Polygons

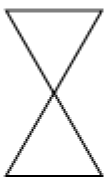
Polygons and Their Attributes

Let us discuss some examples based on polygon.

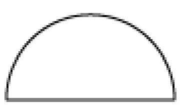
Example 1:

State whether each of the following curves is a polygon or not.

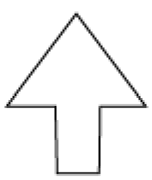
(a)



(b)



(c)



Solution:

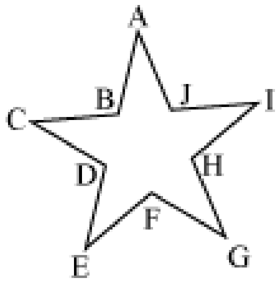
(a) A polygon is always a simple and closed curve, entirely made up of line segments. Since the given curve crosses itself, it is not a simple curve and thus, not a polygon.

(b) The given curve is not entirely made up of line segments. Therefore, it is not a polygon.

(c) The given curve is a simple and closed curve and is entirely made up of line segments. It is, thus, a polygon.

Example 2:

Answer the questions below with respect to the given figure.



(a) Name the vertices of the polygon.

(b) Name the adjacent sides of AJ, GF, and BC.

(c) Name the adjacent vertices of A, G, and E.

Solution:

(a) The point where two sides of a polygon meet is called its vertex. The vertices of the polygon are A, B, C, D, E, F, G, H, I, and J.

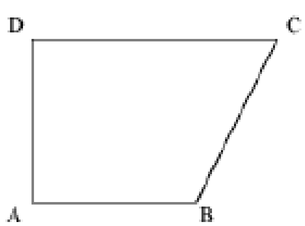
(b) Any two sides of a polygon with a common vertex are called adjacent sides. Thus, the adjacent sides of AJ are AB and IJ; that of GF are GH and EF; and that of BC are AB and CD.

(c) The vertices of a polygon that lie on the same side are called adjacent vertices. Thus, the adjacent vertices of A are B and J; that of G are H and F; and that of E are D and F.

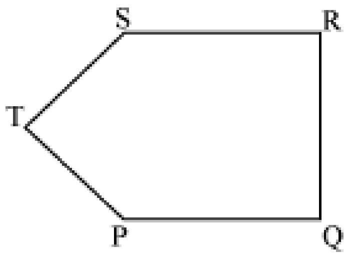
Example 3:

Name all the sides of the following polygons. Also, draw and count all the possible number of diagonals.

(a)

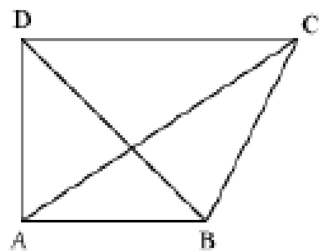


(b)

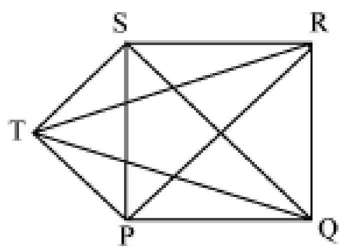


Solution:

(a) The sides of the given polygon are AB, BC, CD, and DA. It has two diagonals, AC and BD.

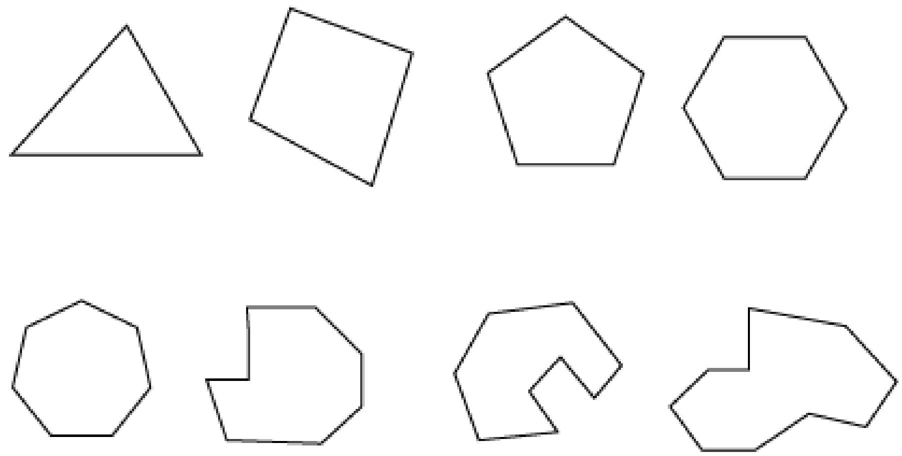


(b) The sides of the given polygon are PQ, QR, RS, ST, and TP. It has five diagonals, PR, PS, QS, QT, and RT.



Classification of Polygons on the Basis of Their Sides

Look at the following figures.



What do we observe in these figures?

We observe that each figure is made up of line segments only and has different number of sides. All these figures are known as **polygons.** We know that polygons with three sides are known as ***triangles*** and polygons with 4 sides are known as ***quadrilaterals.***

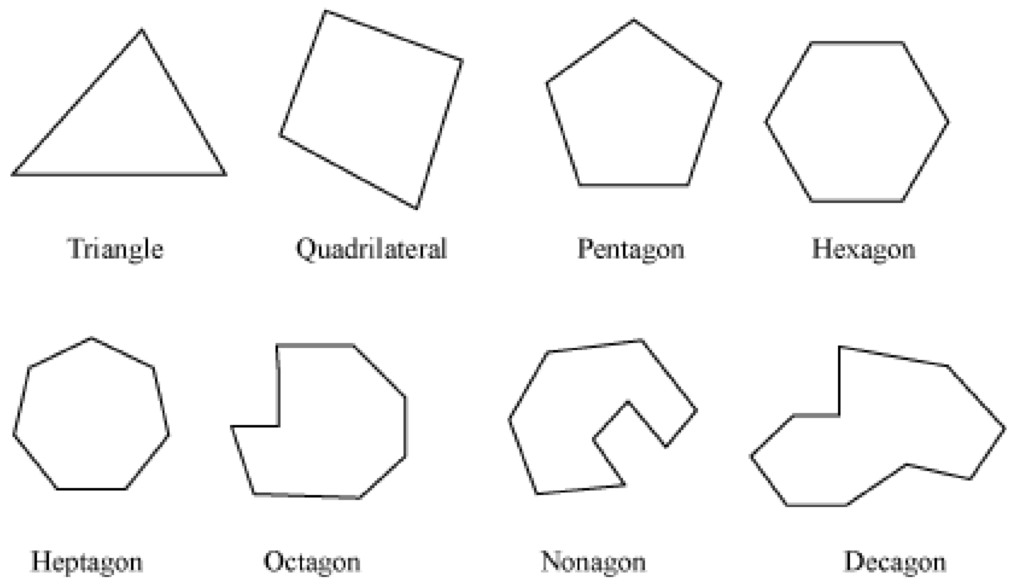
But how do we classify the polygons having more than four sides?

Let us see.

We can classify polygons on the basis of the number of sides as follows:

- **Pentagon:** Polygon having five sides
- **Hexagon:** Polygon having six sides
- **Heptagon:** Polygon having seven sides
- **Octagon:** Polygon having eight sides
- **Nonagon:** Polygon having nine sides
- **Decagon**: Polygon having ten sides

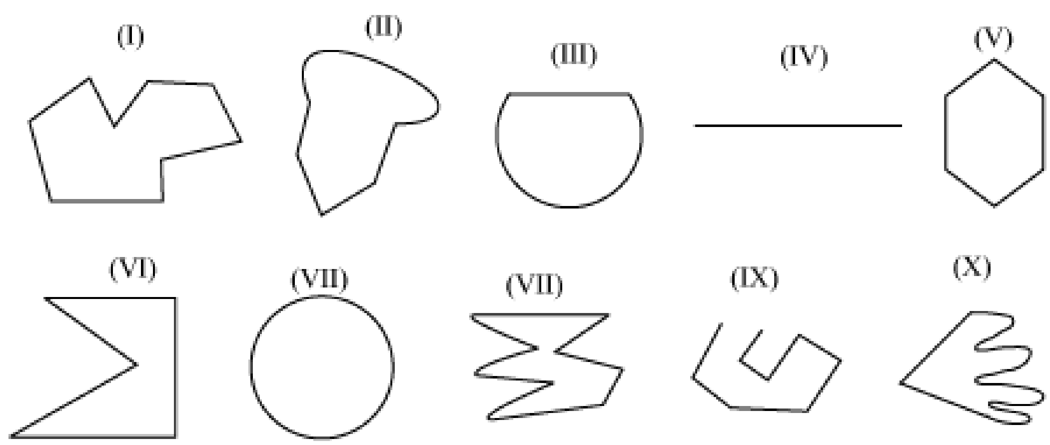
Therefore, now we can classify the polygons in the above given figures.



Let us now look at some more examples to understand this concept better.

Example 1:

Identify and name the polygons out of the following figures.



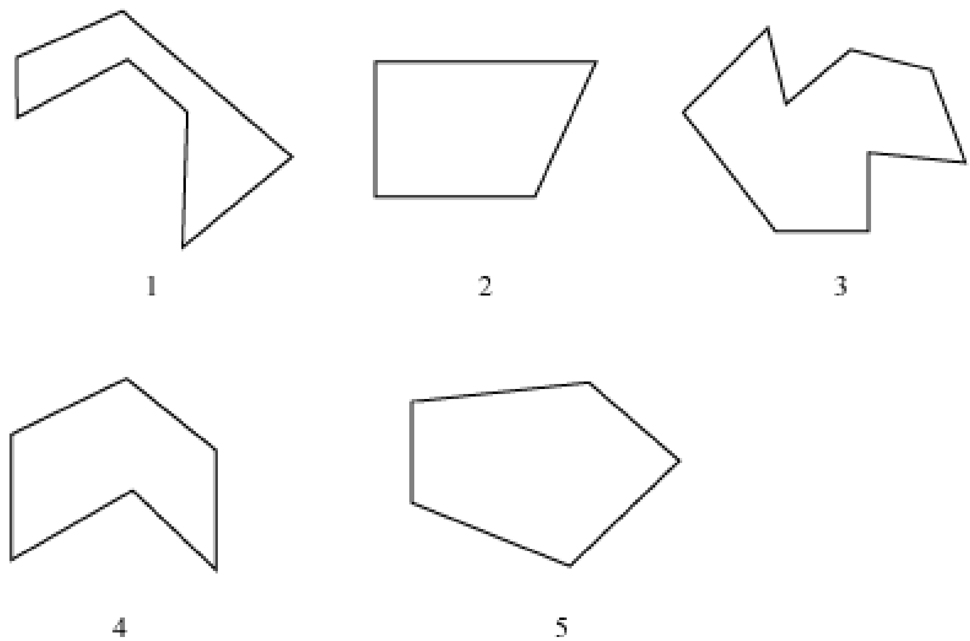
Solution:

1. The closed figure is made of nine line segments. Therefore, it is a nonagon.
2. The closed figure has a curve. Therefore, it is not a polygon.
3. The closed figure has a curve. Therefore, it is not a polygon.
4. The figure has only one-line segment. A polygon should have at least three line segments. Therefore, it is not a polygon.
5. The closed figure is made of six line segments. Therefore, it is a hexagon.
6. The closed figure is made of five line segments. Therefore, it is a pentagon.

7. The closed figure is a curve. Therefore, it is not a polygon.
8. The closed figure is made of nine line segments. Therefore, it is a nonagon.
9. The figure is not closed. Therefore, it cannot be a polygon.
10. The closed figure has curves. Therefore, it is not a polygon.

Example 2:

Name the following polygons.



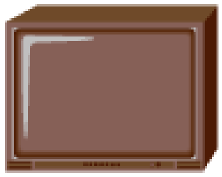
Solution:

1. The given figure has seven sides. Therefore, it is a heptagon.
2. The given figure has four sides. Therefore, it is a quadrilateral.
3. The given figure has nine sides. Therefore, it is a nonagon.
4. The given figure has six sides. Therefore, it is a hexagon.
5. The given figure has five sides. Therefore, it is a pentagon.

Example 3:

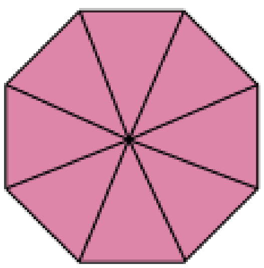
Write the number of sides and the types of polygons represented by the following figures.

(i)



(ii)

(iii)



Solution:

1. This figure has four sides. Therefore, it is a quadrilateral.
2. This figure has three sides. Therefore, it is a triangle.
3. This figure has eight sides. Therefore, it is an octagon.

Angle Sum Property Of Polygons

Let us suppose that we have a quadrilateral and we want to find the sum of all the interior angles made by its sides.

One simple way to find the sum of the angles is to find the measure of the angles and then add them. But how will we find its angles?

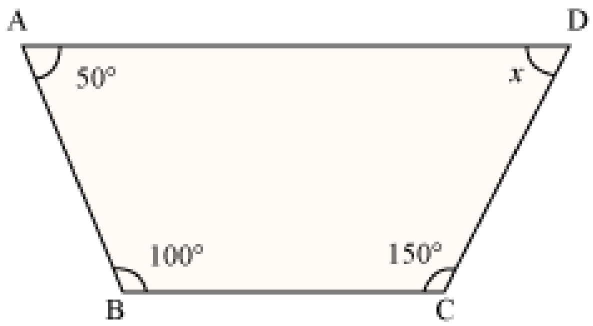
Is it possible to find the sum of all the angles of a quadrilateral without finding the measure of each angle? Is the sum of the interior angles of every quadrilateral same?

Let us find out the answers to these questions.

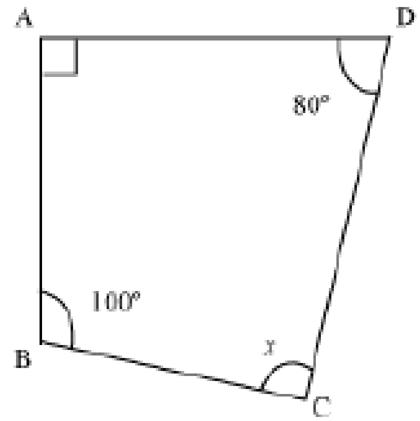
Let us solve some examples now.

Example:

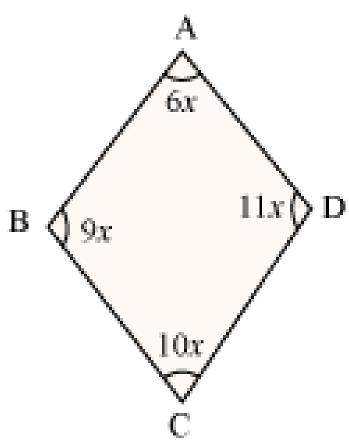
Find the value of *x* in the following figures.



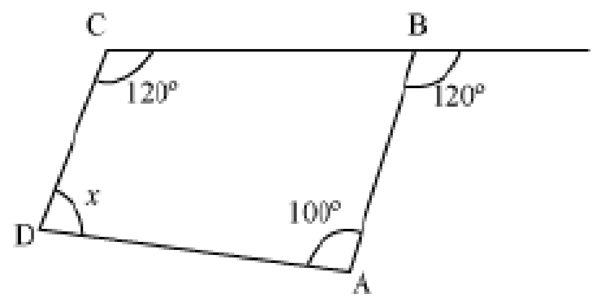
(a)



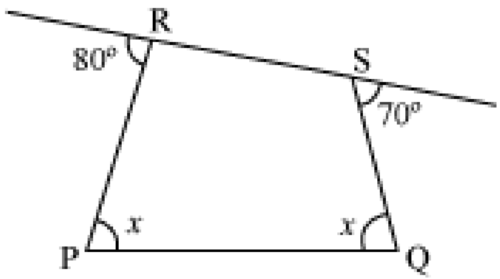
(b)



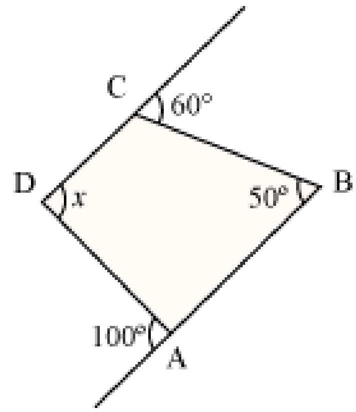
(c)



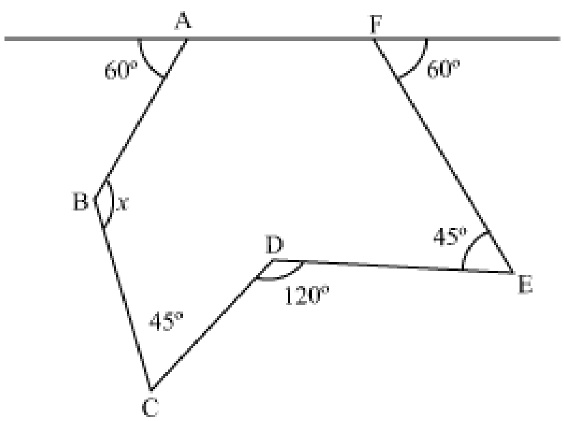
(d)



(e)



(f)



(g)

Solution:

(a) The sum of all the interior angles of a quadrilateral is 360°.

Therefore, from the figure,

$100° + 150° + x + 50° = 360°$

$\Rightarrow 300° + x = 360°$

$\Rightarrow x = 360° - 300°$

$\Rightarrow x = 60°$

(b) The sum of all the interior angles of a quadrilateral is 360°.

Therefore,

$90° + 80° + x + 100° = 360°$

$\Rightarrow 270° + x = 360°$

$\Rightarrow x = 360° - 270°$

$\Rightarrow x = 90°$

(c) The sum of all the interior angles of a quadrilateral is 360°. Therefore, from the figure,

$9x + 6x + 11x + 10x = 360°$

$\Rightarrow 36x = 360°$

On dividing both sides by 36, we obtain $x = 10°$

Thus, the angles of the quadrilateral are 90°, 60°, 110°, and 100°.

(d) The sum of the angles which forms a linear pair is 180°.

$\therefore \angle ABC + 120° = 180°$

$\Rightarrow \angle ABC = 180° - 120° = 60°$

Also, the sum of all the interior angles of a quadrilateral is 360°. Therefore,

$x + 100° + 60° + 120° = 360°$

$\Rightarrow x + 280° = 360°$

$\Rightarrow x = 360° - 280°$

$\Rightarrow x = 80°$

(e) The sum of the adjacent angles on a straight line is 180°.

$\therefore \angle PSR + 80° = 180°$

$\Rightarrow \angle PSR = 180° - 80°$

$\Rightarrow \angle PSR = 100°$

Also, $\angle SRQ + 70° = 180°$

$\Rightarrow \angle SRQ = 180° - 70°$

$\Rightarrow \angle SRQ = 110°$

The sum of all the interior angles of a quadrilateral is 360°. Therefore,

$100° + 110° + x + x = 360°$

$\Rightarrow 210° + 2x = 360°$

$\Rightarrow 2x = 360° - 210°$

$\Rightarrow 2x = 150°$

$\Rightarrow x = 75°$

(f) The sum of the angles which forms a linear pair is 180°.

$\therefore 100° + \angle DAB = 180°$

$\Rightarrow \angle DAB = 180° - 100°$

$\Rightarrow \angle DAB = 80°$

Similarly, $\angle DCB + 60° = 180°$

$\Rightarrow \angle DCB = 180° - 60°$

$\Rightarrow \angle DCB = 120°$

Now, the sum of all the interior angles of a quadrilateral is 360°. Therefore,

$x + 80° + 50° + 120° = 360°$

$\Rightarrow x + 250° = 360°$

$\Rightarrow x = 360° - 250°$

$\Rightarrow x = 110°$

(g) $\angle$BAF = 180° − 60° = 120°

Similarly, $\angle$AFE = 120°

$\angle$CDE = 360° − 120° = 240°

The polygon ABCDEF is a hexagon.

∴ Sum of all the interior angles of a hexagon = 180° × (6 − 2)

= 180° × 4

= 720°

$\therefore x = 720° - (120° + 120° + 45° + 240° + 45°)$

$\Rightarrow x = 720° - 570°$

$\Rightarrow x = 150°$

Exterior Angle Sum Property of Polygons

Let us consider a quadrilateral. We know that the sum of all the interior angles of a quadrilateral is 360°.

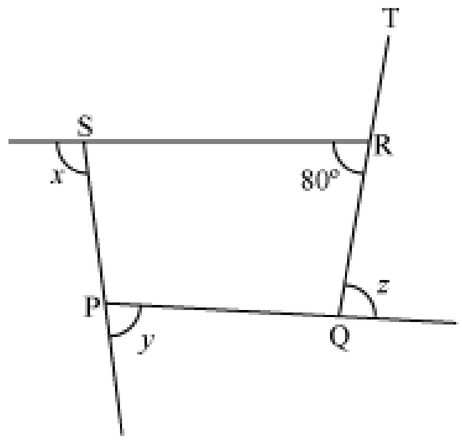
But if we want to find the sum of all the exterior angles of a quadrilateral, then how will we proceed?

In order to understand the answer to this question, let's look at the following video.

Let us look at some examples now.

Example 1:

Find the value of ($x + y + z$) from the following figure.



Solution:

In the given figure, we can see that ∠QRS and ∠SRT form linear pair of angles.

Therefore, their sum should be 180°.

Thus, we obtain

$80° + \angle SRT = 180°$

$\Rightarrow \angle SRT = 180° – 80°$

$= 100°$

The sum of the exterior angles of a quadrilateral is 360°.

$\therefore x + y + z + 100° = 360°$

$\Rightarrow x + y + z = 360° – 100°$

$\Rightarrow x + y + z = 260°$

Example 2:

Find the number of sides of a regular polygon in which each exterior angle has a measure of 40°.

Solution:

The measure of all the exterior angles of a polygon is 360°. It is given that the measure of each exterior angle is 40° and the given polygon is a regular polygon, therefore all the exterior angles are same.

Therefore, number of exterior angles = $\frac{360°}{40°}$ = 9

Thus, the polygon has 9 sides.

Example 3:

Find the measure of each exterior angle of a regular polygon having 12 sides.

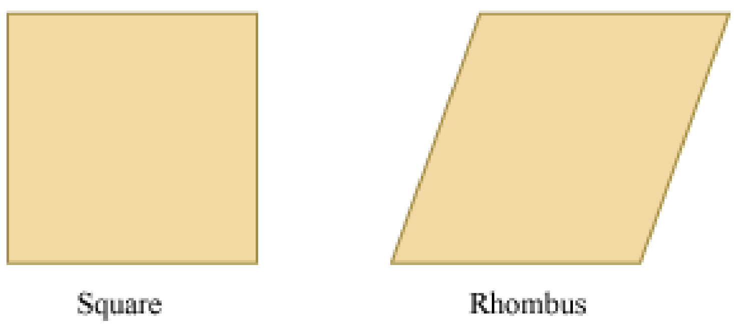
Solution:

The measure of all the exterior angles of a polygon is 360°. It is given that the polygon has 12 sides. Since it is a regular polygon, all its exterior angles are equal.

∴ Measure of each exterior angle = $\frac{360°}{12}$ = 30°

Classification of Polygons as Regular and Irregular

Let us consider a square and a rhombus.



What is the difference between the two figures?

We can see that in a square, all the sides are equal and all the angles are also of equal measure. On the other hand, in a rhombus, all sides are equal; however, the measures of all angles are not equal.

We thus say that a **square is a regular polygon** and a **rhombus is an irregular polygon**.

The **regular** and **irregular polygons** can be defined as follows.

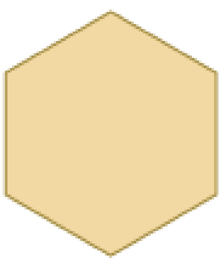
"Polygons in which all sides are of equal length and all interior angles of equal measure are known as regular polygons".

"Polygons in which all sides are not of equal length and all angles are not of equal measure are known as irregular polygons".

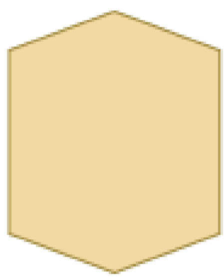
Let us see another example.

A **regular hexagon** has all sides of equal length. Moreover, all the angles are of equal measure 120°.

However, in case of an **irregular hexagon**, all the sides are not of equal length. Also, all the angles are not equal. A regular and an irregular hexagon are shown in the following figure.



Regular hexagon



Irregular hexagon

Formulas Related to Regular Polygons:

(i) The sum of the interior angles of an *n* sided polygon = (2n − 4) × 90°

where each interior angle = $\frac{(2n-4)\times 90^\circ}{n}$

(ii) A regular polygon has all its exterior angles equal.

The sum of its exterior angles = 360º

So, the sum of each exterior angle = 360°n

(iii) Number of sides of a regular polygon, n = $\frac{360^\circ}{\text{exterior angle}}$

Note: For a polygon, regardless of the fact whether it is regular or non-regular, at each vertex the sum of exterior and interior angle = 180°

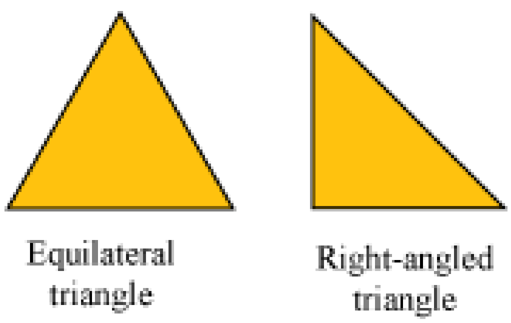
i.e Exterior angle + Interior angle = 180°

Let us now look at some more examples to understand this concept better.

Example 1:

Show that an equilateral triangle is a regular polygon and a right-angled triangle is an irregular polygon.

Solution:



An equilateral triangle is a regular polygon as all the sides of equilateral triangle are of equal length and all angles are of equal measure 60°.

In case of a right-angled triangle, neither all the sides are of equal length nor the measure of all angles are equal. Therefore, right-angled triangle is an example of irregular polygon.

Example 2:

Write the name of a regular polygon having

(i) 3 sides

(ii) 4 sides

Solution:

A regular polygon is a polygon in which all the sides are of equal length and all interior angles are of equal measure.

Therefore, a regular polygon having 3 sides is an equilateral triangle. A regular polygon having 4 sides is a square.