

Ex 1.1

Relations Ex 1.1 Q1(i)

A be the set of human beings.

$R = \{(x, y) : x \text{ and } y \text{ work at the same place}\}$

Reflexive:

Δ x and x works together

Δ $(x, x) \in R$

$\Rightarrow R$ is reflexive

Symmetric: If x and y work at the same place, which implies,
 y and x work at the same place

Δ $(y, x) \in R$

$\Rightarrow R$ is symmetric

Transitive: If x and y work at the same place
then x and y work at the same place and y and z work at the same place

$\Rightarrow (x, z) \in R$ and

Hence,

$\Rightarrow R$ is transitive

Relations Ex 1.1 Q1(ii)

.A be the set of human beings.

$$R = \{(x, y) : x \text{ and } y \text{ lives in the same locality}\}$$

Reflexive: since x and x lives in the same locality

$$\Rightarrow (x, x) \in R$$

$\Rightarrow R$ is reflexive

Symmetric: Let $(x, y) \in R$

$\Rightarrow x$ and y lives in the same locality

$\Rightarrow y$ and x lives in the same locality

$$\Rightarrow (y, x) \in R$$

Transitive: Let $(x, y) \in R$ and $(y, z) \in R$

$$(x, y) \in R$$

$\Rightarrow x$ and y lives in the same locality

and $(y, z) \in R$

$\Rightarrow y$ and z lives in the same locality

$\Rightarrow x$ and z lives in the same locality

$$\Rightarrow (x, z) \in R$$

$\Rightarrow R$ is transitive

Relations Ex 1.1 Q1(iii)

$$R = \{(x, y) : x \text{ is wife of } y\}$$

Reflexive: since x can not be wife of x

$$\therefore (x, x) \notin R$$

$\Rightarrow R$ is not reflexive

Symmetric: Let $(x, y) \in R$

$\Rightarrow x$ is wife of y

$\Rightarrow y$ is husband of x

$$\Rightarrow (y, x) \notin R$$

$\Rightarrow R$ is not symmetric

Transitive: Let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow x$ is wife of y and y is husband of z
which is a contradiction

$$\Rightarrow (x, z) \notin R$$

$\Rightarrow R$ is not transitive

Relations Ex 1.1 Q1(iv)

A be the set of human beings

$R = \{(x, y) : x \text{ is father of } y\}$

Reflexive: since x can not be father of x

$\therefore (x, x) \notin R$

$\Rightarrow R$ is not reflexive

Symmetric: Let $(x, y) \in R$

$\Rightarrow x$ is father of y

$\Rightarrow y$ can not be father of x

$\Rightarrow (y, x) \notin R$

$\Rightarrow R$ is not symmetric

Transitive: Let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow x$ is father of y and y is father of z

$\Rightarrow x$ is grandfather of z

$\Rightarrow (x, z) \notin R$

$\Rightarrow R$ is not transitive

Relations Ex 1.1 Q2

We have, $A = \{a, b, c\}$

$R_1 = \{(a, a)(a, b)(a, c)(b, b)(b, c)(c, a)(c, b)(c, c)\}$

R_1 is reflexive as $(a, a) \in R_1, (b, b) \in R_1$ & $(c, c) \in R_1$

R_1 is not symmetric as $(a, b) \in R_1$ but $(b, a) \notin R_1$

R_1 is not transitive as $(b, c) \in R_1$ and $(c, a) \in R_1$ but $(b, a) \notin R_1$

$R_2 = \{(a, a)\}$

R_2 is not reflexive as $(b, b) \notin R_2$

R_2 is symmetric and transitive.

$R_3 = \{(b, c)\}$

R_3 is not reflexive as $(b, b) \notin R_3$

R_3 is not symmetric

R_3 is not transitive.

$R_4 = \{(a, b)(b, c)(c, a)\}$

R_4 is not reflexive on set A as $(a, a) \notin R_4$

R_4 is not symmetric as $(a, b) \in R_4$ but $(b, a) \notin R_4$

R_4 is not transitive as $(a, b) \in R_4$ and $(b, c) \in R_4$ but $(a, c) \notin R_4$

Relations Ex 1.1 Q3

$$R_1 = \left\{ (x, y), x, y \in Q_0, x = \frac{1}{y} \right\}$$

Reflexivity: Let, $x \in Q_0$

$$\Rightarrow x \neq \frac{1}{x}$$

$$\Rightarrow (x, x) \notin R_1$$

$\therefore R_1$ is not reflexive

Symmetric: Let, $(x, y) \in R_1$

$$\Rightarrow x = \frac{1}{y}$$

$$\Rightarrow y = \frac{1}{x}$$

$$\Rightarrow (y, x) \in R_1$$

$\therefore R_1$ is symmetric

Transitive: Let, $(x, y) \in R_1$ and $(y, z) \in R_1$

$$\Rightarrow x = \frac{1}{y} \text{ and } y = \frac{1}{z}$$

$$\Rightarrow x = z$$

$$\Rightarrow (x, z) \notin R_1$$

$\therefore R_1$ is not transitive

Relations Ex 1.1 Q3(ii)

Reflexivity: Let, $a \in \mathbb{Z}$

$$\Rightarrow |a - a| = 0 \leq 5$$

$\therefore (a, a) \in R_2 \Rightarrow R_2$ is reflexive

Symmetry: Let, $(a, b) \in R_2$

$$\Rightarrow |a - a| \leq 5$$

$$\Rightarrow |b - a| \leq 5$$

$$\Rightarrow |b, a| \in R_2 \Rightarrow R_2 \text{ is symmetric}$$

Transitivity: Let, $(a, b) \in R_2$ and $(b, c) \in R_2$

$$\Rightarrow |a - b| \leq 5 \text{ and } |b - c| \leq 5$$

$$\nRightarrow |a - c| \leq 5$$

$$\Rightarrow R_2 \text{ is not transitive}$$

$$\left[\begin{array}{l} \therefore \text{ if } a = 15, b = 11, c = 7 \\ \Rightarrow |15 - 11| \leq 5 \text{ and } |11 - 7| \leq 5 \\ \text{but } |15 - 7| \geq 5 \end{array} \right]$$

Relations Ex 1.1 Q4

(i) We have, $A = \{1, 2, 3\}$ and
 $R_1 = \{(1,1)(1,3)(3,1)(2,2)(2,1)(3,3)\}$

$\therefore (1,1), (2,2)$ and $(3,3) \in R_1$

$\therefore R_1$ is not Reflexive

Now,

$\therefore (2,1) \in R_1$ but $(1,2) \notin R_1$

$\therefore R_1$ is not Symmetric

Again,

$\therefore (2,1) \in R_1$ and $(1,3) \in R_1$ but $(2,3) \notin R_1$

$\therefore R_1$ is not Transitive

(ii) $R_2 = \{(2,2), (3,1), (1,3)\}$

$\therefore (1,1) \notin R_2$

$\Rightarrow R_2$ is not reflexive

Now, $(1,3) \in R_2$

$\Rightarrow (3,1) \in R_2$

$\Rightarrow R_2$ is symmetric

Again, $(3,1) \in R_2$ and $(1,3) \in R_2$ but $(3,3) \notin R_2$

$\therefore R_2$ is not transitive

(iii) $R_3 = \{(1,3)(3,3)\}$

$\therefore (1,1) \notin R_3$

$\Rightarrow R_3$ is not reflexive

Now, $(1,3) \in R_3$ but $(3,1) \notin R_3$

$\Rightarrow R_3$ is not symmetric

Again, It is clear that R_3 is transitive

Relations Ex 1.1 Q5.

(i) aRb if $a-b > 0$

Let R be the set of real numbers.

Reflexivity: Let $a \in R$

$$\Rightarrow a - a = 0$$

$$\Rightarrow (a, a) \notin R$$

$\therefore R$ is not reflexive

Symmetric: Let aRb

$$\Rightarrow a - b > 0$$

$$\Rightarrow b - a < 0$$

$$\therefore b \not R a$$

$\therefore R$ is not Symmetric

Transitive: Let aRb and bRc

$$\Rightarrow a - b > 0 \text{ and } b - c > 0$$

$$\Rightarrow a - c > 0$$

$$\Rightarrow aRc$$

$\therefore R$ is Transitive

Relations Ex 1.1 Q5(ii)

We have, aRb iff $1 + ab > 0$

Let R be the set of real numbers

Reflexive: Let $a \in R$

$$\Rightarrow 1 + a^2 > 0$$

$$\Rightarrow aRa$$

$\Rightarrow R$ is reflexive

Symmetric: Let aRb

$$\Rightarrow 1 + ab > 0$$

$$\Rightarrow 1 + ba > 0$$

$$\Rightarrow bRa$$

$\Rightarrow R$ is symmetric

Transitive: Let aRb and bRc

$$\Rightarrow 1 + ab > 0 \text{ and } 1 + bc > 0$$

$$\nRightarrow 1 + ac > 0$$

$\Rightarrow R$ is not transitive

Relations Ex 1.1 Q5(iii)

We have, aRb if $|a| \leq b$

Reflexivity: Let $a \in R$

$$\Rightarrow |a| \not\leq a \quad \left[\because \quad |-2| = 2 > -2 \right]$$

$\Rightarrow R$ is not reflexive

Symmetric: Let aRb

$$\Rightarrow |a| \leq b$$

$$\nRightarrow |b| \leq a \quad \left[\because \quad \begin{array}{l} \text{Let } a = 4, \quad b = 6 \\ |4| \leq 6 \text{ but } |6| > 4 \end{array} \right]$$

$\Rightarrow R$ is not symmetric

Transitive: Let aRb and bRc

$$\Rightarrow |a| \leq b \text{ and } |b| \leq c$$

$$\Rightarrow |a| \leq |b| \leq c$$

$$\Rightarrow |a| \leq c$$

$$\Rightarrow aRc$$

$\Rightarrow R$ is transitive

Relations Ex 1.1 Q6.

Let $A = \{1, 2, 3, 4, 5, 6\}$.

A relation R is defined on set A as:

$$R = \{(a, b) : b = a + 1\}$$

$$\text{Therefore, } R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$$

We find $(a, a) \notin R$, where $a \in A$.

For instance, $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \notin R$

Therefore, R is not reflexive.

It can be observed that $(1, 2) \in R$, but $(2, 1) \notin R$.

Therefore, R is not symmetric.

Now, $(1, 2), (2, 3) \in R$

But, $(1, 3) \notin R$

Therefore, R is not transitive

Hence, R is neither reflexive, nor symmetric, nor transitive.

Relations Ex 1.1 Q7.

$$R = \{(a, b) : a \leq b^3\}$$

It is observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$ as $\frac{1}{2} > \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

Therefore, R is not reflexive.

Now, $(1, 2) \in R$ (as $1 < 2^3 = 8$)

But, $(2, 1) \notin R$ (as $2^3 > 1$)

Therefore, R is not symmetric.

We have

$$\left(3, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{6}{5}\right) \in R \text{ as } 3 < \left(\frac{3}{2}\right)^3 \text{ and } \frac{3}{2} < \left(\frac{6}{5}\right)^3.$$

$$\text{But } \left(3, \frac{6}{5}\right) \notin R \text{ as } 3 > \left(\frac{6}{5}\right)^3.$$

Therefore, R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Relations Ex 1.1 Q8

Let A be a set.

Then $I_A = \{(a, a) ; a \in A\}$ is the identity relation on A .

Hence, every identity relation on a set is reflexive by definition.

Converse:

Let $A = \{(a, b, c)\}$ be a set.

Let $R = \{(a, a) (b, b) (c, c) (a, b)\}$ be a relation defined on A .

Clearly R is reflexive on set A , but it is not identity relation on set A as $(a, b) \in R$

Hence, a reflexive relation need not be identity relation.

Relations Ex 1.1 Q9

We have, $A = \{1, 2, 3, 4\}$

(i) $R = \{(1, 1) (2, 2) (3, 3) (4, 4) (1, 2)\}$ is a relation on set A which is reflexive, transitive but not symmetric

(ii) $R = \{(2, 3) (3, 2)\}$ is a relation on set A which is symmetric but neither reflexive nor transitive

(iii) $R = \{(1, 1) (2, 2) (3, 3) (4, 4) (1, 2) (2, 1)\}$ is a relation on set A which is reflexive, symmetric and transitive

Relations Ex 1.1 Q10

We have, $R = \{(x, y) ; x, y \in N, 2x + y = 41\}$

Then Domain of R is $x \in N$, such that

$$2x + y = 41$$

$$\Rightarrow x = \frac{41 - y}{2}$$

Since $y \in N$, largest value that x can take corresponds to the smallest value that y can take.

$$\therefore x = \{1, 2, 3, \dots, 20\}$$

Range of R is $y \in N$ such that

$$2x + y = 41$$

$$\Rightarrow y = 41 - 2x$$

Since, $x = \{1, 2, 3, \dots, 20\}$

$$\therefore y = \{39, 37, 35, 33, \dots, 7, 5, 3, 1\}$$

Since, $(2, 2) \notin R$, R is not reflexive.

Also, since $(1, 39) \in R$ but $(39, 1) \notin R$, R is not symmetric.

Finally, since, $(15, 11) \in R$ and $(11, 19) \in R$ but $(15, 19) \notin R$

$\therefore R$ is not transitive.

Relations Ex 1.1 Q11

No, it is not necessary that a relation which is symmetric and transitive is reflexive as well.

For Example,

Let $A = \{a, b, c\}$ be a set and

$R_2 = \{(a, a)\}$ is a relation defined on A .

Clearly,

R_2 is symmetric and transitive but not reflexive.

Relations Ex 1.1 Q12

It is given that an integer m is said to be relative to another integer n if m is a multiple of n .

In other words

$$R = \{(m, n); \quad m = kn, k \in \mathbb{Z}\}$$

Reflexivity: Let, $m \in \mathbb{Z}$

$$\Rightarrow m = 1.m$$

$$\Rightarrow (m, m) \in R$$

$\therefore R$ is reflexive

Transitive: Let $(a, b) \in R$ and $(b, c) \in R$

$$\Rightarrow a = kb \quad \text{and} \quad b = k'c$$

$$\Rightarrow a = kk'c \quad [\because \quad kk' \in \mathbb{Z}]$$

$$\Rightarrow a = lc \quad [\because \quad l = kk' \in \mathbb{Z}]$$

$$\Rightarrow (a, c) \in R$$

$\therefore R$ is transitive

Symmetric: Let $(a, b) \in R$

$$\Rightarrow a = kb$$

$$\Rightarrow b = \frac{1}{k}a \quad \text{but } \frac{1}{k} \notin \mathbb{Z} \text{ if } k \in \mathbb{Z}$$

$$\therefore (b, a) \notin R$$

$\therefore R$ is not symmetric

Relations Ex 1.1 Q13

We have,

relation $R = " \geq "$ on the set R of all real numbers

Reflexivity: Let $a \in R$

$$\Rightarrow a \geq a$$

$$\Rightarrow " \geq " \text{ is reflexive}$$

Symmetric: Let $a, b \in R$

such that $a \geq b \not\Rightarrow b \geq a$

$\therefore " \geq " \text{ not symmetric}$

Transitivity: Let $a, b, c \in R$

and $a \geq b$ & $b \geq c$

$$\Rightarrow a \geq c$$

$$\Rightarrow " \geq " \text{ is transitive}$$

Relations Ex 1.1 Q14

(i) Let $A = \{4, 6, 8\}$.

Define a relation R on A as:

$$A = \{(4, 4), (6, 6), (8, 8), (4, 6), (6, 4), (6, 8), (8, 6)\}$$

Relation R is reflexive since for every $a \in A$, $(a, a) \in R$ i.e., $(4, 4), (6, 6), (8, 8) \in R$.

Relation R is symmetric since $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in R$.

Relation R is not transitive since $(4, 6), (6, 8) \in R$, but $(4, 8) \notin R$.

Hence, relation R is reflexive and symmetric but not transitive.

(ii) Define a relation R in $\mathbf{R}$ as:

$$R = \{a, b\}: a^3 \geq b^3\}$$

Clearly $(a, a) \in R$ as $a^3 = a^3$.

$$a = a.$$

Therefore, R is reflexive.

Now, $(2, 1) \in R$ (as $2^3 \geq 1^3$)

But, $(1, 2) \notin R$ (as $1^3 < 2^3$)

Therefore, R is not symmetric.

Now, Let $(a, b), (b, c) \in R$.

$$\Rightarrow a^3 \geq b^3 \text{ and } b^3 \geq c^3$$

$$\Rightarrow a^3 \geq c^3$$

$$\Rightarrow (a, c) \in R$$

Therefore, R is transitive.

Hence, relation R is reflexive and transitive but not symmetric.

Hence, relation R is transitive but not reflexive and symmetric.

(iv) Let $A = \{5, 6, 7\}$.

Define a relation R on A as $R = \{(5, 6), (6, 5)\}$.

Relation R is not reflexive as $(5, 5), (6, 6), (7, 7) \notin R$.

Now, as $(5, 6) \in R$ and also $(6, 5) \in R$, R is symmetric.

$$\Rightarrow (5, 6), (6, 5) \in R, \text{ but } (5, 5) \notin R$$

Therefore, R is not transitive.

Hence, relation R is symmetric but not reflexive or transitive.

(v) Consider a relation R in $\mathbf{R}$ defined as:

$$R = \{a, b\}: a < b\}$$

For any $a \in \mathbf{R}$, we have $(a, a) \notin R$ since a cannot be strictly less than a itself. In fact, $a = a$.

Therefore, R is not reflexive.

Now, $(1, 2) \in R$ (as $1 < 2$)

But, 2 is not less than 1 .

Therefore, $(2, 1) \notin R$

Therefore, R is not symmetric.

Now, let $(a, b), (b, c) \in R$.

$$\Rightarrow a < b \text{ and } b < c$$

$$\Rightarrow a < c$$

$$\Rightarrow (a, c) \in R$$

Therefore, R is transitive.

Hence, relation R is transitive but not reflexive and symmetric.

Relations Ex 1.1 Q15

We have,

$$A = \{1, 2, 3\} \text{ and } R = \{(1, 2), (2, 3)\}$$

Now,

To make R reflexive, we will add $(1, 1), (2, 2)$ and $(3, 3)$ to get

$$\therefore R' = \{(1, 2), (2, 3), (1, 1), (2, 2), (3, 3)\} \text{ is reflexive}$$

Again to make R' symmetric we shall add $(3, 2)$ and $(2, 1)$

$$\therefore R'' = \{(1, 2), (2, 3), (1, 1), (2, 2), (3, 3), (3, 2), (2, 1)\} \text{ is reflexive and symmetric}$$

Now,

To make R'' transitive we shall add $(1, 3)$ and $(3, 1)$

$$\therefore R''' = \{(1, 2), (2, 3), (1, 1), (2, 2), (3, 3), (3, 2), (2, 1), (1, 3), (3, 1)\}$$

$$\therefore R''' \text{ is reflexive, symmetric and transitive}$$

Relations Ex 1.1 Q16

We have, $A = \{1, 2, 3\}$ and $R = \{(1, 2) \{1, 1\} \{2, 3\}\}$

To make R transitive we shall add $\{1, 3\}$ only.

$$\therefore R' = \{(1, 2) \{1, 1\} \{2, 3\} \{1, 3\}\}$$

Relations Ex 1.1 Q17

A relation R in A is said to be reflexive if aRa for all $a \in A$

R is said to be transitive if aRb and $bRc \Rightarrow aRc$

for all $a, b, c \in A$.

Hence for R to be reflexive (b, b) and (c, c) must be there in the set R .

Also for R to be transitive (a, c) must be in R because $(a, b) \in R$ and $(b, c) \in R$ so (a, c) must be in R .

So at least 3 ordered pairs must be added for R to be reflexive and transitive.

Relations Ex 1.1 Q18

A relation R in A is said to be reflexive if aRa for all $a \in A$, R is symmetric if $aRb \Rightarrow bRa$, for all $a, b \in A$ and it is said to be transitive if aRb and $bRc \Rightarrow aRc$ for all $a, b, c \in A$.

• $x > y, x, y \in \mathbb{N}$

$(x, y) \in \{(2, 1), (3, 1), \dots, (3, 2), (4, 2), \dots\}$

This is not reflexive as $(1, 1), (2, 2), \dots$ are absent.

This is not symmetric as $(2, 1)$ is present but $(1, 2)$ is absent.

This is transitive as $(3, 2) \in R$ and $(2, 1) \in R$ also $(3, 1) \in R$, similarly this property satisfies all cases.

• $x + y = 10, x, y \in \mathbb{N}$

$(x, y) \in \{(1, 9), (9, 1), (2, 8), (8, 2), (3, 7), (7, 3), (4, 6), (6, 4), (5, 5)\}$

This is not reflexive as $(1, 1), (2, 2), \dots$ are absent.

This only follows the condition of symmetric set as $(1, 9) \in R$ also $(9, 1) \in R$ similarly other cases are also satisfy the condition.

This is not transitive because $\{(1, 9), (9, 1)\} \in R$ but $(1, 1)$ is absent.

• xy is square of an integer, $x, y \in \mathbb{N}$

$(x, y) \in \{(1, 1), (2, 2), (4, 1), (1, 4), (3, 3), (9, 1), (1, 9), (4, 4), (2, 8), (8, 2), (16, 1), (1, 16), \dots\}$

This is reflexive as $(1, 1), (2, 2), \dots$ are present.

This is also symmetric because if $aRb \Rightarrow bRa$, for all $a, b \in \mathbb{N}$.

This is transitive also because if aRb and $bRc \Rightarrow aRc$ for all $a, b, c \in \mathbb{N}$.

• $x + 4y = 10, x, y \in \mathbb{N}$

$(x, y) \in \{(6, 1), (2, 2)\}$

This is not reflexive as $(1, 1), (2, 2), \dots$ are absent.

This is not symmetric because $(6, 1) \in R$ but $(1, 6)$ is absent.

This is not transitive as there are only two elements in the set having no element common.

Ex 1.2

Relations Ex 1.2 Q1

We have,

$$R = \{(a, b) : a - b \text{ is divisible by } 3; a, b, \in \mathbb{Z}\}$$

To prove: R is an equivalence relation

Proff:

Reflexivity: Let $a \in \mathbb{Z}$

$$\Rightarrow a - a = 0$$

$$\Rightarrow a - a \text{ is divisible by } 3$$

$$\Rightarrow (a, a) \in R$$

$$\Rightarrow R \text{ is reflexive}$$

Symmetric: Let $a, b \in \mathbb{Z}$ and $(a, b) \in R$

$$\Rightarrow a - b \text{ is divisible by } 3$$

$$\Rightarrow a - b = 3p \quad \text{For some } p \in \mathbb{Z}$$

$$\Rightarrow b - a = 3 \times (-p)$$

$$\Rightarrow b - a \in R$$

$$\Rightarrow R \text{ is symmetric}$$

Transitive: Let $a, b, c \in \mathbb{Z}$ and such that $(a, b) \in R$ and $(b, c) \in R$

$$\Rightarrow a - b = 3p \quad \text{and} \quad b - c = 3q \quad \text{For some } p, q \in \mathbb{Z}$$

$$\Rightarrow a - c = 3(p + q)$$

$$\Rightarrow a - c \text{ is divisible by } 3$$

$$\Rightarrow (a, c) \in R$$

$$\Rightarrow R \text{ is transitive}$$

Since, R is reflexive, symmetric and transitive, so R is equivalence relation.

Relations Ex 1.2 Q2

We have,

$$R = \{(a,b) : a-b \text{ is divisible by } 2; a,b, \in \mathbb{Z}\}$$

To prove: R is an equivalence relation

Proff:

Reflexivity: Let $a \in \mathbb{Z}$

$$\Rightarrow a - a = 0$$

$$\Rightarrow a - a \text{ is divisible by } 2$$

$$\Rightarrow (a,a) \in R$$

$$\Rightarrow R \text{ is reflexive}$$

Symmetric: Let $a,b \in \mathbb{Z}$ and $(a,b) \in R$

$$\Rightarrow a - b \text{ is divisible by } 2$$

$$\Rightarrow a - b = 2p \quad \text{For some } p \in \mathbb{Z}$$

$$\Rightarrow b - a = 2 \times (-p)$$

$$\Rightarrow b - a \in R$$

$$\Rightarrow R \text{ is symmetric}$$

Transitive: Let $a,b,c \in \mathbb{Z}$ and such that $(a,b) \in R$ and $(b,c) \in R$

$$\Rightarrow a - b = 2p \text{ and } b - c = q \text{ For some } p,q \in \mathbb{Z}$$

$$\Rightarrow a - c = 2(p+q)$$

$$\Rightarrow a - c \text{ is divisible by } 2$$

$$\Rightarrow (a,c) \in R$$

$$\Rightarrow R \text{ is transitive}$$

Relations Ex 1.2 Q3

We have,

$$R = \{(a,b) : (a-b) \text{ is divisible by } 5\} \text{ on } \mathbb{Z}.$$

We want to prove that R is an equivalence relation on $\mathbb{Z}$.

Now,

Reflexivity: Let $a \in \mathbb{Z}$

$$\Rightarrow a - a = 0$$

$$\Rightarrow a - a \text{ is divisible by } 5.$$

$$\therefore (a,a) \in R, \text{ so } R \text{ is reflexive}$$

Symmetric: Let $(a,b) \in R$

$$\Rightarrow a - b = 5p \quad \text{For some } p \in \mathbb{Z}$$

$$\Rightarrow b - a = 5 \times (-p)$$

$$\Rightarrow b - a \text{ is divisible by } 5$$

$$\Rightarrow (b,a) \in R, \text{ so } R \text{ is symmetric}$$

Transitive: Let $(a,b) \in R$ and $(b,c) \in R$

$$\Rightarrow a - b = 5p \text{ and } b - c = 5q \text{ For some } p,q \in \mathbb{Z}$$

$$\Rightarrow a - c = 5(p+q)$$

$$\Rightarrow a - c \text{ is divisible by } 5.$$

$$\Rightarrow R \text{ is transitive.}$$

Thus, R being reflexive, symmetric and transitive on $\mathbb{Z}$.

Hence, R is equivalence relation on $\mathbb{Z}$

Relations Ex 1.2 Q4

$R = \{(a, b) : a - b \text{ is divisible by } n\}$ on $\mathbb{Z}$.

Now,

Reflexivity: Let $a \in \mathbb{Z}$

$$\Rightarrow a - a = 0 \times n$$

$$\Rightarrow a - a \text{ is divisible by } n$$

$$\Rightarrow (a, a) \in R$$

$$\Rightarrow R \text{ is reflexive}$$

Symmetric: Let $(a, b) \in R$

$$\Rightarrow a - b = np \quad \text{For some } p \in \mathbb{Z}$$

$$\Rightarrow b - a = n(-p)$$

$$\Rightarrow b - a \text{ is divisible by } n$$

$$\Rightarrow (b, a) \in R$$

$$\Rightarrow R \text{ is symmetric}$$

Transitive: Let $(a, b) \in R$ and $(b, c) \in R$

$$\Rightarrow a - b = xp \quad \text{and} \quad b - c = xq \quad \text{For some } p, q \in \mathbb{Z}$$

$$\Rightarrow a - c = n(p + q)$$

$$\Rightarrow a - c \text{ is divisible by } n$$

$$\Rightarrow (a, c) \in R$$

$$\Rightarrow R \text{ is transitive}$$

Thus, R being reflexive, symmetric and transitive on $\mathbb{Z}$.

Hence, R is an equivalence relation on $\mathbb{Z}$

Relations Chapter 1 Ex 1.2 Q5

We have, $\mathbb{Z}$ be set of integers and

$R = \{(a,b) : a,b \in \mathbb{Z} \text{ and } a+b \text{ is even}\}$ be a relation on $\mathbb{Z}$.

We want to prove that R is an equivalence relation on $\mathbb{Z}$.

Now,

Reflexivity: Let $a \in \mathbb{Z}$

$$\Rightarrow a+a \text{ is even} \quad \left[\begin{array}{l} \text{if } a \text{ is even} \Rightarrow a+a \text{ is even} \\ \text{if } a \text{ is odd} \Rightarrow a+a \text{ is even} \end{array} \right]$$

$$\Rightarrow (a,a) \in R$$

$$\Rightarrow R \text{ is reflexive}$$

Symmetric: Let $a,b \in \mathbb{Z}$ and $(a,b) \in R$

$$\Rightarrow a+b \text{ is even}$$

$$\Rightarrow b+a \text{ is even}$$

$$\Rightarrow (b,a) \in R,$$

$$\Rightarrow R \text{ is symmetric}$$

Transitivity: Let $(a,b) \in R$ and $(b,c) \in R$ For some $a,b,c \in \mathbb{Z}$

$$\Rightarrow a+b \text{ is even and } b+c \text{ is even}$$

$$\Rightarrow a+c \text{ is even} \quad \left[\begin{array}{l} \text{if } b \text{ is odd, then } a \text{ and } c \text{ must be odd} \Rightarrow a+c \text{ is even,} \\ \text{If } b \text{ is even, then } a \text{ and } c \text{ must be even} \Rightarrow a+c \text{ is even} \end{array} \right]$$

$$\Rightarrow (a,c) \in R$$

$$\Rightarrow R \text{ is transitive}$$

Hence, R is an equivalence relation on $\mathbb{Z}$

Relations Ex 1.2 Q6

Let Z be set of integers

$R = \{(m, n) : m - n \text{ is divisible by } 13\}$ be a relation on Z .

Now,

Reflexivity: Let $m \in Z$

- $\Rightarrow m - m = 0$
- $\Rightarrow m - m$ is divisible by 13
- $\Rightarrow (m, m) \in R,$
- $\Rightarrow R$ is reflexive

Symmetric: Let $m, n \in Z$ and $(m, n) \in R$

- $\Rightarrow m - n = 13p$ For some $p \in Z$
- $\Rightarrow n - m = 13 \times (-p)$
- $\Rightarrow n - m$ is divisible by 13
- $\Rightarrow (n, m) \in R,$
- so
- $\Rightarrow R$ is symmetric

Transitivity: Let $(m, n) \in R$ and $(n, q) \in R$ For some $m, n, q \in Z$

- $\Rightarrow m - n = 13p$ and $n - q = 13s$ For some $p, s \in Z$
- $\Rightarrow m - q = 13(p + s)$
- $\Rightarrow m - q$ is divisible by 13
- $\Rightarrow (m, q) \in R$
- $\Rightarrow R$ is transitive

Hence, R is an equivalence relation on Z

Relations Ex 1.2 Q7

$$(x, y) R (u, v) \Leftrightarrow xv = yu$$

TPT Reflexive $\therefore xy = yx$

$$\therefore (x, y) R (x, y)$$

TPT Symmetric Let $(x, y) R (u, v)$

TPT $(u, v) R (x, y)$

Given $xv = yu$

$$\Rightarrow yu = xv$$

$$\Rightarrow uy = vx$$

$$\therefore (u, v) R (x, y)$$

Transitive Let $(x, y) R (u, v)$ and $(u, v) R (p, q)$ (i)

TPT $(x, y) R (p, q)$

$$\text{TPT } xq = yp$$

from (1) $xv = yu$ & $uq = vp$

$$xvuq = yuvp$$

$$xq = yp$$

$$\therefore R \text{ is transitive}$$

since R is reflexive symmetric & transitive all means it is an equivalence relation.]

Relations Ex 1.2 Q8

We have, $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ be a set and

$R = \{(a, b) : a = b\}$ be a relation on A

Now,

Reflexivity: Let $a \in A$

$$\Rightarrow a = a$$

$$\Rightarrow (a, a) \in R$$

$$\Rightarrow R \text{ is reflexive}$$

Symmetric: Let $a, b \in A$ and $(a, b) \in R$

$$\Rightarrow a = b$$

$$\Rightarrow b = a$$

$$\Rightarrow (b, a) \in R$$

$$\Rightarrow R \text{ is symmetric}$$

Transitive: Let a, b & $c \in A$

and Let $(a, b) \in R$ and $(b, c) \in R$

$$\Rightarrow a = b \text{ and } b = c$$

$$\Rightarrow a = c$$

$$\Rightarrow (a, c) \in R$$

$$\Rightarrow R \text{ is transitive}$$

Since R is being reflexive, symmetric and transitive, so

R is an equivalence relation.

Also, we need to find the set of all elements related to 1.

Since the relation is given by, $R = \{(a, b) : a = b\}$, and 1 is an element of A ,

$$R = \{(1, 1) : 1 = 1\}$$

Thus, the set of all elements related to 1 is 1.

Relations Ex 1.2 Q9

(i) We have, $\mathcal{L}$ is the set of lines.

$R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$ be a relation on $\mathcal{L}$

Now,

Reflexivity: Let $L_1 \in \mathcal{L}$

Since a line is always parallel to itself.

$$\therefore (L_1, L_1) \in R$$

$\Rightarrow R$ is reflexive

Symmetric: Let $L_1, L_2 \in \mathcal{L}$ and $(L_1, L_2) \in R$

$\Rightarrow L_1$ is parallel to L_2

$\Rightarrow L_2$ is parallel to L_1

$$\Rightarrow (L_2, L_1) \in R$$

$\Rightarrow R$ is symmetric

Transitive: Let L_1, L_2 and $L_3 \in \mathcal{L}$ such that $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

$\Rightarrow L_1$ is parallel to L_2 and L_2 is parallel to L_3

$\Rightarrow L_1$ is parallel to L_3

$$\Rightarrow (L_1, L_3) \in R$$

$\Rightarrow R$ is transitive

Since, R is reflexive, symmetric and transitive, so R is an equivalence relation.

(ii) The set of lines parallel to the line $y = 2x + 4$ is

$y = 2x + c$ For all $c \in R$

Where R is the set of real numbers.

Relations Ex 1.2 Q10

$R = \{(P_1, P_2) : P_1 \text{ and } P_2 \text{ have same the number of sides}\}$

R is reflexive since $(P, P) \in R$ as the same polygon has the same number of sides with itself.

Let $(P_1, P_2) \in R$.

$\Rightarrow P_1$ and P_2 have the same number of sides.

$\Rightarrow P_2$ and P_1 have the same number of sides.

$$\Rightarrow (P_2, P_1) \in R$$

$\therefore R$ is symmetric.

Now,

Let $(P_1, P_2), (P_2, P_3) \in R$.

$\Rightarrow P_1$ and P_2 have the same number of sides. Also, P_2 and P_3 have the same number of sides.

$\Rightarrow P_1$ and P_3 have the same number of sides.

$$\Rightarrow (P_1, P_3) \in R$$

$\therefore R$ is transitive.

Hence, R is an equivalence relation.

The elements in A related to the right-angled triangle (T) with sides 3, 4, and 5 are those polygons which have 3 sides (since T is a polygon with 3 sides).

Hence, the set of all elements in A related to triangle T is the set of all triangles.

Relations Ex 1.2 Q11

Let A be set of points on plane.

Let $R = \{(P, Q) : OP = OQ\}$ be a relation on A where O is the origin.

To prove R is an equivalence relation, we need to show that R is reflexive, symmetric and transitive on A .

Now,

Reflexivity: Let $p \in A$

Since $OP = OP \Rightarrow (P, P) \in R$

$\Rightarrow R$ is reflexive

Symmetric: Let $(P, Q) \in R$ for $P, Q \in A$

Then $OP = OQ$

$\Rightarrow OQ = OP$

$\Rightarrow (Q, P) \in R$

$\Rightarrow R$ is symmetric

Transitive: Let $(P, Q) \in R$ and $(Q, S) \in R$

$\Rightarrow OP = OQ$ and $OQ = OS$

$\Rightarrow OP = OS$

$\Rightarrow (P, S) \in R$

$\Rightarrow R$ is transitive

Thus, R is an equivalence relation on A

Relations Ex 1.2 Q12

Given $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{(a, b) : \text{both } a \text{ and } b \text{ are either odd or even number}\}$

Therefore,

$R = \{(1, 1), (1, 3), (1, 5), (1, 6), (3, 3), (3, 5), (3, 7), (5, 5), (5, 7), (7, 7), (7, 5), (7, 3), (5, 3), (6, 1), (5, 1), (3, 1),$
 $(2, 2), (2, 4), (2, 6), (4, 4), (4, 6), (6, 6), (6, 4), (6, 2), (4, 2)\}$

Form the relation R it is seen that R is symmetric, reflexive and transitive also. Therefore R is an equivalent relation.

From the relation R it is seen that $\{1, 3, 5, 7\}$ are related with each other only and $\{2, 4, 6\}$ are related with each other

Relations Ex 1.2 Q13

$S = \{(a, b) : a^2 + b^2 = 1\}$

Now,

Reflexivity: Let $a = \frac{1}{2} \in R$

Then, $a^2 + a^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \neq 1$

$\Rightarrow (a, a) \notin S$

$\Rightarrow S$ is not reflexive

Hence, S is not an equivalence relation on R

Relations Ex 1.2 Q14

We have, Z be set of integers and Z_0 be the set of non-zero integers.

$R = \{(a,b)(c,d) : ad = bc\}$ be a relation on $Z \times Z_0$.

Now,

Reflexivity: $(a,b) \in Z \times Z_0$

$$\Rightarrow ab = ba$$

$$\Rightarrow ((a,b), (a,b)) \in R$$

$$\Rightarrow R \text{ is reflexive}$$

Symmetric: Let $((a,b), (c,d)) \in R$

$$\Rightarrow ad = bc$$

$$\Rightarrow cd = da$$

$$\Rightarrow ((c,d), (a,b)) \in R$$

$$\Rightarrow R \text{ is symmetric}$$

Transitive: Let $(a,b), (c,d) \in R$ and $(c,d), (e,f) \in R$

$$\Rightarrow ad = bc \text{ and } cf = de$$

$$\Rightarrow \frac{a}{b} = \frac{c}{d} \text{ and } \frac{c}{d} = \frac{e}{f}$$

$$\Rightarrow \frac{a}{b} = \frac{e}{f}$$

$$\Rightarrow af = be$$

We have, Z be set of integers and Z_0 be the set of non-zero integers.

$R = \{(a,b)(c,d) : ad = bc\}$ be a relation on Z and Z_0 .

Now,

Reflexivity: $(a,b) \in Z \times Z_0$

$$\Rightarrow ab = ba$$

$$\Rightarrow ((a,b), (a,b)) \in R$$

$$\Rightarrow R \text{ is reflexive}$$

Symmetric: Let $((a,b), (c,d)) \in R$

$$\Rightarrow ad = bc$$

$$\Rightarrow cd = da$$

$$\Rightarrow ((c,d), (a,b)) \in R$$

$$\Rightarrow R \text{ is symmetric}$$

Transitive: Let $(a,b), (c,d) \in R$ and $(c,d), (e,f) \in R$

$$\Rightarrow ad = bc \text{ and } cf = de$$

$$\Rightarrow \frac{a}{b} = \frac{c}{d} \text{ and } \frac{c}{d} = \frac{e}{f}$$

$$\Rightarrow \frac{a}{b} = \frac{e}{f}$$

$$\Rightarrow af = be$$

$$\Rightarrow (a,b)(e,f) \in R$$

$$\Rightarrow R \text{ is transitive}$$

Hence, R is an equivalence relation on $Z \times Z_0$

Relations Ex 1.2 Q15.

R and S are two symmetric relations on set A

(i) To prove: $R \cap S$ is symmetric

Let $(a, b) \in R \cap S$

$$\begin{aligned} \Rightarrow & (a, b) \in R \text{ and } (a, b) \in S \\ \Rightarrow & (b, a) \in R \text{ and } (b, a) \in S \quad [\because R \text{ and } S \text{ are symmetric}] \\ \Rightarrow & (b, a) \in R \cap S \\ \Rightarrow & R \cap S \text{ is symmetric} \end{aligned}$$

To prove: $R \cup S$ is symmetric.

Let $(a, b) \in R \cup S$

$$\begin{aligned} \Rightarrow & (a, b) \in R \text{ or } (a, b) \in S \\ \Rightarrow & (b, a) \in R \text{ or } (b, a) \in S \quad [\because R \text{ and } S \text{ are symmetric}] \\ \Rightarrow & (b, a) \in R \cup S \\ \Rightarrow & R \cup S \text{ is symmetric} \end{aligned}$$

(ii) R and S are two relations on A such that R is reflexive.

To prove: $R \cup S$ is reflexive

Suppose $R \cup S$ is not reflexive.

This means that there is an $a \in R \cup S$ such that $(a, a) \notin R \cup S$

Since $a \in R \cup S$,

$\therefore a \in R$ or $a \in S$

If $a \in R$, then $(a, a) \in R$ $[\because R$ is reflexive]

$\Rightarrow (a, a) \in R \cup S$

Hence, $R \cup S$ is reflexive

Relations Ex 1.2 Q16.

We will prove this by means of an example.

Let $A = \{a, b, c\}$ be a set and

$R = \{(a, a)(b, b)(c, c)(a, b)(b, a)\}$ and

$S = \{(a, a)(b, b)(c, c)(b, c)(c, b)\}$ are two relations on A

Clearly R and S are transitive relation on A

Now, $R \cup S = \{(a, a)(b, b)(c, c)(a, b)(b, a)(b, c)(c, b)\}$

Here, $(a, b) \in R \cup S$ and $(b, c) \in R \cup S$

but $(a, c) \notin R \cup S$

$\therefore R \cup S$ is not transitive