Chapter - Thermodynamics

Topic-1: First Law of Thermodynamics

1 MCQs with One Correct Answer

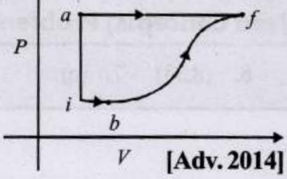
1. In a given process on an ideal gas, $dW = 0$ and $dQ < 0$. Then for the gas **[2001S]**

(a) the temperature will decrease
(b) the volume will increase
(c) the pressure will remain constant
(d) the temperature will increase

2 Integer Value Answer

2. A thermodynamic system is taken from an initial state i with internal energy $U_i = 100$ J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are $W_{af} = 200$ J, $W_{ib} = 50$ J and $W_{bf} = 100$ J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is $U_b = 200$ J and $Q_{iaf} = 500$ J,

The ratio $\frac{Q_{bf}}{Q_{ib}}$ is



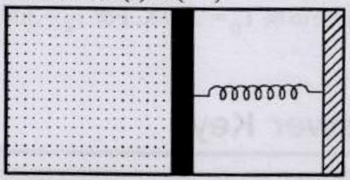
[Adv. 2014]

4 Fill in the Blanks

3. A container of volume 1m^3 is divided into two equal parts by a partition. One part has an ideal gas at 300K and the other part is vacuum. The whole system is thermally isolated from the surroundings. When the partition is removed, the gas expands to occupy the whole volume. Its temperature will now be....... **[1993-1 Mark]**

6 MCQs with One or More than One Correct Answer

4. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is (are) **[Adv. 2015]**



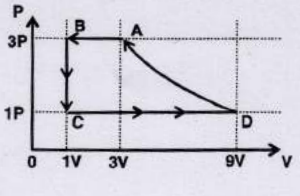
(a) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$
(b) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$
(c) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$
(d) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$

5. During the melting of a slab of ice at 273 K at atmospheric pressure, **[1998S - 2 Marks]**

(a) positive work is done by the ice-water system on the atmosphere.
(b) positive work is done on the ice- water system by the atmosphere.
(c) the internal energy of the ice-water system increases.
(d) the internal energy of the ice-water system decreases.

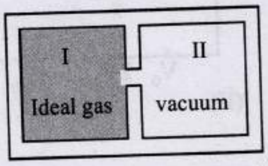
7 Match the Following

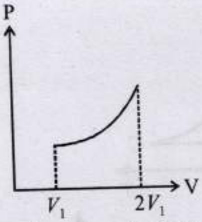
6. One mole of a monatomic gas is taken through a cycle ABCDA as shown in the P-V diagram. Column II give the characteristics involved in the cycle. Match them with each of the processes given in Column I. **[2011]**



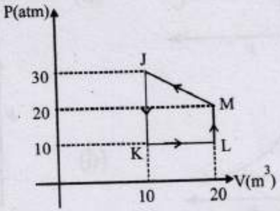
Column I	Column II
(A) Process A $\rightarrow$ B	(p) Internal energy decreases
(B) Process B $\rightarrow$ C	(q) Internal energy increases
(C) Process C $\rightarrow$ D	(r) Heat is lost
(D) Process D $\rightarrow$ A	(s) Heat is gained
	(t) Work is done on the gas

7. Column I contains a list of processes involving expansion of an ideal gas. Match this with Column II describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS. **[2008]**

Column I	Column II
(A) An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the Chamber II has vacuum. The valve is opened. 	(p) The temperature of the gas decreases
(B) An ideal monoatomic gas expands to twice its remains original volume such that its pressure $P \propto 1/V^2$ where V is the volume of the gas	(q) The temperature of the gas increases or constant
(C) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto 1/V^{4/3}$	(r) The gas loses heat where V is its volume
(D) An ideal monoatomic gas expands such that its pressure P and volume V follows the behaviour shown in the graph	(s) The gas gains heat



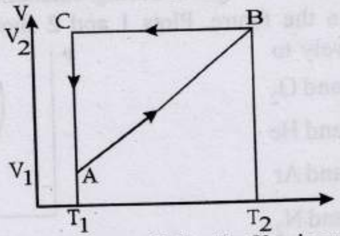
8. Heat given to process is positive, match the following option of Column I with the corresponding option of column II : **[2006 - 6M]**



Column I	Column II
(A) JK	(p) $\Delta W > 0$
(B) KL	(q) $\Delta Q < 0$
(C) LM	(r) $\Delta W < 0$
(D) MJ	(s) $\Delta Q > 0$

10 Subjective Problems

9. A cyclic process *ABCA* shown in the *V-T* diagram (fig) is performed with a constant mass of an ideal gas. Show the same process on a *P-V* diagram **[1981 - 4 Marks]**



(In the figure, *CA* is parallel to the *V*-axis and *BC* is parallel to the *T*-axis)

Topic-2: Specific Heat Capacity and Thermodynamical Processes

1 MCQs with One Correct Answer

1. One mole of an ideal gas expands adiabatically from an initial state (T_A, V_0) to final state (T_f, $5V_0$). Another mole of the same gas expands isothermally from a different initial state (T_B, V_0) to the same final state (T_f, $5V_0$). The ratio of the specific heats at constant pressure and constant volume of this ideal gas is γ. What is the ratio T_A/T_B? **[Adv. 2023]**

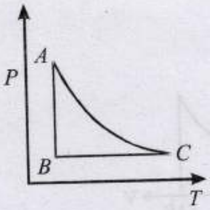
(a) $5^{\gamma-1}$ (b) $5^{1-\gamma}$ (c) 5^{γ} (d) $5^{1+\gamma}$

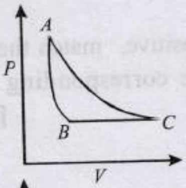
2. An ideal gas is initially at P_1, V_1 is expanded to P_2, V_2 and then compressed adiabatically to the same volume V_1 and pressure P_3. If W is the net work done by the gas in complete process which of the following is true **[2004S]**

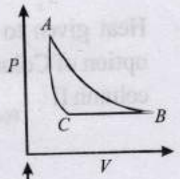
(a) $W > 0\,;\, P_3 > P_1$ (b) $W < 0\,;\, P_3 > P_1$

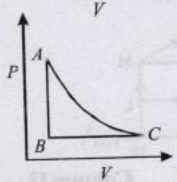
(c) $W > 0\,;\, P_3 < P_1$ (d) $W < 0\,;\, P_3 < P_1$

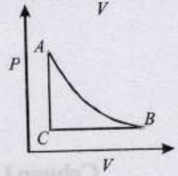
3. The *PT* diagram for an ideal gas is shown in the figure, where *AC* is an adiabatic process, find the corresponding *PV* diagram. **[2003S]**



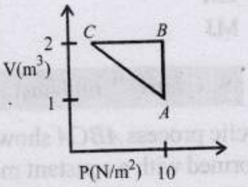
(a)


(b)


(c)


(d)


4. An ideal gas is taken through the cycle $A \to B \to C \to A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5J, the work done by the gas in the process $C \to A$ is **[2002S]**



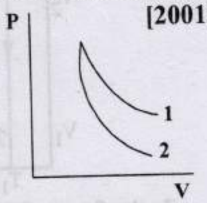
(a) –5 J

(b) –10 J

(c) –15 J

(d) –20 J

5. P-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to **[2001S]**



(a) He and O_2

(b) O_2 and He

(c) He and Ar

(d) O_2 and N_2

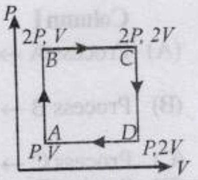
6. A monatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the length of the gas column before and after expansion respectively, then $\frac{T_1}{T_2}$ is given by **[2000S]**

(a) $\left(\frac{L_1}{L_2}\right)^{2/3}$ (b) $\frac{L_1}{L_2}$ (c) $\frac{L_2}{L_1}$ (d) $\left(\frac{L_2}{L_1}\right)^{2/3}$

7. A given quantity of a ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is **[1998S - 2 Marks]**

(a) $\frac{2}{3}P$ (b) P (c) $\frac{3}{2}P$ (d) $2P$

8. An ideal monatomic gas is taken round the cycle $ABCDA$ as shown in the $P-V$ diagram (see Fig.). The work done during the cycle is **[1983 - 1 Mark]**



(a) PV

(b) $2PV$

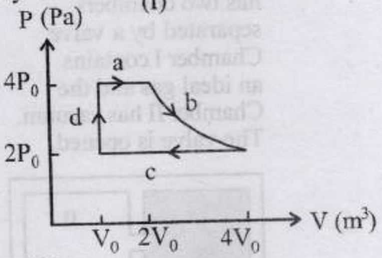
(c) $\frac{1}{2}PV$

(d) zero

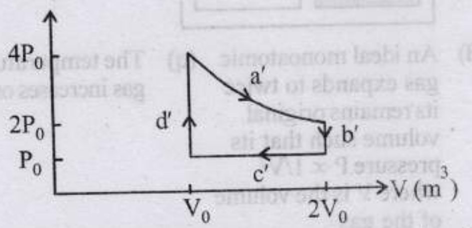
2 Integer Value Answer

9. One mole of an ideal gas undergoes two different cyclic processes I and II, as shown in the P-V diagrams below. In cycle I, processes a, b, c and d are isobaric, isothermal, isobaric and isochoric, respectively. In cycle II, processes a′, b′, c′ and d′ are isothermal, isochoric, isobaric and isochoric, respectively. The total work done during cycle I is W_I and that during cycle II is W_{II}. The ratio W_I/W_{II} is ______. **[Adv. 2023]**

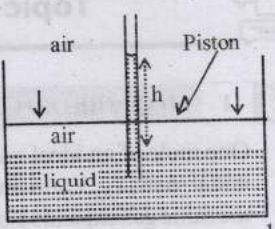
(I)



(II)



10. An incompressible liquid is kept in a container having a weightless piston with a hole. A capillary tube of inner radius 0.1 mm is dipped vertically into the liquid through the airtight piston hole, as shown in the figure. The air in the container is isothermally compressed from its original volume V_0 to $\frac{100}{101}V_0$ with the movable piston. Considering air as an ideal gas, the height (h) of the liquid column in the capillary above the liquid level in cm is ______.
[Given: Surface tension of the liquid is 0.075 N m^{-1}, atmospheric pressure is 10^5 N m^{-2}, acceleration due to gravity (g) is 10 m s^{-2}, density of the liquid is 10^3 kg m^{-3} and contact angle of capillary surface with the liquid is zero] **[Adv. 2023]**



11. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is a T_i, the value of a is **[2010]**

3 Numeric Answer

12. Consider one mole of helium gas enclosed in a container at initial pressure P_1 and volume V_1. It expands isothermally to volume $4V_1$. After this, the gas expands adiabatically and its volume becomes $32V_1$. The work done by the gas during isothermal and adiabatic expansion processes are W_{iso} and W_{adia}, respectively. If the ratio $\frac{W_{iso}}{W_{adia}} = f \ln 2$, then f is ______. **[Adv. 2020]**

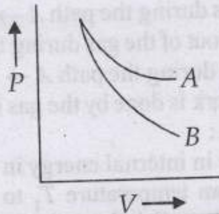
13. A spherical bubble inside water has radius R. Take the pressure inside the bubble and the water pressure to be p_0. The bubble now gets compressed radially in an adiabatic manner so that its radius becomes $(R - a)$. For $a \ll R$ the magnitude of the work done in the process is given by $(4\pi P_0 R a^2)$ X where X is a constant and $\gamma = C_p / C_v = 41/30$. The value of X is ______. **[Adv. 2020]**

4 Fill in the Blanks

14. An ideal gas with pressure P, volume V and temperature T is expanded isothermally to a volume $2V$ and a final pressure P_i. If the same gas is expanded adiabatically to a volume $2V$, the final pressure is P_a. The ratio of the specific heats of the gas is 1.67. The ratio $\frac{P_a}{P_1}$ is ..... **[1994 - 2 Marks]**

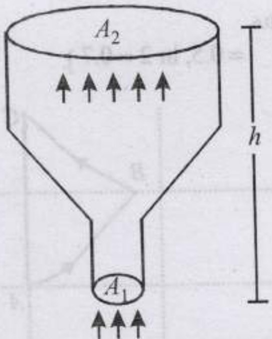
5 True / False

15. The curves A and B in the figure shown P-V graphs for an isothermal and an adiabatic process for an ideal gas. The isothermal process is represented by the curve A. **[1985 - 3 Marks]**



6 MCQs with One or More than One Correct Answer

16. An ideal gas of density $\rho = 0.2$ kg m^{-3} enters a chimney of height h at the rate of $\alpha = 0.8$ kg s^{-1} from its lower end, and escapes through the upper end as shown in the figure. The cross-sectional area of the lower end is $A_1 = 0.1\ m^2$ and the upper end is $A_2 = 0.4$ m^2. The pressure and the temperature of the gas at the lower end are 600 Pa and 300 K, respectively, while its temperature at the upper end is 150 K. The chimney is heat insulated so that the gas undergoes adiabatic expansion. Take $g = 10$ m s^{-2} and the ratio of specific heats of the gas $\gamma = 2$. Ignore atmospheric pressure. **[Adv. 2022]**



Which of the following statement(s) is(are) correct?

(a) The pressure of the gas at the upper end of the chimney is 300 Pa.

(b) The velocity of the gas at the lower end of the chimney is 40 m s^{-1} and at the upper end is 20 m s^{-1}.

(c) The height of the chimney is 590 m.

(d) The density of the gas at the upper end is 0.05 kg m^{-3}.

17. A bubble has surface tension S. The ideal gas inside the bubble has ratio of specific heats $\gamma = \frac{5}{3}$. The bubble is exposed to the atmosphere and it always retains its spherical shape. When the atmospheric pressure is $P_{\alpha 1}$, the radius of the bubble is found to be r_1 and the temperature of the enclosed gas is T_1. When the atmospheric pressure is $P_{\alpha 2}$, the radius of the bubble and the temperature of the enclosed gas are r_1 and T_2, respectively. **[Adv. 2022]**

Which of the following statement(s) is(are) correct?

(a) If the surface of the bubble is a perfect heat insulator, then $\left(\frac{r_1}{r_2}\right)^5 = \frac{P_{\alpha 2} + \frac{2S}{r_2}}{P_{\alpha 1} + \frac{2S}{r_1}}$.

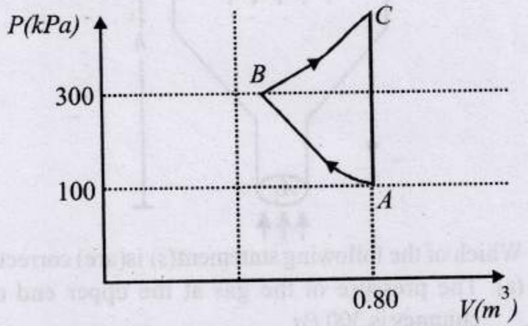
(b) If the surface of the bubble is a perfect heat insulator, then the total internal energy of the bubble including its surface energy does not change with the external atmospheric pressure.

(c) If the surface of the bubble is a perfect heat conductor and the change in atmospheric temperature is negligible, then $\left(\frac{r_1}{r_2}\right)^3 = \frac{P_{\alpha 2} + \frac{4S}{r_2}}{P_{\alpha 1} + \frac{4S}{r_1}}$.

(d) If the surface of the bubble is a perfect heat insulator, then $\left(\frac{T_2}{T_1}\right)^{\frac{5}{2}} = \frac{P_{\alpha 2} + \frac{4S}{r_2}}{P_{\alpha 1} + \frac{4S}{r_1}}$.

18. In the given P-V diagram, a monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is first compressed adiabatically from state A to state B. Then it expands isothermally from state B to state C.

[Given $\left(\frac{1}{3}\right)^{0.6} \simeq 0.5$, $\ln 2 \simeq 0.7$]. **[Adv. 2022]**



Which of the following statement(s) is(are) correct?

(a) The magnitude of the total work done in the process $A \rightarrow B \rightarrow C$ is 144 kJ.

(b) The magnitude of the work done in the process $B \rightarrow C$ is 84 kJ.

(c) The magnitude of the work done in the process $A \rightarrow B$ is 60 kJ.

(d) The magnitude of the work done in the process $C \rightarrow A$ is zero.

19. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0, and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are, (Given $2^{1.2} = 2.3$; $2^{3.2}$ 9.2; R is gas constant) **[Adv. 2019]**

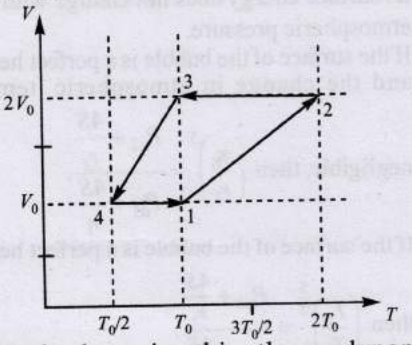
(a) The work |W| done during the process is $13RT_0$

(b) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$

(c) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$

(d) Adiabatic constant of the gas mixture is 1.6

20. One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (*V–T*) diagram. The correct statement(s) is/are: [R is the gas constant **[Adv. 2019]**



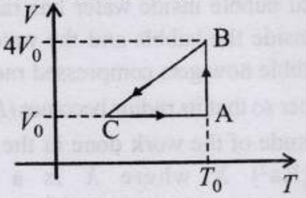
(a) Work done in this thermodynamic cycle $(1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1)$ is $|W| = \frac{1}{2}RT_0$

(b) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(c) The ratio of heat transfer during processes $1 \rightarrow 2$ and $3 \rightarrow 4$ is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{5}{3}$

(d) The ratio of heat transfer during processes $1 \rightarrow 2$ and $3 \rightarrow 4$ is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right| = \frac{1}{2}$

21. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following **[2010]**



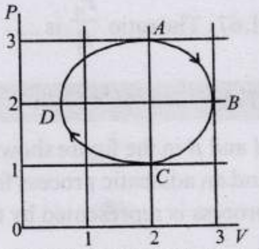
(a) Internal energies at A and B are the same

(b) Work done by the gas in process AB is $P_0V_0 \ln 4$

(c) Pressure at C is $\frac{P_0}{4}$

(d) Temperature at C is $\frac{T_0}{4}$

22. The figure shows the *P-V* plot of an ideal gas taken through a cycle *ABCDA*. The part *ABC* is a semi-circle and *CDA* is half of an ellipse. Then, **[2009]**



(a) the process during the path $A \rightarrow B$ is isothermal

(b) heat flows out of the gas during the path $B \rightarrow C \rightarrow D$

(c) work done during the path $A \rightarrow B \rightarrow C$ is zero

(d) positive work is done by the gas in the cycle *ABCDA*

23. For an ideal gas : **[1989 - 2 Marks]**

(a) the change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nC_v(T_2 - T_1)$, where C_v is the molar specific heat at constant volume and n the number of moles of the gas.

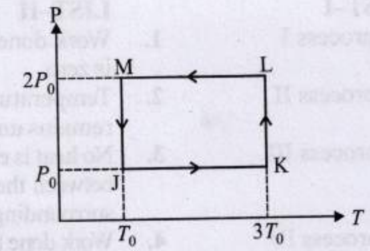
(b) the change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process.

(c) the internal energy does not change in an isothermal process.

(d) no heat is added or removed in an adiabatic process.

7 Match the Following

24. One mole of a monatomic ideal gas undergoes the cyclic process $J \rightarrow K \rightarrow L \rightarrow M \rightarrow J$, as shown in the P-T diagram.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option. [*R* is the gas constant.] **[Adv. 2024]**

List-I	**List-II**
(P) Work done in the complete cyclic process	(1) $RT_0 - 4RT_0 \ln 2$
(Q) Change in the internal energy of the gas in the process JK	(2) 0
(R) Heat given to the gas in the process KL	(3) $3RT_0$
(S) Change in the internal energy of the gas in the process MJ	(4) $-2RT_0 \ln 2$
	(5) $-3RT_0 \ln 2$

(a) $P \to 1; Q \to 3; R \to 5; S \to 4$
(b) $P \to 4; Q \to 3; R \to 5; S \to 2$
(c) $P \to 4; Q \to 1; R \to 2; S \to 2$
(d) $P \to 2; Q \to 5; R \to 3; S \to 4$

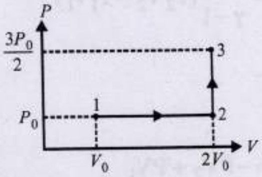
25. List I describes thermodynamic processes in four different systems. List II gives the magnitudes (either exactly or as a close approximation) of possible changes in the internal energy of the system due to the process. **[Adv. 2022]**

List-I	**List-II**
(I) 10^{-3} *kg* of water at 100°C is converted to steam at the same temperature, at a pressure of 10^5 *Pa*. The volume of the system changes from 10^{-6} m^3 to 10^{-3} m^3 in the process. Latent heat of water = 2250 kJ/kg.	(P) 2 kJ
(II) 0.2 moles of a rigid diatomic ideal gas with volume *V* at temperature 500 *K* undergoes an isobaric expansion to volume 3 *V*. Assume $R = 8.0\, J\,mol^{-1} K^{-1}$.	(Q) 7 kJ
(III) One mole of a monatomic ideal gas is compressed adiabatically from volume $V = \frac{1}{3} m^3$ and pressure 2 *kPa* to volume $\frac{V}{8}$.	(R) 4 kJ
(IV) Three moles of a diatomic ideal gas whose molecules can vibrate, is given 9 *kJ* of heat and undergoes isobaric expansion.	(S) 5 kJ
	(T) 3 kJ

Which one of the following options is correct?

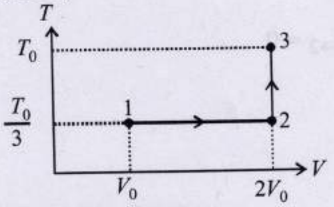
(a) (I) → (T); (II) → (R); (III) → (S); (IV) → (Q)
(b) (I) → (S); (II) → (P); (III) → (T); (IV) → (P)
(c) (I) → (P); (II) → (R); (III) → (T); (IV) → (Q)
(d) (I) → (Q); (II) → (R); (III) → (S); (IV) → (T)

26. If the process carried out on one mole of monatomic ideal gas is shown in the figure PV-diagram with $P_0V_0 = \frac{1}{3} RT_0$, the correct match is,



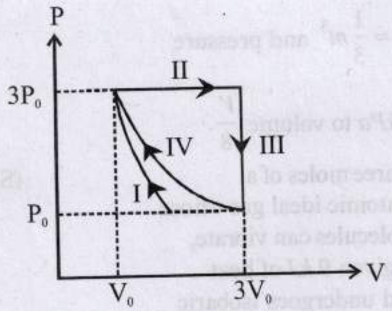
(a) I→Q, II→R, III→S, IV→U
(b) I→S, II→R, III→Q, IV→T
(c) I→Q, II→R, III→P, IV→U
(d) I→Q, II→S, III→R, IV→U

27. If the process carried out on one mole of monatomic ideal gas is as shown in the *TV*-diagram with $P_0V_0 = \frac{1}{3} RT_0$, the correct match is,



(a) I→P, II→T, III→Q, IV→T
(b) I→S, II→T, III→Q, IV→U
(c) I→P, II→R, III→T, IV→P
(d) I→P, II→R, III→T, IV→S

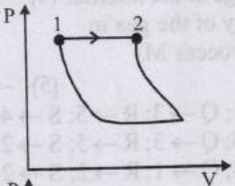
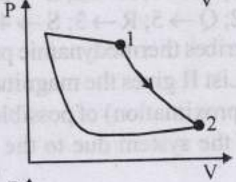
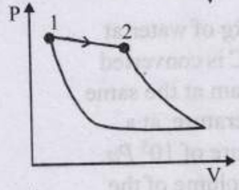
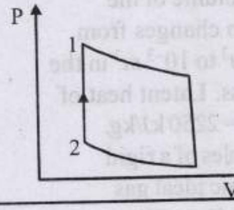
28. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List-II. **[Adv. 2018]**



	LIST–I		LIST–II
P.	In process I	**1.**	Work done by the gas is zero
Q.	In process II	**2.**	Temperature of the gas remains unchanged
R.	In process III	**3.**	No heat is exchanged between the gas and its surroundings
S.	In process IV	**4.**	Work done by the gas is $6P_0V0$

(a) $P \to 4; Q \to 3; R \to 1; S \to 2$
(b) $P \to 1; Q \to 3; R \to 2; S \to 4$
(c) $P \to 3; Q \to 4; R \to 1; S \to 2$
(d) $P \to 3; Q \to 4; R \to 2; S \to 1$

DIRECTIONS Q. No. 29, 30 and 31 : By appropriately matching the information given in the three columns of the following table. An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding P–V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic processes. Here Y is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n. **[Adv. 2017]**

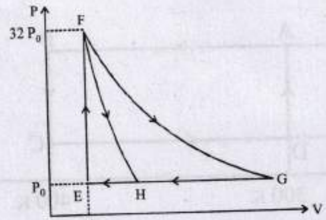
Column 1	Column 2	Column 3
(I) $W_{1\to2} = \frac{1}{\gamma-1}(P_2V_2 - P_1V_1)$	(i) Isothermal	(P) 
(II) $W_{1\to2} = -PV_2 + PV_1$	(ii) Isochoric	(Q) 
(III) $W_{1\to2} = 0$	(iii) Isobaric	(R) 
(IV) $W_{1\to2} = -nRT\ln\left(\frac{V_2}{V_1}\right)$	(iv) Adiabatic	(S) 

29. Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?
(a) (I) (ii) (Q) (b) (IV) (ii) (R)
(c) (III) (iv) (R) (d) (I) (iv) (Q)

30. Which one of the following options is the correct combination?
(a) (IV) (ii) (S) (b) (III) (ii) (S)
(c) (II) (iv) (P) (d) (II) (iv) (R)

31. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?
(a) (II) (iv) (R) (b) (III) (iii) (P)
(c) (II) (iii) (S) (d) (II) (iii) (P)

32. One mole of a monatomic ideal gas is taken along two cyclic processes $E \to F \to G \to E$ and $E \to F \to H \to E$ as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic. **[Adv. 2013]**



Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists.

List I		**List II**	
P.	G→E	1.	$160\,P_0V_0\ln 2$
Q.	G→H	2.	$36\,P_0V_0$
R.	F→H	3.	$24\,P_0V_0$
S.	F→G	4.	$31\,P_0V_0$

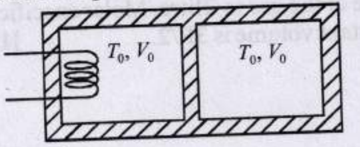
Codes:

	P	Q	R	S
(a)	4	3	2	1
(b)	4	3	1	2
(c)	3	1	2	4
(d)	1	3	2	4

8 Comprehension/Passage Based Questions

Passage

A thermally insulating cylinder has a thermally insulating and frictionless movable partition in the middle, as shown in the figure below. On each side of the partition, there is one mole of an ideal gas, with specific heat at constant volume, $C_v = 2R$. Here, R is the gas constant. Initially, each side has a volume V_0 and temperature T_0. The left side has an electric heater, which is turned on at very low power to transfer heat Q to the gas on the left side. As a result the partition moves slowly towards the right reducing the right side volume to $V_0/2$. Consequently, the gas temperatures on the left and the right sides become T_L and T_R, respectively. Ignore the changes in the temperatures of the cylinder, heater and the partition.



33. The value of $\frac{T_R}{T_0}$ is **[Adv. 2021]**

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 2 (d) 3

34. The value of $\frac{Q}{RT_0}$ is **[Adv. 2021]**

(a) $4(2\sqrt{2}+1)$ (b) $4(2\sqrt{2}-1)$

(c) $(5\sqrt{2}+1)$ (d) $(5\sqrt{2}-1)$

10 Subjective Problems

35. A cylinder of mass 1 kg is given heat of 20,000J at atmospheric pressure. If initially the temperature of cylinder is 20°C, find **[2005 - 6 Marks]**

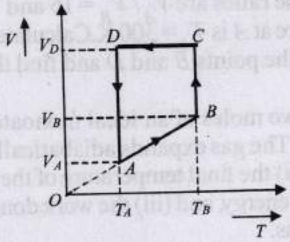
(a) final temperature of the cylinder.

(b) work done by the cylinder.

(c) change in internal energy of the cylinder

(Given that specific heat of cylinder = 400 J kg^{-1} $°C^{-1}$, coefficient of volume expansion = 9×10^{-5} $°C^{-1}$, Atmospheric pressure = 10^5 N/m^2 and Density of cylinder = 9000 kg/m^3)

36. A monoatomic ideal gas of two moles is taken through a cyclic process starting from A as shown in figure. The volume ratios are $\frac{V_B}{V_A} = 2$ and $\frac{V_D}{V_A} = 4$. If the temperature T_A at A is 27°C. **[2001-10 Marks]**



Calculate,

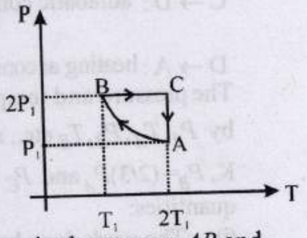
(a) the temperature of the gas at point B,

(b) heat absorbed or released by the gas in each process,

(c) the total work done by the gas during the complete cycle.

Express your answer in terms of the gas constant R.

37. Two moles of an ideal monatomic gas is taken through a cycle $ABCA$ as shown in the P-T diagram. During the process AB, pressure and temperature of the gas vary such that $PT =$ Constant. If $T_1 = 300$ K, calculate **[2000 - 10 Marks]**



(a) the work done on the gas in the process AB and

(b) the heat absorbed or released by the gas in each of the processes.

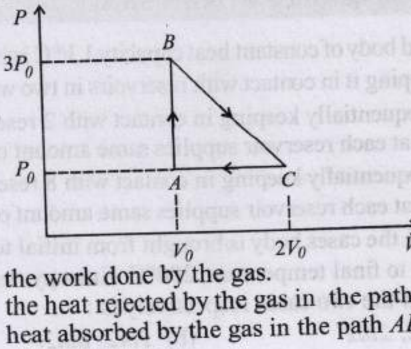
Give answer in terms of the gas constant R.

38. Two moles of an ideal monatomic gas, initially at pressure p_1 and volume V_1, undergo an adiabatic compression until its volume is V_2. Then the gas is given heat Q at constant volume V_2. **[1999 - 10 Marks]**

(a) Sketch the complete process on a $p - V$ diagram.

(b) Find the total work done by the gas, the total change in its internal energy and the final temperature of the gas. [Give your answer in terms of p_1, V_1, V_2, Q and R]

39. One mole of an ideal monatomic gas is taken round the cyclic process $ABCA$ as shown in Figure. Calculate **[1998 - 8 Marks]**



(a) the work done by the gas.

(b) the heat rejected by the gas in the path CA and the heat absorbed by the gas in the path AB;

(c) the net heat absorbed by the gas in the path *BC*;

(d) the maximum temperature attained by the gas during the cycle.

40. One mole of a diatomic ideal gas ($\gamma = 1.4$) is taken through a cyclic process starting from point *A*. The process $A \rightarrow B$ is an adiabatic compression, $B \rightarrow C$ is isobaric expansion, $C \rightarrow D$ is an adiabatic expansion, and $D \rightarrow A$ is isochoric. The volume ratios are $V_A / V_B = 16$ and $V_C / V_B = 2$ and the temperature at *A* is $T_A = 300$ K. Calculate the temperature of the gas at the points *B* and *D* and find the efficiency of the cycle. **[1997 - 5 Marks]**

41. At 27°C two moles of an ideal monoatomic gas occupy a volume *V*. The gas expands adiabaticallly to a volume 2*V*. Calculate (i) the final temperature of the gas, (ii) change in its internal energy, and (iii) the work done by the gas during this process. **[1996 - 5 Marks]**

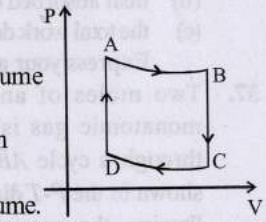
42. One mole of a mono atomic ideal gas is taken through the cycle shown in Fig: **[1993 - 4+4+2 Marks]**

$A \rightarrow B$: adiabatic expansion

$B \rightarrow C$: cooling at constant volume

$C \rightarrow D$: adiabatic compression

$D \rightarrow A$: heating at constant volume.



The pressure and temperature at *A*, *B*, etc. are denoted by P_A, T_A, P_B, T_B etc., respectively. Given that $T_A = 1000$ K, $P_B = (2/3)P_A$ and $P_C = (1/3)P_A$, calculate the following quantities:

(i) The work done by the gas in the process $A \rightarrow B$

(ii) The heat lost by the gas in the process $B \rightarrow C$.

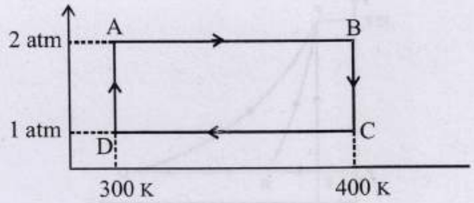
(iii) The temperature T_D. [Given : $(2/3)^{2/5} = 0.85$]

43. Two moles of helium gas undergo a cyclic process as shown in Fig. Assuming the gas to be ideal, calculate the following quantities in this process **[1992 - 8 Marks]**

(a) The net change in the heat energy

(b) The net work done

(c) The net change in internal energy



44. Three moles of an ideal gas $\left(C_p = \frac{7}{2}R\right)$ at pressure, P_A and temperature T_A is isothermally expanded to twice its initial volume. It is then compressed at constant pressure to its original volume. Finally gas is compressed at constant volume to its original pressure P_A. **[1991 - 4 + 4 Marks]**

(a) Sketch *P* - *V* and *P* - *T* diagrams for the complete process.

(b) Calculate the net work done by the gas, and net heat supplied to the gas during the complete process.

45. An ideal gas having initial pressure *P*, volume *V* and temperature *T* is allowed to expand adiabatically until its volume becomes 5.66 *V* while its temperature falls to $\frac{T}{2}$. **[1990 - 7 Mark]**

(i) How many degrees of freedom do the gas molecules have?

(ii) Obtain the work done by the gas during the expansion as a function of the initial pressure *P* and volume *V*.

46. An ideal gas has a specific heat at constant pressure $C_P = \frac{5R}{2}$. The gas is kept in a closed vessel of volume 0.0083 m^3, at a temperature of 300 K and a pressure of $1.6 \times 10^6 \text{ N/m}^2$. An amount of 2.49×10^4 Joules of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas. **[1987 - 7 Mark]**

47. Calculate the work done when one mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 105 N/m^2 and 6 litres respectively. The final volume of the gas is 2litre. Molar specific heat of the gas at constant volume is 3R/2. **[1982 - 8 Marks]**

Topic-3: Carnot Engine, Refrigerators and Second Laws of Thermodynamics

1 MCQs with One Correct Answer

1. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways : **[2012]**

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is :

(a) *ln2, 2ln2*	(b) *2ln2, 8ln2*
(c) *ln2, 4ln2*	(d) *ln2, ln2*

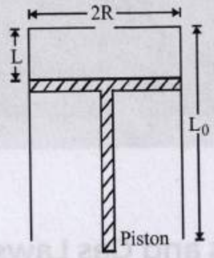
2. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A, and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are found to be ΔP and 1.5 ΔP respectively. Then **[1998S - 2 Marks]**

(a) $4m_A = 9m_B$	(b) $2m_A = 3m_B$
(c) $3m_A = 2m_B$	(d) $9m_A = 4m_B$

8 Comprehension/Passage Based Questions

Passage

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0.



3. The piston is now pulled out slowly and held at a distance 2L from the top. The pressure in the cylinder between its top and the piston will then be **[2007]**

(a) P_0 (b) $\frac{P_0}{2}$

(c) $\frac{P_0}{2}+\frac{Mg}{\pi R^2}$ (d) $\frac{P_0}{2}-\frac{Mg}{\pi R^2}$

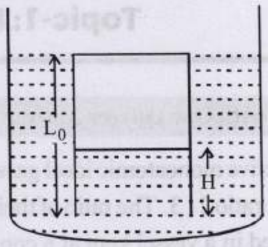
Therefore the pressure inside the cylinder is P_0 throughout the slow pulling process.

4. While the piston is at a distance 2L from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is **[2007]**

(a) $\left(\frac{2P_0\pi R^2}{\pi R^2P_0+Mg}\right)(2L)$ (b) $\left(\frac{P_0\pi R^2-Mg}{\pi R^2P_0}\right)(2L)$

(c) $\left(\frac{P_0\pi R^2+Mg}{\pi R^2P_0}\right)(2L)$ (d) $\left(\frac{P_0\pi R^2}{\pi R^2P_0-Mg}\right)(2L)$

5. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies
[2007]



(a) $\rho g(L_0-H)^2+P_0(L_0-H)+L_0P_0=0$

(b) $\rho g(L_0-H)^2-P_0(L_0-H)-L_0P_0=0$

(c) $\rho g(L_0-H)^2+P_0(L_0-H)-L_0P_0=0$

(d) $\rho g(L_0-H)^2-P_0(L_0-H)+L_0P_0=0$

Answer Key

Topic-1 : First Law of Thermodynamics

1. (a) **2.** (2) **3.** (Constant) **4** (a, b, c) **5.** (b, c)

6. $A\rightarrow p, r, t$; $B\rightarrow p, r$; $C\rightarrow q, s$; $D\rightarrow r, t$ **7.** $A\rightarrow q$; $B\rightarrow p, r$; $D\rightarrow q, s$

Topic-2 : Specific Heat Capacity and Thermodynamical Processes

1. (a) **2.** (b) **3.** (b) **4.** (a) **5.** (b) **6.** (d) **7.** (b) **8.** (a) **9.** (2) **10.** (25)
11. (4) **12.** (1.77 to 1.78) **13.** (2.05) **15.** (True) **16.** (b) **17.** (c, d) **18.** (b, c, d)
19. (a, b, d) **20.** (a, c) **21.** (a, b) **22.** (b, d) **23.** (a, b, c, d) **24.** (b) **25.** (c) **26.** (a) **27.** (c)
28. (c) **29.** (d) **30.** (b) **31.** (d) **32.** (a) **33.** (a) **34.** (b)

Topic-3 : Carnot Engine, Refrigerators and Second Law of Thermodynamics

1. (d) **2.** (c) **3.** (a) **4.** (d) **5.** (c)

Hints & Solutions

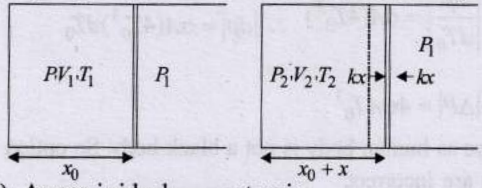
Topic-1: First Law of Thermodynamics

1. **(a)** From the first law of thermodynamics
$dQ = dU + dW$
Here $dW = 0$ (given) $\therefore dQ = dU$
Now since $dQ < 0$ (given)
i.e., dQ is negative so dU decreases.
Internal energy 'U' decrease when temperature T decreases.

2. **(2)** Applying first law of thermodynamics to path iaf
$Q_{iaf} = \Delta U_{iaf} + W_{iaf}$
$500 = \Delta U_{iaf} + 200 \qquad \therefore \Delta U_{iaf} = 300\text{ J}$
Now,
$Q_{ibf} = \Delta U_{ibf} + W_{ib} + W_{bf} = 300 + 50 + 100$
$Q_{ib} + Q_{bf} = 450\text{ J} \qquad$ ...(i)
Also $Q_{ib} = \Delta U_{ib} + W_{ib}$
$\therefore Q_{ib} = 100 + 50 = 150\text{ J}$ ...(ii)
From eq. (i) & (ii) $\dfrac{Q_{bf}}{Q_{ib}} = \dfrac{Q_{ib}f - Q_{ib}}{Q_{ib}} = \dfrac{300}{150} = 2$

3. **(Constant)** In this expansion, no work is done because the gas expands constant in vacuum. $\therefore \Delta W = 0$
Also, $Q = 0$ as the process is adiabatic. Hence from first law of thermodynamics, $\Delta U = 0$ *i.e.*, temperature remains constant.

4. **(a, b, c)**
Let spring is compressed by x on heating the gas.



(a) As gas is ideal monoatomic,
$$\therefore \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \qquad \text{...(i)}$$
Force on spring by gas $= kx$
$$\therefore P_2 = P_1 + \frac{kx}{A} \text{ (A=area of cross-section of piston) ...(ii)}$$
When $V_2 = 2V_1$, $T_2 = 3T_1$
$$\therefore \frac{P_1V_1}{T_1} = \frac{P_2 \times (2V_1)}{3T_1} \Rightarrow P_2 = \frac{3}{2}P_1$$
Putting this value of P_2 in eqn. (ii) we get
$$\frac{3}{2}P_1 = P_1 + \frac{kx}{A} \Rightarrow kx = \frac{P_1A}{2}$$
$$x = \frac{V_2 - V_1}{A} = \frac{2V_1 - V_1}{A} = \frac{V_1}{A}$$
Energy stored in the spring $= \dfrac{1}{2}kx^2 = \dfrac{1}{2}(kx)(x) = \dfrac{P_1V_1}{4}$

(b) Change in internal energy,
$$\Delta U = \frac{f}{2}(P_2V_2 - P_1V_1) = \frac{3}{2}\left(\frac{3}{2}P_1 \times 2V_1 - P_1V_1\right) = 3P_1V_1$$
(c) Again, when $V_2 = 3V_1$ and $T_2 = 4T_1$ then
From eqn. (i),
$$\frac{P_1V_1}{T_1} = \frac{P_2(3V_1)}{4T_1} \Rightarrow P_2 = \frac{4}{3}P_1 \Rightarrow x = \frac{V_2 - V_1}{A} = \frac{2V_1}{A}$$
From eqn. (ii),
$$\frac{4}{3}P_1 = P_1 + \frac{kx}{A} \Rightarrow kx = \frac{P_1A}{3}$$
Work done by gas = Work done by gas on atmosphere + Energy stored in spring.
$$W_g = P_1Ax + \frac{1}{2}kx^2 = P_1(2V_1) + \frac{1}{2}\left(\frac{P_1A}{3}\right)\left(\frac{2V_1}{A}\right)$$
$$= 2P_1V_1 + \frac{1}{3}P_1V_1 = \frac{7}{3}P_1V_1$$
(d) $\Delta Q = W_g + \Delta U = \dfrac{7}{3}P_1V_1 + \dfrac{3}{2}(P_2V_2 - P_1V_1)$
$$= \frac{7}{3}P_1V_1 + \frac{3}{2}\left(\frac{4}{3}P_1 \times 3V_1 - P_1V_1\right)$$
$$= \frac{7}{3}P_1V_1 + 6P_1V_1 - \frac{3}{2}P_1V_1 = P_1V_1\left(\frac{14 + 36 - 9}{6}\right) = \frac{41}{6}P_1V_1$$

5. **(b, c)**
There is a decrease in volume during melting of an ice slab at 273 K. Therefore, negative work is done by ice-water system on the atmosphere or positive work is done on the ice-water system by the atmosphere. Hence, option (b) is correct.
Secondly heat is absorbed during melting (i.e. dQ is positive) and as we have seen, work done by ice-water system is negative (dW is negative.) Therefore, from first law of thermodynamics $dU = dQ - dW$
change in internal energy of ice-water system, dU will be positive or internal energy will increase.

6. **A-p,r,t; B-p,r; C-q,s; D-r,t**
(A) Process A → B
This is an isobaric process, P = constant and volume (V) of the gas decreases. Therefore work is done on the gas.
$W = P(3V - V) = 2PV$
Also V decreases so temeraturе at B decreases
$\therefore$ Internal energy U decreases.
From, $Q = U + W$ as U and W decreases so Q decreases that means heat is lost.
(B) Process B → C
This is an isochoric process V = constant pressure decreases
$P \propto T$ so temperature also decreases.
$W = 0$; ΔU = negative so ΔQ negative

Hence heat is lost.

(C) Process C → D

This is isobaric, Pressure P = constant V increases and $V \propto T$ so T increases. Hence ΔW, ΔU and ΔQ + ve so heat gained by the gas.

(D) Process D → A

Applying $PV = nRT$

for **D** $P(9V) = 1\,RT_D \therefore T_D = \frac{9PV}{R}$

for **A** $3P(3V) = 1RT_A \therefore T_A = \frac{9PV}{R}$

i.e., the process is isothermal $\therefore\ \Delta U = 0$

Now, $\Delta Q = \Delta U + W \quad \therefore \Delta Q = W.$

As volume decreaes in this process so W negative i.e., wom done on the gas aand ΔQ negative hence heat is lost.

7. **(A)-(q) :** As the ideal gas expands in vacuum, and the container is insulated therefore $W = 0$ & $Q = 0$ and according to first law of thermodynamics

$\Delta U = Q + W \Rightarrow \Delta U = 0$

Hence there is no change in the temparature of the gas of T is constant.

(B)-(p, r) : Given $P \propto \frac{1}{V^2}$ or, PV^2 = constant

or, $nRTV$ = constant $\quad \therefore V \times T$ = constant

As the gas expands its volume increases so temperature decreases

We know that $Q = n\,C\,\Delta T$ ...(i)

For a polytropic process

$C = C_v + \frac{R}{1-n}$ and PV^n = constant

Here PV^2 = Constant. $\therefore n = 2$

$$\therefore C = C_v + \frac{R}{1-2} = C_v - R = \frac{3}{2}R - R = \frac{R}{2}$$

$$Q = nC\,\Delta T = n \times \frac{R}{2} \times \Delta T.$$

i.e. ΔT is negative, Q is negative so heat is lost.

As volume increases, so temperature decreases given

$$P \propto \frac{1}{V^{4/3}} \Rightarrow PV^{4/3} = \text{constant}.$$

$$\therefore n = \frac{4}{3}.$$

$$\therefore C = C_v + \frac{R}{1-\frac{4}{3}} = \frac{3}{2}R + \frac{3R}{-1} = \frac{3}{2}R - 3R = \frac{-3R}{2}$$

$$\therefore Q = nC_V\Delta T = n\left(\frac{-3R}{2}\right)\Delta t$$

As ΔT is negative, Q is positive. So gas gains heat.

(D)-(q, s) : From $PV = nRT \Rightarrow T = \frac{PV}{nR}$ PV increase So T increases volume increases so W increases.

From $Q = \Delta U + W$, Q increases.

Hence the gas gains heat.

8. From the given P–V graph, In process $J \to K$ volume, V = constant p is decreasing and $P \propto T$ Therefore, T should also decrease.

$\therefore W = Pdv = 0, \Delta V = \Delta Q < 0$ (negative)

From $\Delta Q = \Delta U + \Delta W$

In process $K \to L$ p = constant = $nC_v\Delta T$ V is increasing So temperature should also increase.

$\therefore \Delta W = Pdv > 0, \Delta U = nC_v\Delta T > 0$ and $Q = mC\Delta T > 0$

In process $L \to M$ V = constant P in creases to T increases

$\therefore W = 0, \Delta U > 0$ and $Q > 0$

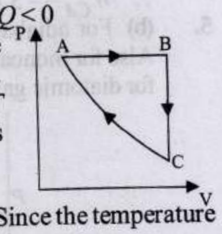
In process $M \to J$

V is decreasing $\therefore \Delta W < 0$

T is also decreasing $\therefore \Delta U < 0$ and $\Delta Q < 0$

9. The corresponding P-V vgraph of the given V-T graph is shown in figure.

Process **A → B** A straight line in V-T graph indicates $V \propto T \therefore$ Pressure is constant. Both V and T are increasing.



Process **B → C** Volume is constant. Since the temperature is decreasing, So pressure will also decrease.

Process **C → A** The temperature is constant but volume decreases. Hence pressure will increase as $P \propto \frac{I}{V}$.

Topic-2: Specific Heat Capacity and Thermodynamical Processes

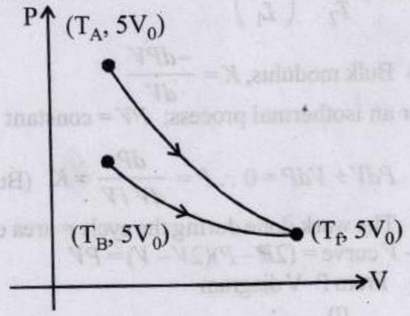
1. **(a)** For adiabatic Process $T\,V^{\gamma-1}$ = constant

$\therefore T_A V_0^{\gamma-1} = T_f (5\,V_0)^{\gamma-1}$ ...(i)

and for isothermal process, no change in temperature

$\therefore T_B = T_f$ ...(ii)

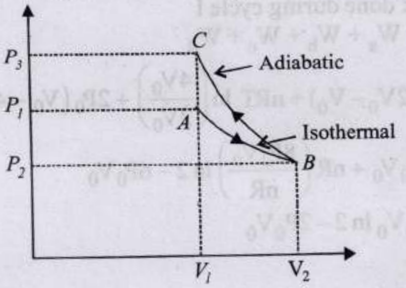
Dividing (i) by (ii)



$$\frac{T_A}{T_B} = 5^{\gamma-1}$$

2. **(b)** In the first process W_{AB} is + ve as ΔV is positive, in the second process W_{BC} is – ve as ΔV is – ve and area under the curve of second process is more

$\therefore$ Net Work < 0 and also $P_3 > P_1$.



3. **(b)** From the $P-T$ graph we find AB to be a isothermal process, AC is adiabatic process given. Also for an expansion process, the slope of adiabatic curve is more (or we can say that the area under the $P-V$ graph for isothermal process is more than adiabatic process for same increase in volume). Hence graph (b) correctly depicts corresponding P – V graph.

4. **(a)** For cyclic process;

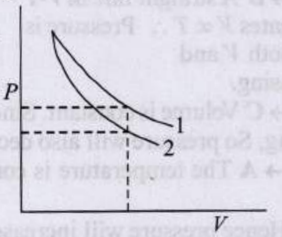
$Q_{\text{cyclic}} = W_{AB} + W_{BC} + W_{CA} = 10\,\text{J} + 0 + W_{CA} = 5\,\text{J}$

$\therefore \quad W_{CA} = -5\,\text{J}$

5. **(b)** For adiabatic process $PV^{\gamma} = \text{constant}$

Also for monoatomic gas like helium, $\gamma = 1.67$

for diatomic gas like oxygen, $\gamma = 1.4$



Since, $\gamma_{\text{diatomic}} < \gamma_{\text{monoatomic}} \therefore P_{\text{diatomic}} > P_{\text{monoatomic}}$

Hence graph 1 is for diatomic *i.e.*, oxygen and graph 2 is for monoatomic *i.e.*, for helium.

6. **(d)** From $TV^{\gamma-1} = \text{constant}$

For monatomic gas $\gamma = \frac{5}{3}$, hence $TV^{2/3} = \text{constant}$

$\therefore \quad T_1 L_1^{2/3} = T_2 L_2^{2/3} \qquad (\because V \propto L)$

Hence, $\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}$

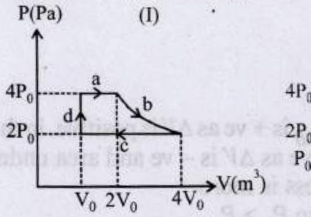
7. **(b)** Bulk modulus, $K = \frac{-dPV}{dV}$

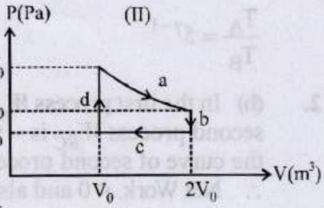
For an isothermal process; $PV = \text{constant}$

$\therefore \quad PdV + VdP = 0 \therefore P = \frac{dP}{dV/V} = K$ (Bulk modulus)

8. **(a)** The work done during the cycle = area enclosed in the $P-V$ curve $= (2P-P)(2V-V) = PV$

9. **(2)** From P–V diagram





Work done during cycle I

$W_I = W_a + W_b + W_c + W_d$

$4P_0(2V_0 - V_0) + nRT\ln\left(\frac{4V_0}{2V_0}\right) + 2P_0(V_0 - 4V_0) + 0$

$= 4P_0V_0 + nR\left(\frac{8P_0V_0}{nR}\right)\ln 2 - 6P_0V_0$

$= 8P_0V_0\ln 2 - 2P_0V_0$

Work done during cycle II

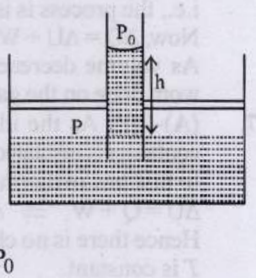
$W_{II} = W_a' + W_b' + W_c' + W_d'$

$= nRT\ln\left(\frac{2V_0}{V_0}\right) + 0 + P_0(V_0 - 2V_0) + 0$

$= nR\left(\frac{4P_0V_0}{nR}\right)\ln 2 - P_0V_0 = 4P_0V_0\ln 2 - P_0V_0$

$\therefore \frac{W_I}{W_{II}} = \frac{8P_0V_0\ln 2 - 2P_0V_0}{4P_0V_0\ln 2 - P_0V_0} = 2$

10. **(25)** $h_0 = \frac{2T\cos\theta}{\rho g r} = \frac{2T\cos 0^\circ}{\rho g r}$



$= \frac{2 \times 0.075 \times 1}{10^3 \times 10 \times 10^{-4}} = 15\text{cm}$

$P_0V_0 = P\frac{100V_0}{101} \Rightarrow P = \frac{101}{100}P_0$

$P_0 - \frac{2T\cos\theta}{r} + \rho gh = P = \frac{101}{100}P_0$

$\Rightarrow -\rho gh_0 + \rho gh = \frac{P_0}{100} \Rightarrow h = h_0 + \frac{P_0}{100\rho g}$

$\therefore h = 15\,\text{cm} + \frac{10^5 \times 100}{100 \times 10^3 \times 10} = 25\text{cm}$

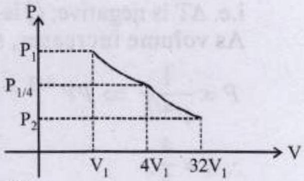
11. **(4)** For an adiabatic process, applying $TV^{\gamma-1} = \text{constant}$

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1}$

$\gamma = 1.4$ (for diatomic gas) $V_2 = \frac{V_1}{32}, T_1 = T_i, T_2 = aT_i$

$\therefore T_i = aT_i\left[\frac{1}{32}\right]^{1.4-1} = aT_i\left[\frac{1}{2^5}\right]^{0.4} = \frac{aT_i}{4} \quad \therefore a = \mathbf{4}$

12. **(1.77 to 1.78)**



For monatomic gas, $\gamma = \frac{5}{3}$

In adiabatic process

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$\Rightarrow \frac{P_1}{4}(4V_1)^{5/3} = P_2(32V_1)^{5/3}$

$\Rightarrow P_2 = \frac{P_1}{4}\left(\frac{1}{8}\right)^{5/3} = \frac{P_1}{128}$

And $W_{\text{adi}} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{P_1V_1 - \frac{P_1}{128}(32V_1)}{\frac{5}{2} - 1}$

$= \frac{P_1V_1(3/4)}{2/3} = \frac{9}{8}P_1V_1$

In isothermal process,

$$W_{iso} = 2.303\ \mu RT\ \log_{10}\left(\frac{V_2}{V_1}\right)$$

$$\Rightarrow W_{iso} = P_1V_1\ \ln\left(\frac{4V_1}{V_1}\right) = 2P_1V_1\ \ln 2$$

$$\therefore \frac{W_{iso}}{W_{adia}} = \frac{2P_1V_1\ \ln 2}{\frac{9}{8}P_1V_1} = \frac{16}{9}\ \ln 2 = f\ln 2$$

$$\text{So, } f = \frac{16}{9} = 1.7778 \approx 1.78$$

13. **(2.05)** Here, we will neglect surface tension.

As $P_{inside} = P_{outside} = P_0$

Here, $\Delta k = 0$

So, $W_{all\ forces} = 0$

$W_{ext} + W_w + W_g = 0$, $W_w \rightarrow$ work done by water

$W_{ext} = -(W_w + W_g) W_g \rightarrow$ work done by gas

$$\text{Now, } W_w = P_0\left[\frac{4}{3}\pi R^3 - \frac{4}{3}\pi(R-a)^3\right]$$

$$= \frac{4}{3}P_0\pi[R-(R-a)][R^2 + R(R-a) + (R-a)^2]$$

$$= \frac{4}{3}P_0\pi[a][R^2 + R^2 + R^2 - 3Ra + a^2]$$

$$= 4P_0\pi[R^2a - Ra^2]$$

and,

$$W_g = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = P_0\frac{4}{3}\pi R^3 - P_0\left(\frac{R}{R-a}\right)^{41/10}\ ;\frac{4}{3}\pi(R-a)^3$$

$$[\because PV^{\gamma} = \text{cons}]$$

$$= \frac{30}{11}P_0 \times \frac{4}{3}\pi R^3\left[1 - \left(\frac{R}{R-a}\right)^{41/10}\left(\frac{R-a}{R}\right)^3\right]$$

$$= \frac{40}{11}P_0\pi R^3\left[1 - \left(1 - \frac{a}{R}\right)^{-11/10}\right]$$

$$= \frac{40}{11}P_0\pi R^3\left[1 - 1 - \frac{11a}{10R} - \frac{\left(-\frac{11}{10}\right)\left(\frac{-21}{10}\right)}{2}\frac{a^2}{R^2}\right]$$

$$= -4P_0\pi R^2a - 4.2P_0\pi Ra^2$$

So, $W_w + W_g = -4P_0\pi R_0^2[1 + 1.05] = -4P_0\pi R_0^2 \times 2.05$

Therefore, $W_{ext} = 4P_0\pi Ra^2 \times 2.05$. So, $X = 5$

14. For adiabatic expansion

$$PV^{\gamma} = P_a \times (2V)^{\gamma} \Rightarrow P_a = \frac{P}{2^{\gamma}} = \frac{P}{2^{1.67}}$$

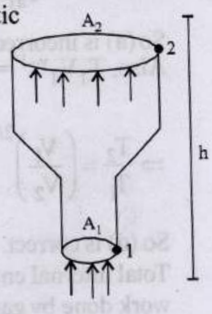
For isothermal expansion

$$P \times V = P_i \times 2V \Rightarrow P_i = \frac{P}{2}$$

$$\therefore \frac{P_a}{P_i} = \frac{P}{2^{1.67}} \times \frac{2}{P} = \frac{2}{2^{1.67}} = 0.628$$

15. **(True)** The slope of *P-V* curve is more for adiabatic process than for isothermal process. From the graph it is clear that slope for *B* is greater than the slope for *A*. So the isothermal process is represented by the curve *A*.

16. **(b)** As expansion of gas is adiabatic



$$\text{so, } PT^{\frac{\gamma}{1-\gamma}} = \text{cons.}$$

$$\Rightarrow 600 \times 300^{-2} = P_2\,150^{-2}$$

$$\Rightarrow P_2 = \frac{600 \times 150^2}{300^2}$$

$$\Rightarrow P_2 = 150\ \text{Pa.}$$

So (a) is incorrect.

$$\text{As } \frac{dm}{dt} = \rho Av$$

At 1.

$0.8 = 0.2 \times 0.1 \times v_1$

$v_1 = 40$ m/s

By ideal gas equation

$$PV = nRT$$

$$\Rightarrow PV = \frac{m}{M_0}RT \Rightarrow PM_0 = \rho RT \Rightarrow \frac{P}{\rho T} = \text{cons.}$$

$$\Rightarrow \frac{P_1}{\rho_1T_1} = \frac{P_2}{\rho_2T_2} \Rightarrow \frac{600}{0.2 \times 300} = \frac{150}{\rho_2 \times 150}$$

$\Rightarrow \rho = 0.1\ \text{kg/m}^3$. So option (d) is incorrect.

$$\text{Again, } \frac{dm}{dt} = \rho Av$$

At 2.

$0.8 = 0.1 \times 0.4 \times v_2$

$\Rightarrow v_2 = 20$ m/s. So option (b) is correct.

Here, $T \neq$ constant. So Bernoulli's theorem is need to be modified. So, we need to add internal energy per unit volume factor.

Therefore, factor of $\frac{nC_VT}{V}$ is added.

$$\text{As } \frac{nC_VT}{V} = \frac{n\frac{f}{2}RT}{V} = \frac{nRT}{V} \times \frac{2}{2} = P$$

So, Bernoulli's theorem becomes $2P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$

$$\Rightarrow 2P_1 + \frac{1}{2}\rho_1v_1^2 + \rho_1g \times 0 \Rightarrow 2P_2 + \frac{1}{2}\rho_2v_2^2 + \rho_2gh$$

$$\Rightarrow 2 \times 600 + \frac{1}{2} \times 0.2 \times 40^2 = 2 \times 150 + \frac{1}{2} \times 0.1 \times 20^2 + 0.1 \times 10 \times h$$

$\Rightarrow h = 1040$ m. So option (c) is incorrect.

17. **(c, d)** If the surface of the bubble is perfect heat insulator, So process is adiabatic so,

$PV^{\gamma} = \text{constant}$

$$\Rightarrow \left(P_{a1} + \frac{4s}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right)^{5/3} = \left(P_{a2} + \frac{4s}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right)^{5/3}$$

$$\Rightarrow\left(\frac{r_1}{r_2}\right)^5=\frac{P_{a2}+\frac{4s}{r_2}}{P_{a1}+\frac{4s}{r_1}}$$

So (a) is incorrect.
Also, $T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1}$

$$\Rightarrow\frac{T_2}{T_1}=\left(\frac{V_1}{V_2}\right)^{2/3}\Rightarrow\frac{T_2}{T_1}=\left(\frac{r_1}{r_2}\right)^2\Rightarrow\frac{T_2}{T_1}=\left(\frac{P_{a2}+\frac{4s}{r_2}}{P_{a1}+\frac{4s}{r_1}}\right)^{2/5}$$

So (d) is correct.
Total internal energy + surface energy will not be same as work done by gas will be there
i.e. $\Delta U=-\Delta W\ (\because \Delta Q=0)$
So, (b) is incorrect.
Now, if bubble is perfect heat conductor, then temperature will remain constant. So, PV = constant

$$\left(P_{a1}+\frac{4s}{r_1}\right)\times\frac{4}{3}\pi r_1^3=\left(P_{a2}+\frac{4s}{r_2}\right)\times\frac{4}{3}\pi r_2^3$$

$$\left(\frac{r_1}{r_2}\right)^3=\left(\frac{P_{a2}+\frac{4s}{r_2}}{P_{a1}+\frac{4s}{r_1}}\right)$$

So (c) is correct.

18. **(b, c, d)** (A) $W_{A\to B\to C}=W_{A\to B}+W_{B\to C}$

$$=\frac{P_AV_A-P_BV_B}{\gamma-1}+P_AV_A\ell n\left(\frac{V_C}{V_B}\right)$$

$$\text{As, } P_AV_A^{\gamma}=P_BV_B^{\gamma}\Rightarrow\left(\frac{V_B}{V_A}\right)^{\gamma}=\frac{P_A}{P_B}$$

$$\Rightarrow V_B=\left(\frac{P_A}{P_B}\right)^{1/\gamma}V_A=0.8\left(\frac{1}{3}\right)^{3/5}=0.8\times0.5=0.4\text{ m}^3$$

$$\text{So, } W_{A\to B\to C}=\frac{100\times0.8-300\times0.4}{\frac{5}{3}-1}+300\times0.4\ell n\left(\frac{0.8}{0.4}\right)$$

$$=\frac{-40}{\frac{2}{3}}+120\ell n2=-60+120\times0.7=-60+84=24\text{kJ}$$

So, (b, c, d) are correct.

19. **(a, b, d)**
Adiabatic constant of the gas mixture,

$$\gamma_m=\frac{n_1C_{p1}+n_2C_{p2}}{n_1C_{v1}+n_2C_{v2}}=\frac{5\times\frac{5R}{2}+1\times\frac{7R}{2}}{5\times\frac{3R}{2}+1\times\frac{5R}{2}}=1.6$$

For an adiabatic process, $P_v\gamma$ = Constant

$$\therefore\ P=P_0\left(\frac{V_0}{V}\right)^{1.6}=P_0(4)^{1.6}$$

$=P_0(2^2)^{1.6}=P_0\,2^{3.2}=9.2\,P_0$
Work done during the process,

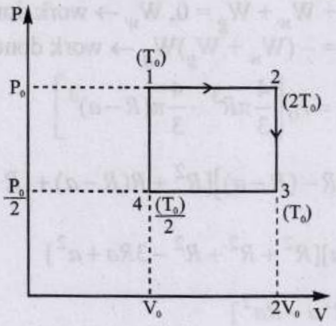
$$W=\frac{P_2V_2-P_1V_1}{1-\gamma}=\frac{9.2P_0\times(V_0/4)-P_0V_0}{1-1.6}=\frac{-13P_0V_0}{6}$$

But $P_0V_0=6RT_0$ (as $n=5+1=6$)

$$\therefore\ W=\frac{-13(6RT_0)}{6}=-13RT_0\ \therefore\ |W|=13\,RT_0$$

The average K.E. of the gas mixture,
$\overline{K.E}=nCV_{mi}\times T_2$
From, $T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1}$
or, $T_2=T_1(2)^{6/5}=23\,T_0$
$\therefore\ \overline{K.E}=nCV_{mi}\times T_2=23RT_0$

20. **(a, c)** The *P–V* graph of the given *V–T* graph is given below.



(a) Work done during cyclic process (1→2→3→4–1) *W* = area enclosed in the loop $=\frac{P_0}{2}V_0$

$$\because\ P_0V_0=nRT_0\quad\therefore\frac{P_0V_0}{2}=\frac{nRT_0}{2}$$

$$\therefore\ \text{Work done } W=\frac{nRT_0}{1}=\frac{RT_0}{2}\quad[\text{as } n=1]$$

(b) Process $\underline{1\to2}$ is isobaric
Process $\underline{2\to3}$ is isochoric
Process $\underline{3\to4}$ is isobaric
Process $\underline{4\to1}$ is isochoric
Hence no adiabatic process is imvolved.

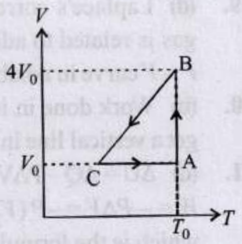
(c) $|\Delta Q_{1\to2}|=|nC_p\,\Delta T|=|nC_p(2T_0-T_0)|=|nC_pT_0|$

$|\Delta Q_{2\to3}|=|\Delta U|=|nC_v\,\Delta T|=|nC_vT_0|$

$$\therefore\left|\frac{\Delta Q_{1\to2}}{\Delta Q_{2\to3}}\right|=\frac{C_p}{C_v}=\frac{5}{3}$$

(d) $|\Delta Q_{3\to4}|=nC_p\frac{T_0}{2}\ \therefore\left|\frac{\Delta Q_{1\to2}}{\Delta Q_{3\to4}}\right|=\frac{nC_pT_0}{nC_p\frac{T_0}{2}}=\frac{2}{1}$

21. **(a, b)** **Process A to B**



Here, $T_A = T_B \therefore U_A = U_B$

Also $P_0 V_0 = P_B \times 4\, V_0$

$\Rightarrow P_B = \dfrac{P_0}{4}$

Work done

$$W_{AB} = nRT_0 \log_e \frac{4V_0}{V_0}$$

$= P_0 V_0 \log_e 4$ $\qquad [\because P_0 V_0 = nRT_0]$

The process BC is not clear. Therefore no judgement can be made at C regarding P and T.

22. **(b, d)**

(a) Process $A-B$ is not isothermal. In case of an isothermal process we get a rectangular hyperbola in a P-V diagram.

(b) In process $B \to C \to D$, ΔU is negative. PV decreases and volume also decreases, therefore W is negative. From first law of thermodynamic, Q is negative i.e., there is a heat loss.

(c) $W_{AB} > W_{BC}$. Therefore work done during path $A \to B \to C$ is positive.

(d) Work done is clockwise cycle in a PV diagram is positive.

23. **(a, b, c, d)**

(a) For all thermal processes, change is internal energy in a constant pressure process

$\Delta U = nC_v \Delta T$ where $\Delta T = (T_2 - T_1)$

(b) According to first law of thermodynamics.

$\Delta Q = \Delta U + \Delta W$

In an adiabatic process $\Delta Q = 0$.

or, $0 = \Delta U + \Delta W$

or, $|\Delta U| = |\Delta W|$

(c) In the isothermal process, $\Delta T = 0 \therefore \Delta U = 0$

$\because \Delta U = nC_v \Delta T$

(d) In the adiabatic process, $\Delta Q = 0$.

24. **(b)** From given $P-T$ diagram,

For process JK : Isobaric $W = nR(3T_o - T_o) = 2RT_o$

and $\Delta u = n\, c_v \Delta T = \dfrac{3R}{2} \times 2T_o = 3RT_o$

For process KL : Isothermal $W = -nR(3T_o)\ln 2$

$W = -3RT_o \ln 2$

$\Delta u = 0$ and $Q = -3RT_o \ln 2$

For process LM: Isobaric

$W = nR(T_o - 3T_o) = -2RT_o$

$\Delta u = nc_v \Delta T = n \times \dfrac{3}{2} R \times (-2T_0)$

$= -3RT_o$

For process MJ : Isothermal

$W = nRT \ln 2 = RT_o \ln 2$

$\Delta u = o$

And work done in the complete cyclic process,

$W_{net} = RT_o \ln 2 + 2RT_o - 3RT_o \ln 2 - 2RT_o$

$= -2RT_o \ln$

So correct option is (b)

25. **(c)** **(i)** By first law of thermodynamics.

$\Delta U = \Delta Q - \Delta W$

$= ML_V - P\Delta V$

$= 10^{-3} \times 2250 \times 10^3 - 10^5 \times (10^{-3} - 10^{-6})$

$= 2250 - 100\ 2150\,J$

$= 2.15$ kJ. So (I) → (P)

(ii) $P = \dfrac{nRT}{V} = \dfrac{0.2 \times 8 \times 500}{V} = \dfrac{800}{V}$ Pa

$$\Delta U = \frac{f}{2} P\Delta V = \frac{5}{2} \times \frac{800}{V} \times 2V = 4000\,J = 4kJ$$

So (II) → (R)

(iii) $PV^\gamma = \text{const} \Rightarrow P_1 V_1{}^\gamma = P_2 V_2{}^\gamma$

$$\Rightarrow \quad 2V^\gamma = P_2 \left(\frac{V}{8}\right)^\gamma \Rightarrow P_2 = 2 \times 8^\gamma = 2 \times 8^{5/3} = 64\,kPa$$

So, $\Delta U = \dfrac{f}{2}(P_2V_2 - P_1V_1) = \dfrac{3}{2}\left(64 \times \dfrac{1}{24} - 2 \times \dfrac{1}{3}\right) \times 10^3$

$= 3kJ.$

So (III) → (T)

(iv) Here $f = 7$

So, $\Delta U = nC_V \Delta T = \dfrac{f}{2} nR\Delta T = \dfrac{7}{2} nR\Delta T$

and, $\Delta Q = nC_V \Delta T = \left(\dfrac{f}{2} + 1\right) nR\Delta T = \dfrac{9}{2} nR\Delta T = \dfrac{9}{2} \times \dfrac{2}{7}$

$$\Delta U = \frac{9}{7} \Delta U$$

So, $\Delta U = \dfrac{7}{9} \Delta Q = \dfrac{7}{9} \times 9 = 7$kJ. So (IV) → (Q)

26. **(a)** **I.** $W_{1\to2\to3} = W_{1\to2} + W_{2\to3}$ $[W_{2\to3} = 0, \Delta V = 0]$

$= P_0 \times V_0 + O$

$= P_0 V_0 = \dfrac{RT_0}{3}$ $\qquad [\because P_0 V_0 = \dfrac{1}{3} RT_0 \text{ given}]$

$\therefore I \to Q$

II. $\Delta U_{1\to2\to3} = \Delta U_{1\to2} + \Delta U_{2\to3}$

$= nC_v \Delta T_{1\to2} + nC_v \Delta T_{2\to3}$

$= 1 \times \dfrac{3R}{2}(T_f - T_i)_{1\to2} + 1 \times \dfrac{3R}{2}[T_f - T_i]_{2\to3}$

$= \dfrac{3}{2}[2P_0V_0 - P_0V_0] + \dfrac{3}{2}\left[\dfrac{3P_0}{2} \times 2V_0 - P_0 \times 2V_0\right]$

$= 3P_0V_0 = 3 \times \dfrac{1}{3} RT_0 = RT_0$

$II \to R$

III. $Q_{1\to2\to3} = Q_{1\to2} + Q_{2\to3}$
$= nC_p\Delta T_{1\to2} + nC_v\Delta T_{2\to3}$

$$= \frac{5}{2}P_0V_0 + 1\times\frac{3}{2}\left[\frac{3P_0}{2}\times 2V_0 - P_2(2V_0)\right]$$

$$= \frac{8}{2}P_0V_0 = \frac{8}{2}\times\frac{RT_0}{3} = \frac{4}{3}RT_0$$

III – S

IV. $Q_{1\to2} = nC_p\Delta T_{1\to2} = nCp(T_f - T_i)$

$$= 1\times\frac{5}{2}R\left[\frac{P_0(2V_0)}{R} - \frac{P_0V_0}{R}\right][\because PV = nRT]$$

$$= \frac{5}{2}P_0V_0 = \frac{5}{2}\left(\frac{RT_0}{3}\right) = \frac{5RT_0}{6}$$

$\therefore$ IV – U

27. **(c)** **I.** $W_{1\to2\to3} = W_{1\to2} + W_{2-3}$

$$= nRT\ln\frac{V_f}{V_i} + Pdv$$

$$= 1R\frac{T_0}{3}\ell n\frac{2V_0}{V_0} + \text{zero}\ \left[\begin{array}{l}\because n = 1, \text{ and } W_{2\to3} \text{ isochroic}\\ \text{no change in volume, } \Delta V = 0\end{array}\right]$$

$$= \frac{RT_0}{3}\ln 2 \qquad \therefore \text{ I} - \text{P}$$

II. $\Delta U_{1\to2\to3} = \Delta U_{1\to2} + \Delta U_{2\to3}$

$$= O + nC_v\Delta T = n\frac{f}{2}R\Delta T\ [\text{Process } 1\to2 \text{ isothermic } \Delta T = 0]$$

$$= 1\times\frac{3}{2}R\left(T_0 - \frac{T_0}{3}\right) = RT_0\ [\text{For monoatomic gas f} = 3]$$

$\therefore$ II – R

III. $Q_{1\to2\to3} = W_{1\to2\to3} + \Delta U_{1\to2\to3}$
From first law of thermodynamics

$$= \frac{RT_0}{3}\ell n2 + RT_0 = \frac{RT_0}{3}[\ell n2 + 3]$$

$\therefore$ III – T

IV. $Q_1\to_2 = W_{1\to2} + \Delta U_{1\to2}$

$$= \frac{RT_0}{3}\ell n2 + 0 = \frac{RT_0}{3}\ell n2\ [\because \Delta U_{1\to2} = 0]$$

IV – P

28. **(c)** Process I is adiabatic therefore $\Delta Q = 0$
Process II is isobaric P = constant therefore $W = P(V_2 - V_1)$
$= 3P_0(3V_0 - V_0) = 6P_0V_0$
Process III is isochoric V = constant therefore
$W = P(V_2 - V_1) = 0$
Process IV is isothermal, temperature T = constant, $\therefore \Delta u = 0$

29. **(d)** Laplace's correction of the speed of sound in ideal gas is related to adiabatic process.
$P - V$ curve in adiabatic process is steeper than isothermal

30. **(b)** Work done in isochoric process is zero for which we get a vertical line in P-V graph, graph (*s*).

31. **(d)** $\Delta U = \Delta Q - P\Delta V$
$W = -P\Delta V = -P(V_2 - V_1) = -PV_2 + PV_1$
which is the formula for isobaric process, so graph (*P*)

32. **(a)** $W_{GE} = P_0(V_0 - 32V_0) = -31P_0V_0$
$W_{GH} = P_0(8V_0 - 32V_0) = -24P_0V_0$

$$(W_{FH})_{\text{adiabatic}} = \frac{P_fV_f - P_iV_i}{1-\gamma} = \frac{P_0(8V_0) - 32P_0(V_0)}{1-\frac{5}{3}} = 36P_0V_0$$

$$(W_{FG})_{\text{ isothermal}} = nRT\ln\left(\frac{V_f}{V_i}\right) = P_0V_0\ln\left(\frac{V_f}{V_i}\right)$$

$$= 1(32P_0V_0)\log_e\frac{32V_0}{V_0}$$

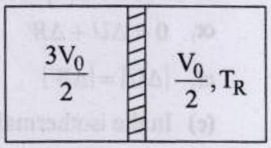
$$= 32P_0V_0\log_e 2^5 = 160P_0V_0\log_e 2$$

(a) is the correct option

33. **(a)** As $C_V = 2R \Rightarrow \frac{R}{\gamma - 1} = 2R \Rightarrow \gamma = \frac{3}{2}$

In adiabatic process, $TV^{\gamma-1}$ = cons

$$\Rightarrow\ T_0V_0^{1/2} = T_R\left(\frac{V_0}{2}\right)^{1/2}$$



$$\Rightarrow\ T_R = \sqrt{2}T_0$$

34. **(b)** $\because\ C_V = \frac{R}{\gamma - 1} = 2R$ or $\gamma - 1 = \frac{1}{2}$

$$\therefore\ \gamma = \frac{3}{2}$$

$T_0V_0^{\gamma-1} = T_R\left(\frac{V_0}{2}\right)^{\gamma-1}$ [$\because$ Final volume of right side cylinder is reduced to $\frac{V_0}{2}$]

$$\Rightarrow\ \frac{T_R}{T_0} = \sqrt{2}$$

From PV^γ = constant

$$P\left(\frac{V_0}{2}\right)^\gamma = P_0V_0^\gamma \Rightarrow P = P_0\times 2^{\frac{3}{2}}$$

$$\frac{PV}{T_L}=\frac{P_0V_0}{T_0}\Rightarrow T_L=2^{\frac{3}{2}}\times\frac{3}{2}T_0=3\sqrt{2}T_0$$

$$Q=nC_V\Delta T_1+nC_V\Delta T_2=2R(T_L-T_0)+2R(T_R-T_0)$$

$[\because\ n=1]$

$$=2R\times\left(3\sqrt{2}-1\right)T_0+2R\times\left(\sqrt{2}-1\right)T_0=4RT_0[2\sqrt{2}-1]$$

$$\therefore\ \frac{Q}{RT_0}=2\left(3\sqrt{2}-1\right)+2\left(\sqrt{2}-1\right)=8\sqrt{2}-4$$

$$=4\left(2\sqrt{2}-1\right)$$

35. (a) From $\Delta Q=ms\Delta\text{T}$

$$\Delta T=\frac{\Delta Q}{ms}=\frac{20000}{1\times 400}=50°C$$

(b) $\Delta T=V\gamma\Delta T=\left(\frac{1}{9000}\right)\left(9\times10^{-5}\right)(50)$

$=5\times10^{-7}\,\text{m}^3$

$\therefore\ W=p\,\Delta V=(10)^5(5\times10^{-7})=0.05\text{J}$

(c) $\Delta U=\Delta Q-W=\left(2000-0.05\right)J=19999.95J$

36. (a) In the given V-T graph, AB is a straight line

$\therefore\ \frac{V}{T}=$ constant (Isobaric process)

$$\therefore\ \frac{V_A}{T_A}=\frac{V_B}{T_B}$$

$$\Rightarrow\ T_B=\frac{V_B}{V_A}\times T_A=2\times300=600\,K\ \left[\because\frac{V_B}{V_A}=2\right]$$

(b) Heat absorbed or released

In process, A to B is a (isobaric process)

$$\therefore\ Q=nC_p\Delta T=2\times\frac{5}{2}R\times300=1500\,R$$

$\left[\because C_p=\frac{5}{2}R\text{ for monoatomic gas}\right]$

Heat is absorbed,

In process, B to C is an (isothermal process).

$\therefore$ Internal energy change $dU=0\ \because\ \Delta T=0$

$\therefore$ From first law of thermodynamics $dQ=dW$

$$\therefore\ Q=2.303\times nRT\log_{10}\frac{V_f}{V_i}$$

$=2.30.\times2\times R\times600\times\log_{10}2$

$=2763.6\times\log_{10}2\times R=831.8\,R$ (absorbed)

In process, C to D is a (isochoric process)

$dW=0\quad\because dV=0$

$$\therefore\ Q=nC_v\Delta T=n\left(\frac{3}{2}R\right)\left(T_A-T_B\right)$$

$=2\times\frac{3}{2}R\times(-300)=-900\,R$ (released).

In process, D to A (isothermal process)

$$\therefore\ Q=2.303\times nRT\log_{10}\frac{V_f}{V_i}$$

$$=2.303\times2\,R\times300\times\log\left(\frac{V_f}{V_i}\right)=-831.8\,R$$

Heat is released

(c) Total work done

$W=Q_{A\to B}+Q_{B\to C}+Q_{C\to D}+Q_{D\to A}$

$=(1500\,R+831.8\,R)-(900\,R+831.8\,R)=600\,R$

37. (a) Number of moles, $n=2$, $T_1=300\,\text{K}$

During the process $\text{A}\to\text{B}$

$pT=$ constant or $p^2V=$ constant $=K$ (say)

$$\therefore\ p=\frac{\sqrt{K}}{\sqrt{V}}$$

$$\therefore\ W_{A\to B}=\int_{V_A}^{V_B}p.\text{dV}=\int_{V_A}^{V_B}\frac{\sqrt{K}}{\sqrt{V}}dV$$

$=2\sqrt{K}\sqrt{V_B}-\sqrt{V_A}]$

$=2[\sqrt{KV_B}-\sqrt{KV_A}]$

$=2[\sqrt{\left(P_B^2V_B\right)V_B}-\sqrt{\left(P_A^2V_A\right)V_A}]$

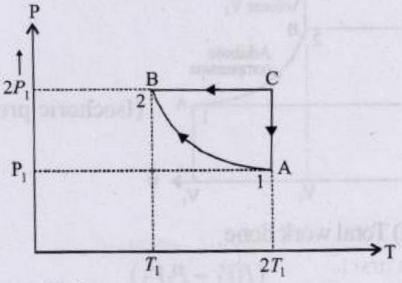
$=2\,[P_BV_B-P_AV_A]$

$=2\,[nRT_B-nRT_A]=2\,nR\,[T_1-2T_1]$

$=(2)(2)(R)\,[300-600]=-1200R$

$\therefore$ Work done on the gas in the process AB is $1200R$.

Alternate method



$pV=nRT$

$\therefore\ pdV+Vdp=nRdT$

or $pdV+\frac{(nRT)}{P}.dp=nRdT$ ...(i)

From the given condition

$pT=$ constant

$pdT+Tdp=0$ ...(ii)

From Eqs. (i) and (ii), we get

$pdV=2nRdT$

$$\therefore\ W_{A\to B}=\int pdV=2nR\int_{T_A}^{T_B}dT=2nR(T_B-T_A)$$

$=2nR\,(T_1-2T_1)=(2)\,(2)\,(R)\,(300-600)$

or $W_{A\to B}=-1200R$

(b) For PV^x = Const., Molar heat capacity

$$C=\frac{R}{\gamma-1}+\frac{R}{1-x}=\frac{R}{\frac{5}{3}-1}+\frac{R}{1-\frac{1}{2}}$$

Here P^2V = constant or $PV^{1/2}$ = constant

$$\therefore x=\frac{1}{2}\Rightarrow C=3.5R$$

$Q_{A\rightarrow B}=nC\,\Delta T=2(3.5\,R)\,(300-600)=-2100\,R$

Process B – C: Process is isobaric

$Q_{B\rightarrow C}=nC_p\,\Delta T=(2)\left(\frac{5}{2}R\right)\,(T_C-T_B)$

$=2\left(\frac{5}{2}R\right)(2T_1-T_1)=(5R)\,(600-300)=1500R$

Heat is absorbed

Process C – A: Process is isothermal

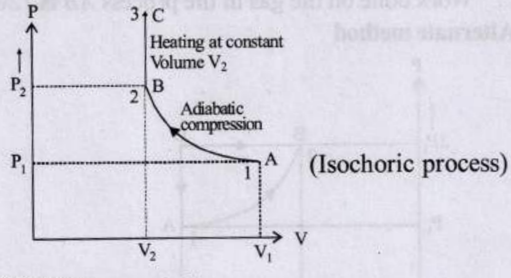
$\Delta T=0\ \therefore\ \Delta U=0$ So, $Q_{C\rightarrow A}=W_{C\rightarrow A}=nRT_C\ln\left(\frac{P_C}{P_A}\right)$

$=nR(2T_1)\ln\left(\frac{2P_1}{P_1}\right)=(2)(R)(600)\ln(2)=1200R\times0.6932$

$Q_{C\rightarrow A}=831.6\,R$ (absorbed)

38. n = no. of moles = 2,

(A) The complete process on P-V diagram is shown below.



(B) (i) Total work done

$$W=W_{AB}+W_{BC}=\frac{(P_1V_1-P_2V_2)}{(\gamma-1)}+0$$

$[\because W_{BC}=P\Delta V=P\times0=0]$

According to Poisson's law, $P_2=P_1\left(\frac{V_1}{V_2}\right)^{\gamma}$

$$\therefore\quad W=\frac{1}{\gamma-1}\left[P_1V_1-P_1\left(\frac{V_1}{V_2}\right)^{\gamma}V_2\right]$$

$$=\frac{1}{\gamma-1}\left[P_1V_1-P_1V_2.\frac{V_1}{V_2}.\left(\frac{V_1}{V_2}\right)^{\gamma-1}\right]\left(\because\gamma=\frac{5}{3}\right)$$

$$W=\frac{3}{2}\left[P_1V_1-P_1V_1\left(\frac{V_1}{V_2}\right)^{2/3}\right]=\frac{3}{2}P_1V_1\left[1-\left(\frac{V_1}{V_2}\right)^{2/3}\right]$$

(ii) Total change in internal energy. $\Delta U=\Delta U_{AB}+\Delta U_{BC}=Q-W$

$$=Q-\frac{3}{2}P_1V_1\left[1-\left(\frac{V_1}{V_2}\right)^{2/3}\right]$$

[according to first law of thermodynamics]

$$\begin{bmatrix}B\rightarrow C & Q=\Delta U_{BC}+0\\ A\rightarrow B & Q=\Delta U_{AB}+W\end{bmatrix}$$

(iii) For process BC : $\Delta U_{BC}=nC_v\Delta T=Q$ $[\because W_{BC}=0]$

For monoatomic gas $C_v=\frac{R}{\gamma-1}=\frac{3}{2}R,$

$$\therefore\quad \Delta U_{BC}=Q=2\times\frac{3R}{2}.\Delta T\Rightarrow\Delta T=\frac{Q}{3R}.$$

According to Poission's Law :

For the process AB, $T_AV_B^{\gamma-1}=T_BV_B^{\gamma-1}$

or $T_B=T_A\left(\frac{V_1}{V_2}\right)^{\gamma-1}=\frac{P_1V_1}{nR}\left(\frac{V_1}{V_2}\right)^{\gamma-1}$

$$\therefore\ T_B=\frac{P_1}{2R}.V_1^{\gamma}.V_2^{1-\gamma}=\frac{P_1V_1^{5/3}V_2^{-2/3}}{2R}$$

Hence, $T_C=T_B+\Delta T=\frac{P_1V_1^{5/3}V_2^{-2/3}}{2R}+\frac{Q}{3R}$

39. Number of mole, $n=1$, For monoatomic gas :

$$C_p=\frac{5R}{2},C_v=\frac{3R}{2}$$

(a) Work done by the gas, w = Area of ΔABC

$$W=\frac{1}{2}\times\text{base}\times\text{height}=\frac{1}{2}V_0\times2P_0=P_0V_0$$

(b) Heat rejected by the gas in the path CA i.e., isobaric process

$\Delta Q_{CA}=nC_p\Delta T=1\times(5R/2)\,(T_A-T_C)$

$$T_C=\frac{2P_0V_0}{I\times R},T_A=\frac{P_0V_0}{I\times R}$$

$$\Delta Q_{CA}=\frac{5R}{2}\left[\frac{P_0V_0}{R}-\frac{2P_0V_0}{R}\right]=-\frac{5}{2}P_0V_0$$

Heat absorbed by the gas in the path AB *i.e.*, isochoric process

$\Delta Q_{AB}=nC_v\Delta T=1\times(3R/2)\,(T_B-T_A)$

$$=\frac{3R}{2}\left[\frac{3P_0V_0}{1\times R}-\frac{P_0V_0}{1\times R}\right]=3P_0V_0$$

(c) As $\Delta U=0$ in cyclic process, hence, from first law of thermodynamics $\Delta Q=\Delta U+\Delta W$

$\therefore\ \Delta Q=\Delta W$

$\Delta Q_{AB}+\Delta Q_{CA}+\Delta Q_{BC}=\Delta W$

$$\Delta Q_{BC}=P_0V_0-\frac{P_0V_0}{2}=\frac{P_0V_0}{2}$$

(d) Equation for line BC $P=-\left[\frac{2P_0}{V_0}\right]V+5P_0$,

$(\because y=mx+c \text{ or}, P=-mV+C)$

$P=\frac{RT}{V}$ [for one mole]

$\therefore \quad RT=-\frac{2P_0}{V_0}V^2+5P_0V \qquad \text{...(i)}$

For maximum; $\frac{dT}{dV}=0, \; -\frac{2P_0}{V_0}\times 2V+5P_0=0;$

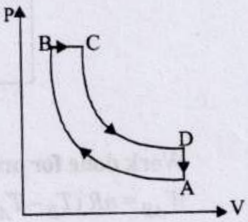
$\therefore \quad V=\frac{5V_0}{4} \qquad \text{...(ii)}$

Hence from equation (i) and (ii)

$$RT_{\max}=\frac{-2P_0}{V_0}\times\left(\frac{5V_0}{4}\right)^2+5P_0\left(\frac{5V_0}{4}\right)$$

$$=-2P_0V_0\times\frac{25}{16}+\frac{25P_0V_0}{4}=\frac{25}{8}P_0V_0 \;\therefore\; T_{\max}=\frac{25}{8}\frac{P_0V_0}{R}$$

40. Given $\frac{V_A}{V_B}=16, \frac{V_C}{V_B}=2$



$T_A=300\,K, T_B=?, T_D=?, \eta=?$

$n=1$, for diatomic gas, $\gamma=1.4$

For adiabatic process $A\to B$

$T_AV_A^{\gamma-1}=T_BV_B^{\gamma-1}$

or $T_B=\left(\frac{V_A}{V_B}\right)^{\gamma-1}T_A=(16)^{2/5}\times 300=909\,K$

$\therefore$ For isobaric process $B\to C$ Pressure = constant

$\frac{V_B}{T_B}=\frac{V_C}{T_C}$ or $T_C=T_B\left(\frac{V_C}{V_B}\right)=909\,[2]=1818\,K$

For adiabatic process $C\to D$:

$\because \frac{V_A}{V_B}=16$ and $\frac{V_C}{V_B}=2; \therefore \frac{V_A}{V_C}=8$

$T_CV_C^{\gamma-1}=T_DV_D^{\gamma-1}$

$$\therefore T_D=T_C\left[\frac{V_C}{V_D}\right]^{\gamma-1}=1818\left[\frac{1}{8}\right]^{2/5}=\frac{1818}{(64)^{1/5}}=791K$$

For $B\to C$ process : Heat absorbed $Q_1=nC_p\,(T_C-T_B)$

$$=n\frac{\gamma R}{\gamma-1}(T_C-T_B)=1\frac{(7/5)R}{(2/5)}(1818-909)$$

$$=\frac{7R}{2}\times 909\cong 3182\,R$$

For $D\to A$ process : Heat released

$$Q_2=nC_v\,(T_D-T_A)=n\frac{R}{\gamma-1}(T_D-T_A)$$

$$=1.\frac{R}{(2/5)}(791-300)=\frac{5R}{2}\times 491$$

($\because$ No heat is exchanged in adiabatic processes).

Now, $W_{AB}=-\frac{nR}{\gamma-1}(T_B-T_A)$

$$=-\frac{R}{(2/5)}(900-300)=-\frac{5R}{2}\times 609$$

$$W_{BC}=-nR\,(T_C-T_B)=1\times R\,(1818-909)=909\,R$$

$$W_{CD}=-\frac{nR}{\gamma-1}(T_C-T_D)=+\frac{R}{(2/5)}(1818-791)$$

$$=\frac{5R}{2}\times 1027$$

$$W_{\text{net}}=909\,R+\frac{5R}{2}\,(1027-609)=909\,R+\frac{5R}{2}\times 418$$

$$=909\,R+1045\,R=1954\,R$$

$\therefore$ Efficiency $=100\times(W_{\text{net}}/Q_1)=100\times\frac{1954\,R}{3182\,R}=61.4\%$

41. (i) $T_i=27+273=300\,K;\ \gamma=\frac{5}{3}$ (for monoatomic gas)

Since the gas expands adiabatically, so using

$T_iV_i^{\gamma-1}=T_fV_f^{\gamma-1}$

$$\Rightarrow T_f=T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1}=300\left[\frac{V}{2V}\right]^{5/3-1}=189\,K$$

(ii) Work done is adiabatic process

$$W=\frac{-nR(T_2-T_1)}{\gamma-1}=\frac{-2\times 8.31(189-300)}{5/3-1}$$

$$=\frac{+8.31\times 111\times 3}{2}=+2767\text{J}$$

According to first law of thermodynamics

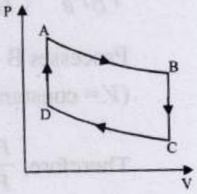
$\Delta Q=\Delta U+\Delta W$ But $\Delta Q=0$

(as the process is adiabatic)

$\therefore \quad \Delta U=-\Delta W=-2767$ J

(iii) Work done by the gas, $W=2767$ J

42. Given $T_A=1000$ K



$$P_B=\frac{2}{3}P_A;\ P_C=\frac{1}{3}P_A$$

No. of moles, $n=1$, $R=8.31$J mol^{-1} k^{-1}, $T_A=1000$ K

$\frac{C_P}{C_V}=\gamma=\frac{5}{3}$ (For mono atomic gas)

(i) W_{AB} (adiabatic process)

$$W_{AB}=\frac{nR[T_A-T_B]}{\gamma-1}$$

From $T^{\gamma} P^{1-\gamma}$ = constant

$$T_A^{\gamma} P_A^{1-\gamma} = T_B^{\gamma} P_B^{1-\gamma} \Rightarrow \left(\frac{P_A}{P_B}\right)^{\gamma-1} = \left(\frac{T_A}{T_B}\right)^{\gamma} \quad \text{... (i)}$$

$$\Rightarrow T_B = T_A\left[\frac{P_A}{P_B}\right]^{\frac{1-\gamma}{\gamma}} = 1000\left[\frac{3}{2}\right]^{\frac{1-5/3}{5/3}} = 850\,\text{K}$$

$$\therefore \quad W_{AB} = \frac{1 \times 8.31[1000-850]}{5/3-1} = 1870\,\text{J}$$

(ii) Heat lost in the process B → C isochoric process.

$Q_{BC} = nC_v\Delta T = nC_v(T_B - T_C)$

To find T_C, we use $\dfrac{P_B}{T_B} = \dfrac{P_C}{T_C}$ (volume V = constant)

$$\Rightarrow T_C = \frac{P_C}{P_B} \times T_B = \frac{1}{2} \times 850 = 425\,K \left[\because \frac{P_C}{P_A} = \frac{\frac{1}{3}P_A}{\frac{2}{3}P_A} = \frac{1}{2}\right]$$

$$\therefore \quad Q = 1 \times \frac{3}{2} \times 8.31\,[425-850] = -5298\,\text{J}$$

(iii) C → D and A → B are adiabatic processes.

$$\therefore P_C^{1-\gamma}T_C^{\gamma} = p_D^{1-\gamma}T_D^{\gamma} \Rightarrow \frac{P_C}{P_D} = \left(\frac{T_D}{T_C}\right)^{\frac{\gamma}{1-\gamma}} \quad \text{...(i)}$$

$$P_A^{1-\gamma}T_A^{\gamma} = p_B^{1-\gamma}T_B^{\gamma}$$

$$\Rightarrow \frac{P_A}{P_B} = \left(\frac{T_B}{T_A}\right)^{\frac{\gamma}{1-\gamma}} \quad \text{...(ii)}$$

Multiplying Eqs. (i) and (ii), we get

$$\frac{P_C P_A}{P_D P_B} = \left(\frac{T_D T_B}{T_C T_A}\right)^{\frac{\gamma}{1-\gamma}}$$

Processes B → C and D → A are isochoric.
(V = constant)

Therefore, $\dfrac{P_C}{P_B} = \dfrac{T_C}{T_B}$ and $\dfrac{P_A}{P_D} = \dfrac{T_A}{T_D}$

Multiplying these two equations, we get

$$\frac{P_C\,P_A}{P_B\,P_B} = \frac{T_C\,T_A}{T_B\,P_B} \quad \text{...(iv)}$$

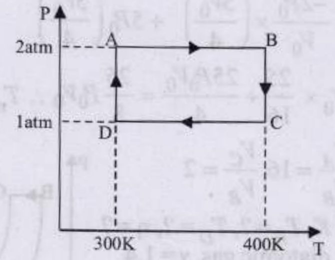
From Eqs. (iii) and (iv), we have

$$\left(\frac{T_D\,T_B}{T_C\,T_A}\right)^{\frac{\gamma}{1-\gamma}} = \left(\frac{T_C\,T_A}{T_B\,T_D}\right) \text{ or } \left(\frac{T_C\,T_A}{T_D\,T_B}\right)^{\frac{\gamma}{1-\gamma}} = \left(\frac{T_C\,T_A}{T_B\,T_D}\right)$$

$$\Rightarrow \frac{T_C\,T_A}{T_D\,T_B} = 1$$

$$\text{or} \quad T_D = \frac{T_C\,T_A}{T_B} = \frac{(425)(1000)}{850} \text{ or } T_D = 500\,\text{K}$$

43. For the cyclic process given



Work done for process A to B (Isobaric process)

$W_{AB} = nR\,(T_B - T_A)$

$= nR \times 100 = 2 \times 200 \times 8.32 = 1664\,\text{J}$

Work done for process C to D (Isobaric process)

$W_{CD} = -W_{AB} = -1664J$

$= -100nR = -200 \times 8.32 = -1664\,\text{J}$

Work done for process B to C (Isothermal process)

$$W_{BC} = 2.303nRT\log_{10}\frac{P_B}{P_C}$$

$$= 2.303nR \times 400\log_{10}\frac{2}{1} = 277.2\,nR$$

$= 554.4 \times 8.32 = 4612.6$

Work done for process D to A (Isothermal process)

$$W_{DA} = 2.303nRT\log_{10}\frac{P_D}{P_A} = 2.303nR \times 300\log_{10}\frac{1}{2}$$

$= -207.9nR$

$= -415.8 \times 8.32 = -3459.5\,\text{J}$

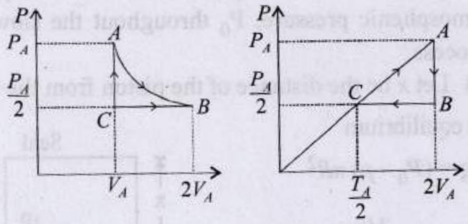
The total work done, $W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$
$= 1153\,\text{J}$

(a) From first law of thermodynamics $\Delta U = Q - W$
For complete cycle $\Delta U = 0$
$\therefore \quad Q = W = 1153\,\text{J}$

(b) Net work done $W = 1153\,\text{J}$

(c) $\Delta U = 0$. Since, the process is cyclic i.e., $\Delta T = 0$

44. (a) The p-V and p-T diagrams are shown below



$$(pV)_C = \frac{(pV)_A}{2} \Rightarrow \therefore T_C = \frac{T_A}{2}$$

(b) Process A-B T = constnat

$\therefore P \propto \frac{1}{V}$, V is doubled so, p will become half.

Also, $V_A = \frac{nRT_A}{P_A} = \frac{3RT_A}{P_A}$

$\Delta U_{AB} = 0$

$\therefore Q_{AB} = W_{AB} = nRT_A \ \text{In}\left(\frac{2V_A}{V_A}\right)$

$= 3RT_A \text{ in}(2) = 2.80RT_A$

Process B-C

$Q_{BC} = nC_P(T_C - T_B)$

$= (3)\left(\frac{7}{2}R\right)\left(\frac{T_A}{2} - T_A\right) = -\frac{21}{4}RT_A = -5.25RT_A$

Process C-A V = constant

$\therefore W_{CA} = 0$

or $Q_{CA} = \Delta U_{CA} = nC_V(T_A - T_C)$

$= (3)\left(\frac{5}{2}R\right)\left(T_A - \frac{T_A}{2}\right) = 3.75RT_A$

In a cyclic process,

$\Delta U = 0$

$\therefore Q_{net} = Q_{AB} + Q_{BC} + Q_{CA} = 0.58\,RT_A$

45. (i) For adiabatic process, $T.V.^{\gamma-1}$ = constant

$TV^{\gamma-1} = \frac{T}{2}(5.66V)^{\gamma-1} \Rightarrow 2 = (5.66)^{\gamma-1}$

Taking log on both sides, $\log 2 = (\gamma - 1)\log 5.66$

$\therefore \gamma = 1.4$

But $\gamma = 1 + \frac{2}{f} \Rightarrow 1.4 = 1 + \frac{2}{f}$

$\Rightarrow f = \frac{2}{0.4} = 5$ (Degree of freedom)

(ii) For adiabatic process now using PV^r = constant

$P_1V_1^\gamma = P_2V_2^\gamma$

$\Rightarrow P_2 = \frac{P}{(5.66)^{1.4}} = \frac{P}{11.32}$

Work done for adiabatic process

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{PV - \left(\frac{P}{11.32}\right)(5.66V)}{1.4 - 1} = 1.25\,PV$$

46. Applying ideal gas equation $PV = nRT$

$\therefore \ n = \frac{PV}{RT} = \frac{1.6 \times 10^6 \times 0.0083}{8.3 \times 300} = \frac{16}{3} = 5.33$ moles

Given: $C_p = \frac{5R}{2} \ \therefore \ C_v = \frac{3R}{2}$

2.49×10^4 J of heat energy is supplied at constant volume hence from

$Q = nC_v\Delta T$

$\therefore \ \Delta T = \frac{Q}{nC_v} = \frac{2.49 \times 10^4}{5.33 \times \frac{3}{2} \times 8.3} = 375\text{ K}$

$\therefore$ final temperature $= 300 + 375 = 675$ K

Now from Gay Lussac's law,

$\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow P_2 = \frac{P_1T_2}{T_1}$

$= \frac{1.6 \times 10^6 \times 675}{300} = 3.6 \times 10^6\text{ Nm}^{-2}$

47. In adiabatic process, work done

$W = \frac{1}{1-\gamma}[P_2V_2 - P_1V_1]$

Here, $P_1 = 10^5\text{ N/m}^2$, $V_1 = 6\,\ell = 6 \times 10^{-3}\text{ m}^3$

$P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma$, $V_2 = 2\,\ell = 2 \times 10^{-3}$

Given $C_v = \frac{3}{2}R \quad \therefore C_p = \frac{5}{2}R \ \ [\because C_p - C_v = R]$

$\therefore \ \gamma = \frac{C_p}{C_v} = 1.67$

$\therefore \ P_2 = 10^5\left[\frac{6}{2}\right]^{1.67} = 10^5 \times (3)^{1.67} = 6.26 \times 10^5\text{ N/m}^2$

$$\therefore\ W=\frac{1}{1-1.67}\left[6.26\times10^{5}\times2\times10^{-3}-10^{5}\times6\times10^{-3}\right]$$

$$W=\frac{1}{-0.67}[1252-600]=-\frac{652}{0.67}=-973.1\ \text{J}$$

Here negative sign indicates that the gas is compressed.

Topic-3: Carnot Engine, Refrigerators and Second Laws of Thermodynamics

1. **(d)** The entropy change of the body in the two cases is same as entropy is a state function.

2. **(c)**

Container A	**Container B**
Mass of gas = m_A	Mass of gas = m_B
Change in pressure = ΔP	Change in pressure = $1.5\Delta P$

$$P_AV=\frac{m_A}{M}RT \qquad P_BV=\frac{m_B}{M}RT$$

$$P'_A(2V)=\frac{m_A}{M}RT \qquad P'_B(2V)=\frac{m_B}{M}RT$$

$$\Rightarrow P_A-P'_A=\frac{m_ART}{MV}-\frac{m_ART}{M(2V)}$$

$$\Rightarrow \Delta P=\frac{m_ART}{2MV}\quad\text{... (i)}$$

$$\text{and}\ P_B-P'_B=\frac{m_BRT}{MV}-\frac{m_BRT}{M(2V)}$$

$$1.5\,\Delta P=\frac{m_BRT}{2MV}\quad\text{... (ii)}$$

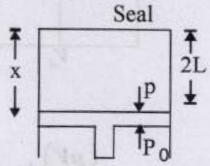
Dividing eq. (ii) by (i)

$$\frac{1.5\Delta P}{\Delta P}=\frac{m_B}{M_A}\Rightarrow\frac{3}{2}=\frac{m_B}{m_A}\Rightarrow 3m_A=2m_B$$

3. **(a)** As the cylinder is open at its bottom and has a smell hole at its tope, hence the pressure inside the cylinder is atmosphenic pressure, P_0 throughout the slow pulling process.

4. **(d)** Let x be the distance of the piston from the top.

At equilibrium



$$Mg=(P_0-p)\,\pi R^2$$

$$\Rightarrow p=\frac{-Mg}{\pi R^2}+P_0$$

Since the cylinder isothermally conducting

$\therefore$ temperature, T = constant

Applying $P_1V_1=P_2V_2$

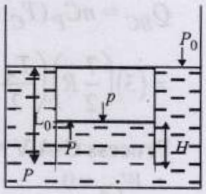
$$P_0\times(2L\times\pi R^2)=px\times\pi R^2\Rightarrow x=\frac{P_0}{p}\times2L$$

$$=\frac{P_0}{\left[P_0-\frac{Mg}{\pi R^2}\right]}\times2L=\left[\frac{P_0\times\pi R^2}{P_0\,\pi R^2-Mg}\right]\times2L$$

5. **(c)** At equilibrium, $p=P$

$$\Rightarrow p=P_0+(L_0-H)\rho g\quad\text{... (i)}$$

Also $P_0\times(\pi R^2L_0)=p\,[\pi R^2(L_0-H)]\quad(\because P_1V_1=P_2V_2)$



$$\Rightarrow p=\frac{L_0P_0}{L_0-H}$$

Substituting this value of E from (i) and (ii)

$$\frac{L_0P_0}{L_0-H}=P_0+(L_0-H)\rho g$$

$$\Rightarrow L_0P_0=P_0(L_0-H)+(L_0-H)^2\rho g$$

$$\Rightarrow \rho g(L_0-H)^2+P_0(L_0-H)-L_0P_0=0$$