CHAPTER

8.5

ELECTROMAGNETIC WAVE PROPAGATION

Statement for Q.1–3:

A y-polarized uniform plane wave with a frequency of 100 MHz propagates in air in the $+x$ direction and impinges normally on a perfectly conducting plane at $x = 0$. The amplitude of incident **E**-field is 6 mV/m.

1. The phasor $\mathbf{H}_s$ of the incident wave in air is

(A) $16e^{-j\frac{2\pi}{3}x}\mathbf{u}_z\ \mu$A/m (B) $-16e^{-j\frac{2\pi}{3}x}\mathbf{u}_z\ \mu$A/m

(C) $16e^{-j\frac{2\pi}{3}x}\mathbf{u}_x\ \mu$A/m (D) $-16e^{-j\frac{2\pi}{3}x}\mathbf{u}_x\ \mu$A/m

2. The **E**-field of total wave in air is

(A) $j12\sin\left(\frac{2\pi}{3}x\right)\mathbf{u}_y$ mV/m

(B) $-j12\sin\left(\frac{2\pi}{3}x\right)\mathbf{u}_y$ mV/m

(C) $12\cos\left(\frac{2\pi}{3}x\right)\mathbf{u}_y$ mV/m

(D) $-12\cos\left(\frac{2\pi}{3}x\right)\mathbf{u}_y$ mV/m

3. The location in air nearest to the conducting plane, where total **E**-field is zero, is

(A) $x = 1.5$ m (B) $x = -1.5$ m

(C) $x = 3$ m (D) $x = -3$ m

4. The phasor magnetic field intensity for a 400 MHz uniform plane wave propagating in a certain lossless material is $(6\mathbf{u}_y - j5\mathbf{u}_z)e^{-j18x}$ A/m . The phase velocity v_p is

(A) 6.43×10^6 m/s (B) 2.2×10^7 m/s

(C) 1.4×10^8 m/s (D) None of the above

Statement for Q.5–6:

A uniform plane wave in free space has electric field $\mathbf{E}_s = (2\mathbf{u}_z + 3\mathbf{u}_y)e^{-j\beta x}$ V/m.

5. The magnetic field phasor $\mathbf{H}_s$ is

(A) $(-5.3\mathbf{u}_y - 8\mathbf{u}_z)e^{-j\beta x}$ mA/m

(B) $(5.3\mathbf{u}_y - 8\mathbf{u}_z)e^{-j\beta x}$ mA/m

(C) $(-5.3\mathbf{u}_y + 8\mathbf{u}_z)e^{-j\beta x}$ mA/m

(D) $(5.3\mathbf{u}_y + 8\mathbf{u}_z)e^{-j\beta x}$ mA/m

6. The average power density in the wave is

(A) 34 mW/m^2 (B) 17 mW/m^2

(C) 22 mW/m^2 (D) 44 mW/m^2

7. The electric field of a uniform plane wave in free space is given by $\mathbf{E}_s = 12\pi(\mathbf{u}_y + j\mathbf{u}_z)e^{-j15x}$. The magnetic field phasor $\mathbf{H}_s$ is

(A) $\frac{12}{\eta_o}(-\mathbf{u}_z + j\mathbf{u}_y)e^{-j15x}$ (B) $\frac{12}{\eta_o}(\mathbf{u}_z + j\mathbf{u}_y)e^{-j15x}$

(C) $\frac{12}{\eta_o}(-\mathbf{u}_z - j\mathbf{u}_y)e^{-j15x}$ (D) $\frac{12}{\eta_o}(\mathbf{u}_z - j\mathbf{u}_y)e^{-j15x}$

Statement for Q.8–9:

A lossy material has $\mu = 5\mu_o$, $\varepsilon = 2\varepsilon_o$. The phase constant is 10 rad/m at 5 MHz.

8. The loss tangent is

(A) 2913 (B) 1823

(C) 2468 (D) 1374

9. The attenuation constant α is

(A) 4.43 (B) 9.99

(C) 5.57 (D) None of the above

Statement for Q.10–11:

At 50 MHz a lossy dielectric material is characterized by $\mu = 2.1\mu_o$, $\varepsilon = 3.6\varepsilon_o$ and $\sigma = 0.08$ S/m. The electric field is $\mathbf{E}_s = 6e^{-j\gamma x}\mathbf{u}_z$ V/m.

10. The propagation constant γ is

(A) $7.43 + j2.46$ per meter

(B) $2.46 + j7.43$ per meter

(C) $6.13 + j5.41$ per meter

(D) $5.41 + j6.13$ per meter

11. The impedance η is

(A) 101.4 Ω (B) 167.4 Ω

(C) 98.3 Ω (D) 67.3 Ω

Statement for Q.12–13:

A non magnetic medium has an intrinsic impedance $360\angle 30°\,\Omega$.

12. The loss tangent is

(A) 0.866 (B) 0.5

(C) 1.732 (D) 0.577

13. The Dielectric constant is

(A) 1.634 (B) 1.234

(C) 0.936 (D) 0.548

Statement for Q.14–15:

The amplitude of a wave traveling through a lossy nonmagnetic medium reduces by 18% every meter. The wave operates at 10 MHz and the electric field leads the magnetic field by 24°.

14. The propagation constant is

(A) $0.198 + j0.448$ per meter

(B) $0.346 + j0.713$ per meter

(C) $0.448 + j0.198$ per meter

(D) $0.713 + j0.346$ per meter

15. The skin depth is

(A) 2.52 m (B) 5.05 m

(C) 8.46 m (D) 4.23 m

16. A 60 m long aluminium ($\sigma = 3.5 \times 10^7$ S/m, $\mu_r = 1$, $\varepsilon_2 = 1$) pipe with inner and outer radii 9 mm and 12 mm carries a total current of $16\sin(10^6 \pi t)$ A. The effective resistance of the pipe is

(A) 0.19 Ω (B) 3.48 Ω

(C) 1.46 Ω (D) 2.43 Ω

17. Silver plated brass wave guide is operating at 12 GHz. If at least the thickness of silver ($\sigma = 6.1 \times 10^7$ S/m, $\mu_r = \varepsilon_r = 1$) is 5δ, the minimum thickness required for wave-guide is

(A) 6.41 μm (B) 3.86 μm

(C) 5.21 μm (D) 2.94 μm

Statement for Q.18–19:

A uniform plane wave in a lossy nonmagnetic media has

$$\mathbf{E}_s = (5\mathbf{u}_x + 12\mathbf{u}_y)e^{-\gamma z}, \quad \gamma = 0.2 + j3.4 \text{ m}^{-1}$$

18. The magnitude of the wave at $z = 4$ m and $t = T/8$ is

(A) 10.34 (B) 5.66

(C) 4.36 (D) 12.60

19. The loss suffered by the wave in the interval $0 < z < 3$ m is

(A) 4.12 dB (B) 8.24 dB

(C) 10.42 dB (D) 5.21 dB

Statement for Q.20–22:

The plane wave $\mathbf{E} = 42\cos(\omega t - z)\,\mathbf{u}_x$ V/m in air normally hits a lossless medium ($\mu_r = 1$, $\varepsilon_r = 4$) at $z = 0$.

20. The SWR s is

(A) 2 (B) 1

(C) $\frac{1}{2}$ (D) None of the above

21. The transmission coefficient τ is

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$

(C) $\frac{1}{3}$ (D) 3

22. The reflected electric field is

(A) $-14\cos(\omega t - z)\,\mathbf{u}_x$ V/m

(B) $-14\cos(\omega t + z)\,\mathbf{u}_x$ V/m

33. The region $z < 0$ is characterized by $\varepsilon_r = \mu_r = 1$ and $\sigma = 0$. The total electric field here is given $\mathbf{E}_s = 150e^{-j10z}\,\mathbf{u}_x + 50\angle 20°\, e^{j10z}\,\mathbf{u}_x$ V/m. The intrinsic impedance of the region $z > 0$ is

(A) $692 + j176\,\Omega$ (B) $193 - j49\,\Omega$

(C) $176 + j692\,\Omega$ (D) $49 - j193\,\Omega$

Statement for Q.34–35:

Region 1, $z < 0$ and region 2, $z > 0$, are both perfect dielectrics. A uniform plane wave traveling in the $\mathbf{u}_z$ direction has a frequency of 3×10^{10} rad/s. Its wavelength in the two region are $\lambda_1 = 5$ cm and $\lambda_2 = 3$ cm.

34. On the boundary the reflected energy is

(A) 6.25% (B) 12.5%

(C) 25% (D) 50%

35. The SWR is

(A) 1.67 (B) 0.6

(C) 2 (D) 1.16

36. A uniform plane wave is incident from region 1 ($\mu_r = 1$, $\sigma = 0$) to free space. If the amplitude of incident wave is one-half that of reflected wave in region, then the value of ε_r is

(A) 4 (B) 3

(C) 16 (D) 9

37. A 150 MHz uniform plane wave is normally incident from air onto a material. Measurements yield a SWR of 3 and the appearance of an electric field minimum at 0.3λ in front of the interface. The impedance of material is

(A) $502 - j641\,\Omega$ (B) $641 - j502\,\Omega$

(C) $641 + j502\,\Omega$ (D) $502 + j641\,\Omega$

38. A plane wave is normally incident from air onto a semi-infinite slab of perfect dielectric ($\varepsilon_r = 3.45$). The fraction of transmitted power is

(A) 0.91 (B) 0.3

(C) 0.7 (D) 0.49

Statement for Q.39–40:

Consider three lossless region :

Region 1 ($z < 0$): $\mu_1 = 4\,\mu$H/m, $\varepsilon_1 = 10$ pF/m

Region 2 ($0 < z < 6$ cm): $\mu_2 = 2\,\mu$H/m, $\varepsilon_2 = 25$ pF/m

Region 3 ($z > 6$ cm): $\mu_3 = 4\,\mu$H/m, $\varepsilon_3 = 10$ pF/m

39. The lowest frequency, at which a uniform plane wave incident from region 1 onto the boundary at $z = 0$ will have no reflection, is

(A) 2.96 GHz (B) 4.38 GHz

(C) 1.18 GHz (D) 590 MHz

40. If frequency is 50 MHz, the SWR in region 1 is

(A) 0.64 (B) 1.27

(C) 2.38 (D) 4.16

41. A uniform plane wave in air is normally incident onto a lossless dielectric plate of thickness $\lambda/8$, and of intrinsic impedance $\eta = 260\,\Omega$. The SWR in front of the plate is

(A) 1.12 (B) 1.34

(C) 1.70 (D) 1.93

42. The **E**-field of a uniform plane wave propagating in a dielectric medium is given by

$$\mathbf{E} = 2\cos\left(10^8 t - \frac{z}{\sqrt{3}}\right)\mathbf{u}_x - \sin\left(10^8 t - \frac{z}{\sqrt{3}}\right)\mathbf{u}_y \text{ V/m}$$

The dielectric constant of medium is

(A) 3 (B) 9

(C) 6 (D) $\sqrt{6}$

43. An electromagnetic wave from an under water source with perpendicular polarization is incident on a water-air interface at angle 20° with normal to surface. For water assume $\varepsilon_r = 81$, $\mu_r = 1$. The critical angle θ_c is

(A) 83.62° (B) 6.38°

(C) 42.6° (D) None of the above

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SOLUTIONS

1. (A) $\omega = 2\pi \times 10^8$ rad/s

$$\beta = \frac{\omega}{c} = \frac{2\pi \times 10^8}{3 \times 10^8} = \frac{2\pi}{3} \text{ rad/m}$$

$$\mathbf{E}_s = 6e^{-j\frac{2\pi}{3}x}\mathbf{u}_y \text{ mV/m}$$

$$\mathbf{u}_E \times \mathbf{u}_H = \mathbf{u}_x, \quad \mathbf{u}_y \times \mathbf{u}_H = \mathbf{u}_x, \quad \mathbf{u}_H = \mathbf{u}_z$$

$$\mathbf{H}_s = \frac{6}{120\pi}e^{-j\frac{2\pi}{3}x}\mathbf{u}_z = 16e^{-j\frac{2\pi}{3}x}\mathbf{u}_z \ \mu\text{A/m}$$

2. (B) For conducting plane $\Gamma = -1$,

$$\mathbf{E}_r = -6e^{j\frac{2\pi}{3}x}\mathbf{u}_y \text{ mV/m},$$

$$\mathbf{E} = \mathbf{E}_i + \mathbf{E}_r = \left(6e^{-j\frac{2\pi}{3}x}\mathbf{u}_y - 6e^{-j\frac{2\pi}{3}x}\mathbf{u}_y\right) \text{mV/m}$$

$$= -j12\sin\left(\frac{2\pi}{3}x\right)\mathbf{u}_y \text{ mV/m}$$

3. (B) The electric field vanish at the surface of the conducting plane at $x = 0$. In air the first null occur at

$$x = -\frac{\lambda_1}{2} = -\frac{\pi}{\beta_1} = -\frac{3}{2} \text{ m}$$

4. (C) $v_p = \frac{\omega}{\beta} = \frac{2\pi \times 400 \times 10^6}{18} = 1.4 \times 10^8$ m/s

5. (C) The wave is propagating in forward x direction. Therefore $\mathbf{u}_E \times \mathbf{u}_H = \mathbf{u}_x$.

For $\mathbf{u}_E = \mathbf{u}_z$, $\mathbf{u}_z \times \mathbf{u}_H = \mathbf{u}_x \Rightarrow \mathbf{u}_H = -\mathbf{u}_y$

For $\mathbf{u}_E = \mathbf{u}_y$, $\mathbf{u}_y \times \mathbf{u}_H = \mathbf{u}_x \Rightarrow \mathbf{u}_H = \mathbf{u}_z$

$$\mathbf{H}_s = \frac{1}{120\pi}(-2\mathbf{u}_y + 3\mathbf{u}_z)e^{-j\beta x} = (-5.3\mathbf{u}_y + 8\mathbf{u}_z)e^{-j\beta x} \text{ mA/m}$$

6. (B) $\mathbf{P}_{avg} = \frac{1}{2}\text{Re}\{\mathbf{E}_s \times \mathbf{H}_s^*\}$

$$\frac{1}{2}\{(5.3)\mathbf{u}_x + 3(8)\mathbf{u}_x\} \times 10^{-3} = 17.3\mathbf{u}_x \text{ mW/m}^2$$

7. (D) Since Pointing vector is in the positive x direction, therefore $\mathbf{u}_E \times \mathbf{u}_H = \mathbf{u}_x$.

For $\mathbf{u}_E = \mathbf{u}_y$, $\mathbf{u}_y \times \mathbf{u}_H = \mathbf{u}_x \Rightarrow \mathbf{u}_H = \mathbf{u}_z$

For $\mathbf{u}_E = \mathbf{u}_z$, $\mathbf{u}_z \times \mathbf{u}_H = \mathbf{u}_x \Rightarrow \mathbf{u}_H = -\mathbf{u}_y$,

$$\mathbf{H}_s = \frac{12}{\eta_o}(\mathbf{u}_z - j\mathbf{u}_y)e^{-j15x}$$

8. (B) Loss tangent $\frac{\sigma}{\omega\varepsilon} = x$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

$$\Rightarrow \quad 10 = \frac{2\pi \times 5 \times 10^6}{3 \times 10^8}\sqrt{\frac{5 \times 2}{2}\left[\sqrt{1+x^2}+1\right]}$$

$$\Rightarrow \quad x = \frac{\sigma}{\omega\varepsilon} = 1823$$

9. (B) $\frac{\alpha}{\beta} = \frac{\sqrt{\sqrt{1+x^2}-1}}{\sqrt{\sqrt{1+x^2}+1}}$

$$\Rightarrow \quad \frac{\alpha}{\beta} = \frac{\sqrt{1822}}{\sqrt{1824}}$$

$$\alpha = 10 \times 0.999 = 9.99$$

10. (D) $\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$

$$\frac{\sigma}{\omega\varepsilon} = \frac{0.08}{3.6 \times 50 \times 10^6 \times 2\pi\varepsilon_o} = 8$$

$$\alpha = \frac{2\pi \times 50 \times 10^6}{3 \times 10^8}\sqrt{\frac{(2.1)(3.6)}{2}(\sqrt{65}-1)} = 5.41$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

$$= \frac{2\pi \times 50 \times 10^6}{3 \times 10^8}\sqrt{\frac{(2.1)(3.6)}{2}(\sqrt{65}+1)} = 6.13$$

$\gamma = \alpha + j\beta = 5.41 + j6.13$ per meter.

11. (A) $|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\left(1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{\frac{1}{4}}} = \frac{120\pi\sqrt{\frac{2.1}{3.6}}}{64^{\frac{1}{4}}} = 101.4$

12. (C) $\frac{\sigma}{\omega\varepsilon} = \tan 2\theta_n = \tan 60° = 1.732$

13. (D) $|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\left(1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{\frac{1}{4}}}$

$$\Rightarrow \quad 360 = \frac{\frac{120\pi}{\varepsilon_r}}{(1+1.732^2)^{\frac{1}{4}}} \quad \Rightarrow \quad \varepsilon_r = 0.548$$

14. (A) $|\mathbf{E}| = E_o e^{-\alpha z}$

$$E_o e^{-\alpha 1} = (1-0.18)E_o$$

$$e^{-\alpha 1} = 0.82 \quad \Rightarrow \quad \alpha = \ln\frac{1}{0.82} = 0.198$$

$$\theta_n = 24° \quad \Rightarrow \quad \tan 2\theta_n = \frac{\sigma}{\omega\varepsilon} = 1.111$$

$$\frac{\alpha}{\beta} = \frac{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1}}{\sqrt{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1}}$$

$$\frac{0.198}{\beta} = \frac{\sqrt{\sqrt{234} - 1}}{\sqrt{\sqrt{234} + 1}} \quad \Rightarrow \quad \beta = 0.448$$

$$\gamma = \alpha + j\beta = 0.198 + j0.448$$

15. (B) $\delta = \frac{1}{\alpha} = \frac{1}{0.198} = 5.05$

16. (A) $\omega = \pi 10^6 \quad \Rightarrow \quad f = 5 \times 10^5$ Hz,

$$\delta = \frac{1}{\sqrt{\pi f \sigma \mu}} = \frac{1}{\sqrt{\pi \times 5 \times 10^5 \times 3.5 \times 10^7 \times \mu_o}} = 120\ \mu\text{m}$$

$$R_{ac} = \frac{l}{\sigma \delta w}$$

Since δ is very small, $w = 2\pi\rho_{outer}$

$$R_{ac} = \frac{60}{3.5 \times 10^7 \times 120 \times 10^{-6} \times 2\pi \times 12 \times 10^{-3}} = 0.19\ \Omega$$

17. (D) $t = 5\delta = \frac{5}{\sqrt{\pi f \mu \sigma}}$

$$= \frac{5}{\sqrt{\pi \times 12 \times 10^9 \times \mu_o \times 6.1 \times 10^7}} = 2.94\ \mu\text{m}$$

18. (B) $\mathbf{E} = \text{Re}\{\mathbf{E}_s e^{j\omega t}\} = (5\mathbf{u}_x + 12\mathbf{u}_y)e^{-0.2z}\cos(\omega t - 3.4z)$

At $z = 4$ m, $t = \frac{T}{8}$

$$\mathbf{E} = (5\mathbf{u}_x + 12\mathbf{u}_y)e^{-0.8}\cos\left(\frac{\pi}{4} - 13.6\right)$$

$$|\mathbf{E}| = 13e^{-0.8}\cos\left(\frac{\pi}{4} - 13.6\right) = 5.66$$

19. (D) Loss $= \alpha\Delta z = 0.2 \times 3 = 0.6 Np$

$1Np = 8.686$ DB, $\quad 0.6Np = 5.21$ dB.

20. (A) $\eta_1 = \eta_o$, $\eta_2 = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o}{2}$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{\eta_o}{2} - \eta_o}{\frac{\eta_o}{2} + \eta_o} = -\frac{1}{3}$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = 2$$

21. (A) $\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2\frac{\eta_o}{2}}{\frac{\eta_o}{2} + 2} = \frac{2}{3}$

22. (A) $E_{or} = \Gamma E_{oi} = -\frac{1}{3}(42) = -14$

$$E_r = -14\cos(\omega t - z)\,\mathbf{u}_x \text{ V/m}$$

23. (C) $\eta_1 = \eta_o$, $\eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_o}{\varepsilon_r} = \frac{\eta_o}{2}$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_{11}} = \frac{\frac{\eta_o}{2} - \eta_o}{\frac{\eta_o}{2} + \eta_o} = -\frac{1}{3}$$

24. (D) $\eta_1 = \eta_o$, $\eta_2 = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}} = \eta_o\sqrt{\frac{\mu_r}{12.5}}$

$$\frac{E_{or}}{E_{oi}} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

But $E_{or} = \eta_1 H_{or} = \Gamma E_{oi}$

$$\eta_1 H_{or} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right)E_{oi} \quad \Rightarrow \quad \eta_1 = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right)\frac{18}{6 \times 10^{-3}}$$

$$\eta_1 = \eta_o \quad \Rightarrow \quad \eta_o = \left(\frac{\eta_2 - \eta_o}{\eta_2 + \eta_o}\right)3000$$

$$\frac{377}{3000} = \frac{\eta_2 - 377}{\eta_2 + 377} \quad \Rightarrow \quad \eta_2 = 485.37 = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$\Rightarrow \quad \varepsilon_r = 12.5, \quad \mu_r = 20.75$$

25. (A) $\eta_1 = \eta_o$, $\eta_2 = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o}{2}$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -\frac{1}{3}$$

$$E_{or} = -\frac{1}{3}(10) = -\frac{10}{3}$$

$$H_{or} = \frac{E_{or}}{\eta_o} = \frac{10}{3 \times 377} = 8.8 \times 10^{-3}$$

$$\mathbf{u}_E \times \mathbf{u}_H = \mathbf{u}_k, \quad -\mathbf{u}_y \times \mathbf{u}_H = -\mathbf{u}_z \quad \Rightarrow \quad \mathbf{u}_H = -\mathbf{u}_x$$

$$\mathbf{H}_r = -8.8\cos(\omega t - z)\,\mathbf{u}_x \text{ mA/m}$$

$$\beta = 1 = \frac{\omega}{v} = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r}$$

$$\omega = \frac{3\times 10^8}{\sqrt{12\times 3}} = 0.5\times 10^8 \text{ rad/s.}$$

26. (D) $\eta_1 = \eta_o$, $\eta_2 = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o}{\sqrt{3}} = 0.58\eta_o$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{0.58\eta_o - \eta_o}{0.58\eta_o + \eta_o} = -0.266$$

$\tau = 1 + \Gamma = 0.734,\qquad E_{ot} = \tau E_{oi} = 7.34$

$\mathbf{E}_t = 7.34\cos(\omega t - z)\,\mathbf{u}_y$ V/m

27. (B) $\mathbf{E}_{Total} = \mathbf{E}_i + \mathbf{E}_r$, $\quad E_{or} = \Gamma E_{oi} = -2.66$

$\mathbf{E}_{Total} = 10\cos(\omega t - z)\,\mathbf{u}_y - 2.66\cos(\omega t + z)\,\mathbf{u}_y$ V/m

28. (B) $\mu_o = \mu_1 = \mu_2$

$$\sin\theta_{t1} = \sqrt{\frac{\varepsilon_o}{\varepsilon_1}}\sin\theta_i \quad\Rightarrow\quad \sin\theta_{t1} = \sqrt{\frac{1}{4.5}}\sin 45° = 0.333$$

$\Rightarrow\quad \theta_{t1} = 19.47°$

29. (B) $\sin\theta_{t2} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\sin\theta_{t1} = \sqrt{\frac{4.5}{2.25}}(0.333) = 0.47$

$\Rightarrow\quad \theta_{t2} = \sin^{-1}0.47 = 28°$

30. (A) Since both media are non magnetic

$$\tan\theta_B = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} = \sqrt{\frac{2.6\varepsilon_o}{\varepsilon_o}} = \sqrt{2.6}$$

But $\cos\theta_t = \frac{\eta_1}{\eta_2}\cos\theta_B = \frac{\eta_o}{\frac{\eta_o}{\sqrt{2.6}}}\cos 58.2° = \sqrt{2.6}\cos 58.2°$

$\Rightarrow\quad \theta_t = 31.8°$

31. (A) $\eta_1 = \eta_o$, $\eta_2 = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o}{\sqrt{5}} = 0.447\eta_o$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.38,\ \tau = 1 + \Gamma = 0.62$$

$E_t = \tau E_i = 92.7\cos(\omega t - 8y)\,\mathbf{u}_z$ V/m

32. (B) $|\Gamma|^2 = 0.2$, $\qquad \Gamma = \pm 0.447$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_o\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} - \eta_o\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}}{\eta_o\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} + \eta_o\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}} = \frac{\sqrt{\frac{\mu_{r2}}{\mu_{r2}^3}} - \sqrt{\frac{\mu_{r1}}{\mu_{r1}^3}}}{\sqrt{\frac{\mu_{r2}}{\mu_{r2}^3}} + \sqrt{\frac{\mu_{r1}}{\mu_{r1}^3}}} = \frac{\mu_{r1} - \mu_{r2}}{\mu_{r1} + \mu_{r2}}$$

$$\Rightarrow\quad \frac{\mu_{r2}}{\mu_{r1}} = \frac{1 \mp 0.447}{1 \pm 0.447} = 0.382,\quad 2.62$$

$$\Rightarrow\quad \frac{\varepsilon_{r1}}{\varepsilon_{r2}} = \left(\frac{\mu_{r2}}{\mu_{r1}}\right)^3 = 0.056,\ 17.9$$

33.(A) $\Gamma = \frac{E_r}{E_i} = \frac{50\angle 20°}{150} = \frac{e^{j20}}{3}$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1},\quad \eta_1 = \eta_o$,

$$\eta_2 = \eta_o\left(\frac{1+\Gamma}{1-\Gamma}\right) = 377\left(\frac{1 + \frac{e^{j20}}{3}}{1 - \frac{e^{j20}}{3}}\right) = 692 + j176\,\Omega$$

34. (A) $\varepsilon_{r1} = \left(\frac{2\pi c}{\lambda_1\omega}\right)^2$, $\varepsilon_{r2} = \left(\frac{2\pi c}{\lambda_2\omega}\right)^2 \quad\Rightarrow\quad \frac{\varepsilon_{r1}}{\varepsilon_{r2}} = \left(\frac{\lambda_2}{\lambda_1}\right)^2$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{\eta_o}{\sqrt{\varepsilon_{r2}}} - \frac{\eta_o}{\sqrt{\varepsilon_{r1}}}}{\frac{\eta_o}{\sqrt{\varepsilon_{r2}}} + \frac{\eta_o}{\sqrt{\varepsilon_{r1}}}} = \frac{\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} - 1}{\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} + 1} = \frac{\frac{\lambda_2}{\lambda_1} - 1}{\frac{\lambda_2}{\lambda_1} + 1}$$

$$\Rightarrow\quad \Gamma = \frac{\lambda_2 - \lambda_1}{\lambda_2 + \lambda_1} = \frac{3-5}{3+5} = -\frac{1}{4}$$

The fraction of the incident energy that is reflected is $\Gamma^2 = \frac{1}{16} = 6.25\%$.

35. (A) $s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \frac{1}{4}}{1 - \frac{1}{4}} = \frac{5}{3}$

36. (D) $\eta_2 = \eta_o$, $\eta_1 = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o}{\sqrt{\varepsilon_r}}$

$$\Gamma = \frac{|\boldsymbol{E}_i|}{|\boldsymbol{E}_r|} = \frac{\mathbf{1}}{\mathbf{2}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\Rightarrow\quad \frac{\eta_o - \frac{\eta_o}{\sqrt{\varepsilon_r}}}{\eta_o + \frac{\eta_o}{\sqrt{\varepsilon_r}}} = \frac{1}{2} \quad\Rightarrow\quad \varepsilon_r = 9$$

37. (C) At minimum $\frac{(\phi + \pi)}{2\beta} = 0.3\lambda$,

$\beta = \frac{2\pi}{\lambda} \quad\Rightarrow\quad \phi = 0.2\pi$

$$|\Gamma| = \frac{s-1}{s+1} = \frac{3-1}{3+1} = \frac{1}{2}$$

$$\Gamma = 0.5e^{j0.2\pi} = \frac{\eta_2 - \eta_o}{\eta_2 + \eta_o}$$

$$\Rightarrow\quad \eta_2 = \eta_o\left(\frac{1 + 0.5e^{j0.2\pi}}{1 - 0.5e^{j0.2\pi}}\right) = 641 + j502\ \Omega$$

38. (A) $\eta_1 = \eta_o$, $\eta_2 = \eta_o\sqrt{\dfrac{\mu_r}{\varepsilon_r}} = \dfrac{\eta_o}{\sqrt{3.45}}$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\dfrac{\eta_o}{\sqrt{3.45}} - \eta_o}{\dfrac{\eta_o}{\sqrt{3.45}} + \eta_o} = -0.3$$

The transmitted fraction is $1 - |\Gamma|^2 = 1 - 0.09 = 0.91$.

39. (C) This frequency gives the condition $\beta_2 d = \pi$

Where $d = 6$ cm, $\beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$

$$\Rightarrow \quad \omega\sqrt{\mu_2\varepsilon_2} = \frac{\pi}{0.06}$$

$$\Rightarrow \quad f = \frac{1}{2 \times 0.06\sqrt{2 \times 10^{-6} \times 25 \times 10^{-12}}} = 1.18 \text{ GHz}$$

40. (B) At 50 MHz,

$$\beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = \mathbf{2\pi \times 50 \times 10^6 \sqrt{2 \times 10^{-6} \times 25 \times 10^{-12}} = 2.2}$$

$$\beta_2 d = 2.22(0.06) = 0.133$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{4 \times 10^{-6}}{10^{-11}}} = 632\,\Omega$$

$$\eta_3 = 632\,\Omega$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{2 \times 10^{-6}}{25 \times 10^{-12}}} = 283\,\Omega$$

The input impedance at the first interface is

$$\eta_{in} = \eta_2\left(\frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)}\right) = 283\left(\frac{632 + j283(0.134)}{283 + j632(0.134)}\right)$$

$$= 590 - j138$$

$$\Gamma = \frac{\eta_{in} - \eta_1}{\eta_{in} + \eta_1} = \frac{590 - j138 - 632}{590 - j138 + 632} = 0.12\angle -100.5°$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.12}{1 - 0.12} = 1.27$$

41. (C) $\beta d = \dfrac{2\pi}{\lambda}\cdot\dfrac{\lambda}{8} = \dfrac{\pi}{4}$, $\tan\dfrac{\pi}{4} = 1$

$\eta_2 = 260$, $\eta_1 = \eta_3 = \eta_o$

$$\eta_{in} = \eta_2\left(\frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)}\right) = 260\left(\frac{377 + j260}{260 + j377}\right)$$

$$= 243 - j92\ \Omega$$

$$\Gamma = \frac{\eta_{in} - \eta_o}{\eta_{in} + \eta_o} = \frac{243 - j92 - 377}{243 - j92 + 377} = 0.26\angle -137°$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1.26}{0.74} = 1.70$$

42. (A) $\omega = 10^8$ rad/s, $\beta = \dfrac{1}{\sqrt{3}}$ rad/m, $v = \dfrac{c}{\sqrt{\varepsilon_r}} = \dfrac{\omega}{\beta}$

$$\Rightarrow \quad \frac{3 \times 10^8}{\sqrt{\varepsilon_r}} = \frac{10^8}{1/\sqrt{3}} \quad \Rightarrow \quad \varepsilon_r = 3.$$

43. (B) $\theta_c = \sin^{-1}\sqrt{\dfrac{\varepsilon_{r2}}{\varepsilon_{r1}}} = \sin^{-1}\sqrt{\dfrac{1}{81}} = 6.38°$

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