a, b are respectively [**EAMCET 2001**]

(a) –6, –12, –18 (b) –6, 4, 9 (c) –6, –4, –9 (d) –6, 12, 18

49. If $A=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$, then $A^n=$ [**Kerala (Engg.) 2001**]

(a) $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & n \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2n \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

50. If matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then $A^{16} =$ **[Karnataka CET 2002]**

(a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

51. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then $A^2 - 5A =$ **[Rajasthan PET 2002]**

(a) I (b) $14I$ (c) 0 (d) None of these

52. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then $A^{100} =$ **[UPSEAT 2002]**

(a) $2^{100} A$ (b) $2^{99} A$ (c) $2^{101} A$ (d) None of these

53. Which is true about matrix multiplication **[UPSEAT 2002]**

(a) It is commutative (b) It is associative (c) Both (a) and (b) (d) None of these

54. If $P = \begin{pmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{pmatrix}$ and $Q = \begin{pmatrix} -i & i \\ 0 & 0 \\ i & -i \end{pmatrix}$, then PQ is equal to **[Kerala (Engg.) 2002]**

(a) $\begin{pmatrix} -2 & 2 \\ 1 & -1 \\ 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & -2 \\ -1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

55. $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ is equal to

(a) $\begin{bmatrix} 43 \\ 44 \end{bmatrix}$ (b) $\begin{bmatrix} 43 \\ 45 \end{bmatrix}$ (c) $\begin{bmatrix} 45 \\ 44 \end{bmatrix}$ (d) $\begin{bmatrix} 44 \\ 45 \end{bmatrix}$

56. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, then AB is **[MP PET 2003]**

(a) $\begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 11 & 4 & 3 \\ 1 & 2 & 3 \\ 0 & 3 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 8 & 4 \\ 2 & 9 & 6 \\ 0 & 2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 2 \\ 5 & 4 & 3 \\ 1 & 8 & 2 \end{bmatrix}$

57. For 2×2 matrices A, B and I, if $A + B = I$ and $2A - 2B = I$, then A equals

(a) $\begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{3}{4} & 0 \\ 0 & \frac{3}{4} \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

58. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $A^2 + 2A$ equals **[AMU 1988]**

(a) A (b) $2A$ (c) $3A$ (d) $4A$

59. If $A=\begin{bmatrix}1 & -6 & 2\\0 & -1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}2\\2\\1\end{bmatrix}$, then AB equals **[AMU 1987]**

(a) $\begin{bmatrix}-8 & 3\end{bmatrix}$ (b) $\begin{bmatrix}-8\\3\end{bmatrix}$ (c) $\begin{bmatrix}2 & -12 & 2\\0 & -2 & 5\end{bmatrix}$ (d) $\begin{bmatrix}2 & 12 & 4\\0 & -2 & -10\end{bmatrix}$

60. Let $A=\begin{bmatrix}1 & 0\\2 & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 0\\1 & 12\end{bmatrix}$, then **[DCE 1999]**

(a) $AB=O, BA=O$ (b) $AB=O, BA\neq O$ (c) $AB\neq O, BA=O$ (d) $AB\neq O, BA\neq O$

61. If *A, B* are square matrices of order $n\times n$, then $(A-B)^2$ is equal to

(a) A^2-B^2 (b) $A^2-2BA+B^2$ (c) $A^2-AB-BA+B^2$ (d) $A^2-2AB+B^2$

62. If $A=\begin{bmatrix}3 & 5\\-2 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & -3\\2 & -7\end{bmatrix}$, then $2A-3B$ is equal to **[Rajasthan PET 1989,90]**

(a) $\begin{bmatrix}3 & -19\\10 & 29\end{bmatrix}$ (b) $\begin{bmatrix}3 & 19\\-10 & 29\end{bmatrix}$ (c) $\begin{bmatrix}-3 & 19\\10 & 29\end{bmatrix}$ (d) None of these

63. If $A=\begin{bmatrix}2 & 4\\0 & 3\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\0 & 5\end{bmatrix}$, then $4A-3B$ is equal to **[Rajasthan PET 1993]**

(a) $\begin{bmatrix}1 & 2\\0 & -2\end{bmatrix}$ (b) $\begin{bmatrix}7 & 14\\0 & 7\end{bmatrix}$ (c) $\begin{bmatrix}5 & 10\\0 & -3\end{bmatrix}$ (d) $\begin{bmatrix}-1 & -2\\0 & -12\end{bmatrix}$

64. If $A=\begin{pmatrix}1 & -2\\3 & 4\end{pmatrix}$, $B=\begin{pmatrix}2 & 1\\-4 & 3\end{pmatrix}$ and $C=\begin{pmatrix}2 & 12\\-17 & 9\end{pmatrix}$, then $5A-3B+C$ equals **[Rajasthan PET 1993]**

(a) $\begin{pmatrix}1 & 10\\-1 & 20\end{pmatrix}$ (b) $\begin{pmatrix}1 & -1\\10 & 20\end{pmatrix}$ (c) $\begin{pmatrix}1 & 1\\-10 & 20\end{pmatrix}$ (d) $\begin{pmatrix}-1 & 1\\1 & 20\end{pmatrix}$

65. If $A=\begin{bmatrix}2 & -1\\4 & 2\end{bmatrix}$, $B=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$ and $C=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, then $A+B-C$ equals

(a) $\begin{bmatrix}3 & 2\\5 & 3\end{bmatrix}$ (b) $\begin{bmatrix}5 & 2\\5 & 5\end{bmatrix}$ (c) $\begin{bmatrix}5 & 2\\3 & 5\end{bmatrix}$ (d) $\begin{bmatrix}3 & 2\\5 & 5\end{bmatrix}$

66. If $\begin{bmatrix}x & y\\-2 & 2\end{bmatrix}+\begin{bmatrix}x & 1\\4 & 0\end{bmatrix}=\begin{bmatrix}2 & 3\\y & 2\end{bmatrix}$, then x and y are **[Rajasthan PET 1994]**

(a) 1, 1 (b) 1, 2 (c) 2, 2 (d) 2, 1

67. If $X=\begin{bmatrix}1 & a\\0 & 1\end{bmatrix}$ and $3X-\begin{bmatrix}2 & 3\\0 & 2\end{bmatrix}=\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}$, then the value of a is

(a) –2 (b) 0 (c) 2 (d) 1

68. If $A=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$, then **[Rajasthan PET 1985]**

(a) $AB=BA$ (b) $AB=B^2$ (c) $AB=-BA$ (d) None of these

69. If $A=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3 & 2\\2 & 3 & 4\end{bmatrix}$, then AB is equal to **[Rajasthan PET 1989, 90, 98]**

(a) $\begin{bmatrix} 8 & 15 & 16 \\ 5 & 9 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 15 & 9 \\ 16 & 10 \end{bmatrix}$ (c) $\begin{bmatrix} 8 & 5 \\ 15 & 9 \end{bmatrix}$ (d) None of these

70. If $A = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$, then AB equals [**Rajasthan PET 1991**]

(a) $\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

71. $A \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 7 & 7 \end{bmatrix}$, then A equals [**EAMCET 1996**]

(a) $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$

72. If $A = \begin{bmatrix} 4 & 6 & -1 \\ 3 & 0 & 2 \\ 1 & -2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ 0 & 1 \\ -1 & 2 \end{bmatrix}$ $\begin{bmatrix} 2 & 4 \\ 0 & 1 \\ -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$, then which of the following is not defined

(a) AB (b) $B'C$ (c) CC' (d) $A^2 + 2B - 2A$

73. If a matrix B is obtained by multiplying each element of a matrix A of order 2×2 by 3, then relation between A and B *is*

[**Rajasthan PET 1986**]

(a) $A = 3B$ (b) $3A = B$ (c) $9A = B$ (d) $A = 9B$

Advance Level

74. For each real number x such that $-1 < x < 1$, let $A(x)$ be the matrix $(1-x)^{-1} \begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}$ and $z = \frac{x+y}{1+xy}$. Then

(a) $A(z) = A(x) + A(y)$ (b) $A(z) = A(x)[A(y)]^{-1}$ (c) $A(z) = A(x)A(y)$ (d) $A(z) = A(x) - A(y)$

75. If $A = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$, then the value of A^{40} is [**Rajasthan PET 1999**]

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$

76. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$, then [**Kurukshetra CEE 2002**]

(a) $A^3 + 3A^2 + A - 9I_3 = 0$ (b) $A^3 - 3A^2 + A + 9I_3 = 0$ (c) $A^3 + 3A^2 - A + 9I_3 = 0$ (d) $A^3 - 3A^2 - A + 9I_3 = 0$

77. If $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ and I is the unit matrix of order 2, then A^2 equals

(a) $4A - 3I$ (b) $3A - 4I$ (c) $A - I$ (d) $A + I$

78. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, then $A^n =$

(a) $\begin{bmatrix} na & 0 & 0 \\ 0 & nb & 0 \\ 0 & 0 & nc \end{bmatrix}$ (b) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ (c) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{bmatrix}$ (d) None of these

79. If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then which of following statement is true

(a) $A_\alpha . A_\beta = A_{\alpha\beta}$ and $(A_\alpha)^n = \begin{bmatrix} \cos^n\alpha & \sin^n\alpha \\ -\sin^n\alpha & \cos^n\alpha \end{bmatrix}$ (b) $A_\alpha . A_\beta = A_{\alpha\beta}$ and $(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

(c) $A_\alpha . A_\beta = A_{\alpha+\beta}$ and $(A_\alpha)^n = \begin{bmatrix} \cos^n\alpha & \sin^n\alpha \\ -\sin^n\alpha & \cos^n\alpha \end{bmatrix}$ (d) $A_\alpha . A_\beta = A_{\alpha+\beta}$ and $(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

Properties of Matrices

Basic Level

80. $AB = 0$, if and only if [**MNR 1981; Karnataka CET 1993**]

(a) $A \neq 0, B \neq 0$ (b) $A = 0, B \neq 0$ (c) $A = 0$ or $B = 0$ (d) None of these

81. If $A \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$, then the order of A is

(a) 1×1 (b) 2×1 (c) 1×2 (d) 2×2

82. If $AB = C$, then matrices *A, B, C* are [**MP PET 1991**]

(a) $A_{2\times3}, B_{3\times2}, C_{2\times3}$ (b) $A_{3\times2}, B_{2\times3}, C_{3\times2}$ (c) $A_{3\times3}, B_{2\times3}, C_{3\times3}$ (d) $A_{3\times2}, B_{2\times3}, C_{3\times3}$

83. $A = [a_{ij}]_{m\times n}$ is a square matrix, if

(a) $m < n$ (b) $m > n$ (c) $m = n$ (d) None of these

84. If $A = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ -2 & 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & -4 & 0 \end{bmatrix}$, then the element of 3rd row and third column in *AB* will be

(a) –18 (b) 4 (c) –12 (d) None of these

85. If *A* and *B* be symmetric matrices of the same order, then $AB - BA$ will be a

(a) Symmetric matrix (b) Skew-symmetric matrix (c) Null matrix (d) None of these

86. If *A* and *B* are square matrices of order 2, then $(A+B)^2 =$

(a) $A^2 + 2AB + B^2$ (b) $A^2 + AB + BA + B^2$ (c) $A^2 + 2BA + B^2$ (d) None of these

87. If the order of the matrices *A* and *B* be 2×3 and 3×2 respectively, then the order of $A + B$ will be

(a) 2×2 (b) 3×3 (c) 2×3 (d) None of these

88. In a lower triangular matrix element $a_{ij} = 0$, if

(a) $i \leq j$ (b) $i \geq j$ (c) $i > j$ (d) $i < j$

89. If A is a square matrix of order *n* and $A = kB$, where k is a scalar, then $|A| =$ [**Karnataka CET 1992**]

(a) $|B|$ (b) $k|B|$ (c) $k^n|B|$ (d) $n|B|$

90. Let $A = \begin{bmatrix} 4 & 6 & -1 \\ 3 & 0 & 2 \\ 1 & -2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ 0 & 1 \\ -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$. The expression which is not defined is

(a) $B'B$ (b) CAB (c) $A+B'$ (d) A^2+A

91. If $A=[a \quad b]$, $B=[-b \quad -a]$ and $C=\begin{bmatrix} a \\ -a \end{bmatrix}$, then the correct statement is **[AMU 1987]**

(a) $A=-B$ (b) $A+B=A-B$ (c) $AC=BC$ (d) $CA=CB$

92. If A and B are two matrices and $(A+B)(A-B)=A^2-B^2$, then

(a) $AB=BA$ (b) $A^2+B^2=A^2-B^2$ (c) $A'B'=AB$ (d) None of these

93. If A and B are square matrices of same order, then **[Roorkee 1995]**

(a) $A+B=B+A$ (b) $A+B=A-B$ (c) $A-B=B-A$ (d) $AB=BA$

94. Which of the following is incorrect

(a) $A^2-B^2=(A+B)(A-B)$ (b) $(A^T)^T=A$

(c) $(AB)^n=A^nB^n$, where A,B commute (d) $(A-I)(I+A)=0 \Leftrightarrow A^2=I$

95. Which of the following is/are incorrect

(i) Adjoint of a symmetric matrix is symmetric,

(ii) Adjoint of unit matrix is a unit matrix,

(iii) $A(adj\,A)=(adj\,A)A=|A|\,I$ and

(iv) Adjoint of a diagonal matrix is a diagonal matrix

(a) (i) (b) (ii) (c) (iii) and (iv) (d) None of these

96. Let $A=[a_{ij}]_{n\times n}$ be a square matrix and let c_{ij} be cofactor of a_{ij} in A. If $C=[c_{ij}]$, then

(a) $|C|=|A|$ (b) $|C|=|A|^{n-1}$ (c) $|C|=|A|^{n-2}$ (d) None of these

97. *A, B* are *n*-rowed square matrices such that $AB=0$ and B is non-singular. Then

(a) $A\neq 0$ (b) $A=0$ (c) $A=I$ (d) None of these

98. If A and B are two matrices such that $AB=B$ and $BA=A$, then $A^2+B^2=$ **[EAMCET 1994]**

(a) $2AB$ (b) $2BA$ (c) $A+B$ (d) AB

99. If A and B are two matrices such that $A+B$ and AB are both defined, then **[Pb. CET 1990]**

(a) A and B are two matrices not necessarily of same order

(b) A and B are square matrices of same order

(c) Number of columns of A= number of rows of B

(d) None of these

100. If $A=\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the identity matrix, then x = **[EAMCET 1993]**

(a) 1 (b) 2 (c) 3 (d) 0

101. If $A=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, then $(A+B)^2$ equals **[Rajasthan PET 1994]**

(a) A^2+B^2 (b) A^2+B^2+2AB (c) $A^2+B^2+AB-BA$ (d) None of these

102. If $A=\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ and $B=\begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$, then $(A+B)(A-B)$ is equal to **[Rajasthan PET 1994]**

(a) A^2-B^2 (b) A^2+B^2 (c) $A^2-B^2+BA+AB$ (d) None of these

103. If A is 3×4 matrix and B is a matrix such that $A'B$ and BA' are both defined. Then B is of the type **[Himachal Pradesh**

(a) 3×4 (b) 3×3 (c) 4×4 (d) 4×3

104. Which of the following is not true

(a) Every skew-symmetric matrix of odd order is non-singular

(b) If determinant of a square matrix is non-zero, then it is non-singular

(c) Adjoint of a symmetric matrix is symmetric

(d) Adjoint of a diagonal matrix is diagonal

105. Which one of the following statements is true [**MP PET 1996**]

(a) Non-singular square matrix does not have a unique inverse (b) Determinant of a non-singular matrix is zero

(c) If $A' = A$, then A is a square matrix (d) If $|A| \neq 0$, then $|A.adj\,A| = |A|^{(n-1)}$, where $A = (a_{ij})_{n\times n}$

106. If matrix $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, then

(a) $A' = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (b) $A^{-1} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

(c) $A.\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = 2I$ (d) $\lambda A = \begin{bmatrix} \lambda & -\lambda \\ 1 & 1 \end{bmatrix}$, where λ is a non-zero scalar

107. If $A = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}$, then [**MP PET 1996**]

(a) $A^2 = A$ (b) $B^2 = B$ (c) $AB \neq BA$ (d) $AB = BA$

108. Which one of the following is correct [**Kurukshetra CEE 1998**]

(a) Skew-symmetric matrix of odd order is non-singular. (b) Skew-symmetric matrix of odd order is singular

(c) Skew-symmetric matrix of even order is always singular (d) None of these

109. Choose the correct answer

(a) Every identity matrix is a scalar matrix

(b) Every scalar matrix is an identity matrix

(c) Every diagonal matrix is an identity matrix

(d) A square matrix whose each element is 1 is an identity matrix.

110. If A and B are two square matrices such that $B = -A^{-1}BA$, then $(A+B)^2 =$ [**EAMCET 2000**]

(a) 0 (b) $A^2 + B^2$ (c) $A^2 + 2AB + B^2$ (d) $A + B$

111. For a matrix A, $AI = A$ and $AA^T = I$ is true for [**Rajasthan PET 2000**]

(a) If A is a square matrix (b) If A is a non singular matrix (c)If A is a symmetric matrix

112. If two matrices A and B are of order $p \times q$ and $r \times s$ respectively, can be subtracted only, if

(a) $p = q$ (b) $p = q,\ r = s$ (c) $p = r,\ q = s$ (d) None of these

113. The set of all 2×2 matrices over the real numbers is not a group under matrix multiplication because

(a) Identity element does not exist (b) Closure property is not satisfied

(c) Association property is not satisfied (d) Inverse axiom may not be satisfied

114. If the matrix $AB = O$, then [**Pb. CET 2000; Kurukshetra CEE 1998; Rajasthan PET 2001**]

(a) $A = O$ or $B = O$ (b) $A = O$ and $B = O$

(c) It is not necessary that either $A = O$ or $B = O$ (d) $A \neq O,\ \ B \neq O$

115. If $a_{ij} = \frac{1}{2}(3i - 2j)$ and $A = [a_{ij}]_{2\times 2}$, then A is equal to [**Rajasthan PET 2001**]

(a) $\begin{bmatrix} 1/2 & 2 \\ -1/2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1/2 & -1/2 \\ 2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 1 \\ 1/2 & -1/2 \end{bmatrix}$ (d) None of these

116. Assuming that the sums and products given below are defined, which of the following is not true for matrices [**Karnat**

(a) $A+B=B+A$ (b) $AB=AC$ does not imply $B=C$

(c) $AB=O$ implies $A=O$ or $B=O$ (d) $(AB)'=B'A'$

117. Which of the following is true for matrix AB [**Rajasthan PET 2003**]

(a) $(AB)^{-1}=A^{-1}B^{-1}$ (b) $(AB)^{-1}=B^{-1}A^{-1}$ (c) $AB=BA$ (d) All of these

118. If A and B are 3×3 matrices such that $AB=A$ and $BA=B$, then

(a) $A^2=A$ and $B^2\neq B$ (b) $A^2\neq A$ and $B^2=B$ (c) $A^2=A$ and $B^2=B$ (d) $A^2\neq A$ and $B^2\neq B$

119. If A and B are symmetric matrices of order $n(A\neq B)$, then

(a) $A+B$ is skew symmetric (b) $A+B$ is symmetric

(c) $A+B$ is a diagonal matrix (d) $A-B$ is a zero matrix

120. The possible number of different order which a matrix can have when it has 24 elements is

(a) 6 (b) 8 (c) 4 (d) 10

121. If $A=\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ and $A^n=0$, then minimum value of n is

(a) 2 (b) 4 (c) 5 (d) 3

122. If *A, B, C* are square matrices of the same order, then which of the following is true [**JMIEE 1997**]

(a) $AB=AC$ (b) $(AB)^2=A^2B^2$ (c) $AB=0\Rightarrow A=0$ or $B=0$ (d) $AB=I\Rightarrow AB=BA$

123. If a matrix has 13 elements, then the possible dimensions (order) it can have are [**MNR 1985**]

(a) $1\times13,\ 13\times1$ (b) $1\times26,\ 26\times1$ (c) $2\times13,\ 13\times2$ (d) None of these

Transpose of Matrices

Basic Level

124. If *A, B, C* are three $n\times n$ matrices, then $(ABC)'=$ [**MP PET 1988**]

(a) $A'B'C'$ (b) $C'B'A'$ (c) $B'C'A'$ (d) $B'A'C'$

125. If $A=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $AA'=$ [**MP PET 1992**]

(a) 14 (b) $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ (d) None of these

126. If $A=\begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$, then $A+A^T$ equals [**Rajasthan PET 1994**]

(a) $\begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -4 \\ 10 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 4 \\ -10 & 6 \end{bmatrix}$ (d) None of these

127. If $A=\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$, then $(AB)^T=$ [**Rajasthan PET 1996, 2001**]

(a) $\begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 10 \\ 7 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

128. If $A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{pmatrix}$, then $(AB)^T$ is equal to [**Rajasthan PET 2001**]

(a) $\begin{pmatrix} -3 & -2 \\ 10 & 7 \end{pmatrix}$ (b) $\begin{pmatrix} -3 & 10 \\ -2 & 7 \end{pmatrix}$ (c) $\begin{pmatrix} -3 & 7 \\ 10 & 2 \end{pmatrix}$ (d) None of these

129. If $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 5 \\ 2 & 5 & 0 \end{bmatrix}$, then [**MNR 1982**]

(a) $A' = A$ (b) $A' = -A$ (c) $A' = 2A$ (d) None of these

130. Transpose of a row matrix is a

(a) Row matrix (b) Column matrix (c) A square matrix (d) A scalar matrix

131. If $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}$, then correct statement is [**Rajasthan PET 1987**]

(a) $AB = BA$ (b) $AA^T = A^2$ (c) $AB = B^2$ (d) None of these

132. If matrix *A* is of order $m \times n$ and *B* is of order $n \times p$, then order of $(AB)^T$ is equal to

(a) Order of *AB* (b) Order of *BA* (c) Order of $A^T B^T$ (d) Order of $B^T A^T$

133. If $A = \begin{pmatrix} 4 & 2 & 7 \\ 6 & 0 & 8 \end{pmatrix}$, then AA^T is [**Rajasthan PET 1991**]

(a) $\begin{pmatrix} 69 & 80 \\ 80 & 100 \end{pmatrix}$ (b) $\begin{pmatrix} 69 & 80 \\ 100 & 69 \end{pmatrix}$ (c) $\begin{pmatrix} 69 & 80 \\ 80 & 69 \end{pmatrix}$ (d) $\begin{pmatrix} 69 & 100 \\ 100 & 69 \end{pmatrix}$

134. Let *A* is a skew-symmetric matrix and *C* is a column matrix, then $C^T AC$ is [**Rajasthan PET 1995**]

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $[0]$ (c) $[1]$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

135. If *A* and *B* are matrices of suitable order and *k* is any number, then correct statement is

(a) $(AB)^T = A^T B^T$ (b) $(A+B)^T = A^T + B^T$ (c) $(AB)^{-1} = A^{-1}B^{-1}$ (d) $(kA)^T \neq kA^T$

136. If *A* and *B* are matrices of suitable order, then wrong statement is

(a) $(AB)^T = A^T B$ (b) $(A^T)^T = A$ (c) $(A-B)^T = A^T - B^T$ (d) $(A^T)^{-1} = (A^{-1})^T$

137. If *A* is a square matrix such that $|A| = 2$, then $|A'|$, where *A'* is transpose of *A*, is equal to

(a) 0 (b) –2 (c) 1/2 (d) 2

Special types of Matrices

Basic Level

138. An orthogonal matrix is

(a) $\begin{bmatrix} \cos\alpha & 2\sin\alpha \\ -2\sin\alpha & \cos\alpha \end{bmatrix}$ (b) $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ (c) $\begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

139. Matrix $\begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & -5 \\ -1 & 5 & 0 \end{bmatrix}$ is

(a) Orthogonal (b) Idempotent (c) Skew-symmetric (d) Symmetric

140. The inverse of a symmetric matrix is

(a) Symmetric (b) Skew-symmetric (c) Diagonal matrix (d) None of these

141. If A is a symmetric matrix and $n \in N$, then A^n is

(a) Symmetric (b) Skew-symmetric (c) A diagonal matrix (d) None of these

142. If A is a skew-symmetric matrix and n is a positive integer, then A^n is

(a) A symmetric matrix (b) Skew-symmetric matrix (c) Diagonal matrix (d) None of these

143. If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric, then $x =$ [**Karnataka CET 1994**]

(a) 3 (b) 5 (c) 2 (d) 4

144. If A is a square matrix, then $A + A^T$ is [**Rajasthan PET 2001**]

(a) Non-singular matrix (b) Symmetric matrix (c) Skew-symmetric matrix (d) Unit matrix

145. For any square matrix A, AA^T is a [**Rajasthan PET 2000**]

(a) Unit matrix (b) Symmetric matrix (c) Skew-symmetric matrix (d) Diagonal matrix

146. If A is a square matrix for which $a_{ij} = i^2 - j^2$, then A is [**Rajasthan PET 1999**]

(a) Zero matrix (b) Unit matrix (c) Symmetric matrix (d) Skew-symmetric matrix

147. If A is a square matrix and $A + A^T$ is symmetric matrix, then $A - A^T =$

(a) Unit matrix (b) Symmetric matrix (c) Skew-symmetric matrix (d) Zero matrix

148. The value of a for which the matrix $A = \begin{pmatrix} a & 2 \\ 2 & 4 \end{pmatrix}$ is singular if

(a) $a \neq 1$ (b) $a = 1$ (c) $a = 0$ (d) $a = -1$

149. The matrix $A = \begin{bmatrix} i & 1-2i \\ -1-2i & 0 \end{bmatrix}$ is which of the following [**Kurukshetra CEE 2002**]

(a) Symmetric (b) Skew-symmetric (c) Hermitian (d) Skew-hermitian

150. The matrix, $A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$ is nilpotent of index [**Kurukshetra CEE 2002**]

(a) 2 (b) 3 (c) 4 (d) 6

151. If $\begin{bmatrix} x & y \\ u & v \end{bmatrix}$ is symmetric matrix, then

(a) $x + v = 0$ (b) $x - v = 0$ (c) $y + u = 0$ (d) $y - u = 0$

152. The matrix $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is a

(a) Non-singular (b) Idempotent (c) Nilpotent (d) Orthogonal

153. For any square matrix A, which statement is wrong

(a) $(adj\, A)^{-1} = adj(A^{-1})$ (b) $(A^T)^{-1} = (A^{-1})^T$ (c) $(A^3)^{-1} = (A^{-1})^3$ (d) None of these

154. If $A=\begin{bmatrix}1&0&0&0\\2&3&0&0\\4&5&6&0\\7&8&9&10\end{bmatrix}$, then A is

(a) An upper triangular matrix (b) A null matrix
(c) A lower triangular matrix (d) None of these

155. If A is a square matrix, then A will be non-singular if

(a) $|A|=0$ (b) $|A|>0$ (c) $|A|<0$ (d) $|A|\neq 0$

156. The matrix $\begin{bmatrix}a&h&g\\h&b&f\\g&f&c\end{bmatrix}$ is

(a) Symmetric (b) Skew-symmetric (c) Scalar (d) None of these

157. If $A=\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}$, then A^2 is **[MNR 1980]**

(a) Null matrix (b) Unit matrix (c) A (d) $2A$

158. If A is a symmetric matrix, then matrix $M'AM$ is **[MP PET 1990]**

(a) Symmetric (b) Skew-symmetric (c) Hermitian (d) Skew-Hermitian

159. If A is a square matrix, then which of the following matrices is not symmetric

(a) $A+A'$ (b) AA' (c) $A'A$ (d) $A-A'$

160. Square matrix $[a_{ij}]_{n\times n}$ will be an upper triangular matrix, if

(a) $a_{ij}\neq 0$ for $i>j$ (b) $a_{ij}=0$ for $i>j$ (c) $a_{ij}=0$ for $i<j$ (d) None of these

161. If the matrix $\begin{bmatrix}0&1&-2\\-1&0&3\\\lambda&-3&0\end{bmatrix}$ is singular, then $\lambda=$ **[MP PET 1989]**

(a) –2 (b) –1 (c) 1 (d) 2

162. In order that the matrix $\begin{bmatrix}1&2&3\\4&5&6\\3&\lambda&5\end{bmatrix}$ be non-singular, λ should not be equal to **[Kurukshetra CEE 1998]**

(a) 1 (b) 2 (c) 3 (d) 4

163. If A is involutory matrix and and I is unit matrix of same order, then $(I-A)(I+A)$ is

(a) Zero matrix (b) A (c) I (d) $2A$

164. If $A=\begin{bmatrix}1&0&1\\0&1&1\\1&0&0\end{bmatrix}$, then A is

(a) Symmetric (b) Skew-symmetric (c) Non-singular (d) Singular

165. If $A=\begin{bmatrix}1&0&0\\0&1&0\\a&b&-1\end{bmatrix}$, then $A^2=$ **[MNR 1980; Pb. CET 1990]**

(a) Unit matrix (b) Null matrix (c) A (d) – A

166. If the matrix $\begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$ is singular, then $\lambda =$ [**MP PET 1990; Pb. CET 2000**]

(a) –2 (b) 4 (c) 2 (d) –4

167. Out of the following a skew-symmetric matrix is [**MP PET 1992**]

(a) $\begin{bmatrix} 0 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 & 5 \\ -4 & 1 & -6 \\ -5 & 6 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 4 & 5 \\ -4 & 2 & -6 \\ -5 & 6 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} i+1 & 4 & 5 \\ -4 & i & -6 \\ -5 & 6 & i \end{bmatrix}$

168. If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 4 \\ 3 & 5 & 6 \end{bmatrix}$, then A is

(a) Singular (b) Non-singular (c) Unitary (d) Symmetric

169. If *A, B, C* are three square matrices such that $AB = AC$ implies $B = C$, then the matrix A is always [**MP PET 1989; Karnataka CET 1992**]

(a) A singular matrix (b) A Non-singular matrix (c) An orthogonal matrix (d) A diagonal matrix

170. The matrix $A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$ is [**MP PET 1988**]

(a) Unitary (b) Orthogonal (c) Nilpotent (d) Involutary

171. If a matrix A is symmetric as well as skew symmetric, then

(a) A is a diagonal matrix (b) A is a null matrix (c) A is a unit matrix (d)

172. A and B are any two square matrices. Which one of the following is a skew symmetric matrix

(a) $\frac{A+A'}{2}$ (b) $\frac{A+B}{2}$ (c) $\frac{A'-A}{2}$ (d) None of the above.

173. Choose the correct answer

(a) Every scalar matrix is an identity matrix

(b) Every identity matrix is a scalar matrix

(c) Every diagonal matrix is an identity matrix

(d) A Square matrix whose each element is 1 is an identity matrix

174. For a square matrix *A,* it is given that $AA' = I$, then A is a [**DCE 1998**]

(a) Orthogonal matrix (b) Diagonal matrix (c) Symmetric matrix (d) None of these

175. A square matrix can always be expressed as a [**DCE1998**]

(a) Sum of a symmetric matrix and a skew-symmetric matrix (b) Sum of a diagonal matrix and a symmetric matrix

(c) Skew matrix (d) Skew- symmetric matrix

176. If A is a skew-symmetric matrix and n is odd positive integer, then A^n is

(a) A symmetric matrix (b) A skew-symmetric matrix (c) A diagonal matrix (d) None of these

177. If *A*, B symmetric matrices of the same order then $AB - BA$ is

(a) Symmetric matrix (b) Skew-symmetric matrix (c) Null matrix (d) Unit matrix

Adjoint of Matrix

Basic Level

178. If k is a scalar and I is a unit matrix of order 3, then $adj(kI) =$

(a) $k^3 I$ (b) $k^2 I$ (c) $-k^3 I$ (d) $-k^2 I$

179. If $A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $adj\, A =$

(a) A (b) I (c) O (d) A^2

180. If A is a $n \times n$ matrix, then $adj(adj\, A) =$

(a) $|A|^{n-1} A$ (b) $|A|^{n-2} A$ (c) $|A|^n n$ (d) None of these

181. Adjoint of the matrix $N = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ is **[MP PET 1989]**

(a) N (b) $2N$ (c) $-N$ (d) None of these

182. If A is a non-singular matrix, then $A(adj\, A) =$

(a) A (b) I (c) $|A|\, I$ (d) $|A|^2\, I$

183. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ and $A\, adj\, A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then k is equal to

(a) 0 (b) 1 (c) $\sin\alpha \cos\alpha$ (d) $\cos 2\alpha$

184. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ -1 & 6 & 1 \end{bmatrix}$, then the adjoint of A is **[MNR 1982]**

(a) $\begin{bmatrix} 2 & -5 & 32 \\ 0 & 1 & -6 \\ 0 & 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 & 0 \\ -5 & -2 & 0 \\ 1 & -6 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 0 & 0 \\ -5 & -2 & 0 \\ 1 & -6 & -1 \end{bmatrix}$ (d) None of these

185. If $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$, then $A(adj\, A) =$ **[MP PET 1995; Rajasthan PET 1997]**

(a) $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 10 & 1 \\ 1 & 10 \end{bmatrix}$ (d) None of these

186. If A is a singular matrix, then $adj\, A$ is **[Karnataka CET 1993]**

(a) Singular (b) Non-singular (c) Symmetric (d) Not defined

187. The adjoint of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ is

(a) $\begin{bmatrix} 3 & -9 & -5 \\ -4 & 1 & 3 \\ -5 & 4 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$ (d) None of these

188. $Adj.(AB) - (Adj.B)(Adj.A) =$ **[MP PET 1997]**

(a) $Adj.A - Adj.B$ (b) I (c) O (d) None of these

189. If $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$, then $adj\,A =$ **[Rajasthan PET 1996]**

(a) $\begin{pmatrix} 1 & 4 & -2 \\ -2 & 1 & 4 \\ 4 & -2 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2 & 4 \\ -4 & 1 & 2 \\ -4 & -2 & 1 \end{pmatrix}$ (d) None of these

190. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$, then the value of $|adj\,A|$ is **[Rajasthan PET 1999]**

(a) 36 (b) 72 (c) 144 (d) None of these

191. If A is a matrix of order 3 and $|A| = 8$, then $|adj\,A| =$ **[DCE 1999; Karnataka CET 2002]**

(a) 1 (b) 2 (c) 2^3 (d) 2^6

192. If A and B are non-singular square matrices of same order, then $adj(AB)$ is equal to **[AMU 1999]**

(a) $(adj\,A)(adj\,B)$ (b) $(adj\,B)(adj\,A)$ (c) $(adj\,B^{-1})(adj\,A^{-1})$ (d) $(adj\,A^{-1})(adj\,B^{-1})$

193. If d is the determinant of a square matrix A of order n, then the determinant of its adjoint is

(a) d^n (b) d^{n-1} (c) d^{n+1} (d) d

194. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, then $adj\,A$ is equal to **[Rajasthan PET 2001]**

(a) $\begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$

195. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$, then $A.(adj\,A) =$ **[Rajasthan PET 2002]**

(a) I (b) $|A|$ (c) $|A|\,I$ (d) None of these

196. If $A = \begin{bmatrix} -2 & 6 \\ -5 & 7 \end{bmatrix}$, then $adj(A)$ is

(a) $\begin{bmatrix} 7 & -6 \\ 5 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -6 \\ 5 & -7 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & -5 \\ 6 & -2 \end{bmatrix}$ (d) None of these

197. The adjoint matrix of $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ is **[MP PET 2003]**

(a) $\begin{bmatrix} 4 & 8 & 3 \\ 2 & 1 & 6 \\ 0 & 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ (c) $\begin{bmatrix} 11 & 9 & 3 \\ 1 & 2 & 8 \\ 6 & 9 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 3 \\ -2 & 3 & -3 \end{bmatrix}$

198. If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then $A.(adj(A)) =$ **[Rajasthan PET 2003]**

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

199. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of $|A||Adj\;A|$ is **[AMU 1987]**

(a) a^3 (b) a^6 (c) a^9 (d) a^{27}

200. If $A=\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of $|adj\ A|$ is [**AMU 1989**]

(a) a^3 (b) a^6 (c) a^9 (d) a^{27}

201. If $A=\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then determinant $(adj(adj\,A))$ is

(a) $(14)^1$ (b) $(14)^2$ (c) $(14)^3$ (d) $(14)^4$

202. If A is a square matrix, then $adj(A')-(adj\,A)'$ is equal to

(a) $2|A|$ (b) $2|A|\,I$ (c) Null matrix (d) Unit matrix

203. If $A=\begin{bmatrix} 1 & -2 & 3 \\ 4 & 0 & -1 \\ -3 & 1 & 5 \end{bmatrix}$, then $(adj\,A)_{23}$ is equal to [**Rajasthan PET 1984**]

(a) 13 (b) – 13 (c) 5 (d) – 5

204. For a third order non-singular matrix *A*, $|A(adj\ A)|$ equals

(a) $|A|$ (b) $|A|^2$ (c) $|A|^3$ (d) None of these

Advance Level

205. If A be a square matrix of order n and if $|A|=D$ and $|adj\ A|=D'$, then [**Rajasthan PET 2000**]

(a) $DD'=D^2$ (b) $DD'=D^{n-1}$ (c) $DD'=D^n$ (d) None of these

Inverse of a Matrix

Basic Level

206. Inverse of the matrix $\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ is [**MP PET 1990**]

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 3 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -3 & 5 \\ 7 & 4 & 6 \\ 4 & 2 & 7 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & -4 \\ 8 & -4 & -5 \\ 3 & 5 & 2 \end{bmatrix}$

207. If A and B are non-singular matrices, then [**MP PET 1991; Kurukshetra CEE 1998**]

(a) $(AB)^{-1}=A^{-1}B^{-1}$ (b) $AB=BA$ (c) $(AB)'=A'B'$ (d) $(AB)^{-1}=B^{-1}A^{-1}$

208. If $A=\begin{bmatrix} i & 0 \\ 0 & i/2 \end{bmatrix}$, $(i=\sqrt{-1})$, then $A^{-1}=$ [**MP PET 1992**]

(a) $\begin{bmatrix} i & 0 \\ 0 & i/2 \end{bmatrix}$ (b) $\begin{bmatrix} -i & 0 \\ 0 & -2i \end{bmatrix}$ (c) $\begin{bmatrix} i & 0 \\ 0 & 2i \end{bmatrix}$ (d) $\begin{bmatrix} 0 & i \\ 2i & 0 \end{bmatrix}$

209. If $A=\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A^{-1}=$

(a) $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ (b) $\begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ (c) $\begin{bmatrix} -\cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{bmatrix}$ (d) None of these

210. If $A = \begin{bmatrix} a & c \\ d & b \end{bmatrix}$, then $A^{-1} =$ **[MP PET 1988]**

(a) $\frac{1}{ab-cd}\begin{bmatrix} b & -c \\ -d & a \end{bmatrix}$ (b) $\frac{1}{ad-bc}\begin{bmatrix} b & -c \\ -d & a \end{bmatrix}$ (c) $\frac{1}{ab-cd}\begin{bmatrix} b & d \\ c & a \end{bmatrix}$ (d) None of these

211. The element of second row and third column in the inverse of $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is **[MP PET 1992]**

(a) – 2 (b) – 1 (c) 1 (d) 2

212. The inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is **[MP PET 1989; Pb. CET 1989, 93]**

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

213. The inverse of $\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$ is

(a) $\frac{-1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ (b) $\frac{-1}{8}\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$ (c) $\frac{1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ (d) $\frac{1}{8}\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$

214. The inverse of the matrix $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$ is **[MP PET 1994]**

(a) $\begin{bmatrix} \frac{4}{14} & \frac{2}{14} \\ \frac{-1}{14} & \frac{3}{14} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{3}{14} & \frac{-2}{14} \\ \frac{1}{14} & \frac{4}{14} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{4}{14} & \frac{-2}{14} \\ \frac{1}{14} & \frac{3}{14} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{3}{14} & \frac{2}{14} \\ \frac{1}{14} & \frac{4}{14} \end{bmatrix}$

215. If a matrix A is such that $3A^3 + 2A^2 + 5A + I = 0$, then its inverse is

(a) $-(3A^2 + 2A + 5I)$ (b) $3A^2 + 2A + 5I$ (c) $3A^2 - 2A - 5I$ (d) None of these

216. If $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $G(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$, then $[F(\alpha)\, G(\beta)]^{-1} =$

(a) $F(\alpha) - G(\beta)$ (b) $-F(\alpha) - G(\beta)$ (c) $[F(\alpha)]^{-1}[G(\beta)]^{-1}$ (d) $[G(\beta)]^{-1}[F(\alpha)]^{-1}$

217. If $A = \begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}$ and $AB = I$, then $B =$ **[MP PET 1995, 98]**

(a) $\cos^2\frac{\theta}{2}.A$ (b) $\cos^2\frac{\theta}{2}.A^T$ (c) $\cos^2\frac{\theta}{2}.\, I$ (d) None of these

218. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then the matrix $A =$

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

219. If A is an invertible matrix, then which of the following is correct

(a) A^{-1} is multivalued (b) A^{-1} is singular (c) $(A^{-1})^T \neq (A^T)^{-1}$ (d) $|A| \neq 0$

220. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, then $A^{-1} =$ **[AMU 1988]**

(a) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ (b) $\begin{bmatrix} -a & 0 & 0 \\ 0 & -b & 0 \\ 0 & 0 & -c \end{bmatrix}$ (c) $\begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$ (d) None of these

221. $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}^{-1} =$ **[EAMCET 1994; DCE 1999]**

(a) $\begin{bmatrix} 10 & 3 \\ 3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -3 \\ -3 & -10 \end{bmatrix}$

222. If $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$, then $A^{-1} =$ **[EAMCET 1988]**

(a) $\begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -2 \\ -3 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

223. $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}^{-1} =$ **[Karnataka CET 1994]**

(a) $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -5 \\ -7 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & -5 \\ 7 & -6 \end{bmatrix}$

224. The inverse of matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is **[Karnataka CET 1993]**

(a) A (b) A^T (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

225. The inverse of $\begin{bmatrix} 3 & 5 & 7 \\ 2 & -3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ is **[EAMCET 1989]**

(a) $\begin{bmatrix} 7 & 3 & -26 \\ 3 & 1 & -11 \\ -5 & -2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 3 & -26 \\ 3 & 1 & 11 \\ -5 & -2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 & 11 \\ 7 & 3 & -26 \\ -5 & 2 & 1 \end{bmatrix}$ (d) None of these

226. The inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ (d) None of these

227. The matrix $\begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is invertible, if **[Kurukshetra CEE 1996]**

(a) $\lambda \neq -15$ (b) $\lambda \neq -17$ (c) $\lambda \neq -16$ (d) $\lambda \neq -18$

228. If $A = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$, then $(A^{-1})^3$ is equal to **[MP PET 1997]**

(a) $\frac{1}{27}\begin{pmatrix}1 & -26\\0 & 27\end{pmatrix}$ (b) $\frac{1}{27}\begin{pmatrix}-1 & 26\\0 & 27\end{pmatrix}$ (c) $\frac{1}{27}\begin{pmatrix}1 & -26\\0 & -27\end{pmatrix}$ (d) $\frac{1}{27}\begin{pmatrix}-1 & -26\\0 & -27\end{pmatrix}$

229. The matrix $\begin{bmatrix}1 & a & 2\\1 & 2 & 5\\2 & 1 & 1\end{bmatrix}$ is not invertible, if 'a' has the value

(a) 2 (b) 1 (c) 0 (d) – 1

230. Inverse matrix of $\begin{bmatrix}4 & 7\\1 & 2\end{bmatrix}$ **[Rajasthan PET 1996, 2001]**

(a) $\begin{bmatrix}2 & -7\\-1 & 4\end{bmatrix}$ (b) $\begin{bmatrix}2 & -1\\-7 & 4\end{bmatrix}$ (c) $\begin{bmatrix}-2 & 7\\1 & -4\end{bmatrix}$ (d) $\begin{bmatrix}-2 & 1\\7 & -4\end{bmatrix}$

231. If the multiplicative group of 2×2 matrices of the form $\begin{pmatrix}a & a\\a & a\end{pmatrix}$, for $a\neq 0$ and $a\in R$, then the inverse of $\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix}$ is **[Karnataka CET 1999]**

(a) $\begin{pmatrix}\frac{1}{8} & \frac{1}{8}\\\frac{1}{8} & \frac{1}{8}\end{pmatrix}$ (b) $\begin{pmatrix}\frac{1}{4} & \frac{1}{4}\\\frac{1}{4} & \frac{1}{4}\end{pmatrix}$ (c) $\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\\frac{1}{2} & \frac{1}{2}\end{pmatrix}$ (d) Does not exist

232. The element in the first row and third column of the inverse of the matrix $\begin{bmatrix}1 & 2 & -3\\0 & 1 & 2\\0 & 0 & 1\end{bmatrix}$ is

(a) – 2 (b) 0 (c) 1 (d) 7

233. If I_3 is the identity matrix of order 3, then I_3^{-1} is **[Pb. CET 2000]**

(a) 0 (b) $3I_3$ (c) I_3 (d) Does not exist

234. If a matrix A is such that $4A^3+2A^2+7A+I=O$, then A^{-1} equals

(a) $(4A^2+2A+7I)$ (b) $-(4A^2+2A+7I)$ (c) $-(4A^2-2A+7I)$ (d) $(4A^2+2A-7I)$

235. If $A=\begin{bmatrix}2 & 2\\-3 & 2\end{bmatrix}$ and $B=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$, then $(B^{-1}A^{-1})^{-1}=$ **[EAMCET 2001]**

(a) $\begin{bmatrix}2 & -2\\2 & 3\end{bmatrix}$ (b) $\begin{bmatrix}3 & -2\\2 & 2\end{bmatrix}$ (c) $\frac{1}{10}\begin{bmatrix}2 & 2\\-2 & 3\end{bmatrix}$ (d) $\frac{1}{10}\begin{bmatrix}3 & 2\\-2 & 2\end{bmatrix}$

236. If $A^2-A+I=0$, then $A^{-1}=$ **[Kerala (Engg.) 2001]**

(a) A^{-2} (b) $A+I$ (c) $I-A$ (d) $A-I$

237. If $A=\begin{bmatrix}2 & 3\\4 & 6\end{bmatrix}$, then $A^{-1}=$ **[Karnataka CET 2001]**

(a) $\begin{bmatrix}1 & 2\\-3/2 & 3\end{bmatrix}$ (b) $\begin{bmatrix}2 & -3\\4 & 6\end{bmatrix}$ (c) $\begin{bmatrix}-2 & 4\\-3 & 6\end{bmatrix}$ (d) Does not exist

238. If for the matrix A, $A^3=I$, then $A^{-1}=$ **[Rajasthan PET 2002]**

(a) A^2 (b) A^3 (c) A (d) None of these

239. For two invertible matrices A and B of suitable orders, the value of $(AB)^{-1}$ is **[Rajasthan PET 2000, 02; Karnataka CET 2001]**

(a) $(BA)^{-1}$ (b) $B^{-1}A^{-1}$ (c) $A^{-1}B^{-1}$ (d) $(AB')^{-1}$

240. If $A=\begin{bmatrix}-1 & 2\\2 & -1\end{bmatrix}$ and $B=\begin{bmatrix}3\\1\end{bmatrix}$, $AX=B$, then $X=$ **[MP PET 2002]**

(a) $[5\ \ 7]$ (b) $\frac{1}{3}\begin{bmatrix}5\\7\end{bmatrix}$ (c) $\frac{1}{3}[5\ \ 7]$ (d) $\begin{bmatrix}5\\7\end{bmatrix}$

241. If $A=\begin{bmatrix}1 & 2\\ 3 & -5\end{bmatrix}$, then $A^{-1}=$

(a) $\begin{bmatrix}-5 & -2\\ -3 & 1\end{bmatrix}$ (b) $\begin{bmatrix}\frac{5}{11} & \frac{2}{11}\\ \frac{3}{11} & -\frac{1}{11}\end{bmatrix}$ (c) $\begin{bmatrix}-\frac{5}{11} & -\frac{2}{11}\\ -\frac{3}{11} & -\frac{1}{11}\end{bmatrix}$ (d) $\begin{bmatrix}5 & 2\\ 3 & -1\end{bmatrix}$

242. If $A=\begin{bmatrix}0 & 3\\ 2 & 0\end{bmatrix}$ and $A^{-1}=\lambda(adj(A))$, then $\lambda=$ **[UPSEAT 2002]**

(a) $\frac{-1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{-1}{3}$ (d) $\frac{1}{6}$

243. The multiplicative inverse of matrix $\begin{bmatrix}2 & 1\\ 7 & 4\end{bmatrix}$ is **[DCE 2002]**

(a) $\begin{bmatrix}4 & -1\\ -7 & -2\end{bmatrix}$ (b) $\begin{bmatrix}-4 & -1\\ 7 & -2\end{bmatrix}$ (c) $\begin{bmatrix}4 & -7\\ 7 & 2\end{bmatrix}$ (d) $\begin{bmatrix}4 & -1\\ -7 & 2\end{bmatrix}$

244. The inverse matrix of $\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$ is **[MP PET 2003]**

(a) $\begin{bmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\ -4 & 3 & -1\\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2}\end{bmatrix}$ (b) $\begin{bmatrix}\frac{1}{2} & -4 & \frac{5}{2}\\ 1 & -6 & 3\\ 1 & 2 & -1\end{bmatrix}$ (c) $\frac{1}{2}\begin{bmatrix}1 & 2 & 3\\ 3 & 2 & 1\\ 4 & 2 & 3\end{bmatrix}$ (d) $\frac{1}{2}\begin{bmatrix}1 & -1 & -1\\ -8 & 6 & -2\\ 5 & -3 & 1\end{bmatrix}$

245. Inverse of the matrix $\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}$ is

(a) $\frac{1}{10}\begin{bmatrix}4 & 2\\ -3 & 1\end{bmatrix}$ (b) $\frac{1}{10}\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}$ (c) $\frac{1}{10}\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}$ (d) $\begin{bmatrix}4 & 2\\ -3 & 1\end{bmatrix}$

246. If A is an orthogonal matrix, then A^{-1} is equal to

(a) A (b) A' (c) A^2 (d) None of these

247. The multiplicative inverse of the matrix $A=\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}$ is

(a) $\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & 1 & -1\\ 0 & -1 & 0\\ -1 & 0 & 0\end{bmatrix}$ (c) $\begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 1 & 0 & -1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}$

248. Let A be an invertible matrix. Which of the following is not true

(a) $A^{-1} = |A|^{-1}$ (b) $(A^2)^{-1}=(A^{-1})^2$ (c) $(A')^{-1}=(A^{-1})'$ (d) None of these

249. Inverse of $\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 3 & 4 & 6\end{bmatrix}$ is **[Kurukshetra CEE 1995]**

(a) $\begin{bmatrix}2 & 0 & -1\\ 0 & -3 & 2\\ -1 & 2 & -1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 3 & 4 & 6\end{bmatrix}$ (c) $\begin{bmatrix}-2 & 0 & 1\\ 0 & 3 & -2\\ 1 & -2 & 1\end{bmatrix}$ (d) None of these

250. If $A=\begin{bmatrix}1 & 2\\ -4 & -1\end{bmatrix}$, then $A^{-1}=$ **[Karnataka CET 1997]**

(a) $\frac{1}{7}\begin{bmatrix}1 & 2\\ -4 & -1\end{bmatrix}$ (b) $\begin{bmatrix}-1 & -2\\ 4 & 1\end{bmatrix}$ (c) $\frac{1}{9}\begin{bmatrix}1 & 2\\ 4 & 1\end{bmatrix}$ (d) $\frac{1}{7}\begin{bmatrix}-1 & -2\\ 4 & 1\end{bmatrix}$

251. If $A=\begin{bmatrix}0 & -1 & 2\\ 2 & -2 & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 1 & 0\\ 1 & 1\end{bmatrix}$ and $M=AB$, then M^{-1} is equal to

(a) $\begin{bmatrix}2 & -2\\ 2 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1/3 & 1/3\\ -1/3 & 1/6\end{bmatrix}$ (c) $\begin{bmatrix}1/3 & -1/3\\ 1/3 & 1/6\end{bmatrix}$ (d) $\begin{bmatrix}1/3 & -1/3\\ -1/3 & 1/6\end{bmatrix}$

252. If for a square matrix A, $AA^{-1}=I$, then A is [**DCE 1998**]

(a) Orthogonal matrix (b) Symmetric matrix (c) Diagonal matrix (d) Invertible matrix

253. If matrix $\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & \lambda & 5\end{bmatrix}$ is invertible, then [**Kurukshetra CEE 1998**]

(a) $\lambda \neq 4$ (b) $\lambda \neq 3$ (c) $\lambda \neq 2$ (d) $\lambda \neq 0$

254. If $\begin{bmatrix}1 & -\tan\theta\\ \tan\theta & 1\end{bmatrix}\begin{bmatrix}1 & \tan\theta\\ -\tan\theta & 1\end{bmatrix}^{-1}=\begin{bmatrix}a & -b\\ b & a\end{bmatrix}$, then

(a) $a=1, b=1$ (b) $a=\cos 2\theta, b=\sin 2\theta$ (c) $a=\sin 2\theta, b=\cos 2\theta$ (d) None of these

Advance Level

255. If $A=\begin{bmatrix}1 & -2 & 3\\ 0 & -1 & 4\\ -2 & 2 & 1\end{bmatrix}$, then $(A')^{-1}=$

(a) $\begin{bmatrix}-9 & -8 & -2\\ 8 & 7 & 2\\ -5 & -4 & -1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 2\\ 3 & 4 & 1\end{bmatrix}$ (c) $\begin{bmatrix}-9 & 8 & -5\\ -8 & 7 & -4\\ 2 & 2 & -1\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

256. If ω is a cube root of unity and $A=\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{bmatrix}$, then $A^{-1}=$

(a) $\begin{bmatrix}1 & \omega & \omega^2\\ \omega^2 & 1 & \omega\\ \omega & \omega^2 & 1\end{bmatrix}$ (b) $\frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \omega^2 & \omega\\ 1 & \omega & \omega^2\end{bmatrix}$ (c) $\begin{bmatrix}1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\\ 1 & 1 & 1\end{bmatrix}$ (d) $\frac{1}{2}\begin{bmatrix}1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\\ 1 & 1 & 1\end{bmatrix}$

257. If $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$, then $A^{-1}=$ [**DCE 1999**]

(a) A (b) A^2 (c) A^3 (d) A^4

258. If $D=\text{diag}(d_1, d_2, d_3, ..., d_n)$, where $d_i \neq 0$ for all $i=1,2,...,n$, then D^{-1} is equal to

(a) D (b) $\text{diag}(d_1^{-1}, d_2^{-1}, ...d_n^{-1})$ (c) I (d) None of these

259. If $A=\text{diag}(d_1, d_2, d_3, ..., d_n)$, then A^n is equal to

(a) diag $(d_1^{n-1}, d_2^{n-1}, d_3^{n-1}, ...d_n^{n-1})$ (b) diag $(d_1^n, d_2^n, d_3^n, ...d_n^n)$

(c) A (d) None of these

Relation between Determinants and Matrices

Basic Level

260. If A is a square matrix of order 3, then true statement is (where I is unit matrix) **[MP PET 1992]**

(a) det $(-A) = -\det A$ (b) det A = 0 (c) det $(A+I) = 1 + \det A$ (d) det $2A$ = 2 det A

261. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$, then $|AB|$ is equal to **[Rajasthan PET 1995]**

(a) 4 (b) 8 (c) 16 (d) 32

262. If A and B are square matrices of order 3 such that $|A| = -1, |B| = 3$, then $|3AB| =$ **[IIT 1988; MP PET 1995, 99]**

(a) – 9 (b) – 81 (c) – 27 (d) 81

263. Which of the following is correct

(a) Determinant is a square matrix (b) Determinant is a number associated to a matrix

(c) Determinant is a number associated to a square matrix (d) None of these

264. Let A be a skew-symmetric matrix of odd order, then $|A|$ is equal to

(a) 0 (b) 1 (c) –1 (d) None of these

265. Let A be a skew-symmetric matrix of even order, then $|A|$

(a) Is a perfect square (b) Is not a perfect square (c) Is always zero (d) None of these

266. For any 2×2 matrix A, if $A(adj.A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then $|A| =$ **[MP PET 1999]**

(a) 0 (b) 10 (c) 20 (d) 100

267. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, then determinant of $A^2 - 2A$ is **[EAMCET 2000]**

(a) 5 (b) 25 (c) – 5 (d) – 25

268. If $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix, then x is **[Kerala (Engg.) 2001]**

(a) $\frac{13}{25}$ (b) $-\frac{25}{13}$ (c) $\frac{5}{13}$ (d) $\frac{25}{13}$

269. The product of a matrix and its transpose is an identity matrix. The value of determinant of this matrix is

(a) – 1 (b) 0 (c) ± 1 (d) 1

270. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, then det A =

(a) 2 (b) 3 (c) 4 (d) 5

271. If $A \neq O$ and $B \neq O$ are $n \times n$ matrix such that $AB = O$, then

(a) $Det\,(A) = 0$ or $Det\,(B) = 0$ (b) $Det(A) = 0$ and $Det(B) = 0$

(c) $Det\,(A) = Det\,(B) \neq 0$ (d) $A^{-1} = B^{-1}$

272. If A is a square matrix such that $A^2 = A$, then det (A) equals

(a) 0 or 1 (b) – 2 or 2 (c) – 3 or 3 (d) None of these

273. If A is a square matrix such that $|A| = 2$, then for any +ve integer n, $|A^n|$ is equal to

(a) 0 (b) $2n$ (c) 2^n (d) n^2

274. If A is a square matrix of order 3 and entries of A are positive integers, then $|A|$ is

(a) Different from zero (b) 0 (c) Positive (d) An arbitrary integer.

275. If A and B are any 2×2 matrix, then det $(A+B) = 0$ implies

(a) $DetA + DetB = 0$ (b) $DetA = 0$ or $DetB = 0$ (c) $DetA = 0$ and $DetB = 0$ (d) None of these

System of Linear Equations

Basic Level

276. If $\begin{bmatrix} x+y+z \\ x+y \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$, then (x, y, z) =

(a) (4, 3, 2) (b) (3, 2, 4) (c) (2, 3, 4) (d) None of these

277. The solution of the equation $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ is (x, y, z) =

(a) (1, 1, 1) (b) (0, –1, 2) (c) (–1, 2, 2) (d) (–1, 0, 2)

278. If $AX = B$, $B = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 3 & -\frac{1}{2} & -\frac{1}{2} \\ -4 & \frac{3}{4} & \frac{5}{4} \\ 2 & -\frac{1}{4} & -\frac{3}{4} \end{bmatrix}$, then X is equal to

(a) $\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ (b) $\begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} -4 \\ 2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 3 \\ \frac{3}{4} \\ -\frac{3}{4} \end{bmatrix}$

Rank of a Matrix

Basic Level

279. If A is a non-zero column matrix of order $m \times 1$ and B is a non-zero row matrix of order $1 \times n$, then rank of AB is equal to

(a) m (b) n (c) 1 (d) None of these

280. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then

(a) $\rho(A) = 2$ (b) $\rho(A) = 1$ (c) $\rho(A) = 3$ (d) None of these

281. If I_n is the identity matrix of order n, then rank of I_n is

(a) 1 (b) n (c) 0 (d) None of these

282. The rank of a null matrix is

(a) 0 (b) 1 (c) Does not exist (d) None of these

283. If A is a non-singular square matrix of order n, then the rank of A is

(a) Equal to n (b) Less then n (c) Greater then n (d) None of these

284. If A and B are two matrices such that rank of $A = m$ and rank of $B = n$, then

(a) rank $(AB) = mn$ (b) rank $(AB) \geq$ rank (A)

(c) rank $(AB) \geq$ rank (B) (d) rank $(AB) \leq$ min (rank A, rank B

285. If A is an inevitable matrix and B is a matrix, then

(a) rank (AB) = rank (A) (b) rank (AB) = rank (B) (c) rank (AB) > rank (A) (d) rank (AB) > rank (B)

Advance Level

286. If the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are collinear, then the rank of the matrix $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$ will always be less than

[Orissa JEE 2003]

(a) 3 (b) 2 (c) 1 (d) None of these

287. If A is a matrix such that there exists a square submatrix of order r which is non-singular and every square submatrix of order $r+1$ or more is singular, then

(a) rank (A) = r +1 (b) rank (A) = r (c) rank (A) > r (d) rank (A) $\geq$ r +1

288. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 3 (d) 4

289. The system of equations $AX = B$ of n equations in n unknown has infinitely many solutions if

(a) $\det A \neq 0$ (b) $\det A \neq 0, (adj\, A)B = 0$ (c) $\det A = 0, (adj\, A)B \neq 0$ (d) $\det A = 0, (adj\, A)B \neq 0$

290. The trace of skew symmetric matrix of order $n \times n$ is

(a) 0 (b) 1 (c) n (d) n^2

Miscellaneous Problems

Basic Level

291. If $A = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$ and $f(x) = 2x^2 - 3x$,then $f(A)$ equals

(a) $\begin{bmatrix} 14 & 1 \\ 0 & -9 \end{bmatrix}$ (b) $\begin{bmatrix} -14 & 1 \\ 0 & 9 \end{bmatrix}$ (c) $\begin{bmatrix} 14 & -1 \\ 0 & 9 \end{bmatrix}$ (d) $\begin{bmatrix} -14 & -1 \\ 0 & -9 \end{bmatrix}$

292. The construction of 3×4 matrix A whose element a_{ij} is given by $\frac{(i+j)^2}{2}$ is **[IIT 1988]**

(a) $\begin{bmatrix} 2 & 9/2 & 8 & 25 \\ 9 & 4 & 5 & 18 \\ 8 & 25 & 18 & 49 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 9/2 & 25/2 \\ 9/2 & 5/2 & 5 \\ 25 & 18 & 25 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 9/2 & 8 & 25/2 \\ 9/2 & 8 & 25/2 & 18 \\ 8 & 25/2 & 18 & 49/2 \end{bmatrix}$ (d) None of these

293. If A is a square matrix of order n such that its elements are polynomial in x and its r-rows become identical for $x = k$, then

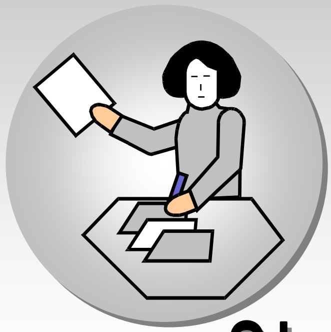
(a) $(x-k)^r$ is a factor of $|A|$ (b) $(x-k)^{r-1}$ is a factor of $|A|$

(c) $(x-k)^{r+1}$ is a factor of $|A|$ (d) $(x-k)^r$ is a factor of A

294. If $A = [a_{ij}]$ is a scalar matrix of order $n \times n$ such that $a_{ij} = k$ for all i, then trace of A is equal to

(a) nk (b) $n+k$ (c) n/k (d) None of these

***



Answer Sheet

Matrices **_Assignment (Basic and Advance Level)_**

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
b	b	c	d	d	d	c	c	a	b	b	b	b	c	c	b	a	b	a	c
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
b	c	d	c	b	d	a	b	c	b	a	d	d	b	c	b	b	b	b	b
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
d	c	a,b	d	a	c	c	c	a	d	b	b	b	b	a	a	c	c	b	b
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
c	b	c	b	a	b	c	c	a	c	c	d	b	c	b	d	a	c	d	d
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
c	d	c	b	b	b	d	d	c	c	c	a	a	a	d	b	b	c	b	d
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
a	a	a	a	c	c	c	b	a	b	a	c	d	c	b	c	b	c	b	b
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
a	d	a	b	c	a	b	b	b	b	d	d	a	b	b	a	d	b	c	a
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
a	d	b	b	b	c,d	c	b	d	a	d	b	d	c	d	a	b	a	d	b
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
d	d	a	c	a	b	a	a	b	c	b	c	b	a	a	b	b	b	a	b
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
a	c	b	d	a	a	b	c	b	c	d	b	b	b	c	a	b	a	c	b
201	**202**	**203**	**204**	**205**	**206**	**207**	**208**	**209**	**210**	**211**	**212**	**213**	**214**	**215**	**216**	**217**	**218**	**219**	**220**
d	c	a	c	c	c	d	b	a	a	b	b	a	a	a	d	b	a	d	c
221	**222**	**223**	**224**	**225**	**226**	**227**	**228**	**229**	**230**	**231**	**232**	**233**	**234**	**235**	**236**	**237**	**238**	**239**	**240**
b	b	a	a	d	b	b	a	b	a	d	d	c	b	a	c	d	a	b	b
241	**242**	**243**	**244**	**245**	**246**	**247**	**248**	**249**	**250**	**251**	**252**	**253**	**254**	**255**	**256**	**257**	**258**	**259**	**260**
b	a	d	a	a	b	d	a	c	d	c	d	a	b	a	b	c	b	b	a
261	**262**	**263**	**264**	**265**	**266**	**267**	**268**	**269**	**270**	**271**	**272**	**273**	**274**	**275**	**276**	**277**	**278**	**279**	**280**
c	b	c	a	a	b	b	b	c	a	a	a	c	d	d	c	d	a	c	c
281	**282**	**283**	**284**	**285**	**286**	**287**	**288**	**289**	**290**	**291**	**292**	**293**	**294**						
b	c	a	d	b	b	b	c	c	a	c	c	a	a						