Inverse Trignometric Function

Mathematics may not Teach us How to add or Subtract..... But it gives us every Reason to Hope that every problem has a Salution

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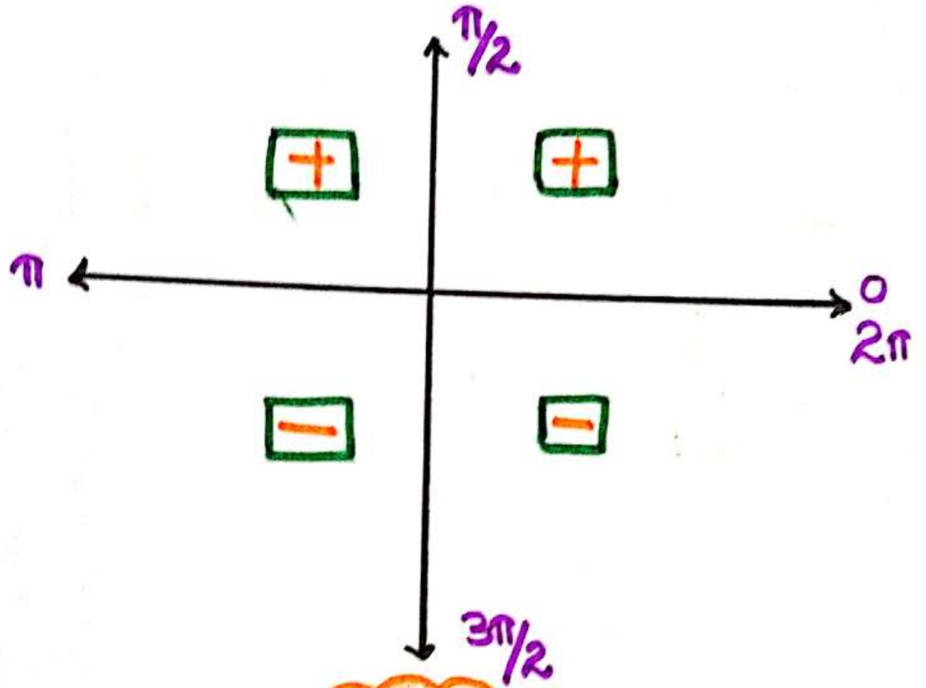
Inverse Trignometric Function

Art →

Let $y = \sin x$



Degree → Radian

$0^\circ \rightarrow 0$

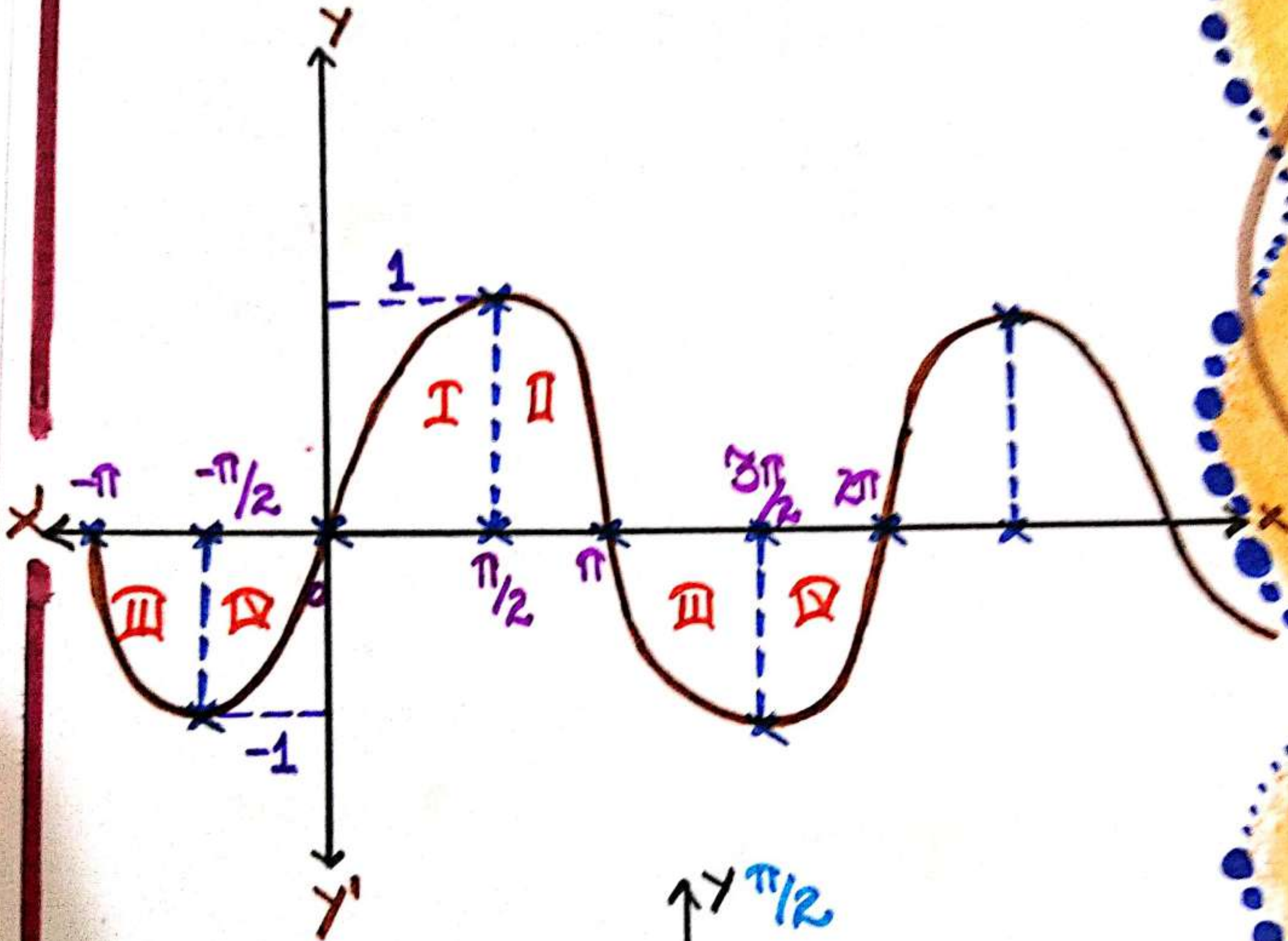
$30^\circ \rightarrow \pi/6$

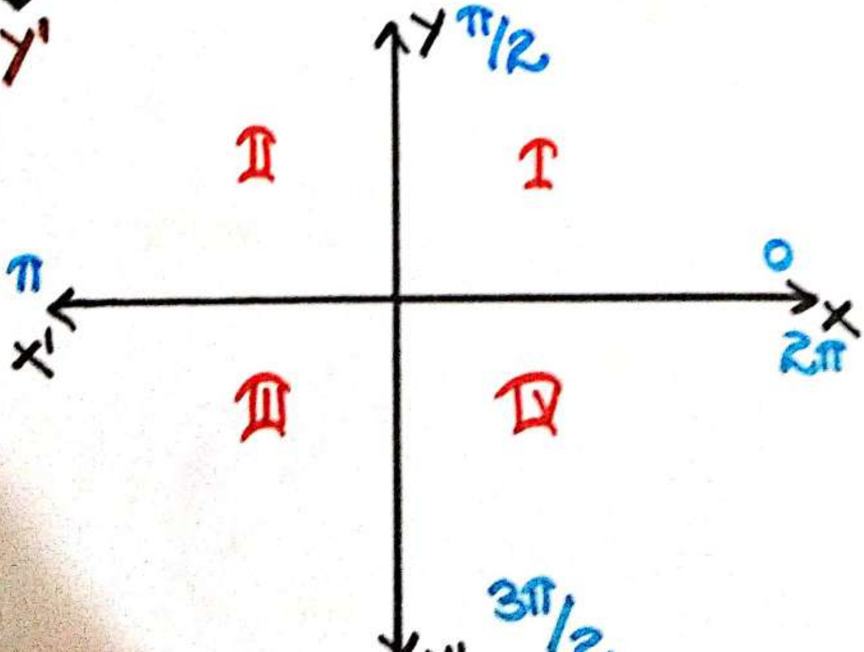
$45^\circ \rightarrow \pi/4$

$60^\circ \rightarrow \pi/3$

$90^\circ \rightarrow \pi/2$

Graph of Sin x





Let $y = f(x)$

$\therefore x = f^{-1}(y)$

$\sin(0) = 0$ $\quad$ $\sin(-\pi/2) = -1$

$\sin(\pi) = 0$

$\sin(+3\pi/2) = -1$

$\sin(2\pi) = 0$

$\sin(-\pi) = 0$

★ This function is many one......

Let $y = f(x)$

$y = x^2$

put, $x = 2$

$y = (2)^2 = 4$ → Range

Domain

put, $x = 3$

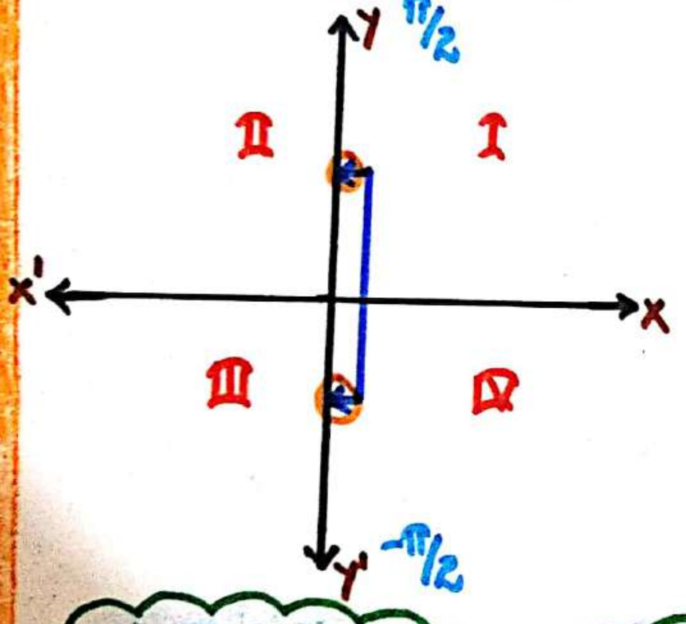
$y = (3)^2 = 9$ → Range

Domain

How to Remember the Properties

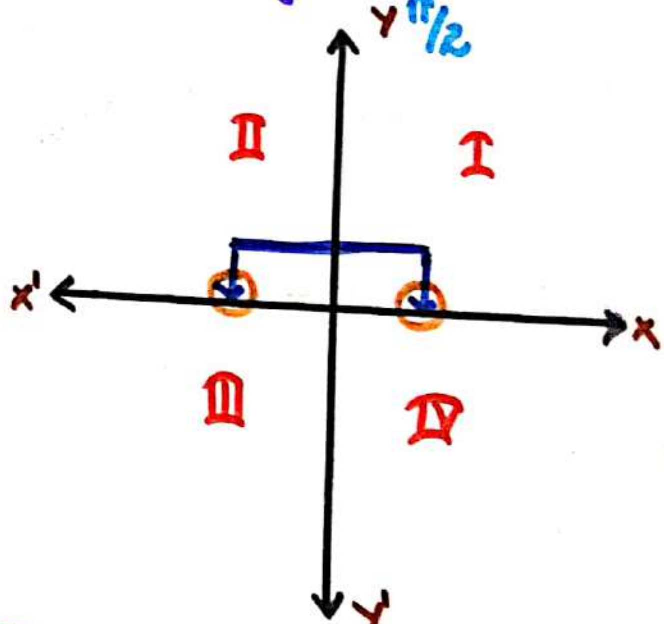
Family (I, IV)

	Domain	Range
(1) $\sin^{-1}x$	$[-1,1]$	$[-\pi/2, \pi/2]$
(2) $\operatorname{cosec}^{-1}x$	$\mathbb{R}-(-1,1)$	$[-\pi/2, \pi/2]-\{0\}$
(3) $\tan^{-1}x$	$\mathbb{R}$	$(-\pi/2, \pi/2)$



Family (I, II)

	Domain	Range
(1) $\cos^{-1}x$	$[-1,1]$	$[0,\pi]$
(2) $\sec^{-1}x$	$\mathbb{R}-(-1,1)$	$[0,\pi]-\{\pi/2\}$
(3) $\cot^{-1}x$	$\mathbb{R}$	$(0,\pi)$



For Group (I)

$\sin^{-1}(-x) = -\sin^{-1}(x)$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x)$

$\tan^{-1}(-x) = -\tan^{-1}(x)$

for Group (II)

$\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

$\sec^{-1}(-x) = \pi - \sec^{-1}(x)$

$\cot^{-1}(-x) = \pi - \cot^{-1}(x)$

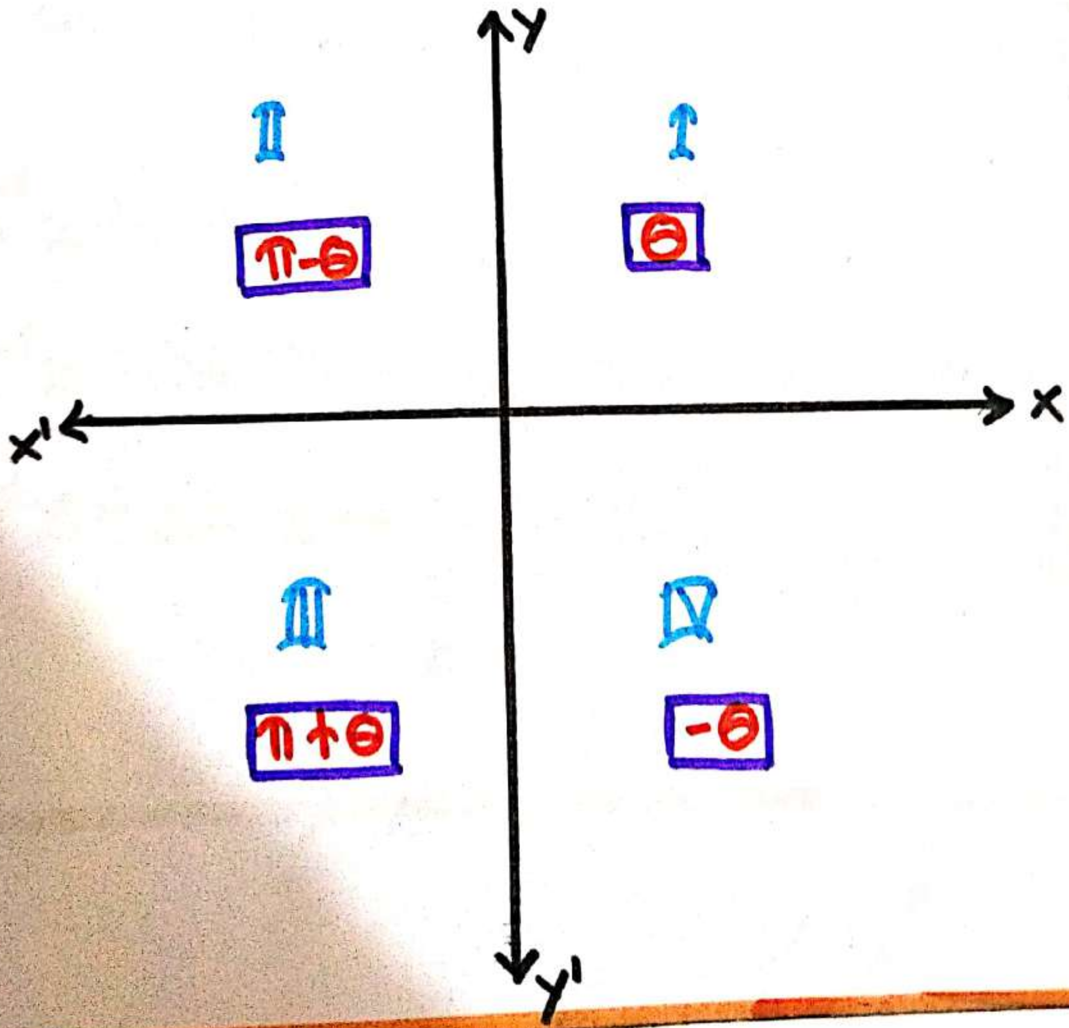
For Corresponding Members

(1) $\sin^{-1}(x) + \cos^{-1}(x) = \pi/2$

(2) $\tan^{-1}(x) + \cot^{-1}(x) = \pi/2$

(3) [illegible]

θ	0°	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not defined



NOTE

(1) $\sin^{-1}(-x) \rightarrow -\sin^{-1}x$

(2) $\operatorname{cosec}^{-1}(-x) \rightarrow -\operatorname{cosec}^{-1}x$

(3) $\tan^{-1}(-x) \rightarrow -\tan^{-1}x$

(1) $\cos^{-1}(-x) \rightarrow \pi - \cos^{-1}x$

(2) $\sec^{-1}(-x) \rightarrow \pi - \sec^{-1}x$

(3) $\cot^{-1}(-x) \rightarrow \pi - \cot^{-1}x$

(1) $\sin^{-1}x + \cos^{-1}x \rightarrow \pi/2$ $\quad x \in [-1, 1]$

(2) $\sec^{-1}x + \operatorname{cosec}^{-1}x \rightarrow \pi/2$ $\quad x \in R-(-1,1)$ or $|x| \geq 1$

(3) $\tan^{-1}x + \cot^{-1}x \rightarrow \pi/2$ $\quad x \in R$

Find The principal Value Of:-

(1) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) \rightarrow \pi/4$

(2) $\sin^{-1}\left(-\sqrt{3}/2\right) \rightarrow -\sin^{-1}\left(\sqrt{3}/2\right) = -\pi/3$

(3) $\sin^{-1}\left(1/2\right) \rightarrow \pi/6$

(4) $\cos^{-1}\left(\sqrt{3}/2\right) \rightarrow \pi/6$

(5) $\sin^{-1}\left(-1/2\right) \rightarrow -\sin^{-1}\left(1/2\right) = -\pi/6$

(6) $\cos^{-1}\left(-1/2\right) \rightarrow \pi - \cos^{-1}\left(1/2\right)$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

(7) $\sin^{-1}\left(-\sqrt{3}/2\right) \rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\pi/3$

(8) $\tan^{-1}\left(\sqrt{3}\right) \rightarrow \pi/3$

(9) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) \rightarrow \pi/4$

(10) $\cos^{-1}(-1/\sqrt{2}) \rightarrow \pi - \cos^{-1}(1/\sqrt{2})$

$\therefore \cos(-1/\sqrt{2}) = \pi - \pi/4$

$\therefore \cos^{-1}(-1/\sqrt{2}) = 3\pi/4$

$\therefore \cos^{-1}(-1/\sqrt{2}) = 3\pi/4$

Qus Write the value of:→

$\tan^{-1}\left[2\sin\left(2\cos^{-1}\left(\sqrt{3}/2\right)\right)\right]$

Solⁿ→ Let $y = \cos^{-1}(\sqrt{3}/2)$

$\cos y = \sqrt{3}/2$

$\therefore y = \pi/6$

$\therefore = \tan^{-1}\left[2\sin\left(2\cos^{-1}\left(\sqrt{3}/2\right)\right)\right]$

$\therefore = \tan^{-1}\left[2\sin\left(\cancel{2}\times\pi/\cancel{6}_{3}\right)\right]$

$\therefore = \tan^{-1}\left[2\sin \pi/3\right]$

$\therefore = \tan^{-1}\left[\cancel{2}\times\sqrt{3}/\cancel{2}\right]$

$\therefore = \tan^{-1}[\sqrt{3}]$

$\therefore = \pi/3$

$\therefore \tan\left[2\sin\left(2\cos^{-1}\left(\sqrt{3}/2\right)\right)\right] = \pi/3.$

Qus Write the principal value of:→

$\tan^{-1}(1) + \cos^{-1}(-1/2)$

Solⁿ→ Consider $\tan^{-1}(1) + \cos^{-1}(-1/2)$

$\therefore \tan^{-1}(1) + \left(\pi - \cos^{-1}(1/2)\right)$

$\therefore = \pi/4 + (\pi - \pi/3)$

$\therefore = \pi/4 + 2\pi/3$

$\therefore = \dfrac{3\pi + 8\pi}{12}$

$\therefore = \dfrac{11\pi}{12}$

$\therefore \tan^{-1}(1) + \cos^{-1}(-1/2) = 11\pi/12$

Qus Evaluate

$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

Sol^n → Consider, $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

$\therefore = \tan^{-1}(\sqrt{3}) - [\pi - \sec^{-1}(2)]$

$\therefore = \tan^{-1}(\sqrt{3}) - [\pi - \cos^{-1}(1/2)]$

$\therefore = \pi/3 - [\pi - \pi/3]$ $\quad [\sec^{-1}x = \cos^{-1}(1/x)]$

$\therefore = \pi/3 - 2\pi/3 = -\pi/3$

Qus find the value of the following:→

(1) $\tan^{-1}(1) + \cos^{-1}(-1/2) + \sin^{-1}(-1/2)$

Sol^n → $\tan^{-1}(1) + \cos^{-1}(-1/2) + \sin^{-1}(-1/2)$

$= \tan^{-1}(1) + (\pi - \cos^{-1}(1/2)) + (-\sin^{-1}(1/2))$

$\therefore = \pi/4 + (\pi - \pi/3) + (-\pi/6)$

$\therefore = 3\pi + 8\pi - 2\pi / 12$

$\therefore \cancel{9}^{3}\pi / \cancel{12}_{4}$

$\therefore = 3\pi/4$

(2) $\cos^{-1}(1/2) + 2\sin^{-1}(1/2)$

Sol^n → $\cos^{-1}(1/2) + 2\sin^{-1}(1/2)$

$\therefore = \pi/3 + \cancel{2}^{1}(\pi/\cancel{6}_{3})$

$\therefore \pi/3 + \pi/3 = \pi + \pi / 3$

$\therefore = 2\pi/3$

(3) $\cos^{-1}(1/2) - 2\sin^{-1}(-1/2)$

Sol^n $\rightarrow$ $\cos^{-1}(1/2) - 2\sin^{-1}(-1/2)$

$\therefore = \pi/3 - 2\left(-\sin^{-1}(1/2)\right)$

$\therefore = \pi/3 - \cancel{2}\left(-\pi/\cancel{6}_{3}\right)$

$\therefore = \pi/3 - (-\pi/3)$

$\therefore = \pi/3 + \pi/3$

$\therefore = \pi + \pi/3$

$\therefore 2\pi/3$

(4) $\cos^{-1}(-1/2) + 2\sin^{-1}(1/2)$

Sol^n $\rightarrow$ $\cos^{-1}(-1/2) + 2\sin^{-1}(1/2)$

$\therefore = \pi - \cos^{-1}(1/2) + \cancel{2}(\pi/\cancel{6}_{3})$

$\therefore = \pi - \pi/3 + \pi/3$

$\therefore = \dfrac{3\pi - \cancel{\pi} + \cancel{\pi}}{3} = \cancel{3}\pi/\cancel{3} = \pi$

(5) $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$

Sol^n $\rightarrow$ $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$

$\therefore = \tan^{-1}(\sqrt{3}) - [\pi - \cot^{-1}(\sqrt{3})]$

$\therefore = \tan^{-1}(\sqrt{3}) - [\pi - \tan^{-1}(\frac{1}{\sqrt{3}})]$

$\therefore = \pi/3 - [\pi - \pi/6]$

$\therefore = \pi/3 - 5\pi/6$

$\therefore = 2\pi - 5\pi/6 = -\cancel{3}\pi/\cancel{6}\,2$

$\therefore = -\pi/2$

(6) $\tan^{-1}(1) + \sin^{-1}(-1/2)$

Sol^n $\rightarrow$ $\pi/4 + (-\sin^{-1}(1/2))$

$\therefore = \pi/4 - \pi/6$

$\therefore = 3\pi - 2\pi/12$

$\therefore = \pi/12$

Range — Let $y = \sin x$ — Domain

then x belongs to $[-1,1]$

y belongs to $[-\pi/2, \pi/2]$

Art → properties of Inverse Trignometric function

1 prove that :→

* $\sin(\sin^{-1}(x)) = x \quad x \in [-1,1]$

	D^+	R^+
$\sin^{-1}x$	$[-1,1]$	$[-\pi/2, \pi/2]$
$\sin x$	$[-\pi/2, \pi/2]$	$[-1,1]$

$\sin^{-1}(\sin x) = x$ $\quad x \in [-\pi/2, \pi/2]$

$\cos(\cos^{-1} x) = x$ $\quad x \in [-1, 1]$

$\cos^{-1}(\cos x) = x$ $\quad x \in [0, \pi]$

Qus Find the value of the following:→

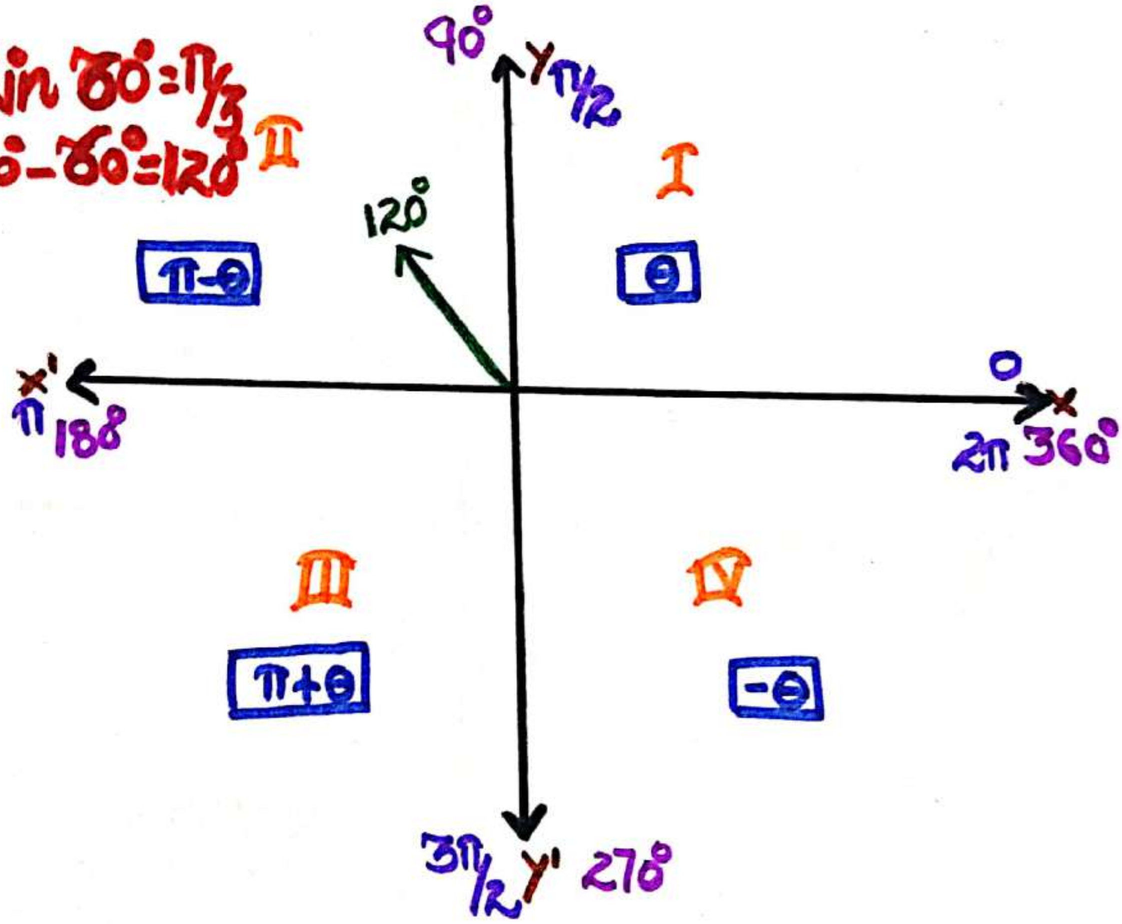
(1) $\sin^{-1}(\sin(2\pi/3))$

Sol^n → $\sin^{-1}(\sin(2\pi/3))$ $\Big|$ $\frac{2\times\cancel{180}^{60^\circ}}{\cancel{3}} = 120^\circ$

$\therefore \sin^{-1}(\sin(\pi/3)) = \pi/3$.

$\sin 60^\circ = \pi/3$

$180^\circ - 60^\circ = 120^\circ$



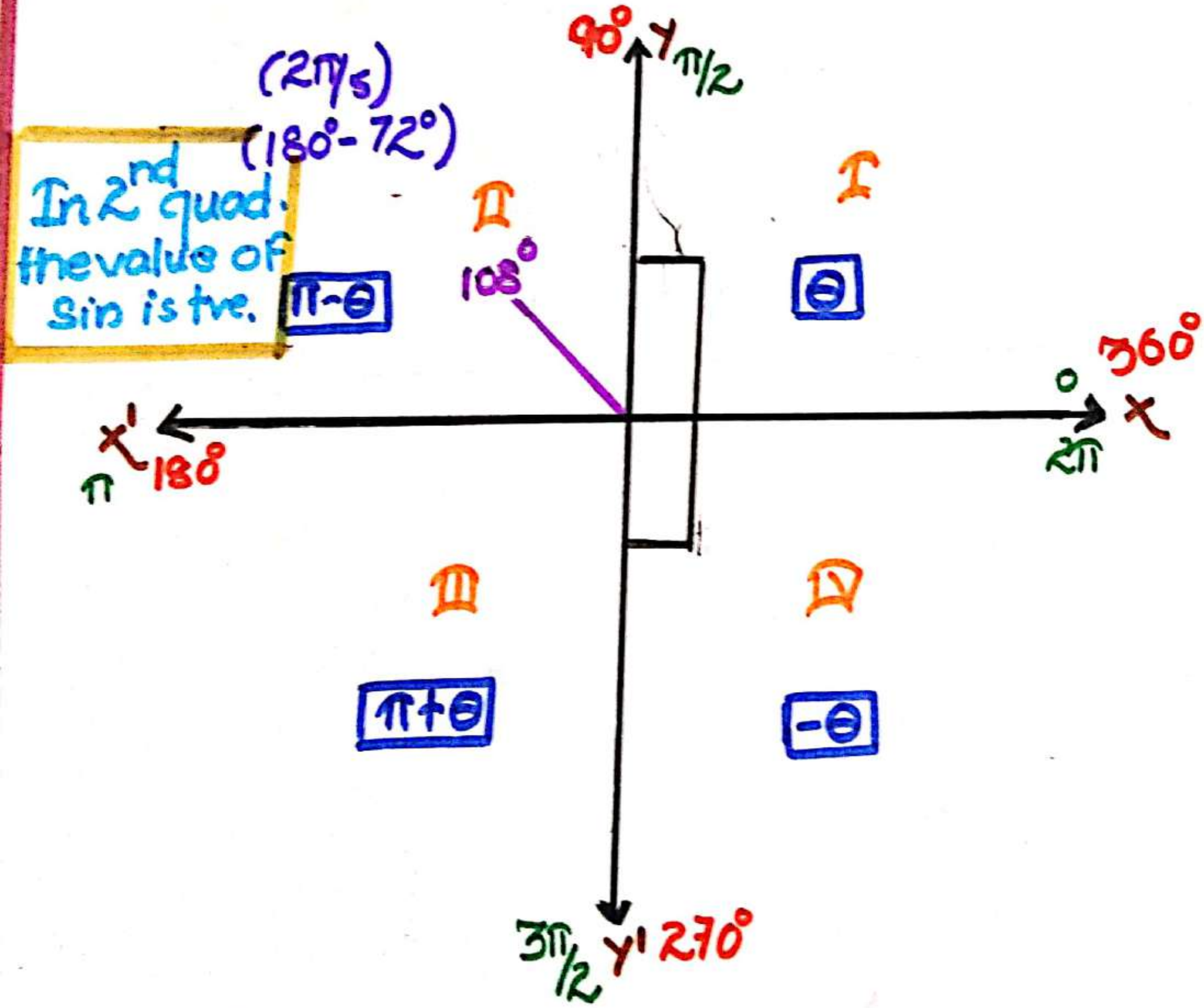
Qus Find the principal value of the following:-

(2) $\sin^{-1}(\sin(3\pi/5))$

Sol^n → $\sin^{-1}(\sin(3\pi/5))$ | $3\times \cancel{180}^{36}/\cancel{5} = 108°$

$\therefore = \sin^{-1}(\sin(2\pi/5))$

$\therefore = 2\pi/5$



(3) $\cos^{-1}(\cos(7\pi/6))$

Sol^n → $\cos^{-1}(\cos(7\pi/6))$ | $7\pi/6 \rightarrow 7\times \cancel{180}^{30°}/\cancel{6} = 210°$

$\therefore = \cos^{-1}(\cos(5\pi/6))$

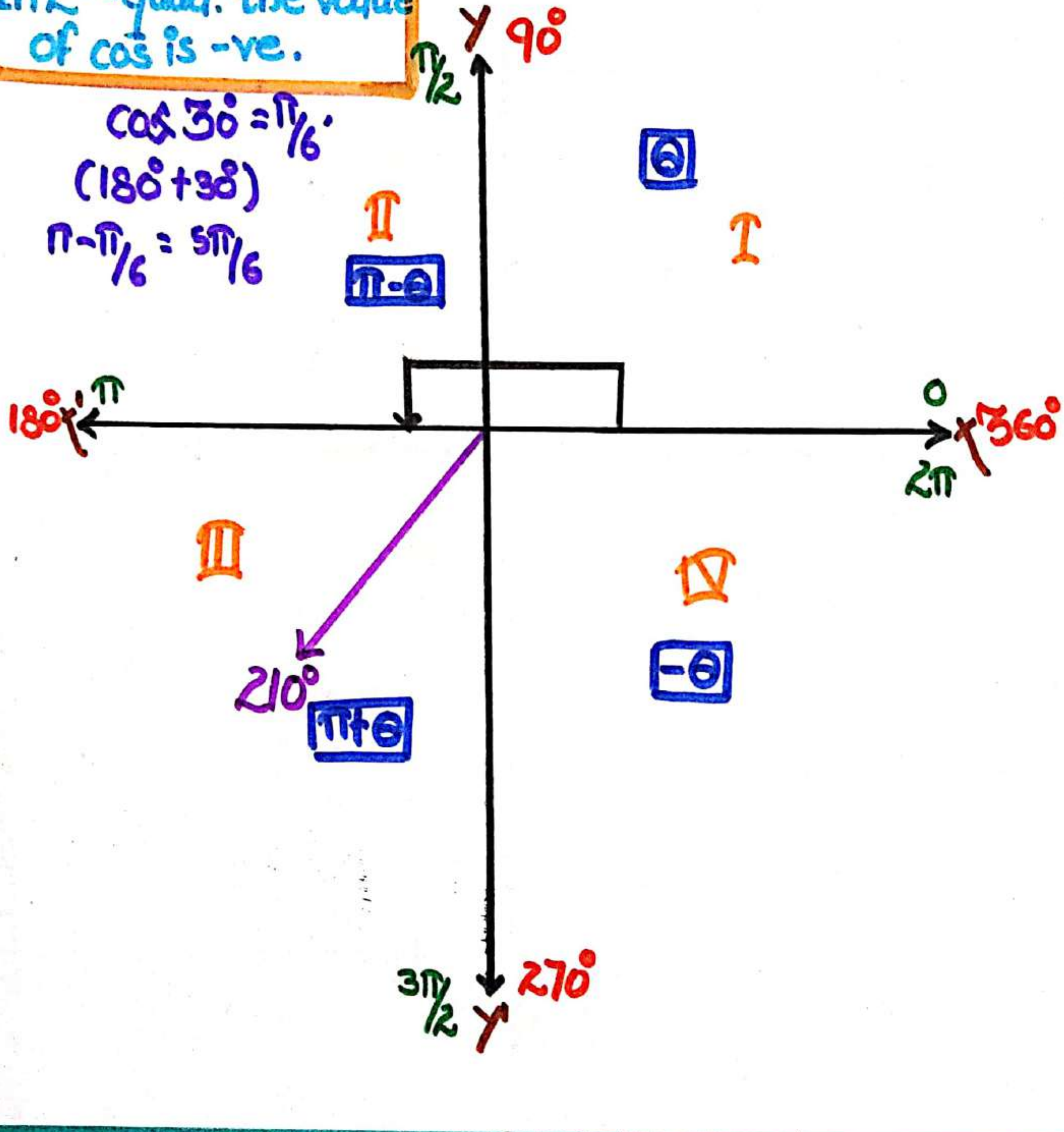
$\therefore = 5\pi/6$

In 2nd quad. the value of cos is -ve.

$\cos 30° = \pi/6$.

$(180° + 30°)$

$\pi - \pi/6 = 5\pi/6$

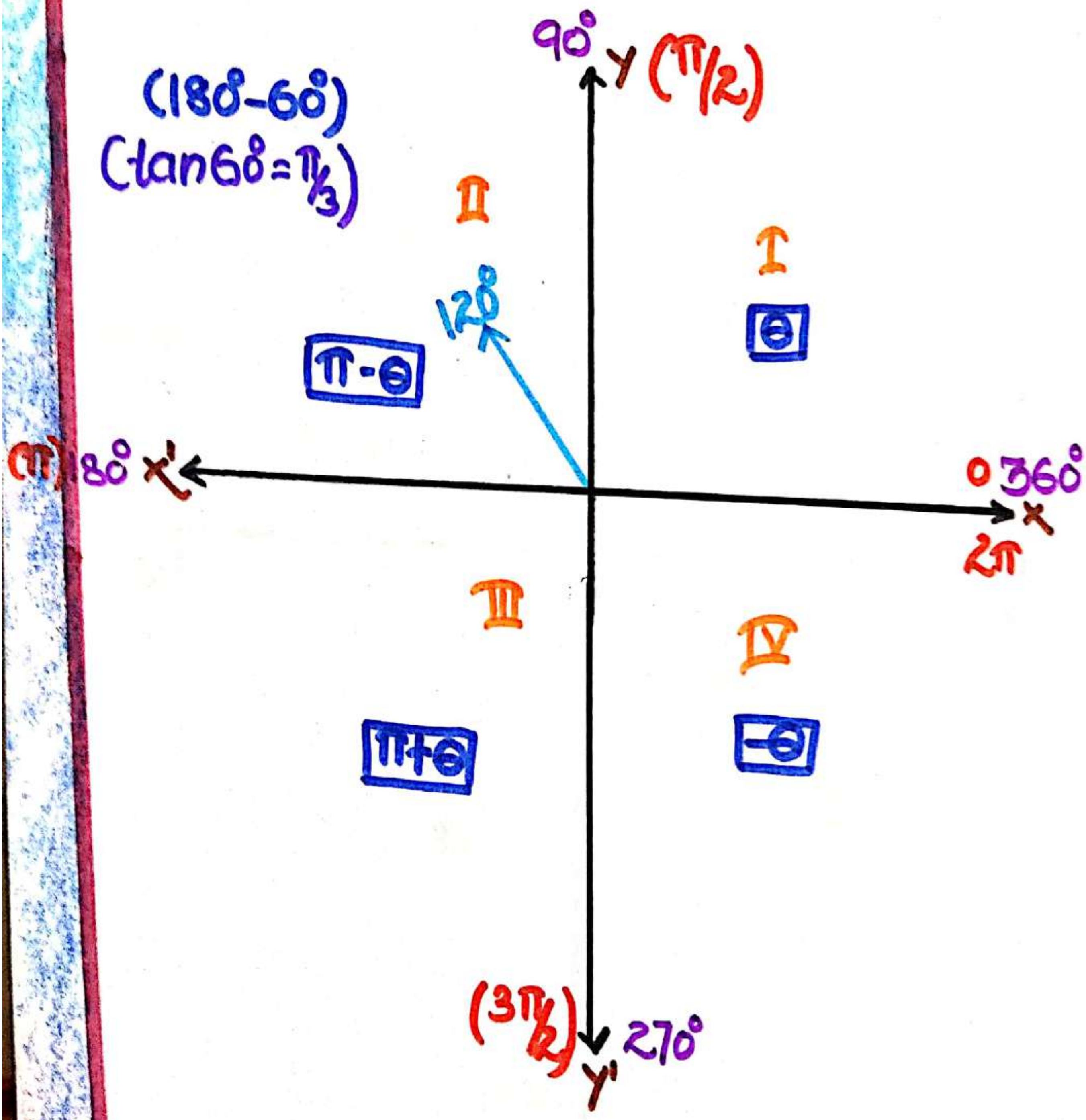


(4) $\tan^{-1}(\tan(2\pi/3)$

Sol^n $\rightarrow$ $\tan^{-1}(\tan(2\pi/3))$ $\quad\Big|\; 2\pi/3 = \dfrac{2\times\cancel{180^\circ}\,60^\circ}{\cancel{3}} = 120^\circ$

$\therefore = \tan^{-1}(\tan(\pi/3))$

$\therefore = \pi/3$



(5) $\tan^{-1}(\tan(3\pi/4))$

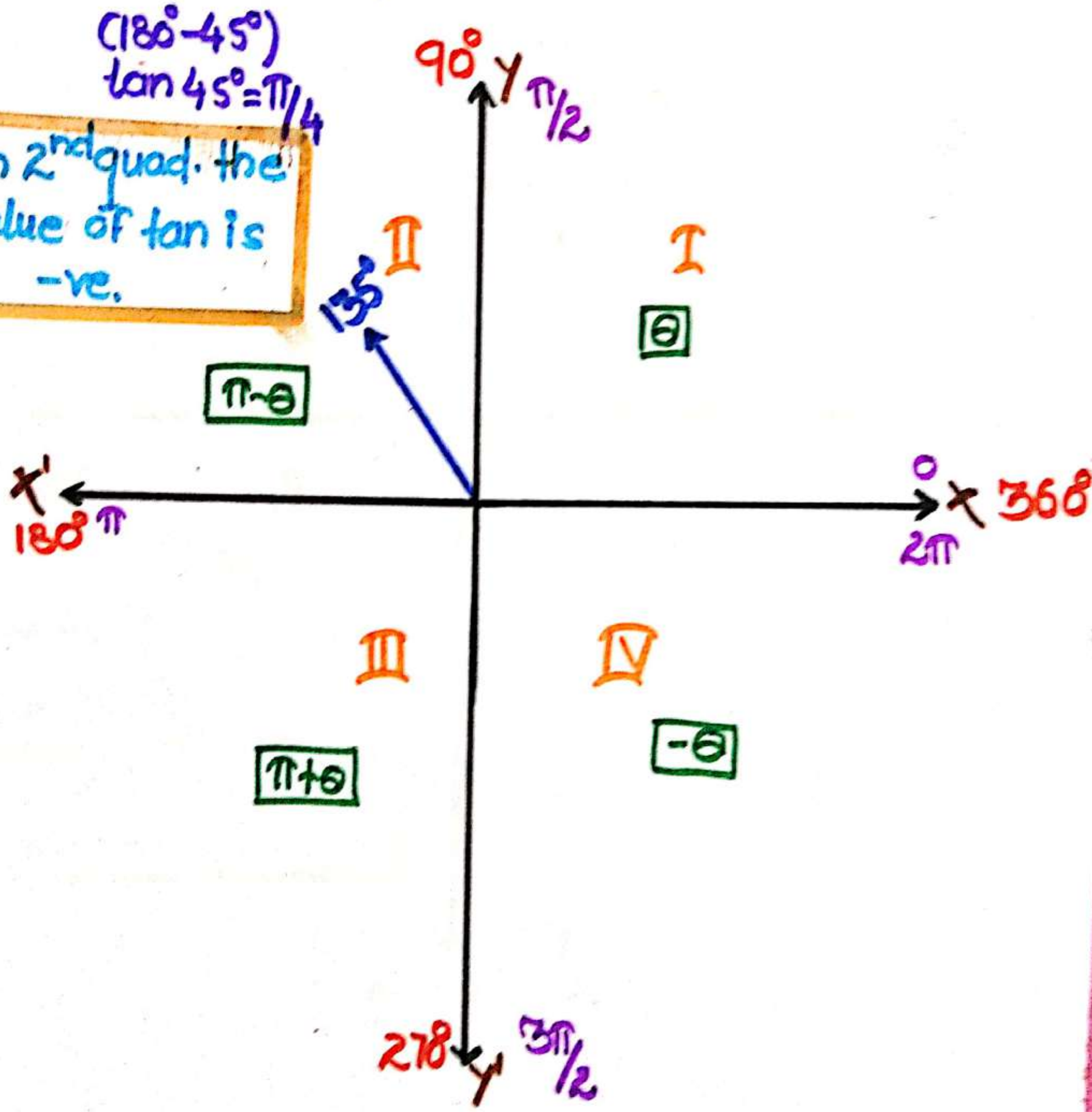
Sol^n → $\tan^{-1}(\tan(3\pi/4))$ | $\frac{3\times\cancel{180}^{45}}{\cancel{4}} = 135°$.

$\therefore = \tan^{-1}(\tan(-\pi/4))$

$\therefore = -\pi/4$

$(180°-45°)$
$\tan 45° = \pi/4$

In 2nd quad. the value of tan is -ve.

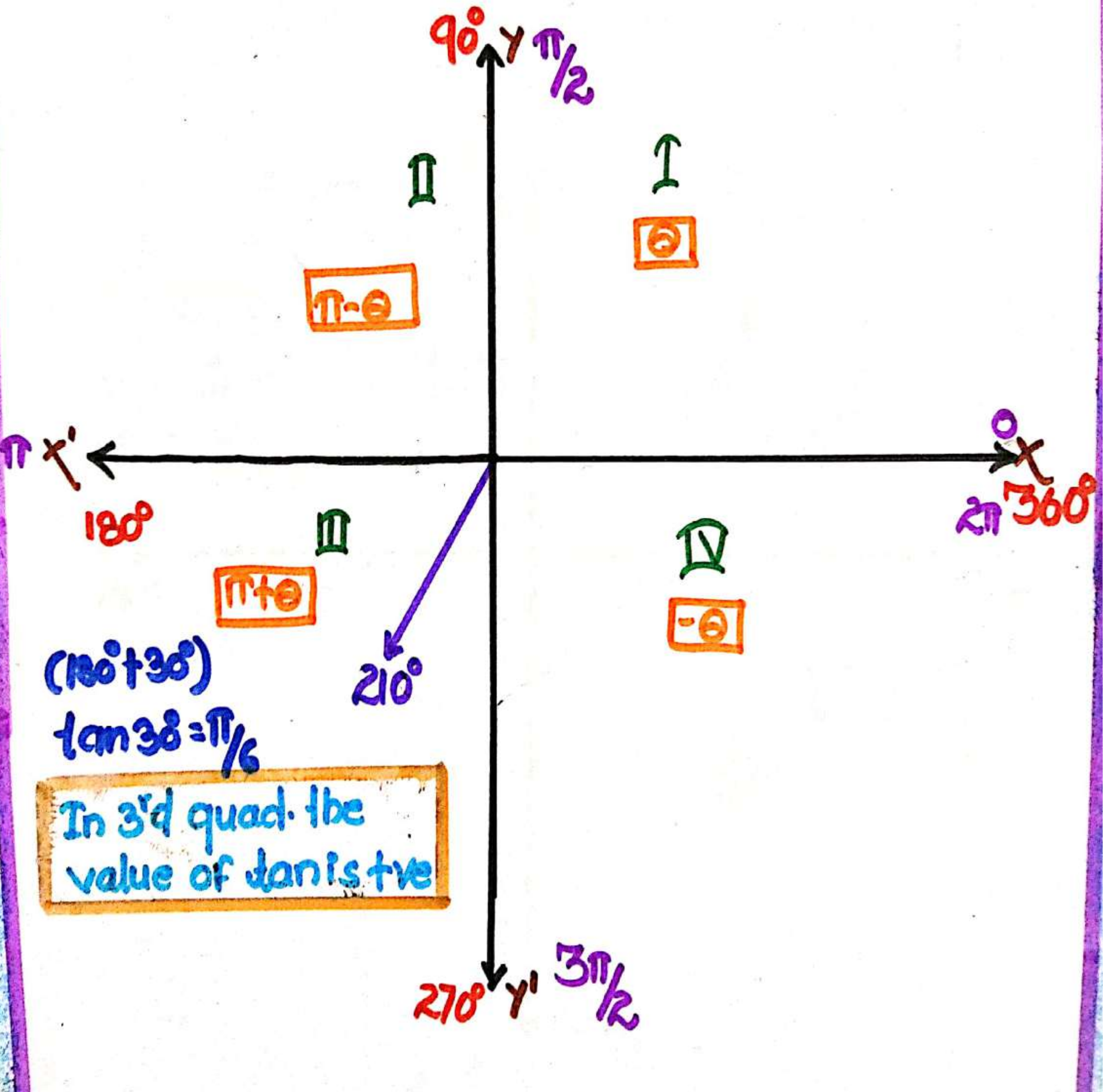


(6) $\tan^{-1}(\tan(7\pi/6))$

Sol^n $\rightarrow$ $\tan^{-1}(\tan(7\pi/6))$ | $7 \times \cancel{180}^{30°} / \cancel{6} = 210°$

$\therefore = \tan^{-1}(\tan(\pi/6))$

$\therefore = \pi/6$



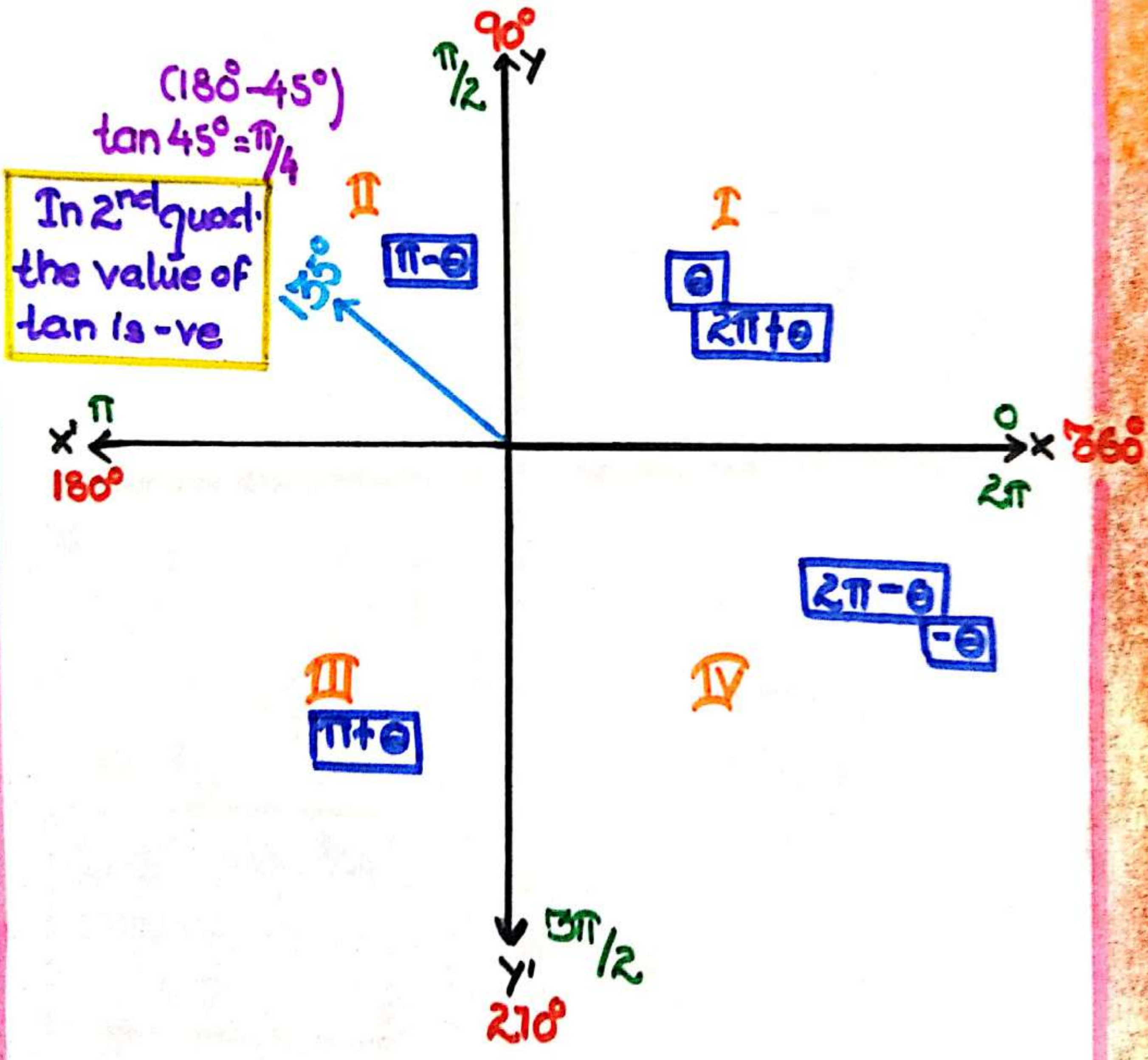
Qus Find the value of each of the expression.

(1) $\tan^{-1}(\tan(3\pi/4))$

Sol^n $\rightarrow$ $\tan^{-1}(\tan(3\pi/4))$ | $3\times \cancel{180^\circ}^{45^\circ}/\cancel{4} = 135^\circ$

$\therefore \tan^{-1}(\tan(-\pi/4))$

$\therefore = -\pi/4$

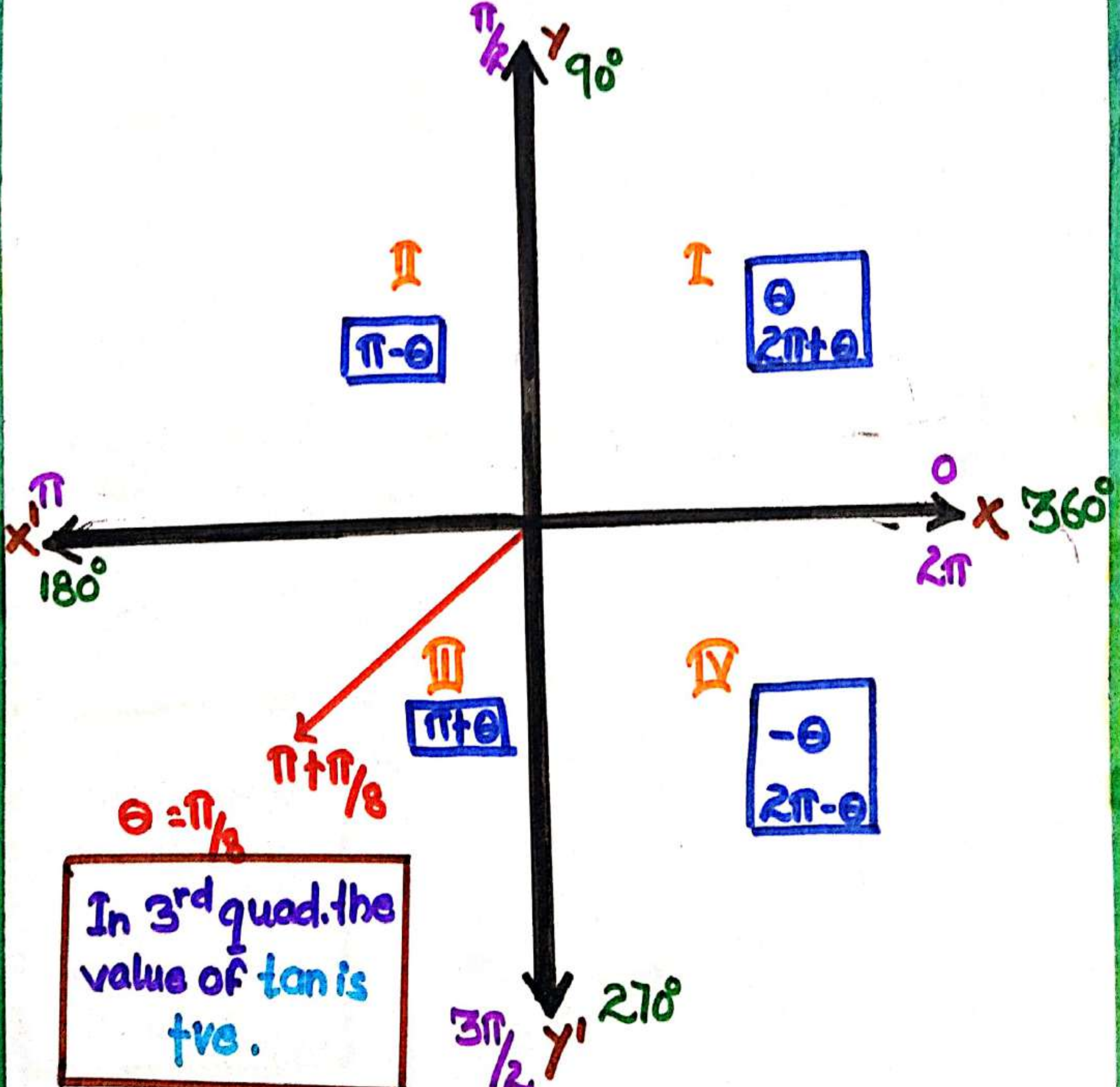


(2) $\tan^{-1}(\tan(9\pi/8))$

Sol^n → $\tan^{-1}(\tan(9\pi/8))$ | $9\pi/8 = \pi + \pi/8$

$\therefore = \tan^{-1}(\tan(\pi/8))$

$\therefore = \pi/8$



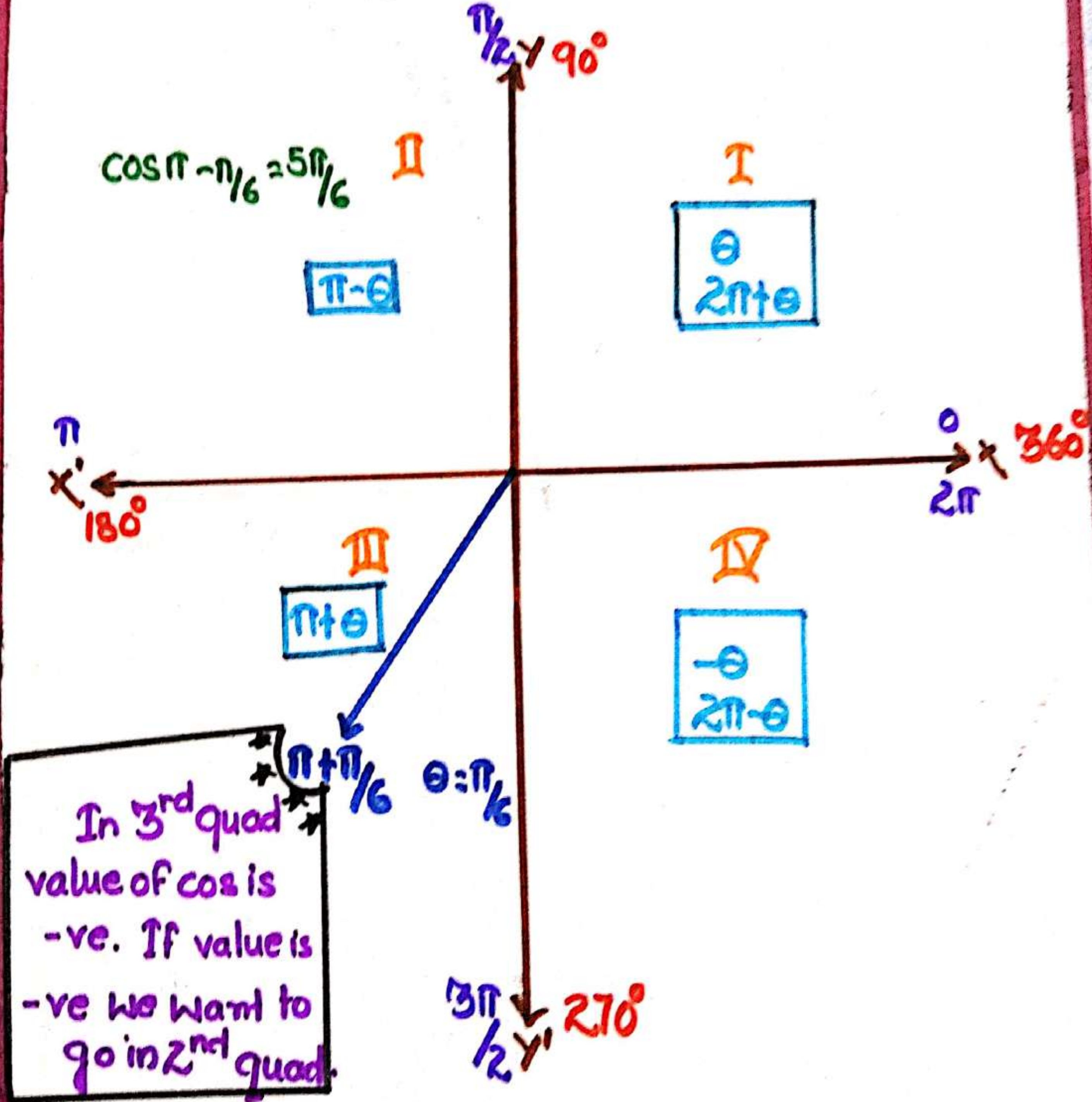
In 3rd quad. the value of tan is +ve.

(3) $\cos^{-1}(\cos(7\pi/6))$

Sol^n → $\cos^{-1}\left(\cos\left(7\pi/6\right)\right)$ | $7\pi/6 = \pi + \pi/6$

$\therefore = \cos^{-1}\left(\cos\left(5\pi/6\right)\right)$

$\therefore = 5\pi/6$



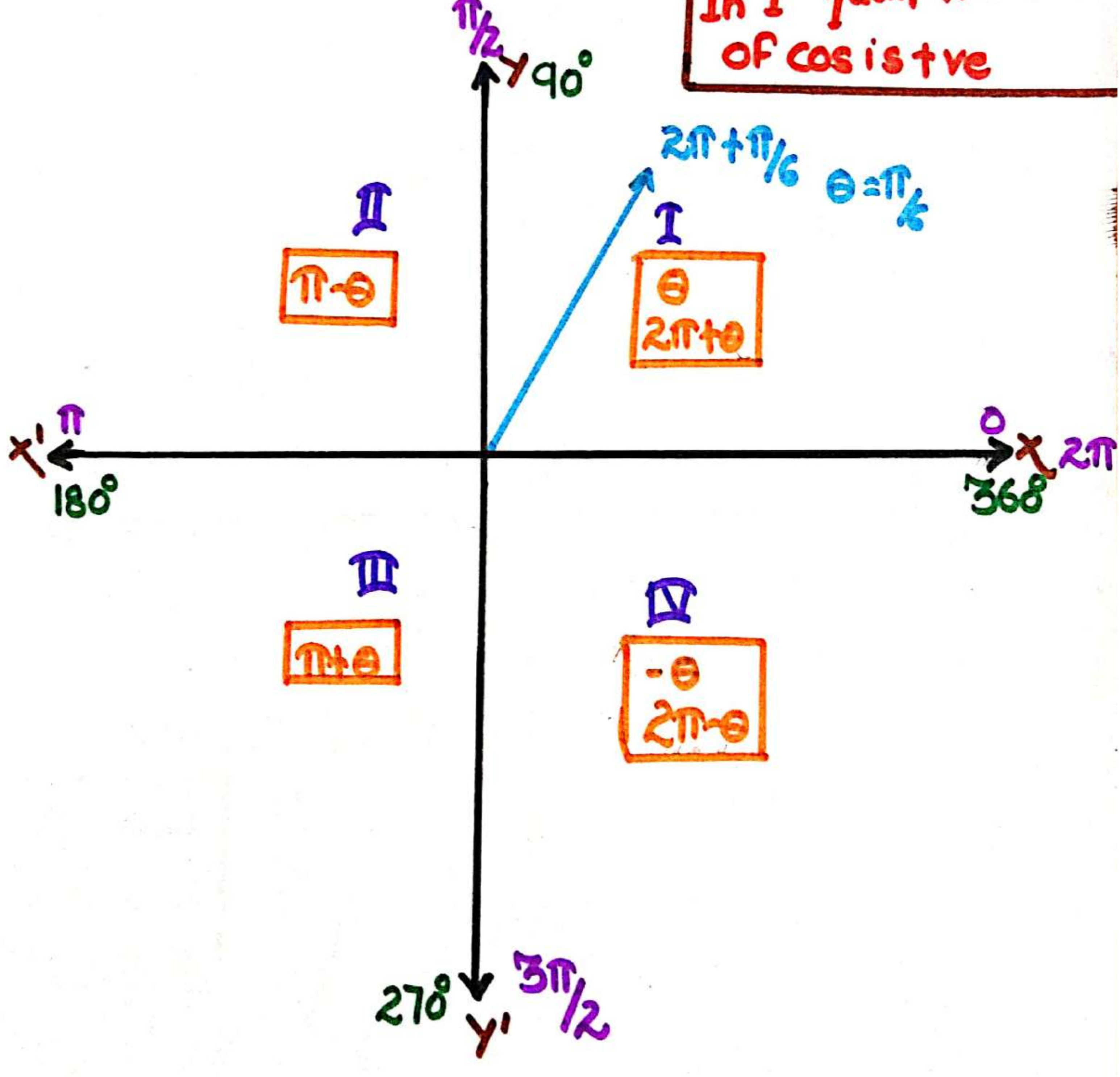
4) $\cos^{-1}\left(\cos\left(\frac{13\pi}{6}\right)\right)$

Sol^n → $\cos^{-1}\left(\cos\left(\frac{13\pi}{6}\right)\right)$ | $\frac{13\pi}{6} = 2\pi + \frac{\pi}{6}$

$\therefore = \cos^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right)$

$\therefore = \frac{\pi}{6}$

In Ist quad. the value of cos is +ve

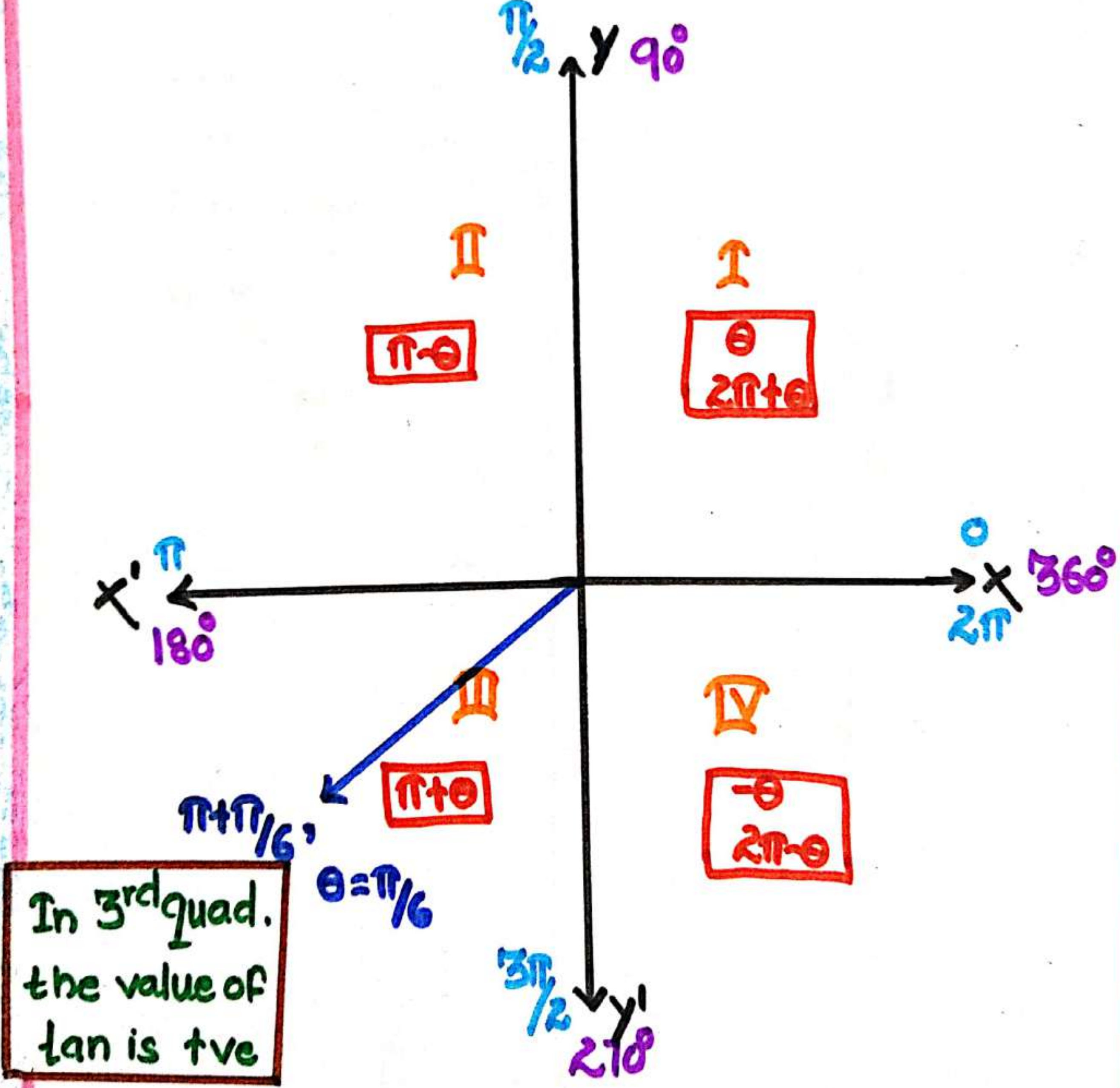


(5) $\tan^{-1}(\tan(7\pi/6))$

Sol^n → $\tan^{-1}(\tan(7\pi/6))$ | $7\pi/6 = \pi + \pi/6$

$\therefore = \tan^{-1}(\tan(\pi/6))$

$\therefore = \pi/6$



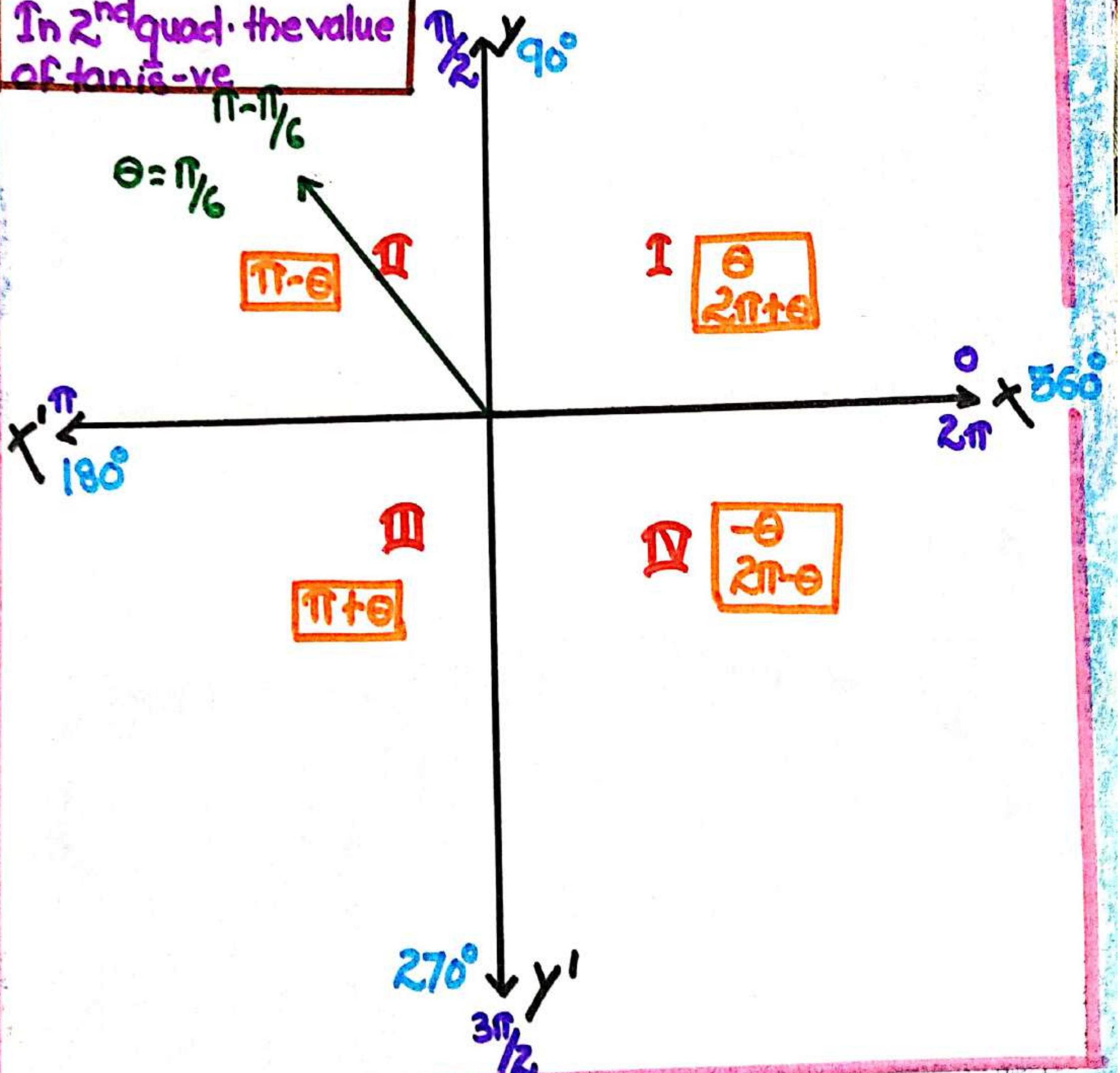
(6) $\tan^{-1}\left(\tan\left(5\pi/6\right)\right)$

Sol → $\tan^{-1}\left(\tan\left(5\pi/6\right)\right)$ | $5\pi/6 = \pi - \pi/6$

$\therefore = \tan^{-1}\left(\tan\left(-\pi/6\right)\right)$

$\therefore = -\dfrac{\pi}{6}$

In 2^{nd} quad. the value of tan is -ve



Art. that →

prove that,

$$\sin^{-1}x + \cos^{-1}x = \pi/2 \quad x \in [-1,1]$$

$$\tan^{-1}x + \cot^{-1}x = \pi/2 \quad x \in \mathbb{R}$$

$$\operatorname{cosec}^{-1}x + \sec^{-1}x = \pi/2 \quad x \in \mathbb{R} - (-1,1)$$

or $|x| \geq 1$

Qus If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find the value of x.

Sol^n → The given eq^n is,

$$\therefore \sin\left(\sin^{-1}\left(\frac{1}{5}\right)+\cos^{-1}x\right)=1$$

$$\therefore \sin\left(\sin^{-1}\left(\frac{1}{5}\right)+\cos^{-1}x\right)=1$$

$$\therefore \sin^{-1}\left(\frac{1}{5}\right)+\cos^{-1}x=\sin^{-1}(1)$$

$$\therefore \sin^{-1}\left(\frac{1}{5}\right)+\cos^{-1}x=\frac{\pi}{2} \rightarrow (1)$$

We know that,

$$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2} \longrightarrow (2)$$

$\therefore$ from (1) & (2)

$$\sin^{-1}\left(\frac{1}{5}\right)=\sin^{-1}(x)$$

$$\therefore x=\frac{1}{5}$$

or

$\therefore (1) \Rightarrow$

$$\sin^{-1}\left(\frac{1}{5}\right)=\frac{\pi}{2}-\cos^{-1}(x)$$

$$\therefore \sin^{-1}\left(\frac{1}{5}\right)=\sin^{-1}(x)$$

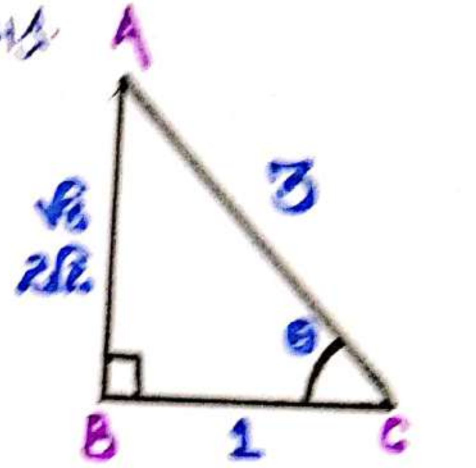
$$\therefore \boxed{x=\frac{1}{5}}$$

Que prove that $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$.

Sol^n → $L.H.S. = \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)$

$\therefore = \frac{9}{4}\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{3}\right)\right]$

$\therefore = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) = R.H.S.$



Let $\cos^{-1}\left(\frac{1}{3}\right) = \theta$

$\therefore \cos\theta = \frac{1}{3}$

$\therefore \sin\theta = \frac{2\sqrt{2}}{3}$

$\therefore \theta = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

$\therefore \cos^{-1}\left(\frac{1}{3}\right) = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

$$\boxed{\therefore \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)}$$

Art. → Properties of Inverse Trignometric Function

* Prove that :→

(1) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, $xy < 1$

(2) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, $xy > -1$

Qus If $\tan^{-1}x + \tan^{-1}y = \pi/4$, Where $xy<1$, find the value of $x+y+xy$.

Sol^n → We are given,

$$\tan^{-1}x + \tan^{-1}y = \pi/4$$

$$\therefore \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \pi/4$$

$$\therefore \frac{x+y}{1-xy} = \tan^{-1}\pi/4$$

$$\therefore \frac{x+y}{1-xy} = 1$$

$$\left\{\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$$

$$\therefore x+y = 1-xy$$

$$\therefore x+y+xy = 1$$

The value of $x+y+xy$ is 1

Qus Show that

$$\tan^{-1}(1/2) + \tan^{-1}(2/11) = \tan^{-1}(3/4)$$

Sol^n → L·H·S → $\tan^{-1}(1/2) + \tan^{-1}(2/11)$

$$\therefore = \tan^{-1}\left(\frac{1/2 + 2/11}{1-(1/2)(2/11)}\right)$$

$\therefore = \tan^{-1}\left(11+4/22-2\right)$

$\therefore = \tan^{-1}\left(\cancel{15}^{3}/\cancel{20}_{4}\right)$

$$\left\{\therefore \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$$

$\therefore = \tan^{-1}\left(3/4\right) = R.H.S.$

Qus prove that

$\tan^{-1}(1/5) + \tan^{-1}(1/7) + \tan^{-1}(1/3) + \tan^{-1}\left(\frac{1}{8}\right) = \pi/4.$

Sol^n → $L.H.S = \tan^{-1}(1/5) + \tan^{-1}(1/7) + \tan^{-1}(1/3) + \tan^{-1}(1/8)$

$\therefore = \left(\tan^{-1}(1/5) + \tan^{-1}(1/7)\right) + \left(\tan^{-1}(1/3) + \tan^{-1}(1/8)\right)$

$\therefore = \left(\tan^{-1}\left(\frac{1/5+1/7}{1-(1/5)(1/7)}\right) + \tan^{-1}\left(\frac{1/3+1/8}{1-(1/3)(1/8)}\right)\right)$

$\therefore = \tan^{-1}\left(7+5/35-1\right) + \tan^{-1}\left(8+3/24-1\right)$

$\therefore = \tan^{-1}\left(\cancel{12}^{6}/\cancel{34}_{17}\right) + \tan^{-1}\left(11/23\right)$

$\therefore = \tan^{-1}\left(6/17 + 11/23 \Big/ 1-(6/17)(11/23)\right)$

$\therefore = \tan^{-1}\left(\frac{138+187}{391-66}\right)$

$\therefore = \tan^{-1}\left(\cancel{325}^{1}/\cancel{325}\right)$

$\therefore = \tan^{-1}(1) = \pi/4 = R.H.S$

L.H.S = R.H.S , Henced proved.

V.Imp Qus Solve $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$:

Sol^n → The given eq^n is,

$\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$,

$\therefore \tan^{-1}\left[\frac{2x+3x}{1-(2x)(3x)}\right] = \pi/4$

provided $(2x)(3x) < 1$

$\therefore \tan^{-1}\left[5x/1-6x^2\right] = \pi/4$,

provided $\underline{6x^2 < 1}$

$\therefore \frac{5x}{1-6x^2} = \tan(\pi/4)$,

$\therefore 5x/1-6x^2 = 1$

$\therefore 5x = 1-6x^2$

$\therefore 6x^2+5x-1=0$

$\therefore 6x^2+6x-x-1=0$

$\therefore 6x(x+1)-1(x+1)=0$

$\therefore (6x-1)(x+1)=0$

$\therefore 6x-1=0$ | $x+1=0$

$x=1/6$ | $x=-1$

$\therefore x=-1$ as $x=-1$ does not satisfy $6x^2<1 \therefore x=1/6$

$x=1/6$

$6(-1)^2=6<1$ (False) ✗

$6(1/6)^2=1/6<1$

(True) ✓

$\therefore$ rejecting $x=-1$ as $x=-1$, does not satisfy $6x^2<1$

$\boxed{x=1/6}$

Que prove that

$$2\tan^{-1}(1/3) + \tan^{-1}(1/7) = \pi/4.$$

Sol^n $\rightarrow$ $L.H.S. = 2\tan^{-1}(1/3) + \tan^{-1}(1/7)$

$\therefore = \tan^{-1}(1/3) + \tan^{-1}(1/3) + \tan^{-1}(1/7)$

$\therefore = \tan^{-1}(1/3) + [\tan^{-1}(1/3) + \tan^{-1}(1/7)]$

$\therefore = \tan^{-1}(1/3) + \tan^{-1}\left[\dfrac{1/3 + 1/7}{1-(\frac{1}{3})(\frac{1}{7})}\right]$

$\therefore = \tan^{-1}(1/3) + \tan^{-1}\left[\dfrac{7+3}{21-1}\right]$

$\therefore = \tan^{-1}(1/3) + \tan^{-1}[\cancel{10}/\cancel{20}]$

$\therefore = \tan^{-1}(1/3) + \tan^{-1}[1/2]$

$\therefore = \tan^{-1}\left[\dfrac{1/3 + 1/2}{1-(1/3)(1/2)}\right]$

$\therefore = \tan^{-1}\left[\dfrac{2+3}{6-1}\right]$

$\therefore = \tan^{-1}\left[\cancel{5}/\cancel{5}^{\,1}\right]$

$\therefore = \tan^{-1}(1) = \pi/4 = R.H.S.$

$L.H.S. = R.H.S.$

Hence, proved.

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Qus Solve the following x :→

$$\tan^{-1}x + 2\cot^{-1}x = 2\pi/3$$

Sol^n → The given eq^n is,

$$\tan^{-1}x + 2\cot^{-1}x = 2\pi/3$$

$$\therefore \tan^{-1}x + 2\tan^{-1}(1/x) = 2\pi/3$$

$$\therefore \tan^{-1}x + (\tan^{-1}(1/x) + \tan^{-1}(1/x)) = 2\pi/3$$

$$\therefore \tan^{-1}x + \tan^{-1}\left[1/x + 1/x \Big/ 1-(1/x)(1/x)\right] = 2\pi/3$$

$$\therefore \tan^{-1}x + \tan^{-1}\left[2/x \Big/ x^2-1/x^2\right] = 2\pi/3$$

$$\therefore \tan^{-1}x + \tan^{-1}\left[2/x \times x^2 / x^2-1\right] = 2\pi/3$$

$$\therefore \tan^{-1}\left[\frac{x + \frac{2x}{x^2-1}}{1-(x)\left(\frac{2x}{x^2-1}\right)}\right] = 2\pi/3$$

$$\therefore \tan^{-1}\left[\frac{x(x^2-1)+2x}{(x^2-1)-2x^2}\right] = 2\pi/3$$

$$\therefore \tan^{-1}\left[\frac{x^3-x+2x}{x^2-1-2x}\right] = 2\pi/3$$

$$\therefore \tan^{-1}\left[\frac{x^2+x}{-x^2-1}\right] = 2\pi/3$$

$$\therefore \tan^{-1}\left[\frac{x\cancel{(x^2+1)}}{-\cancel{(x^2+1)}}\right] = 2\pi/3$$

$$\therefore \tan^{-1}(-x) = 2\pi/3$$

$$\therefore -x = \tan(2\pi/3)$$

$$\therefore -x = \tan(\pi - \pi/3)$$

$$+x = +\tan \pi/3$$

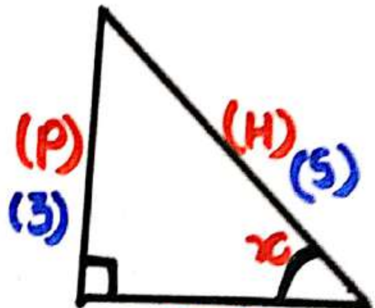
$$\therefore \boxed{x = \sqrt{3}}$$

Qus prove that

$$\cos^{-1}(4/5) + \cos^{-1}(12/13) = \cos^{-1}(33/65)$$

Sol^n → put $\cos^{-1}(4/5) = x$

$$\therefore \cos x = 4/5$$

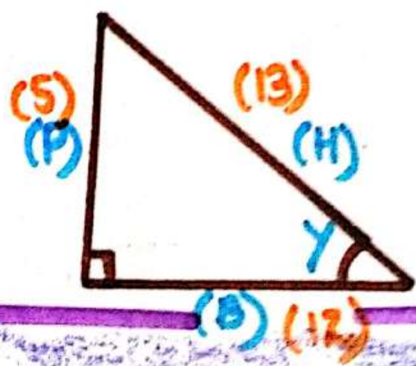


$$\therefore \tan x = 3/4$$

$$x = \tan^{-1}(3/4)$$

$$\therefore \cos^{-1}(4/5) = \tan^{-1}(3/4)$$

put $\cos^{-1}(12/13) = y$

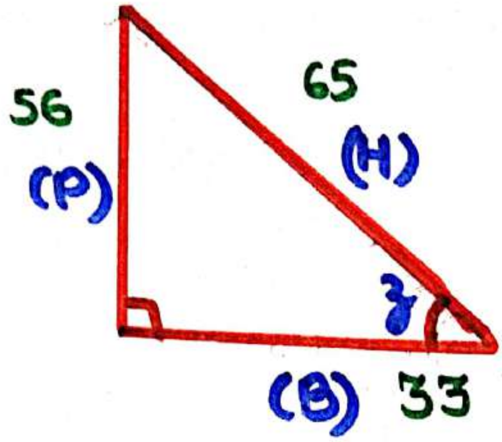


$\therefore \cos y = 12/13$

$\therefore \tan y = 5/12$

$\therefore y = \tan^{-1}(5/12)$ $\quad \therefore \cos^{-1}(12/13) = \tan^{-1}(5/12)$

put $\cos^{-1}(33/65) = z$



$\therefore \cos z = 33/65$

$\therefore \tan z = 56/33$

$\therefore z = \tan^{-1}(56/33)$

$\therefore \cos^{-1}(33/65) = \tan^{-1}(56/33)$

$\therefore L.H.S. = \cos^{-1}(4/5) + \cos^{-1}(12/13)$

$\therefore = \tan^{-1}(3/4) + \tan^{-1}(5/12)$

$\therefore = \tan^{-1}\left(\dfrac{3/4 + 5/12}{1 - (3/4)(5/12)}\right)$

$\therefore = \tan^{-1}(36+20 / 48-15)$

$\therefore = \tan^{-1}(56/36)$

$$\therefore \cos^{-1}(33/65) = R.H.S$$

$$L.H.S = R.H.S,$$

Hence, proved.

Qus prove that,

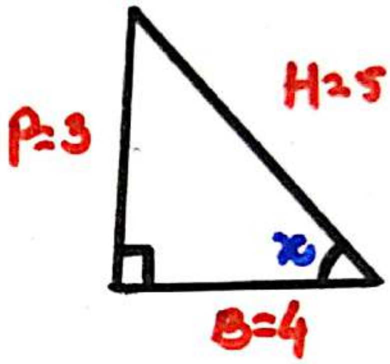
$$\sin^{-1}(3/5) - \sin^{-1}(8/17) = \cos^{-1}(84/85)$$

Sol^n $\rightarrow$ put $\sin^{-1}(3/5) = x$

$\therefore \tan x = (3/4)$

$\therefore x = \tan^{-1}(3/4)$

$\therefore \sin^{-1}(3/5) = \tan^{-1}(3/4)$

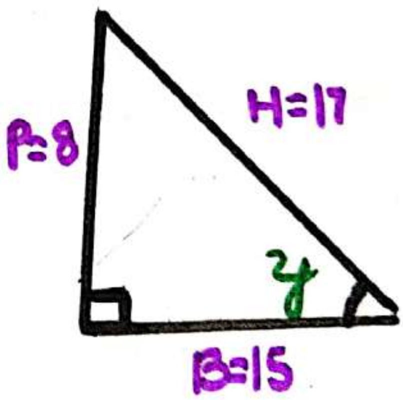


puts $-\sin^{-1}(8/17) = y$

$\therefore -\tan y = -\tan^{-1}(8/15)$

$\therefore y = -\tan^{-1}(8/15)$

$\therefore -\sin^{-1}(8/17) = \tan^{-1}(8/15)$

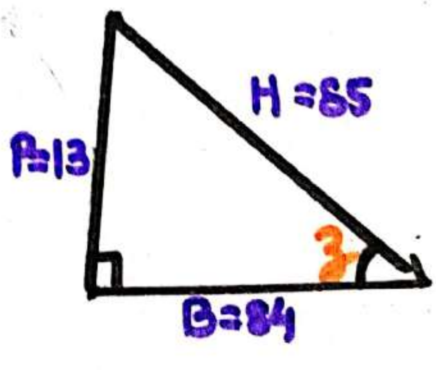


put $\cos^{-1}(84/85) = z$

$\therefore \tan z = (13/84)$

$\therefore z = \tan^{-1}(13/84)$

$\therefore \cos^{-1}(84/85) = \tan^{-1}(13/84)$



$\therefore$ L.H.S = $\tan^{-1}(3/4) - \tan^{-1}(8/15)$

$\therefore = \tan^{-1}\left(\dfrac{3/4 - 8/15}{1-(3/4)(8/15)}\right)$

$\therefore = \tan^{-1}\left(\dfrac{13/60}{60+24/60}\right)$

$\therefore = \tan^{-1}(13/84)$

$\therefore = \cos^{-1}(84/85)$ = R.H.S.

L.H.S = R.H.S

Hence, proved.

Art prove that

(1) $\sin 2x = 2\sin x \cos x$

$$= \frac{2\tan x}{1+\tan^2 x}.$$

(2) $\cos 2x = \cos^2 x - \sin^2 x$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$= \frac{1-\tan^2 x}{1+\tan^2 x}.$$

(3) $\tan 2x = 2\tan x / 1-\tan^2 x$

Art prove that

(1) $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$

(2) $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

(3) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$$* \; 2\tan^{-1}x = \begin{cases} \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) \\ \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right) \\ \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) \end{cases}$$

Qus Solve that

(1) $\tan^{-1}(1/2) + \tan^{-1}(1/5) = 1/2 \cos^{-1}(16/65)$

Sol^n → $L.H.S. = \tan^{-1}(1/2) + \tan^{-1}(1/5)$

$$\therefore = \tan^{-1}\left[\frac{1/2 + 1/5}{1-(1/2)(1/5)}\right]$$

$$\therefore = \tan^{-1}\left[5+2/10-1\right]$$

$$\therefore = \tan^{-1}\left[7/9\right]$$

$$\therefore = 1/2 \times 2\tan^{-1}\left[7/9\right]$$

$$\therefore = 1/2 \times \cos^{-1}\left(\frac{1-\left(\frac{7}{9}\right)^2}{1+\left[\frac{7}{9}\right]^2}\right) \quad \because \left\{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$$

$$\therefore = 1/2 \times \cos^{-1}\left[\frac{1-49/81}{1+49/81}\right]$$

$$\therefore = 1/2 \times \cos^{-1}\left[\frac{81-49}{81+49}\right]$$

$$\therefore = 1/2 \times \cos^{-1}\left[\cancel{32}^{16}/\cancel{130}_{65}\right]$$

$$\therefore = 1/2 \cos^{-1}\left[16/65\right]$$

L.H.S. = R.H.S.

Hence, proved.

Qus 2) $\tan^{-1}(1/4) + \tan^{-1}(2/9) = \frac{1}{2}\cos^{-1}(3/5)$

Sol^n → L.H.S. → $\tan^{-1}(1/4) + \tan^{-1}(2/9)$

$$\therefore = \tan^{-1}\left(\frac{1/4 + 2/9}{1-(1/4)(2/9)}\right)$$

$$\therefore = \tan^{-1}\left(\frac{\frac{9+8}{36}}{1-\frac{2}{36}}\right)$$

$$\therefore = \tan^{-1}\left(\frac{17}{36-2}\right)$$

$$\therefore = \tan^{-1}\left(\cancel{17}^{1}/\cancel{34}_{2}\right)$$

$$\therefore = \tan^{-1}(1/2)$$

$$\therefore = 1/2 \times 2\tan^{-1}(1/2)$$

$$\therefore = 1/2 \times \cos^{-1}\left(\frac{1+(1/2)^2}{1-(1/2)^2}\right) \qquad \because \left\{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$$

$$\therefore = 1/2 \times \cos^{-1}\left(\frac{1+1/4}{1-1/4}\right)$$

$$\therefore = 1/2 \times \cos^{-1}\left(\frac{4+1}{4-1}\right)$$

$$\therefore = 1/2 \times \cos^{-1}(5/3) = \text{R.H.S.}$$

L.H.S. = R.H.S.

Hence, proved

Qus $\tan^{-1}(1/2) + \tan^{-1}(2/11) = 1/2 \sin^{-1}(24/25)$

Sol^n → L.H.S. → $\tan^{-1}(1/2) + \tan^{-1}(2/11)$

$$\therefore = \tan^{-1}\left(\frac{1/2 + 2/11}{1-(1/2)(2/11)}\right)$$

$$\therefore = \tan^{-1}\left(\frac{11+4}{22-2}\right)$$

$$\therefore = \tan^{-1}(15/20)$$

$$\therefore = \tan^{-1}\left(\cancel{15}^{3}/\cancel{20}_{4}\right)$$

$$\therefore = \tan^{-1}(3/4)$$

$$\therefore = 1/2 \times [2\tan^{-1}(3/4)]$$

$$\therefore = 1/2 \times \sin^{-1}\left(2(3/4)/1+(3/4)^2\right) \quad \because \left\{\sin^{-1} = \left(\frac{2x}{1+x^2}\right)\right\}$$

$$\therefore = 1/2 \times \sin^{-1}(3/2) / 1+(9/16)$$

$$\therefore = 1/2 \times \sin^{-1} \frac{3 \times \cancel{16}\,8}{25 \times \cancel{2}}$$

$$\therefore = 1/2 \sin^{-1}(24/25) = \text{R.H.S.}$$

L.H.S. = R.H.S.

Hence, proved.

Que prove that

(1) $\tan^{-1}(1/2) + \tan^{-1}(1/5) + \tan^{-1}(1/8) = \pi/4$

Soln → $L.H.S. = \tan^{-1}(1/2) + \tan^{-1}(1/5) + \tan^{-1}(1/8)$

$$\therefore = \left[\tan^{-1}(1/2) + \tan^{-1}(1/5) + \tan^{-1}(1/8)\right]$$

$$\therefore = \left[\tan^{-1}\left[\frac{1/2 + 1/5}{1-(1/2)(1/5)}\right]\right] + \tan^{-1}(1/8)$$

$$\therefore = \tan^{-1}\left[5+2/10-1\right] + \tan^{-1}(1/8)$$

$$\therefore = \tan^{-1}\left[7/9\right] + \tan^{-1}(1/8)$$

$$\therefore = \tan^{-1}\left[\frac{7/9 + 1/8}{1-(7/9)(1/8)}\right]$$

$$\therefore = \tan^{-1}\left[\frac{56+9}{76-7}\right]$$

$$\therefore = \tan^{-1}\left[\frac{\cancel{65}}{\cancel{65}}\right]1$$

$$\therefore = \tan^{-1}[1] = \pi/4 = R.H.S.$$

$L.H.S = R.H.S$

Hence, proved.

(2) $\cot^{-1}(7)+\cot^{-1}(8)+\cot^{-1}(18)=\cot^{-1}(3)$

Sol^n → L.H.S. $=\cot^{-1}(7)+\cot^{-1}(8)+\cot^{-1}(18)$

$$\therefore = \tan^{-1}(1/7)+\tan^{-1}(1/8)+\tan^{-1}(1/18)$$

$$\therefore = \tan^{-1}\left[\frac{1/7+1/8}{1-(1/7)(1/8)}\right]+\tan^{-1}(1/18)$$

$$\therefore = \tan^{-1}\left[\frac{8+7}{56-1}\right]+\tan^{-1}(1/18)$$

$$\therefore = \tan^{-1}\left[\frac{\cancel{15}\,3}{\cancel{55}\,11}\right]+\tan^{-1}(1/18)$$

$$\therefore = \tan^{-1}\left[\frac{3/11+1/18}{1-(3/11)(1/18)}\right]$$

$$\therefore = \tan^{-1}\left[\frac{54+11}{198-3}\right]$$

$$\therefore = \tan^{-1}\left[\frac{\cancel{65}\,\cancel{5}\,1}{\cancel{195}\,\cancel{15}\,3}\right] = \tan^{-1}(1/3)$$

$$\therefore = \cot^{-1}(3)$$

L.H.S. = R.H.S.

Henced, proved.

Que. Solve for x : $\tan^{-1}(x+2)+\tan^{-1}(x-2)=\pi/4$, $(x>0)$

Sol^n → The given eq^n,

$$\tan^{-1}(x+2)+\tan^{-1}(x-2)=\pi/4$$

$$\therefore \tan^{-1}\left(\frac{(x+2)+(x-2)}{1-(x+2)(x-2)}\right)=\pi/4$$

$$\therefore \tan^{-1}\left(\frac{2x}{(1-(x^2-4))}\right)=\pi/4$$

$$\therefore \tan^{-1}\left(\frac{2x}{1-x^2+4}\right)=\pi/4$$

$$\therefore 2x/5-x^2=\tan\pi/4$$

$$\therefore \frac{2x}{5-x^2}=1$$

$$\therefore 2x=5-x^2$$

$$\therefore x^2+2x-5=0$$

$$\therefore x=\frac{-2\pm\sqrt{4+20}}{2\times1}$$

$$\therefore x=\frac{-2\pm\sqrt{24}}{2}$$

$\therefore x = \dfrac{-2 \pm 2\sqrt{6}}{2}$

$\therefore x = -1 \pm \sqrt{6}$

$\therefore x = -1+\sqrt{6}, -1-\sqrt{6}.$

Rejecting $\rightarrow x = -1-\sqrt{6}$, as $x > 0$

$\therefore x = -1+\sqrt{6}.$

Que Solve for x:

$\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1}(8/79), x > 0.$

Sol^n $\rightarrow$ The given eq^n is,

$$\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1}(8/79)$$

$$\therefore \tan^{-1}\left[\frac{(x+2)+(x-2)}{1-(x+2)(x-2)}\right] = \tan^{-1}(8/79)$$

$$\therefore \tan^{-1}\left[\frac{2x}{1-(x^2-4)}\right] = \tan^{-1}(8/79)$$

$$\therefore \tan^{-1}\left[\frac{2x}{1-(x^2-4)}\right] = \tan^{-1}(8/79)$$

$$\therefore \frac{2x}{1-x^2+4} = (8/79)$$

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$$\therefore \frac{2x}{5-x^2} = 8/79$$

$$\therefore (\cancel{2}x) \times 79 = \cancel{8} \times (5-x^2)$$

$$\therefore 79x = 20 - 4x^2$$

$$\therefore 4x^2 + 79x - 20 = 0$$

$$\therefore 4x^2 + 80x - x - 20 = 0$$

$$\therefore 4x(x+20) - 1(x+20) = 0$$

$$\therefore (4x-1)(x+20) = 0$$

$$\therefore x = 1/4, -20.$$

rejecting $x = -20$, as $x > 0$.

$$\therefore \boxed{x = 1/4}$$

reject, $x = -20$, $x > 0$

So, $x = 1/4$

Qus. If $\cos^{-1}\left(\frac{x}{a}\right)+\cos^{-1}(y/b)=\alpha$,

prove that $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+y^2/b^2=\sin^2\alpha$.

Sol.$\rightarrow$ We are given

$$\cos^{-1}\left(\frac{x}{a}\right)+\cos^{-1}(y/b)=\alpha$$

$$\therefore \cos^{-1}\left[(x/a)(y/b)-\sqrt{1-(x/a)^2}\cdot\sqrt{1-(y/b)^2}\right]$$

$$\therefore \frac{xy}{ab}-\sqrt{\left(1-\frac{x^2}{a^2}\right)}\sqrt{1-\frac{y^2}{b^2}}=\cos\alpha$$

$$\left(\cos^{-1}x\pm\cos^{-1}y=\cos^{-1}\left[xy\mp\sqrt{1-x^2}\sqrt{1-y^2}\right]\right)$$ using this formula

$$\therefore \frac{xy}{ab}-\cos\alpha=\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}$$

(Sq. B. S)

$$\left(\frac{xy}{ab}-\cos\alpha\right)^2=\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)$$

$$\therefore \cancel{\frac{x^2y^2}{a^2b^2}}+\cos^2\alpha-\frac{2xy}{ab}\cos\alpha=1-\frac{y^2}{b^2}-\frac{x^2}{a^2}+\cancel{\frac{x^2y^2}{a^2b^2}}$$

$$\boxed{1-\cos^2\alpha=\sin^2\alpha}$$

$$\therefore \frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + y^2/b^2 = 1-\cos^2\alpha$$

$$\therefore \frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + y^2/b^2 = \sin^2\alpha.$$

Ans:→

Some Important Subsitutions ....

In case of,

(1) a^2+x^2, put $x=a\tan\theta$

(2) $\sqrt{a^2-x^2}$, put $x=a\sin\theta$

(3) $\sqrt{x^2-a^2}$, put $x=a\sec\theta$

Que Verify the following in the simplest form.

(1) $\tan^{-1}\left[\sqrt{\dfrac{1-\cos x}{1+\cos x}}\right], -\pi < x < \pi$

(2) $\tan^{-1}\left[\dfrac{\cos x}{1+\sin x}\right], -\pi/2 < x < \pi/2$

(3) $\tan^{-1}\left[\dfrac{\cos x - \sin x}{\cos x + \sin x}\right], -\pi/4 < x < \pi/4$

Note:

(1) $1-\cos 2x = 2\sin^2 x$

$1-\cos x = 2\sin^2\left(\dfrac{x}{2}\right)$

(2) $1+\cos 2x = 2\cos^2 x$

$1+\cos x = 2\cos^2(x/2)$

Sol^n → (1) consider $\tan^{-1}\left[\sqrt{\dfrac{1-\cos x}{1+\cos x}}\right]$

$\therefore \tan^{-1}\left[\sqrt{\dfrac{\cancel{2}\sin^2(x/2)}{\cancel{2}\cos^2(x/2)}}\right]$

$$\therefore = \tan^{-1}\left[\tan\left(\frac{x}{2}\right)\right]$$

$$\therefore = \frac{x}{2}$$

Sol^n → (2) consider $\tan^{-1}\left[\frac{\cos x}{1+\sin x}\right]$

$$\therefore = \tan^{-1}\left[\frac{\cos^2(x/2) - \sin^2(x/2)}{\cos^2(x/2) + \sin^2(x/2) + 2\sin(x/2)\cos(x/2)}\right]$$

$$\therefore = \tan^{-1}\left[\frac{(\cos(x/2) - \sin(x/2))\cancel{(\cos(x/2) + \sin(x/2))}}{(\cos(x/2) + \sin(x/2))^{\cancel{2}}}\right]$$

$$\therefore = \tan^{-1}\left[\frac{\cos(x/2) - \sin(x/2)}{\cos(x/2) + \sin(x/2)}\right]$$

$$\therefore = \tan^{-1}\left[\frac{1 - \tan(x/2)}{1 + \tan(x/2)}\right]$$

$$\therefore = \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$\because \tan\left(\frac{\pi}{4} \mp \theta\right) = \frac{1 \mp \tan\theta}{1 \pm \tan\theta}$$

$$\therefore = \left(\frac{\pi}{4} - \frac{x}{2}\right)$$

Sol^n (3) →

Consider, $\tan^{-1}\left[\frac{\cos x - \sin x}{\cos x + \sin x}\right]$

$\therefore = \tan^{-1}\left[\frac{1 - \tan x}{1 + \tan x}\right]$

$\therefore = \tan^{-1}\left[\tan\left(\pi/4 - x\right)\right]$

$\therefore = \pi/4 - x.$

V. Imp

Qus prove that:

$$\tan^{-1}\left[\frac{x}{1 + \sqrt{1^2 - x^2}}\right] = \frac{1}{2}\sin^{-1}x.$$

Sol^n → consider that,

$\tan^{-1}\left[\frac{x}{1 + \sqrt{1^2 - x^2}}\right]$

$\left[\begin{array}{l} \text{put } x = \sin\theta \\ \therefore \theta = \sin^{-1}(x) \end{array}\right]$

$\therefore = \tan^{-1}\left[\frac{\sin\theta}{1 + \sqrt{1 - \sin^2\theta}}\right] \therefore = \tan^{-1}\left[\frac{\sin\theta}{1 + \cos\theta}\right]$

$\therefore = \tan^{-1}\left[\frac{\cancel{2}\sin(\theta/2)\cancel{\cos(\theta/2)}}{\cancel{2}\cos^{\cancel{2}}\left(\frac{\theta}{2}\right)}\right]$

$$\therefore = \tan^{-1}\left[\tan\left(\theta/2\right)\right]$$

$$\therefore = \theta/2 = 1/2 \sin^{-1} x$$

$$\therefore = \tan^{-1} x \left[\frac{x}{1+\sqrt{1-x^2}}\right] = 1/2 \sin^{-1} x.$$

Que Write $\tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$, $|x| > 1$, in the simplest form.

Soln → Consider $\tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$

put $x = \sec\theta$

$$\therefore \theta = \sec^{-1}(x)$$

$$\therefore \tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{\sec^2\theta-1}}\right)$$

$$\therefore = \tan^{-1}\left(1/\tan\theta\right)$$

$$\therefore = \tan^{-1}(\cot\theta)$$

$$\therefore = \tan^{-1}\left[\tan\left(\pi/2-\theta\right)\right]$$

$$\therefore = \pi/2-\theta$$

$$\therefore = \pi/2 - \sec^{-1}(x)$$

Que Solve that

$$\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \pi/_4 + 1/_2 \cos^{-1}(x^2) \quad , -1 < x < 1$$

Sol^n → $L.H.S. = \tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$

put $x^2 = \cos\theta$

$\therefore \theta = \cos^{-1}(x^2)$

$$\therefore L.H.S. = \tan^{-1}\left[\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}\right]$$

$$\therefore = \tan^{-1}\left[\frac{\sqrt{2\cos^2(\theta/2)}+\sqrt{2\sin^2(\theta/2)}}{\sqrt{2\cos^2(\theta/2)}-\sqrt{2\sin^2(\theta/2)}}\right]$$

$$\therefore = \tan^{-1}\left[\frac{\cos(\theta/2)+\sin(\theta/2)}{\cos(\theta/2)-\sin(\theta/2)}\right]$$

$$\therefore = \tan^{-1}\left[\frac{1+\tan(\theta/2)}{1-\tan(\theta/2)}\right]$$

$$\therefore = \tan^{-1}\left[\tan\left(\pi/_2 + \theta/_2\right)\right]$$

$$\therefore = \pi/_4 + \theta/_2$$

$$\therefore = \pi/_4 + 1/_2 \cos^{-1}(x^2) = R.H.S.$$

Hence, proved.

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Que prove that:→

(1) $\cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\pi/2$ if $\pi/2<x<\pi$

(2) $\tan^{-1}\left[\dfrac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right]=\pi/4-x/2$ if $\pi<x<3\pi/2$.

Sol^n → (1)

Note →

$\therefore 1+\sin = \cos^2(x/2)+\sin^2(x/2)+2\sin(x/2)\cos(x/2)$

$\therefore 1+\sin x = (\cos(x/2)+\sin(x/2))^2$

$\therefore 1-\sin x = \cos^2(x/2)+\sin^2(x/2)-2\sin(x/2)\cos(x/2)$

$\therefore 1-\sin x = (\cos(x/2)-\sin(x/2))^2$

$\therefore \cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$

$$\therefore \cot^{-1}\left[\frac{(\cos(x/2)+\sin(x/2)+\cos(x/2)-\sin(x/2)}{(\cos(x/2)+\sin(x/2)-\cos(x/2)-\sin(x/2)}\right]$$

$$\therefore =\cot^{-1}\left[\frac{\cancel{2}\cos(x/2)}{\cancel{2}\sin(x/2)}\right]$$

$$\therefore =\cot^{-1}\left[\cot(x/2)\right]$$

$$\therefore = x/2.$$

Sol^n → (2)

Note →

$$\boxed{\begin{aligned} 1+\cos x &= 2\cos^2(x/2) \\ 1-\cos x &= 2\sin^2(x/2) \end{aligned}}$$

$$\therefore =\tan^{-1}\left[\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right]$$

$$\therefore =\tan^{-1}\left[\frac{\sqrt{2\cos^2(x/2)}+\sqrt{2\sin^2(x/2)}}{\sqrt{2\cos^2(x/2)}-\sqrt{2\sin^2(x/2)}}\right]$$

$$\therefore = \tan^{-1}\left[\frac{2\cos(x/2) + 2\sin(x/2)}{2\cos(x/2) - 2\sin(x/2)}\right]$$

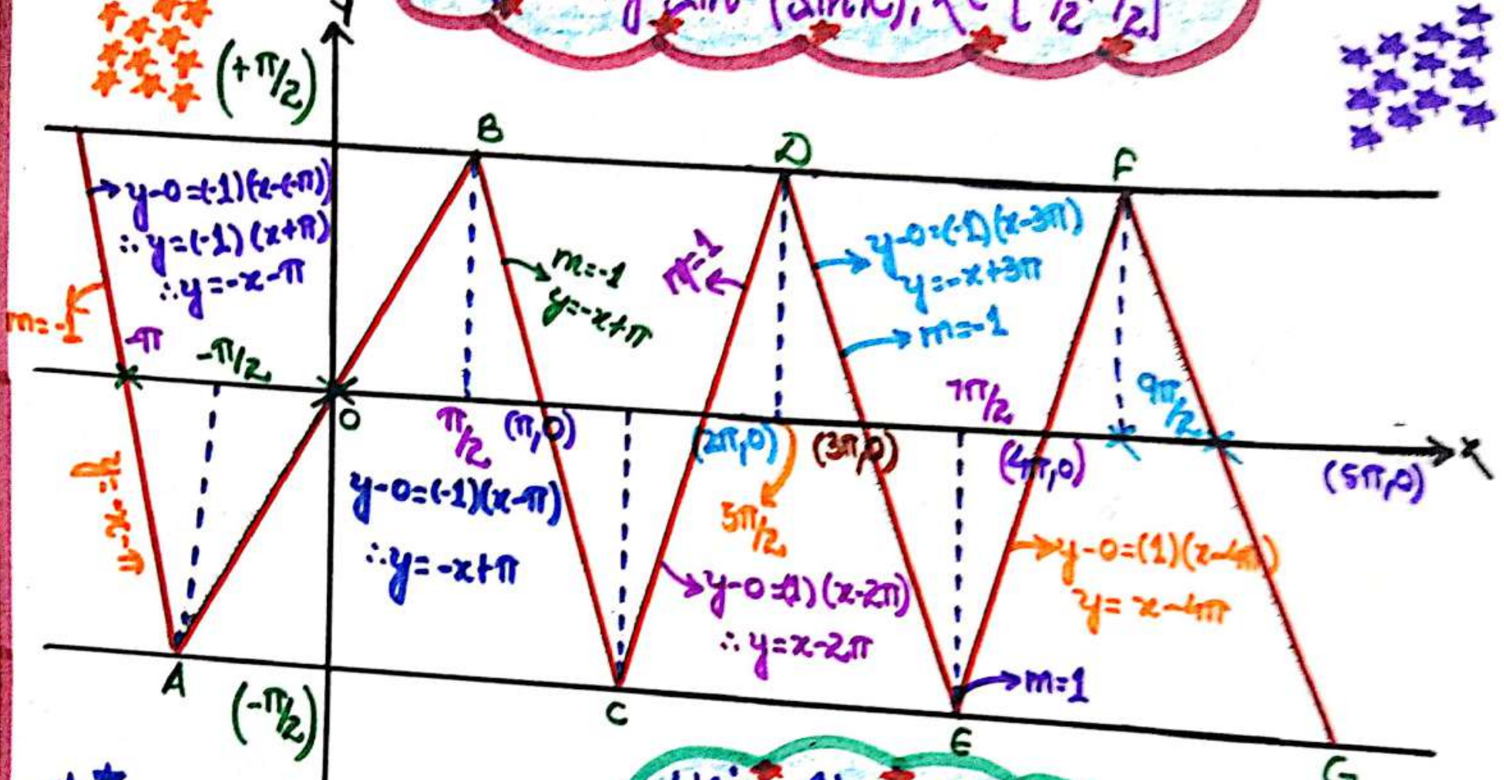
$$\therefore = \tan^{-1}\left[\frac{\cancel{2}(\cos(x/2) + \sin(x/2))}{\cancel{2}(\cos(x/2) - \sin(x/2))}\right]$$

$$\therefore = \tan^{-1}\left[\frac{1 + \tan(x/2)}{1 - \tan(x/2)}\right]$$

$$\therefore = \pi/4 + x/2 = R.H.S. \quad (\text{If } \pi < x < 3\pi/2.)$$

Graphical Representation

$y = \sin^{-1}(\sin x),\ x \in [-\pi/2, \pi/2]$



Using Slope Form

$y - y_1 = m(x - x_1)$

Graphical Representation

$f(x) = \cos^{-1}(\cos x)\ [0, \pi]$

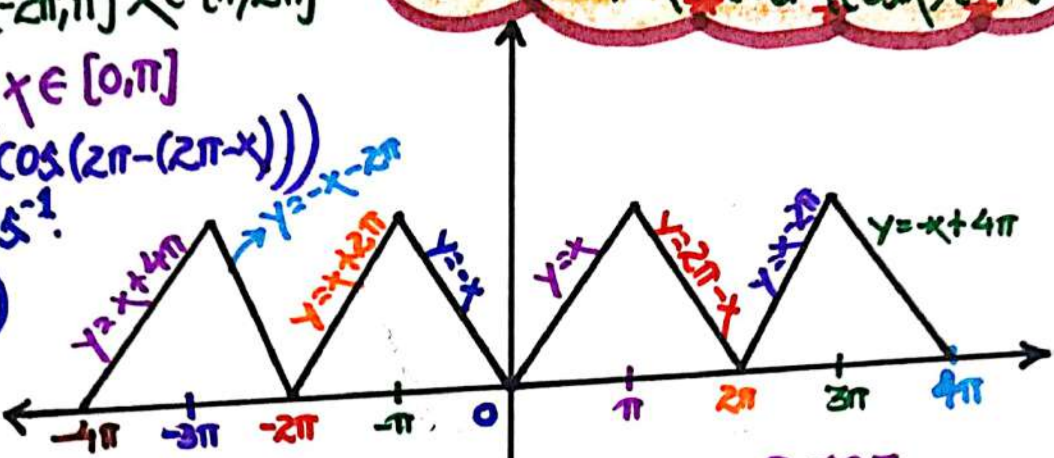
$-x \in [-2\pi, \pi]$ $x \in [\pi, 2\pi]$

$2\pi - x \in [0, \pi]$

$\cos^{-1}(\cos(2\pi - (2\pi - x)))$

$2\pi - x = \cos^{-1}$

$(\cos(2\pi - x))$



$x = n\pi, n \in Z$

Using point slope form

$y - y_1 = m(x - x_1)$

Qus Evaluate the foll. values:→

(1) $\sin^{-1}(\sin(2\pi/3))$ (2) $\sin^{-1}(\sin(3\pi/5))$

(3) $\sin^{-1}(\sin(4\pi/5))$ (4) $\sin^{-1}(\sin(10))$

Solⁿ→ (1) Let $y = \sin^{-1}(\sin x) = x$

$(y=x)$

$$\left[-\pi/2, \pi/2\right]$$

$$\sin^{-1}(\sin(2\pi/3)) \neq 2\pi/3$$

$120^\circ = 180^\circ - 60^\circ$

$2^{nd} = \pi/3$

$$2\pi/3 = 120^\circ \notin \left[-\pi/2, \pi/2\right]$$

Solⁿ (1)→ $\sin^{-1}(\sin(2\pi/3)) = -2\pi/3 + \pi = \pi/3$

120° $\left[-\pi/2, \pi/2\right]$ $[-90^\circ, 90^\circ]$

Solⁿ(2)→ $\sin^{-1}(\sin(3\pi/5))$

$$= -3\pi/5 + \pi = 2\pi/5$$

Solⁿ(3) → $\sin^{-1}(\sin(4\pi/5))$

$= -4\pi/5 + \pi = \pi/5$.

Solⁿ(4) → $\sin^{-1}(\sin 10)$

$= -10 + 3\pi$

$\pi = 3.14$

$3\pi = 3.14 \times 3 = 9.42$.

Important points........

(1) let $y = \sin^{-1}(\sin x) = x$

$\downarrow$

$x \in [-\pi/2, \pi/2]$

(2) $y = \sin^{-1}(\sin x) = -x + \pi$

$\downarrow$

$x \in [\pi/2, 3\pi/2]$

(3) $y = \sin^{-1}(\sin x) = -x + 3\pi$

$\downarrow$

$x \in [3\pi/2, 5\pi/2]$

(4) $y = \sin^{-1}(\sin x) = -x + 3\pi$

$\downarrow$

$x \in [5\pi/2, 7\pi/2]$

(5) $y = \sin^{-1}(\sin x) = x - 4\pi$

$\downarrow$

$x \in [7\pi/2, 9\pi/2]$

Qus $\sin^{-1}(\sin 8)$

Sol^n → $\sin^{-1}(\sin 8)$

$= -8 + 3\pi$.

Qus $\sin^{-1}(\sin 7\pi/4)$

Sol^n → $7\pi/4 - 2\pi$

$\frac{7\pi - 8\pi}{4} = -\pi/4$

$3\pi - \pi/2 = 7.85$

$5\pi/2$ 3π

↓

$7\pi/4$

$=$.

Qus If $\sin(\sin^{-1}(1/5)+\cos^{-1}(x))=1$,
then find the value of x.

Sol^n → We are given,

$$\sin[\sin^{-1}(1/5)+\cos^{-1}(x)]=1$$

$$\therefore \sin^{-1}(1/5)+\cos^{-1}(x)=\sin^{-1}(1)$$

$$\therefore \sin^{-1}(1/5)+\cos^{-1}(x)=\pi/2 \rightarrow (1)$$

We know that,

$$\sin^{-1}(x)+\cos^{-1}(x)=\pi/2.$$

$$\therefore \boxed{x=1/5}$$

Qus find the value of x.
$\cos(2\tan^{-1}x)=1/2$.

Sol^n → The given eq^n is,

$$\cos[2\tan^{-1}(x)]=1/2$$

$$\therefore 2\tan^{-1}(x)=\cos^{-1}(1/2)$$

$$\therefore 2\tan^{-1}(x)=\cos^{-1}(1/2)$$

$$\therefore 2\tan^{-1}(x)=\pi/3$$

$$\therefore \tan^{-1}(x)=\pi/6$$

$$\therefore x=1/\sqrt{3}$$

$$\boxed{x=1/\sqrt{3}}$$

Qus If $\sin^{-1}(x/5) + \operatorname{cosec}^{-1}(5/4) = \pi/2$, then the value of x.

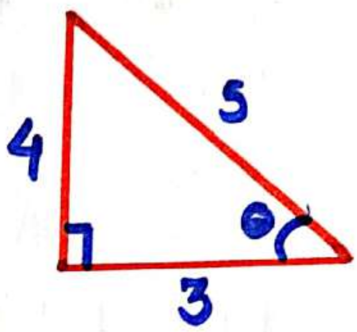
Soln → The given eqn is,

$\sin^{-1}(x/5) + \operatorname{cosec}^{-1}(5/4) = \pi/2$

Let $\operatorname{cosec}^{-1}(5/4) = \theta$

$\therefore 5/4 = \operatorname{cosec}\theta$

$\therefore \sin\theta = 4/5$



$\therefore \cos\theta = 3/5$

$\therefore \theta = \cos^{-1}(3/5)$

$\therefore \operatorname{cosec}^{-1}(5/4) = \cos^{-1}(3/5)$

$\therefore \sin^{-1}(x/5) + \cos^{-1}(3/5) = \pi/2$

$\therefore x/5 = 3/5$

$\therefore x = \frac{3 \times \cancel{5}}{\cancel{5}}$

$\therefore \boxed{x = 3}$

(1) $\sin^{-1}(\sin 2)$

Sol^n → $\sin^{-1}(\sin 2) = -2+\pi$

$\pi/2 \to 1.57$, $\pi \to 3.14$

$\pi/2$ π

1.57 3.14

(2) $\sin^{-1}(\sin 4)$

Sol^n → $\sin^{-1}(\sin 4) = -4+\pi$

$\pi/2 \to 1.57$ $\pi \to 3.14$

$\pi + \pi/2$

$\underbrace{3.14+1.57}_{4} \to 4.71$

(3) $\sin^{-1}(\sin 3)$

Sol^n → $\sin^{-1}(\sin 3) = -3+\pi$

$\pi/2 \to 1.57$

$\pi \to 3.14$

$\pi/2$ π

1.57 3.14

3

Qus $\sin^{-1}(\sin 5)$

Sol^n → $\sin^{-1}(\sin 5) = 5 - 2\pi$

$3\pi/2 \quad 2\pi$

$4.71 \quad 6.28$

$\underbrace{\qquad\qquad}_{5}$

Qus $\sin^{-1}(\sin 6)$

Sol^n → $\sin^{-1}(\sin 6) = 6 - 2\pi$

$3\pi/2 \quad 2\pi$

$\underbrace{\qquad\qquad}_{6}$

Qus Solve for x.

$\tan^{-1}(x+2) + \tan^{-1}(x-2) = \pi/4,\ x>0$

$(\tan^{-1}x + \tan^{-1}y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right) < 1$

Sol^n → We are given,

$$\tan^{-1}(x+2) + \tan^{-1}(x-2) = \pi/4$$

$$\therefore \tan^{-1}\left[\frac{(x+2)+(x-2)}{1-(x+2)(x-2)}\right] = \pi/4$$

$$\therefore \frac{2x}{1-(x^2-4)} = \tan \pi/4$$

$$\therefore \frac{2x}{1-(x^2-4)} = 1$$

$$\therefore \frac{2x}{5-x^2} = 1$$

$$2x = 5 - x^2$$

$$\therefore x^2 + 2x - 5 = 0$$

$$\therefore x^2 + 2x - 5 = 0$$

$$\therefore x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-5)}}{2 \times 1}$$

$$\therefore x = -2 \pm \sqrt{24} / 2$$

$$\therefore x = -2 \pm 2\sqrt{6}/2$$

$$\therefore x = -1 \pm \sqrt{6}$$

$$\therefore x = -1+\sqrt{6}, \underline{-1-\sqrt{6}} \rightarrow \text{Reject}$$

$$\therefore \boxed{x = -1+\sqrt{6}.}$$

Qus prove that

$$\tan^{-1}(1/2) + \tan^{-1}(1/5) = 1/2 \cos^{-1}(16/65).$$

Sol^n → L.H.S. → $\tan^{-1}(1/2) + \tan^{-1}(1/5)$

$$\therefore = \tan^{-1}\left(\frac{1/2 + 1/5}{1-(1/2)(1/5)}\right)$$

$\because \tan^{-1}x + \tan^{-1}y$

$\therefore = \tan^{-1}\left(\frac{x+y}{1-xy}\right), xy<1$

$$\therefore = \tan^{-1}\left(\frac{5+2/\cancel{10}}{10-1/\cancel{10}}\right)$$

$$\therefore = \tan^{-1}(7/9)$$

Note:-

$$2\tan^{-1}x \rightarrow \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\rightarrow \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\rightarrow \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\therefore = \frac{1}{2}\left\{2\tan^{-1}(7/9)\right\}$$

$$\therefore = \frac{1}{2}\cos^{-1}\left(\frac{(1-(7/9)^2}{1+(7/9)^2}\right)$$

$$\therefore = \frac{1}{2}\cos^{-1}\left(16/65\right) = R.H.S.$$

Qus Find the value of the foll-

$\tan^{-1}\left[2\cos\left(2\sin^{-1}(1/2)\right)\right]$

Sol^n → consider, $\tan^{-1}\left[2\cos\left(2\sin^{-1}(1/2)\right)\right]$

$$\therefore \tan^{-1}\left[2\cos\left(\cancel{2}\times\pi/\cancel{6}_3\right)\right]$$

Put $\sin^{-1}(1/2)=\theta$

$\sin\theta = 1/2$

$\theta = \pi/6$

$$\therefore = \tan^{-1}\left[2\cos\pi/3\right]$$

$$\therefore = \tan^{-1}\left[2\times 1/2\right]$$

$$\therefore = \tan^{-1}(1)\ \pi/4$$

$\therefore = \tan^{-1}\left[2\cos\left(2\sin^{-1}(1/2)\right)\right] = \pi/4$

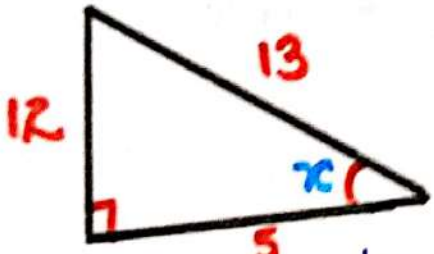
Qus Show that $\sin^{-1}(12/13) + \cos^{-1}(4/5) + \tan^{-1}(63/16) = \pi$

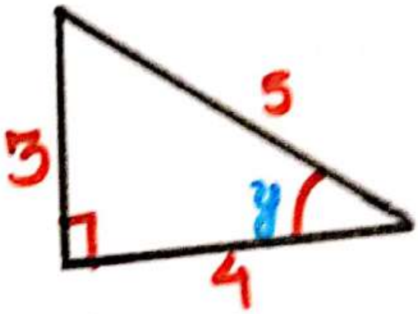
Sol^n $\rightarrow$ Let $\sin^{-1}(12/13) = x$

$\cos^{-1}(4/5) = y$

$\tan^{-1}(63/16) = z$

$\sin x = 12/13,\ \cos y = 4/5,\ \tan^{-1}(63/16)$





$\tan x = 12/5,\ \tan y = 3/4,\ \tan z = 63/16$

$$\therefore \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\therefore \tan(x+y) = \frac{12/5 + 3/4}{1 - (12/5)(3/4)}$$

$$\therefore \tan(x+y) = 48+15/20-36.$$

$\therefore \tan(x+y) = -63/16$ $\left[\begin{array}{l}-\tan x = \tan -x \\ \text{or } \tan(\pi - x\end{array}\right]$

$\therefore \tan(x+y) = -\tan z$

$\therefore \tan(x+y) = \tan(-z)$ or $\tan(\pi - z)$

$\therefore x+y = -z$ or $x+y = \pi - z$.

But

$x+y \neq -z$ [$\because x, y, z$ are $+ve$]

$\therefore x+y+z = \pi$.

$$\boxed{\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi.}$$


Thank you!
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