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Probability

18.01 Introduction

In our daily life many such incidents take place which have more than one result. It is natural that everyone has a curiosity to know the result. Persons try to take advantage by anticipating the result of these incidents. The science of knowing the result of an incident on the basis of prior information and circumstances is called probability.

The theory of probability was first propounded in the 17th century in Europe. The gamblers and match fixers made attempts to know the results of their respective games before-hand in order to have maximum advantages. These people put this problem before their contemporary Mathematicians like Galilio, Pascal, Farma Kardeno etc. These mathematicians developed certain mathematical methods to solve these problems, and consequently this branch of Mathematics came to exit. Prominent mathematicians of the 18th and 19th centuries: Laplace, Gauss, Bernouli, etc. developed this principal further.

In the 20th century, sampling theory, decision theory etc. based on probability theory were developed and credit of these got to R.S. Fisser and Karl Pearson.

In modern times the theory of probability is applied in various fields where decisions pertaining to future have to be taken. For example, in preparing the budget of any state or country, theory of probability is used. Insurance companies prepare death tables and make inference as to how long a person of particular age group in likely to survive and defence experts frame their strategies with the help of this theory. Many important policies in the fields of society, state administration, commerce and science are determined broadly on the basis of probability. First of all, we will attempt to define certain important terms used in the study of probability.

18.02 Some Definitions

1. **Random Experiment :** When all the possible result of an experiment are already known and no inference of any particular result is possible, it is called a random experiment.

   For example, the two result of tossing of a coin, either head or tail are already known. No definite result can be forecast, therefore toss of a coin is a random experiment.

2. **Trial and Event :** When out of many possible results (outcomes) of a random experiment is called a trial and the possible results are called events. For example:

   (i) Tossing of a coin is trial and getting head (H) or tail (T) are events.

   (ii) Throwing a dice is trial and getting any of 1, 2, 3, 4, 5 and 6 is event.

   (iii) Appearing in the examination of a candidate is trial and pass or fail is event.

3. **Simple Event :** When in a trial, only one event takes place at a time, it is a simple event.

   For example : Drawing a ball from a bag containing a few black and white balls is a simple event.

4. **Exhaustive Events or Total Number of Cases :** All possible results of a trails are called Exhaustive events or total number of cases of that trail.

   For Example :

(i) Tossing of a coin is a trial and head or tail can occur. So in this trail exhaustive events are 2.

(ii) On throwing of a dice 1, 2, 3, 4, 5 or 6 can occur so in this trail exhaustive events are 6.

5. **Favourable events or cases :** The number of cases favourable to a particular event in a trial is the number of outcomes which entail the happening of the particular event. For example :

(i) In throwing a die, the number of cases favourable to getting an even number is 2 ,4, 6 i.e., 3.

(ii) On drawing two cards from a pack of card, the number of cases favourable to getting king is 6.

(ii) In throwing of two dice, the number of cases favourable to getting a sum 5 is (1,4), (4, 1), (2,3), (3, 2), i.e. 4.

6. **Independent and dependent events :**

(i) Independent events : Two or more events are called independent events if the happening or not happening of any one does not depend on the happening or non happening of the other. For example, on tossing a coin and throwing a dice, the outcomes getting head on coin and 4 on dice are independent events.

(ii) Dependent event : Two or more events are called dependent events if the happening of any one does depend on the happening of the other. For example : A card drawn from an ordinary pack of card should be a heart card , after that without replacing it in pack, again a drawn card should be a spade card, both are dependent events.

7. **Mutually exclusive or disjoint events :** Two or more events are said to be mutually exclusive or disjoint events if no two or more occur simultaneously in the same trial i.e., if the occurrence of any one of them prevents the occurrence of all others. For example :

(i) On tossing of a coin occurring of head or tail are mutually exclusive events.

(ii) A card is drawn from a pack of card, it being a king or a queen are mutually exclusive events.

8. **Equally likely events :** If in an experiment, possibility of happening of all events is same then such events are called equally likely events. For example likely events.

(i) In tossing a coin, getting of head or tail are equally

(ii) In drawing a card from a pack of card it will be red or black card, is equally likely events.

9. **Compound events :** If two or more events happen at a time then they are, called compound events or joint event.

For example : In two bags, there are some blue and some red balls. Selection of a beg and then drawing a ball from it is a compound event because selection of one bag from two bags and then a ball is drawn from selected bag is happening at a time.

10. **Sample point and sample space :** Each outcome of a trial is called sample point and set of all sample point of a trial is called its sample space. It is generally denoted by S.

(i) The sample point in tossing of two coins are

(H,H), (H,T), (T,H), (T,T)

and $S = \{(H,H), (H,T), (T,H), (T,T)\}$ is sample space.

(ii) Two children are selected from 3 boys and 2 girls. The sample space of this trial is (boys B_1, B_2, B_3, girls G_1, G_2) :

$S = \{B_1B_2, B_2B_3, B_3B_1, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}$

18.03 Mathematical definition of Probability :

If the results of a trial *n* are equally likely, mutually exclusive and exhaustive cases and m of them are favourable to the happening of an event A, then the probability of A is defined as the ratio *m/n* and is denoted by P(A).

$$\text{Thus } P(A) = \frac{\text{favourable cases of A}}{\text{exhaustive cases of A}} = \frac{m}{n}, \text{ (numerical measure)}$$

If in a trial, happening of event A is sure then $m = n$ and

$$P(A) = \frac{n}{n} = 1,$$

If happening of event A is impossible then $m = 0$ and

$$P(A) = \frac{0}{n} = 0,$$

Therefore, for any event A, $0 \le P(A) \le 1$

i.e., Probability of any event can not be less than 0 and greater than 1 and limit of probability is from 0 to 1. Probability of non happening of event A is denoted as $P(\bar{A})$

$$\text{So,} \quad P(\bar{A}) = \frac{\text{unfavourable cases of event A}}{\text{exhaustive cases of event A}} = \frac{n - m}{n} = 1 - \frac{m}{n}$$

$$\Rightarrow \quad P(\bar{A}) = 1 - P(A)$$

18.04 Notation :

(1) P(A) = Probability of happening an event A

(2) P(A) = Probability of non happening an event A.

Illustrative Examples

Example 1 : Find the probability of getting in throwing an a dice.

Solution : In a throw of a dice, 6 types of numbers can occur. Hence the number of exhaustive events = 6, even number 2, 4, 6 will occur for the required event, which is in number 3. So number of favourable events = 3.

$$\therefore \quad \text{Required probability} = \frac{3}{6} = \frac{1}{2}$$

Example 2 : In a single throw of two dice, determine probability of getting a total of 7.

Solution : On throwing two dice 6x 6 = 36 output can be obtained. So exhaustive cases for required event = 36

The following pairs are possible for a total of 7.

(1,6), (2,5), (3,4), (4,3), (5,2), (6,1) which number is 6.

So favourable number of events = 6

$$\therefore \quad \text{Required probability} = \frac{6}{36} = \frac{1}{6}$$

Example 3 : Find the probability that a leap year, selected at random, will contain 53 Mondays.

Solution : We know that in a leap year there are 366 days. So 52complete week and 2 days are remaining. The seven possibilities of these 2 days are as follows :

(1) Sunday and Monday (2) Monday and Tuesday (3) Tuesday and Wednesday

(4) Wednesday and Thursday

(5) Thursday and Friday

(6) Friday and Saturday

(7) Saturday and Sunday.

So exhaustive cases for required event = 7

Out of seven possible cases in two cases, Monday occurs.

So favourable cases for required event = 2

$\therefore$ Required probability $= \frac{2}{7}$

Example 4 : From 12 tickets marked 1 to 12, if one ticket is selected at random. Find the probability that the number on it is a multiple of 2 or 3.

Solution : The multiple of 2 or 3, in number 1 to 12 is 2, 3, 4, 6,8 9 10, 12

So out of 12 equally likely cases 8 are favourable.

$\therefore$ Required probability $= \frac{8}{12} = \frac{2}{3}$

Example 5 : A coin is tossed once. Find the probability of getting tail.

Solution : In tossing a coin possible outcome are two, head (H) and tail (T). Let E be event of getting tail, then favourable event for E is 3,

$$\text{So,} \quad P(E) = P(T) = \frac{\textit{Favourable cases} \text{ of events E}}{\textit{Total} \text{ no. of cases}} = \frac{1}{2}$$

Example 6 : A bag contains one white ball, one black ball and one red ball of same size. Savita taken out one ball without looking in the bag. What will be the probability that drawn ball is red?

Solution : Drawing a ball by Savita is equally likely event and total number of balls (events) is 3. Let drawing red ball is event R, then favourable outcome of the event R is 1.

$$\text{So,} \quad P(R) = \frac{1}{3}$$

Example 7: A dice is thrown once. What will be the probability that the number on the dice will be 5 or less than 5.

Solution : In throwing a dice possible outcomes are 1, 2, 3, 4, 5, 6. Let to obtain the number less than or equal to 5 is event E than favourable out comes will be 1, 2, 3, 4, and 5.

$$\therefore \quad P(E) = \frac{5}{6}$$

Example 8 : A card is drawn at random form a well-shuffled deck of 52 cards. Find the probability of its being a king.

Solution : In a deck of 52 cards there are 4 kings. Let a king card is an event E, so number of favourable events is 4. Total number of events are 52.

$$\therefore \quad P(E) = \frac{4}{52} = \frac{1}{13}$$

Example 9 : Two players Ram and Shyam plays a chess match. It is given that probability of winning the match by Ram is $\frac{4}{5}$. What will be the probability of winning the match by Shyam?

Solution : Let R and S are the event of winning the match by Ram and Shyam respectively.

$\therefore$ Probability of winning Ram = $P(R) = \frac{4}{5}$

$\therefore$ Probability of winning Shyam = $P(S) = 1 - P(R)$

$$= 1 - \frac{4}{5}$$

$$= \frac{1}{5}$$

Example 10 : A coin is tossed twice. Find the probability of getting at least one head.

Solution : Possible outcomes in tossing a coin twice are (H,H), (H,T), (T,H), (T,T). Let E be the event of getting at least one head. Therefore the favourable outcomes are (H,T), (T,H), (T,T).

Total number of favourable cases = 3.

$\therefore$ $P(E) = \frac{3}{4}$

Example 11 : Two dices are thrown together. What is the probability that sum of two numbers on the faces is 7 ?

Solution : On throwing two dice together, the possible 36 outcomes are -

{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5) (6,6)}

Let E be the event when sum on the faces is 7.

Favourable events are = {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}

Total number of favourable events = 6

$\therefore$ $P(E) = \frac{6}{36} = \frac{1}{6}$

Exercise 18.1

1. In a throw of a die, determine the probability of getting a number more than four.
2. A coin is tossed twice. Find the probability of getting tails both times.
3. One number is selected at random from natural number from 1 to 17. Find the probability that the number is prime.
4. Find the probability of throwing head or tail alternatively in 3 successive tossing of a coin.
5. Find the probability that a non-leap year should have only 52 Sunday.
6. If P (A) = 0.65 than what will be the probability of "not A".
7. On tossing two coins find the probability of getting at most one tail.
8. A dice is thrown twice. What will be the probability that the sum on the faces is-

   (i) 9 (ii) 13
9. A bag contains 5 red and 3 white balls. From this bag one ball is drawn randomly. What will be the probability that the drawn ball is

   (i) White (ii) Not White

10. Due to some reason 12 faulty pens are mixed in 132 good pens. By inspection are can not judge that which pen is faulty. If one pen is drawn at random then what is the probability that this pen is good one?

11. A card is drawn from a well shuffled deck of 52 card. Find the probability of getting the following

(i) Jack of red colour (ii) Card of red colour (iii) Ace of heart

(iv) Queen of diamond (v) Card of spade

IMPORTANT POINTS

1. Trail and event: In any reference out of all the possible result (out comes) of a random experiment is called a trial and the possible results are called events.

2. Exhaustive events or total number of cases: All possible results of a trials are called exhaustive events or total number of cases of that trail.

3. Favourable events or cases: The numbers of cases favourable to a particular event in a trial is the number of outcomes which entail the happening of the particular event.

4. Probability :

Probability of happening event A is

$$P(A) = \frac{\text{favourable cases of A}}{\text{exhaustive cases of A}} = \frac{m}{n}$$

Probability of not happening the event A is

$$P(\bar{A}) = \frac{\text{unfavourable cases of event A}}{\text{exhaustive cases of event A}} = \frac{n-m}{n} = 1 - \frac{m}{n}$$

or $P(\bar{A}) = 1 - P(A)$

or $P(A) + P(A) \leq 1$

Limit of Probability is $0 \leq P(A) \leq 1$

Answers

Excercise 18.1

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{7}{17}$ (4) $\frac{1}{4}$ (5) $\frac{6}{7}$ (6) 0.35 (7) $\frac{1}{2}$

(8) (i) $\frac{1}{9}$ (ii) 0 (9) (i) $\frac{3}{8}$ (ii) $\frac{5}{8}$ (10) $\frac{11}{12}$

(11) (i) $\frac{1}{52}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{52}$ (iv) $\frac{1}{52}$ (v) $\frac{1}{4}$