Chapter

5

Ratio & Proportion

RATIO

Ratio is strictly a mathematical term to compare two similar quantities expressed in the same units.

The ratio of two terms 'x' and 'y' is denoted by x : y.

In general, the ratio of a number x to a number y is defined as the quotient of the numbers x and y.

COMPARISON OF TWO OR MORE RATIOS

Two or more ratios may be compared by reducing the equivalent fractions to a common denominator and then comparing the magnitudes of their numerator. Thus, suppose 2 : 5, 4 : 3 and 4 : 5 are three ratios to be compared then the fractions $\frac{2}{5}, \frac{4}{3}$ and $\frac{4}{5}$ are reduced to equivalent fractions with a common denominator. For this, the denominator of each is changed to 15 equal to the L.C.M. their denominators Hence the given ratios are expressed $\frac{6}{15}, \frac{20}{15}$ and $\frac{12}{15}$ or 2 : 5, 4 : 3, 4 : 5 according to magnitude.

REMEMBER

- ★ In the ratio of two quantities the two quantities must be of the same kind and in same unit.
- ★ The ratio is a pure number, i.e., without any unit of measurement.
- ★ The ratio would stay unaltered even if both the numerator and the denominator are multiplied or divided by the same number.

COMPOUND RATIO

Ratios are compounded by multiplying together the numerators for a new denominator and the denominator for a new denominator.

The compound ratio of a : b and c : d is $\frac{a \times c}{b \times d}$, i.e., ac : bd.

☞ Shortcut Approach

- The duplicate ratio of x : y is $x^2 : y^2$.
  The triplicate ratio of x : y is $x^3 : y^3$.

  The subduplicate ratio of x : y is $\sqrt{x} : \sqrt{y}$.

  The subtriplicate ratio of x : y is $\sqrt[3]{x} : \sqrt[3]{y}$.

  Reciprocal ratio of a : b is $\frac{1}{a} : \frac{1}{b}$ or b : a

- **Inverse ratio**
  Inverse ratio of x : y is y : x.

See Example : Refer ebook Solved Examples/Ch-5

PROPERTIES OF RATIOS

1. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$, i.e., the inverse ratios of two equal ratios are equal. The property is called **Invertendo**.

2. If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$, i.e., the ratio of antecedents and consequents of two equal ratios are equal. This property is called **Alternendo**.

3. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$. This property is called **Componendo**.

4. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$. This property is called **Dividendo**.

5. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. This property is called **Componendo and Dividendo.**

6. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ...$ Then,

   Each ratio $= \frac{\text{sum of Numerators}}{\text{sum of Denominators}}$

   i.e. $\frac{a}{b} = \frac{c}{d} = \frac{a+c+e+...}{b+d+f+...}$

If we have two equations containing three unknowns as

$a_1x + b_1y + c_1z = 0$ and ... (i)

$a_2x + b_2y + c_2z = 0$ ... (ii)

then, the values of x, y and z cannot be resolved without having a third equation.

However, in the absence of a third equation, we can find the ratio x : y : z.

This will be given by

$b_1c_2 - b_2c_1 : c_1a_2 - c_2a_1 : a_1b_2 - a_2b_1$.

Shortcut Approach

To divide a given quantity into a given ratio.

Suppose any given quantity a, is to be divided in the ratio m : n.

Let one part of the given quantity be x then the other part will be a – x.

$\therefore \quad \frac{x}{a-x} = \frac{m}{n}$ or $nx = ma - mx$ or $(m+n)x = ma$

$\therefore$ one part is $\frac{ma}{m+n}$ and the other part will be

$$a - \frac{ma}{m+n} = \frac{na}{m+n}$$

If A : B = a : b and B : C = m : n, then A : B : C = am : mb : nb and A : C = am : bn

If A : B = a : b, B : C = c : d and C : D = e : f, then A : B : C : D = ace : bce : bde : bdf

Two numbers are in ratio a : b and x is subtracted from the numbers, then the ratio becomes c : d. The two numbers will be $\frac{xa(d-c)}{ad-bc}$ and $\frac{xb(d-c)}{ad-bc}$, respectively.

See Example : Refer ebook Solved Examples/Ch-5

☞ Shortcut Approach

- In any 2-dimensional figures, if the corresponding sides are in the ratio x : y, then their areas are in the ratio $x^2 : y^2$.
- In any two 3-dimensional similar figures, if the corresponding sides are in the ratio x : y, then their volumes are in the ratio $x^3 : y^3$.

☞ Shortcut Approach

- If the ratio between two numbers is a : b and if each number is increased by x, the ratio becomes c : d. Then, two numbers are given as $\frac{xa(c-d)}{ad-bc}$ and $\frac{xb(c-d)}{ad-bc}$
- If the sum of two numbers is A and their difference is a, then the ratio of numbers is given by A + a : A – a.

☞ Shortcut Approach

- Let a vessel contains Q unit of mixture of ingredients A and B. From this, R unit of mixture is taken out and replaced by an equal amount of ingredient B only.
  If this process is repeated n times, then after n operations
  $$\frac{\text{Quantity of A left}}{\text{Quantity of A originally present}} = \left(1 - \frac{R}{Q}\right)^n$$
  and Quantity of B left = Q – Quantity of A Left
- In a container, milk and water are present in the ratio a : b. If x L of water is added to this mixture, the ratio becomes a : c. Then,
- Quantity of milk in original mixture $= \frac{ax}{c-b}$ L
  and quantity of water in original mixture $= \frac{bx}{c-b}$ L
- A container has milk and water in the ratio a : b, a second container has milk and water in the ratio c : d. If both the mixtures are emptied into a third container, then the ratio of milk of water in third container is given by

$$\left(\frac{a}{a+b}+\frac{c}{c+d}\right):\left(\frac{b}{a+b}+\frac{d}{c+d}\right)$$

See Example : Refer ebook Solved Examples/Ch-5

PROPORTION

When two ratios are equal, the four quantities composing them are said to be in proportion.

If $\frac{a}{b}=\frac{c}{d}$, then a, b, c, d are in proportions.

This is expressed by saying that 'a' is to 'b' as 'c' is to 'd' and the proportion is written as

$$a : b :: c : d \qquad \text{or} \qquad a : b = c : d$$

The terms a and d are called the extremes while the terms b and c are called the means.

REMEMBER

★ If four quantities are in proportion, the product of the extremes is equal to the product of the means.
Let a, b, c, d be in proportion, then
$\frac{a}{b}=\frac{c}{d} \Rightarrow ad = bc.$

★ If three quantities a, b and c are in continued proportion, then a : b = b : c
$\therefore\ ac = b^2$
b is called mean proportional.

DIRECT PROPORTION

If on the increase of one quantity, the other quantity increases to the same extent or on the decrease of one, the other decreases to the same extent, then we say that the given two quantities are directly proportional. If A and B are directly proportional then we denote it by $A \propto B$.

Some Examples :

1. Work done $\propto$ number of men
2. Cost $\propto$ number of Articles
3. Work $\propto$ wages
4. Working hour of a machine $\propto$ fuel consumed
5. Speed $\propto$ distance to be covered

INDIRECT PROPORTION (OR INVERSE PROPORTION)

If on the increase of one quantity, the other quantity decreases to the same extent or vice versa, then we say that the given two quantities are indirectly proportional. If A and B are indirectly proportional then we denote it by $A \propto \frac{1}{B}$.

Also, $A = \frac{k}{B}$ (k is a constant)

$\Rightarrow \quad AB = k$

If b_1, b_2 are the values of B corresponding to the values a_1, a_2 of A respectively, then

$$a_1 b_1 = a_2 b_2$$

Some Examples :

1. More men, less time
2. Less men, more time
3. More speed, less taken time to be covered distance

RULE OF THREE

In a problem on simple proportion, usually three terms are given and we have to find the fourth term, which we can solve by using Rule of three. In such problems, two of given terms are of same kind and the third term is of same kind as the required fourth term.

First of all we have to find whether given problem is a case of direct proportion or indirect proportion.

For this, write the given quantities under their respective headings and then mark the arrow in increasing direction. If both arrows are in same direction then the relation between them is direct otherwise it is indirect or inverse proportion. Proportion will be made by either head to tail or tail to head.

The complete procedure can be understand by the examples.

PARTNERSHIP

A partnership is an association of two or more persons who invest their money in order to carry on a certain business.

A partner who manages the business is called the **working partner** and the one who simply invests the money is called the **sleeping partner.**

Partnership is of two kinds :

(i) Simple (ii) Compound.

Simple partnership :

If the capitals is of the partners are invested for the same period, the partnership is called simple.

Compound partnership :

If the capitals of the partners are invested for different lengths of time, the partnership is called compound.

Shortcut Approach

- If the period of investment is the same for each partner, then the profit or loss is divided in the ratio of their investments.
  If A and B are partners in a business, then

$$\frac{\text{Investment of A}}{\text{Investment of B}} = \frac{\text{Profit of A}}{\text{Profit of B}} = \frac{\text{Loss of A}}{\text{Loss of B}}$$

- If A, B and C are partners in a business, then
  Investment of A : Investment of B : Investment of C
  = Profit of A : Profit of B : Profit of C, or
  = Loss of A : Loss of B : Loss of C

- When the amount of capital invested by different partners is same (say ₹ x) for differend time periods, $t_1, t_2, t_3, \ldots$, then
- Ratio of profit/loss = Ratio of time period for which the capital is invested

$$P_1 : P_2 : P_3 : \ldots = t_1 : t_2 : t_3 : \ldots$$

See Example : Refer ebook Solved Examples/Ch-5

MONTHLY EQUIVALENT INVESTMENT

It is the product of the capital invested and the period for which it is invested.
If the period of investment is different, then the profit or loss is divided in the ratio of their Monthly Equivalent Investment.

$$\frac{\text{Monthly Equivalent Investment of A}}{\text{Monthly Equivalent Investment of B}}$$

$$= \frac{\text{Profit of A}}{\text{Profit of B}} \text{ or } \frac{\text{Loss of A}}{\text{Loss of B}}$$

i.e., $$\frac{\text{Investment of A} \times \text{Period of Investment of A}}{\text{Investment of B} \times \text{Period of Investment of B}}$$

$$= \frac{\text{Profit of A}}{\text{Profit of B}} \text{ or } \frac{\text{Loss of A}}{\text{Loss of B}}$$

☞ Shortcut Approach

- If A, B and C are partners in a business, then
  Monthly Equivalent Investment of A : Monthly Equivalent Investment of B : Monthly Equivalent Investment of C
  = Profit of A : Profit of B : Profit of C.
  = Loss of A : Loss of B : Loss of C.
- When capital invested by the partners is givne as $X_1, X_2, X_3, \ldots$ for different time period $t_1, t_2, t_3, \ldots$. in a business, then
  Ratio of their profits $P_1 : P_2 : P_3 : \ldots = X_1t_1 : X_2t_2 : X_3t_3 : \ldots$
- If $P_1 : P_2 : P_3 :$ ---- is the ratio of profit and $t_1 : t_2 : t_3 : \ldots$ is the ratio of time $\frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3}$ ..... periods, then ratio of investments is given by

See Example : Refer ebook Solved Examples/Ch-5

MIXTURE

Simple Mixture : When two different ingredients are mixed together, it is known as a simple mixture.

Compound Mixture : When two or more simple mixtures are mixed together to form another mixture, it is known as a compound mixture.

Alligation : Alligation is nothing but a faster technique of solving problems based on the weighted average situation as applied to the case of two groups being mixed together.

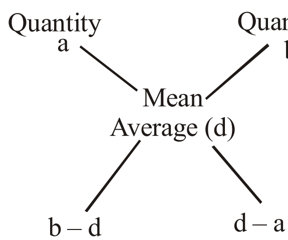
The word 'Alligation' literally means 'linking'.

ALLIGATION RULE

It states that when different quantities of the same or different ingredients of different costs are mixed together to produce a mixture of a mean cost, the ratio of their quantities is inversely proportional to the difference in their cost from the mean cost.

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{Price of Dearer} - \text{Mean Price}}{\text{Mean Price} - \text{Price of Cheaper}}$$

Graphical representation of Alligation Rule :



$$\frac{\text{Quantity of a}}{\text{Quantity of b}} = \frac{b-d}{d-a}$$

Applications of Alligation Rule :

(i) To find the mean value of a mixture when the prices of two or more ingredients, which are mixed together and the proportion in which they are mixed are given.

(ii) To find the proportion in which the ingredients at given prices must be mixed to produce a mixture at a given price.

☞ *Shortcut Approach*

Price of the Mixture :

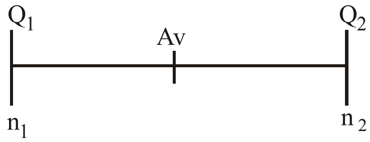
When quantities Q_i of ingredients M_i's with the cost C_i's are mixed then cost of the mixture C_m is given by

$$C_m = \frac{\sum C_i Q_i}{\sum Q_i}$$

See Example : Refer ebook Solved Examples/Ch-5

STRAIGHT LINE APPROACH OF ALLIGATION

Let Q_1 and Q_2 be the two quantities, and n_1 and n_2 are the number of elements present in the two quantities respectively,



where Av is the average of the new group formed then

n_1 corresponds to Q_2 – Av, n_2 corresponds to Av – Q_1 and $(n_1 + n_2)$ corresponds to $Q_2 - Q_1$.

Let us consider the previous example.

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