

Chapter 12
Heron's Formula
Exercise No. 12.1

Multiple Choice Questions:

1. An isosceles right triangle has area 8 cm^2 . The length of its hypotenuse is

- (A)** $\sqrt{32} \text{ cm}$
- (B)** $\sqrt{16} \text{ cm}$
- (C)** $\sqrt{48} \text{ cm}$
- (D)** $\sqrt{24} \text{ cm}$

Solution:

Given: An isosceles right triangle has area 8 cm^2 .

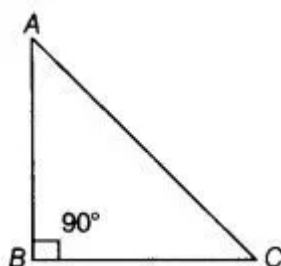
$$\text{Area of an isosceles right triangle} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$\text{So, } 8 = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$(\text{Base})^2 = 16 \quad [\text{Base} = \text{height, as triangle is an isosceles}]$$

$$\text{Base} = \sqrt{16}$$

$$\text{Base} = 4 \text{ cm}$$



See the triangle ABC, using Pythagoras theorem:

$$AC^2 + AB^2 + BC^2 = 4^2 + 4^2$$

$$= 16 + 16$$

$$AC^2 = 32$$

$$AC = \sqrt{32}$$

Therefore, the length of its hypotenuse is $\sqrt{32}$.

Hence, the correct option is (A).

2. The perimeter of an equilateral triangle is 60 m. The area is

- (A)** $10\sqrt{3} \text{ m}^2$
- (B)** $15\sqrt{3} \text{ m}^2$
- (C)** $20\sqrt{3} \text{ m}^2$
- (D)** $100\sqrt{3} \text{ m}^2$

Solution:

Given: The perimeter of an equilateral triangle is 60 m.

Suppose that each side of an equilateral be a.

$$a + a + a = 60\text{m}$$

$$3a = 60\text{m}$$

$$a = 20\text{m}$$

$$\begin{aligned}\text{Area of an equilateral triangle} &= \frac{\sqrt{3}}{4} \times (\text{Side})^2 \\ &= \frac{\sqrt{3}}{4} \times (20)^2 \\ &= 100\sqrt{3}\text{m}^2\end{aligned}$$

Therefore, the area of the triangle is $100\sqrt{3}\text{m}^2$.

3. The sides of a triangle are 56 cm, 60 cm and 52 cm long. Then the area of the triangle is

(A) 1322 cm^2

(B) 1311 cm^2

(C) 1344 cm^2

(D) 1392 cm^2

Solution:

The sides of a triangle are $a = 56\text{cm}$, $b = 60\text{cm}$ and $c = 52\text{cm}$.

So, semi-perimeter of a triangle will be:

$$\begin{aligned}s &= \frac{a + b + c}{2} \\ &= \frac{56 + 60 + 52}{2} \\ &= \frac{168}{2} \\ &= 84\text{cm}\end{aligned}$$

$$\begin{aligned}\text{Area of the triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \quad [\text{By heron's formula}] \\ &= \sqrt{84(84-56)(84-60)(84-52)} \\ &= \sqrt{84 \times 28 \times 24 \times 32} \\ &= \sqrt{4 \times 7 \times 3 \times 4 \times 7 \times 4 \times 2 \times 3 \times 4 \times 4 \times 2} \\ &= \sqrt{4^6 \times 7^2 \times 3^2} \\ &= 4^3 \times 7 \times 3 \\ &= 1344\text{cm}^2\end{aligned}$$

Hence, the correct option is (C).

4. The area of an equilateral triangle with side $2\sqrt{3}$ cm is

(A) 5.196 cm^2

(B) 0.866 cm^2

(C) 3.496 cm^2

(D) 1.732 cm^2

Solution:

Given: The side of an equilateral triangle is $2\sqrt{3}$ cm.

$$\begin{aligned}\text{Now, area of an equilateral triangle} &= \frac{\sqrt{3}}{4} (\text{Side})^2 \\ &= \frac{\sqrt{3}}{4} \times (2\sqrt{3})^2 \\ &= \frac{\sqrt{3}}{4} \times 4 \times 3 \\ &= 3\sqrt{3} \\ &= 3 \times 1.732 \\ &= 5.196 \text{ cm}^2\end{aligned}$$

Hence, the area of an equilateral triangle is 5.196 cm^2 .

Hence, the correct option is (A).

5. The length of each side of an equilateral triangle having an area of $9\sqrt{3} \text{ cm}^2$ is

(A) 8 cm

(B) 36 cm

(C) 4 cm

(D) 6 cm

Solution:

Given: area of an equilateral triangle = $9\sqrt{3} \text{ cm}^2$

$$\text{Area of an equilateral triangle} = \frac{\sqrt{3}}{4} \times (\text{Side})^2$$

$$\frac{\sqrt{3}}{4} \times (\text{Side})^2 = 9\sqrt{3}$$

$$(\text{Side})^2 = 9 \times 4$$

$$\text{Side} = \sqrt{9 \times 4}$$

$$\text{Side} = 3 \times 2$$

$$\text{Side} = 6 \text{ cm}$$

Therefore, the length of an equilateral triangle is 6 cm.

Hence, the correct option is (D).

6. If the area of an equilateral triangle is $16\sqrt{3} \text{ cm}^2$, then the perimeter of the triangle is

- (A) 48 cm**
- (B) 24 cm**
- (C) 12 cm**
- (D) 306 cm**

Solution:

Given: The area of an equilateral triangle is $16\sqrt{3} \text{ cm}^2$.

$$\text{Area of equilateral triangle} = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$16\sqrt{3} = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$\begin{aligned} (\text{Side})^2 &= \frac{16\sqrt{3} \times 4}{\sqrt{3}} \\ &= 64 \end{aligned}$$

$$\text{Side} = \sqrt{64}$$

$$\text{Side} = 8\text{cm}$$

Therefore, the perimeter of triangle $8 + 8 + 8 = 24\text{cm}$

Hence, the correct option is (B).

7. The sides of a triangle are 35 cm, 54 cm and 61 cm, respectively. The length of its longest altitude

- (A) $16\sqrt{5} \text{ cm}$**
- (B) $10\sqrt{5} \text{ cm}$**
- (C) $24\sqrt{5} \text{ cm}$**
- (D) 28 cm**

Solution:

Given: The sides of a triangle are $a = 35 \text{ cm}$, $b = 54 \text{ cm}$ and $c = 61 \text{ cm}$, respectively. So, semi-perimeter of a triangle is:

$$s = \frac{a+b+c}{2} = \frac{35+54+61}{2} = \frac{150}{2} = 75$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\begin{aligned}
&= \sqrt{75(75-35)(75-54)(75-61)} \\
&= \sqrt{75 \times 40 \times 21 \times 14} \\
&= \sqrt{5 \times 5 \times 3 \times 2 \times 2 \times 2 \times 5 \times 3 \times 7 \times 7 \times 2} \\
&= 5 \times 3 \times 2 \times 2 \times 7\sqrt{5} \\
&= 420\sqrt{5}
\end{aligned}$$

As know that,

$$\text{Area of triangle ABC} = \frac{1}{2} \times \text{Base} \times \text{Altitude}$$

$$\frac{1}{2} \times 35 \times \text{Altitude} = 420\sqrt{5}$$

$$\text{Altitude} = \frac{420\sqrt{5} \times 2}{35}$$

$$\text{Altitude} = 24\sqrt{5}$$

Therefore, the length of altitude is $24\sqrt{5}$.

Hence, the correct option is (C).

8. The area of an isosceles triangle having base 2 cm and the length of one of the equal sides 4 cm, is

(A) $\sqrt{15} \text{ cm}^2$

(B) $\sqrt{\frac{15}{2}} \text{ cm}^2$

(C) $2\sqrt{15} \text{ cm}^2$

(D) $4\sqrt{15} \text{ cm}^2$

Solution:

Given: The length of side be $a = 2\text{cm}$ and $b = 4\text{cm}$.

As we know that,

$$\begin{aligned}
\text{Area of an isosceles triangle} &= \frac{a}{4} \sqrt{4b^2 - a^2} \\
&= \frac{2\sqrt{4 \times (4)^2 - 2^2}}{4} \\
&= \frac{\sqrt{64 - 4}}{2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\sqrt{60}}{2} \\
&= \frac{2\sqrt{15}}{2} \\
&= \sqrt{15}\text{cm}^2
\end{aligned}$$

Hence, the correct option is (A).

9. The edges of a triangular board are 6 cm, 8 cm and 10 cm. The cost of painting it at the rate of 9 paise per cm² is

(A) Rs 2.00

(B) Rs 2.16

(C) Rs 2.48

(D) Rs 3.00

Solution:

Given: The edges of a triangular board are a=6 cm, b=8 cm and c=10 cm.

Now, semi-perimeter of a triangular board will be:

$$\begin{aligned}
s &= \frac{a+b+c}{2} \\
&= \frac{6+8+10}{2} \\
&= \frac{24}{2} \\
&= 12\text{cm}
\end{aligned}$$

Now, by Heron's formula:

$$\begin{aligned}
\text{Area of a triangle board} &= \sqrt{s(s-a)(s-b)(s-c)} \\
&= \sqrt{12(12-6)(12-8)(12-10)} \\
&= \sqrt{12 \times 6 \times 4 \times 2} \\
&= \sqrt{12^2 \times 2^2} \\
&= 12 \times 2 \\
&= 24\text{cm}^2
\end{aligned}$$

As, the cost of painting for area 1 cm² = Rs. 0.09

So, Cost of paint for area 24 cm² = 0.09 × 24 = Rs. 2.16

Therefore, the cost of a triangular board is Rs. 2.16.

Hence, the correct option is (B).

Exercise No. 12.2

Short Answer Questions with Reasoning:

Write True or False and justify your answer:

1. The area of a triangle with base 4 cm and height 6 cm is 24 cm².

Solution:

Given: The base and height of a triangle are 4 cm and 6 cm respectively.

$$\begin{aligned}\text{As we know that, area of a triangle} &= \frac{1}{2} \times \text{Base} \times \text{Height} \\ &= \frac{1}{2} \times 4 \times 6 \\ &= 12\text{cm}^2\end{aligned}$$

Hence, the given statement is false.

2. The area of $\triangle ABC$ is 8 cm² in which $AB = AC = 4$ cm and $\angle A = 90^\circ$.

Solution:

$$\begin{aligned}\text{Area of a triangle} &= \frac{1}{2} \times \text{Base} \times \text{Height} \\ &= \frac{1}{2} \times 4 \times 4 \\ &= 8\text{cm}^2\end{aligned}$$

Hence, the given statement is true.

3. The area of the isosceles triangle is $\frac{5}{4}\sqrt{11}$ cm², if the perimeter is 11 cm and the base is 5 cm.

Solution:

Suppose that side of isosceles triangle be a.

Now, perimeter of an isosceles triangle:

$$2s = 5 + a + a \quad [2s = a + b + c]$$

$$11 = 5 + 2a$$

$$2a = 11 - 5$$

$$2a = 6$$

$$a = 3$$

$$\text{Now, the formula of an area of isosceles triangle} = \frac{a}{4} \sqrt{4b^2 - a^2}$$

$$\begin{aligned}
 \text{So, area of an isosceles triangle} &= \frac{5\sqrt{4 \times (3)^2 - (5)^2}}{4} \\
 &= \frac{5\sqrt{4 \times 9 - 25}}{4} \\
 &= 5 \times \frac{\sqrt{36 - 25}}{4} \\
 &= \frac{5\sqrt{11}}{4} \text{ cm}^2
 \end{aligned}$$

Hence, the given statement is true.

4. The area of the equilateral triangle is $20\sqrt{3} \text{ cm}^2$ whose each side is 8 cm.

Solution:

Given, side of an equilateral triangle be 8 cm.

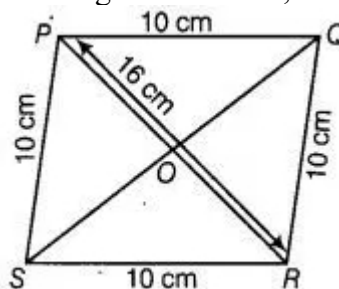
$$\begin{aligned}
 \text{Area of the equilateral triangle} &= \frac{\sqrt{3}}{4} (\text{Side})^2 \\
 &= \frac{\sqrt{3}}{4} \times (8)^2 \\
 &= \frac{64}{4} \sqrt{3} [\because \text{side} = 8 \text{ cm}] \\
 &= 16 \sqrt{3} \text{ cm}^2
 \end{aligned}$$

Hence, the given statement is false.

5. If the side of a rhombus is 10 cm and one diagonal is 16 cm, the area of the rhombus is 96 cm^2 .

Solution:

Let PQRS be the rhombus whose one diagonal is 16 cm, the area of the rhombus is 10 cm.



As we know that diagonal of a rhombus bisect each other at right angles. So,
 $OA = OC = 8 \text{ cm}$ and $OB = OD$

Now, in triangle AOB, $\angle AOB = 90^\circ$

So, $AB^2 = OA^2 + OB^2$ [By Pythagoras theorem]

$$AB^2 = OA^2 + OB^2$$

$$OB^2 = AB^2 - OA^2$$

$$= (10)^2 - 8^2$$

$$= 100 - 64$$

$$= 36$$

$$\text{So, } OB = \sqrt{36} = 6$$

Also,

$$OB = 2(OA) = 2 \times 6$$

$$= 12\text{cm}$$

Therefore, area of rhombus = $\frac{1}{2} \times \text{Products of diagonals}$

$$= \frac{1}{2} \times 16 \times 12$$

$$= 96\text{cm}^2$$

Hence, the given statement is true.

6. The base and the corresponding altitude of a parallelogram are 10 cm and 3.5 cm, respectively. The area of the parallelogram is 30 cm².

Solution:

Given, parallelogram in which base = 10 cm and altitude = 3.5 cm

Area of a parallelogram = Base x Altitude

$$= 10 \times 3.5$$

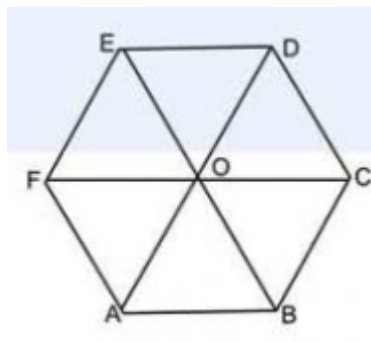
$$= 35\text{ cm}^2$$

Hence, the given statement is false.

7. The area of a regular hexagon of side 'a' is the sum of the areas of the five equilateral triangles with side a.

Solution:

Given: The side of a regular hexagon is 'a'.



As we know that the regular hexagon is divided into six equilateral triangles. So,

Area of regular hexagon = Sum of area of the six equilateral triangles.
Hence, the given statement is false.

8. The cost of levelling the ground in the form of a triangle having the sides 51 m, 37 m and 20 m at the rate of Rs 3 per m² is Rs 918.

Solution:

Given: The sides of the ground are $a = 51\text{m}$, $b = 37\text{cm}$, and $c = 20\text{cm}$. Now, the semi-parameter(s) of ground is:

$$2s = a + b + c$$

$$2s = 51\text{m} + 37\text{m} + 20\text{m}$$

$$2s = 108\text{m}$$

$$s = \frac{108\text{m}}{2}$$

$$s = 54\text{m}$$

$$\begin{aligned}\text{Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{54(54-51)(54-37)(54-20)} \\ &= \sqrt{54 \times 3 \times 17 \times 34} \\ &= \sqrt{3 \times 3 \times 3 \times 2 \times 3 \times 17 \times 17 \times 2} \\ &= 3 \times 3 \times 17 \times 2 \\ &= 306\text{m}^2\end{aligned}$$

The cost of levelling of 1 m² area is Rs. 3.

So, cost of levelling the ground of 306 m² area = Rs. 3×306 = Rs. 918

Hence, the given statement is true.

9. In a triangle, the sides are given as 11 cm, 12 cm and 13 cm. The length of the altitude is 10.25 cm corresponding to the side having length 12 cm.

Solution:

Given: The length of the altitude is 10.25. And in a triangle, the sides are $a=11\text{cm}$, $b=12\text{cm}$ and $c = 13\text{cm}$.

So, semi-perimeter(s) will be:

$$2s = a + b + c$$

$$2s = 11\text{cm} + 12\text{cm} + 13\text{cm}$$

$$2s = 36\text{cm}$$

$$s = \frac{36}{2}$$

$$s = 18\text{cm}$$

$$\begin{aligned}
 \text{So, area of triangle} &= \frac{2 \times \text{Area of } \Delta}{\text{Base}} \\
 &= \frac{2 \times 6\sqrt{105}}{12} \\
 &= \sqrt{105} \\
 &= 10.25
 \end{aligned}$$

Hence, the given statement is true.

Exercise No. 12.3

Short Answer Questions:

1 Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs 7 per m².

Solution:

Given: The sides of the ground are $a = 50\text{m}$, $b = 65\text{m}$, and $c = 65\text{m}$. Now, the semi-parameter(s) of the cost of levelling is:

$$2s = a + b + c$$

$$2s = 50\text{m} + 65\text{m} + 65\text{m}$$

$$2s = 180\text{m}$$

$$s = \frac{180\text{m}}{2}$$

$$s = 90\text{m}$$

$$\begin{aligned}\text{Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{90(90-50)(90-65)(90-65)} \\ &= \sqrt{90 \times 40 \times 25 \times 25} \\ &= 3 \times 2 \times 10 \times 25 \\ &= 6 \times 250 \\ &= 1500\text{m}^2\end{aligned}$$

The cost of laying grass 1 m² area is Rs. 7.

Therefore, the cost of levelling grass per 1500m² = Rs. 7×1500 = Rs. 10500

2 The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m and 15 m. The advertisements yield an earning of Rs 2000 per m² a year. A company hired one of its walls for 6 months. How much rent did it pay?

Solution:

Let the sides of a triangular walls are $a = 13\text{m}$, $b = 14\text{m}$ and $c = 15\text{m}$.

Now, the semi-perimeter of triangular side wall,

$$s = \frac{a+b+c}{2}$$

$$= \frac{13+14+15}{2}$$

$$= 21\text{m}$$

Now, area of triangular wall = $\sqrt{s(s-a)(s-b)(s-c)}$ [By Heron's formula]

$$\begin{aligned}
&= \sqrt{21(21-13)(21-14)(21-15)} \\
&= \sqrt{21 \times (21-13) \times (21-14) \times (21-15)} \\
&= \sqrt{21 \times 8 \times 7 \times 6} \\
&= \sqrt{21 \times 4 \times 2 \times 7 \times 3 \times 2} \\
&= \sqrt{21^2 \times 4^2} \\
&= 21 \times 4 \\
&= 84\text{m}^2
\end{aligned}$$

The advertisement yield earning per year for 1 m^2 area is Rs. 2000.

Therefore, advertisement yield earning per year on $84\text{ m}^2 = 2000 \times 84 = \text{Rs. } 168000$.

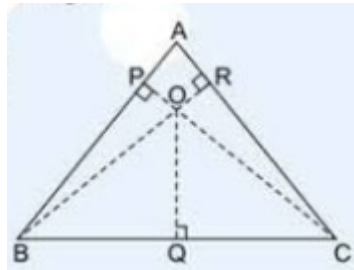
According to the question, the company hired one of its walls for 6 months, therefore company pay the rent $= \frac{1}{2} \times 168000 = \text{Rs. } 84000$.

Hence, the company paid rent Rs. 84000.

3 From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.

Solution:

Let ABC be an equilateral triangle, O be the interior point and $OP=14\text{cm}$, $OQ = 10\text{cm}$ and $OR = 6\text{cm}$. Also, sides of an equilateral triangle be $a\text{ m}$.



$$\begin{aligned}
\text{Area of triangle OAB} &= \frac{1}{2} \times AB \times OP \quad [\text{Area of a triangle} = \frac{1}{2} \times (\text{Base} \times \text{Height})] \\
&= \frac{1}{2} \times a \times 14 \\
&= 7a\text{cm}^2
\end{aligned}$$

$$\begin{aligned}
\text{Similarly, Area of triangle OBC} &= \frac{1}{2} \times BC \times OQ \\
&= \frac{1}{2} \times a \times 10 \\
&= 5a\text{cm}^2
\end{aligned}$$

$$\begin{aligned}
 \text{Again, area of triangle OAC} &= \frac{1}{2} \times AC \times OR \\
 &= \frac{1}{2} \times a \times 6 \\
 &= 3a\text{cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{See the given figure, area of equilateral triangle ABC} &= \text{Area of } (\triangle OAB + \triangle OBC + \triangle OAC) \\
 &= (7a + 5a + 3a)\text{cm}^2 \\
 &= 15a\text{cm}^2
 \end{aligned}$$

Now, semi-perimeter of triangle ABC is:

$$\begin{aligned}
 s &= \frac{a + a + a}{2} \\
 s &= \frac{3a}{2}\text{cm}
 \end{aligned}$$

$$\begin{aligned}
 \text{As, area of equilateral triangle ABC} &= \sqrt{s(s-a)(s-b)(s-c)} \quad [\text{By Heron's formula}] \\
 &= \sqrt{\frac{3a}{2} \left(\frac{3a}{2} - a \right) \left(\frac{3a}{2} - a \right) \left(\frac{3a}{2} - a \right)} \\
 &= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}} \\
 &= \frac{\sqrt{3}}{4} a^2 \dots (\text{II})
 \end{aligned}$$

According to the equation (I) and (II), get:

$$\begin{aligned}
 \frac{\sqrt{3}}{4} a^2 &= 15a \\
 a &= \frac{15 \times 4}{\sqrt{3}} \\
 a &= \frac{60}{\sqrt{3}} \\
 a &= 20\sqrt{3}
 \end{aligned}$$

Putting $a = 20\sqrt{3}$ in equation (II), get:

$$\begin{aligned}
 \text{Area of triangle ABC} &= \frac{\sqrt{3}}{4} \times (20\sqrt{3})^2 \\
 &= \frac{\sqrt{3}}{4} \times 400 \times 3 \\
 &= 300\sqrt{3}\text{cm}^2
 \end{aligned}$$

Hence, the area of an equilateral triangle is $300\sqrt{3}\text{cm}^2$.

4 The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to its base is 3 : 2. Find the area of the triangle.

Solution:

Given: Perimeter of triangle = 32cm

The ratio of the equal side to its base of an isosceles triangle is 3 : 2. Let sides of an isosceles triangle be $3x$, $3x$ and $2x$.

So, perimeter of the triangle = $3x + 3x + 2x = 8x$

$$32 = 8x$$

$$x = \frac{32}{8}$$

$$x = 4$$

Since, the sides of the isosceles triangle are $3 \times 4 = 12$, $3 \times 4 = 12$ and $2 \times 4 = 8\text{cm}$.

Now, semi-perimeter of triangle will be:

$$s = \frac{12 + 12 + 8}{2}$$

$$= \frac{32}{2}$$

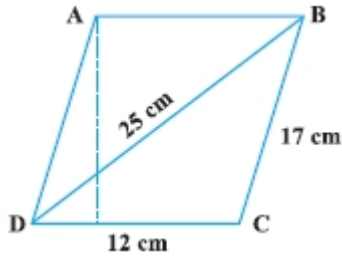
$$= 16\text{cm}$$

$$\begin{aligned}\text{Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{16(16-12)(16-12)(16-8)} \\ &= \sqrt{16 \times 4 \times 4 \times 8} \\ &= 4 \times 4 \times 2\sqrt{2}\text{cm}^2 \\ &= 32\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Therefore, the area of an isosceles triangle ABC} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{16(16-12)(16-12)(16-8)} \\ &= \sqrt{16 \times 4 \times 4 \times 8} \\ &= 32\sqrt{2}\text{cm}^2\end{aligned}$$

Therefore, the area of an isosceles triangle is $32\sqrt{2}\text{cm}^2$.

5 Find the area of a parallelogram given in Fig. Also find the length of the altitude from vertex A on the side DC.



Solution:

Let the sides of a triangle BCD are $a = 12\text{cm}$, $b = 17\text{ cm}$ and $c = 25\text{ cm}$ and altitude of a parallelogram is h .

Area of parallelogram, $ABCD = 2 \text{ (Area of triangle BCD) } \dots(\text{I})$

Now, semi-perimeter(s) of triangle BCD will be:

$$\begin{aligned} s &= \frac{a+b+c}{2} \\ &= \frac{12+17+25}{2} \\ &= \frac{54}{2} \\ &= 27\text{cm} \end{aligned}$$

$$\begin{aligned} \text{Area of triangle BCD} &= \sqrt{s(s-a)(s-b)(s-c)} \quad [\text{By heron's formula}] \\ &= \sqrt{27(27-12)(27-17)(27-25)} \\ &= \sqrt{27 \times 15 \times 10 \times 2} \\ &= \sqrt{9 \times 3 \times 3 \times 5 \times 5 \times 2 \times 2} \\ &= 3 \times 3 \times 5 \times 2\text{cm}^2 \\ &= 90\text{cm}^2 \end{aligned}$$

$$\begin{aligned} \text{So, area of parallelogram ABCD} &= 2 \times \text{Area of triangle BCD} \\ &= 2 \times 90\text{cm}^2 \\ &= 180\text{cm}^2 \quad \dots(\text{II}) \end{aligned}$$

As, Area of parallelogram $ABCD = \text{Base} \times \text{Altitude}$

$$180 = DC \times h$$

$$180 = 12 \times h$$

$$h = \frac{180}{12}$$

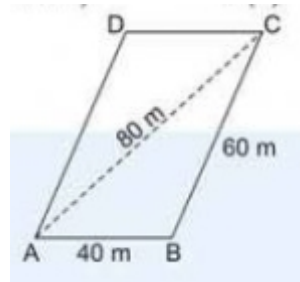
$$h = 15\text{cm}$$

Therefore, the area of parallelogram is 180 cm^2 and the length of altitude is 15 cm .

6 A field in the form of a parallelogram has sides 60 m and 40 m and one of its diagonals is 80 m long. Find the area of the parallelogram.

Solution:

Given: Let a field in the form of a parallelogram ABCD has sides 60 m and 40 m and one of its diagonals is 80 m long.



See the figure, in triangle ABC, let $a = 40\text{m}$, $b = 60\text{m}$ and $c = 80\text{m}$.

Now, semi perimeter(s) of triangle ABC:

$$\begin{aligned} s &= \frac{a+b+c}{2} \\ &= \frac{40+60+80}{2} \\ &= \frac{180}{2} \\ &= 90\text{m} \end{aligned}$$

$$\begin{aligned} \text{So, area of triangle ABC will be} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{90(90-40)(90-60)(90-80)} \\ &= \sqrt{90 \times 50 \times 30 \times 10} \\ &= \sqrt{3 \times 30 \times 5 \times 10 \times 30 \times 10} \\ &= 300\sqrt{15} \\ &= 1161.895\text{m}^2 \end{aligned}$$

Now, from equation (I),

$$\text{Area of parallelogram ABCD} = 2 \times 1161.895\text{m}^2 = 2323.79\text{m}^2.$$

Therefore, the area of parallelogram ABCD is 2323.79m^2 .

7 The perimeter of a triangular field is 420 m and its sides are in the ratio 6 : 7 : 8. Find the area of the triangular field.

Solution:

Given: The perimeter of a triangular field is 420 m and its sides are in the ratio 6 : 7 : 8.

According to the question, Let the sides in meters are $a = 6x$, $b = 7x$ and $c = 8x$.

So, perimeter of the triangle $= 6x + 7x + 8x$

$$420 = 21x$$

$$x = \frac{420}{21}$$

$$x = 20$$

Since, the sides of the triangular field are $a = 6 \times 20\text{cm} = 120\text{m}$, $b = 7 \times 20\text{m} = 140\text{m}$ and $c = 8 \times 20\text{m} = 160\text{m}$.

Now, semi-perimeter(s) of triangle will be:

$$\begin{aligned} s &= \frac{1}{2} \times 420\text{m} \\ &= 210\text{m} \end{aligned}$$

$$\begin{aligned} \text{Area of the triangle field} &= \sqrt{s(s-a)(s-b)(s-c)} \quad [\text{Using Heron's formula}] \\ &= \sqrt{210(210-120)(210-140)(210-160)} \\ &= \sqrt{210 \times 90 \times 70 \times 50} \\ &= 100\sqrt{7 \times 3 \times 3^2 \times 7 \times 5} \\ &= 100 \times 7 \times 3 \times \sqrt{15} \\ &= 2100\sqrt{15} \end{aligned}$$

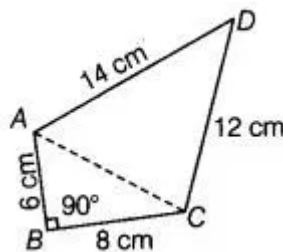
Therefore, the area of the triangular field is $2100\sqrt{15}$.

8 The sides of a quadrilateral ABCD are 6 cm, 8 cm, 12 cm and 14 cm (taken in order) respectively, and the angle between the first two sides is a right angle. Find its area.

Solution:

Given: The sides of a quadrilateral ABCD are AB = 6 cm, BC = 8 cm, and CD = 12 cm and DA = 14 cm.

Construction: Join AC.



In the right triangle ABC, whose angle B is right angle. So,

$$AC^2 = AB^2 + BC^2 \quad [\text{By Pythagoras theorem}]$$

$$AC^2 = 6^2 + 8^2$$

$$AC^2 = 36 + 64$$

$$AC = \sqrt{100}$$

$$AC = 10$$

Area of quadrilateral ABCD = Area of triangle ABC + Area of triangle ACD

$$\begin{aligned}\text{Now, area of triangle ABC} &= \frac{1}{2} \times AB \times AC \\ &= \frac{1}{2} \times 6 \times 8 \\ &= 24\text{cm}^2\end{aligned}$$

In triangle ACD, let AC = a = 10 cm, CD = b = 12 cm, and DA = c = 14 cm.

Now, semi-perimeter of triangle ACD will be:

$$\begin{aligned}s &= \frac{a+b+c}{2} \\ &= \frac{10+12+14}{2} \\ &= \frac{36}{2} \\ &= 18\text{cm}\end{aligned}$$

$$\begin{aligned}\text{So, area of triangle ACD} &= \sqrt{s(s-a)(s-b)(s-c)} \quad [\text{By heron's formula}] \\ &= \sqrt{18(18-10)(18-12)(18-14)} \\ &= \sqrt{18 \times 8 \times 6 \times 4} \\ &= \sqrt{(3)^2 \times 2 \times 4 \times 2 \times 3 \times 2 \times 4} \\ &= 3 \times 4 \times 2\sqrt{3 \times 2} \\ &= 24\sqrt{6}\text{cm}^2\end{aligned}$$

Hence, the area of the quadrilateral ABCD is $24\sqrt{6}\text{cm}^2$.

9 A rhombus shaped sheet with perimeter 40 cm and one diagonal 12 cm, is painted on both sides at the rate of Rs 5 per m². Find the cost of painting.

Solution:

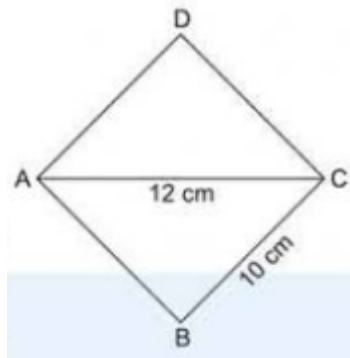
Given: One diagonal = 12 cm, Perimeter of rhombus = 40 cm

So,

$$4 \times \text{Side} = 40$$

$$\text{side} = \frac{40}{4}$$

$$\text{Side} = 10\text{cm}$$



In triangle ABC, let $a = 10$ cm, $b = 10$ cm, and $c = 12$ cm.

As we know that rhombus is also a parallelogram, so its diagonal divide it into two congruent triangles of equal area. So,

Area of rhombus = 2 (Area of triangle ABC)

Now, Semi-perimeter of triangle ABC will be:

$$\begin{aligned} s &= \frac{a+b+c}{2} \\ &= \frac{10+10+12}{2} \\ &= \frac{32}{2} \\ &= 16\text{cm} \end{aligned}$$

$$\begin{aligned} \text{So, area of triangle ABC} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{16(16-10)(16-10)(16-12)} \\ &= \sqrt{16 \times 6 \times 6 \times 4} \\ &= \sqrt{2304} \\ &= 48\text{cm}^2 \end{aligned}$$

Since, area of rhombus = 2 (Area of triangle ABC)

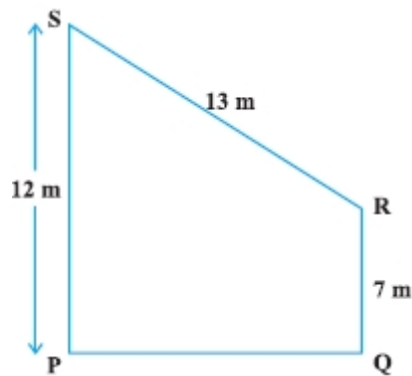
$$= 2 \times 48\text{cm}^2$$

$$= 96\text{cm}^2$$

The cost of painting of the sheet is Rs. 5 per m^2 .

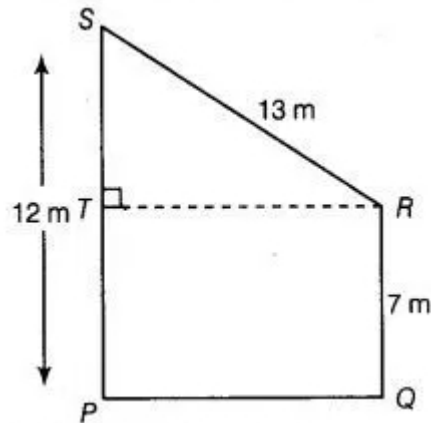
Therefore, cost of painting both sides of rhombus shaped sheet ABCD =
Rs. $(2 \times 5 \times 96) = \text{Rs. } 960$.

10 Find the area of the trapezium PQRS with height PQ given in Fig.



Solution:

Let PQRS is a trapezium, in which draw a line RT perpendicular to PS.



See the figure, $ST = PS - TP = 12 - 7 = 5\text{cm}$

So, in right triangle STR,

$$(SR)^2 = (ST)^2 + (TR)^2 \text{ [By using Pythagoras theorem]}$$

$$(13)^2 = (5)^2 + (TR)^2$$

$$(TR)^2 = 169 - 25$$

$$(TR)^2 = 144$$

$$TR = 12\text{m}$$

$$\text{Now, area of triangle STR} = \frac{1}{2} \times TR \times TS \text{ [area of triangle} = \frac{1}{2} \times \text{Base} \times \text{Height]}$$

$$= \frac{1}{2} \times 12 \times 5$$

$$= 30\text{m}^2$$

$$\text{As, area of rectangle PQRT} = PQ \times RQ = 12 \times 7 = 84\text{m}^2$$

$$\text{Now, area of trapezium} = \text{Area of DSTR} + \text{Area of rectangle PQRT}$$

$$= 30 + 84$$

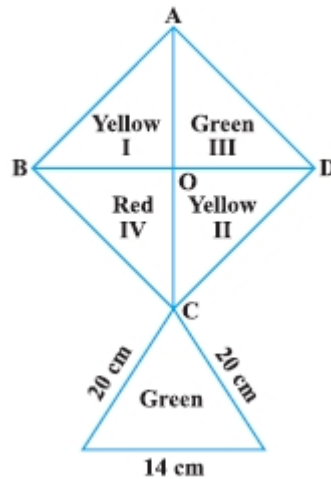
$$= 114\text{m}^2$$

Therefore, the area of trapezium is 114m^2 .

Exercise No. 12.4

Long Answer Questions:

1. How much paper of each shade is needed to make a kite given in Fig., in which ABCD is a square with diagonal 44 cm?



Solution:

Given: Diagonal of square ABCD = 44 cm

Also, ABCD is a square. So, AB = BC = CD = DA

Now, in triangle ACD,

$$AC^2 = AD^2 + DC^2$$

$$44^2 = AD^2 + AD^2$$

$$2AD^2 = 44 \times 44$$

$$2AD^2 = 22 \times 2 \times 22 \times 2$$

$$AD^2 = 22 \times 2 \times 22$$

$$AD = \sqrt{22 \times 2 \times 22}$$

$$AD = 22\sqrt{2}$$

Now, area of square ABCD = Side \times Side = $22\sqrt{2} \times 22\sqrt{2} = 968\text{cm}^2$

Since, area of square is divided into four parts.

Now, the area of paper of Red shade needed to make the kite is: $= \frac{1}{4} \times 968\text{cm}^2 = 242\text{cm}^2$

Also, area of green portion is:

$$= \frac{1}{4} \times 968\text{cm}^2$$

$$= 242\text{cm}^2$$

Similarly, area of yellow portion is:

$$= \frac{1}{2} \times 968 \text{ cm}^2 = 484 \text{ cm}^2$$

In triangle PCQ, Let PC = a = 20cm, CQ = b = 20cm, and PQ = c = 14cm.

Now, semi-perimeter of triangle PCQ will be:

$$\begin{aligned} s &= \frac{a+b+c}{2} \\ &= \frac{20+20+14}{2} \\ &= \frac{54}{2} \\ &= 27 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{So, area of triangle PCQ} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{27 \times (27-20) \times (27-20) \times (27-14)} \\ &= \sqrt{27 \times 7 \times 7 \times 13} \\ &= \sqrt{3 \times 3 \times 3 \times 7 \times 7 \times 13} \\ &= 21\sqrt{39} \\ &= 21 \times 6.24 \\ &= 131.04 \text{ cm}^2 \end{aligned}$$

Since, the total area of green portion = $242 \text{ cm}^2 + 131.04 \text{ cm}^2 = 373.04 \text{ cm}^2$

Therefore, the paper required for each shade to make a kite is red paper = 242 cm^2 , yellow paper = 484 cm^2 , and green paper = 373.04 cm^2 .

2. The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

Solution:

Given: the perimeter of a triangle is 50 cm.

Now, semi-perimeter(s) of the triangle is $= \frac{\text{Perimeter of triangle}}{2} = \frac{50}{2} = 25$

Suppose that the smaller side of the triangle be a = x cm. So, the second side will be b = (x+4) cm and 3rd side will be c = (2x-6)cm.

Now, perimeter of triangle = a + b + c = x + (x+4) + (2x-6)

$$50 \text{ cm} = (4x - 2) \text{ cm}$$

$$50 = 4x - 2$$

$$4x = 50 + 2$$

$$4x = 52$$

$$x = \frac{52}{4}$$

$$x = 13$$

Since, the three side of the triangle are:

$$a = x = 13,$$

$$b = x + 4 = 13 + 4 = 17$$

$$c = 2x - 6 = 2 \times 13 - 6 = 26 - 6 = 20.$$

$$\begin{aligned}\text{So, area of the triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{25 \times (25-13) \times (25-17) \times (25-20)} \\ &= \sqrt{25 \times 12 \times 8 \times 5} \\ &= \sqrt{5 \times 5 \times 4 \times 3 \times 4 \times 2 \times 5} \\ &= 5 \times 4 \times 2 \sqrt{30} \text{ cm}^2 \\ &= 20\sqrt{30} \text{ cm}^2\end{aligned}$$

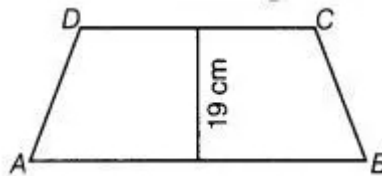
Therefore, the area of triangle is $20\sqrt{30} \text{ cm}^2$.

3. The area of a trapezium is 475 cm^2 and the height is 19 cm. Find the lengths of its two parallel sides if one side is 4 cm greater than the other.

Solution:

Given:

Area of a trapezium = 475 cm^2 and Height = 19 cm.



According to the question, let one sides of trapezium is x . So, another side will be $x + 4$.

Now, Area of trapezium = $\frac{1}{2} \times (\text{Sum of the parallel sides}) \times \text{Height}$

$$475 = \frac{1}{2} \times (x + x + 4) \times 19 \text{ cm}$$

$$2x + 4 = \frac{950}{19}$$

$$= 50$$

$$2x = 50 - 4$$

$$2x = 46$$

$$x = 23$$

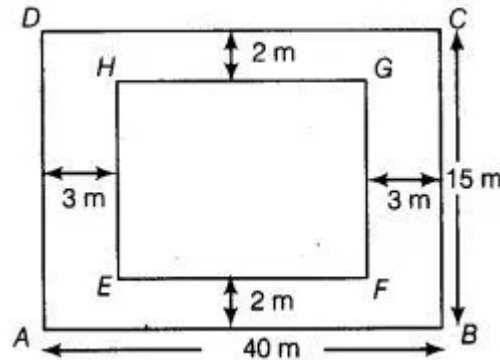
Therefore, the length of the parallel side of trapezium are $x = 23 \text{ cm}$ and $x + 4 = 23 + 4 = 27 \text{ cm}$.

4. A rectangular plot is given for constructing a house, having a measurement of 40 m long and 15 m in the front. According to the laws, a

minimum of 3 m, wide space should be left in the front and back each and 2 m wide space on each of other sides. Find the largest area where house can be constructed.

Solution:

Given: Let a rectangular plot ABCD is constructing a house, having a measurement of 40 m long and 15 m in the front.



According to the question,

Length of inner-rectangle (EF) = $40 - 3 - 3 = 34\text{m}$

And breadth of inner rectangle (FG) = $15 - 2 - 2 = 11\text{m}$

$$\begin{aligned}\text{Now, area of inner rectangle (EFGH) will be} &= \text{Length} \times \text{Breadth} \\ &= EF \times FG \\ &= 34 \times 11\text{m}^2 \\ &= 374\text{m}^2\end{aligned}$$

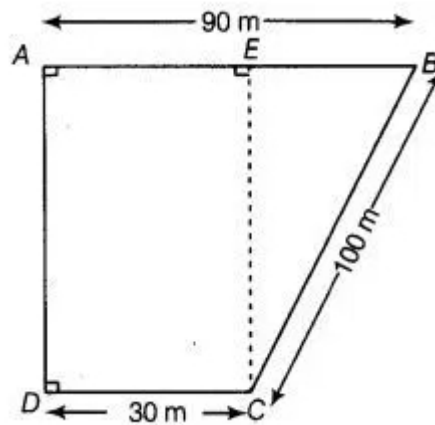
Therefore, the largest area where house can be constructed = 374m^2 .

5. A field is in the shape of a trapezium having parallel sides 90 m and 30 m. These sides meet the third side at right angles. The length of the fourth side is 100 m. If it costs Rs 4 to plough 1m^2 of the field, find the total cost of ploughing the field.

Solution:

Given: In the trapezium ABCD, the two parallel sides are $AB = 90\text{ m}$, $CD = 30\text{ m}$, and $EC \perp AB$.

So, $EB = AB - EA = 90\text{ m} - 30\text{ m} = 60\text{m}$



Now, in triangle BEC,

$$(BC)^2 = (BE)^2 + (EC)^2$$

$$100^2 = 60^2 + (EC)^2$$

$$(EC)^2 = 10000 - 3600$$

$$(EC)^2 = 6400$$

$$EC = \sqrt{6400}$$

$$EC = 80m$$

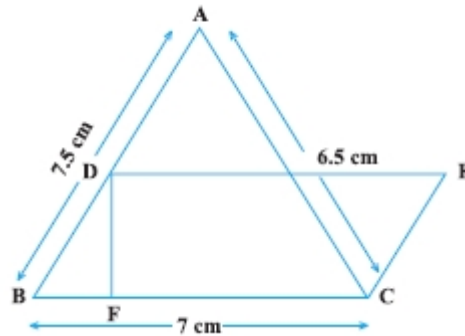
$$\begin{aligned}
 \text{Now, area of trapezium } ABCD &= \frac{1}{2} \times (\text{Sum of parallel sides}) \times (\text{Distance between parallel sides}) \\
 &= \frac{1}{2} \times (AB + DC) \times EC \\
 &= \frac{1}{2} \times (90 + 30) \times 80 \\
 &= \frac{1}{2} \times 120 \times 80 \\
 &= 4800m^2
 \end{aligned}$$

The cost of ploughing the field of $1m^2$ is Rs. 4.

Now, The cost of ploughing the field of $4800m^2$ area = $4800 \times \text{Rs. } 4 = \text{Rs. } 19200$.

Therefore, the total cost of ploughing the field is Rs. 19200.

6. In Fig., $\triangle ABC$ has sides $AB = 7.5$ cm, $AC = 6.5$ cm and $BC = 7$ cm. On base BC a parallelogram $DBCE$ of same area as that of $\triangle ABC$ is constructed. Find the height DF of the parallelogram.



Solution:

Given: in triangle ABC, the sides are $AB = a = 7.5$ cm, $BC = b = 7$ cm, and $CA = c = 6.5$ cm.
Now, semi-perimeter of a triangle will be:

$$\begin{aligned} s &= \frac{a+b+c}{2} \\ &= \frac{7.5+7+6.5}{2} \\ &= \frac{21}{2} \\ &= 10.5 \end{aligned}$$

$$\begin{aligned} \text{So, area of triangle ABC} &= \sqrt{s(s-a)(s-b)(s-c)} \quad [\text{By heron's formula}] \\ &= \sqrt{10.5 \times (10.5 - 7.5)(10.5 - 7)(10.5 - 6.5)} \\ &= \sqrt{10.5 \times 3 \times 3.5 \times 4} \\ &= \sqrt{441} \\ &= 21 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Also, the area of parallelogram BCED will be} &= \text{Base} \times \text{Height} \\ &= BC \times DF \\ &= 7 \times DF \end{aligned}$$

Now, according to the question,

$$\text{Area of triangle ABC} = \text{Area of parallelogram BCED}$$

$$21 = 7 \times DF$$

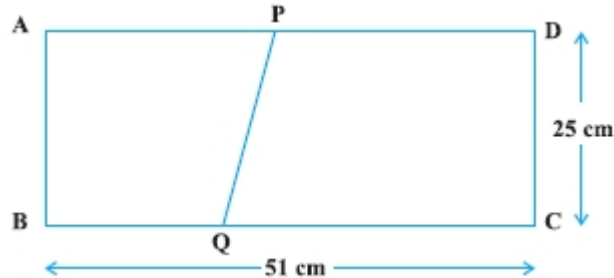
$$DF = \frac{21}{7}$$

$$DF = 3 \text{ cm}$$

Hence, the height of parallelogram BCED is 3 cm.

7. The dimensions of a rectangle ABCD are 51 cm × 25 cm. A trapezium PQCD with its parallel sides QC and PD in the ratio 9 : 8, is cut off from the

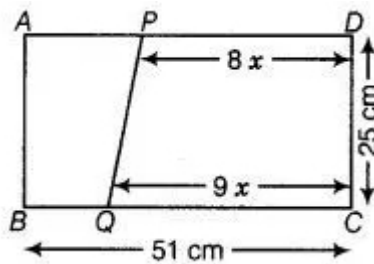
rectangle as shown in the Fig. If the area of the trapezium PQCD is $\frac{5}{6}$ th part of the area of the rectangle, find the lengths QC and PD.



Solution:

Given: ABCD is a rectangle, where AB = 51 cm and BC = 25 cm.

The parallel sides QC and PD of the trapezium PQCD are in the ratio of 9 : 8. Let QC = 9x and PD = 8x.



Now, the area of trapezium PQCD:

$$\begin{aligned}
 &= \frac{1}{2} \times (\text{Sum of parallel sides}) \times (\text{Distance between parallel sides}) \\
 &= \frac{1}{2} \times (9x + 8x) \times 25 \text{ cm}^2 \\
 &= \frac{1}{2} \times 17x \times 25
 \end{aligned}$$

Again, area of rectangle ABCD = $BC \times CD = 51 \times 25$

Now, according to the question,

$$\text{Area of trapezium PQCD} = \frac{5}{6} \times \text{Area of rectangle ABCD}$$

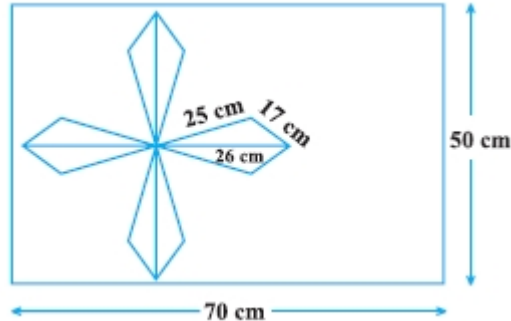
$$\frac{1}{2} \times 17x \times 25 = \frac{5}{6} \times 51 \times 25$$

$$x = \frac{5}{6} \times 51 \times 25 \times 2 \times \frac{1}{17 \times 25}$$

$$x = 5$$

Therefore, the length of the trapezium PQCD, QC = $9x = 9 \times 5 = 45 \text{ cm}$ and, PD = $8x = 8 \times 5 = 40 \text{ cm}$.

8. A design is made on a rectangular tile of dimensions $50\text{ cm} \times 70\text{ cm}$ as shown in Fig. The design shows 8 triangles, each of sides 26 cm , 17 cm and 25 cm . Find the total area of the design and the remaining area of the tile.



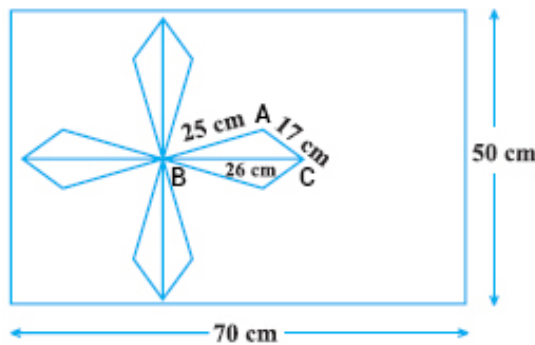
Solution:

Given: the dimension of the rectangular tile are $50\text{ cm} \times 70\text{ cm}$.

So, area of the rectangular tile = $50\text{ cm} \times 70\text{ cm} = 3500\text{ cm}^2$.

See the given figure in the question, the sides of the triangle ABC be:

$a = 25\text{ cm}$, $b = 17\text{ cm}$, and $c = 26\text{ cm}$



Since, semi-parameter(s) of triangle be:

$$\begin{aligned} s &= \frac{a+b+c}{2} \\ &= \frac{25+17+26}{2} \\ &= \frac{68}{2} \\ &= 34 \end{aligned}$$

$$\text{So, area of triangle ABC} = \sqrt{s(s-a)(s-b)(s-c)} \quad [\text{By heron's formula}]$$

$$\begin{aligned} \text{Since, Total area of eight triangle} &= 8 \times \text{Area of triangle ABC} \\ &= 204 \times 8 \\ &= 1632\text{ cm}^2 \end{aligned}$$

The area of the design will be equal to the area of eight triangle that is 1632 cm^2 .

Now, remaining area of the tile = Area of the rectangle – Area of the design =
 $3500\text{cm}^2 - 1632\text{cm}^2 = 1868\text{cm}^2$

Therefore, total area of the design is 1632cm^2 and the remaining area of the tile is 1868cm^2 .