Chapter 4

Percentage

PER CENT

The word "per cent" is derived from the latin words "per centum", which means "per hundred".

A **percentage** is a fraction with denominator hundred.

It is denoted by the symbol %.

Numerator of the fraction is called the **rate per cent**.

VALUE OF PERCENTAGE

Value of percentage always depends on the quantity to which it refers.

Consider the statement :

"65% of the students in this class are boys". From the context, it is understood that boys form 65% of the total number of students in the class. To know the value of 65% of the total number of students in the class, the value of the total number of boys student should be known.

If the total number of students is 200, then, the number of boys

$= \frac{200 \times 65}{100} = 130$; It can also be written as $(200) \times (0.65) = 130$.

If the total number of students is 500, then the number of boys

$$= \frac{500 \times 65}{100} = 325$$

NOTE *that the expressions 6%, 63%, 72%, 155% etc. do not have any value to themselves. Their values depend on the quantities to which they refer.*

Some Quick Results:

5% of a number $= \frac{\text{Number}}{20}$, $\quad 10\%$ of a number $= \frac{\text{Number}}{10}$

$12\frac{1}{2}\%$ of a number $= \frac{\text{Number}}{8}$, 20% of a number $= \frac{\text{Number}}{5}$

25% of a number $= \frac{\text{Number}}{4}$, 50% of a number $= \frac{\text{Number}}{2}$

To express the fraction equivalent to % :

Express the fraction with the denominator 100, then the numerator is the answer.

Fractional Equivalents of % (Percentage)

$1\% = \frac{1}{100}$	$33\frac{1}{3}\% = \frac{1}{3}$
$2\% = \frac{1}{50}$	$40\% = \frac{2}{5}$
$4\% = \frac{1}{25}$	$50\% = \frac{1}{2}$
$5\% = \frac{1}{20}$	$66\frac{2}{3}\% = \frac{2}{3}$
$6\frac{1}{4}\% = \frac{1}{16}$	$60\% = \frac{3}{5}$
$10\% = \frac{1}{10}$	$75\% = \frac{3}{4}$
$11\frac{1}{3}\% = \frac{17}{150}$	$80\% = \frac{4}{5}$
$12\frac{1}{2}\% = \frac{1}{8}$	$96\% = \frac{24}{25}$
$16\% = \frac{4}{25}$	$100\% = 1$
$16\frac{2}{3}\% = \frac{1}{6}$	$115\% = \frac{23}{20}$
$20\% = \frac{1}{5}$	$133\frac{1}{3}\% = \frac{4}{3}$
$25\% = \frac{1}{4}$	

EXPRESSING ONE QUANTITY AS A PER CENT WITH RESPECT TO OTHER

To express a quantity as a per cent with respect to other quantity following formula is used.

$$\frac{\text{The quantity to be expressed in per cent}}{\text{2nd quantity (in respect of which the per cent has to be obtained)}} \times 100\%$$

Note : *To apply this formula, both the quantities must be in same unit.*

PERCENTAGE INCREASE OR DECREASE OF A VALUE

$$\text{Increase \%} = \frac{\text{Increase value}}{\text{Original value}} \times 100\%$$

$$\text{Decrease \%} = \frac{\text{Decrease value}}{\text{Original value}} \times 100\%$$

☞ Shortcut Approach

When a number x is increased or decreased by y%, then the new number will be $\frac{100 \pm y}{100} \times x$.

NOTE : 1. '+' *sign is used in case of increase*.
2. '–' *sign is used in case of decrease*.

- If x is a% more than y, then y is $\left(\frac{a}{100+a} \times 100\right)$% less than x.
- If x is a% less than y, then y is $\left(\frac{a}{100-a} \times 100\right)$% more than x.

See Example : Refer ebook Solved Examples/Ch-4

Shortcut Approach

- If A's income is r % more than that of B, then B's income is less than that of A by

$$\left(\frac{r}{100+r}\times 100\right)\%$$

- If A's income is r % less than that of B, then B's income is more than that of A by

$$\left(\frac{r}{100-r}\times 100\right)\%$$

Shortcut Approach

- If A is x % of C and B is y % of C, then A is $\frac{x}{y}\times 100$ % of B.
- x % of a quantity is taken by the first, y % of the remaining is taken by the second and z % of the remaining is taken by third person. Now, if A is left in the fund, then the initial amount

$$=\frac{A\times 100\times 100\times 100}{(100-x)(100-y)(100-z)}$$ in the begining.

- x % of a quantity is added. Again, y % of the increased quantity is added. Again z % of the increased quantity is added. Now it becomes A, then the initial amount

$$=\frac{A\times 100\times 100\times 100}{(100+x)(100+y)(100+z)}$$

Shortcut Approach

- If the value of a number is first increased by a% and later decreased by a%, then the net effect is always a decrease which is equal to a% of a and is written as $\frac{a^2}{100}\%$ or $\left(\frac{a^2}{10}\right)\%$.

See Example : Refer ebook Solved Examples/Ch-4

☞ Shortcut Approach

- If the price of a commodity increases by r %, then reduction in consumption, so as not to increase the expenditure is $\left(\frac{r}{100+r}\times 100\right)\%$.
- If the price of a commodity decreases by r %, then the increase in consumption so as not to decrease the expenditure is $\left(\frac{r}{100-r}\times 100\right)\%$

☞ Shortcut Approach

- If due to r% decrease in the price of an item, a person can buy A kg more in ₹ x, then

  Actual price of that item = ₹ $\frac{r\,x}{(100-r)\,A}$ per kg

☞ Shortcut Approach

Population Formula

- If the original population of a town is P, and the annual increase is r %, then the population after n years is $P\left(1+\frac{r}{100}\right)^n$ and

  population before n years = $\frac{P}{\left(1+\frac{r}{100}\right)^n}$

- If the annual decrease be r %, then the population after n years is $P\left(1-\frac{r}{100}\right)^n$ and

  Population before n years = $\frac{P}{\left(1-\frac{r}{100}\right)^n}$

See Example : Refer ebook Solved Examples/Ch-4

Shortcut Approach

First Increase and then decrease :

If the value is first increased by x % and then decreased by y % then there is $\left(x - y - \frac{xy}{100}\right)\%$ increase or decrease, according to the + ve or – ve sign obtained respectively.

Average percentage rate of change over a period.

$= \frac{(\text{New Value} - \text{Old Value})}{\text{Old Value}} \times \frac{100}{n}\%$ where n = period.

The percentage error = $\frac{\text{The Error}}{\text{True Value}} \times 100\%$

Shortcut Approach

Successive increase or decrease

If the value is increased **successively by x % and y %** then the final **increase** is given by

$$\left(x + y + \frac{xy}{100}\right)\%$$

If the value is decreased **successively by x % and y %** then the final **decrease** is given by

$$\left(-x - y + \frac{xy}{100}\right)\%$$

Shortcut Approach

Student and Marks

The percentage of passing marks in an examination is x%. If a candidate who scores y marks fails by z marks, then the maximum marks $M = \frac{100(y+z)}{x}$

A candidate scoring x % in an examination fails by 'a' marks, while another candidate who scores y% marks gets 'b' marks more than the minimum required passing marks. Then the maximum marks $M = \frac{100(a+b)}{y-x}$.

See Example : Refer ebook Solved Examples/Ch-4

☞ Shortcut Approach

- If two candidates contested in an election and one candidate got x% of total votes and still lose by y votes, then

- Total number of votes casted = $\frac{100 \times y}{100 - 2x}$

2–dimensional figure and area

- If the sides of a triangle, square, rhombus or radius of a circle are increased by a%, its area is increased by $\frac{a(a+200)}{100}\%$

See Example : Refer ebook Solved Examples/Ch-4

ebooks Reference		*Page No.*
Solved Examples	–	S-12-15
Exercises with Hints & Solutions	–	E-29-34
Chapter Test	–	7-8
Past Solved Papers		