Rational Numbers

NOTES

FUNDAMENTALS

- Natural numbers (N): 1, 2, 3, 4, 5.......etc., are called natural numbers.
- Whole numbers (W): 0, 1, 2, 3, 4, etc.., are called whole numbers.
- Integers (Z): 1......-4, -3, -2, -1, 0, 1, 2, 3, 4 etc.., are called integers. (denoted by I or Z) 1, 2, 3, 4, ...

...etc., are called positive integers denoted by $Z^{\scriptscriptstyle +}$ or $I^{\scriptscriptstyle +}$.

 $-1, -2, -3, -4, \dots$ etc.., are called negative integers denoted by Z^- or I^- .

Note: 0 is neither positive nor negative numbers.

• Fractions: The numbers of the form $\frac{x}{y}$, where x and y c are natural numbers, are known as fractions. e.g.,

 $\frac{2}{5}, \frac{3}{1}, \frac{1}{122}, \dots$ etc.

Elementary questions:

Identify which of the following number is a whole number as well as a fraction?

(a)
$$\frac{3}{36}$$
 (b) $\frac{36}{3}$ (c) $\frac{20}{8}$ (d) $\frac{8}{20}$

Ans. (b) $\frac{36}{3} = 12$ which can be expressed as a fraction $\left(\frac{12}{1}\right)$ as well as a whole number (=12).

Rational numbers (Q):

A number of the form $\frac{p}{q}$ ($q \neq 0$). where p and q are integers is called a rational number.

e.g.,
$$\frac{-3}{6}$$
, $-\frac{1}{12}$, $\frac{10}{13}$, $\frac{12}{17}$, etc.

Note: 0 is rational number, since $0 = \frac{0}{1}$.

• A rational number $\frac{p}{q}$ is positive if p and q are either both positive or both negative.

e.g.
$$\frac{6}{11}, \frac{-8}{-16}$$

• A rational number $\frac{p}{q}$ is negative if either of p and q is positive and the other term (q or p) is negative.

e.g.,
$$\frac{-4}{7}, \frac{8}{-23}$$

Note: 0 is neither a positive nor a negative rational number.

Representation of Rational numbers on a number line:

We can mark rational numbers on a number line just as we do for integer. The negative rational numbers are marked to the left of 0 and the positive rational numbers are marked to the right of 0.

Thus, $\frac{1}{6}$ and $-\frac{1}{6}$ would be at an equal distance from 0 but on its either side of zero.

Similarly, other rational numbers with different denominators can also be represented on the number line. In general, any rational number is either of the following two types.

(a)
$$\frac{p}{q}$$
 where $p < q$
(b) $\frac{p}{q}$ where $p > q$
e.g., $\frac{1}{8}, \frac{2}{9}, \frac{16}{17}$ etc.
e.g., $\frac{8}{1}, \frac{9}{2}, \frac{17}{16}$ etc.

Representation of $\frac{p}{q}$ on the number line where p < q:

The rational number $\frac{4}{6}$ (4<6) is represented on the number line as shown.



Representation of $\frac{p}{q}$ on the number line where p > q:

Consider the rational number $\frac{13}{6}$

Let us convert the rational number $\frac{13}{6}$ into a mixed fraction $= 2\frac{1}{6}$ and then mark it on the number line. i.e.



• Standard form of a rational number:

A rational number $\frac{p}{q}$ is said to be in standard form if q is a positive integer and the integer p and q have no common

factor other than 1.

Additive Inverse:

$$\frac{-p}{q} \text{ is the additive inverse of } \frac{p}{q} \text{ and } \frac{p}{q} \text{ is the additive inverse of } \frac{-p}{q}.$$

e.g.,
$$\frac{-13}{6} + \frac{13}{6} = 0 = \frac{13}{6} + \left(\frac{-13}{6}\right).$$

Reciprocal of a rational number:

If the product of two rational numbers is 1, then each rational number is called the reciprocal of the other.

Thus, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$ and we write, $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$ Note (a); Clearly, (a) Reciprocal of 0 does not exist. (b) Reciprocal of 1 is 1. (c) Reciprocal of-lis-1.

Elementary question:

Find the reciprocal of $\frac{5}{6}$ and also its additive increase. **Ans**. Reciprocal of $\frac{5}{6}$

Let 'a' be reciprocal of 5/6

Then $a \times \frac{5}{6} = 1 \Longrightarrow a = \frac{6}{5}$

Let 'x' be addition inverse

Then,
$$x + \frac{5}{6} = 0$$

 $\therefore x = \left(-\frac{5}{6}\right)$