
CBSE Sample Paper -05 (solved)
SUMMATIVE ASSESSMENT -I
Class – X Mathematics

Time allowed: 3 hours

Maximum Marks: 90

General Instructions:

- a) All questions are compulsory.
- b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
- c) Questions 1 to 4 in section A are one mark questions. These are MCQs. Choose the correct option.
- d) Questions 5 to 10 in section B are two marks questions.
- e) Questions 11 to 20 in section C are three marks questions.
- f) Questions 21 to 31 in section D are four marks questions.
- g) There is no overall choice in the question paper. Use of calculators is not permitted.

SECTION - A

1. Explain why $7 \times 11 \times 13 + 13$ is a composite number.
2. Find whether the given system of equations has a unique solution, no solution or infinitely many solutions:
$$x + y = 3, \quad 2x + 5y = 12$$
3. Evaluate $\frac{\sin 18^\circ}{\cos 72^\circ}$.
4. Find the mode of the following data:
120, 110, 130, 110, 120, 140, 130, 120, 140, 120
5. The perimeters of two similar triangles are 30 cm and 20 cm. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.

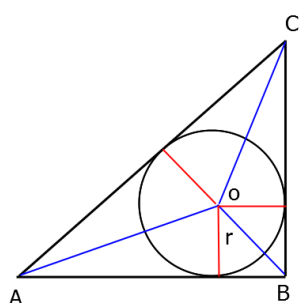
SECTION - B

6. Prove that the polynomial $x^2 + 2x + 5$ has no zero.
7. The areas of two similar triangles ABC and PQR are 64 cm^2 and 121 cm^2 respectively. If QR = 15.4 cm, find BC.
8. For any positive real number x, prove that there exists an irrational number y such that $0 < y < x$.
9. Given that $\sin(A + B) = \sin A \cos B + \cos A \sin B$, find the value of $\sin 75^\circ$.
10. Find the values of α and β for which the following system of linear equations has infinite number of solutions.

$$2x + 3y = 7, \quad 2\alpha x + (\alpha + \beta)y = 28$$

SECTION - C

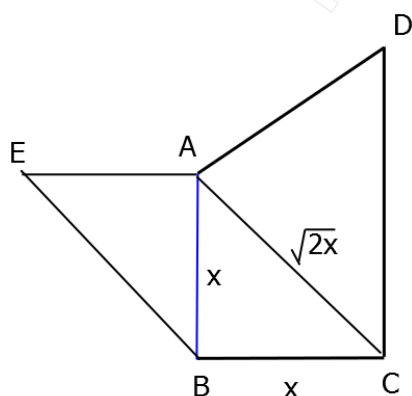
11. Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.
12. Find the condition that the zeros of the polynomial $f(x) = x^3 - px^2 + qx - r$ may be in arithmetic progression.
13. ABC is a right-angled triangle right angled at A. A circle is inscribed in it the lengths of two sides containing the right angle are 6 cm and 8 cm. Find the radius of the circle.



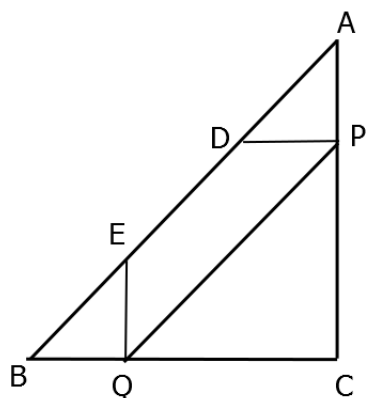
14. Find the four angles of a cyclic quadrilateral ABCD in which $\angle A = (2x - 5)^\circ$, $\angle B = (y + 5)^\circ$, $\angle C = (2y + 15)^\circ$ and $\angle D = (4x - 7)^\circ$.
15. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data.

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

16. In a $\triangle ABC$, right angled at B, if $AB = 4$ and $BC = 3$, find all the six trigonometric ratios of $\angle A$.
17. ABC is an isosceles triangle right-angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of $\triangle ABE$ and $\triangle ACD$.



18. I am 3 times as old as my son. 5 years later, I shall be two and a half times as old as my son. How old am I and how old is my son?
19. Prove $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$.
20. Let ABC be a triangle and D and E be two points on side AB such that AD = BE. If DP || BC and EQ || AC, then prove that PQ || AB.



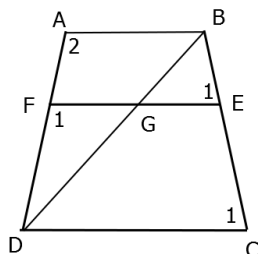
SECTION - D

21. The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6, then the denominator becomes 12 times the numerator. Determine the fraction.
22. If $\operatorname{cosec} A = 2$, find the value of $\frac{1}{\tan A} + \frac{\sin A}{1 + \cos A}$.
23. If $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$ and $x \sin \theta = y \cos \theta$, prove that $x^2 + y^2 = 1$.
24. A frequency distribution of the life times of 400 T.V. picture tubes tested in a company is given below. Find the average life of a tube.

Life time (in hours)	Frequency	Life time (in hours)	Frequency
300-399	14	800-899	62
400-499	46	900-999	48
500-599	58	1000-1099	22
600-699	76	1100-1199	6
700-799	68		

25. What must be added to $f(x) = 4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is divisible by $g(x) = x^2 + 2x - 3$?

26. In trapezium ABCD, $AB \parallel DC$ and $DC = 2AB$. A line EF drawn parallel to AB cuts AD in F and BC in E such that $\frac{BE}{EC} = \frac{3}{4}$. Diagonal DB intersects EF at G. Prove that $7FE = 10AB$.



27. Solve the following system of linear equations graphically.

$$x - y = 1$$

$$2x + y = 8$$

Shade the area bounded by these two lines and y-axis. Also, determine this area.

28. Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.
29. Following is the age distribution of a group of students. Draw the cumulative frequency polygon, cumulative frequency curve (less than type) and hence obtain the median value.

Age	Frequency	Age	Frequency
5-6	40	11-12	92
6-7	56	12-13	80
7-8	60	13-14	64
8-9	66	14-15	44
9-10	84	15-16	20
10-11	96	16-17	8

30. Prove $\frac{(1 + \cot A + \tan A)(\sin A - \cos A)}{\sec^3 A - \operatorname{cosec}^3 A} = \sin^2 A \cos^2 A$

31. In a housing society, people decided to do rainwater harvesting. Rainwater is collected in the underground tank at the rate of $30 \text{ cm}^3/\text{sec}$. Taking volume of water collected in x seconds as $y \text{ cm}^3$.
- Form a linear equation.
 - Write it in standard form as $ax + by + c = 0$.
 - Which values are promoted by the members of this society?

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ANSWERS

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SECTION - A

1. **Solution:**

$$7 \times 11 \times 13 + 13 = 1001 + 13 = 1014$$

$$1014 = 2 \times 3 \times 13 \times 13$$

i.e., it is the product of prime factors.

$\therefore 7 \times 11 \times 13 + 13$ is a composite number.

2. **Solution:**

Here, $a_1 = 1, a_2 = 2, b_1 = 1, b_2 = 5, c_1 = -3$ and $c_2 = -12$

We have,

$$\frac{a_1}{a_2} = \frac{1}{2} \quad \text{and} \quad \frac{b_1}{b_2} = \frac{1}{5}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

\therefore The system has a unique **solution**.

3. **Solution:**

$$\begin{aligned} \frac{\sin 18^\circ}{\cos 72^\circ} &= \frac{\sin 18^\circ}{\cos(90^\circ - 72^\circ)} && [\text{Using } \cos \theta = \sin(90^\circ - \theta)] \\ &= \frac{\sin 18^\circ}{\cos 18^\circ} \\ &= 1 \end{aligned}$$

4. **Solution:**

Let us first form the frequency table for the given data as given below:

Value (x_i)	110	120	130	140
Frequency (f_i)	2	4	2	2

We observe that the value 120 has the maximum frequency.

Thus, the mode is 120.

5. **Solution:**

Let $\triangle ABC$ and $\triangle DEF$ be two similar triangles of perimeters P_1 and P_2 respectively. Also, let $AB = 12$ cm, $P_1 = 30$ cm and $P_2 = 20$ cm. Then,

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{P_1}{P_2}$$

$$\Rightarrow \frac{AB}{DE} = \frac{P_1}{P_2}$$

$$\Rightarrow \frac{12}{DE} = \frac{30}{20} \quad \Rightarrow \quad DE = \frac{12 \times 20}{30} = 8 \text{ cm}$$

Thus, the corresponding side of the second triangle is 8 cm.

SECTION - B

6. **Solution:**

$$\begin{aligned}\text{Let } f(x) &= x^2 + 2x + 5 \\ &= x^2 + 2x + 1 + 4 \\ &= (x+1)^2 + 4\end{aligned}$$

Now, for every real value of x , $(x+1)^2 \geq 0$

$$\Rightarrow \text{For every real value of } x, (x+1)^2 + 4 \geq 4$$

\therefore For every real value of x , $f(x) \geq 4$ and hence it has no zero.

7. **Solution:**

Given that $\triangle ABC \sim \triangle PQR$

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\Rightarrow \frac{8 \times 15.4}{11} = 11.2 \text{ cm}$$

$$\Rightarrow BC = \frac{8}{11} = \frac{BC}{15.4}$$

8. **Solution:**

If x is irrational, then $y = \frac{x}{2}$ is also an irrational number such that $0 < y < x$.

If x is rational, then $\frac{x}{\sqrt{2}}$ is an irrational number such that $\frac{x}{\sqrt{2}} < x$ as $\sqrt{2} > 1$.

$\therefore y = \frac{x}{\sqrt{2}}$ is an irrational number such that $0 < y < x$.

9. **Solution:**

Putting $A = 45^\circ$ and $B = 30^\circ$ in $\sin(A+B) = \sin A \cos B + \cos A \sin B$, we get

$$\sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$\begin{aligned}\Rightarrow \sin 75^\circ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \\ &= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}\end{aligned}$$

10. Solution:

The given system of equations will have infinite number of solutions, if

$$\begin{aligned}\frac{2}{2\alpha} &= \frac{3}{\alpha+\beta} = \frac{7}{28} \\ \Rightarrow \frac{1}{\alpha} &= \frac{3}{\alpha+\beta} = \frac{1}{4} \\ \Rightarrow \frac{1}{\alpha} &= \frac{1}{4} \text{ and } \frac{3}{\alpha+\beta} = \frac{1}{4} \\ \Rightarrow \alpha &= 4 \text{ and } \alpha + \beta = 12 \\ \Rightarrow \alpha &= 4 \text{ and } \beta = 12 - 4 = 8\end{aligned}$$

SECTION - C

11. Solution:

It is given that on dividing 398 by the required number, there is a remainder of 7. This means that $398 - 7 = 391$ is exactly divisible by the required number. In other words, required number is a factor of 391.

Similarly, required positive integer is a factor of $436 - 11 = 425$ and $542 - 15 = 527$.

Clearly, required number is the HCF of 391, 425 and 527.

Using the factor tree, the prime factorisations of 391, 425 and 527 are as follows:

$$391 = 17 \times 23$$

$$425 = 5^2 \times 17$$

$$527 = 17 \times 31$$

\therefore HCF of 391, 425 and 527 is 17.

Thus, the required number is 17.

12. Solution:

Let $a - d$, a and $a + d$ be the zeros of the polynomial $f(x)$. Then,

$$\text{Sum of the zeros} = \frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\Rightarrow (a-d) + a + (a+d) = -\frac{(-p)}{1}$$

$$\Rightarrow 3a = p \quad \Rightarrow \quad a = \frac{p}{3}$$

Since a is a zero of the polynomial $f(x)$. Therefore,

$$f(a) = 0$$

$$\Rightarrow a^3 - pa^2 + qa - r = 0$$

$$\Rightarrow \left(\frac{p}{3}\right)^3 - p\left(\frac{p}{3}\right)^2 + q\left(\frac{p}{3}\right) - r = 0 \quad \left[\because a = \frac{p}{3} \right]$$

$$\Rightarrow p^3 - 3p^3 + 9pq - 27r = 0$$

$$\Rightarrow 2p^3 - 9pq + 27r = 0$$

13. Solution

Using Pythagoras theorem in $\triangle BAC$, we have

$$BC^2 = AB^2 + AC^2$$

$$\Rightarrow BC^2 = 6^2 + 8^2 = 100$$

$$\Rightarrow BC = 10 \text{ cm}$$

Now,

$$\text{Area of } \triangle ABC = \text{Area of } \triangle OAB + \text{Area of } \triangle OBC + \text{Area of } \triangle OCA$$

$$\Rightarrow \frac{1}{2} AB \times AC = \frac{1}{2} AB \times r + \frac{1}{2} BC \times r + \frac{1}{2} CA \times r$$

$$\Rightarrow \frac{1}{2} 6 \times 8 = \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r + \frac{1}{2} \times 8 \times r$$

$$\Rightarrow 48 = 24r$$

$$\Rightarrow r = 2 \text{ cm}$$

14. Solution:

We know that the sum of the opposite angles of a cyclic quadrilateral is 180° . In the cyclic quadrilateral ABCD, angles A and C and angles B and D form pairs of opposite angles.

$$\therefore \angle A + \angle C = 180^\circ \text{ and } \angle B + \angle D = 180^\circ$$

$$\Rightarrow 2x - 1 + 2y + 15 = 180^\circ \text{ and } y + 5 + 4x - 7 = 180^\circ$$

$$\Rightarrow 2x + 2y = 166^\circ \text{ and } 4x + y = 182^\circ$$

$$\Rightarrow x + y = 83^\circ \quad \dots(i)$$

$$\text{And, } 4x + y = 182^\circ \quad \dots(ii)$$

Subtracting equation (i) from equation (ii), we get

$$3x = 99 \quad \Rightarrow \quad x = 33$$

Substituting $x = 33$ in equation (i), we get $y = 50$

$$\text{Hence, } \angle A = (2x - 1)^\circ = (2 \times 33 - 1)^\circ = 65^\circ, \angle B = (y + 5)^\circ = (50 + 5)^\circ = 55^\circ$$

$$\angle C = (2y + 15)^\circ = (2 \times 50 + 15)^\circ = 115^\circ \text{ and } \angle D = (4x - 7)^\circ = (4 \times 33 - 7)^\circ = 125^\circ$$

15. **Solution:**

The modal class is 40-50, since it has the maximum frequency.

$$l = 40, f_1 = 20, f_0 = 12, f_2 = 11, h = 10$$

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 40 + \left(\frac{20 - 12}{2 \times 20 - 12 - 11} \right) \times 10 \\ &= 40 + \left(\frac{8}{17} \right) \times 10 \\ &= 40 + 4.71 \\ &= 44.71 \text{ cars} \end{aligned}$$

16. **Solution :**

We have $AB = 4$ and $BC = 3$

By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC = \sqrt{AB^2 + BC^2}$$

$$\Rightarrow AC = \sqrt{4^2 + 3^2}$$

$$\Rightarrow AC = \sqrt{25} = 5$$

When we consider the t-ratios of $\angle A$, we have

Base = $AB = 4$, Perpendicular = $BC = 3$ and Hypotenuse = $AC = 5$

$$\therefore \sin A = \frac{BC}{AC} = \frac{3}{5}, \cos A = \frac{AB}{AC} = \frac{4}{5}, \tan A = \frac{BC}{AB} = \frac{3}{4}$$

$$\operatorname{cosec} A = \frac{AC}{BC} = \frac{5}{3}, \sec A = \frac{AC}{AB} = \frac{5}{4} \text{ and } \cot A = \frac{AB}{BC} = \frac{4}{3}$$

17. **Solution:**

Let $AB = BC = x$.

It is given that $\triangle ABC$ is a right-angled at B.

$$\therefore AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = x^2 + x^2$$

$$\Rightarrow AC = \sqrt{2}x$$

It is given that

$$\triangle ABE \sim \triangle ACD$$

$$\begin{aligned}\Rightarrow \frac{\text{Area}(\triangle ABE)}{\text{Area}(\triangle ACD)} &= \frac{AB^2}{AC^2} \\ &= \frac{x^2}{(\sqrt{2}x)^2} \\ &= \frac{1}{2}\end{aligned}$$

18. **Solution**

Suppose my age is x years and my son's age is y years. Then,

$$x = 3y \quad \dots(i)$$

5 years later, my age will be $(x + 5)$ years and my son's age will be $(y + 5)$ years.

$$\therefore x + 5 = \frac{5}{2}(y + 5)$$

$$\Rightarrow 2x - 5y - 15 = 0 \quad \dots(ii)$$

Putting $x = 3y$ in equation (ii), we get

$$6y - 5y - 15 = 0 \quad \Rightarrow \quad y = 15$$

Putting $y = 15$ in equation (i), we get

$$x = 45$$

19. **Solution**

We have,

$$\begin{aligned}\text{LHS} &= \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} \\ &= \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta}}\end{aligned}$$

$$\begin{aligned}
&= \sqrt{\frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta}} \\
&= \sqrt{\frac{(1 - \sin \theta)^2}{\cos^2 \theta}} \\
&= \sqrt{\left(\frac{1 - \sin \theta}{\cos \theta}\right)^2} \\
&= \frac{1 - \sin \theta}{\cos \theta} \\
&= \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} = \sec \theta - \tan \theta = \text{RHS}
\end{aligned}$$

20. Solution

In $\triangle ABC$, we have

$DP \parallel BC$ and $EQ \parallel AC$

$$\therefore \frac{AD}{DB} = \frac{AP}{PC} \text{ and } \frac{BE}{EA} = \frac{BQ}{QC}$$

$$\Rightarrow \frac{AD}{DB} = \frac{AP}{PC} \text{ and } \frac{AD}{DB} = \frac{BQ}{QC} \quad [\because EA = ED + DA = ED + BE = BD, \therefore AD = BE]$$

$$\Rightarrow \frac{AP}{PC} = \frac{BQ}{QC}$$

\Rightarrow In a $\triangle ABC$, P and Q divide sides CA and CB respectively in the same ratio.

$\Rightarrow PQ \parallel AB$.

SECTION - D

21. Solution

Let the numerator and denominator of the fraction be x and y respectively.

Then,

$$\text{Fraction} = \frac{x}{y}$$

It is given that

$$\text{Denominator} = 2(\text{Numerator}) + 4$$

$$\Rightarrow y = 2x + 4$$

$$\Rightarrow 2x - y + 4 = 0$$

According to the given condition, we have

$$y - 6 = 12(x - 6)$$

$$\Rightarrow y - 6 = 12x - 72$$

$$\Rightarrow 12x - y - 66 = 0$$

Thus, we have the following system of equations

$$2x - y + 4 = 0 \quad \dots(i)$$

$$12x - y - 66 = 0 \quad \dots(ii)$$

Subtracting equation (i) from equation (ii), we get

$$10x - 70 = 0$$

$$\Rightarrow x = 7$$

Putting $x = 7$ in equation (i), we get

$$14 - y + 4 = 0$$

$$\Rightarrow y = 18$$

$$\text{Hence, required fraction} = \frac{7}{18}$$

22. **Solution:**

We have,

$$\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{2}{1}$$

So, we draw a right triangle, right angled at B such that

Perpendicular = BC = 1, Hypotenuse = AC = 2

By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow 2^2 = AB^2 + 1^2$$

$$\Rightarrow AB^2 = 4 - 1$$

$$\Rightarrow AB^2 = 3$$

$$\Rightarrow AB = \sqrt{3}$$

Now,

$$\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}, \sin A = \frac{BC}{AC} = \frac{1}{2} \text{ and } \cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\begin{aligned}
\therefore \frac{1}{\tan A} + \frac{\sin A}{1 + \cos A} &= \frac{1}{\frac{1}{\sqrt{3}}} + \frac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}} \\
&= \frac{\sqrt{3}}{1} + \frac{\frac{1}{2}}{\frac{2 + \sqrt{3}}{2}} \\
&= \frac{\sqrt{3}}{1} + \frac{1}{2 + \sqrt{3}} \\
&= \sqrt{3} + \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} \\
&= \sqrt{3} + \frac{2 - \sqrt{3}}{2^2 - (\sqrt{3})^2} \\
&= \sqrt{3} + \frac{2 - \sqrt{3}}{4 - 3} \\
&= \sqrt{3} + (2 - \sqrt{3}) = 2
\end{aligned}$$

23. **Solution:**

We have,

$$\begin{aligned}
&x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta \\
\Rightarrow &x \sin \theta (\sin^2 \theta) + y \cos \theta (\cos^2 \theta) = \sin \theta \cos \theta \\
\Rightarrow &x \sin \theta (\sin^2 \theta) + x \sin \theta (\cos^2 \theta) = \sin \theta \cos \theta \quad [\because x \sin \theta = y \cos \theta] \\
\Rightarrow &x \sin \theta (\sin^2 \theta + \cos^2 \theta) = \sin \theta \cos \theta \\
\Rightarrow &x \sin \theta = \sin \theta \cos \theta \\
\Rightarrow &x = \frac{\sin \theta \cos \theta}{\sin \theta} = \cos \theta
\end{aligned}$$

Now,

$$\begin{aligned}
&x \sin \theta = y \cos \theta \\
\Rightarrow &\cos \theta \sin \theta = y \cos \theta \quad [\because x = \cos \theta] \\
\Rightarrow &y = \frac{\cos \theta \sin \theta}{\cos \theta} = \sin \theta
\end{aligned}$$

$$\therefore x^2 + y^2 = \sin^2 \theta + \cos^2 \theta = 1$$

24. **Solution:**

Here, the class intervals are formed by exclusive method. If we make the series an inclusive, one of the mid-values remain same. So, there is no need to convert the series into an inclusive form.

Let the assumed mean be $A = 749.5$ and $h = 100$.

Calculation of mean

Life time (in hours)	Frequency f_i	Mid-values x_i	$d_i = x_i - A$ $= x_i - 749.5$	$u_i = \frac{x_i - A}{h}$ $u_i = \frac{x_i - 749.5}{100}$	$f_i u_i$
300-399	14	349.5	-400	-4	-56
400-499	46	449.5	-300	-3	-138
500-599	58	549.5	-200	-2	-116
600-699	76	649.5	-100	-1	-76
700-799	68	749.5	0	0	0
800-899	62	849.5	100	1	62
900-999	48	949.5	200	2	96
1000-1099	22	1049.5	300	3	66
1100-1199	6	1149.5	400	4	24
$N = \sum f_i = 400$					$\sum f_i u_i = -138$

We have $N = 400$, $A = 749.5$, $h = 100$ and $\sum f_i u_i = -138$

$$\begin{aligned}
 \therefore \bar{X} &= A + h \left\{ \frac{1}{N} \sum f_i u_i \right\} \\
 &= 749.5 + 100 \times \left(\frac{-138}{400} \right) \\
 &= 749.5 - \frac{138}{4} \\
 &= 749.5 - 34.5 = 715
 \end{aligned}$$

Thus, the average life time of a tube is 715 hours.

25. **Solution:**

By division algorithm, we have

$$f(x) = g(x) \times q(x) + r(x)$$

$$\Rightarrow f(x) - r(x) = g(x) \times q(x)$$

$$\Rightarrow f(x) + \{-r(x)\} = g(x) \times q(x)$$

Clearly, RHS is divisible by $g(x)$. Therefore, LHS is also divisible by $g(x)$. Thus, if we add $-r(x)$ to $f(x)$, then the resulting polynomial is divisible by $g(x)$. Let us now find the remainder when $f(x)$ is divided by $g(x)$.

$$\begin{array}{r}
 \overline{4x^2-6x+22} \\
 x^2+2x-3 \overline{) 4x^4+2x^3-2x^2+x-1} \\
 \underline{4x^4+8x^3-12x^2} \\
 -6x^3+10x^2+x-1 \\
 \underline{-6x^3-12x^2+18x} \\
 22x^2-17x-2 \\
 \underline{22x^2+44x-66} \\
 -61x+65
 \end{array}$$

$$\therefore r(x) = -61x + 65$$

Thus, we should add $-r(x) = 61x - 65$ to $f(x)$ so that the resulting polynomial is divisible by $g(x)$.

26. **Solution:**

In $\triangle DFG$ and $\triangle DAB$, we have

$$\angle 1 = \angle 2 \quad [\because AB \parallel DC \parallel EF, \therefore \angle 1 \text{ and } \angle 2 \text{ are corresponding angles}]$$

$$\angle FDG = \angle ADB \quad [\text{Common}]$$

So, by AA-criterion of similarity, we have

$$\triangle DFG \sim \triangle DAB \quad \Rightarrow \quad \frac{DF}{DA} = \frac{FG}{AB} \quad \dots(i)$$

In trapezium ABCD, we have

$$EF \parallel AB \parallel DC$$

$$\therefore \frac{AF}{DF} = \frac{BE}{EC}$$

$$\Rightarrow \frac{AF}{DF} = \frac{3}{4}$$

$$\left[\because \frac{BE}{EC} = \frac{3}{4} \text{ (Given)} \right]$$

$$\Rightarrow \frac{AF}{DF} + 1 = \frac{3}{4} + 1$$

[Adding 1 on both sides]

$$\Rightarrow \frac{AF + DF}{DF} = \frac{7}{4}$$

$$\Rightarrow \frac{AD}{DF} = \frac{7}{4} \quad \Rightarrow \quad \frac{DF}{AD} = \frac{4}{7} \quad \dots \text{(ii)}$$

From (i) and (ii), we get

$$\frac{FG}{AB} = \frac{4}{7} \quad \Rightarrow \quad FG = \frac{4}{7} AB \quad \dots \text{(iii)}$$

In $\triangle BEG$ and $\triangle BCD$, we have

$$\angle BEG = \angle BCD \quad \text{[Corresponding angles]}$$

$$\angle B = \angle B \quad \text{[Common]}$$

$$\therefore \triangle BEG \sim \triangle BCD \quad \text{[By AA-criterion of similarity]}$$

$$\Rightarrow \frac{BE}{EC} = \frac{EG}{CD}$$

$$\Rightarrow \frac{3}{7} = \frac{EG}{CD} \quad \left[\because \frac{BE}{EC} = \frac{3}{4} \Rightarrow \frac{EC}{BE} = \frac{4}{3} \Rightarrow \frac{EC}{BE} + 1 = \frac{4}{3} + 1 \Rightarrow \frac{BC}{BE} = \frac{7}{3} \right]$$

$$\Rightarrow EG = \frac{3}{7} CD = \frac{3}{7} \times 2AB = \frac{6}{7} AB \quad \dots \text{(iv)}$$

Adding (iii) and (iv), we get

$$FG + EG = \frac{4}{7} AB + \frac{6}{7} AB$$

$$\Rightarrow EF = \frac{10}{7} AB$$

$$\Rightarrow 7EF = 10AB$$

27. **Solution:**

We have,

$$x - y = 1$$

$$2x + y = 8$$

Graph of the equation $x - y = 1$:

We have,

$$x - y = 1 \Rightarrow y = x - 1 \text{ and } x = y + 1$$

Putting $x = 0$, we get $y = -1$

Putting $y = 0$, we get $x = 1$

Thus, we have the following table for the points on the line $x - y = 1$:

x	0	1
y	-1	0

Graph of the equation $2x + y = 8$:

We have,

$$2x + y = 8 \Rightarrow y = 8 - 2x \text{ and } x = \frac{8 - y}{2}$$

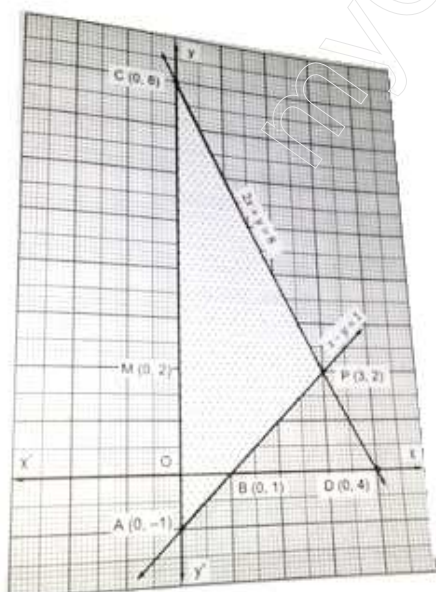
Putting $x = 0$, we get $y = 8$

Putting $y = 0$, we get $x = 4$

Thus, we have the following table for the points on the line $2x + y = 8$:

x	0	8
y	8	0

Plotting points $A(0, -1)$, $B(1, 0)$ on the graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $x - y = 1$.



Plotting points C(0, 8), D(4, 0) on the same graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $2x + y = 8$.

Clearly, the two lines intersect at P(3, 2). The area enclosed by the lines represented by the given equations and the y-axis is shaded.

Now, required area = Area of the shaded region

$$= \text{Area of } \triangle PAC$$

$$= \frac{1}{2}(\text{Base} \times \text{Height})$$

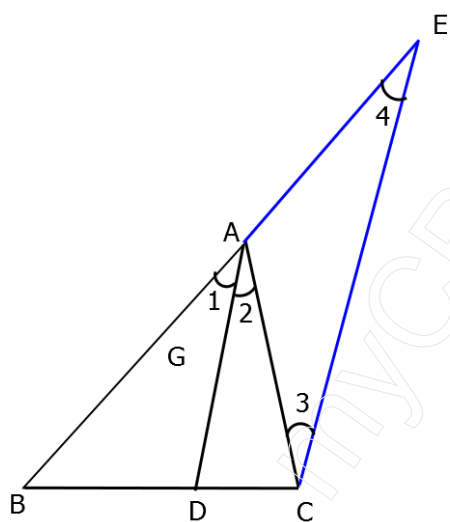
$$= \frac{1}{2}(AC \times PM) \quad [\because PM = x\text{-coordinate of } P = 3]$$

$$= \frac{1}{2}(9 \times 3) = 13.5 \text{ sq. units}$$

28. **Solution:**

Given: A $\triangle ABC$ in which AD is the internal bisector of $\angle A$ and meets BC in D.

To prove: $\frac{BD}{DC} = \frac{AB}{AC}$



Construction: Draw $CE \parallel DA$ to meet BA produced in E.

Proof: Since $CE \parallel DA$ and AC cuts them,

$$\therefore \angle 2 = \angle 3 \quad [\text{Alternate angles}] \quad \dots(i)$$

$$\text{And, } \angle 1 = \angle 4 \quad [\text{Corresponding angles}] \quad \dots(ii)$$

$$\text{But, } \angle 1 = \angle 2 \quad [\because AD \text{ is the bisector of } \angle A]$$

From (i) and (ii), we get

$$\angle 3 = \angle 4$$

Thus, in $\triangle ACE$, we have

$$\angle 3 = \angle 4$$

$$\Rightarrow AE = AC$$

[Sides opposite to equal angles are equal] ...(iii)

Now, in $\triangle BCE$, we have

$$DA \parallel CE$$

$$\Rightarrow \frac{BD}{DC} = \frac{BA}{AE}$$

[Using Basic Proportionality Theorem]

$$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$$

[$\because BA = AB$ and $AE = AC$ (From (iii))]

$$\text{Thus, } \frac{BD}{DC} = \frac{AB}{AC}$$

29. **Solution:**

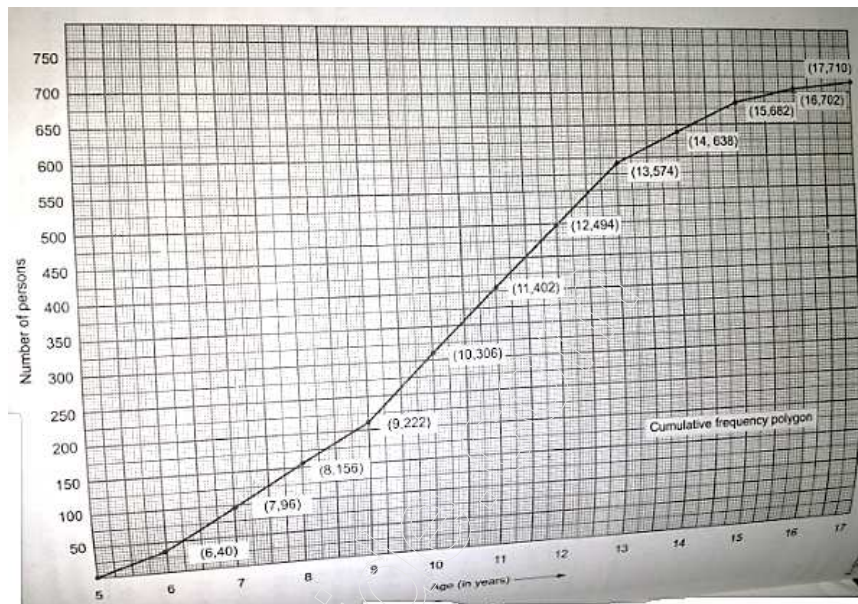
We first prepare the cumulative frequency table by less than method as given below:

Age	Frequency	Age less than	Cumulative frequency
5-6	10	6	40
6-7	56	7	96
7-8	60	8	156
8-9	66	9	222
9-10	84	10	306
10-11	96	11	402
11-12	92	12	494
12-13	80	13	574
13-14	64	14	638
14-15	44	15	682
15-16	20	16	702
16-17	8	17	710

Other than the given class intervals, we assume a class 4-5 before the first class-interval 5-6 with zero frequency.

Now, we mark the upper class limits (including the imagined class) along x-axis on a suitable scale and the cumulative frequencies along y-axis on a suitable scale.

Thus, we plot the points (5, 0), (6, 40), (7, 96), (8, 156), (9, 222), (10, 306), (11, 402), (12, 494), (13, 574), (14, 638), (15, 682), (16, 702) and (17, 710). These points are marked and joined by line segments to obtain the cumulative frequency polygon as shown in the figure.

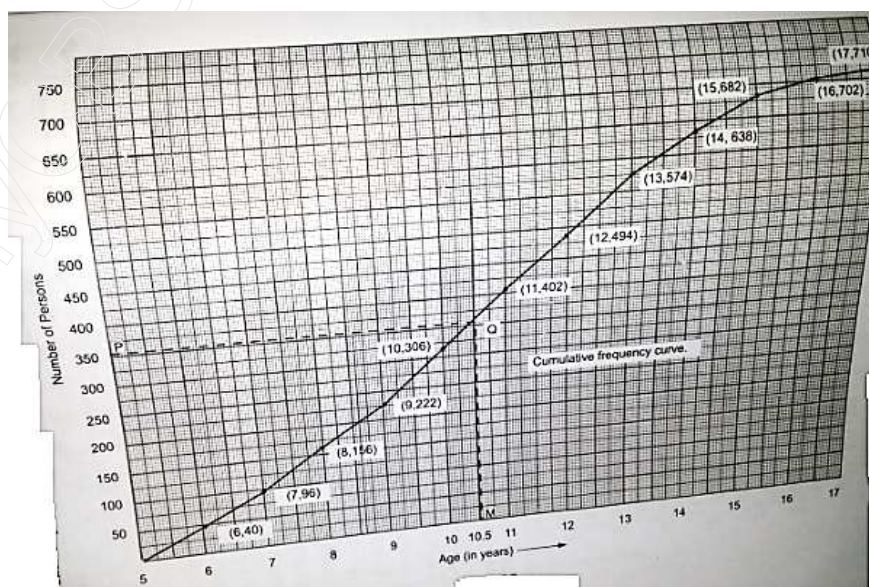


In order to obtain the cumulative frequency curve, we draw a smooth curve passing through the points discussed above.

The graph given below shows the total number of students as 710. The median is the age corresponding to

$$\frac{N}{2} = \frac{710}{2} = 355 \text{ students.}$$

In order to find the median, we first locate the point corresponding to 355th student on y-axis. Let the point be P. From this point, draw a line parallel to the x-axis cutting the curve at Q.



From this point Q, draw a line parallel to y-axis and meeting x-axis at the point M. The x-coordinate of M is 10.5.

Hence, median is 10.5.

30. **Solution:**

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{(1 + \cot A + \tan A)(\sin A - \cos A)}{\sec^3 A - \operatorname{cosec}^3 A} \\
 &= \frac{\left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A)}{\left(\frac{1}{\cos^3 A} - \frac{1}{\sin^3 A}\right)} \\
 &= \frac{\left(1 + \frac{\cos^2 A + \sin^2 A}{\sin A \cos A}\right)(\sin A - \cos A)}{\left(\frac{\sin^3 A - \cos^3 A}{\sin^3 A \cos^3 A}\right)} \\
 &= \frac{\sin A \cos A + \cos^2 A + \sin^2 A}{\sin A \cos A} \times \frac{\sin^3 A \cos^3 A}{\sin^3 A - \cos^3 A} (\sin A - \cos A) \\
 &= \frac{\sin A \cos A + 1}{\sin^3 A - \cos^3 A} \times (\sin^2 A \cos^2 A)(\sin A - \cos A) \\
 &= \frac{(\sin A \cos A + 1)(\sin^2 A \cos^2 A)(\sin A - \cos A)}{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)} \quad \left[\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)\right] \\
 &= \frac{(\sin A \cos A + 1)\sin^2 A \cos^2 A}{1 + \sin A \cos A} \\
 &= \sin^2 A \cos^2 A = \text{RHS}
 \end{aligned}$$

31. **Solution:**

Rate at which rainwater is collected in the tank = $30 \text{ cm}^3/\text{sec}$

Time for which water is collected = x seconds

Total amount of water collected = $y \text{ cm}^3$

- According to the given condition, linear equation formed is $y = 30x$
- The equation in standard form is $30x - y + 0 = 0$
- Values promoted by the members of the society are environmental protection and co-operation.