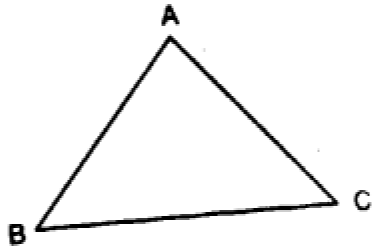
Triangles

POINTS TO REMEMBER

1. Definition of a triangle : A closed figure, having 3 sides, is called a triangle and is usually denoted by the Greek letter Δ (delta).

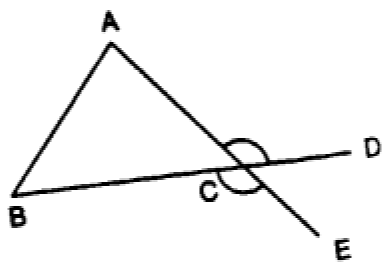


The figure, given alongside, shows a triangle ABC (ΔABC) bounded by three sides AB, BC and CA.
Hence it has six elements : 3 angles and 3 sides.

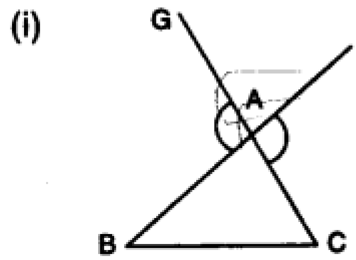
2. Vertex : The point, where any two sides of a triangle meet, is called a vertex. Clearly, the given triangle has three vertices; namely : A, B and C. [Vertices is the plural of vertex]

3. Interior angles : In ΔABC (given above), the angles BAC, ABC and ACB are called its interior angles as they lie inside the Δ ABC. The sum of interior angles of a triangle is always 180°.

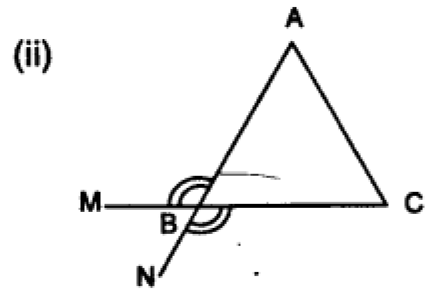
4. Exterior angles : When any side of a triangle is produced the angle so formed, outside the triangle and at its vertex, is called its **exterior angle.**



e.g. if side BC is produced to the point D; then ∠ACD is its exterior angle. And, if side AC is produced to the point E, then the exterior angle would be ∠BCE.
Thus. at every vertex, two exterior angles can be formed and that these two angles being vertically opposite angles, are always equal.
Make the following figures clear :

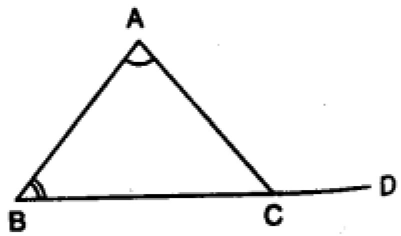


[Ext. ∠BAG = Ext. ∠CAF]



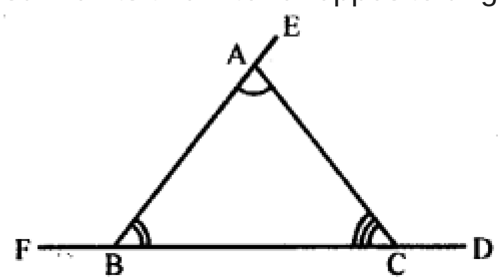
[Ext. ∠ABM = Ext. ∠CBN]

5. **Interior opposite angles :** When any side of a triangle is produced; an exterior angle is formed. The two interior angles of this triangle, that are opposite to the exterior angle formed; are called its **interior opposite angles.**



In the given figure, side BC of ∆ABC is produced to the point D, so that the exterior ∠ACD is formed. Then the two interior opposite angles are ∠B AC and ∠ABC.

6. **Relation between exterior angle and interior opposite angles :**
Exterior angle of a triangle is always equal to the
sum of its two interior opposite angles.

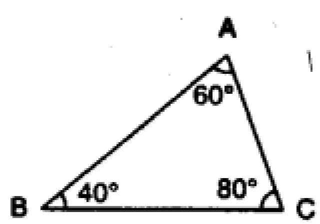


In ∆ABC,
Ext. ∠ACD = ∠A + ∠B

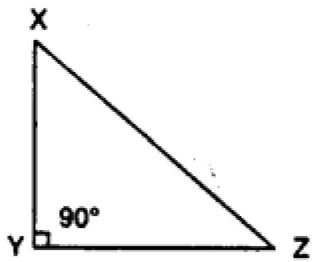
7. CLASSIFICATION OF TRIANGLES

(A) With regard to their angles :

1. Acute angled triangle : It is a triangle, whose each angle is acute i.c. each angle is less than 90°.

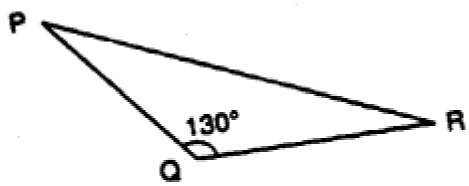


2. Right angled triangle : It is a triangle, whose one angle is a right angle i.e. equal to 90”.
The figure, given alongside, shows a right angled triangle XYZ as ∠XYZ = 90°
Note : (i) One angle of a right triangle is 90° and the other two angles of it are acute; such that their sum is always 90”.

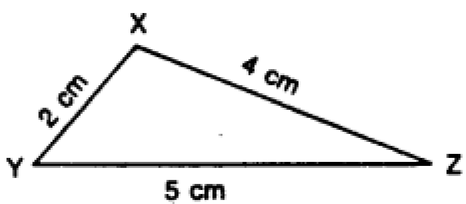


In ∆XYZ, given above, ∠Y = 90° and each of ∠X and ∠Z is acute such that ∠X + ∠Z = 90°. .
(ii)In a right triangle, the side opposite to the right angle is largest of all its sides and is called the**hypotenuse**. In given right angled ∆ XYZ side XZ is its hypotenuse

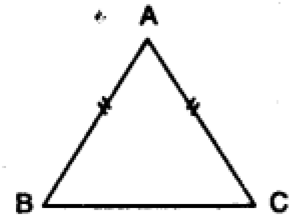
3.Obtuse angled triangle : If one angle of a triangle is 1
obtuse, it is called an obtuse angled triangle.
Note : In case of an obtuse angled triangle, each of the other two angles is always acute and their sum is less than 90”.



(B) With regard to their sides :
(1) Scalene triangle: If all the sides of a triangle are unequal, it is called a **scalene triangle.**
In a scalene triangle; all its angles are also unequal.

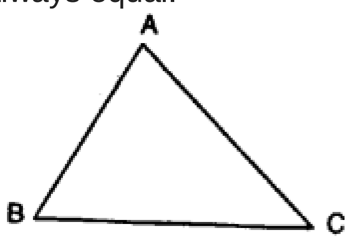


(2) Isosceles triangle : If atleast two sides of a triangle are equal, it is called an **isosceles triangle.**
In ∆ ABC, shown alongside, side AB = side AC.
∴∆ ABC is an isosceles triangle.



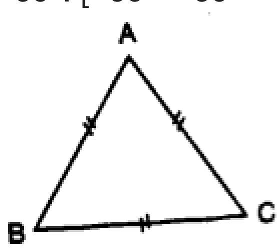
Note : (i) The angle contained by equal sides i.e. ∠BAC is called the **vertical angle** or the **angle of vertex.**
(ii) The third side (i.e. the unequal side) is called the **base** of the isosceles triangle.
(iii) The two other angles (i.e. other than the angle of vertex) are called the **base angles** of the triangle.

IMPORTANT PROPERTIES OF AN ISOSCELES TRIANGLE
The base angles i.e. the angles opposite to equal sides of an isosceles triangle are always equal.



In given triangle ABC,
(i) If side AB = side BC; then angle opposite to AB = angle opposite to BC i.e. ∠C = ∠A.
(ii) If side BC = side AC; then angle opposite to BC = angle opposite to AC i.e. ∠A = ∠B and so on.
Conversely : If any two angles of a triangle are equal; the sides opposite to these angles are also equal i.e. the triangle is isosceles.
Thus in Δ ABC,
(i) If ∠B = ∠C => side opposite to ∠B = side opposite to ∠C i.e. side AC = side AB.
(ii) If ∠A = ∠B => side BC = side AC and so on.

(3) Equilateral triangle :
If all the sides of a triangle are equal, it is called an equilateral triangle.
In the given figure, A ABC is equilateral, because AB = BC = CA.
Also, all the angles of an equilateral triangle are equal to each other and so each angle = 60°. [∵60° + 60° + 60° = 180°]



Since, all the angles of an equilateral triangle are equal, it is also known as equiangular triangle. Note : An equilateral triangle is always an isosceles triangle, but its converse is not always true.

(4) Isosceles right angled triangle : If one angle of an isosceles triangle is 90°, it is called an isosceles right angled triangle.
In the given figure, Δ ABC is an isosceles right angled triangle, because : ∠ ACB = 90° and AC = BC.
Here, the base is AB, the vertex is C and the base angles are ∠BAC and ∠ABC, which are equal.
Since, the sura of the angles of a triangle = 180″
∴∠ABC = ∠BAC = 45 [∵45° + 45° + 90° = 180°]

EXERCISE 15 (A)

Question 1.
Stale, if the triangles are possible with the following angles :
(i) 20°, 70° and 90°
(ii) 40°, 130° and 20°
(iii) 60°, 60° and 50°
(iv) 125°, 40° and 15°
Solution:

We know that, the sum of three angles of a triangle is 180°, therefore

(*i*) Sum of 20°, 70° and 90°

$$= 20^\circ + 70^\circ + 90^\circ = 180^\circ.$$

Since the sum is 180°. Hence it is possible.

(*ii*) Sum of 40°, 130° and 20°

$$= 40^\circ + 130^\circ + 20^\circ = 190^\circ.$$

Since the sum is not 180°, therefore it is not possible.

(*iii*) Sum of 60°, 60° and 50°

$$= 60^\circ + 60^\circ + 50^\circ = 170^\circ.$$

Since the sum is not 180°, therefore it is not possible.

(*iv*) Sum of 125°, 40° and 15°

$$= 125^\circ + 40^\circ + 15^\circ = 180^\circ.$$

Since the sum is 180°, therefore it is possible.

Question 2.
If the angles of a triangle are equal, find its angles.
Solution:

Since the three angles of a triangle are equal and their sum is $180°$, therefore each angle will be $\frac{180°}{3} = 60°$.

Question 3.
In a triangle ABC, ∠A = 45° and ∠B = 75°, find ∠C.
Solution:

Since the sum of angles of a triangle is $180°$

$\therefore \angle A + \angle B + \angle C = 180°$

$\Rightarrow 45° + 75° + \angle C = 180°$

$\Rightarrow 120° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 120° = 60°$

Question 4.
In a triangle PQR, ∠P = 60° and ∠Q = ∠R, find ∠R.
Solution:

Let $\angle Q = \angle R = x$, $\angle P = 60°$

But $\angle P + \angle Q + \angle R = 180°$

$\therefore 60° + x + x = 180°$

$\Rightarrow 60° + 2x = 180°$

$\Rightarrow 2x = 180° - 60° = 120°$

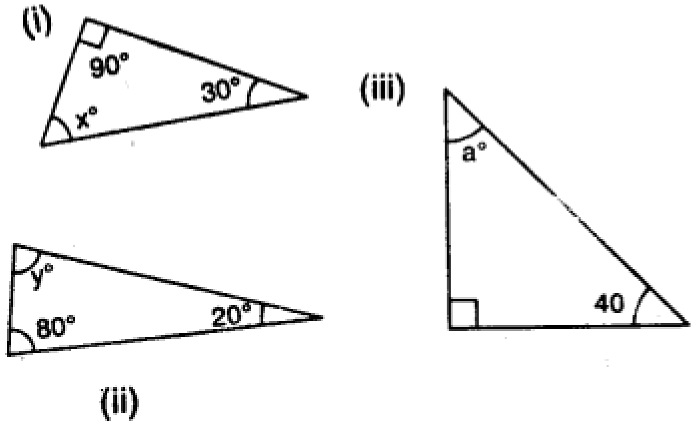
$\Rightarrow x = \frac{120°}{2} = 60°$

$\therefore \angle Q = \angle R = 60°$

Hence, $\angle R = 60°$

Question 5.

Calculate the unknown marked angles in each figure :



Solution:

We know that, the sum of three angles of a triangle is 180°, therefore

(*i*) In figure (*i*),

$90° + 30° + x = 180°$

$\Rightarrow 120° + x = 180°$

$\Rightarrow \quad x = 180° - 120° = 60°$

Hence $x = 60°$

(*ii*) In figure (*ii*),

$y + 80° + 20° = 180°$

$\Rightarrow y + 100° = 180°$

$\Rightarrow \quad y = 180° - 100° = 80°$

Hence $y = 80°$

(*iii*) In figure (*iii*),

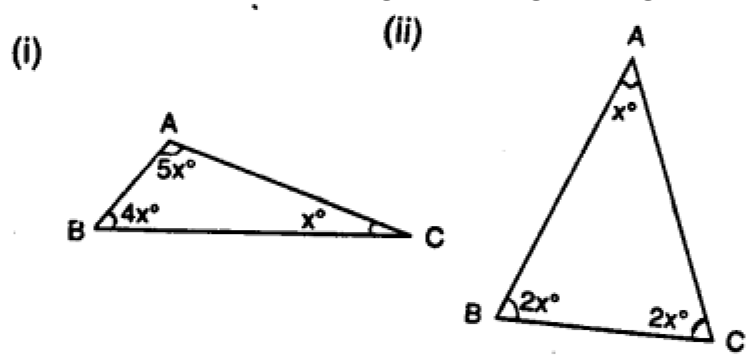
$a + 90° + 40° = 180°$

$\Rightarrow a + 130° = 180°$

$\Rightarrow \quad a = 180° - 130° = 50°$

Hence $a = 50°$

Question 6.
Find the value of each angle in the given figures:



Solution:

(*i*) In the figure (*i*),

$\angle A + \angle B + \angle C = 180°$ (Sum of angles of a triangle)

$\Rightarrow 5x° + 4x° + x° = 180°$

$\Rightarrow 10x° = 180° \quad \Rightarrow x = \frac{180°}{10} = 18°$

$\therefore \angle A = 5x° = 5 \times 18° = 90°$

$\angle B = 4x = 4 \times 18° = 72°$

and $\angle C = x = 18°$

(*ii*) In figure (*ii*),

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow x° + 2x° + 2x° = 180°$

$\Rightarrow 5x° = 180° \quad \Rightarrow x° = \frac{180°}{5} = 36°$

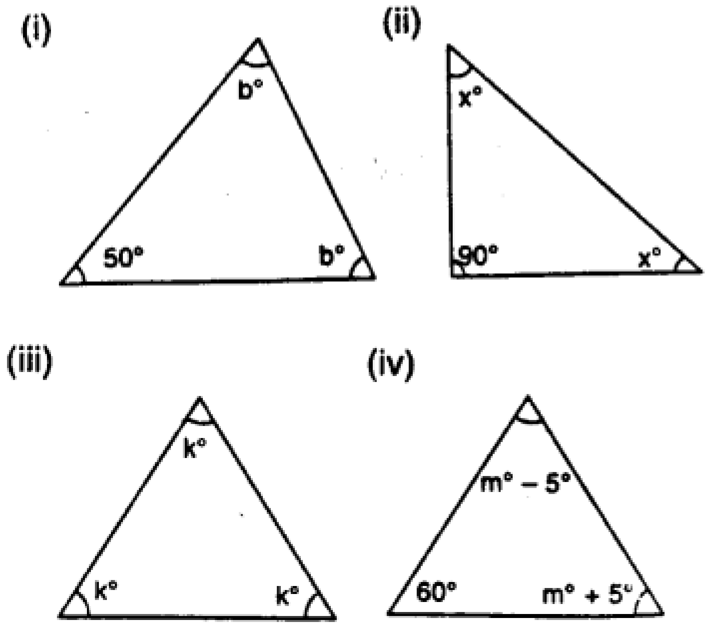
$\therefore \angle A = x° = 36°$

$\angle B = 2x° = 2 \times 36° = 72°$

and $\angle C = 2x° = 2 \times 36° = 72°$

Question 7.

Find the unknown marked angles in the given figure:



Solution:

(*i*) In the figure (*i*),

$\angle A + \angle B + \angle C = 180°$

(Sum of angles of a triangle)

$b° + 50° + b° = 180°$

$\Rightarrow 2b° + 50° = 180°$

$\Rightarrow \quad 2b° = 180° - 50° = 130°$

$$\Rightarrow \quad b° = \frac{130°}{2} = 65°$$

Hence $\angle A = b° = 65°$

and $\quad \angle C = b° = 65°$

(*ii*) In the figure (*ii*)

$\angle A + \angle B + \angle C = 180°$

(Sum of angles of a triangle)

$x° + 90° + x° = 180°$

$2x° + 90° = 180°$

$2x° = 180° - 90°$

$2x° = 90°$

$$x° = \frac{90°}{2} = 45°$$

Hence $\angle A = x° = 45°$

and $\angle C = x° = 45°$

(*iii*) In the figure (*iii*)

$\angle A + \angle B + \angle C = 180°$

(Sum of angles of a triangle)

$k° + k° + k° = 180°$

$3k° = 180°$

$$k° = \frac{180°}{3} = 60°$$

Hence $\angle A = k° = 60°$, $\angle B = k° = 60°$

and $\angle C = k° = 60°$

(*iv*) In the figure (*iv*)

$\angle A + \angle B + \angle C = 180°$

(Sum of Angles of a triangle)

$(m° - 5°) + 60° + (m° + 5°) = 180°$

$m° - 5° + 60° + m° + 5° = 180°$

$2m° = 180° - 65 + 5$

$2m° = 120°$

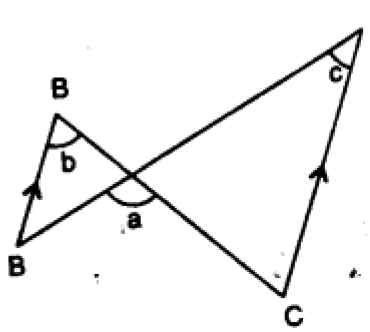
$$\therefore m° = \frac{120°}{2} = 60°$$

Hence $\angle A = m° - 5° = 60° - 5° = 55°$

and $\angle C = m° + 5° = 60° + 5° = 65°$

Question 8.

In the given figure, show that: ∠a = ∠b + ∠c



(i) If ∠b = 60° and ∠c = 50° ; find ∠a.
(ii) If ∠a = 100° and ∠b = 55° : find ∠c.
(iii) If ∠a = 108° and ∠c = 48° ; find ∠b.

Solution:

$\because$ AB || CD

$\therefore$ $b = c$ and $\angle A = \angle C$ (Alternate angles)

Now in Δ PCD,

Ext. $\angle APC = \angle C + \angle D$

$\Rightarrow a = b + c$

(*i*) If $b = 60°$, $c = 50°$, then

$a = b + c = 60° + 50° = 110°$

(*ii*) If $a = 100°$ and $b = 55°$,

then $a = b + c \Rightarrow 100° = 55° + c$

$\Rightarrow c = 100° - 55° = 45°$

(*iii*) If $a = 108°$ and $c = 48°$,

then $a = b + c \Rightarrow 108° = b + 48°$

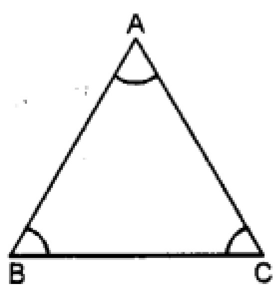
$\Rightarrow b = 108° - 48° = 60°$

Question 9.

Calculate the angles of a triangle if they are in the ratio 4 : 5 : 6.

Solution:

We know that sum of angles of a triangle is 180°



$\therefore \angle A + \angle B + \angle C = 180°$

But $\angle A : \angle B : \angle C = 4 : 5 : 6$

Let $\angle A = 4x$, $\angle B = 5x$ and $\angle C = 6x$,

then $4x + 5x + 6x = 180°$

$\Rightarrow 15x = 180° \quad \Rightarrow x = \frac{180°}{15} = 12°$

$\therefore \angle A = 4x = 4 \times 12° = 48°$

$\angle B = 5x = 5 \times 12° = 60°$

$\angle C = 6x = 6 \times 12° = 72°$.

Question 10.

One angle of a triangle is 60°. The, other two angles are in the ratio of 5 : 7. Find the two angles.

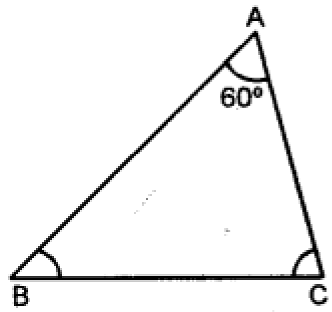
Solution:

In Δ ABC,

Let $\angle A = 60°$ and then $\angle B : \angle C = 5 : 7$

But $\angle A + \angle B + \angle C = 180°$

(Angles of a triangle)



$\Rightarrow 60° + \angle B + \angle C = 180°$

$\Rightarrow \angle B + \angle C = 180° - 60° = 120°$

Let $\angle B = 5x$ and $\angle C = 7x$

$\therefore\ 5x + 7x = 120°$

$\Rightarrow 12x = 120° \qquad \Rightarrow x = \frac{120°}{12} = 10°$

$\therefore\ \angle B = 5x = 5 \times 10° = 50°$

$\angle C = 7x = 7 \times 10 = 70°$

Question 11.

One angle of a triangle is 61° and the other two angles are in the ratio $1\frac{1}{2} : 1\frac{1}{3}$. Find these angles.

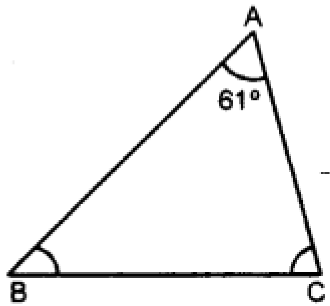
Solution:

In Δ ABC,

Let $\angle A = 61°$

But $\angle A + \angle B + \angle C = 180°$

(Angles of a triangle)



$\Rightarrow 61° + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle B + \angle C = 180° - 61° = 119°$

But $\angle B : \angle C = 1\frac{1}{2} : 1\frac{1}{3} = \frac{3}{2} : \frac{4}{3}$

$= \frac{9:8}{6} = 9:8$

Let $\angle B = 9x$ and $\angle C = 8x$,

then, $9x + 8x = 119°$

$\Rightarrow 17x = 119°$

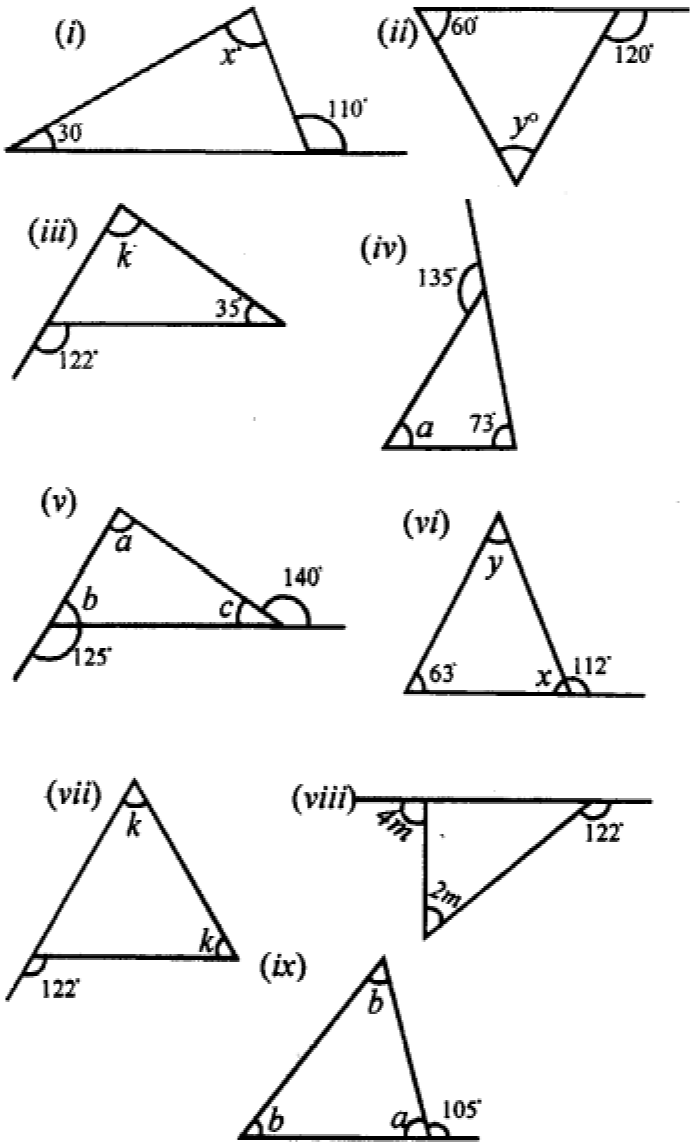
$\Rightarrow \quad x = \frac{119°}{17} = 7°$

$\therefore \angle B = 9x = 9 \times 7° = 63°$

$\angle C = 8x = 8 \times 7° = 56°$

Question 12.

Find the unknown marked angles in the given figures :



Solution:

We know that in a triangle, if one side of it is produced, then

Exterior angle = sum of its interior opposite angles.

(*i*) In Fig. (*i*), $110° = x° + 30°$

$\Rightarrow x° = 110° - 30° = 80°$

(*ii*) In Fig. (*ii*), $120° = y° + 60°$

$\Rightarrow y° = 120° - 60° = 60°$

(*iii*) In Fig. (*iii*), $122° = k° + 35°$

$\Rightarrow k° = 122° - 35° = 87°$

(*iv*) In Fig. (*iv*), $135° = a° + 73°$

$\Rightarrow a° = 135° - 73° = 62°$

(*v*) In Fig. (*v*), $125° = a + c$ ...(*i*)

and $140° = a + b$ ...(*ii*)

Adding, we get

$a + c + a + b = 125° + 140°$

$\Rightarrow a + a + b + c = 265°$

But $a + b + c = 180°$

(Sum of angles of a triangle)

$\therefore \quad a + 180° = 265°$

$\Rightarrow a = 265° - 180° = 85°$

But $a + b = 140°$

$\Rightarrow 85° + b = 140°$

$\Rightarrow \quad b = 140° - 85° = 55°$

and $a + c = 125° \Rightarrow 85° + c = 125°$

$\Rightarrow \quad c = 125° - 85° = 40°$

Hence $a = 85°$, $b = 55°$ and $c = 40°$

(*vi*) In Fig. (*vi*),

$112° + x° = 180°$ (Linear pair)

$\Rightarrow \quad x = 180° - 112° = 68°$

and $112° = y + 63°$

$\Rightarrow \quad y = 112° - 63° = 49°$

Hence $x = 68°$, $y = 49°$

(*vii*) In fig. (*vii*),

$120° = a + a \quad \Rightarrow \quad 2a = 120°$

$\Rightarrow a = \frac{120°}{2} = 60°$

$\therefore a = 60°$ **Ans.**

(*viii*) In fig. (*viii*),

$140° + a = 180°$ (Linear pair)

$\Rightarrow \quad a = 180° - 140° = 40°$

Now $4m = 2m = a \Rightarrow 4m - 2m = a$

$\Rightarrow 2m = 40° \Rightarrow m = \frac{40°}{2} = 20°$

Hence $m = 20°$

(*ix*) In fig. (*ix*),

$105° = b + b \quad \Rightarrow 2b = 105°$

$\Rightarrow \; b = \frac{105°}{2} = 52{\cdot}5°$

But $a + 105° = 180°$ (Linear pair)

$\Rightarrow \quad a = 180° - 105° = 75°$

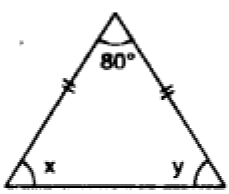
Hence $a = 75°, b = 52{\cdot}5°$

EXERCISE 15 (B)

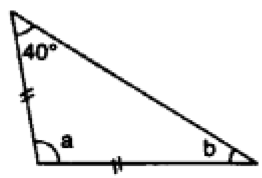
Question 1.

Find the unknown angles in the given figures:

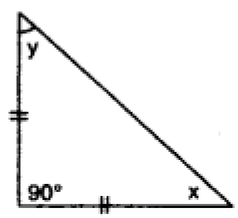
(i)



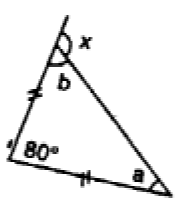
(ii)



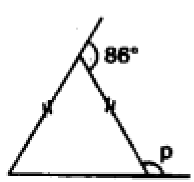
(iii)



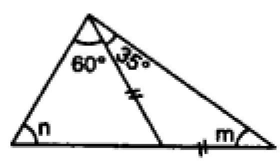
(iv)



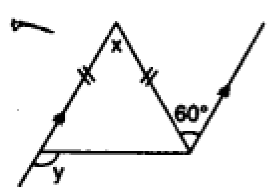
(v)



(vi)



(vii)



Solution:

(*i*) In Fig (*i*),

$x = y$ (Angles opposite to equal sides)

But $x + y + 80° = 180°$ (Angles of a triangle)

$\Rightarrow x + x + 80° = 180°$

$\Rightarrow 2x + 80° = 180°$

$\Rightarrow 2x = 180° - 80° = 100°$

$\Rightarrow x = \frac{100°}{2} = 50° \quad \therefore y = x = 50°$

Hence $x = 50°, y = 50°$

(*ii*) In Fig. (*ii*),

$b = 40°$ (Angles opposite to equal sides)

But $a + b + 40° = 180°$

(Angles of a triangle)

$\Rightarrow a + 40° + 40° = 180°$

$\Rightarrow a + 80° = 180°$

$\Rightarrow a = 180° - 80° = 100°$

Hence $a = 100°, b = 40°$

(*iii*) In Fig. (*iii*),

$x = y$ (Angles opposite to equal sides)

But $x + y + 90° = 180°$

(Angles of a triangle)

$\Rightarrow x + x + 90° = 180°$

$\Rightarrow \quad 2x + 90° = 180°$

$\Rightarrow \quad 2x = 180° - 90° = 90°$

$\therefore\ x = \dfrac{90°}{2} = 45° \quad \therefore\ y = x = 45°$

Hence $x = 45°, y = 45°$

(*iv*) In Fig. (*iv*),

$a = b$ (Angles opposite to equal sides)

But $a + b + 80° = 180°$

(Angles of a triangle)

$\Rightarrow a + a + 80° = 180°$

$\Rightarrow \quad 2a + 80° = 180°$

$\Rightarrow \quad 2a = 180° - 80° = 100°$

$\Rightarrow \quad a = \dfrac{100°}{2} = 50° \quad \therefore\ b = a = 50°$

$x = a + 80°$

(Exterior angle of a triangle is equal to sum of its opposite interior angles)

$= 50° + 80° = 130°$

Hence $a = 50°$, $b = 50°$ and $x = 130°$

(*v*) In Fig. (*v*),

Let each equal angle of an isosceles triangle be x,

then $x + x = 86° \quad \Rightarrow 2x = 86°$

$\Rightarrow x = \dfrac{86°}{2} = 43°$

But $p + x = 180°$ (Linear pair)

$p + 43° = 180°$

$\Rightarrow \quad p = 180° - 43° \quad \Rightarrow \quad p = 137°$

Hence $p = 137°$

(*vi*) In Fig. (*vi*),

$m = 35°$ (Angles opposite to equal sides)

But $m + n + (60° + 35°) = 180°$

(Angles of a triangle)

$\Rightarrow \quad m + n + 95° = 180°$

$\Rightarrow 35° + n + 95° = 180°$

$\Rightarrow \quad n + 130° = 180°$

$\Rightarrow \quad n = 180° - 130° = 50°$

Hence $m = 35°, n = 50°$

(*vii*) In Fig. (*vii*),

$x = 60°$ (Alternate angles)

Let each equal angle of an isosceles triangle be a then $a + a + x = 180°$

(Angles of a triangle)

$2a + x = 180° \Rightarrow 2a + 60° = 180°$

$\Rightarrow 2a = 180° - 60° = 120°$

$$\Rightarrow \quad a = \frac{120°}{2} = 60°$$

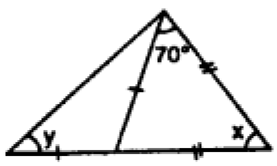
$\therefore y = x + a = 60° + 60° = 120°$

Hence $x = 60°$ and $y = 120°$

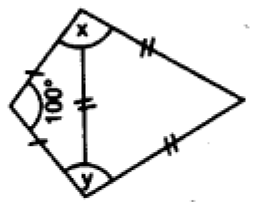
Question 2.

Apply the properties of isosceles and equilateral triangles to find the unknown angles in the given figures :

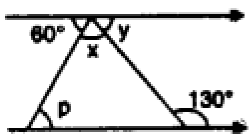
(i)



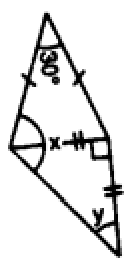
(ii)



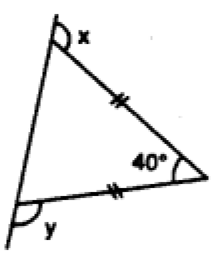
(iii)



(iv)



(v)



(vi)

Solution:

$a = 70°$ (Angles opposite to equal sides)

But $a + 70° + x = 180°$ (Angles of a triangle)

$\Rightarrow 70° + 70° + x = 180°$

$\Rightarrow \quad 140° + x = 180°$

$\Rightarrow \quad x = 180° - 140° = 40°$

$y = b$ (Angles opposite to equal sides)

But $a = y + b$

(Exterior angle of a triangle is equal to sum of its interior opposite angles)

$\Rightarrow 70° = y + y \quad \Rightarrow 2y = 70°$

$$\Rightarrow \quad y = \frac{70°}{2} = 35°$$

Hence $x = 40°, y = 35°$

(*ii*) In Fig. (*ii*),

In an equilateral triangle.

each angle $= 60°$

In isosceles triangle.,

Let each base angle $= a$

$\therefore a + a + 100° = 180°$

$\Rightarrow \quad 2a + 100° = 180°$

$\Rightarrow \quad 2a = 180° - 100° = 80°$

$$\therefore a = \frac{80°}{2} = 40° \quad \therefore x = 60° + 40° = 100°$$

and $y = 60° + 40° = 100°$

(*iii*) In Fig. (*iii*), $130° = x + p$

(Exterior angle of a triangle is equal to the sum of its interior opposite angles)

$\because$ Lines are parallel (Given)

$\therefore \quad p = 60°$ (Alternate angle)

and $y = a$

But $a + 130° = 180°$ (Linear pair

$\Rightarrow a = 180° - 130° = 50° \quad \therefore y = 50°$

and $x + p = 130°$

$\Rightarrow x + 60° = 130° \Rightarrow x = 130° - 60° = 70$

Hence $x = 70°, y = 50°$ and $p = 60°$

(*iv*) In Fig. (*iv*),

$x = a + b$

But $b = y$ (Angles opposite to equal sides)

Similarly $a = c$

But $a + c + 30° = 180°$

$\Rightarrow a + a + 30° = 180° \Rightarrow 2a + 30° = 180°$

$\Rightarrow\ 2a = 180° - 30° = 150°$

$\Rightarrow\ a = \dfrac{150°}{2} = 75°$ and $b + y = 90°$

$\Rightarrow y + y = 90°$ $\qquad\qquad \Rightarrow 2y = 90°$

$\Rightarrow\quad y = \dfrac{90°}{2} = 45° \qquad \Rightarrow\ b = 45°$

Hence $x = a + b = 75° + 45° = 120°$

and $\quad y = 45°$

(*v*) In Fig. (*v*),

$a + b + 40° = 180°$ (Angles of a triangle)

$\Rightarrow a + b = 180° - 40° = 140°$

But $a = b$ (Angles opposite to equal sides)

$\therefore\ a = b = \dfrac{140°}{2} = 70°$

$\therefore\ x = b + 40° = 70° + 40° = 110°$

(Exterior angle of a triangle is equal to the sum of its interior opposite angles)

Similarly $y = a + 40°$

$= 70° + 40° = 110°$

Hence $x = 110°, y = 110°$

(*vi*) In the Fig. (*vi*),

$a = b$ (Angles opposite to equal sides)

$\therefore\ y = 120°$

But $a + 120° = 180°$ (Linear pair)

$\Rightarrow a = 180° - 120° = 60°$

$\therefore\ b = 60°$

But $x + a + b = 180°$ (Angles of a triangle)

$\Rightarrow x + 60° + 60° = 180°$

$\Rightarrow \qquad x + 120° = 180°$

$\therefore\ x = 180° - 120° = 60°$

$b = z + 25$

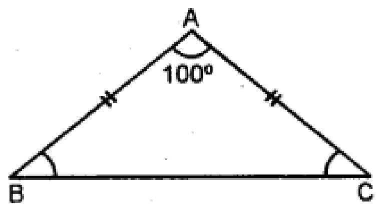
(Exterior angle of a triangle is equal to the sum of its interior opposite angles)

$\Rightarrow 60° = z + 25°$

$\Rightarrow \quad z = 60° - 25° = 35°$

Hence $x = 60°, y = 120°$ and $z = 35°$

Question 3.
The angle of vertex of an isosceles triangle is 100°. Find its base angles.
Solution:
In ΔABC,

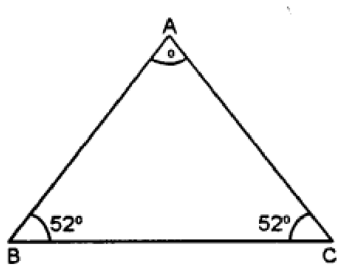


$\because \quad AB = AC.$
$\therefore \quad \angle B = \angle C$
But $\angle A = 100°$
and $\angle A + \angle B + \angle C = 180°$
(Angles of a triangle)
$\Rightarrow 100° + \angle B + \angle B = 180°$
$\Rightarrow 2\angle B = 180° - 100° \quad \Rightarrow 2\angle B = 80°$

$$\therefore \quad \angle B = \frac{80°}{2} = 40°$$

Hence $\angle B = \angle C = 40°$

Question 4.
One of the base angles of an isosceles triangle is 52°. Find its angle of vertex.
Solution:
Each of the base angles of an isosceles $\Delta ABC = 52°$
$\therefore \angle B = \angle C = 52°$



But $\angle A + \angle B + \angle C = 180°$
(Angles of a triangle)
$\Rightarrow \angle A + 52° + 52° = 180°$
$\Rightarrow \quad \angle A + 104° = 180°$
$\Rightarrow \quad \angle A = 180° - 104° = 76°$
Hence $\angle A = 76°$

Question 5.
In an isosceles triangle, each base angle is four times of its vertical angle. Find all the angles of the triangle.
Solution:
Let vertical angle of an isosceles triangle = x
$\therefore$ Each base angle = $4x$
$\therefore$ $x + 4x + 4x = 180°$
(Sum of angles of a triangle)
$\Rightarrow 9x = 180° \quad \Rightarrow \quad x = \frac{180°}{9} = 20°$
$\therefore$ Vertical angle = $20°$
and each of the base angle = $4x$
$= 4 \times 20° = 80°$

Question 6.
The vertical angle of an isosceles triangle is 15° more than each of its base angles. Find each angle of the triangle.
Solution:
Let each angle of the base of the isosceles triangle = $x°$
Then vertical angle = $x + 15°$
Now $x + x + x + 15° = 180°$
(Sum of angles of a triangle)
$\Rightarrow 3x + 15° = 180°$
$\Rightarrow \quad 3x = 180° - 15° = 165°$
$\therefore \; x = \frac{165°}{3} = 55°$
Hence each base angle = $55°$
and vertical angle = $55° + 15° = 70°$

Question 7.
The base angle of an isosceles triangle is 15° more than its vertical angle. Find its each angle.
Solution:

Let vertical angle of the isosceles triangle $= x°$

$\therefore$ Each base angle $= x + 15°$

$\therefore x + 15° + x + 15° + x° = 180°$

(Sum of angles of a triangle)

$\Rightarrow 3x + 30° = 180°$

$\Rightarrow \quad 3x = 180° - 30° = 150°$

$\therefore \ x = \dfrac{150°}{3} = 50°$

Hence vertical angle $= 50°$

and each base angle $= 50° + 15° = 65°$

Question 8.
The vertical angle of an isosceles triangle is three times the sum of its base angles. Find each angle.
Solution:

Let each base angle of an isosceles, triangle $= x$

then its vertical $= 3\ (x + x) = 3 \times 2x = 6x.$

$\therefore \ 6x + x + x = 180°$

(Sum of angles of a triangle)

$\Rightarrow 8x = 180° \qquad \Rightarrow \ x = \dfrac{180°}{8} = 22{\cdot}5°$

$\therefore$ Each base angle $= 22{\cdot}5°$

and vertical angle $= 3 \times (22{\cdot}5 + 22{\cdot}5)$

$= 3 \times 45 = 135°$

Question 9.
The ratio between a base angle and the vertical angle of an isosceles triangle is 1 : 4. Find each angle of the triangle.

Solution:

Ratio between base angle and vertical angle of an isosceles triangle = 1 : 4

Let each base angle = x

then vertical angle = $4x$

$\therefore x + x + 4x = 180°$

(Sum of angles of a triangle)

$\Rightarrow 6x = 180°$ $\Rightarrow$

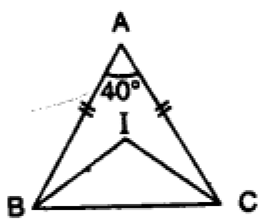
$x = \frac{180°}{6} = 30°$

$\therefore$ Each base angle = $x = 30°$

and vertical angle = $4x$

$= 4 \times 30° = 120°$

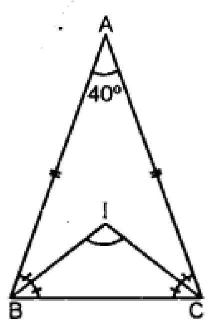
Question 10.
In the given figure, BI is the bisector of∠ABC and CI is the bisector of ∠ACB. Find ∠BIC.



Solution:

In Δ ABC,

BI is the bisector of ∠ABC and CI is the bisector of ∠ACB.



$\because AB = AC$

$\therefore \angle B = \angle C$

(Angles opposite to equal sides)

But $\angle A = 40°$

and $\angle A + \angle B + \angle C = 180°$

(Angles of a triangle)

$\Rightarrow 40° + \angle B + \angle B = 180°$

$\Rightarrow \quad 40° + 2\angle B = 180°$

$\Rightarrow \quad 2\angle B = 180° - 40° = 140°$

$\Rightarrow \quad \angle B = \frac{140°}{2} = 70°$

$\therefore \angle ABC = \angle ACB = 70°$

But BI and CI are the bisectors of $\angle ABC$ and $\angle ACB$ respectively.

$\therefore \angle IBC = \frac{1}{2}\angle ABC = \frac{1}{2}(70°) = 35°$

and $\angle ICB = \frac{1}{2}\angle ACB = \frac{1}{2} \times 70° = 35°$

Now in Δ IBC,

$\angle BIC + \angle IBC + \angle ICB = 180°$

(Angles of a triangle)

$\Rightarrow \angle BIC + 35° + 35° = 180°$

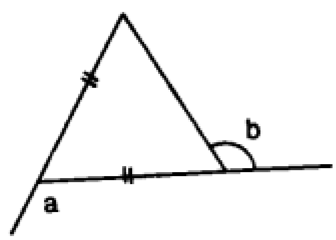
$\Rightarrow \angle BIC + 70° = 180°$

$\Rightarrow \quad \angle BIC = 180° - 70° = 110°$

Hence $\quad \angle BIC = 110°$

Question 11.

In the given figure, express a in terms of b.



Solution:

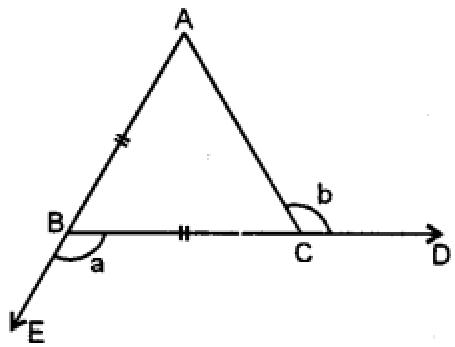
In Δ ABC,

$BC = BA$

$\therefore \angle BCA = \angle BAC$

and Ext. $\angle CBE = \angle BCA + \angle BAC$

$\Rightarrow \quad a = \angle BCA + \angle BCA$



$\Rightarrow a = 2\angle BCA$ ...(i)

But $\angle ACB = 180° - b$

($\because \angle ACD$ and $\angle ACB$ are linear pair)

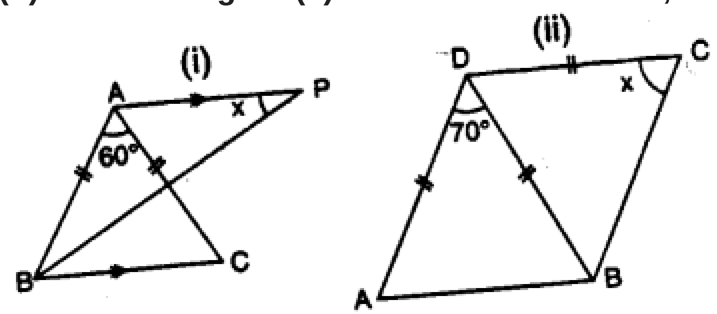
$\Rightarrow \angle BCA = 180° - b$ ...(ii)

$\therefore a = 2\angle BCA = 2(180° - b)$

$= 360° - 2b$

Question 12.

(a) In Figure (i) BP bisects ∠ABC and AB = AC. Find x.
(b) Find x in Figure (ii) Given: DA = DB = DC, BD bisects ∠ABC and∠ADB = 70°.



Solution:

(a) In figure (i),

AB = AC, and BP bisects ∠ABC

AP||BC is drawn.

Now $\angle PBC = \angle PBA$

($\because$ PB is the bisector of ∠ABC)

$\because$ AP||BC

$\therefore \angle APB = \angle PBC$ (Alternate angles)

$\Rightarrow x = \angle PBC$ ...(i)

In Δ ABC, $\angle A = 60°$

and $\angle B = \angle C$ ($\because$ AB = AC)

But $\angle A + \angle B + \angle C = 180°$

(Angles of a triangle)

$\Rightarrow 60° + \angle B + \angle C = 180°$

$\Rightarrow \quad 60^\circ + \angle B + \angle B = 180^\circ$

$\Rightarrow 2\angle B = 180^\circ - 60^\circ = 120^\circ$

$\therefore \quad \angle B = \dfrac{120^\circ}{2} = 60^\circ$

$\Rightarrow \dfrac{1}{2}\angle B = \dfrac{60^\circ}{2} = 30^\circ \quad \Rightarrow \quad \angle PBC = 30^\circ$

$\therefore$ From (*i*),

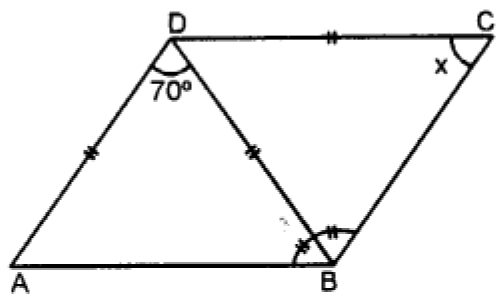
$x = 30^\circ$

(*b*) In the figure (*ii*),

DA = DB = DC

BD bisects $\angle ABC$

and $\angle ADB = 70^\circ$



But $\angle ADB + \angle DAB + \angle DBA = 180^\circ$

(Angles of a triangle)

$\Rightarrow 70^\circ + \angle DBA + \angle DBA = 180^\circ$

($\because$ DA = DB)

$\Rightarrow 70^\circ + 2\angle DBA = 180^\circ$

$\Rightarrow \quad 2\angle DBA = 180^\circ - 70^\circ$

$= 110^\circ$

$\therefore \angle DBA = \dfrac{110^\circ}{2} = 55^\circ$

$\because$ BD is the bisector of $\angle ABC$,

$\therefore \angle DBA = \angle DBC, = 55^\circ$

But in Δ DBC,

DB = DC

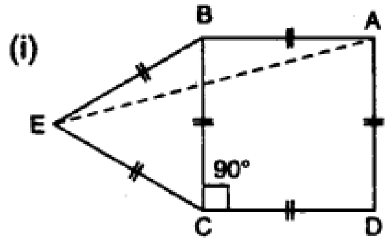
$\therefore \angle DCB = \angle DBC$

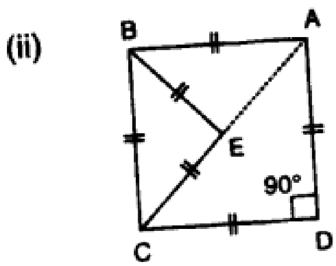
$\Rightarrow \quad x = 55^\circ$.

Question 13.

In each figure, given below, ABCD is a square and △ BEC is an equilateral triangle.

Find, in each case : (i) ∠ABE(ii) ∠BAE





Solution:

We know that the sides of a square are equal and each angle is of 90°

Three sides of an equilateral triangle are equal and each angle is of 60°. Therefore,

In fig. (*i*), ABCD is a square and Δ BEC is an equilateral triangle.

(*i*) $\angle ABE = \angle ABC + \angle CBE$

$= 90° + 60° = 150°$

(*ii*) But in Δ ABE

$\angle ABE + \angle BEA + \angle BAE = 180°$

(Angles of a triangle)

$\Rightarrow 150° + \angle BAE + \angle BAE = 180°$

$(\because AB = BE)$

$\Rightarrow 150° + 2\angle BAE = 180°$

$\Rightarrow \quad 2\angle BAE = 180° - 150° = 30°$

$\therefore \angle BAE = \frac{30°}{2} = 15°$

In figure (*ii*),

∵ ABCD is a square and Δ BEC is an equilateral triangle,

(*i*) $\therefore \angle ABE = \angle ABC - \angle CBE$

$= 90° - 60° = 30°$

(*ii*) In Δ ABE, $\angle ABE + \angle AEB + \angle BAE = 180°$

(Angles of a triangle)

$\Rightarrow 30° + \angle BAE + \angle BAE = 180°$

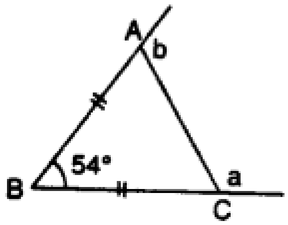
$(\because AB = BE)$

$\Rightarrow 30° + 2\angle BAE = 180°$

$\Rightarrow 2\angle BAE = 180° - 30° = 150°$

$$\Rightarrow \quad \angle BAE = \frac{150°}{2} = 75°$$

Question 14.
In $\triangle$ ABC, BA and BC are produced. Find the angles a and h. if AB = BC.



Solution:

In Δ ABC, sides BA and BC are produced

$\angle ABC = 54°$; AB = BC

Now in Δ ABC,

$\angle BAC + \angle BCA + \angle ABC = 180°$

(Angles of a triangle)

$\Rightarrow \angle BAC + \angle BAC + 54° = 180°$ ($\because$ AB = BC)

$\Rightarrow 2\angle BAC = 180° - 54°$

$\Rightarrow 2\angle BAC = 126°$

$\therefore \quad \angle BAC = \frac{126°}{2} = 63°$ and $\angle BCA = 63°$

$\angle BAC + b = 180°$ (Linear pair)

$\Rightarrow 63° + b = 180°$

$\Rightarrow \quad b = 180° - 63° = 117°$

and $\angle BCA + a = 180°$ (Linear pair)

$\therefore 63° + a = 180°$

$\Rightarrow a = 180° - 63° = 117°$

Hence $a = 117°$, $b = 117°$

EXERCISE 15 (C)

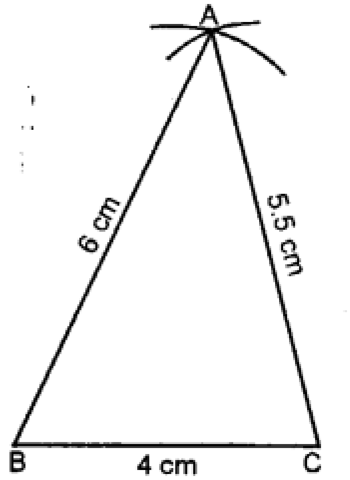
Question 1.

Construct a ∆ABC such that:
(i) AB = 6 cm, BC = 4 cm and CA = 5.5 cm
(ii) CB = 6.5 cm, CA = 4.2 cm and BA = 51 cm
(iii) BC = 4 cm, AC = 5 cm and AB = 3.5 cm

Solution:

(i) Steps of Construction :

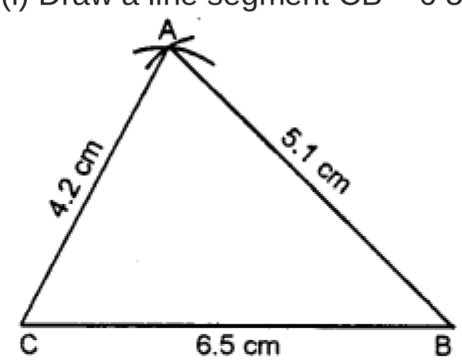
(i) Draw a line segment BC = 4 cm.



(ii) With centre B and radius 6 cm draw an arc.
(iii) With centre C and radius 5.5 cm, draw another arc intersecting the First are at A.
(iv) Join AB and AC. ∆ABC is the required triangle.

(ii) Steps of Construction :

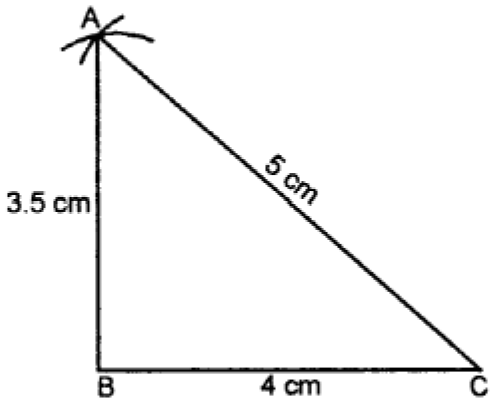
(i) Draw a line segment CB = 6 5 cm



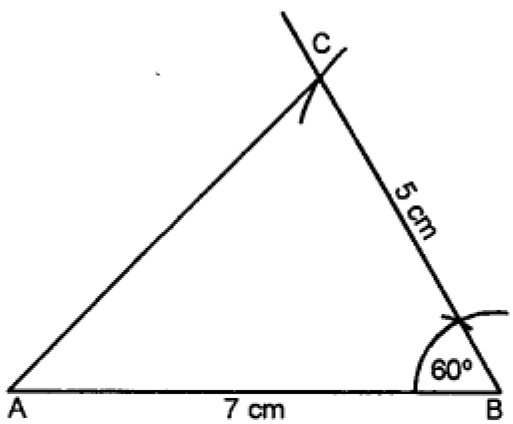
(ii) With centre C and radius 4.2 cm draw an arc.
(iii) With centre B and radius 5.1 cm draw another arc intersecting the first arc at A.
(iv) Join AC and AB.
∆ ABC is the required triangle.

(iii) Steps of Construction :

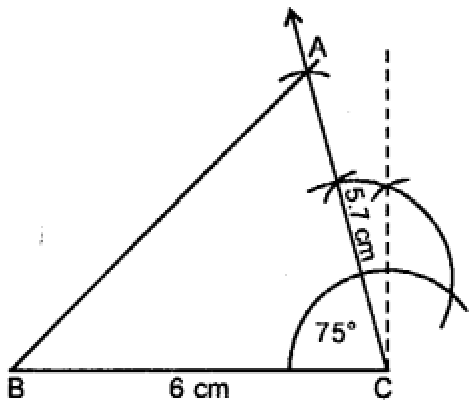
(i) Draw a line segment BC = 4 cm.
(ii) With centre B and radius 3.5 cm, draw an arc
(iii) With centre C and radius 5 cm, draw another arc which intersects the first arc at A.



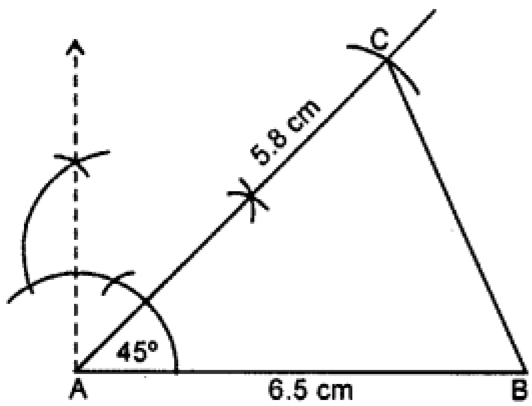
(iv) Join AB and AC.
∆ ABC is the required triangle.

Question 2.
Construct a A ABC such that:
(i) AB = 7 cm, BC = 5 cm and ∠ABC = 60°
(ii) BC = 6 cm, AC = 5.7 cm and ∠ACB = 75°
(iii) AB = 6.5 cm, AC = 5.8 cm and ∠A = 45°
Solution:
(i) Steps of Construction :
(i) Draw a line segment AB = 7 cm.


(ii) At B, draw a ray making an angle of 60° and cut off BC = 5 cm
(iii) Join AC,
∆ABC is the required triangle.
(ii) Steps of Construction :
(i) Draw a line segment BC = 6 cm.
(ii) At C, draw a ray making an angle of 75° and cut off CA = 5.7 cm.
(iii) JoinAB
∆ ABC is the required triangle.

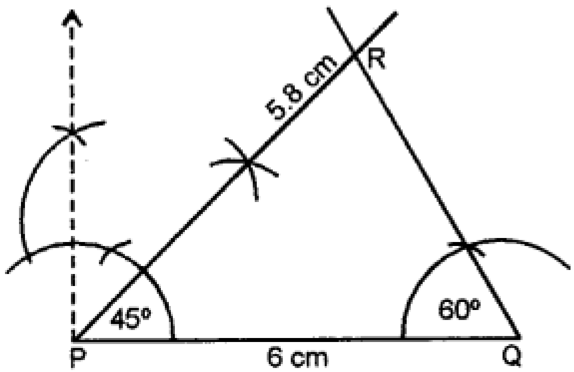


(iii) Steps of Construction :
(i) Draw a line segment AB = 6.5 cm



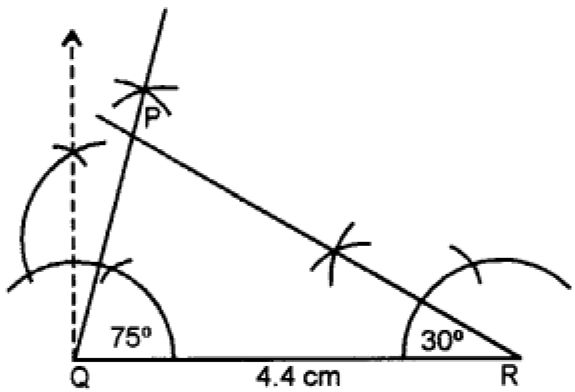
(ii) At A, draw a ray making an angle of 45° and cut off AC = 5.8 cm
(iii) JoinCB.
Δ ABC is the required triangle.

Question 3.
Construct a Δ PQR such that :
(i) PQ = 6 cm, ∠Q = 60° and ∠P = 45°. Measure ∠R.
(ii) QR = 4.4 cm, ∠R = 30° and ∠Q = 75°. Measure PQ and PR.
(iii) PR = 5.8 cm, ∠P = 60° and ∠R = 45°.
Measure ∠Q and verify it by calculations
Solution:
(i) Steps of Construction:
(i) Draw a line segment PQ = 6 cm.
(ii) At P, draw a ray making an angle of 45°
(iii) At Q, draw another ray making an angle of 60° which intersects the first ray at R.
Δ PQR is the required triangle.
On measuring ∠R, it is 75°.



(ii) Steps of Construction :

(i) Draw a line segment QR = 44 cm.



(ii) At Q, draw a ray making an angle of 75°

(iii) At R, draw another arc making an angle of 30° ; which intersects the first ray at R

∆ PQR is the required triangle.

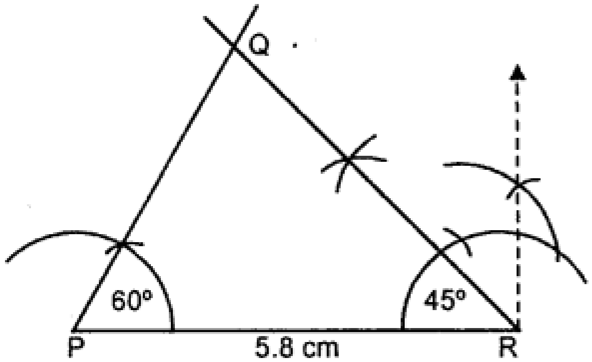
On measuring the lengths of PQ and PR, PQ = 2.1 cm and PR = 4. 4 cm.

(iii) Steps of Construction :

(i) Draw a line segment PR = 5.8 cm

(ii) At P, construct an angle of 60°

(iii) At R, draw another angle of 45° meeting each other at Q.



∆ PQR is the required triangle. On measuring ∠Q, it is 75°

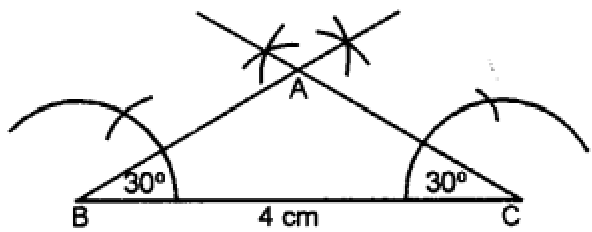
Verification : We know that sum of angles of a triangle is 180°

$\therefore \angle P + \angle Q + \angle R = 180°$

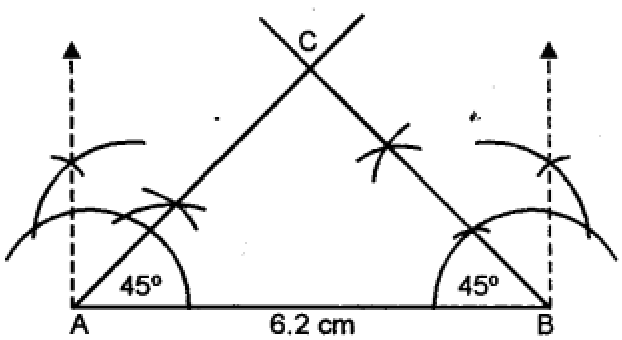
$\Rightarrow 60° + \angle Q + 45° = 180°$

$\Rightarrow \angle Q + 105° = 180°$
$\Rightarrow \angle Q = 180° – 105° = 75°$.

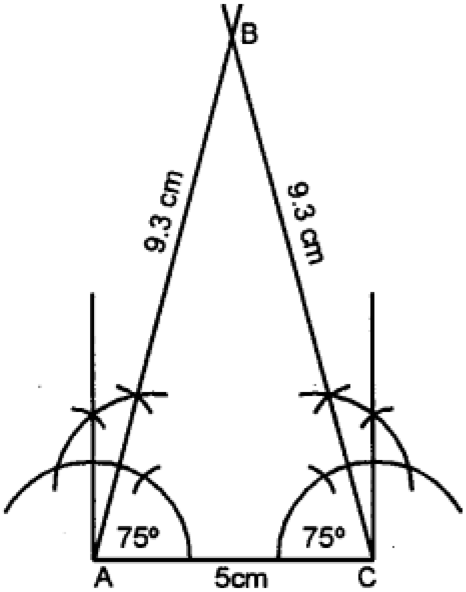
Question 4.
Construct an isosceles A ABC such that:
(i) base BC = 4 cm and base angle = 30°
(ii) base AB = 6-2 cm and base angle = 45°
(iii) base AC = 5 cm and base angle = 75°.
Measure the other two sides of the triangle.
Solution:
(i) Steps of Construction :
We know that in an isosceles triangle base angles are equal.



(i) Draw a line segment BC = 4 cm.
(ii) At B and C, draw rays making an angle of 30° each intersecting each other at A.
Δ ABC is the required triangle.
On measuring the equal sides each is 2.5 cm (approx.) in length.
(ii) Steps of Construction :
We know that in an isosceles triangle, base angles are equal.

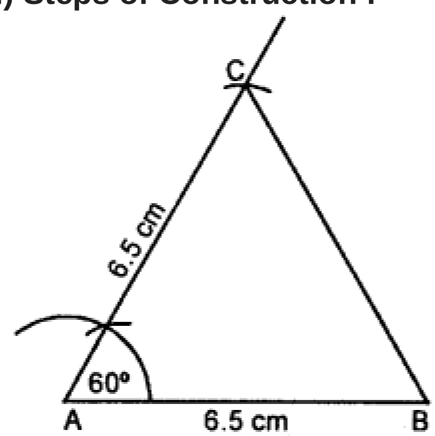


(i) Draw a line segment AB = 6.2 cm
(ii) At A and B, draw rays making an angle of 45° each which intersect each other at C.
ΔABC is the required triangle.
On measuring the equal sides, each is 4.3 cm (approx.) in length.
(iii) Steps of Construction :
We know that base angles of an isosceles triangles are equal.



(i) Draw a line segment AC = 5cm.
(ii) At A and C, draw rays making an angle of 75° each which intersect each other at B.
Δ ABC is the required triangle.
On measuring the equal sides, each is 9.3 cm in length.

Question 5.
Construct an isosceles ΔABC such that:
(i) AB = AC = 6.5 cm and ∠A = 60°
(ii) One of the equal sides = 6 cm and vertex angle = 45°. Measure the base angles.
(iii) BC = AB = 5-8 cm and ZB = 30°. Measure ∠A and ∠C.
Solution:
(i) Steps of Construction :

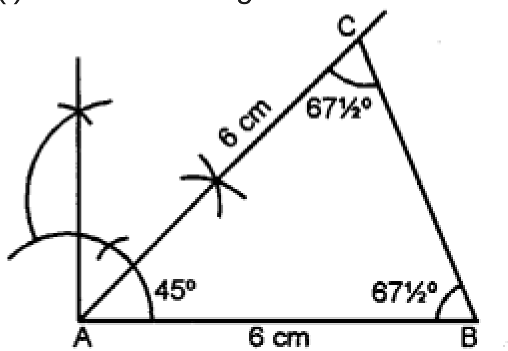


(i) Draw a line segment AB = 6.5 cm.
(ii) At A, draw a ray making an angle of 60°.
(iii) Cut off AC = 6.5 cm
(iv) JoinBC.

∆ABC is the required triangle.

(ii) Steps of Construction :

(i) Draw a line segment AB = 6 cm



(ii) At A, construct an angle equal to 45°

(iii) Cut off AC = 6 cm

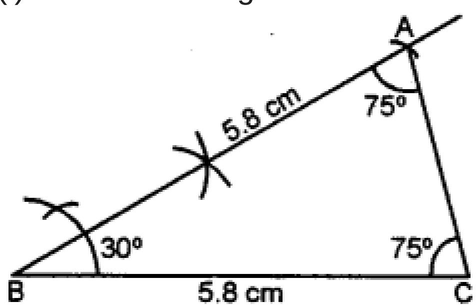
(iv) JoinBC.

∆ ABC is the required triangle.

On measuring, ∠B and ∠C, each is equal 1° to, $67\frac{1}{2}$°

(iii) Steps of Construction :

(i) Draw a line segment BC = 5.8 cm



(ii) At B, draw a ray making an angle of 30°.

(iii) Cut off BA = 5.8 cm

(iv) Join AC.

∆ ABC is the required triangle On measuring ∠C and ∠A, each is equal to 75°.

Question 6.

Construct an equilateral A ABC such that:

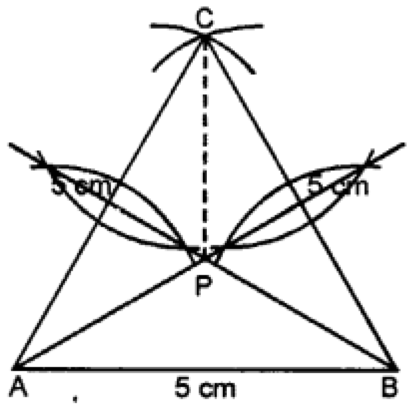
(i) AB = 5 cm. Draw the perpendicular bisectors of BC and AC. Let P be the point of intersection of these two bisectors. Measure PA, PB and PC.

(ii) Each side is 6 cm.

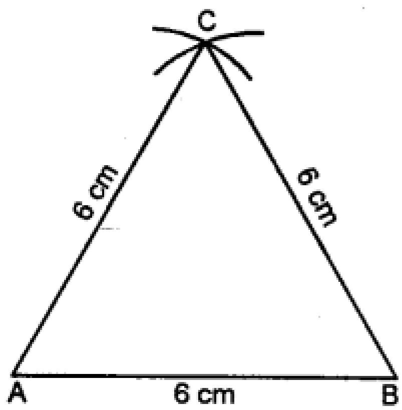
Solution:

(i) Steps of Construction :

(i) Draw a line segment AB = 5 cm.



(ii) With centres A and B and radius 5 cm each, draw two arcs intersecting each other at C.
(iii) Join AC and BC ΔABC is the required triangle.
(iv) Draw the perpendicular bisectors of sides AC and BC which intersect each other at P-
(v) Join PA, PB and PC.
On measuring, each is 2.8 cm.
(ii) Steps of Construction :
(i) Draw a line segment AB = 6 cm.

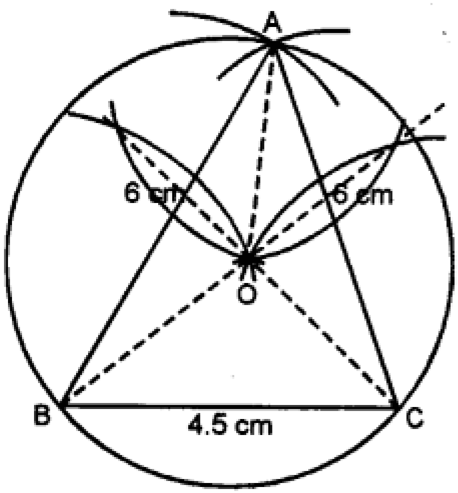


(ii) At A and B as centre and 6 cm as radius draw two arcs intersecting each other at C.
(iii) Join AC and BC.
ΔABC is the required triangle.

Question 7.
(i) Construct a Δ ABC such that AB = 6 cm, BC = 4.5 cm and AC = 5.5 cm. Construct a circumcircle of this triangle.
(ii) Construct an isosceles ΔPQR such that PQ = PR = 6.5 cm and $\angle PQR = 75°$. Using ruler and compasses only construct a circumcircle to this triangle.
(iii) Construct an equilateral triangle ABC such that its one side = 5.5 cm. Construct a circumcircle to this triangle.
Solution:

(i) Steps of Construction :

(*i*) Draw a line segment BC = 4·5 cm



(*ii*) With centre B and radius 6 cm, draw are arc

(*iii*) With centre C and radius 5·5 cm, draw another arc intersecting the first arc at A.

(*iv*) Join AB and AC.

Δ ABC is the required triangle.

(*v*) Draw the perpendicular bisectors of AB and AC. Which intersect each other at O.

(*vi*) Join OB, OC and OA.

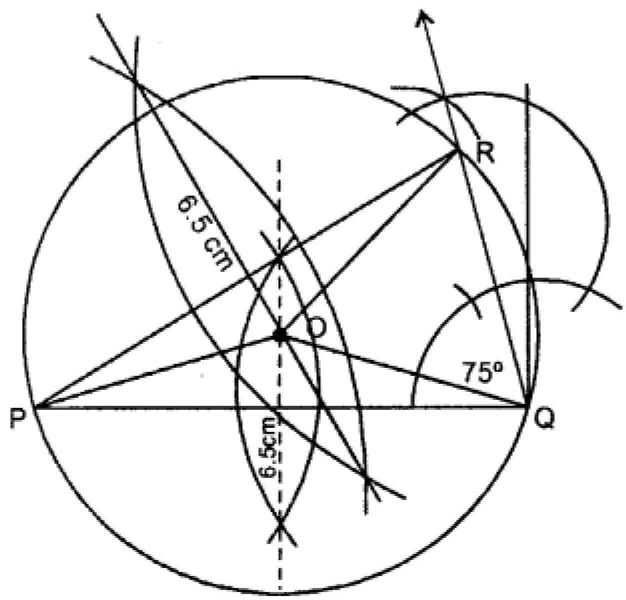
(*vii*) With centre O, and radius OA, draw a circle which passes through A, B and C.

This is the required circum circle of Δ ABC.

(ii) Steps of Construction :

(*i*) Draw a line segment PQ = 6.5 cm

(*ii*) At Q, draw a ray making an angle of 75°.



(*iii*) Through P, with a radius of 6.5 cm, draw an arc which intersects the angle ray at R.

(*iv*) Join PR,

Δ PQR is the required triangle.

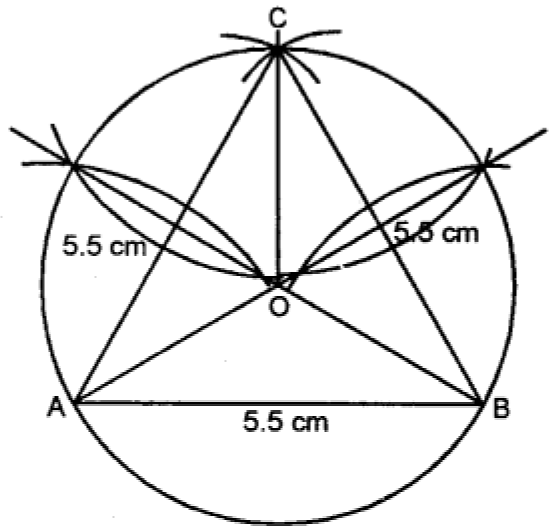
(*v*) Draw the perpendicular bisectors of sides PQ and PR intersecting each other at O.

(*vi*) Join OP, OQ and OR.

(*vii*) With centre O and radius equal to OP or OQ or OR draw a circle which passes through P,Q and R. This is the required circum circle of Δ PQR

(*iii*) Steps of Construction :

(*i*) Draw a line segment AB = 5·5 cm



(*ii*) With centres A and B and radius 5·5 cm, draw two arcs intersecting each other at C.

(*iii*) Join AC and BC.

ΔABC is the required triangle.

(*iv*) Draw perpendicular bisectors of sides AC and BC which intersect each other at O.

(*v*) Join OA, OB and OC.

(*vi*) With centre O and radius OA or OB or OC, draw a circle which passes through A, B and C. This is the required circumcircle.

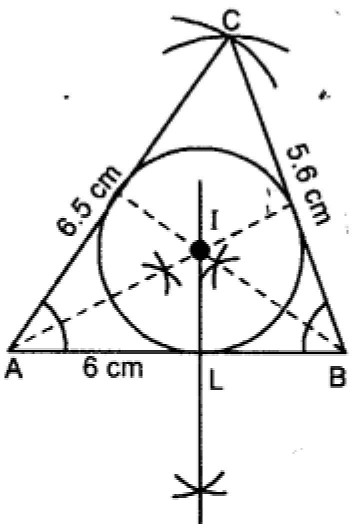
Question 8.

(i) Construct a △ABC such that AB = 6 cm, BC = 5.6 cm and CA = 6.5 cm. Inscribe a circle to this triangle and measure its radius.
(ii) Construct an isosceles △ MNP such that base MN = 5.8 cm, base angle MNP = 30°. Construct an incircle to this triangle and measure its radius.
(iii) Construct an equilateral △DEF whose one side is 5.5 cm. Construct an incircle to this triangle.
(iv) Construct a △ PQR such that PQ = 6 cm, ∠QPR = 45° and angle PQR = 60°. Locate its incentre and then draw its incircle.

Solution:

(*i*) Steps of Construction :

(*i*) Draw a line segment AB = 6 cm.



(*ii*) With centre A and radius 6·5 cm and with centre B and radius 5·6 cm, draw arcs intersecting each other at C.

(*iii*) Join AC and BC.

(*iv*) Draw the angle bisector of ∠A and ∠B intersecting each other at I.

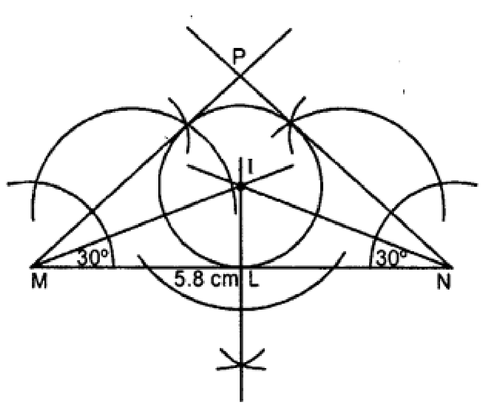
(*v*) From I, draw I L ⊥ AB

(*vi*) With centre I and radius IL, draw a circle which touches the sides of Δ ABC internally.

On measuring the required incircle whose radius is 1.6 cm.

(*ii*) Steps of Construction :

(*i*) Draw a line segment MN = 5.8 cm.



(ii) At M and N, draw two rays making an angle of 30° each which intersect each other at P.

(iii) Now draw the angle bisectors of $\angle$M and $\angle$N which intersect each other at I.

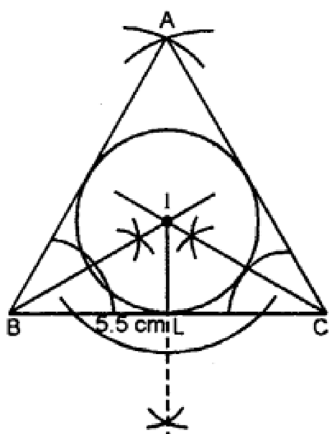
(iv) From I, draw perpendicular IL on MN.

(v) With centre I and radius IL, draw a circle which touches the sides of the Δ PMN internally.

On measuring the required incircle and its radius is 0·6 cm.

(iii) Steps of Construction :

(i) Draw a lines segment BC = 5·5 cm.



(ii) With centres B and C and radius 5·5 cm each draw two arcs intersecting each other at A.

(iii) Join AB and AC.

(iv) Draw the perpendicular bisectors of $\angle$B and $\angle$C intersecting each other at I.

(v) From I, draw IL $\perp$ BC

(vi) With centre I and radius IL, draw a circle which touches the sides of the Δ ABC internally.

This is the required incircle.

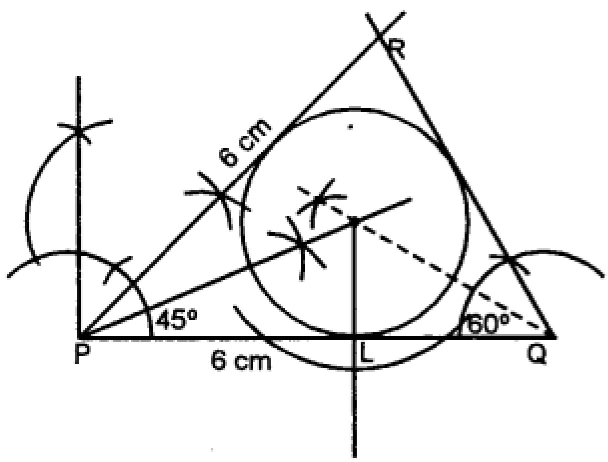
(iv) Steps of Construction :

(*i*) Draw a line segment PQ = 6 cm.

(*ii*) At P draw a rays making an angle of 45° and at Q, making an angle of 60°, intersecting each other at R.

(*iii*) Draw the bisectors of ∠P and ∠Q intersecting each other at I.

(*iv*) From I, draw IL ⊥ PQ.



(*v*) With centre I and radius IL, draw a circle which touches the sides of Δ PQR internally. This is the required incircle whose I is incentre.