CHAPTER 25

Triangles

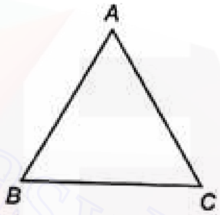
Triangle

A plane (closed) figure bounded by three line segments is called a triangle.

Triangles are denoted by Δ

A ΔABC has

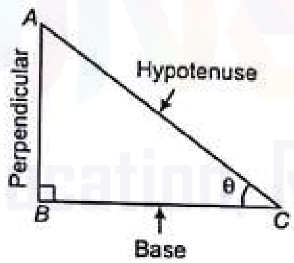
- three vertices, namely A, B and C.
- three sides, namely AB, BC and CA.
- three angles, namely $\angle A$, $\angle B$ and $\angle C$.



A triangle has six parts—three sides and three angles.

Types of Triangles on the Basis of Angles

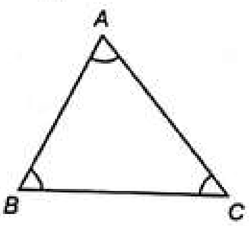
1. **Right Angled Triangle** A triangle in which one of the angles measures 90° is called a right angled triangle. The side opposite to the right angle is called its hypotenuse and the remaining two sides are called as perpendicular and base depending upon conditions.



Here, ΔABC has $\angle B = 90°$ and AC is hypotenuse.

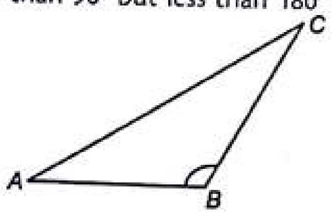
2. **Acute Angled Triangle** A triangle in which every angle measures more than 0° and is less than 90° is called an acute angled triangle.

Here, ΔABC is acute angled triangle.



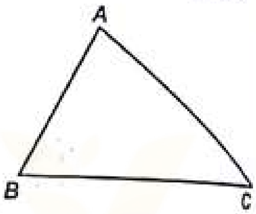
3. **Obtuse Angled Triangle** A triangle in which one of the angles measures more than 90° but less than 180° is called an obtuse angled triangle.

Here, ΔABC is an obtuse angled and $\angle ABC$ is the obtuse angle.



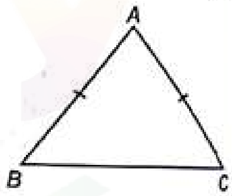
Types of Triangles on the Basis of Sides

1. **Scalene Triangle** A triangle in which all the sides are of different lengths is called a scalene triangle. ΔABC is a scalene triangle as $AB \neq BC \neq AC$.



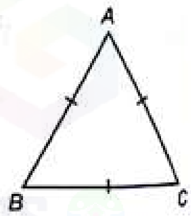
2. **Isosceles Triangle** A triangle in which two sides are equal is called an isosceles triangle. Here, ΔABC is an isosceles triangle as $AB = AC$.
   - Angle opposite to equal sides are equal.

i.e., $\angle B = \angle C$



3. **Equilateral Triangle** A triangle having all sides equal is called an equilateral triangle. Here is $\Delta ABC, AB = BC = AC$.

All angles are equal and are of measures 60°.



Perimeter of a Triangle

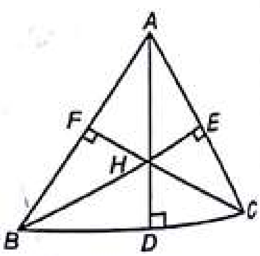
The sum of the lengths of three sides of a triangle is called its perimeter.

So, in ΔABC perimeter $= AB + BC + AC$

Some Terminologies Related to a Triangle

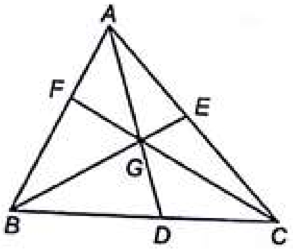
Altitudes The altitude of a triangle is a line segment perpendicular drawn from vertex to the side opposite to it. The side on which the perpendicular is being drawn is called its base.

Here, AD, BE and FC are altitudes drawn on BC, AC and AB, respectively.



- Altitudes of a triangle are concurrent.
- The point of intersection of all the three altitudes of a triangle is called its orthocentre.

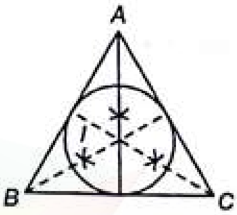
Medians A line segment joining the mid-point of that side with the opposite vertex.



Here, AD, BE and CF are medians.

- The medians of a triangle are concurrent.
- The point of intersection of all the three medians is called its centroid.
- Centroid is denoted by G.

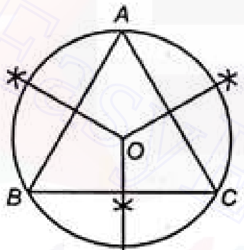
Incentre of a Triangle The point of intersection of all the three angle bisector of a triangle is called its incentre.



- The circle with centre I is called as incircle and radius is called as inradius denoted by 'r'.

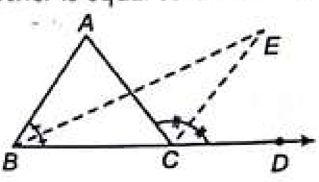
Circumcentre of a Triangle The point of intersection of the perpendicular bisectors of the sides of a triangle is called its a circumcentre.

Circle through it passing through A, B and C is called circumcircle. Radius of circumcircle is called circumradius denoted by R.



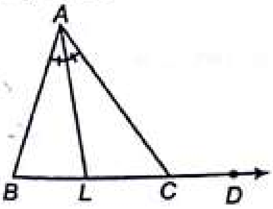
Some Useful Results on Triangles

- The sum of the angles of a triangle is 180°.
- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.
- An exterior angle of a triangle is greater than either of the interior opposite angles.
- The internal bisector of one base angle and the external bisector of the other is equal to one half of the vertical angle.

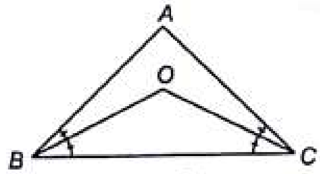


Here, $\angle E = \frac{1}{2} \angle A$

- The side BC of ΔABC is produced to D. The bisector of $\angle A$ meets BC in L. Then, $\angle ABC + \angle ACD = 2\angle ALC$.



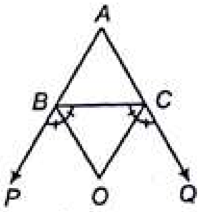
- In a ΔABC the bisector of $\angle B$ and $\angle C$ intersect each other at a point O.



$$\angle BOC = 90° + \frac{1}{2} \angle A$$

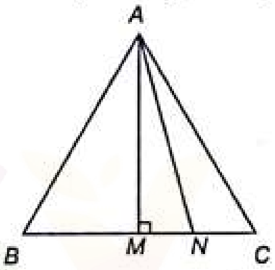
- In a ΔABC, the side AB and AC are produced to P and Q, respectively. The bisectors of $\angle PBC$ and $\angle QCB$ intersect at a point O.

Then, $\angle BOC = 90° - \frac{1}{2} \angle A$

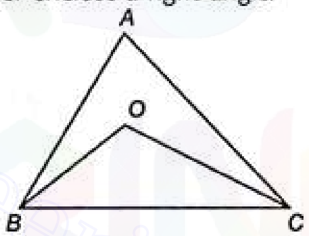


- In ΔABC, $\angle B > \angle C$. If AN is the bisector of $\angle BAC$ and $AM \perp BC$, then

$$\angle MAN = \frac{1}{2} \angle B - \angle C$$



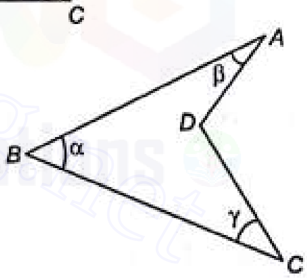
- The bisectors of the base angles of a triangle can never enclose a right angle.



Here, OB and OC are the bisectors of $\angle B$ and $\angle C$.

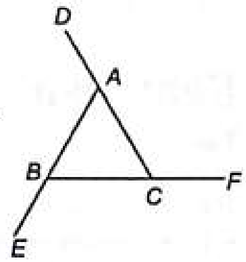
But $\angle BOC \neq 90°$.

- In figure $\angle ADC = \alpha + \beta + \gamma$



- If the three sides of a triangle be produced in order, then the sum of all the exterior angles so formed is 360°.

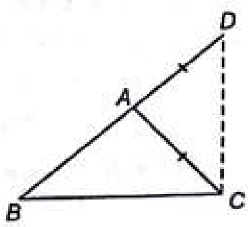
So, $\angle DAB + \angle EBC + \angle ACF = 360°$



- The sum of any two sides of a triangle is greater than its third side.

Here, in ΔABC

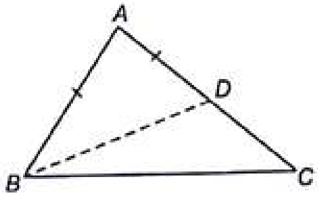
$$AB + AC > BC$$
$$AB + BC > AC$$
$$BC + AC > AB$$



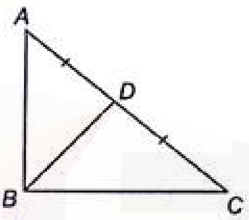
- The difference between any two sides of a triangles is less than its third side.

  Here, in ΔABC

  $$AC - AB < BC$$
  $$BC - AC < AB$$
  $$BC - AB < AC$$



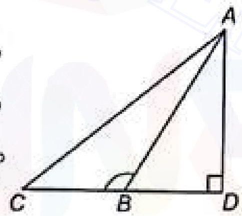
- If the bisector of the vertical angle of a triangle bisects the base, then that triangle is isosceles.
- If the altitude from one vertex of a triangle bisects the opposite side, then the triangle is isosceles.
- The perpendiculars drawn from the vertices of equal angles of an isosceles triangle to the opposite sides are equal.
- Median of equilateral triangle are equal.
- If D is the mid-point of the hypotenuse AC of a right angled ΔABC. Then, $BD = \frac{1}{2} AC$



- The sum of any two sides of a triangle is greater than twice the median drawn to the third side.
- Perimeter of a triangle is greater than the sum of its three medians.

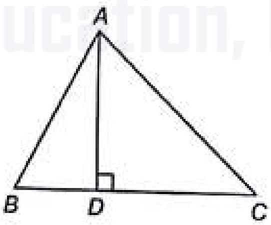
  So, $AB + BC + AC > AD + BE + CF$.
- In a ΔABC, $\angle B = 90°$, an obtuse angle or an acute angle according it

  $$AC^2 = AB^2 + BC^2, \text{ if } \angle B = 90°$$
  $$AC^2 > AB^2 + BC^2, \text{ if } \angle B > 90°$$
  $$AC^2 < AB^2 + BC^2, \text{ if } \angle B < 90°$$



- In ΔABC, if $\angle B$ is obtuse, then

  $$AC^2 = AB^2 + BC^2 + 2BC \cdot AD$$
- In ΔABC, if $\angle B$ is acute, then $AB^2 = BC^2 + AC^2 - 2BC \cdot CD$ and $AD \perp BC$



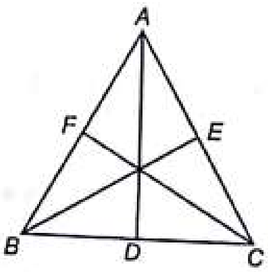
Congruent Triangles

Two triangles are said to be congruent, if both are exactly of same size *i.e.*, all angles and sides are equal to corresponding angles and sides of other.

- Every triangle is congruent to itself

  $\Delta ABC \cong \Delta ABC$
- If $\Delta ABC \cong \Delta DEF$, then

  $\Delta DEF \cong \Delta ABC$
- If $\Delta ABC \cong \Delta DEF$ and $\Delta DEF \cong \Delta PQR$, then $\Delta ABC \cong \Delta PQR$



Sufficient Conditions For Congruence of Two Triangles

Theorem 1 If two triangles have two sides and the included angle of the one equal to the corresponding sides and the included angle of the other. (SAS)

Theorem 2 If two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangles. (ASA)

Theorem 3 If three sides of one are respectively equal to the three sides of the other. (SSS)

Theorem 4 If the hypotenuse and other side of one triangle are respectively equal to the hypotenuse and the corresponding side of the other triangle. (RHS)

Some Inequality Relation in a Triangle

- Angle opposite to two equal sides of a triangle are equal.
- If two angles of a triangle are equal, then the sides opposite to them are also equal.
- If two sides of a triangle are unequal the longer side has greater angle opposite to it.
- In a triangle, the greater angle has the longer side opposite to it.

Congruent Figures

The geometrical figures having the same shape and size are known as congruent figures.

Congruent figures are just like photostat copies, which are alike in every respect.

Similar Figures

Geometric figures having the same shape but different sizes are known as similar figures.

- The congruent figures are always similar but two similar figures need not be congruent.

e.g., Any two circles are similar.

Any two rectangles are similar.

Similar Triangles

Two triangles are said to be similar to each other, if

- their corresponding sides are proportional.
- their corresponding angles are equal.

Results on Similar Triangles

Theorem 1 If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides these sides in the same ratio.

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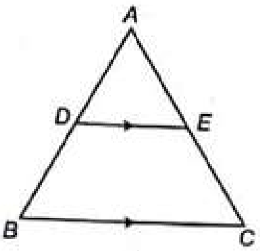
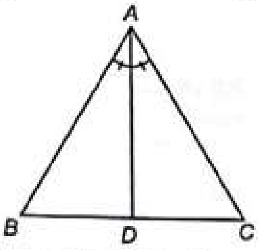
Here, $DE \parallel BC$, then

$$\frac{AD}{DB} = \frac{AE}{EC} \text{ or } \frac{AD}{AB} = \frac{AE}{AC} \text{ or } \frac{AB}{BD} = \frac{AC}{EC}$$



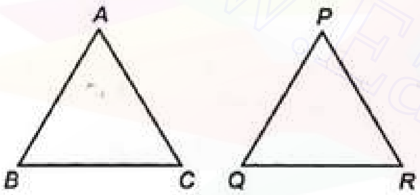
Theorem 2 The internal bisector of an angle of a triangle divides the opposite sides internally in the ratio of the sides containing the angle.

Here, AD is internal bisector of $\angle A$, then $\frac{AB}{AC} = \frac{BD}{DC}$



Theorem 3 The line segment joining the mid-points of any two sides of a triangle is parallel to the third side.

Theorem 4 The ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

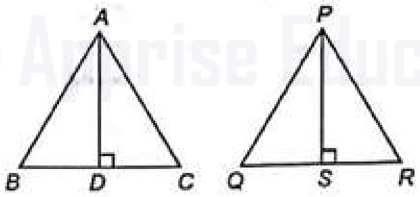


Here, $\Delta ABC \sim \Delta PQR$

$$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

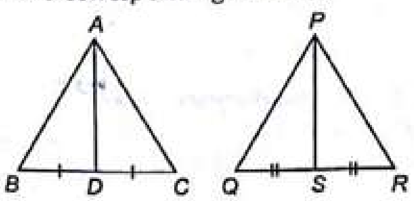
Theorem 5 The areas of two similar triangles are in the ratio of the squares of corresponding altitudes.

Here, $\Delta ABC \sim \Delta PQR$



then $$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \frac{AD^2}{PS^2}$$

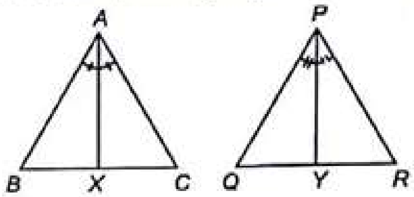
Theorem 6 The areas of two similar triangle are in the ratio of the squares of the corresponding medians.



Here, $\Delta ABC \sim \Delta PQR$,

then $$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \frac{AD^2}{PS^2}$$

Theorem 7 The areas of two similar triangles are in the ratio of squares of the corresponding angle bisector segments.



Here, $\Delta ABC \sim \Delta PQR$

So, here $$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \frac{AX^2}{PY^2}$$

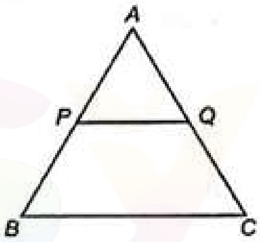
Theorem 8 If the areas of two similar triangles are equal, then the triangles are congruent.

or

Equal and similar triangles are congruent.

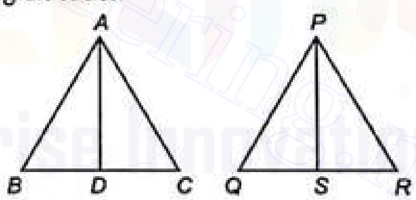
Theorem 9 The line joining the mid-points of any two sides of a triangle is parallel to the third side and is half of the third side.

Here, P and Q are mid-point of AB and AC. So, $PQ = \frac{1}{2} BC$.



Results on Ratio of Sides of Two Similar Triangles

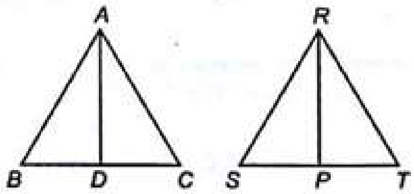
Theorem 1 If two triangles are equiangular, then the ratio of their corresponding sides is the same as the ratio of the corresponding altitudes.



Here, $\Delta ABC \sim \Delta PQR$ and AD and PS are altitude on BC and QR, respectively,

then $$\frac{BC}{QR} = \frac{AD}{PS}$$

Theorem 2 If two triangles are equiangular, then the ratio of the corresponding sides is the same as the ratio of the corresponding angle bisector segments.



Here, ΔABC and ΔRST are equiangular/similar and AD, RP are the angle bisectors of $\angle A$ and $\angle R$.

Then, $$\frac{BC}{ST} = \frac{AD}{RP}$$

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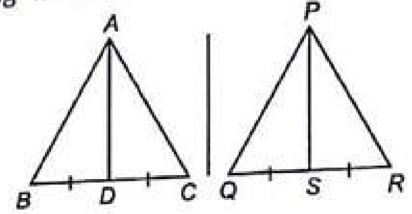
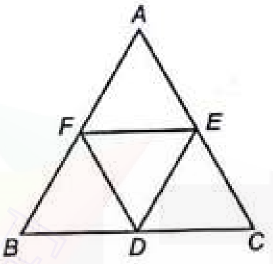
Theorem 3 If two triangles are equiangular, then the ratio of the corresponding sides is the same as the ratio of the corresponding medians.



Here, ΔABC and ΔPQR are equiangular and AD, PS are the medians, then

$$\frac{BC}{QR}=\frac{AD}{PS}$$

Theorem 4 The line segment joining the mid-points of the sides of a triangle form four triangles, each of which is similar to the original triangle.



Here, D, E and F are mid-point of BC, AC and AB. Then, here $\Delta AFE, \Delta FBD, \Delta EDC$ and ΔDEF is similar to ΔABC.

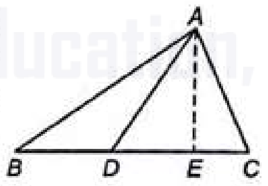
Here, also

$$\frac{\text{area}(\Delta DEF)}{\text{area}(\Delta ABC)}=\frac{DE^2}{AB^2}=\frac{\left(\frac{1}{2}AB\right)^2}{AB^2}=\frac{1}{4}$$

So, area (ΔDEF) : area $(\Delta ABC) = 1:4$

Some Other Useful Facts

- The area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.
- In any triangle, the sum of the square of any two sides is equal to twice the square of half of the third side together with twice the square of the median which bisect the third side.



Here, AD is median, so

$$AB^2+AC^2=2AD^2+2\left(\frac{1}{2}BC\right)^2$$

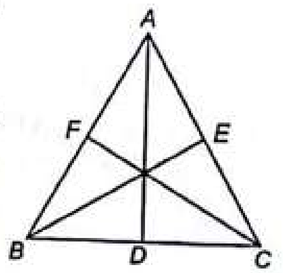
- In a rhombus $ABCD$,

$$AB^2+BC^2+CD^2+AD^2=AC^2+BD^2$$

- If 'O' be a point in the exterior of a rectangle $ABCD$ is joined with each of the vertices A, B, C and D, then

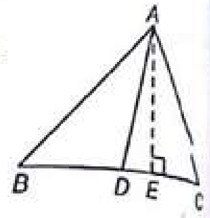
$$OA^2+OC^2=OB^2+OD^2$$

- In a ΔABC, three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.



So, in triangle if AD, BE, FC are the medians, then

$$3(AB^2+BC^2+AC^2)=4(AD^2+BE^2+CF^2)$$



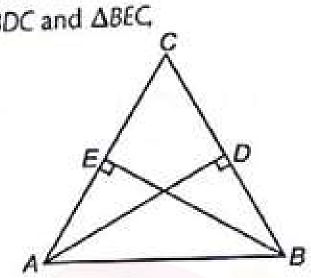
- In an equilateral ΔABC, the side BC is trisected at D. Then,

$$9\,AD^2=7\,AC^2$$

Example 1. In a ΔABC, the altitudes BD and CE are equal and $\angle A = 36°$. What is the value of the $\angle B$?

(a) 72° (b) 84° (c) 18° (d) 36°

Sol. (*a*) For the ΔBDC and ΔBEC,

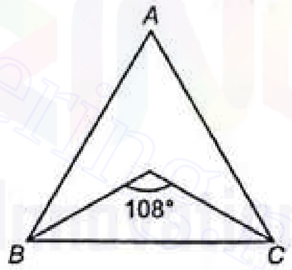


$BD = EC, BC = BC$ and $\angle BEC = \angle BDC = 90°$

Thus, $\Delta BEC \cong \Delta BDC$

$\therefore$ $\angle B = \angle C = \dfrac{180°-36°}{2} = 72°$ each

Example 2. The measure of angle A in the figure given below is



(a) 54° (b) 18°
(c) 36° (d) None of these

Sol. (c) Here, I is the incentre of the ΔABC

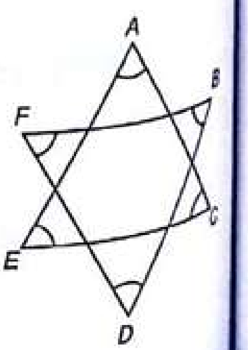
$\therefore BI$ and CI are the bisectors of $\angle B$ and $\angle C$, then

we know that, $\angle BIC = 90° + \frac{1}{2}\angle A$ or $108° = 90° + \frac{1}{2}\angle A$

or $\frac{1}{2}\angle A = 108° - 90° = 18°$

$\angle A = 36°$

Example 3. In the diagram given below what is the sum of all the angles $\angle A, \angle B, \angle C, \angle D, \angle E$ and $\angle F$?



(a) 120°
(b) 180°
(c) 290°
(d) 360°

Sol. (d) Since, sum of the angles of a triangle is 180°.

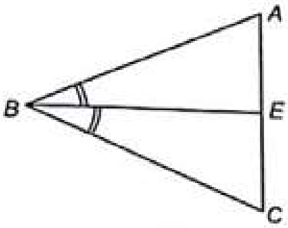
InΔAEC, $\angle A + \angle C + \angle E = 180°$ ...(i)

and In ΔBDF, $\angle B + \angle D + \angle F = 180°$ ...(ii)

On adding the Eqs. (i) and (ii), we get

$$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 180° + 180° = 360°$$

Example 4. If $AB = 4.7$, $BC = 8.9$, $CA = 11.5$, then EA is



(a) 6.07 (b) 3.97
(c) 2.37 (d) None of these

Sol. (b) Since, BE is the bisector of $\angle ABC$.

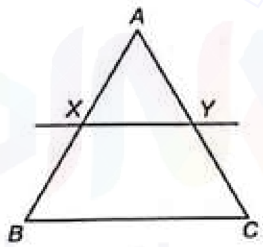
$$\therefore \quad \frac{AB}{BC} = \frac{AE}{EC} \text{ or } \frac{4.7}{8.9} = \frac{AE}{11.5 - AE}$$

or $\quad AE = 3.97$

Example 5. In a triangle, a line XY is drawn parallel to BC meeting AB in X and AC in Y. The area of the ΔABC is 2 times the area of the ΔAXY. In what ratio X divides AB?

(a) $1:\sqrt{2}$ (b) $\sqrt{2}:1$
(c) $(\sqrt{2}-1):1$ (d) $1:(\sqrt{2}-1)$

Sol. (d)



$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta AXY)} = \frac{AB^2}{AX^2}$$

or $$\frac{2\,\text{Area}(\Delta AXY)}{\text{Area}(\Delta AXY)} = \frac{AB^2}{AX^2}$$

or $$\frac{2}{1} = \frac{AB^2}{AX^2}, \frac{AB}{AX} = \sqrt{2}$$

or $$AB = \sqrt{2}\,AX$$

or $$AX + BX = \sqrt{2}\,AX$$

$$\therefore \quad BX = A \times (\sqrt{2} - 1) \Rightarrow \frac{AX}{BX} = \frac{1}{\sqrt{2} - 1}$$

$\therefore$ X divides AB in $1 : (\sqrt{2} - 1)$

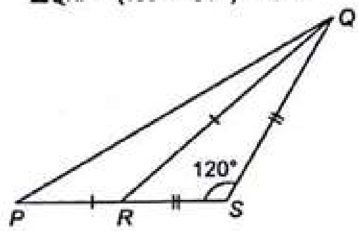
Example 6. In a ΔPQS, R is a point on PS such that $PR = QR$ and $QS = RS$. If $\angle RSQ = 120°$, what is the measure of $\angle QPR$?

(a) 30° (b) 15°
(c) 45° (d) None of these

Sol. (b) $\because \quad RS = SQ$

$$\therefore \quad \angle QRS = \frac{1}{2}(180° - 120°) = 30°$$

$$\therefore \quad \angle QRP = (180° - 30°) = 150°$$

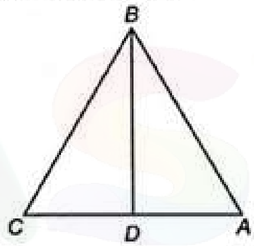


Hence, $\quad \angle QPR = \frac{1}{2}(180° - 150°) = 15° \quad (\because PR = RQ)$

Example 7. ABC is a right angled triangle, where $\angle B = 90°$. BD is drawn perpendicular to AC. If $AD = 9$ cm and $DC = 16$ cm, what is the measure of AB?

(a) 15 cm (b) 18 cm
(c) 16 cm (d) 9.5 cm

Sol. (a) $BD^2 = AD \times DC = 9 \times 16 = 144$



$\therefore \quad BD = 12$

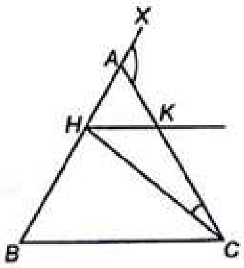
Now, $AB^2 = BD^2 + AD^2 = 144 + 81 = 225$

$\therefore \quad AB = 15$ cm

Example 8. ABC is an isosceles triangle in which $AB = AC$, $CH = CB$ and HK is parallel to BC. If the exterior $\angle CAX = 137°$, then what is the measure of $\angle HCK$?

(a) $68\frac{1}{2}°$ (b) 43°
(c) $25\frac{1}{2}°$ (d) 137°

Sol. (c) $\angle CAX = 137°$



$$\therefore \quad \angle ABC = \frac{1}{2}(137°) = 68\frac{1}{2}°$$

$\therefore$ Again, $\quad BC = CH$ and $\angle ABC = 68\frac{1}{2}°$

Therefore, $\quad \angle CHB = 68\frac{1}{2}°$

Therefore, $\angle HCB = 43°$

Hence, $\quad \angle HCX = 68\frac{1}{2}° - 43° = 25\frac{1}{2}°$

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Exercise

1. A point P lying inside a triangle a equidistant from the vertices of the triangle. Then, the triangle has P its
   (a) centroid (b) incentre
   (c) orthocentre (d) circumcentre

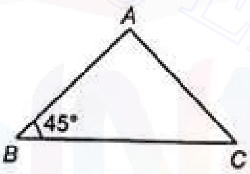
2. If the bisector of an angle of a triangle bisects the opposite side, then the triangle is
   (a) equilateral (b) isosceles
   (c) scalene (d) right angled triangle

3. The line segments joining the mid-points of the sides of a triangle form four triangles each of which is
   (a) similar to the original triangle
   (b) congruent to the original triangle
   (c) an equilateral triangle
   (d) an isosceles triangle

4. The triangle formed by joining the mid-points of the sides of an equilateral triangle is
   (a) a right angled triangle (b) an obtuse angled triangle
   (c) a scalene triangle (d) an equilateral triangle

5. In a ΔABC, BD and CE are perpendicular on AC and AB, respectively. If $BD = CE$, then the ΔABC is
   (a) equilateral (b) isosceles
   (c) right angled (d) scalene

6. $\angle ABC$ is equal to 45° as shown in the adjoining figure. If $\frac{AC}{AB} = \sqrt{2}$, then $\angle BAC$ is equal to



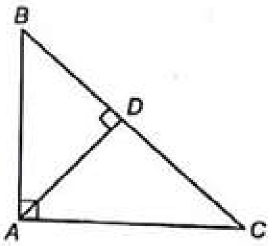
   (a) 95° (b) 100°
   (c) 105° (d) 110°

7. If PL, QM and RN are the altitudes of ΔPQR whose orthocentre is O, then P is the orthocentre of
   (a) ΔPQO (b) ΔPQL (c) ΔQLO (d) ΔQRO

8. If the length of hypotenuse of a right angled triangle is 5 cm and its area is 6 sq cm, then the length of the remaining sides are
   (a) 1 cm and 3 cm (b) 3 cm and 2 cm
   (c) 3 cm and 4 cm (d) 4 cm and 2 cm

9. ΔABC is such that $AB = 3$ cm, $BC = 2$ cm and $AC = 2.5$ cm. ΔDEF is similar to ΔABC. If $EF = 4$ cm, then the perimeter of ΔDEF is
   (a) 5 cm (b) 7.5 cm (c) 15 cm (d) 18 cm

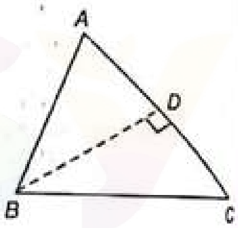
10. Which of the following is true in the given figure where AD is the altitude to the hypotenuse of a right angled ΔABC?



   I. $\Delta ABD \sim \Delta CAD$
   II. $\Delta ABD \cong \Delta CDA$
   III. $\Delta ADB \sim \Delta CAB$
   Of these statements the correct ones are combinations of
   (a) I and II (b) I and III
   (c) II and III (d) I, II and III

11. A soldier goes to a warfield and runs in the following manner.
   From the starting point, he goes West 25 m, then due North 60 m, then due East 80 m, and finally due South 12 cm. The distance between the starting point and the finishing point is
   (a) 177 m (b) 103 m
   (c) 83 m (d) 73 m

12. Let ABC be an isosceles triangle in which $AB = AC$ and $BD \perp AC$. Then, $BD^2 - CD^2$ is equal to



   (a) $2DC \cdot AD$
   (b) $2AD \cdot BC$
   (c) $3DC \cdot AD$
   (d) $\frac{1}{2} AD \cdot DC$

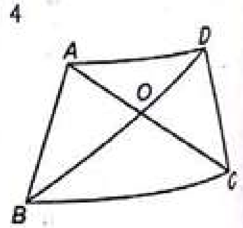
13. D and E are the points on the sides AB and AC respectively of a ΔABC and $AD = 8$ cm, $DB = 12$ cm, $AE = 6$ cm and $EC = 9$ cm, then BC is equal to
   (a) $\frac{2}{5} DE$ (b) $\frac{5}{2} DE$ (c) $\frac{3}{2} DE$ (d) $\frac{2}{3} DE$

14. A vertical stick 15 m long casts a shadow 12 m long on the ground. At the same time, a tower casts a shadow 50 m long on the ground. The height of the tower is
   (a) 60 m (b) 62 m (c) 62.5 m (d) 63 m

15. The areas of two similar triangles are 81cm^2 and 49cm^2, respectively. The ratio of their corresponding heights is
   (a) 9 : 7 (b) 7 : 9
   (c) 6 : 5 (d) 81 : 49

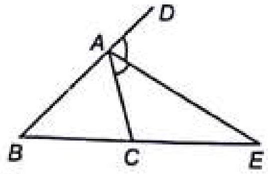
16. If D and E are points on the sides AB and AC, respectively of a ΔABC such that $DE \parallel BC$. If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$. The value of x is
   (a) 2.5 (b) 2
   (c) 3 (d) 4

17. In the adjoining figure, $ABCD$ is a trapezium in which $BC \parallel AD$ and its diagonals intersect at O. If $AO = (3x - 1)$, $OC = (5x - 3)$, $BO = (2x + 1)$ and $OD = (6x - 5)$, then x is equal to



   (a) 1 (b) 2
   (c) 3 (d) 4

18. In the adjoining figure, AE is the bisector of exterior $\angle CAD$ meeting BC produced in E. If $AB = 10$ cm, $AC = 6$ cm and $BC = 12$ cm, then CE is equal to



(a) 6 cm (b) 12 cm
(c) 18 cm (d) 20 cm

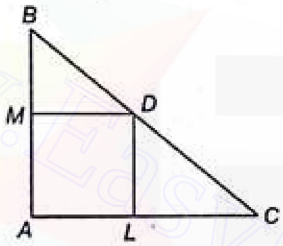
19. If D, E and F are respectively the mid-points of sides BC, AC and AB of a ΔABC. If $EF = 3$ cm, $FD = 4$ cm and $AB = 10$ cm, then DE, BC and CA, respectively will be equal to

(a) 6, 8 and 20 cm (b) $\frac{10}{3}$, 9 and 12 cm
(c) 4, 6 and 8 cm (d) 5, 6 and 8 cm

20. In ΔPQR, $\angle Q = 3a$, $\angle P = a$, $\angle R = b$ and $3b - 5a = 30$, then the triangle is

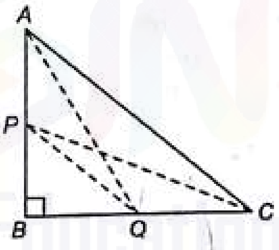
(a) scalene (b) isosceles
(c) equilateral (d) right angled

21. In ΔABC show in the figure $\angle A = 90°$. Let D be a point on BC such that $BD : DC = 1 : 3$. If DM and DL, respectively are perpendicular on AB and AC, then DM and LC are in the ratio of



(a) 1 : 3
(b) 1 : 2
(c) 1 : 1
(d) 4 : 1

22. In a right angled ΔABC, right angled at B, if P and Q are points on the sides AB and AC respectively, then

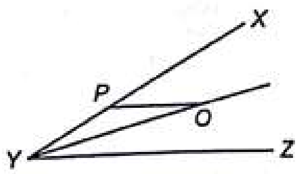


(a) $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$
(b) $2(AQ^2 + CP^2) = AC^2 + PQ^2$
(c) $AQ^2 + CP^2 = AC^2 + PQ^2$
(d) $AQ + CP = \frac{1}{2}(AC + PQ)$

23. If S is the circumcentre of a ΔABC, then

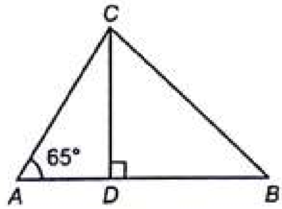
(a) S is equidistant from its sides
(b) S is equidistant from its vertices
(c) SA, SB, SC are the angular bisector
(d) AS, BS, CS produced are the altitudes on the opposite sides

24. O is any point on the bisector of the acute angle $\angle XYZ$. The line OP is parallel to ZY. Then, ΔYPO is **(CDS 2010 II)**



(a) scalene
(b) isosceles but not right angled
(c) equilateral
(d) right angled and isosceles

25. In ΔABC, $\angle C = 90°$ and $CD \perp AB$, also $\angle A = 65°$, then $\angle CBA$ is equal to

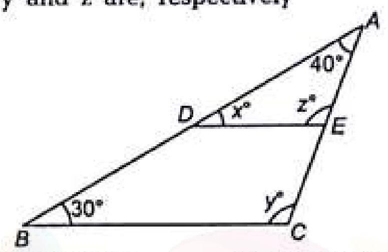


(a) 25°
(b) 35°
(c) 65°
(d) 40°

26. The angles of a triangle are as 2 : 3 : 4. The angles of triangle are, respectively

(a) 30°, 60°, 90° (b) 40°, 60°, 80°
(c) 60°, 40°, 80° (d) 20°, 60°, 80°

27. In figure, D and E are points on sides AB, AC of ΔABC such that $DE \parallel BC$. If $\angle B = 30°$ and $\angle A = 40°$, then x, y and z are, respectively



(a) 30°, 110°, 110° (b) 30°, 105°, 105°
(c) 30°, 85°, 85° (d) 30°, 95°, 95°

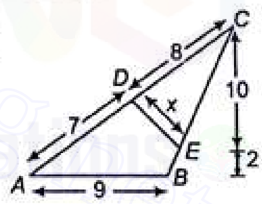
28. Consider the statements

I. Two of the angles are obtuse.
II. Two of the angles are acute.
III. Each angle is less than 60°.
IV. Each angle is equal to 60°.

In which case/cases is it possible to have a triangle?

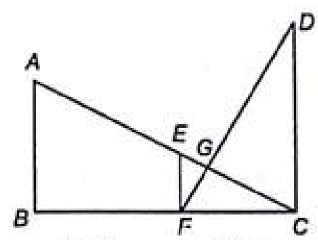
(a) II and IV only (b) I only
(c) I and III only (d) All of these

29. If $\angle A = \angle CED$ and $\Delta CAB \sim \Delta CED$, then the value of x is



(a) 4 cm
(b) 5 cm
(c) 6 cm
(d) 7 cm

30. In figure, AB, EF and CD are parallel lines. Given that $GE = 5$ cm, $GC = 10$ cm and $DC = 18$ cm, then EF is equal to



(a) 11 cm (b) 5 cm (c) 6 cm (d) 9 cm

31. If $ABCD$ is a rhombus, then $AB^2 + BC^2 + CD^2 + AD^2$ is equal to

(a) $AD^2 + BC^2$ (b) $AO^2 + OC^2$
(c) $AC^2 + BD^2$ (d) $2(AO^2 + OB^2)$

32. A point O in the interior of a rectangle is joined with each of the vertices A, B, C and D, then

(a) $OB^2 + OD^2 = OC^2 + OA^2$ (b) $AO^2 - OD^2 = OC^2 - OA^2$

(c) $AO^2 + OD^2 = BO^2 + OC^2$ (d) $AO^2 + OB^2 = AC^2 - BD^2$

33. If AD, BE, CF are the medians of a ΔABC, then the correct relation between the sum of the squares of sides to the sum of the squares of median is

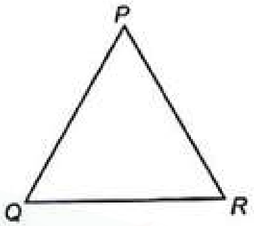
(a) $2(AB^2 + BC^2 + AC^2) = 3(AD^2 + BE^2 + CF^2)$

(b) $4(AB^2 + BC^2 + AC^2) = 3(AD^2 + BE^2 + CF^2)$

(c) $3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$

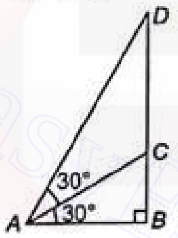
(d) None of the above

34. In ΔPQR length of the side QR is less than twice the length of the side PQ by 2 cm. Length of the side PR exceeds the length of the side PQ by 10 cm. The perimeter is 40 cm. The length of the smallest side of the ΔPQR is



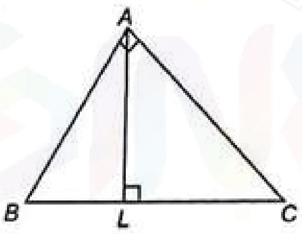
(a) 6 cm (b) 8 cm (c) 7 cm (d) 10 cm

35. In the adjoining figure which of the following statements is true?



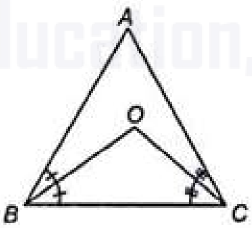
(a) $AB = BC$
(b) $AC = CD$
(c) $BC = CD$
(d) $AB < CD$

36. In a ΔABC, $\angle A = 90°$, AL is drawn perpendicular to BC. Then, $\angle BAL$ is equal to



(a) $\angle ALC$
(b) $\angle ACB$
(c) $\angle BAC$
(d) $\angle B - \angle BAL$

37. OB and OC are respectively the bisectors of $\angle ABC$ and $\angle ACB$. Then, $\angle BOC$ is equal to



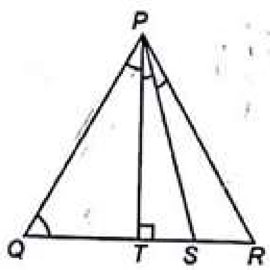
(a) $90° - \frac{1}{2}\angle A$

(b) $90° + \angle A$

(c) $90° + \frac{1}{2}\angle A$

(d) $180° - \frac{1}{2}\angle A$

38. In ΔPQR, PS is the bisector of $\angle P$ and $PT \perp QR$, then $\angle TPS$ is equal to



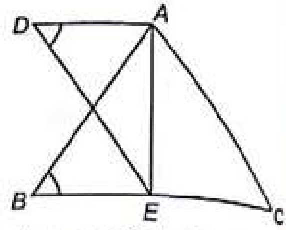
(a) $\angle Q + \angle R$

(b) $90 + \frac{1}{2}\angle Q$

(c) $90 - \frac{1}{2}\angle R$

(d) $\frac{1}{2}(\angle Q - \angle R)$

39. In figure, ΔADE and $\angle ABC$ are similar, if $AC : BC = 3 : 2$, then the ratio $\frac{DE}{AE}$ is

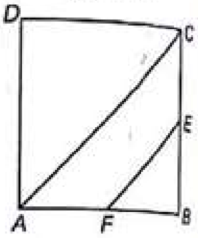


(a) 3 : 2 (b) 2 : 3
(c) 1 : 3 (d) 1 : 2

40. The ratio of the area of the equilateral triangle described on the side of a square to the area of the equilateral triangle described on its diagonal is

(a) 1 : 3 (b) $\sqrt{3} : 4$ (c) 1 : 2 (d) 1 : 4

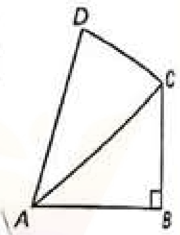
41. In figure, $ABCD$ is a square. F is the mid-point of AB, BE is one-third of BC. If the area of the ΔFBE is 108 sq cm. Then, the length of AC is



(a) $36\sqrt{3}$ cm (b) $35\sqrt{2}$ cm

(c) $12\sqrt{3}$ cm (d) $36\sqrt{2}$ cm

42. In a quadrilateral $ABCD$, $\angle B = 90°$, and $AD^2 = AB^2 + BC^2 + CD^2$. Then, ΔADC is



(a) equilateral triangle
(b) isosceles
(c) right angled triangle
(d) None of the above

43. In ΔABC, $AD \perp BC$, then

(a) $AB^2 - BD^2 = AC^2 - CD^2$ (b) $AB^2 + BD^2 = AC^2 - CD^2$

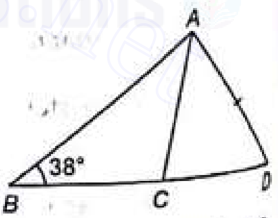
(c) $AB^2 + BD^2 = AC^2 + CD^2$ (d) $AB^2 + AC^2 = BD^2 + DC^2$

44. ΔABC is a right angled at C and P is the length of the perpendicular from C to AB. If $BC = a$, $AC = b$, $AB = c$, then

(a) $\frac{a}{b} = \frac{p}{c}$ (b) $pc = ab$

(c) $\frac{1}{a} + \frac{1}{b} = \frac{1}{ab}$ (d) None of these

45. In the figure, $\angle B = 38°$, $AC = BC$ and $AD = CD$. What is the value $\angle D$?



(CDS 2011 II)

(a) 26° (b) 28°
(c) 38° (d) 52°

46. Let D, E be the points on sides AB and AC, respectively at a ΔABC such that DE is parallel to BC. Let $AD = 2$ cm, $DB = 1$ cm, $AE = 3$ cm and area of $\Delta ADE = 3 \text{ cm}^2$. What is the value of EC?

(a) 1.5 cm (b) 1.6 cm
(c) 1.8 cm (d) 2.1 cm

47. Consider the following statements

Statement I Let PQR be a triangle in which $PQ = 3$ cm, $QR = 4$ cm and $RP = 5$ cm. If D is a point in the plane of the ΔPQR such that D is either outside it or inside it, then

$$DP + DQ + DR > 6 \text{ cm}$$

Statement II PQR is a right angled triangle.

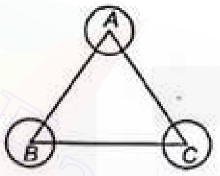
Which one of the following is correct in respect of the above two statements? **(CDS 2010 I)**

(a) Both statements I and II are individually true and statement II is the correct explanation of statement I.
(b) Both statements I and II are individually true but statement II is not the correct explanation of statement I.
(c) Statement I is true and statement II is false.
(d) Statement I is false and statement II is true.

48. ΔPQR is a right angled at Q. If X and Y are the mid-points of the sides PQ and QR respectively, then which one of the following is not correct?

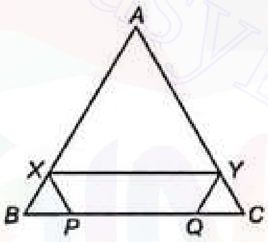
(a) $RX^2 + PY^2 = 5XY^2$
(b) $RX^2 + PY^2 = XY^2 + PR^2$
(c) $4(RX^2 + PY^2) = 5PR^2$
(d) $RX^2 + PY^2 = 3(PQ^2 + QR^2)$

49. In the figure given below, what is the sum of the angles formed around A, B, C except the angles of the ΔABC? **(CDS 2010 II)**



(a) 360° (b) 720°
(c) 900° (d) 1000°

50. In the given figure, ABC is an equilateral triangle of side length 30 cm. XY is parallel to BC, XP is parallel to AC and YQ is parallel to AB. If $(XY + XP + YQ)$ is 40 cm, then what is PQ equal to? **(CDS 2010 II)**



(a) 5 cm (b) 12 cm
(c) 15 cm (d) None of these

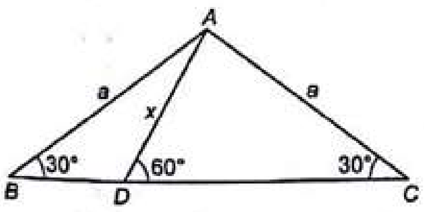
51. Consider the following statements

I. If two triangles are equiangular, then they are similar.
II. If two triangles have equal area, then they are similar.

Which of the statements given above is/are correct?

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

52. In the figure, what is the value of x?



(a) $\frac{a}{3}$ (b) $\frac{a}{2}$ (c) $\frac{a}{\sqrt{3}}$ (d) $\frac{a}{\sqrt{2}}$

53. **Statement I** Let LMN be a triangle. Let P, Q be the mid-points of the sides LM, LN, respectively. If $PQ^2 = MP^2 + NQ^2$, then LMN is a right angled triangle at L.

Statement II If in a ΔABC, $AB^2 > BC^2 + CA^2$, then $\angle ACB$ is obtuse.

Which of the following is correct in the light of the above statements? **(CDS 2010 I)**

(a) Both statements I and II are individually true and statement II is the correct explanation of statement I.
(b) Both statements I and II are individually true but statement II is not the correct explanation of statement I.
(c) Statement I is true and statement II is false.
(d) Statement I is false and statement II is true.

54. ABC is a triangle. The internal bisector of $\angle ABC$ and the external bisector of $\angle ACB$ meet at D. Which one of the following is correct?

(a) $\angle BDC = \angle BAC$ (b) $\angle BDC = \frac{1}{2}\angle BAC$
(c) $\angle BDC = \angle DBC$ (d) $\angle BDC = \frac{1}{2}\angle ABC$

55. Consider the following statements in respect of any triangle.

I. The three medians of a triangle divide it into six triangles of equal area.
II. The perimeter of a triangle is greater than the sum of the lengths of its three medians.

Which of the statement given above is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

56. Consider the following is respect of the given figure

I. $\Delta DAC \sim \Delta EBC$
II. $CA/CB = CD/CE$
III. $AD/BE = CD/CE$

Which of the above are correct? **(CDS 2010 I)**

(a) I, II and III (b) I and II (c) I and III (d) II and III

57. The median BD of the ΔABC meets AC at D. If $BD = \frac{1}{2}AC$, then which one of the following is correct?

(a) $\angle ACB = 1$ right angle (b) $\angle BAC = 1$ right angle
(c) $\angle ABC = 1$ right angle (d) None of these

58. The three sides of a triangle are 10, 100 and x. Which one of the following is correct? **(CDS 2010 I)**

(a) $10 < x < 100$ (b) $90 < x < 110$
(c) $90 \le x \le 100$ (d) $90 \le x < 110$

59. ABC is a triangle, X is a point outside the ΔABC such that $CD = CX$, where D is the point of intersection of BC and AX and $\angle BAX = \angle XAC$. Which one of the following is correct? **(CDS 2009 II)**

(a) ΔABD and ΔACX are similar
(b) $\angle ABD < \angle ACD$
(c) $AC = CX$
(d) $\angle ADB > \angle DXC$

60. Consider the following statements

I. Congruent triangles are similar.

II. Similar triangles are congruent.

III. If the hypotenuse and a side of one right triangle are equal to the hypotenuse and a side of another right triangle respectively, then the two right triangles are congruent.

Which of the statement given above is/are correct?

(a) Only I (b) Only II (c) II and III (d) I and III

61. ABC is a triangle and the perpendicular drawn from A meets BC in D. If $AD^2 = BC \cdot DC$, then which one of the following is correct?

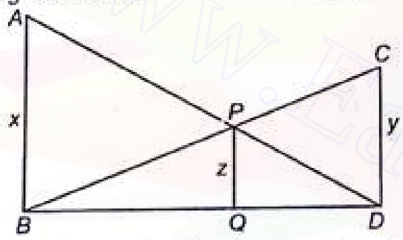
(a) ABC must be an obtuse angled triangle

(b) ABC must be an acute angled triangle

(c) Either $\angle B \geq 45°$ or $\angle C \geq 45°$

(d) $BC^2 = AB^2 + AC^2$

62. In the figure given, $\angle ABD = \angle PQD = \angle CDQ = \frac{\pi}{2}$. If $AB = x$, $PQ = z$ and $CD = y$, then which one of the following is correct? **(CDS 2009 I and 2007 I)**



(a) $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ (b) $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$ (c) $\frac{1}{z} + \frac{1}{y} = \frac{1}{x}$ (d) $\frac{1}{x} + \frac{1}{y} = \frac{2}{z}$

63. ΔPQR is right angled at Q, $PR = 5$ cm and $QR = 4$ cm. If the lengths of sides of another ΔABC are 3 cm, 4 cm and 5 cm, then which one of the following is correct? **(CDS 2009 I)**

(a) Area of ΔPQR is double that of ΔABC

(b) Area of ΔABC is double that of ΔPQR

(c) $\angle B = \frac{\angle Q}{2}$

(d) Both triangles are congruent

64. If C_1 and C_2 and r_1 and r_2 are respectively the centroids and radii of incircles of two congruent triangles, then which one of the following is correct?

(a) C_1 and C_2 are the same points and $r_1 = r_2$

(b) C_1 and C_2 are not necessarily the same point and $r_1 = r_2$

(c) C_1 and C_2 are the same point and r_1 is not necessarily equal to r_2

(d) C_1 and C_2 are not necessarily the same point and r_1 is not necessarily equal to r_2

Directions (Q.Nos. 70-73) *The following four questions consists of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the codes given below* **(CDS 2009 I)**

(a) Both A and R are individually true and R is the correct explanation of A.

(b) Both A and R are individually true but R is not the correct explanation of statement A.

(c) A is true and R is false.

(d) A is false and R is true.

65. Assertion (A) If two triangles have same perimeter, then they are congruent.

Reason (R) If under a given correspondence, the three sides of one triangle are equal to the three sides of the other triangle, then the two triangles are congruent.

66. ABC is a triangle. Let D, E denote the mid-points of BC, CA, respectively. Let AD and BE intersect at G. Let O be a point on AD such that $AO : OD = 2 : 7$.

Assertion (A) $AO = \frac{(2\,GD)}{3}$

Reason (R) $OD = \frac{(2\,AG)}{3}$

67. ABC is a triangle. AD, BE and CF are altitudes of ΔABC.

Assertion (A) $(AB^2 + BC^2 + CA^2) > (AD^2 + BE^2 + CF^2)$

Reason (R) $(AE^2 - AF^2) + (BF^2 - BD^2) + (CD^2 - CE^2) = 0$

68. Assertion (A) Triangles on the same base and between the same parallel lines are equal in area.

Reason (R) The distance between two parallel lines is same everywhere.

69. ABC is a right angled triangle, right angled at C and p is the length of the perpendicular from C on AB. If a, b and c are the sides of the triangle, then which one of the following is correct? **(CDS 2008 II)**

(a) $(a^2 + b^2)p^2 = a^2b^2$ (b) $a^2 + b^2 = a^2b^2p^2$

(c) $p^2 = a^2 + b^2$ (d) $p^2 = a^2 - b^2$

70. If ABC is a triangle, right angled at B and M, N are mid-points of AB and BC respectively, then what is the value of $4(AN^2 + CM^2)$? **(CDS 2008 II)**

(a) $3\,AC^2$ (b) $4\,AC^2$ (c) $5\,AC^2$ (d) $6\,AC^2$

71. If A is the area of a right angled triangle and b is one of the sides containing the right angle, then what is the length of the altitude on the hypotenuse?

(a) $\frac{2\,Ab}{\sqrt{b^4 + 4A^2}}$ (b) $\frac{2\,A^2b}{\sqrt{b^4 + 4A^2}}$ (c) $\frac{2Ab^2}{\sqrt{b^4 + 4A^2}}$ (d) $\frac{2\,A^2b^2}{\sqrt{b^4 + A^2}}$

72. Assertion (A) If two triangles are congruent, then their corresponding angles are equal.

Reason (R) Two congruent triangles have same area. **(CDS 2008 II)**

(a) Both A and R are individually true and R is the correct explanation of A.

(b) Both A and R are individually true but R is not the correct explanation of A.

(c) A is true and R is false.

(d) A is false and R is true.

73. Consider the following statements.

A triangle can be constructed, if its

I. two sides and the included angles are given.

II. three angles are given.

III. two angles and the included side are given.

Which of the statements given above are correct?

(a) I and II (b) I and III (c) II and III (d) I, II and III

74. **Assertion (A)** ABC is a triangle and AD is its angular bisector. If $AB = 6$ cm, $BC = 7$ cm, $AC = 8$ cm, then $BD = 3$ cm and $CD = 4$ cm.

Reason (R) The angular bisector AD of a triangle cuts the base BC in the ratio $AB : AC$. **(CDS 2007 II)**

(a) Both A and R are individually true and R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true and R is false.
(d) A is false and R is true.

75. ABC is triangle with $\angle A = 90°$. From A, a perpendicular AD is drawn on BC. Which one of the following is correct?

(a) $\Delta ABC \sim \Delta DAC$ only
(b) $\Delta DAC \sim \Delta DBA$ only
(c) $\Delta ABC \sim \Delta DBA \sim \Delta DAC$
(d) $\Delta ABC \sim \Delta DAB$ only

Where ~ stands for the notation of similarity.

76. In ΔPQR, $PQ = 4$ cm, $QR = 3$ cm and $RP = 3.5$ cm ΔDEF is similar to ΔPQR. If $EF = 9$ cm, them what is the perimeter to ΔDEF? **(CDS 2007 I)**

(a) 10.5 cm
(b) 21 cm
(c) 31.5 cm
(d) Cannot be determined as data is insufficient

Answers

1. (d)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (c)	**10.** (d)
11. (d)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (a)	**16.** (d)	**17.** (b)	**18.** (c)	**19.** (d)	**20.** (d)
21. (a)	**22.** (c)	**23.** (b)	**24.** (b)	**25.** (a)	**26.** (b)	**27.** (a)	**28.** (a)	**29.** (c)	**30.** (d)
31. (c)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (a)	**36.** (b)	**37.** (c)	**38.** (d)	**39.** (b)	**40.** (c)
41. (d)	**42.** (c)	**43.** (a)	**44.** (b)	**45.** (b)	**46.** (a)	**47.** (a)	**48.** (d)	**49.** (c)	**50.** (d)
51. (a)	**52.** (c)	**53.** (b)	**54.** (b)	**55.** (c)	**56.** (a)	**57.** (c)	**58.** (b)	**59.** (a)	**60.** (d)
61. (d)	**62.** (a)	**63.** (d)	**64.** (a)	**65.** (d)	**66.** (c)	**67.** (b)	**68.** (a)	**69.** (a)	**70.** (c)
71. (a)	**72.** (a)	**73.** (d)	**74.** (a)	**75.** (c)	**76.** (c)				

Hints and Solutions

1. Circumcentre of a triangle may be outside the triangle.

2. Let ABC is any triangle and AD is the bisector of angle A.

Also, $BD = DC$

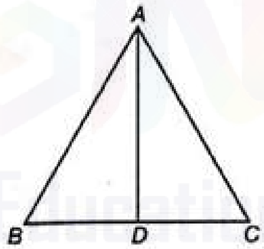
$\therefore$ In ΔABD and ΔADC,

$\angle BAD = \angle DAC$ and $BD = DC$

$\therefore$ $\Delta ABD \cong \Delta ADC$

$\therefore$ $AB = AC$

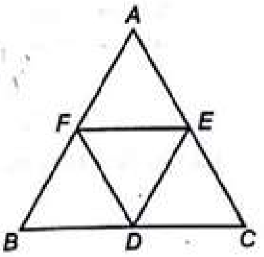
Hence, triangle is isosceles.



3. The line segments joining the mid-points of the sides of a triangle form four triangles each of which is similar to the original triangle.

Here, $\Delta BDF \sim \Delta ABC$

Also, $\Delta DEC, \Delta DEF, \Delta AFE \sim \Delta ABC$



4. The sides of triangle formed will be half of the sides of the original triangle.

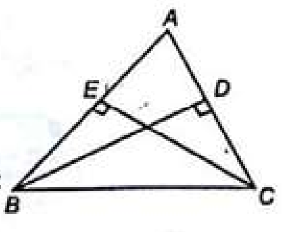
5. As $BD = EC, \angle AEC = \angle BDA = 90°$ each

Also, $\angle A = \angle A$ (common)

$\therefore$ $\Delta BDA \cong \Delta AEC$

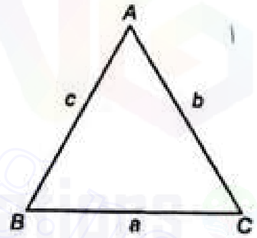
$\Rightarrow$ $AB = AC$ by concept

$\therefore$ Triangle is an isosceles triangle.



6. $\dfrac{AC}{AB} = \sqrt{2}$

By Sine formula $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$



$$\frac{AC}{\sin B} = \frac{AB}{\sin C}$$

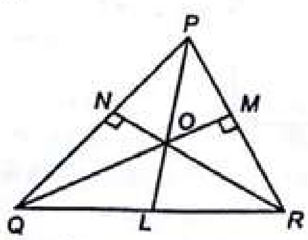
$$\Rightarrow \quad \frac{AC}{AB} = \frac{\sin B}{\sin C} \qquad (\because B = 45°)$$

$$\Rightarrow \quad \frac{\sin 45°}{\sin C} = \frac{\sqrt{2}}{1} \Rightarrow \frac{\frac{1}{\sqrt{2}}}{\sin C} = \frac{\sqrt{2}}{1}$$

$$\Rightarrow \quad \sin C = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2} = \sin 30° \Rightarrow C = 30°$$

$$\therefore \quad \angle BAC = 180° - (\angle B + \angle C) = 180° - (45° + 30° = 105°$$

7. Clearly, QRO as



$QP \perp QR$ and $PR \perp QM$ and $OL \perp QR$

$\therefore P$ is point of intersection of altitude virtually.

8. Let the other side by b and p.

$\therefore \quad \frac{1}{2} b \times p = 6 \Rightarrow b \times p = 12 \Rightarrow b = \frac{12}{p}$

Also, by Pythagorus theorem $H^2 = B^2 + P^2$

$$5^2 = \left(\frac{12}{p}\right)^2 + p^2 \Rightarrow 25 = \frac{144}{p^2} + p^2$$

$25p^2 = 144 + p^4 \Rightarrow p^4 - 25p^2 + 144 = 0$

$\Rightarrow \quad p^4 - 16p^2 - 9p^2 + 144 = 0$

$\Rightarrow \quad p^2(p^2 - 16) - 9(p^2 - 16) = 0$

$\Rightarrow \quad (p^2 - 9)(p^2 - 16) = 0 \Rightarrow p = 3$ or $p = 4$

$\because$ Other sides are 3 cm and 4 cm.

9. As $\Delta ABC \sim \Delta DEF \Rightarrow \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

$\Rightarrow \quad$ But $\frac{BC}{EF} = \frac{2}{4} = \frac{1}{2}$

$\therefore \quad DE = 2AB = 2 \times 3 = 6$ cm

$DF = 2 \times AC = 2 \times 2.5 = 5$ cm

$\therefore$ Perimeter of $\Delta DEF = (6 + 5 + 4) = 15$ cm

Shortcut method

$$\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \text{Ratio of corresponding sides}$$

$\therefore \quad \frac{(3 + 2 + 2.5)}{\text{Perimeter of } \Delta DEF} = \frac{1}{2}$

$\therefore$ Perimeter of $\Delta DEF = 2(7.5) = 15$ cm.

10. I. In ΔABD and ΔCAD,

$\because \quad \angle ADB = \angle ADC = 90°$ each

$\angle BAD = \angle ACD \quad (\because$ each $= 90° - \angle DAC)$

$\therefore \quad \Delta ADB \sim \Delta CAD$

II. In ΔABD and ΔCDA,

$\angle ADB = \angle ADC = 90°$ each

$\angle BAD = \angle ACD = 90° - \angle DAC$ (each)

and $\quad AD = AD$ (common)

$\therefore \quad \Delta ABD \cong \Delta CDA$

III. In ΔADB and ΔCAB

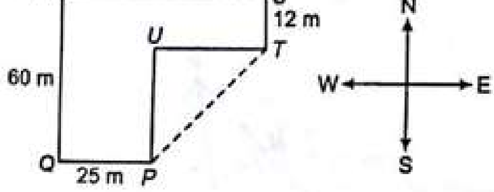
$\angle ADB = \angle BAC \; 90°$ each

$\angle B = \angle B$ (common)

$\therefore \quad \Delta ADB \sim \Delta CAB$

Here, I, II and III are correct statements.

11. Let P be the starting point of his run, then PT is the distance between the starting and the finishing point.



$\therefore \quad PU = RQ - ST = 60 - 12 = 48$ m

$TU = RS - QP = 80 - 25 = 55$ m

$\therefore$ In ΔPUT,

$PT^2 = (PU)^2 + (TU)^2$

$\therefore \quad PT = \sqrt{(48)^2 + (55)^2} = \sqrt{5329} = 73$ m

12. As ADB is a right angled triangle.

So, $\quad AB^2 = AD^2 + BD^2$

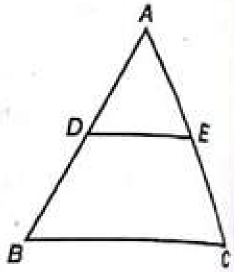
$\Rightarrow \quad AC^2 = AD^2 + BD^2 \quad (\because AB = AC)$

$\Rightarrow \quad (AD + DC)^2 = AD^2 + BD^2$

$\Rightarrow \quad AD^2 + DC^2 + 2AD \cdot DC = AD^2 + BD^2$

$\Rightarrow \quad BD^2 - CD^2 = 2CD \cdot AD$

13. As in ΔADE and ΔABC,



$\frac{AD}{AB} = \frac{8}{20} = \frac{2}{5}, \frac{AE}{EC} = \frac{6}{15} = \frac{2}{5}$

So, $\quad \frac{AD}{AB} = \frac{AE}{EC}$

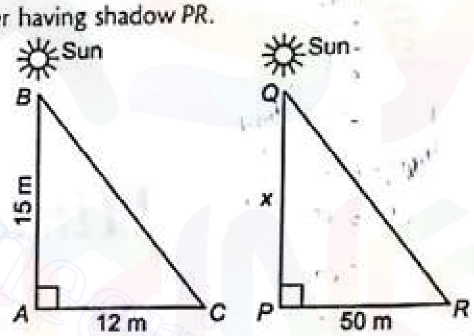
and $\quad \angle A = \angle A'$ (common)

$\Delta ADE \sim \Delta ABC$

$\therefore \quad \frac{DE}{BC} = \frac{AD}{AB} \Rightarrow \frac{DE}{BC} = \frac{2}{5}$

$\Rightarrow \quad BC = \frac{5}{2} DE$

14. Let AB be a vertical stick and AC be its shadow. Also, let PQ be a tower having shadow PR.



As, $\quad \Delta ABC \sim \Delta PQR$

$\therefore \quad \frac{AB}{PQ} = \frac{AC}{PR}$

$\Rightarrow \quad \frac{15}{x} = \frac{12}{50} \Rightarrow x = \frac{15 \times 50}{12} = 62.5$ m

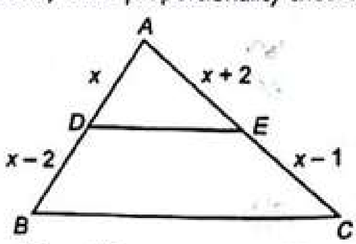
Hence, height of the tower is 62.5 m.

15. Let the ratio of their corresponding height be $h_1 : h_2$.

But the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding heights.

$\therefore \quad \frac{h_1^2}{h_2^2} = \frac{81}{49} \Rightarrow h_1 : h_2 = 9 : 7$

16. As $DE || BC$ so by basic proportionality theorem,



$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{x}{x - 2} = \frac{x + 2}{x - 1}$

$\Rightarrow \quad x(x - 1) = (x + 2)(x - 2)$

$\Rightarrow \quad x^2 - x = x^2 - 4 \Rightarrow x = 4$

17. As $BC || AD$ and the diagonals of a trapezium divide each other proportionally.

So, $$\frac{AO}{OC}=\frac{BO}{OD} \Rightarrow \frac{3x-1}{5x-3}=\frac{2x+1}{6x-5}$$

$$(3x-1)(6x-5)=(5x-3)(2x+1)$$

$\Rightarrow \quad 18x^2-15x-6x+5=10x^2+5x-6x-3$

$\Rightarrow \quad 8x^2-20x+8=0 \Rightarrow 4x^2-10x+4=0$

$\Rightarrow 4x^2-8x-2x+4=0 \Rightarrow 4x(x-2)-2(x-2)=0$

$\Rightarrow \quad (4x-2)(x-2)=0 \Rightarrow x=\frac{1}{2}$ or 2

But as $x=\frac{1}{2}$ will make OC negative.

$\therefore \quad x=2$

18. $\because \frac{BE}{CE}=\frac{AB}{AC}$ as AE is a exterior angle bisector.

Let $\quad CE=x,\ BE=BC+EC=12+x \Rightarrow \frac{12+x}{x}=\frac{10}{6}$

$\Rightarrow \quad (12+x)6=10x \Rightarrow 72+6x=10x$

$\Rightarrow \quad 4x=72 \Rightarrow x=18$ cm

19. As the line joining the mid-points of any two sides or a triangle is parallel to the third side and is half of the third side.

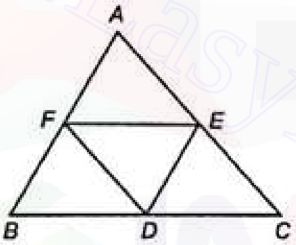
$\therefore \quad DE=\frac{1}{2}AB=\frac{1}{2}\times 10=5$ cm

$EF=\frac{1}{2}BC \Rightarrow BC=2EF$

$=2\times 3=6$ cm

$DF=\frac{1}{2}AC \Rightarrow AC=2\times DF$

$=2\times 4=8$ cm



20. As $\angle P+\angle Q+\angle R=180°$

$\Rightarrow \quad a+3a+b=180° \quad 4a+b=180°$

$-5a+3b=30°$

Solving above equation, $a=30°$ and $b=60°$

$\therefore \quad \angle P=30°, \angle Q=90°$ and $\angle R=60°$

$\therefore \Delta PQR$ is right angled triangle.

21. Consider ΔBMD and ΔDLC

As $\angle BMD=\angle DLC=90°$ each

Also, $\angle BDM=\angle DCL$ corresponding angle

$\therefore \quad \Delta BMD \sim \Delta DLC$

$\therefore \quad \frac{BD}{DC}=\frac{DM}{LC}=\frac{BM}{DL}$

$\frac{BD}{DC}=\frac{DM}{LC}=\frac{1}{3}$

$\therefore \quad DM:LC=1:3$

22. In ΔABC by Pythagorus theorem,

$$AC^2=AB^2+BC^2 \qquad ...(i)$$

And in ΔPBQ,

$$PQ^2=PB^2+BQ^2 \qquad ...(ii)$$

Adding Eqs. (i) and (ii),

$$AC^2+PQ^2=(AB^2+BC^2)+PB^2+BQ^2$$
$$=(AB^2+BQ^2)+(PB^2+BC^2)$$
$$AC^2+PQ^2=AQ^2+PC^2$$

$\because \Delta ABQ$ and ΔPBC are right triangles.

23. Circumcentre is the point of intersection of perpendicular bisectors of sides of the triangle. Hence, it is equidistant form the vertices of the triangle.

24. As $OP || YZ$

$\Rightarrow \quad \angle POY=\angle OYZ$

$\Rightarrow \quad \angle PYO=\angle POY$ ($\because OY$ is angle bisector of $\angle Y$)

$\therefore \quad PY=PO$

As $\angle XYZ$ is an acute angle.

$\therefore \quad \frac{1}{2}\angle XYZ<45°$

$\therefore \quad \angle POY=\angle PYO<45°$

$\therefore \quad \angle YPO>90°$

Hence, ΔPYO is isosceles triangle but not a right angled triangle.

25. In $\Delta BCD, \angle BCD=65°$ and $\angle BDC=90°$

$$\angle CBD=180°-(\angle BCD+\angle CDB)$$
$$=180°-(65°+90°)=180°-155°=25°$$

26. Let angles of triangle be $2x, 3x, 4x$, then

$$2x+3x+4x=180°$$
$$9x=180° \Rightarrow x=20°$$

So, angles are $2x=40°$

$3x=60°,\ 4x=80°$

27. In $\Delta ABC, \quad \angle A+\angle B+y=180°$

$y=180°-(40+30)°=110°$

$\angle ADE=\angle ABC$ (pair of corresponding angles)

$x=30°$

Similarly, $y=z=110°$

28. I. It is not possible to have a triangle in which sum of the two angles is greater than 180°.

II. and IV. cases the sum of the three angles will be 180°.

III. In this case, sum of the three angles will be less than 180°.

29. As $\Delta CAB \sim \Delta CED$

$\therefore \quad \frac{CA}{CD}=\frac{CE}{DE}=\frac{CB}{CD}$

So, $\quad \frac{AB}{DE}=\frac{CB}{CD}$

$\therefore \quad \frac{9}{x}=\frac{10+2}{8} \Rightarrow x=\frac{8\times 9}{12}=6$ cm

30. In ΔGEF and ΔGCD, we have

$\angle EFG=\angle GDC$ (alternate angles)

$\angle EGF=\angle CGD$ (vertically opposite angles)

$\Delta GEF \sim \Delta GCD$

$\therefore \quad \frac{GE}{CG}=\frac{EF}{CD} \Rightarrow \frac{5}{10}=\frac{EF}{18}$

$\Rightarrow \quad EF=\frac{5\times 18}{10}=9$ cm

31. As diagonals of a rhombus bisect each other at right angles.

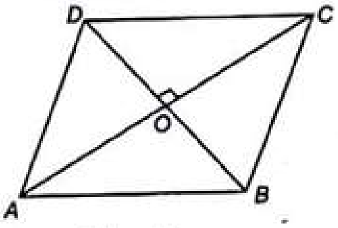
$\Rightarrow \quad AO=OC$ and $BO=OD$

Applying Pythagorus theorem to $\Delta AOB, \Delta AOD, \Delta DOC, \Delta BOC$ and on adding,

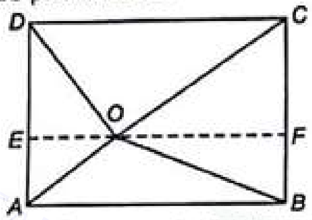
$$AB^2+BC^2+CD^2+AD^2=2[AO^2+OC^2+BO^2+DO^2]$$
$$=2[2AO^2+2BO^2]$$

$= 4[AO^2 + OB^2]$ $\left[\because AO = \frac{AC}{2}, BO = \frac{BD}{2}\right]$

$= AC^2 + BD^2$



32. Draw a line OE parallel to AB.



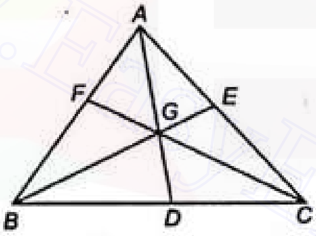
Apply Pythagorus theorem to ΔAOE, ΔDOE, ΔFOC and $\angle OFB$.
Adding equation and equating, we get

$$OB^2 + OD^2 = OC^2 + OA^2$$

33. Let G be the centroid of ΔABC.

In ΔABC

[$\because$ the sum of the squares of any two sides is equal to twice the square of half of the third side together with the square of the median bisecting the third side]



$\therefore$ $AB^2 + AC^2 = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$ ...(i)

or $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$

$BC^2 + AB^2 = 2BE^2 + \frac{1}{2}AC^2$ ...(ii)

$BC^2 + AC^2 = 2CF^2 + \frac{1}{2}AB^2$ ...(iii)

Adding Eqs. (i), (ii) and (iii), we get

$$2(AB^2 + BC^2 + AC^2) = 2(AD^2 + BE^2 + CF^2) + \frac{1}{2}(AB^2 + BC^2 + AC^2)$$

$\therefore$ $3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$

34. In ΔPQR,

Now by given condition,

Here, $QR + 2 = 2PQ$

or $QR = 2PQ - 2$ ...(i)

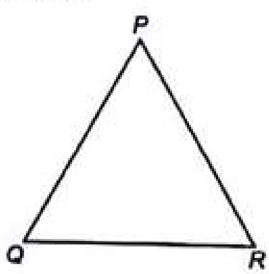
$PR = PQ + 10$ ...(ii)

$PQ + QR + RP = 40$ ...(iii)

Put Eqs. (i) and (ii) in Eq. (iii)

$PQ + 2PQ - 2 + PQ + 10 = 40$

$4PQ = 32$ or $PQ = 8$ cm



35. Sides opposite to equal angles are equal. Here, $\angle ADC = \angle CAD = 30°$.

So, $AC = CO$

36. $\angle BAL + \angle B + 90° = 180°$

$\Rightarrow$ $\angle BAL + \angle B = 90°$

$\angle BAL = 90° - \angle B$ ...(i)

Now in ΔABC, $\angle ACB + \angle B + \angle A = 180°$

$\Rightarrow$ $\angle ACB + \angle B = 180° - 90°$

$\angle ACB + \angle B = 90°$

$\angle ACB = 90° - \angle B$ ...(ii)

From Eqs. (i) and (ii) $\angle BAL = \angle ACB$

37. In ΔBOC,

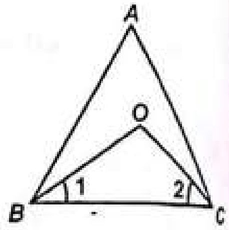
$\angle 1 + \angle 2 + \angle BOC = 180°$ ...(i)

$\angle A + \angle B + \angle C = 180°$

$\frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90°$

$\frac{1}{2}(\angle A) + \angle 1 + \angle 2 = 90°$

$\angle 1 + \angle 2 = 90° - \frac{1}{2}\angle A$



Put $\angle 1 + \angle 2$ in Eq. (i),

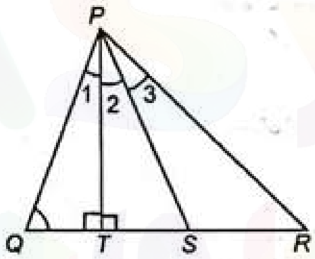
$\angle BOC = 180° - \left(90° - \frac{1}{2}\angle A\right)$

$= 90° + \frac{1}{2}\angle A$

38. $\angle 1 + \angle 2 = \angle 3$ ...(i)

$\angle Q = 90° - \angle 1$

$\angle R = 90° - \angle 2 - \angle 3$



So, $\angle Q - \angle R = (90° - \angle 1) - (90° - \angle 2 - \angle 3)$

$\angle Q - \angle R = \angle 2 + \angle 3 - \angle 1$ [from Eq (i)]

$= \angle 2 + (\angle 1 + \angle 2) - \angle 1$

$\angle Q - \angle R = 2\angle 2$

$\frac{1}{2}(\angle Q - \angle R) = \angle TPS$

39. As $\Delta ADE \sim \Delta ABC$

So, $\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$

$\frac{DE}{AE} = \frac{BC}{AC} = \frac{2}{3}$

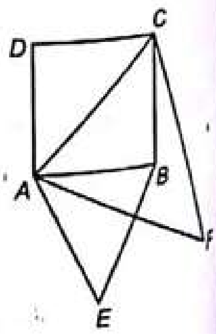
Hence, $DE : AE = 2 : 3$

40. Here, $AC^2 = 2AB^2$

As ΔABE and ΔABC are equiangular, so

$\Delta ABE \sim \Delta ABC$

[The ratio of the areas of two similar triangle is equal to the ratio of their corresponding sides]



$$\frac{\text{Area of } (\Delta ABE)}{\text{Area of } (ACF)} = \frac{AB^2}{AC^2} = \frac{AB^2}{2AB^2} = \frac{1}{2}$$

41. Area of $\Delta FBE = 108$ cm^2

Let each side be $6x$.

$\therefore$ $BE = \frac{1}{3}BC = \frac{1}{3} \times 6x = 2x$

$$BF = \frac{1}{2} AB = \frac{1}{2} \times 6x = 3x$$

Area of $\Delta FBE = \frac{1}{2} 3x \times 2x = 3x^2$

$\therefore$ $3x^2 = 108$

$x^2 = 36 \Rightarrow x = 6$ cm

$\therefore AC^2 = AB^2 + BC^2 = 2AB^2 = 2(36)^2$ $(\because AB = 36)$

$AC = 36\sqrt{2}$ cm

42. Here, $AD^2 = AB^2 + BC^2 + CD^2$

$AD^2 = AC^2 + DC^2$

$\therefore \Delta ACD$ is a right angled triangle.

43. Here, in ΔADC

$AB^2 = AD^2 + BD^2$ ...(i)

In right angled ΔACD, we have

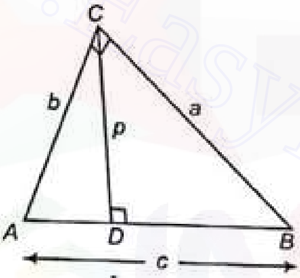
$AC^2 = AD^2 + CD^2$ ...(ii)

Subtracting Eq. (ii) from Eq. (i),

$AB^2 - AC^2 = BD^2 - CD^2$

$AB^2 - BD^2 = AC^2 - CD^2$

44. $\because$ C is the base and p is the altitude of ΔABC



Here, area of

$\Delta ABC = \frac{1}{2} pc$ ...(i)

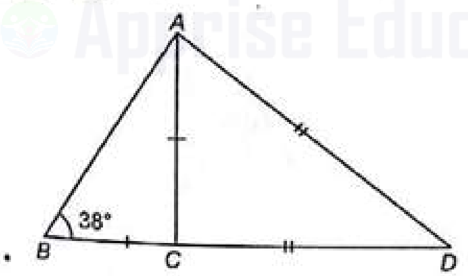
Again area of

$\Delta ABC = \frac{1}{2} ab$ ...(ii)

From Eqs. (i) and (ii), we get

$\frac{1}{2} pc = \frac{1}{2} ab \Rightarrow pc = ab$

45. Given, $AC = BC$



$\therefore$ $\angle B = \angle A$

$\Rightarrow$ $\angle A = \angle B = 38°$

In ΔABC,

$\angle C = 180° - (\angle A + \angle B)$

$= 180° - (38° + 38°) = 180° - 76° = 104°$

In ΔACD, $\angle C = 180° - 104° = 76°$

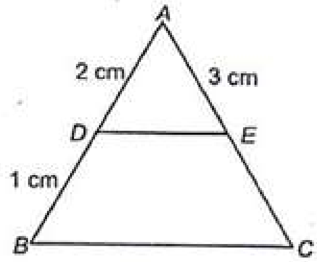
and $\angle A = \angle C = 76°$ $(\because CD = AD)$

$\therefore \angle D = 180° - (\angle A + \angle C) = 180° - (76° + 76°) = 28°$

46. In ΔADE and ΔABC,

$\angle A = \angle A$

$DE || BC$



$\therefore$ $\Delta ADE \sim \Delta ABC$

$\therefore$ $\frac{AD}{BD} = \frac{AE}{EC}$

$\frac{2}{1} = \frac{3}{EC} \Rightarrow EC = \frac{3}{2} = 1.5$ cm

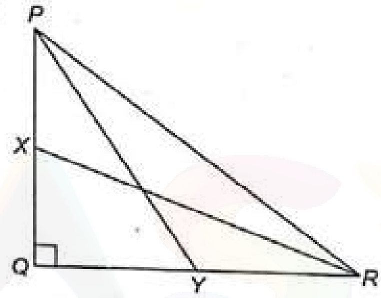
47. Given, $PQ = 3$ cm, $QR = 4$ cm and $RP = 5$ cm

Here, $RP^2 = PQ^2 + QR^2$

So, PQR is a right angle triangle.

Both statements I and II are individually true and statement II is the correct explanation of statement I.

48. In ΔPQY,



$PY^2 = PQ^2 + QY^2$

$\Rightarrow$ $PY^2 = PQ^2 + \left(\frac{QR}{2}\right)^2$ ...(i)

And in ΔXQR,

$RX^2 = QX^2 + QR^2$

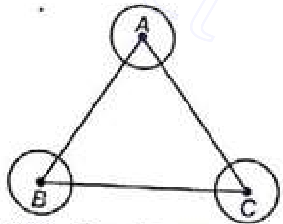
$\Rightarrow$ $RX^2 = \left(\frac{PQ}{2}\right)^2 + QR^2$ ...(ii)

On adding Eqs. (i) and (ii),

$PY^2 + RX^2 = \frac{5PQ^2}{4} + \frac{5QR^2}{4}$

$\Rightarrow$ $4(PY^2 + RX^2) = 5(PR^2)$

49. $\therefore \angle A = 360° - \text{Ext } \angle A$



Similarly, $\angle B = 360° - \text{Ext } \angle B$

and $\angle C = 360° - \text{Ext } \angle C$

We know that,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 360° - \text{Ext } \angle A + 360° - \text{Ext } \angle B + 360° - \text{Ext } \angle C = 180°$

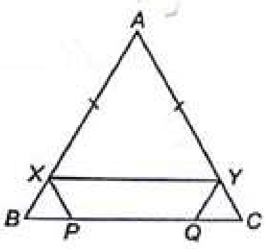
$\Rightarrow$ $\text{Ext } \angle A + \text{Ext } \angle B + \text{Ext } \angle C = 1080° - 180° = 900°$

50. Since, $XP || AC$, $YQ || AB$

$\therefore \angle XBP = \angle YQC$ and

$\angle XPB = \angle YCQ$

$\therefore$ ΔXBP and ΔYCQ are equilateral triangles.



Now, $XY || BC$

$\therefore$ $\frac{AX}{AB} = \frac{XY}{BC}$

$\Rightarrow$ $AX = XY$ $(\because AB = BC = 30 \text{ cm})$

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Also, $XY + XP + YQ = 40$

$AX + XB + YQ = 40$ $(\because XY = AX, XP = XB)$

$\Rightarrow AB + YQ = 40$

$\Rightarrow YQ = 40 - 30 = 10$ cm

$\Rightarrow YQ = XP = 10$ cm

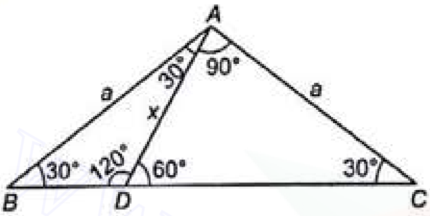
$\therefore BP = CQ = 10$ cm

$\therefore PQ = 30 - BP - CQ = 30 - 10 - 10 = 10$ cm

$\therefore$

51. We know that, if two triangles are equiangular, then they are similar but it need not to be, if two trianlges have equal area, then they are similar.

52. In ΔADC



$$\angle DAC = 180° - 60° - 30° = 90°$$

Again in right ΔDAC,

$$\tan 60° = \frac{AC}{AD} \quad \text{(say)}$$

$$\Rightarrow \sqrt{3} = \frac{a}{x} \Rightarrow x = \frac{a}{\sqrt{3}}$$

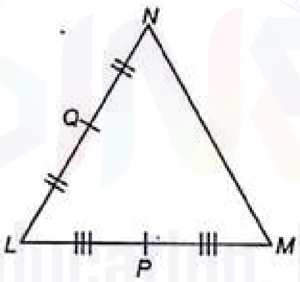
53. Statement I Given,

$PQ^2 = MP^2 + NQ^2$

$\Rightarrow PQ^2 = LP^2 + LQ^2$

$\Rightarrow \angle NLP = 90°$

It means, ΔNLM be a right angled.

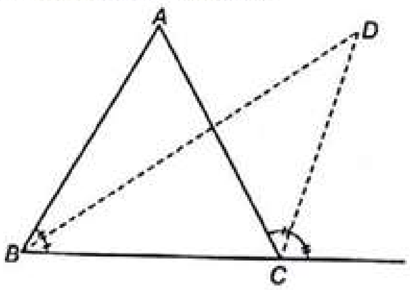


Statement II It also true that if in a ΔABC

$AB^2 > BC^2 + CA^2$, then $\angle ACB$ is obtuse.

Hence, both statements are individually true but statement II is not the correct explanation of statement I.

54. By using the properties of triangle



$$BDC = \frac{1}{2} BAC$$

55. I. It is true that the three medians of a triangle divide it into six triangles of equal area.

II. It is also true that, the perimeter of a triangle is greater than the sum of its three medians.

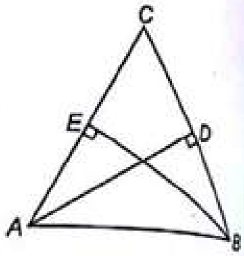
56. In ΔCAD and ΔCEB,

$\angle C = \angle C$ (common)

$\angle CEB = \angle ADC$ (each 90°)

$\angle CAD = \angle CBE$ (rest angle)

$\therefore \Delta CAD \sim \Delta CEB$

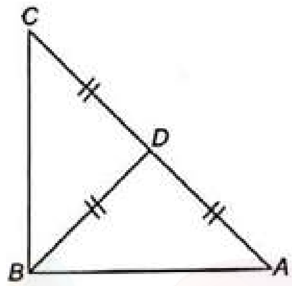


Sides will be in same proportion.

$$\frac{CA}{CB} = \frac{CD}{CE} \text{ and } \frac{AD}{BE} = \frac{CD}{CE}$$

Hence, all three statements are correct.

57. Here, we see that



$$CD = BD = DA$$

This is possible only when ABC is right angled triangle.

58. We know, the sum of two sides is always greater than third sides.

$\therefore 10 + 100 > x, 10 + x > 100$ and $100 + x > 10$

$\Rightarrow 110 > x, x > 90$

and $x > -110$, but x cannot be negative.

$\therefore 90 < x < 110$

59. In ΔDCX,

$CD = CX$ (given)

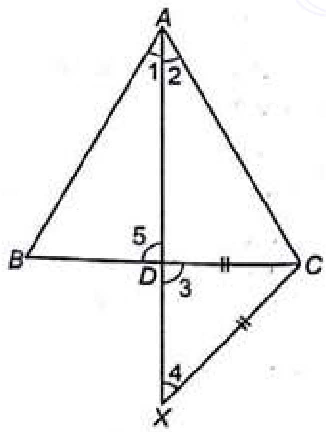
$\angle 3 = \angle 4$ (opposite angle of same sides)

But $\angle 3 = \angle 5$

So, $\angle 4 = \angle 5$

In ΔABD and ΔACX,

$\angle 1 = \angle 2$ (given)



$\angle 4 = \angle 5$

$\angle B = \angle ACX$ (rest angle)

$\therefore \Delta ABD \sim \Delta ACX$

60. It is true that congruent triangles are similar but converse is not true. Also, statement III is true.

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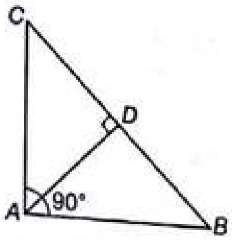
61. Hence, $AD^2 = BD \cdot DC$

$$\Rightarrow \quad \frac{AD}{BD} = \frac{DC}{AD}$$

$\therefore \Delta ABC$ must be right angled triangle.



62. Since, $\angle ABD = \angle PQD = 90°$

So, $\Delta ABD \sim \Delta PQD$

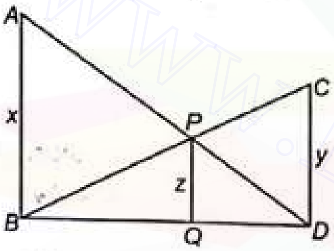
Then, $\frac{x}{z} = \frac{BD}{QD}$ (by Thales theorem) ...(i)

Since, $\angle CDB = \angle PQB = 90°$

So, $\Delta BCD \sim \Delta BPQ$.

$\therefore \quad \frac{z}{y} = \frac{BQ}{BD}$ (by Thales theorem)

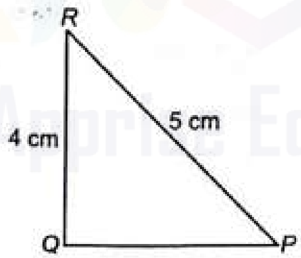
$$\Rightarrow \quad \frac{z}{y} = \frac{BD - QD}{BD} \Rightarrow \frac{z}{y} = 1 - \frac{QD}{BD}$$



$$\Rightarrow \quad \frac{z}{y} = 1 - \frac{z}{x} \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad \frac{z}{x} + \frac{z}{y} = 1 \Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

63. In right angle ΔPQR,



$QP^2 = (5)^2 - (4)^2 = 9 \Rightarrow QP = 3$ cm

In second ΔABC whose sides are 3 cm, 4 cm and 5 cm. So, the sides of both triangle are same, hence they are congruent.

64. Since, in congruent triangles corresponding sides and angles of one triangle are equal to that of other triangle. So, there medians also will be equal and intersect at the same point, so $G_1 = G_2$.

Since, both the triangles have equal so there angle bisectors will be equal and intersect at the same point. Hence, the radii of incircles will be equal.

So,

$$r_1 = r_2$$

65. (A) If two triangles have same perimeter, then it is not necessary that they have same area. So, they need not be congruent.

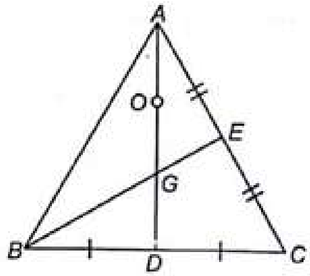
(R) This condition is true, because two triangles are congruent by (SSS) property.

66. Given, $AO : OD = 2 : 7$

$$OA = \frac{2}{9}AD, OD = \frac{7}{9}AD$$

We know that, centroid makes a ratio 2 : 1 on the median.

So, $AG = \frac{2}{3}AD, GD = \frac{1}{3}AD$ ...(i)



(A) $OA = \frac{2}{9}AD$

$$OA = \left(2 \cdot \frac{1}{3}AD\right)\frac{1}{3} = \frac{2GD}{3} \quad \text{[from Eq. (i)]}$$

(R) $OD = \frac{7}{9}AD = \left(7 \cdot \frac{2}{3}AD\right) \cdot \frac{1}{3 \times 2}$

$$= \frac{7AG}{6}$$

So, A is true but R is false.

67. (A) We know that in a right angled triangle, hypotenuse is a largest side.

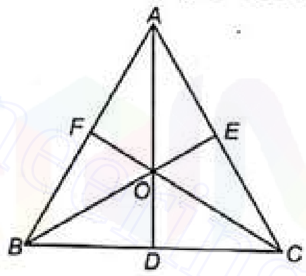
In ΔABD, $AB^2 > AD^2$ ...(i)

In ΔBEC, $BC^2 > BE^2$ ...(ii)

In ΔACF, $AC^2 > CF^2$ ...(iii)

On adding Eq. (i), (ii) and (iii), we get

$$(AB^2 + BC^2 + AC^2 0) > (AO^2 + BE^2 + CF^2)$$



(R) Now,

$$(AE^2 - AF^2) + (BF^2 - BD^2) + (CD^2 - CE^2)$$
$$= (OA^2 - OE^2) - (OA^2 - OF^2) + (OB^2 - OF^2)$$
$$- (OB^2 - OD^2) + (OC^2 - OD^2) - (OC^2 - OE^2) = 0$$

Hence, both A and R are individually true but R is not the correct explanation of A.

68. (A) By the properties of triangle, it its true.

(R) It is also true, that the distance between two parallel lines is same everywhere.

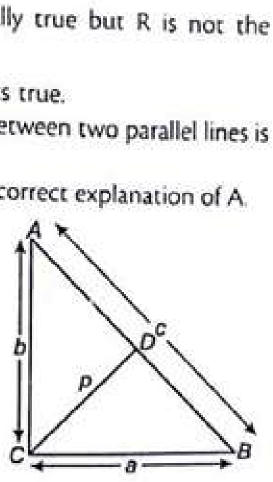
Hence, A and R are true and R is the correct explanation of A.

69. In right ΔABC,

$$\text{Area} = \frac{1}{2} \times a \times b$$

Again, in right ΔABC,

$$\text{Area} = \frac{1}{2} \times AB \times DC$$



$$\Rightarrow \quad \frac{1}{2}ab = \frac{1}{2} \times c \times p$$

$$\Rightarrow \quad ab = p(\sqrt{a^2 + b^2}) \quad (\because c^2 = a^2 + b^2)$$

$$\Rightarrow \quad a^2b^2 = p^2(a^2 + b^2)$$

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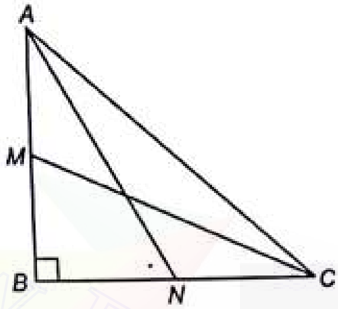
70. In right angled ΔABC,



In ΔABN,

$$AN^2 = AB^2 + BN^2$$

$$= AB^2 + \frac{BC^2}{4} \quad \text{...(i)}$$

In ΔCBM,

$$CM^2 = BC^2 + BM^2$$

$$= BC^2 + \frac{AB^2}{4} \quad \text{...(ii)}$$

From Eqs. (i) and (ii)

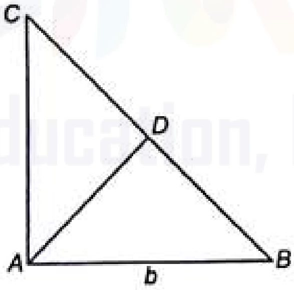
$$AN^2 + CM^2 = AB^2 + \frac{AB^2}{4} + BC^2 + \frac{BC^2}{4}$$

$$= \frac{5(AB^2 + BC^2)}{4}$$

$$4(AN^2 + CM^2) = 5\,AC^2$$

71. In ΔABC,

$$A = \frac{1}{2} \times \text{base} \times \text{altitude} = \frac{1}{2} b \times AC$$



$$AC = \frac{2A}{b}$$

Using Pythagorus theorem,

$$AC^2 + AB^2 = BC^2$$

$$\Rightarrow \quad BC = \sqrt{\frac{4A^2}{b^2} + b^2}$$

Again in $\Delta ABC, A = \frac{1}{2} \times BC \times AD$

$$\Rightarrow \quad AD = \frac{2A}{\sqrt{\frac{4A^2 + b^4}{b^2}}} = \frac{2Ab}{\sqrt{4A^2 + b^4}}$$

72. Both A and R are individually true and R is the correct explanation of A. (by property)

73. A triangle can be constructed by given all three statements.

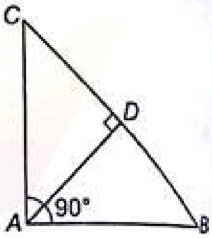
74. Given, $BD : CD = 3 : 4$

and $AB : AC = 6 : 8$ or $3 : 4$

So, in $\Delta ABC, AD$ is a angular bisector.

Hence, both A and R are true and R is the correct explanation of A.

75. Let $\angle C = \theta$, then $\angle B = 90° - \theta$



In ΔADC,

$$\angle D = 90°, \angle C = \theta$$

and $\angle CAD = 90° - \theta$

In ΔADB,

$$\angle D = 90°, \angle B = 90° - \theta, \angle DAB = \theta$$

and in ΔABC,

$$\angle C = \theta, \angle B = 90° - \theta, \angle A = 90°$$

76. Given that, $PQ = 4$ cm, $QR = 3$ cm, $RP = 3.5$ cm and $EF = 9$ cm

$$\because \quad \Delta PQR \sim \Delta DEF$$

$$\therefore \quad \frac{PQ}{DE} = \frac{QR}{EF} = \frac{RP}{FD}$$

$$\Rightarrow \quad \frac{PQ}{DE} = \frac{QR}{EF} \Rightarrow \frac{4}{DE} = \frac{3}{9}$$

$$\Rightarrow \quad DE = 12\text{cm and } \frac{PQ}{DE} = \frac{RP}{FD}$$

$$\Rightarrow \quad \frac{4}{12} = \frac{3.5}{FD} \Rightarrow FD = 10.5 \text{ cm}$$

$\therefore$ Perimeter of $\Delta DEF = DE + EF + FD$

$$= 12 + 9 + 10.5 = 31.5 \text{ cm}$$

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