Sample Paper 15

Class- X Exam - 2022-23

Mathematics - Standard

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A **20 marks**

(Section A consists of 20 questions of 1 mark each.)

1. If $x = 2^2 \times 3^3 \times 7^2$, $y = 2^3 \times 3^2 \times 5 \times 7$, then HCF (x, y) is:

(a) 250 (b) 252

(c) 160 (d) 140 1

2. If the lines represented by $3x + 2py = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of p is:

(a) $\frac{15}{4}$ (b) 13

(c) 12 (d) $\frac{19}{2}$ 1

3. The value of x and y, $x + y = 3$ and $7x + 6y = 2$ are:

(a) $x = 16, y = 9$ (b) $x = 8, y = 5$

(c) $x = 2, y = 15$ (d) $x = -16, y = 19$ 1

4. Form a quadratic polynomial, whose zeros are –3 and 5.

(a) $x^2 - 2x - 15$ (b) $3x^2 + 5$

(c) $x^2 - 3x - 14$ (d) $x^2 + 2x + 15$ 1

5. The distance between the points (0, 5) and (–5, 0) is:

(a) $5\sqrt{2}$ (b) $3\sqrt{2}$

(c) $\sqrt{2}$ (d) 7 1

6. For what values of 'a', does the quadratic equation $x^2 - ax + 1 = 0$ not have real roots?

(a) (2, 1) (b) (0, 1)

(c) (– 2, 2) (d) (– 1, 1) 1

7. The perimeter of the triangle AOB with vertices A(4, 0), O(0,0) and B(0, 3) is:

(a) 10 units (b) 5 units

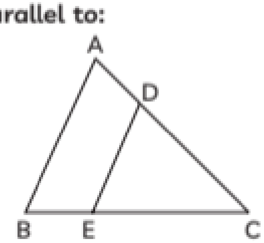
(c) 12 units (d) 4 units 1

8. If p and q are the roots of the quadratic equation $x^2 + px - q = 0$, then find the values of p and q.

(a) $p = 4, q = 5$ (b) $p = 3, q = -2$

(c) $p = 1, q = 1$ (d) $p = -1, q = 2$ 1

9. In the figure, if $\angle A = \angle B$ and AD = BE. Then DE is Parallel to:



(a) BE (b) EC

(c) AC (d) AB 1

10. The distance between two parallel tangents to a circle of radius 5 cm is:

(a) 5 cm (b) 8 cm

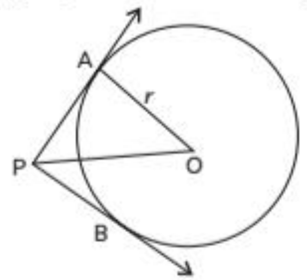
(c) 10 cm (d) 9 cm 1

11. The point (5, -3) lies in :

(a) 1^{st} quadrant (b) 2^{nd} quadrant

(c) 3^{rd} quadrant (d) 4^{th} quadrant 1

12. In the figure, $\angle APB = 90°$. The length of OP is:



(a) 2r (b) $\sqrt{3}r$

(c) 4r (d) $\sqrt{2}r$ 1

13. $\Delta ABC \sim \Delta DEF$ such that DE = 3 cm, EF = 2 cm, DF = 2.5 cm and BC = 4 cm. The perimeter of ΔABC is:

(a) 15 cm (b) 10 cm

(c) 9 cm (d) 8 cm 1

14. The angle of depression of a vehicle from 125 m high towers is 45°. How far is the vehicle from the tower then?

(a) 125 m (b) 60 m

(c) 75 m (d) 95 m 1

15. If cosec A – cot A = 13, then the value of cosec A + cot A is:

(a) $\frac{1}{12}$ (b) $\frac{2}{13}$

(c) $\frac{1}{14}$ (d) $\frac{1}{13}$ 1

16. If ΔABC is right-angled at C, then the value of cos (A + B) is:

(a) 0 (b) 2

(c) 1 (d) 4 1

17. A wire is in the shape of a circle of radius 21 cm. It is bent to form a square. The length of its side is:

(a) 30 cm (b) 15 cm

(c) 33 cm (d) 14 cm 1

18. If the areas of three adjacent faces of a cuboid are X, Y and Z respectively, then the volume of the cuboid is:

(a) XYZ (b) $\sqrt{XYZ}$

(c) $X\sqrt{YZ}$ (d) $Y\sqrt{XZ}$ 1

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of reason (R).
Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

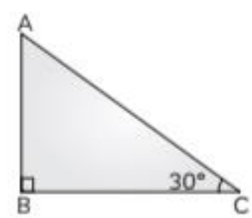
(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. **Statement A (Assertion):** The point (0, 6) lies on y-axis.

Statement R (Reason): The x-coordinate of the point on y-axis is zero. 1

20. **Statement A (Assertion):** Height AB in the figure is 11.56 metres if BC is 20 metres.



Statement R (Reason):

$$\tan \theta = \frac{AB}{BC} = \frac{\text{Perpendicular}}{\text{base}}$$

where θ the angle $\angle ACB$. 1

SECTION - B

10 marks

(Section B consists of 5 questions of 2 marks each.)

21. Given that $\sqrt{5}$ is an irrational number, prove that $3 + \sqrt{5}$ is irrational.

OR

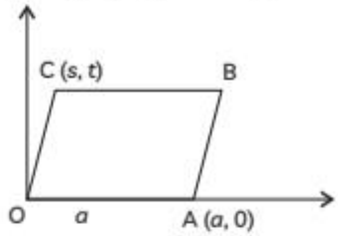
Show that, points (*a*, *b* + *c*), (*b*, *c* + *a*) and (*c*, *a* + *b*) are collinear. 2

22. Two circles with radii of 20 cm and 7 cm, respectively. Find the radius of the circle whose circumference equals the sum of the circumference of the two circles.

OR

The two opposite vertices of a square are (–1, 2) and (3, 2). Find the coordinates of the other two vertices. 2

23. In the figure, OABC is a rhombus, where O is the origin. The coordinates of A and C are $(a, 0)$ and (s, t) respectively.

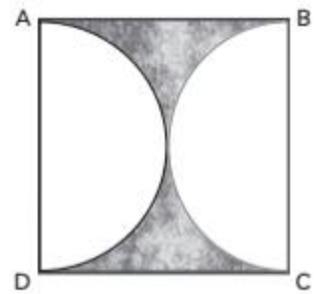


(A) Write down the coordinates of B in terms of a, s and t.

(B) Find the length of OC in terms of s and t. 2

24. What is the total surface area of a cuboid with dimensions of 6 cm in length, 2 cm in width, and 2 cm in height? 2

25. Find the area of the shaded region in the figure, if ABCD is a square of side 14 cm and APD and BPC are semi-circles.



2

SECTION - C **18 marks**

(Section C consists of 6 questions of 3 marks each.)

26. Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

OR

Which term of the AP: −2, −7, −12, ..., will be −77? Find the sum of this AP up to the term −77. 3

27. Determine the number of real roots of the equation: $(x^2 + 1)^2 - x^2 = 0$ 3

28. 5 books and 7 pens together cost ₹ 434, whereas 7 books and 5 pens together cost ₹ 550, find the total cost of 1 book and 2 pens. 3

29. Prove that:

$$\frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A} = 2\operatorname{cosec} A$$

OR

If $\sin\theta = \frac{12}{13}$, find the value of $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} - \frac{1}{\tan^2\theta}$. 3

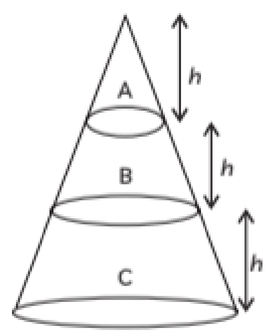
30. A carpenter cuts a wooden cone into three parts A, B and C by two planes parallel to the base as shown in the figure.

The heights of the three parts are equal.

(A) Find the ratio of the volumes of parts A, B and C

(B) Find the ratio of the base areas of parts A, B and C

(C) If the volume of the original cone is 540 cu cm, find the volume of part B.



3

31. Find the mean and median of the following data:

Age (in years)	less than 10	less than 20	less than 30	less than 40	less than 50
Frequency	5	15	20	24	30

3

SECTION - D **20 marks**

(Section D consists of 4 questions of 5 marks each.)

32. If the zeroes of polynomial $p(x) = ax^3 + 3bx^2 + 3cx + d$ are in A.P., then prove that $2b^3 - 3abc + a^2d = 0$.

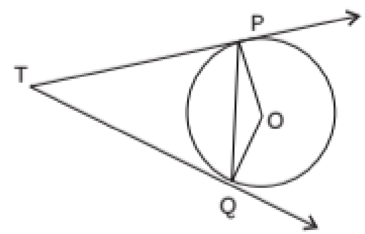
OR

From the top of a tower 'h' metres high, angle of depression of two objects which are in line with foot of the tower are a and b ($b > a$). Find the distance between the two objects. 5

33. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that:

(A) TP = TQ

(B) $\angle PTQ = 2\angle OPQ$



OR

Prove that the area of the semi-circle drawn on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the semi-circles drawn on the other two-sides of the triangle. 5

34. The following quadratic polynomials have zeroes; find them and confirm the relationship between the zeroes and the coefficients.

(A) $x^2 - 2x - 8$

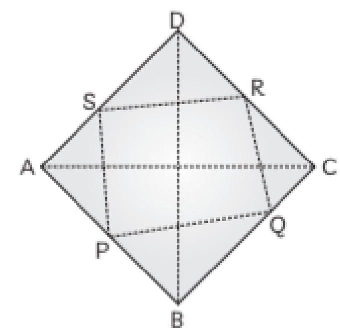
(B) $4s^2 - 4s + 1$

(C) $6x^2 - 3 - 7x$

(D) $4u^2 + 8u$

(E) $t^2 - 15$ 5

35. The midpoints of the sides AB, BC, CD, and DA in the quadrilateral ABCD are P,Q,R, and S, respectively. Show that PQRS is a parallelogram.



5

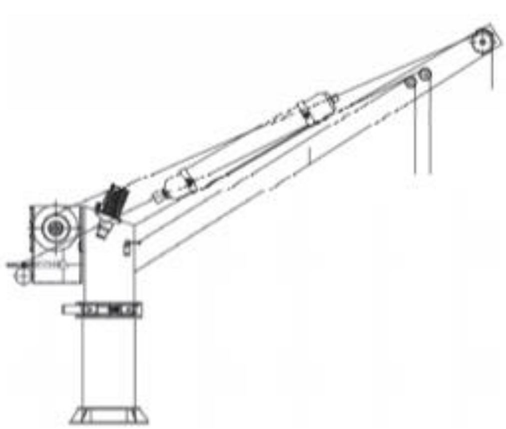
SECTION - E

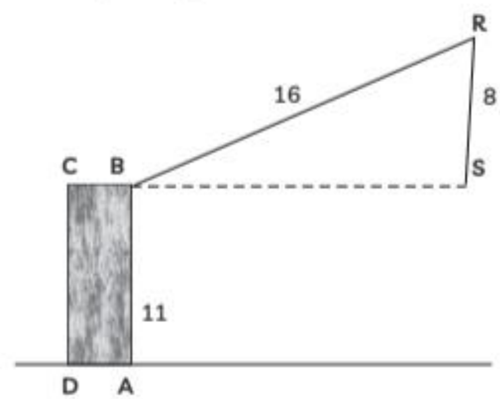
12 marks

(Case Study Based Questions)

(Section E consists of 3 questions. All are compulsory.)

36. *A crane stands on a level ground. It is represented by a tower ABCD, of height 11 m and a jib BR. The jib is of length 20 m and can rotate in a vertical plane about B. A vertical cable, RS, carries a load S. The diagram shows current position of the jib, cable and load.*



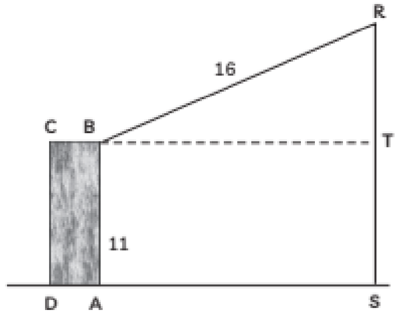


On the basis of the above information, answer the following questions:

(A) Find the length BS. 1

(B) Find the angle that the jib, BR, makes with the horizontal. 1

(C) In the diagram below, the jib BR, has been rotated and the length RS is increased. The load S is now on the ground at a point 8 m from A. Find the measure of the angle BRS and also find the angle through which the jib has been rotated.



OR

Find the length by which RS has increased. 2

37. *NITI aayog has tasked their statistical officer to create a model for farmers to be able to predict their produce output based on various factors.*

To test the model out, the officer picked a local farmer who sells apples to check various factors like weight, bad apples, half-cooked, green vs red etc.

A box containing 250 apples was opened and each apple was weighed.

The distribution of the masses of the apples is given in the following table:

Mass (in grams)	Frequency
80-100	20
100-120	60
120-140	70
140-160	p
160-180	60

On the basis of the above information, answer the following questions:

(A) Find the value of p. 1

(B) Find the mean mass of the apples. 1

(C) Find the upper limit of the median class.

OR

Find the modal mass of the apples. 2

38. *A ticket machine in a car park takes ₹ 1 coin and ₹ 2 coin. A ticket cost ₹ 3. The probability that the machine will accept a particular ₹ 1 coin is 0.9 and that it will accept a particular ₹ 2 coin is 0.8*

On the basis of the above information, answer the following questions:

(A) Find the probability that jayant will not get a ticket. 1

(B) Pritam has one ₹ 1 coins and two ₹ 2 coins. Find the probability that Pritam will get a ticket. 1

(C) Urmila put one ₹ 1 coin and one ₹ 2 coin into the machine. Find the probability that the machine will not accept either of these coins.

OR

Jayant only has three ₹ 1 coins. Find the probability that the machine accept all these coins. Also find the probability that the machine will not accept a particular ₹ 1 coin. 2

SOLUTION

SECTION - A

1. *(b) 252*

Explanation: HCF$(x, y) = 2^2 \times 3^2 \times 7$ i.e. 252

Caution

➥ *While calculating HCF, take all the common factors from the given numbers.*

2. *(a)* $\frac{15}{4}$

Explanation: Since the given lines are parallel,

So, $\frac{3}{2} = \frac{2p}{5} \neq \frac{2}{1}$

$\Rightarrow \quad p = \frac{15}{4}$

3. *(d)* $x = -16, y = 19$

Explanation: $x + y = 3$ gives, $y = 3 - x$ ...(i)

So, $7x + 6y = 2$ gives $7x + 6(3 - x) = 2$

$\Rightarrow \quad 7x + 18 - 6x = 2$

i.e. $\quad x = -16$

From (i), $\quad y = 3 + 16 = 19$

Thus, $x = -16$ and $y = 19$ is the required solution.

Caution

➥ *Derive the value of either x or y, but do which is more convenient and don't mess up the process.*

4. *(a)* $x^2 - 2x - 15$

Explanation: Since, $\alpha = -3$ and $\beta = 5$

Then, $\alpha + \beta = -3 + 5 = 2$

$\alpha\beta = -3 \times 5 = -15$

Hence, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - 2x - 15 = 0,$

is the required quadratic polynomial.

5. *(a)* $5\sqrt{2}$

Explanation:

Distance $= \sqrt{(-5 - 0)^2 + (0 - 5)^2}$

$= \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$

6. *(c) (–2, 2)*

Explanation: Given, equation is $x^2 - ax + 1 = 0$

On comparing the equation with $Ax^2 + Bx + C = 0$, we get

$A = 1, B = -a$ and $C = 1$

Then, $D = B^2 - 4AC$

$= (-a)^2 - 4 \times 1 \times 1$

$= a^2 - 4$

For no real roots

$D < 0$

Then, $a^2 - 4 < 0$

$a^2 < 4$

$-2 < a < 2$

7. *(c) 12 units*

Explanation: Given, coordinates of ΔAOB as $A(4, 0)$, $O(0, 0)$ and $B(0, 3)$

Perimeter of ΔAOB

= Length of AB + length of OA + length of OB

$= \sqrt{(4-0)^2+(0-3)^2} + \sqrt{(4-0)^2+(0-0)^2}$

$+\sqrt{(0-0)^2+(3-0)^2}$

$= \sqrt{16+9} + \sqrt{4^2} + \sqrt{3^2}$

$= 5 + 4 + 3 = 12$ units

8. *(d)* $p = -1, q = 2$

Explanation: Given, quadratic equation is

$x^2 + px - q = 0$

Here, sum of roots i.e,

$p + q = \frac{-p}{1}$

$\Rightarrow \quad 2p = -q$

and product of roots,

$pq = \frac{-q}{1}$

$\Rightarrow \quad p = -1$

Then, $q = 2$

Hence, the values of p and q are -1 and 2 respectively.

9. *(d) AB*

Explanation: In ΔABC, $\angle A = \angle B$ gives, $BC = CA$

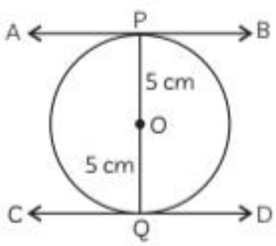
As $AD = BE$ so, $DC = CE$

$$\Rightarrow \quad \frac{CD}{AD} = \frac{CE}{BE}$$

Hence, by the converse of Thales' theorem $DE \parallel AB$.

10. *(c) 10 cm*

Explanation: Here, AB and CD are 2 parallel tangents to a circle with centre O.



Then, distance between two parallel tangents

$$= OP + OQ$$
$$= 5 + 5 = 10 \text{ cm}$$

11. *(d) 4^{th} quadrant*

Explanation:

First quadrant, X = positive and Y = positive

Second quadrant, X = negative and Y = positive

Third quadrant, X = negative and Y = negative

Fourth quadrant, X = positive and Y = negative

(5,–3) has X = 5, positive and Y = 3, negative

$\therefore$ The point lies in the fourth quadrant.

12. *(d) $\sqrt{2}r$*

Explanation: ΔOAP is a right triangle, right-angled at A and $\angle APO = 45°$

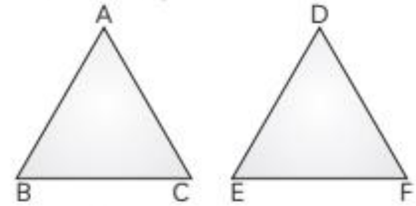
So, $$\frac{OA}{OP} = \sin 45°$$

$$\left[\because \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}\right]$$

$$\Rightarrow \quad OP = \frac{r}{\frac{1}{\sqrt{2}}} = \sqrt{2}\, r$$

13. *(a) 15 cm*

Explanation: Since, $\Delta ABC \sim \Delta DEF$



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\Rightarrow \quad \frac{AB}{3} = \frac{4}{2} = \frac{AC}{2.5}$$

Then, $AB = 6$ cm

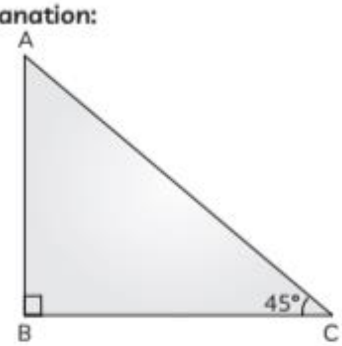
and $AC = 5$ cm

Then, perimeter of ΔABC

$$= AB + BC + AC$$
$$= 6 + 4 + 5$$
$$= 15 \text{ cm}$$

14. *(a) 125 m*

Explanation:



AB is the height of the tower which is 125 m.

C is the point at which the vehicle standing.

In ΔABC

$$\tan 45° = \frac{AB}{BC}$$

$$1 = \frac{AB}{BC}$$

$$AB = BC = 125 \text{ m}$$

Distance between Tower and vehicle is 125 m.

15. *(d) $\frac{1}{13}$*

Explanation: We know that:

$\operatorname{cosec}^2 A - \cot^2 A = 1$

i.e. $(\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A) = 1$

$\Rightarrow \operatorname{cosec} A + \cot A$

$$= \frac{1}{\operatorname{cosec} A - \cot A} = \frac{1}{13}$$

16. *(a) 0*

Explanation: In ΔABC,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + \angle B + 90^\circ = 180^\circ$$

$$\Rightarrow \angle A + \angle B = 90^\circ$$

Now, $\cos(A + B) = \cos 90^\circ = 0$

17. *(c) 33 cm*

Explanation: Circumference of circle = Perimeter of the square.

So, $2\pi r = 4a$

$$\Rightarrow 4a = 2 \times \frac{22}{7} \times 21$$

$$\Rightarrow 4a = 132$$

$$\Rightarrow a = 33$$

Hence, the length of each side of square is 33 cm.

OR

18. *(b)* $\sqrt{XYZ}$

Explanation:

Here, $X = lb$

$Y = bh$

$Z = hl$

where, l = length, b = breadth and h = height of cuboid.

Then, volume, $V = lbh$ ...(i)

Since, $XYZ = lb \times bh \times hl$

$= l^2b^2h^2$

$= (lbh)^2$

or $lbh = \sqrt{XYZ}$

Then $V = \sqrt{XYZ}$ [using (i)]

19. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)*

Explanation: We know that the if the point lies on y-axis, its x-coordinate is 0..

The x co-ordinate of the point (0, 6) is zero.

So, Point (0, 6) lies on y-axis.

20. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)*

Explanation: $\tan 30^\circ = \frac{AB}{BC} = \frac{AB}{20}$

$$AB = \frac{1}{\sqrt{3}} \times 20 = \frac{20}{1.73}$$

$$= 11.56 \text{ m}$$

SECTION - B

21. Suppose $3 + \sqrt{5}$ is a rational number.

Let $3 + \sqrt{5} = p$

$$\therefore \quad 3 = p - \sqrt{5}$$

$$\Rightarrow 9 = (p - \sqrt{5})^2 = p^2 + 5 - 2\sqrt{5}p$$

$$\Rightarrow 2\sqrt{5}p = p^2 - 4$$

$$\Rightarrow \sqrt{5} = \frac{p^2 - 4}{2p},$$

which is a contradiction because left hand side is an irrational number and right hand side is a rational number.

Hence, $3 + \sqrt{5}$ is an irrational number.

OR

We need to show:

$a[(c + a) - (a + b)] + b[(a + b) - (b + c)] + c[(b + c) - (c + a)] = 0$

Consider,

$$a[(c + a) - (a + b)] + b[(a + b) - (b + c)] + c[(b + c) - (c + a)]$$

$$= a[c - b] + b[a - c] + c[b - a]$$

$$= ac - ab + ba - bc + cb - ca$$

$$= 0$$

Thus, the given points are collinear.

Caution

Remember that three or more points are said to be collinear if they lie on the same straight line.

22. Let R be the radius of the circle which has circumference equal to the sum of circumferences of the two circles.

Then according to question,

Radius (r_1) of the 1st circle = 20 cm

Radius (r_2) of the 2nd circle = 7 cm

Let the radius of the 3rd circle be R.

$$2\pi R = 2\pi(20) + 2\pi(7)$$

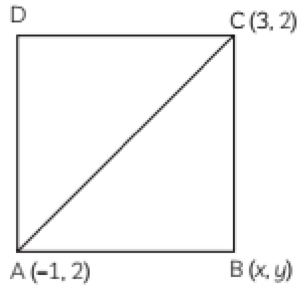
$$R = 20 + 7$$

$$R = 27 \text{ cm}$$

OR

Let ABCD be a square and let A(– 1, 2) and C(3, 2) be the given two vertices.

Also, let B(x, y) be the unknown vertex.



Then, AB = BC

$\Rightarrow (x+1)^2 + (y-2)^2 = (3-x)^2 + (2-y)^2$

$\Rightarrow x^2 + 2x + 1 + y^2 - 4y + 4 = 9 - 6x + x^2 + 4 + y^2 - 4y$

$\Rightarrow 2x + 1 - 4y + 4 = -6x + 9 - 4y + 4$

$\Rightarrow \quad 8x = 8$

$\Rightarrow \quad x = 1$

Further, in ΔABC,

$$AC^2 = AB^2 + BC^2$$

$\Rightarrow (3+1)^2 + (2-2)^2 = (x+1)^2 + (y-2)^2 + (3-x)^2 + (2-y)^2$

$\Rightarrow 16 = 2x^2 + 2y^2 + 2x + 1 - 4y + 4 - 6x + 9 - 4y + 4$

Using $x = 1$, we get:

$16 = 2 + 2y^2 + 2 + 1 - 4y + 4 - 6 + 9 - 4y + 4$

$\Rightarrow \quad 2y^2 - 8y = 0$

$\Rightarrow \quad 2y(y-4) = 0$

$\Rightarrow \quad y = 0$ or $y = 4$

Thus, the other two vertices are (1, 0) and (1, 4).

23. (A) Since OC || AB and CB || OA, the coordinates of B are $(s + a, t)$

(B) Length of OC

$$= \sqrt{(s-0)^2 + (t-0)^2}$$

$$= s^2 + t^2$$

Caution

➥ *The distance formula,*

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

It gives the same answer.

24. Given, the dimensions of cuboid are:

Length, $l = 6$ cm

Width, $b = 2$ cm

Height, $h = 2$ cm

Surface area of cuboid $= 2(lb + bh + lh)$

$= 2\,(6 \times 2 + 2 \times 2 + 2 \times 6)$

$= 2\,(12 + 4 + 12)$

$= 56$ sq.cm.

25. Area of the shaded region

= Area of square ABCD – 2 × area of semi-circle

$= (14 \times 14) - 2 \times \left[\frac{\pi}{2} \times (7)^2\right]$

$= (196 - 2 \times 77)$ cm^2

$= (196 - 154)$ cm^2

$= 42$ cm^2

Caution

➥ *For calculating the area of shaded region, first calculate the area of square and area of semi circle, then subtract.*

SECTION - C

26. Given, number is 12^n, where $n \perp N$

Now, $\quad 12^n = (2^2 \times 3)^n$

Now, for 12^n to end with 0, it should have 2 as well as 5 in its prime factors. Also to end with 5, it requires atleast a single multiple of 5 in its prime factors, so 12^n cannot end with the digit 0 or 5.

OR

Here, $a = -2$ and $d = -5$

Let nth term of the A.P. be –77. Then,

$$-77 = (-2) + (n-1)(-5)$$

$\Rightarrow \quad 5(n-1) = 75$

$\Rightarrow \quad n - 1 = 15$, or $n = 16$

$\Rightarrow$ Sum up to the term (–77)

$= \frac{n}{2}[a + l] = \frac{16}{2}[(-2) + (-77)]$

$= 8 \times (-79)$

$= -632$

27. $(x^2+1)^2 - x^2 = 0$

$x^4 + 2x^2 + 1 - x^2 = 0$

$x^4 + x^2 + 1 = 0$

Put $t = x^2$. Then

$$t^2 + t + 1 = 0$$

Comparing this equation with $ax^2 + bx + c = 0$, we get,

$$a = 1, b = 1, c = 1$$

So, $\quad D = b^2 - 4ac = 1 - 4 = -3 < 0$

which shows that given equation has no real roots.

28. Let, the cost of 1 book be ₹ x and cost of 1 pen be ₹ y.

$\therefore \quad 5x + 7y = 434 \quad$ ...(i)

$7x + 5y = 550 \quad$ ...(ii)

Add equations (i) and (ii) we get

$$12x + 12y = 984$$

$$x + y = 82 \quad \text{...(iii)}$$

Subtract equation (ii) from (i), we get

$$2x - 2y = 116$$

$$x - y = 58 \quad \text{...(iv)}$$

Now, add equations (iii) and (iv), we get

$$2x = 140$$

$$x = 70$$

$\therefore \quad y = 70 - 58 = 12$

Hence, the cost of 1 book is ₹ 70 and the cost of 2 pens is ₹ 24.

29. $\dfrac{\tan A}{1 + \sec A} - \dfrac{\tan A}{1 - \sec A}$

$$= \tan A\left(\frac{1}{1 + \sec A} - \frac{1}{1 - \sec A}\right)$$

$$= \tan A\left(\frac{1 - \sec A - 1 - \sec A}{1 - \sec^2 A}\right)$$

$$= \tan A\left(\frac{-2\sec A}{-\tan^2 A}\right)$$

$$= \left(\frac{2\sec A}{\tan A}\right)$$

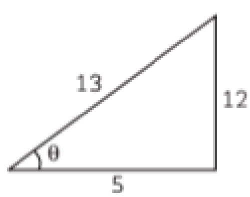
$$= 2 \times \frac{1}{\cos A} \times \frac{\cos A}{\sin A}$$

$$= \frac{2}{\sin A}$$

$= 2 \operatorname{cosec} A$ **Hence Proved.**

OR

$\sin\theta = \dfrac{12}{13}$ gives $\tan\theta = \dfrac{12}{5}$ and $\cos\theta = \dfrac{5}{13}$



So, $\dfrac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} - \dfrac{1}{\tan^2\theta}$

$$= \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)} - \frac{1}{\left(\frac{12}{5}\right)^2}$$

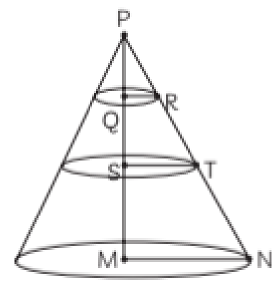
$$= \frac{\frac{144-25}{169}}{\frac{120}{169}} - \frac{25}{144}$$

$$= \frac{119}{120} - \frac{25}{144}$$

$$= \frac{714 - 125}{720}$$

$$= \frac{589}{720}$$

30.



From the figure, we easily find that

$$\frac{QR}{h} = \frac{ST}{2h} = \frac{MN}{3h}$$

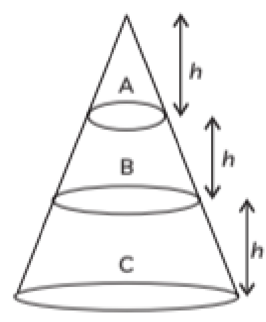
Let $\quad QR = r$, then, $ST = 2r$ and $MN = 3r$

So, Volume of cone PMN $= \dfrac{1}{3}\pi(3r)^2(3h)$

$$= 9\pi r^2 h$$

Volume of cone PST $= \dfrac{1}{3}\pi(2r)^2(2h) = \dfrac{8}{3}\pi r^2 h$

Volume of cone PQR $= \dfrac{1}{3}\pi(r)^2(h) = \dfrac{1}{3}\pi r^2 h$



$\Rightarrow$ Volume of part A $= \dfrac{1}{3}\pi r^2 h$

Volume of part B $= \dfrac{8}{3}\pi r^2 h - \dfrac{1}{3}\pi r^2 h = \dfrac{7}{3}\pi r^2 h$

Volume of part C $= 9\pi r^2 h - \dfrac{8}{3}\pi r^2 h = \dfrac{19}{3}\pi r^2 h$

(A) Ratio of volumes of parts A, B and C

$= \frac{1}{3} : \frac{7}{3} : \frac{19}{3}$ i.e. 1 : 7 : 19

(B) Ratio of Base area of part A, B and C are $\pi r^2 : \pi(2r)^2 : \pi(3r)^2$ i.e. 1 : 4 : 9

(C) Volume of cone PMN = $9\pi r^2 h = 540$

$\Rightarrow \quad \pi r^2 h = 60$

So, volume of part B = $\frac{7}{3}\pi r^2 h = \frac{7}{3} \times 60$ cm^3

= 140 cm^3

31. Put $x^2 = y$ to make partial fractions

Mass	Frequency (f_i)	x_i	f_ix_i	c.f.
0-10	5	5	25	5
10-20	10	15	150	15
20-30	5	25	125	20
30-40	4	35	140	24
40-50	6	45	270	30
	$\Sigma f_i = 30$		710	

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{710}{30}$$

= 23.67 (approx)

For median $\frac{N}{2} = \frac{30}{2} = 15$

Cumulative frequency just greater than 15 is 20, which belongs to class 20-30.

∴ Median class is 20-30

Now, $\text{Median} = l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times h$

$= 20 + \frac{(15-15)}{5} \times 10$

$= 20 + 0$

$= 20$

SECTION - D

32. Let, the zeros of the given polynomial be p, q, r. As the roots are in A.P., then it can be assumed as $p - k$, p, $p + k$, where k is the common difference.

$$p - k + p + p + k = \frac{-3b}{a}$$

$$p = \frac{-b}{a} \quad \text{...(i)}$$

and $(p - k)(p)(p + k) = \frac{-d}{a}$

$\Rightarrow \quad p(p^2 - k^2) = \frac{-d}{a}$

$\Rightarrow \quad \frac{-b}{a}(p^2 - k^2) = \frac{-d}{a}$ [from (i)]

$\Rightarrow \quad p^2 - k^2 = \frac{d}{b}$ ....(ii)

and, $p(p - k) + p(p + k) + (p - k)(p + k) = \frac{3c}{a}$

$\Rightarrow \quad 2p^2 + p^2 - k^2 = \frac{3c}{a}$

$\Rightarrow \quad \frac{2b^2}{a^2} + \frac{d}{b} = \frac{3c}{a}$ [from (i) and (ii)]

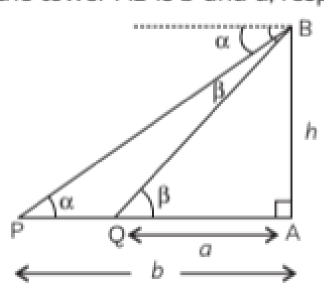
$\Rightarrow \quad \frac{2b^3 + a^2 d}{a^2 b} = \frac{3c}{a}$

$\Rightarrow \quad 2b^3 - 3abc + a^2 d = 0$ **Hence proved.**

OR

Let AB represent the tower of height 'h' metres.

Also, let P and Q be the positions of the two objects on the ground such that their distances from the tower AB is b and a, respectively.



Let ∠BPA = α and ∠BQA = β

We need to determine the distance PQ.

From ΔQAB, we have

$$\tan\beta = \frac{h}{QA} \Rightarrow QA = h\cot\beta$$

From ΔPAB, we have

$$\tan\alpha = \frac{h}{PA} \Rightarrow PA = h\cot\alpha$$

Further

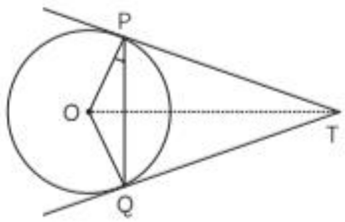
$$PQ = PA - QA$$

$= h \cot \alpha - h \cot \beta$, or $h(\cot \alpha - \cot \beta)$

Thus, the distance between two objects is, $h(\cot \alpha - \cot \beta)$.

33. (A) Join OT.

Consider Δs OPT and OQT.



Here,

$\angle P = \angle Q = 90°$ (each)

$OP = OQ$ (radii)

$OT = OT$ (common)

So, by RHS congruence criterion,

$$\Delta OPT \equiv \Delta OQT$$

$$\Rightarrow \quad TP = TQ$$

(B) In ΔOPQ,

$$\angle OQP = \angle OPQ \qquad (\because OP = OQ)$$

$$= \frac{1}{2}[180° - \angle POQ]$$

$$= \frac{1}{2}[\angle PTQ]$$

[$\because$ In quad. OPTQ, $\angle P = \angle Q = 90°$

So, $\angle POQ + \angle PTQ = 180°$]

$$\Rightarrow \quad \angle PTQ = 2\angle OPQ$$

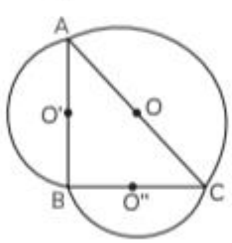
OR

Given: ΔABC, right angled at B.

Here, O, O' and O'' are the mid-points of AC, AB and BC, respectively

Area of semicircle at AC

$$= \frac{1}{2} \times \pi (OA)^2$$



$$= \frac{1}{2}\pi\left(\frac{AC}{2}\right)^2$$

$$= \frac{1}{2}\pi \times \frac{AC^2}{4}$$

Area of semicircle at AB

$$= \frac{1}{2} \neq (O'A)^2$$

$$= \frac{1}{2}\pi\left(\frac{AB}{2}\right)^2$$

$$= \frac{1}{2}\pi \times \frac{AB^2}{4}$$

Area of semicircle at BC

$$= \frac{1}{2} \neq (O''B)^2$$

$$= \frac{1}{2}\pi\left(\frac{BC}{2}\right)^2$$

$$= \frac{1}{2}\pi \times \frac{BC^2}{4}$$

Now, sum of areas of semicircles at AB and BC

$$= \frac{1}{2}\pi \frac{(AB^2)}{4} + \frac{1}{2}\pi \frac{(BC^2)}{4}$$

$$= \frac{1}{2}\pi \times \frac{1}{4}(AB^2 + BC^2)$$

$$= \frac{1}{2}\pi \times \frac{1}{4}(AC^2)$$

$$[\because AC^2 = AB^2 + BC^2]$$

= Area of semicircle at AC

Hence proved.

34. (A) $x^2 - 2x - 8$

Comparing given polynomial with general form $ax^2 + bx + c$,

We get $a = 1, b = -2$ and $c = -8$

We have, $x^2 - 2x - 8$

$$= x^2 - 4x + 2x - 8$$

$$= x(x - 4) + 2(x - 4)$$

$$= (x - 4)(x + 2)$$

Equating this equal to 0 will find value of 2 zeroes of this polynomial, $(x - 4)(x + 2) = 0$

$\Rightarrow x = 4, -2$ are two zeroes.

Sum of zeroes $= 4 - 2 = 2$

$$\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-b}{a} = \frac{-(-2)}{1}$$

Product of zeroes $= 4x - 2 = -8$

$$\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{c}{a} = \frac{-8}{1}$$

(B) $4s^2 - 4s + 1$

We have, $4s^2 - 4s + 1$

$= 4s^2 - 2s - 2s + 1$

$= 2s(2s - 1) - 1(2s - 1)$

$= (2s - 1)(2s - 1)$

Equating this equal to 0 will values of 2 zeroes of this polynomial.

$\Rightarrow (2s - 1)(2s - 1) = 0$

$\Rightarrow \quad s = \frac{1}{2}, \frac{1}{2}$

Therefore, two zeroes of this polynomial are $\frac{1}{2}, \frac{1}{2}$

Sum of zeroes $= \frac{1}{2} + \frac{1}{2} = 1$

$= \frac{-(-1)}{1} \times \frac{4}{4} = \frac{-(-4)}{4}$

$= = \frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of Zeroes $= \frac{1}{2} \times \frac{1}{2} - \frac{1}{4} = \frac{c}{a}$

$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(C) $6x^2 - 3 - 7x$

We have, $6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3) = (2x - 3)(3x + 1)$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$\Rightarrow \quad (2x - 3)(3x + 1) = 0$

$\Rightarrow \quad x = \frac{3}{2}, \frac{-1}{3}$

Therefore, two zeroes of this polynomial are $\frac{3}{2}, \frac{-1}{3}$

Sum of zeroes $= \frac{3}{2} + \frac{-1}{3} = \frac{9-2}{6} = \frac{7}{6}$

$= \frac{-(-7)}{6} = \frac{-b}{a} = \frac{-\text{ Coefficient of } x}{\text{Coefficient of } x^2}$

Product of Zeroes $= \frac{3}{2} \times \frac{-1}{3} = \frac{-1}{2} = \frac{c}{a}$

$= \frac{\text{Coefficient term}}{\text{Coefficient of } x^2}$

(D) $4u^2 + 8u$

Here, $a = 4$, $b = 8$ and $c = 0$

$4u^2 + 8u = 4u(u + 2)$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$\Rightarrow \quad 4u(u + 2) = 0$

$\Rightarrow \quad u = 0, -2$

Therefore, two zeroes of this polynomial are $0, -2$

Sum of zeroes $= 0 - 2 = -2$

$= \frac{-2}{1} \times \frac{4}{4} = \frac{-8}{4} = \frac{-b}{a}$

$= \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of Zeroes $= 0x - 2 = 0$

$= \frac{0}{4} = \frac{c}{a} = \frac{\text{Coefficient term}}{\text{Coefficient of } x^2}$

(E) $t^2 - 15$

We have, $t^2 - 15$

$\Rightarrow \quad t^2 = 15$

$\Rightarrow \quad t = \pm\sqrt{15}$

Therefore, two zeroes of this polynomial are $\sqrt{15}, -\sqrt{15}$

Sum of zeroes $= \sqrt{15} + (-\sqrt{15}) = 0$

$= \frac{0}{1} = \frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of Zeroes $= \sqrt{15} + (-\sqrt{15}) = -15$

$= \frac{-15}{1} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

35. To Prove: PQRS is a parallelogram

Proof : In ΔDAC,

$\frac{DS}{SA} = \frac{DR}{RC} = 1$

[$\because$ S and R are mid-points of AD and DC]

$\Rightarrow \quad SR \parallel AC$ ....(i) [by converse of B.P.T.]

In ΔBAC, $\frac{PB}{AP} = \frac{BQ}{QC} = 1$

[$\because$ P and Q are mid-points of AB and BC]

$\Rightarrow \quad PQ \parallel AC$ ....(ii) [by converse of B.P.T.]

From (i) and (ii), we get

$\Rightarrow \quad SR \parallel PQ$ ....(iii)

Similarly, join B to D and $PS \parallel QR$

$\Rightarrow \therefore$ PQRS is a parallelogram.

SECTION - E

36. (A) In ΔBSR, using Pythagoras theorem, we have

$$BR^2 = BS^2 + SR^2$$
$$(16)^2 = BS^2 + 8^2$$
$$BS^2 = 256 - 64$$
$$= 192$$
$$BS = 13.9 \text{ m}$$

(B) In ΔBRS,

$$\sin B = \frac{SR}{BR} = \frac{8}{16}$$
$$\angle B = 30°$$

(C) In ΔBRS,

$$\angle BRS = 180° - 90° - 30°$$
$$= 180° - 120°$$
$$= 60°$$

In ΔBRT,

$$\cos \angle BRT = \frac{BR}{BT}$$
$$= \frac{8}{16} = \frac{1}{2}$$
$$\angle BRT = 60°$$

$\therefore$ Angle of rotation $= 60° - 30° = 30°$

OR

In ΔBRT, using Pythagoras theorem, we have

$$BR^2 = BT^2 + RT^2$$
$$\Rightarrow \quad (16)^2 = BT^2 + 8^2$$
$$\Rightarrow \quad BT^2 = 256 - 64 = 192$$
$$\Rightarrow \quad BT = 13.9$$

Hence, increase in length

$$= 13.9 - 8 = 5.9 \text{ cm}$$

From all the given options, the value nearest to 5.9 m is 6 m.

37. (A) Since, total apples are 250.

Then

$$20 + 60 + 70 + p + 60 = 250$$
$$p = 250 - 210 = 40$$

(B)

Marks	f_i	x_i	f_ix_i
80-100	20	90	1800
100-120	60	110	6600
120-140	70	130	9100
140-160	40	150	6000
160-180	60	170	10200
	250		33700

Then, mean $= \frac{33700}{250} = 134.8$ g

(C)

Marks	Frequency	c.f.
80-100	20	20
100-120	60	80
120-140	70	150
140-160	40	190
160-180	60	250
	N = 250	

Then, $\frac{N}{2} = \frac{250}{2} = 125$

So, cumulative frequency just greater than 125 is 150, which belong to class 120-140.

So. median class is 120-140.

Hence, upper limit of median class is 140.

OR

Here, modal class is 120 – 140

Then, $\quad l = 120, h = 20$

$f_0 = 60, f_1 = 70, f_2 = 40$

Then, $\quad M_0 = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$$= 120 + \left(\frac{70-60}{140-60-40}\right) \times 20$$
$$= 120 + \left(\frac{200}{40}\right)$$
$$= 125 \text{ g}$$

Caution

Remember about the sequence of the frequency i.e., carefully take f_0, f_1, f_2.

38. (A) P(He will not get ticket)

$$= 1 - 0.729 = 0.271$$

(B) P(getting a ticket) $= 0.9 \times 0.8 = 0.72$

(C) P(it will not accept either coins)

$= $ P(will not accept ₹ 1 coin)

$\times$ P(will not accept ₹ 2 coin)

$= $ [1 – P(will accept ₹ 1 coin)]

$\times$ [1 – P(will accept ₹ 2 coin)]

$= (1 - 0.9) \times (1 - 0.8)$

$= 0.1 \times 0.2 = 0.02$

OR

P(All three coins will be accepted)

$$= 0.9 \times 0.9 \times 0.9 = 0.729$$

P(will not accept ₹ 1 coin)

$= 1 - $ P(will accept ₹ 1 coin)

$= 1 - 0.9 = 0.1$