





- c) A is true but R is false. d) A is false but R is true.
14. **Assertion (A):** A gas has a unique value of specific heat. [1]  
**Reason (R):** Specific heat is defined as the amount of heat required to raise the temperature of unit mass of the substance through unit degree.
- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false. d) A is false but R is true.
15. **Assertion:** An artificial satellite is moving in a circular orbit of the earth. If the gravitational pull suddenly disappears, then it moves with the same speed tangential to the original orbit. [1]  
**Reason:** The orbital speed of a satellite decreases with the increase in radius of the orbit.
- a) Assertion and reason both are correct statements and reason is correct explanation for assertion. b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
- c) Assertion is correct statement but reason is wrong statement. d) Assertion is wrong statement but reason is correct statement.
16. **Assertion (A):** Two particle of different mass, projected with same velocity, the maximum height attained by both the particle will be same. [1]  
**Reason (R):** The maximum height of projectile is independent of particle mass. is equal to maximum height attained by projectile.
- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false. d) A is false but R is true.

### Section B

17. Define the term wave motion. Give four important characteristics of wave motion. [2]
18. Assuming that the critical velocity  $v_c$  of a viscous liquid flowing through a capillary tube depends only upon the radius  $r$  of the tube, density  $\rho$  and the coefficient of viscosity  $\eta$  of the liquid, find the expression for critical velocity. [2]
19. Calculate the dimensions of force and impulse taking velocity, density and frequency as basic quantities. [2]
20. A rocket with a lift-off mass 20,000 kg is blasted upwards with an initial acceleration of  $5.0 \text{ ms}^{-2}$ . Calculate the initial thrust (force) of the blast. [2]
21. Calculate the energy required to move an earth satellite of mass  $10^3 \text{ kg}$  from a circular orbit of radius  $2R$  to that of radius  $3R$ . Given mass of the earth,  $M = 5.98 \times 10^{24} \text{ kg}$  and radius of the earth,  $R = 6.37 \times 10^6 \text{ m}$ . [2]

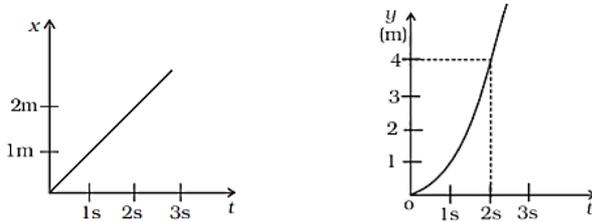
OR

If the mass of the sun is  $2 \times 10^{30} \text{ kg}$ , the distance of the earth from the sun is  $1.5 \times 10^{11} \text{ m}$  and period of revolution of the earth around the sun is one year ( $= 365.3 \text{ days}$ ), calculate the value of gravitational constant.

### Section C

22. What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature? Surface tension of mercury at that temperature ( $20^\circ\text{C}$ ) is  $4.65 \times 10^{-1} \text{ Nm}^{-1}$ . The atmospheric pressure is  $1.01 \times 10^5 \text{ Pa}$ . Also give the excess pressure inside the drop. [3]

23. A 20 cm thick layer of ice has been formed on the surface of a freshwater lake during extreme winter. The temperature of the air is  $-10^{\circ}\text{C}$ . Find how long will it take for another 1mm layer of water to freeze? Thermal conductivity of ice =  $2.1 \text{ W m}^{-1} \text{ K}^{-1}$ , latent heat of fusion of water =  $3.36 \times 10^5 \text{ J kg}^{-1}$  and density of ice =  $10^3 \text{ kg m}^{-3}$ . [3]
24. A stone is dropped from the top of a cliff and is found to travel 44.1m during the last second before it reaches the ground. What is the height of the cliff?  $g = 9.8\text{m/s}^2$ . [3]
25. Figure shows (x, t), (y, t) diagram of a particle moving in 2-dimensions. [3]

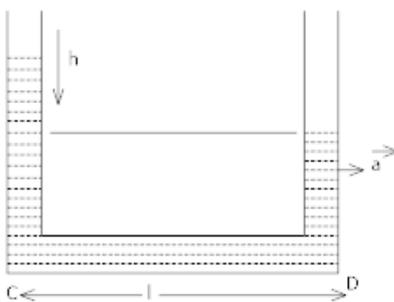


If the particle has a mass of 500 g, find the force (direction and magnitude) acting on the particle.

26. The efficiency of a Carnot engine is  $1/2$ . If the sink temperature is reduced by  $100^{\circ}\text{C}$ , then engine efficiency becomes  $2/3$ . Find [3]
- sink temperature
  - source temperature
  - Explain, why a Carnot engine cannot have 100% efficiency?
27. A batsman deflects a ball by an angle of  $45^{\circ}$  without changing its initial speed which is equal to 54 km/h. What is the impulse imparted to the ball? (Mass of the ball is 0.15 kg.) [3]
28. If a number of little droplets of water, each of radius  $r$ , coalesce to form a single drop of radius  $R$ , show that the rise in temperature will be given by  $\Delta\theta = \frac{3\sigma}{J} \left( \frac{1}{r} - \frac{1}{R} \right)$  where  $\sigma$  is the surface tension of water and  $J$  is the mechanical equivalent of heat. [3]

OR

A liquid stands at the same level in the U - tube when at rest. If  $A$  is the area of cross section of tube and  $g$  is the acceleration due to gravity, what will be the difference in height of the liquid in the two limbs when the system is given acceleration 'a'?

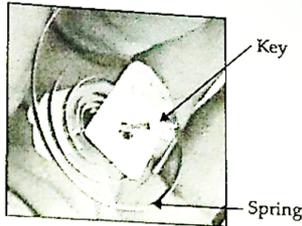


#### Section D

29. Read the text carefully and answer the questions: [4]
- Clockwork refers to the inner workings of mechanical clock or watch (where it is known as "movement") and different types of toys which work using a series of gears driven by a spring. Clockwork device is completely mechanical and its essential parts are:
- A key (or crown) which you wind to add energy
  - A spiral spring in which the energy is stored

- A set of gears through which the spring's energy is released. The gears control how quickly (or slowly) a clockwork machine can do things. Such as in mechanical clock/watch the mechanism is the set of hands that sweep around the dial to tell the time. In a clockwork car toy, the gears drive the wheels.

Winding the clockwork with the key means tightening a sturdy metal spring, called the mainspring. It is the process of storing potential energy. Clockwork springs are usually twists of thick steel, so tightening them (forcing the spring to occupy a much smaller space) is actually quite hard work. With each turn of the key, fingers do work and potential energy is stored in the spring. The amount of energy stored depends on the size and tension of the spring. Harder a spring is to turn and longer it is wound, the more energy it stores.



While the spring uncoils, the potential energy is converted into kinetic energy through gears, cams, cranks and shafts which allow wheels to move faster or slower. In an ancient clock, gears transform the speed of a rotating shaft so that it drives the second hand at one speed, the minute hand at  $\frac{1}{60}$  of that speed, and the hour hand at  $\frac{1}{3600}$  of that speed. Clockwork toy cars often use gears to make themselves race along at surprising speed.

- (a) What is the meaning of **movement** of old age mechanical clocks?
- |  |  |
|--|--|
| a) The pendulum of the clock   | b) The gears which move the hands of the clock |
| c) A spring and combination of gears which move the hands of the clock | d) The hands of the clock                      |
- (b) What type of energy is stored in the spring while winding it?
- |                               |            |
|-------------------------------|------------|
| a) Potential                  | b) Heat    |
| c) Both kinetic and potential | d) Kinetic |
- (c) When the spring of a clockwork uncoils
- |  |  |
|--|--|
| a) Kinetic energy is converted into potential energy               | b) Potential energy is converted into kinetic                    |
| c) Potential energy is converted into heat, light and sound energy | d) Kinetic energy is converted into heat, light and sound energy |

**OR**

In clockwork devices, \_\_\_\_\_ transform the speed of a rotating \_\_\_\_\_ to drive wheels slower or faster.

- |                  |                 |
|------------------|-----------------|
| a) Shaft, spring | b) shaft, gear  |
| c) Gear, Shaft   | d) Spring, gear |
- (d) More energy is stored in a spring if the
- |   |   |
|---|---|
| a) Spring is larger, harder and wound fur a longer time | b) Spring is smaller, harder and wound for a shorter time |
| c) Spring is larger, harder and wound for a             | d) Spring is larger, softer and wound for a               |



Draw a graph to show the variation of P.E., K.E. and total energy of a simple harmonic oscillator with displacement.

32. A projectile is fired horizontally with a velocity of  $98 \text{ ms}^{-1}$  from the hill 490 m high. Find [5]
- time taken to reach the ground
  - the distance of the target from the hill and
  - the velocity with which the projectile strikes the ground.

OR

A projectile is projected horizontally with a velocity  $u$ . Show that its trajectory is parabolic. And obtain expressions for:

- Time of flight
  - Horizontal range
  - Velocity at any instant  $t$ .
33. Obtain an expression for the linear acceleration of a cylinder rolling down an inclined plane and hence find the condition for the cylinder to roll down without slipping. [5]

OR

Two particles each of mass  $m$  and speed  $v$  travel in opposite direction along parallel lines, separated by a distance  $d$ . Show that vector angular momentum of the two particles system is same whatever be the point about which angular momentum is taken.

# Solution

## Section A

- (c) Derived units  
**Explanation:** Derived units are units which may be expressed in terms of base units by means of mathematical symbols of multiplication and division.
- (a)  $3 \times \frac{330}{4}$  Hz  
**Explanation:** Frequency of second note of a closed pipe,  $\nu = \frac{3v}{4L}$   
 $= \frac{3 \times 330}{4 \times 1}$  Hz
- (d) 1.0  
**Explanation:** Moment of inertia  
 $I = mk^2$   
 $m = 2$  Kg  
 $k = 0.5$  m  
 $I = 2 \times 0.5 \times 0.5 = 0.5 \text{Kg m}^2$   
angular momentum  
 $L = I\omega$   
 $\omega = 2$  rad/s  
 $L = 0.5 \times 2 = 1.0 \text{Kg m}^2/\text{s}$
- (b)  $90^\circ$   
**Explanation:**  $h = \frac{2\sigma \cos \theta}{r\rho g}$   
When  $\theta = 90^\circ$ ,  $h = 0$
- (a)  $\text{KE} = \frac{GmM_e}{2(R_e+h)}$   
**Explanation:** KE of satellite  $= \frac{1}{2}mv^2$   
 $= \frac{1}{2}m \left( \sqrt{\frac{GM_e}{(R_e+h)}} \right)^2$   
 $= \frac{1}{2} \frac{GmM_e}{(R_e+h)}$
- (c)  $\lambda = 18$  m  
**Explanation:** Given equation is  $y = 4 \sin \pi \left[ \frac{t}{5} - \frac{x}{9} + \frac{\pi}{6} \right]$   
 $= 4 \sin \left[ \frac{\pi t}{5} - \frac{\pi x}{9} + \frac{\pi^2}{6} \right]$   
Standard equation of wave is  
 $y = r \sin \left[ \frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \phi_0 \right]$   
comparing these equations we get,  $\frac{2\pi}{\lambda} = \frac{-\pi}{9}$   
or  $\lambda = 18$ m
- (b) 50 m/s  
**Explanation:**  $a = \frac{dv}{dt} = 2t + 5$   
 $\int_0^v dv = \int_0^5 (2t + 5) dt$   
 $v = [t^2]_0^5 + 5[t]_0^5$   
 $= (25 - 0) + 5(5 - 0)$   
 $= 50$  m/s

8.

(c)  $\sqrt{T}$

**Explanation:**  $v = \sqrt{\frac{T}{m}}$

$\therefore v \propto \sqrt{T}$

9.

(b) Statement (ii) is correct.

**Explanation:** Both velocity and direction of flow remain same.

10.

(c)  $\frac{1}{11}m$

**Explanation:** At  $\frac{1}{11}m$  from smaller body the intensity of gravitation field is zero.

11.

(c) disc

**Explanation:** Loss in (translational K.E. + rotational K.E.) = Gain in P.E.

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh_{\max}$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = mg \times \frac{3v^2}{4g} = \frac{3}{4}mv^2$$

$$\Rightarrow I = \frac{1}{2}mR^2$$

12.

(d)  $35^\circ\text{C}$

**Explanation:**  $C = \frac{5}{9}(F - 32)$

$$= \frac{5}{9}(95 - 32) = 35^\circ\text{C}$$

13.

(c) A is true but R is false.

**Explanation:** Work done over a closed path is always zero since the displacement is zero. Hence the assertion is true. To move a particle force is always required, whether conservative or non-conservative. So, the reason is false.

14.

(d) A is false but R is true.

**Explanation:** This is because a gas can be heated under different conditions of pressure and volume. The amount of heat required to raise the temperature of unit mass through unit degree is different under different conditions of heating.

15.

(b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

**Explanation:** Assertion and reason both are correct statements but reason is not correct explanation for assertion.

16.

(a) Both A and R are true and R is the correct explanation of A.

**Explanation:** The maximum height to which a projectile rises above the point of projection is,  $H = \frac{u^2 \sin^2 \theta}{2g}$ , which is independent of mass.

### Section B

17. Wave motion is a form of disturbance which travels through a medium due to repeated periodic motion of the particles of the medium about their mean positions.

Four important characteristics of wave motion are:

- i. Wave motion is a form of disturbance which travels through a medium due to the vibrations of the particles of the medium.
- ii. It is the disturbance which travels in the forward direction and not the particles. The particles simply vibrate about their mean position.
- iii. The energy of a particle is wholly kinetic at the mean position and wholly potential at the extreme position.
- iv. The motion of each particle begins a little later than that of its predecessor. In other words, there is always a constant phase difference between any two neighbouring particles. The wave always advances in that direction in which it meets particles with decreasing phase.

18. Velocity of viscous liquid is flowing through a capillary tube depends on radius  $r$  of tube, density  $\rho$ , and coefficient of viscosity of the liquid.

Let the critical velocity is proportional to  $r^a, \rho^b, \eta^n$ .

Then,

$V = kr^a, \rho^b, \eta^n$ ,  $k$  is any dimension less constant.

Thus, Dimension of  $V = LT^{-1}$

Dimension of  $r = L$

Dimension of  $\rho = ML^{-3}$

Dimension of  $\eta = ML^{-1}T^{-1}$

Now, Putting dimension on both sides of equation,

$$[LT^{-1}] = [L]^a[ML^{-3}]^b[ML^{-1}T^{-1}]^n$$

$$[M^0LT^{-1}] = [M^{b+n}][L^{a-n-3b}][T^{-n}]$$

On comparing,

$$n = 1, a - n - 3b = 1 \text{ and } b + n = 0$$

$$b = -n = -1$$

Now, in  $a - n - 3b = 1$ ,

$$a - 1 - 3(-1) = 1$$

$$a + 2 = 1$$

$$a = -1$$

$$V = \frac{k\eta}{r^2\rho}$$

19.  $v = LT^{-1}, \rho = ML^{-3}, \nu = T^{-1}$

Solving for  $M, L$  and  $T$  in terms of  $v, \rho$  and  $\nu$ , we get

$$T = v^{-1}, L = v\nu^{-1}, M = \rho v^3 \nu^{-3}$$

$$[\text{Force}] = MLT^{-2} = \rho v^3 \nu^{-3} v \nu^{-1} v^2 = \rho v^4 \nu^{-2}$$

$$[\text{Impulse}] = \text{Force} \times \text{time} = \rho v^4 \nu^{-2} v^{-1} = \rho v^4 \nu^{-3}$$

20. Here, lift-off mass  $m = 20,000$  kg and  $a = 5.0$  m s<sup>-2</sup> vertically upward.

Using Newton's second of motion, the net force (thrust) acting on the rocket is given by the relation:

$$F - mg = ma$$

$$F = mg + ma = m(g + a)$$

$$= 20,000(10 + 5)$$

$$= 20,000 \times 15$$

$$= 3 \times 10^5 \text{ N}$$

21. Total energy of a satellite,

$$E = \text{P.E.} + \text{K.E.} = -\frac{GMm}{r} + \frac{1}{2}mv^2$$

$$= -\frac{GMm}{r} + \frac{1}{2}m \cdot \frac{GM}{r} = -\frac{GMm}{2r}$$

$$W = E_f - E_i = -\frac{GMm}{2 \times 3R} + \frac{GMm}{2 \times 2R} = \frac{GMm}{12R}$$

$$= \frac{6.67 \times 10^{11} \times 5.98 \times 10^{24} \times 10^3}{12 \times 6.37 \times 10^6}$$

$$= 5.02 \times 10^9 \text{ J}$$

OR

$$G \text{ (gravitational constant)} = 6.673 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

$$M \text{ (mass of sun)} = 2 \times 10^{30} \text{ kg}$$

$$m \text{ (mass of earth)} = 6 \times 10^{24} \text{ kg}$$

$$d \text{ (average distance between sun and earth)} = 1.5 \times 10^8 \text{ km} = 1.5 \times 10^8 \times 1000 = 1.5 \times 10^{11} \text{ m}$$

$$F = \frac{GMm}{d^2}$$

$$= \frac{6.673 \times 10^{-11} \times 2 \times 10^{30} \times 6 \times 10^{24}}{(1.5 \times 10^{11})^2}$$

$$= \frac{6.673 \times 10^{-11} \times 2 \times 10^{30} \times 6 \times 10^{24}}{1.5 \times 1.5 \times 10^{22}} (10^{24+30-11} = 10^{43})$$

$$= \frac{6.673 \times 2 \times 6 \times 10^{43}}{1.5 \times 1.5 \times 10^{22}} (10^{43-22} = 10^{21})$$

$$= \frac{6.673 \times 2 \times 6 \times 10^{21}}{1.5 \times 1.5}$$

$$= \frac{80.076 \times 10^{21}}{1.5 \times 1.5}$$

$$= 35.59 \times 10^{21} \text{ N} = 3.57 \times 10^{22} \text{ N}$$

$$\text{Force of gravity} = 3.57 \times 10^{22} \text{ N}$$

Section C

22. Radius of mercury drop  $r = 3.00 \text{ mm} = 3 \times 10^{-3} \text{ m}$

surface tension of mercury  $S = 4.65 \times 10^{-1} \text{ Nm}^{-1}$

atmospheric pressure  $P_{\text{atm}} = 1.01 \times 10^5 \text{ pa}$

Total pressure ( $P_{\text{total}}$ ) inside the mercury drop = excess pressure inside mercury + atmospheric pressure

equation for excess pressure inside mercury  $P = p_i - p_0 = \frac{2S}{r}$

thus  $P_{\text{total}} = P + P_{\text{atm}} = \frac{2S}{r} + P_{\text{atm}}$

$$P_{\text{total}} = \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} + (1.01 \times 10^5)$$

$$P_{\text{total}} = 1.0131 \times 10^5 \text{ Pa}$$

$$\text{excess pressure } P = \frac{2S}{r} = \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}}$$

$$P = 310 \text{ Pa}$$

23. Given: Thickness of ice layer formed,  $l = 20 \text{ cm} = 0.2 \text{ m}$ ,

Temperature just below ice layer (in the lake)  $T_1 = 0^\circ\text{C}$ ,

Temperature of air (above ice layer)  $T_2 = -10^\circ\text{C}$ ,

Thermal conductivity of ice  $K = 2.1 \text{ W m}^{-1}\text{K}^{-1}$ ,

Latent heat of fusion of water  $L_f = 3.36 \times 10^5 \text{ J kg}^{-1}$  and ice density  $\rho = 10^3 \text{ kg m}^{-3}$ .

Let  $A$  be the surface area of the lake that takes time  $t$  for the formation of ice layer by  $\Delta l = 1 \text{ mm} = 10^{-3} \text{ m}$

$\therefore$  Mass of water formed in time  $t$ ,  $m = A \cdot \Delta l \cdot \rho = A \times 10^{-3} \times 10^3 = A \text{ kg}$

From the heat energy relation,  $H = \frac{\Delta Q}{\Delta t} = \frac{KA(T_1 - T_2)}{l}$ , we have

$$\frac{mL_f}{t} = \frac{AL_f}{t} = \frac{KA(T_1 - T_2)}{l}$$

$$\text{or } t = \frac{L_f \cdot l}{K(T_1 - T_2)} = \frac{3.36 \times 10^5 \times 0.2}{2.1 \times [0 - (-10)]} = \frac{3.36 \times 10^5 \times 0.2}{2.1 \times 10}$$

$$= 3200 \text{ s} = 53 \text{ min } 20 \text{ s.}$$

So it would take 53 min 20 sec to form the layer of ice.

24. Let  $h$  be the height of the cliff and  $n$  be the total time taken by the stone while falling. As

$$u = 0$$

$$a = g = 9.8 \text{ m/s}^2$$

$$S_{nth} = u + \frac{a}{2}(2n - 1)$$

$$44.1 = 0 + \frac{9.8}{2}(2n - 1)$$

$$88.2 = 9.8(2n - 1)$$

$$2n - 1 = 9$$

$$n = \frac{10}{2} = 5 \text{ s}$$

for Height of the cliff using equation

$$h = ut + \frac{1}{2}at^2$$

$$h = un + \frac{1}{2}gn^2$$

$$h = 0 \times 5 + \frac{1}{2} \times 9.8 \times (5)^2$$

$$h = 4.9 \times 25$$

$$h = 122.5 \text{ m}$$

25. From graph (a)  $x = t$

$$v_x = \frac{dx}{dt} = 1 \text{ ms}^{-1}$$

$$a_x = \frac{d^2x}{dt^2} = \frac{dv_x}{dt} = 0$$

From graph (b)  $y = t^2$

$$v_y = \frac{dy}{dt} = 2t$$

$$a_y = \frac{dv_y}{dt} = 2$$

$$a_x = 0, a_y = 2 \text{ m s}^{-1}$$

Here,  $m = 500 \text{ g} = 0.5 \text{ kg}$

$$F_x = 0.5 \times 0 = 0 \text{ N}$$

$$F_y = ma_y = 0.5 \times 2 = 1 \text{ N toward Y -axis,}$$

$$\text{Hence, resultant force acting on particle is } F = \sqrt{F_x^2 + F_y^2} = \sqrt{0^2 + 1^2} = \sqrt{1}$$

$\therefore F = 1\text{N}$  along y-axis.

26. i. Efficiency,  $\eta = 1 - \frac{T_2}{T_1}$

where,  $T_2$  = sink temperature

$T_1$  = source temperature.

$$1 - \frac{T_2}{T_1} = \frac{1}{2} \dots\dots\dots(i)$$

$$1 - \left(\frac{T_2-100}{T_1}\right) = \frac{2}{3} \dots\dots(ii)$$

From Eq. (i),  $\frac{T_2}{T_1} = \frac{1}{2}$  and Eq. (ii)

$$\frac{T_2-100}{T_1} = \frac{1}{3}$$

On dividing,

$$\frac{T_2}{T_2-100} = \frac{3}{2} \Rightarrow T_2 = 300\text{K}$$

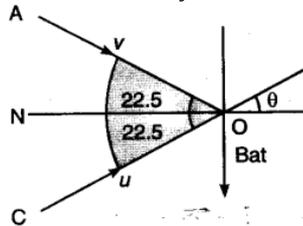
ii. Substituting in eq(i),  $T_1 = 600\text{K}$

iii. As efficiency,  $\eta_2 \Rightarrow 1 - \frac{T_2}{T_1}$

It equals to 1 only when  $\frac{T_2}{T_1} = 0$  or  $T_2 = 0\text{K}$

But absolute zero is not possible.

27. The ball struck by the bat is deflected back such that the total angle is  $45^\circ$ .



Now, initial momentum of ball =  $mu \cos \theta$

$$= \frac{0.15 \times 54 \times 1000 \times \cos 22.5}{3600}$$

$$= 0.15 \times 15 \times 0.9239 \text{ along ON}$$

Final momentum of ball =  $mucos \theta$  along ON

Impulse = change in momentum

$$= mucos \theta - (-mucos \theta)$$

$$= 2mucos \theta$$

$$= 2 \times 0.15 \times 15 \times 0.9239$$

$$\text{i.e., Impulse} = 4.16 \text{ kg ms}^{-1}$$

28. Let n be the number of little droplets which coalesce to form a single drop. Then

The volume of n little droplets = Volume of a single drop

$$\text{or } n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \text{ or } nr^3 = R^3$$

Decrease in surface area =  $n \times 4\pi r^2 - 4\pi R^2$

$$= 4\pi [nr^2 - R^2] = 4\pi \left[\frac{nr^3}{r} - R^2\right]$$

$$= 4\pi \left[\frac{R^3}{r} - R^2\right] = 4\pi R^3 \left[\frac{1}{r} - \frac{1}{R}\right] \quad [\because nr^3 = R^3]$$

Energy evolved,

$W$  = Surface tension  $\times$  decrease in surface area

$$= 4\pi\sigma R^3 \left[\frac{1}{r} - \frac{1}{R}\right]$$

Heat produced,

$$Q = \frac{W}{J} = \frac{4\pi\sigma R^3}{J} \left[\frac{1}{r} - \frac{1}{R}\right]$$

But  $Q = ms\Delta\theta$

$$= \text{Volume of single drop} \times \text{density of water} \times \text{specific heat of water} \times \Delta\theta$$

$$= \frac{4}{3}\pi R^3 \times 1 \times 1 \times \Delta\theta$$

Hence

$$\frac{4}{3}\pi R^3 \Delta\theta = \frac{4\pi\sigma R^3}{J} \left[ \frac{1}{r} - \frac{1}{R} \right]$$

$$\text{or } \Delta\theta = \frac{3\sigma}{J} \left[ \frac{1}{r} - \frac{1}{R} \right]$$

OR

Let  $l$  = Length of the horizontal portion of tube. Mass of liquid in the portion CD = Volume  $\times$  Density

Let  $P$  = Density of water

Volume = Area  $\times$  Length

$A$  = Area of cross – section of tube.

=  $a \times l$

So, Mass of liquid in portion CD =  $(Al) \times \rho = Alp$

Force on the above Mass towards left =  $M \times \bar{a}$

$\bar{a}$  = acceleration

Force =  $Alp \times \bar{a}$  ... (i)

Also, due to difference in height of liquid, the downward force exerted on liquid in the horizontal portion CD

$\Rightarrow$  Pressure =  $\frac{\text{force}}{\text{Area}}$

Pressure =  $h\rho g$

$h$  = height;  $\rho$  = Density;  $g$  = acceleration due to gravity

So, Force = Pressure  $\times$  Area

Force =  $h\rho g \times A$  ... (ii)

Equating equation (i) and equation (ii) for force on C D :

$Alp \times a = h\rho g \times A$

$h = \frac{al}{g}$

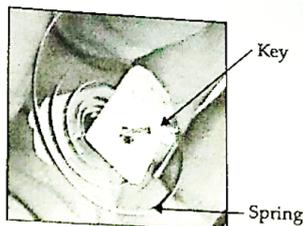
### Section D

#### 29. Read the text carefully and answer the questions:

Clockwork refers to the inner workings of mechanical clock or watch (where it is known as "movement") and different types of toys which work using a series of gears driven by a spring. Clockwork device is completely mechanical and its essential parts are:

- A key (or crown) which you wind to add energy
- A spiral spring in which the energy is stored
- A set of gears through which the spring's energy is released. The gears control how quickly (or slowly) a clockwork machine can do things. Such as in mechanical clock/watch the mechanism is the set of hands that sweep around the dial to tell the time. In a clockwork car toy, the gears drive the wheels.

Winding the clockwork with the key means tightening a sturdy metal spring, called the mainspring. It is the process of storing potential energy. Clockwork springs are usually twists of thick steel, so tightening them (forcing the spring to occupy a much smaller space) is actually quite hard work. With each turn of the key, fingers do work and potential energy is stored in the spring. The amount of energy stored depends on the size and tension of the spring. Harder a spring is to turn and longer it is wound, the more energy it stores.



While the spring uncoils, the potential energy is converted into kinetic energy through gears, cams, cranks and shafts which allow wheels to move faster or slower. In an ancient clock, gears transform the speed of a rotating shaft so that it drives the second hand at one speed, the minute hand at  $\frac{1}{60}$  of that speed, and the hour hand at  $\frac{1}{3600}$  of that speed. Clockwork toy cars often use gears to make themselves race along at surprising speed.

- (i) (c) A spring and combination of gears which move the hands of the clock

**Explanation:** Movement refers to the inner workings of mechanical clock using a series of gears driven by a spring.

- (ii) (a) Potential

**Explanation:** Winding the spring means tightening a sturdy metal spring. It is the process of storing potential energy (forcing the spring to occupy a much smaller space) is actually quite hard work. With each turn of the key, fingers do work and potential energy is stored in the spring.

- (iii) (b) Potential energy is converted into kinetic

**Explanation:** When the spring uncoils, the potential energy is converted into kinetic energy through gears, cams, cranks and shafts which allow wheels to move faster or slower.

OR

- (c) Gear, Shaft

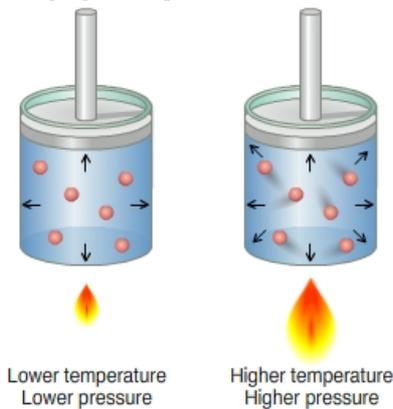
**Explanation:** In an ancient clock, gears transform the speed of a rotating shaft so that it drives the second hand at one speed, the minute hand at  $\frac{1}{60}$  of that speed, and the hour hand at  $\frac{1}{3600}$  of that speed. Clockwork toy cars often use gears to make themselves race along at surprising speed.

- (iv) (a) Spring is larger, harder and wound for a longer time

**Explanation:** With each turn of the key, fingers do work and potential energy is stored in the spring. The amount of energy stored depends on the size and tension of the spring. Harder a spring is to turn and longer it is wound, the more energy it stores.

**30. Read the text carefully and answer the questions:**

In a gas the particles are always in a state of random motion, all the particles move at different speed constantly colliding and changing their speed and direction, as speed increases it will result in an increase in its kinetic energy.



- (i) (b) becomes double

**Explanation:** becomes double

- (ii) (d) Zero

**Explanation:** Zero

- (iii) (d) remains same

**Explanation:** remains same

- (iv) (a) 1:1

**Explanation:** 1:1

OR

- (c) 4.08 v

**Explanation:** 4.08 v

**Section E**

31. Mass of the automobile is given by,  $m = 3000 \text{ kg}$

Displacement in the suspension system is given by,  $x = 15 \text{ cm} = 0.15 \text{ m}$

There are 4 springs in parallel to the support of the mass of the automobile.

The equation for the restoring force for the system is given by:

$$F = -4kx = mg$$

Where,  $k$  is the spring constant of the suspension system

$$\text{Time period, } T = 2\pi\sqrt{\frac{m}{4k}}$$

$$\text{And } k = \frac{mg}{4x} = \frac{3000 \times 10}{4 \times 0.15} = 5000 = 5 \times 10^4 \text{ N/m}$$

$$\text{Spring constant, } k = 5 \times 10^4 \text{ N/m}$$

- a. Each wheel supports a mass is given by,  $M = \frac{3000}{4} = 750 \text{ kg}$

For damping factor  $b$ , the equation for displacement is written as:

$$x = x_0 e^{-bt/2M}$$

The amplitude of oscillation decreases by 50%.

$$\begin{aligned} \therefore x &= \frac{x_0}{2} \\ \frac{x_0}{2} &= x_0 e^{-bt/2M} \\ \log_e 2 &= \frac{bt}{2M} \\ \therefore b &= \frac{2M \log_e 2}{t} \end{aligned}$$

Where,

$$\text{Time period is given by, } t = 2\pi \sqrt{\frac{m}{4k}} = 2\pi \sqrt{\frac{3000}{4 \times 5 \times 10^4}} = 0.7691 \text{ s}$$

$$\begin{aligned} \therefore b &= \frac{2 \times 750 \times 0.693}{0.7691} \\ &= 1351.58 \text{ kg/s} \end{aligned}$$

Therefore, the damping constant of the spring is given by 1351.58 kg/s.

OR

The potential energy (PE) of a simple harmonic oscillator is

$$PE = \frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 x^2 \dots (i)$$

When, PE is plotted against displacement x, we will obtain a parabola.

When x = 0, PE = 0

$$\begin{aligned} \text{When } x &= \pm A, PE = \text{maximum} \\ &= \frac{1}{2} m\omega^2 A^2 \end{aligned}$$

The kinetic energy (KE) of a simple harmonic oscillator  $KE = \frac{1}{2} mv^2$

$$\text{But velocity of oscillator } v = \omega \sqrt{A^2 - x^2}$$

$$\Rightarrow KE = \frac{1}{2} m[\omega \sqrt{A^2 - x^2}]^2$$

$$\text{or } KE = \frac{1}{2} m\omega^2 (A^2 - x^2) \dots (ii)$$

This is also parabola, if we plot KE against displacement x

i.e. KE = 0 at x = ±A

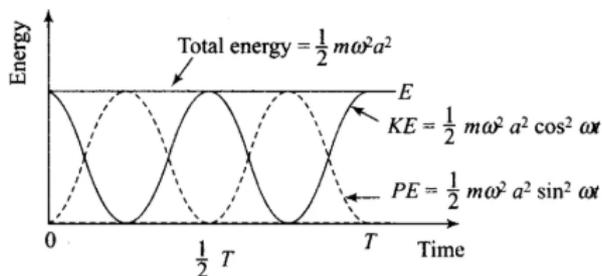
$$\text{and } KE = \frac{1}{2} m\omega^2 A^2 \text{ at } x = 0$$

Now, total energy of the simple harmonic oscillator = PE + KE [using Eqs. (i) and (ii)]

$$\begin{aligned} &= \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m\omega^2 (A^2 - x^2) \\ &= \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m\omega^2 A^2 - \frac{1}{2} m\omega^2 x^2 \\ TE &= \frac{1}{2} m\omega^2 A^2 = \text{constant} \end{aligned}$$

which is a constant and independent of x.

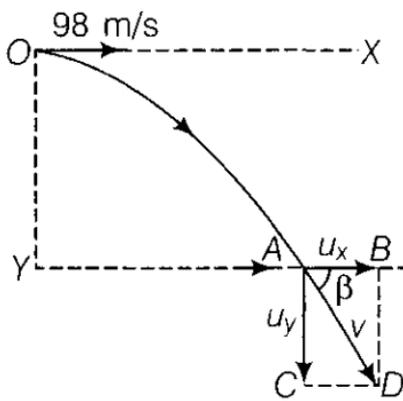
Plotting under the above guidelines KE, PE and TE versus displacement x-graph as follows:



**Important point:** From the graph, we note that potential energy or kinetic energy completes two vibrations in a time during which S.H.M. completes one vibration.

Thus the frequency of potential energy or kinetic energy is double that of S.H.M.

32. From the given figure, YO = 490 m. A body projected horizontally from O with velocity  $u = 98 \text{ ms}^{-1}$  hits the ground at position A following a parabolic path as shown in the figure.



i. Let  $T$  be the time of flight of the projectile.

Taking vertical downward motion of projectile from  $O$  to  $A$ , we have

$$y_0 = 0, y = 490 \text{ m}, u_y = 0, a_y = 9.8 \text{ m/s}^2, t = T$$

$$\text{From equation of kinematics, } y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow 490 = 0 + 0 \times T + \frac{1}{2} \times 9.8 \times T^2 = 4.9T^2$$

$$\text{or } T = \sqrt{\frac{490}{4.9}} = 10 \text{ s}$$

ii. Taking horizontal motion (i.e. motion along  $OX$  axis) of projectile from  $O$  to  $A$ , we have

$$x_0 = 0, x = R \text{ (say)}, u_x = 98 \text{ m/s}, t = T = 10 \text{ s}, a_x = 0 \text{ (as there is no acceleration along horizontal)}$$

$$\text{As, } x = x_0 + u_x t + \frac{1}{2} a_x t^2$$

$$\therefore R = 0 + 98 \times 10 + \frac{1}{2} \times 0 \times 10^2 = 980 \text{ m}$$

iii. Let  $v_x, v_y$  be the horizontal and vertical component velocity of the projectile at point  $A$ .

Using the relation,  $v_x = u_x + a_x t = 98 + 0 \times 10 = 98 \text{ m/s}$ , which is represented by  $AB$ .

Similarly,  $v_y = u_y + a_y t = 0 + 9.8 \times 10 = 98 \text{ m/s}$  as represented by  $AC$

$\therefore$  The magnitude of the resultant velocity is given by

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{98^2 + 98^2} = 98\sqrt{2} \text{ m/s}$$

And the direction of the resultant velocity is given by

$$\tan \beta = \frac{v_y}{v_x} = \frac{98}{98} = 1 \text{ or } \beta = 45^\circ \text{ with the horizontal.}$$

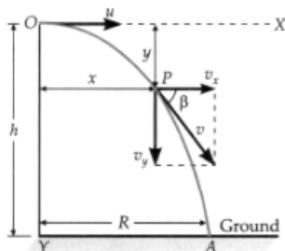
OR

**Projectile fired parallel to horizontal.** As shown in figure, suppose a body is projected horizontally with velocity  $u$  from a point  $O$  at a certain height  $h$  above the ground level. The body is under the influence of two simultaneous independent motions:

i. Uniform horizontal velocity  $u$ .

ii. Vertically downward accelerated motion with constant acceleration  $g$ .

Under the combined effect of the above two motions, the body moves along the path  $OPA$ .



**Horizontal projection of a projectile.**

Trajectory of the projectile. After the time  $t$ , suppose the body reaches the point  $P(x, y)$ .

The horizontal distance covered by the body in time  $t$  is

$$x = ut \therefore t = \frac{x}{u}$$

The vertical distance travelled by the body in time  $t$  is given by

$$s = ut + \frac{1}{2} at^2$$

$$\text{or } y = 0 \times t + \frac{1}{2} gt^2 = \frac{1}{2} gt^2 \text{ [For vertical motion, } u = 0 \text{]}$$

$$\text{or } y = \frac{1}{2} g \left( \frac{x}{u} \right)^2 = \left( \frac{g}{2u^2} \right) x^2 \text{ [} \therefore t = \frac{x}{u} \text{]}$$

or  $y = kx^2$  [Here  $k = \frac{g}{2u^2} = \text{a constant}$ ]

As  $y$  is a quadratic function of  $x$ , so the trajectory of the projectile is a parabola.

**Time of flight.** It is the total time for which the projectile remains in its flight (from O to A). Let  $T$  be its time of flight.

For the vertical downward motion of the body, we use

$$s = ut + \frac{1}{2}at^2 \text{ or } h = 0 \times T + \frac{1}{2}gT^2$$

$$\text{or } T = \sqrt{\frac{2h}{g}}$$

**Horizontal range.** It is the horizontal distance covered by the projectile during its time of flight. It is equal to  $OA = R$ . Thus

$$R = \text{Horizontal velocity} \times \text{time of flight} = u \times T$$

$$\text{or } R = u\sqrt{\frac{2h}{g}}$$

**Velocity of the projectile at any instant.** At the instant  $t$  (when the body is at point P), let the velocity of the projectile be  $v$ . The velocity  $v$  has two rectangular components:

Horizontal component of velocity,  $v_x = u$

Vertical component of velocity,  $v_y = 0 + gt = gt$

$\therefore$  The resultant velocity at point P is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2t^2}$$

If the velocity  $v$  makes an angle  $\beta$  with the horizontal, then

$$\tan \beta = \frac{v_y}{v_x} = \frac{gt}{u}$$

$$\text{or } \beta = \tan^{-1}\left(\frac{gt}{u}\right)$$

33. Let  $M \rightarrow$  mass of the solid cylinder  $R \rightarrow$  Radius of the cylinder,  $\theta \rightarrow$  Angle of inclination of the plane,  $\omega \rightarrow$

Angular acceleration of the cylinder.

Various forces acting on the cylinder are:

$R - mg \cos \omega$ ,  $F =$  Frictional force acting upwards

$\omega =$  Net force in the downward directions  $= f = ma = mg \sin \omega - F$

The torque required for the rolling motion of the cylinder is due to frictional forces, which acts tangential to the surface.

$$\tau = r \times F \text{ Also } \tau = I\alpha = \frac{Ia}{r} [\because a = r\alpha], \frac{Ia}{r} = rF \therefore F = \frac{Ia}{r^2}$$

$$ma - mg \sin \theta - \frac{Ia}{r^2} \rightarrow a \left( m + \frac{I}{r^2} \right) - mg \sin \theta$$

$$\rightarrow a - \frac{mg \sin \theta}{m + \frac{I}{r^2}}$$

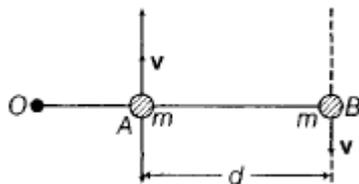
$$\text{For a solid cylinder, M.I. about the axis } I = \frac{Mr^2}{2} \therefore a = \frac{mg \sin \theta}{m + \frac{mr^2}{2} \times \frac{1}{r^2}} = \frac{2}{3}g \sin \theta$$

$$\text{Frictional force} = F = \frac{Ia}{r^2} = \frac{I}{r^2} \cdot \frac{2}{3}g \sin \theta = \frac{1}{3}mg \sin \theta$$

$$\text{Coefficient of friction} = \frac{F}{R} = \frac{\frac{1}{3}mg \sin \theta}{mg \cos \theta} \Rightarrow \mu = \frac{1}{3} \tan \theta$$

OR

Suppose, O be the origin chosen.



Then, angular momentum of particle at A is

$$I_1 = OA \times p = OA \times mv$$

$$= m(OA \times v)$$

and angular momentum of particle at B is

$$I_2 = OB \times p = OB \times (-mv)$$

$$= -m(OB \times v)$$

so, total angular momentum of the system of particles is

$$L = I_1 + I_2$$

$$= m(OA \times v) - m(OB \times v)$$

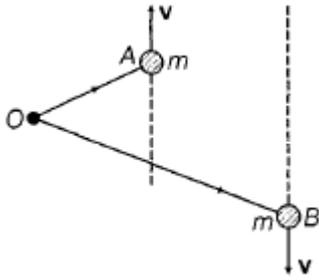
$$= m(OA - OB) \times v$$

$$= m(BA) \times v$$

$$= m(BA) \times v$$

{As, BA = position vector of A - position vector of B}

Above expression is independent of choice of origin.



This is true even when particles are not in a straight line.

$$L_i(I_1 + I_2 = m(OA \times v - OB \times v))$$

$$= m(BA) \times v$$

Which is the same as a previous result. So, the angular momentum of the system is independent of the choice of origin.