Test

Engineering Mathematics

Time: 60 Minutes

1. If λ is an eigen value of an orthogonal matrix A then which of the following is always TRUE?

P: $\frac{1}{\lambda}$ is also an eigen value of A

Q: λ is a non-zero eigen value.

(A) P only (B) Q only

(C) Both P and Q (D) Neither P nor Q

2. A fair die is rolled independently four times. The probability that a non-composite number turns up for atleast 3 times is _____.

(A) $\frac{16}{27}$ (B) $\frac{11}{27}$

(C) $\frac{1}{27}$ (D) $\frac{5}{27}$

3. The value of $I = \int_1^2 x^3 \ln x dx$ is _____.

(A) $4\ln 16 - 15$ (B) $4\ln 16 + 15$

(C) $\ln 16 + \frac{15}{16}$ (D) $\ln 16 - \frac{15}{16}$

4. If $x = \ln \frac{y}{x}$, then y has

(A) a local maximum at $x = -1$

(B) a local minimum at $x = -1$

(C) a local maximum at $x = 1$

(D) a local minimum at $x = 1$

5. The complete solution of the initial value problem $\frac{d^2y}{dx^2} + 4y = 0$; $y(0) = 4$ and $y'(0) = 8$ is _____.

(A) $y = 4(\cos 2x + 2\sin 2x)$

(B) $y = 4\cos 2x$

(C) $y = 4\sin 2x$

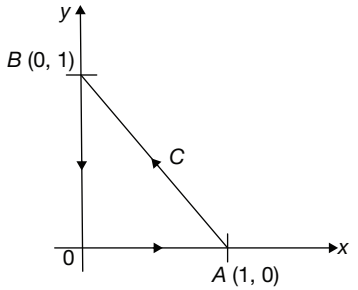
(D) None of these

6. The value of $\lim_{n\to 0}\left(\frac{1+2n}{1+3n}\right)^{1/n}$ is _____

(A) e (B) e^{-1}

(C) e^{-2} (D) 1

7. The value of $\oint_c \bar{F} \cdot \overline{dr}$, where $\bar{F} = xy^2\bar{i} - x^2y\bar{j}$ over the path shown in the figure is _____.



(A) $\frac{1}{3}$ (B) $\frac{-1}{3}$

(C) $\frac{1}{6}$ (D) $\frac{-1}{6}$

8. If the eigen values of a matrix $A = \begin{bmatrix} 7 & a \\ 5 & b \end{bmatrix}$ are 10 and 2, then the values of 'a' and 'b' respectively are _____.

(A) $a = 3, b = -5$ (B) $a = -3, b = -5$

(C) $a = 3, b = 5$ (D) $a = 3, b = -5$

9. The Taylor's series expansion of $\frac{\cos x}{\frac{3\pi}{2} - x}$ at $x = \frac{3\pi}{2}$ is given by

(A) $1 - \frac{\left(x - \frac{3\pi}{2}\right)^2}{2!} + \frac{\left(x - \frac{3\pi}{2}\right)^4}{4!} \cdots \infty$

(B) $\left(x - \frac{3\pi}{2}\right) - \frac{\left(x - \frac{3\pi}{2}\right)^3}{3!} + \frac{\left(x - \frac{3\pi}{2}\right)^5}{5!} \cdots \infty$

(C) $1 - \frac{\left(x - \frac{3\pi}{2}\right)^2}{3!} + \frac{\left(x - \frac{3\pi}{2}\right)^4}{5!} \cdots \infty$

(D) $-1 + \frac{\left(x - \frac{3\pi}{2}\right)^2}{3!} - \frac{\left(x - \frac{3\pi}{2}\right)^4}{5!} + \cdots \infty$

10. If a scalar field f and a vector field $\bar{V}$ are related by $f = \text{Div } \bar{V}$, which of the following is TRUE?

Here S is a closed surface, enclosing a volume V_S.

(A) $\iint_S \text{curl}\,\bar{V} \cdot d\bar{s} = \iiint_{V_S} f \, dv$

(B) $\iint_S \bar{V} \cdot d\bar{s} = \iiint_{V_S} f \, dv$

(C) $\iiint_S f \, dv = \iiint_{V_S} \text{Div}\,\bar{V} \, d\bar{s}$

(D) $\iint_S \text{curl}\,\bar{V} \times d\bar{s} = \iiint_{V_S} f \, dv$

11. The curl of the vector field $2xy\bar{i} + xy^2z\bar{j} - 6xz^3\bar{k}$ at the point (0, 2, 3) is _____

(A) $2\bar{i} + 5\bar{j} - 6\bar{k}$ (B) $5\bar{i} + 2\bar{j}$

(C) $162\bar{j} + 12\bar{k}$ (D) None of these

12. Let S denote the set of all possible arrangements of letters of the word 'AUTHORISED' without repetitions. If an element in S is selected at random, then what is

the probability that, the letter 'I' appears at a later position, than all other vowels?

(A) $\frac{1}{5}$ (B) $\frac{1}{5!}$

(C) $\frac{4!}{5!}\times 2$ (D) $\frac{4!\times 5!}{10!}$

13. In the process of finding a solution to the equation $f(x) = 4x^2 = 4x - 15 = 0$ by Newton–Raphson method, with initial solution as $x_0 = 1.6$, the method converges to actual solution after _____ interaction, when the calculator is fixed to four decimal places.

(A) 2nd (B) 4th

(C) 6th (D) 8th

14. In the process of finding a root for $x^3 - 3x^2 - 5x + 6 = 0$ in between $a = 3$ and $b = 4$, by Regula Falsi method, the value of the root in the first iteration is _____.

(A) 3.8182 (B) 3.5

(C) 3.6235 (D) 3.3218

15. For the system of linear equations

$x + 2y + 3z = 4$

$2x + 3y + (a - 4)z = b$

$4x + 7y - z = 5$

has infinite number of solutions, then the values of 'a' and 'b' are _____

(A) $a = b = 3$ (B) $a = -b = 3$

(C) $a = b = -3$ (D) $a = -b = -3$

16. Evaluate $\int_0^{\pi/3} f(x)dx$ by Simpson's $\frac{3}{8}$ Rule using the following table

x	0	$\frac{\pi}{18}$	$\frac{\pi}{9}$	$\frac{\pi}{6}$	$\frac{2\pi}{9}$	$\frac{5\pi}{18}$	$\frac{\pi}{3}$
$y = f(x)$	0	0.1762	0.3638	0.5770	0.8385	1.1907	1.7299

(A) 0.5862 (B) 0.6929

(C) 0.5928 (D) 0.7234

17. If 'r' is the standard deviation of $a_1, a_2, a_3 \ldots a_n$, then the standard deviation of $ka_1 + 1, ka_2 + 1, ka_3 + 1, \ldots, ka_n + 1$ is

(A) kr (B) $kr + 1$

(C) $\frac{r}{k} - 1$ (D) $\frac{r}{k}$

18. If $\bar{a} = \bar{i} + \bar{j} - 2\bar{k}$ and $\bar{b} = \bar{i} - 2\bar{j} + \bar{k}$, then determinant of the matrix $\begin{bmatrix} \bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} \\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} \end{bmatrix}$, where $\bar{a}\cdot\bar{b}$ denotes the dot product of the vector $\bar{a}$ and $\bar{b}$ is _____.

(A) 9 (B) 27

(C) 18 (D) 54

19. If x and y are two random variables, a and b are any two constants and $E(x)$ and var(x) denote the expectation and the variance of the random variable x, then which of the following is INCORRECT?

(A) var $(ax + b) = a^2$ var(x)

(B) $E(ax + b) = aE(x) + b$

(C) $E(x + y) = E(x) + E(y)$

(D) $E(x^2) = \text{var}(x)$

20. Which of the following pairs of vectors are orthonormal?

(A) $\begin{bmatrix}1\\1\\1\end{bmatrix}; \begin{bmatrix}-1\\2\\-1\end{bmatrix}$ (B) $\begin{bmatrix}\frac{1}{\sqrt{4}}\\\frac{1}{\sqrt{4}}\\\frac{1}{\sqrt{2}}\end{bmatrix}; \begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{4}}\\\frac{-1}{\sqrt{4}}\end{bmatrix}$

(C) $\begin{bmatrix}1\\2\\3\end{bmatrix}; \begin{bmatrix}3\\2\\1\end{bmatrix}$ (D) $\begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\\0\end{bmatrix}; \begin{bmatrix}\frac{1}{\sqrt{3}}\\\frac{1}{\sqrt{3}}\\\frac{-1}{\sqrt{3}}\end{bmatrix}$

21. An integrating factor of the non-exact differential equation $(x^2 + 2xy - 2y^2)dx + (y^2 + 2xy - 2x^2)dy = 0$ is

(A) $x^3 + y^3$

(B) $\frac{1}{x^3 + y^3}$

(C) $x^3 - y^3$

(D) $\frac{1}{x^3 - y^3}$

22. The solution of the differential equation $x\frac{dy}{dx} + y = 3x^2$, $y(1) = 2$ is

(A) $y = x + \frac{1}{x}$

(B) $y = x + \frac{1}{x^2}$

(C) $y = x^3$

(D) $y = x^2 + \frac{1}{x}$

23. If $x = \sqrt{y + \sqrt{y + \sqrt{y + \cdots \infty}}}$, then the value of $\frac{dy}{dx}$ at $x = 1$ is _____.

(A) 0 (B) 1

(C) 4 (D) Undefined

24. Let $f(x) = \begin{cases} 2x-3 \text{ ; for } x \geq \frac{3}{2} \\ 3-2x \text{ ; for } x < \frac{3}{2} \end{cases}$

Then which of the following is true?

(A) $f(x)$ is continuous and differentiable for all real values of x.

(B) $f(x)$ is not continuous at $x = \frac{3}{2}$.

(C) $f(x)$ is continuous for real values of x, except $x = \frac{3}{2}$.

(D) $f(x)$ is continuous for every x and differentiable for all values of x, except $x = \frac{3}{2}$.

25. If $L[f(t)] = \frac{2s+3}{s^2+5s+6}$, then the initial value of $f(t)$ is _____.

(A) 1 (B) 2
(C) 3 (D) 6

Answer Keys

1. C	**2.** A	**3.** D	**4.** B	**5.** D	**6.** B	**7.** D	**8.** C	**9.** D	**10.** B
11. C	**12.** A	**13.** B	**14.** A	**15.** C	**16.** B	**17.** A	**18.** B	**19.** D	**20.** D
21. B	**22.** D	**23.** B	**24.** D	**25.** B					