Chapter 5

Curvilinear Motion

CHAPTER HIGHLIGHTS

- *Kinematics of Curvilinear Translation*
- *Projectile Motion*
- *Equations of the Path of Projectile*
- *Motion of a Projectile on an Inclined Plane*
- *Kinematics of Rotation*
- *Angular Displacement and Angular Velocity*
- *Angular Acceleration*
- *Equations of Motion Along a Circular Path*
- *Curvilinear and Rotary Motion Kinetics of Curvilinear and Rotary Motion*
- *Laws for Rotary Motion*
- *Angular Momentum or Moment of Momentum*
- *Conservation of Angular Momentum*
- *Simple Harmonic Motion and Free Vibrations*
- *Oscillation, Amplitude, Frequency and Period*
- *Velocity and Acceleration*
- *Frequency of Vibration of a Spring Mass System*
- *Oscillations of a Simple Pendulum*

Kinematics of Curvilinear Translation

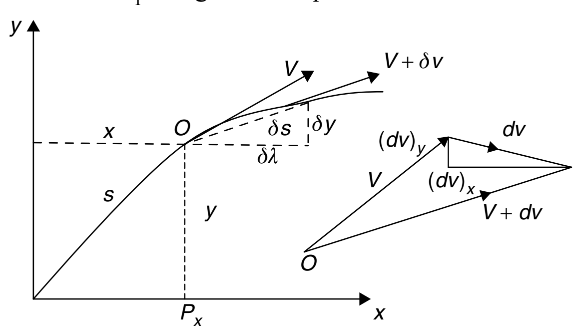
Motion of a particle describing a curved path is called as curvilinear motion,

Velocity and Acceleration

The curvilinear motion of a body P may be imagined as the resultant of two rectilinear motions of its projections P_x and P_y on O_x and O_y axis respectively.

Velocity

Let us consider a body moving through a distance δs from position P to P_1 along a curved path in time δt.



Consider PP_1 as a chord instead of an arc, we have

$$V_{av} = \frac{\delta s}{\delta t}$$

Its projections on the x and y co-ordinates are $(V_{av})_x = \frac{\delta s}{\delta t}\frac{\delta x}{\delta s} = \frac{\delta x}{\delta t}$

$$(v_{av})_y = \frac{\delta s}{\delta t}\frac{\delta y}{\delta s} = \frac{\delta y}{\delta t}$$

Now $\frac{\delta x}{\delta t}$ and $\frac{\delta y}{\delta t}$ are the average velocities of the projections P_x and P_y respectively in the direction of their respective co-ordinates.

If δt approaches zero, v_{av} becomes the instantaneous velocity. Instantaneous velocity at P, $v = \lim\limits_{\delta t \to 0} \frac{\delta_s}{\delta_t} = \frac{d_s}{d_t}$ and its direction will be tangential to the path at P.

Similarly $v_x = \frac{dx}{dt}$, $v_y = \frac{dy}{dt}$

Total velocity $\boxed{v = \sqrt{v_x{}^2 + v_y{}^2}}$

Acceleration

The average acceleration during the interval t is $a_{av} = \frac{\delta v}{\delta t}$

The direction will be same as that of the change of velocity δv.

The projections of a_{av} on x and y co-ordinates will be $\frac{\delta v_x}{\delta_t}$ and $\frac{\delta v_y}{dt}$ respectively.

When δt approaches zero, the instantaneous acceleration,

$$a = \lim_{\delta t \to 0} \frac{\delta v}{\delta t} = \frac{dv}{dt}$$

$$a = \frac{d}{dt}\frac{ds}{dt} = \frac{d^2 s}{dt^2}$$

Similarly the components of the instantaneous acceleration a are,

$$a_x = \frac{d^2 x}{dt^2},$$

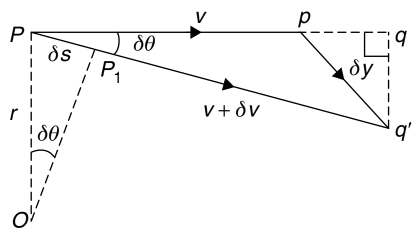
$$a_y = \frac{d^2 y}{dt^2}$$

We get,

$$\boxed{a = \sqrt{a_x^2 + a_y^2}}$$

Tangential and Normal Acceleration

A particle moves on a curved path and from position P, moves a distance δs to position P_1, in the time interval δt, such that at P the instantaneous velocity is v and that at P_1 it is $(v + \delta v)$



Resolving the acceleration into two components:

1. Tangential to the path at the position P.
2. Normal to the path at position P.

Let r be the radius of the curved path PP_1 and $\delta\theta$, the angle subtended at the centre O.

Let θ be the angle included between the normals at P_1 and P.

From the figure we see that P_p = instantaneous velocity v at P.

Resolving δV into two components (pq) in the direction tangential at P and qq' in the direction normal at P as shown.

Tangential acceleration

$$a = \lim_{\delta_t \to 0} \frac{\text{tangential change in velocity}}{\delta t} = \lim_{\delta_t \to 0} \frac{pq}{\delta t}$$

From the triangle Pqq';

$pq = Pq - Pp = (v + \delta v)\cos\delta\theta - v = v + \delta v - v = \delta v$

($\delta\theta$ being very small, $\cos\delta\theta = 1$)

Then $a_t = \lim_{\delta_t \to 0} \frac{\delta v}{\delta t} = \frac{dv}{dt}$

Now normal acceleration $a_n = \lim_{\delta t \to 0} \frac{qq'}{\delta t}$

$qq' = pq \sin\delta\theta = (v + \delta v)\,\delta\theta$

($\delta\theta$ being small $\delta\theta = \delta\theta$ in radians) $= v\delta v + \delta v\delta\theta = v\delta\theta$

($\delta\theta$ and δv being very small, their product will be negligible)

From above figure OPP_1

$$\delta\theta = \frac{PP_1}{r} = \frac{\delta s}{r}$$

$$qq' = \frac{v\,\delta s}{r}$$

Substituting qq' in equation we have

$$a_n = \lim_{\delta t \to 0} \frac{v\,\delta s}{r\,\delta_t}$$

$$a_n = \frac{v}{r} \times \frac{ds}{dt},$$

$$\text{But } \frac{ds}{dt} = v$$

$$\therefore a_n = \frac{v^2}{r}$$

Normal acceleration is also known as 'centripetal acceleration'.

NOTE

During the motion of a particle along a curved path there is a change in the direction of its velocity from instant to instant with or without any change in magnitude. When both magnitude and direction of velocity change, the particle has the tangential and normal acceleration. When there is only change in the direction of velocity, the particle has only normal acceleration.

Solved Examples

Example 1: The equation of motion of a particle moving on a circular path, radius 400 m, is given by $S = 18t + 3t^2 + 2t^3$.

Where S is the total distance covered from the starting point, in metres, till the position reached at the end of t seconds.

(i) The acceleration at the start is

(A) 6m/s^2 (B) 5m/s^2

(C) 10m/s^2 (D) 7m/s^2

(ii) The time when the particle reaches its maximum velocity is

(A) 0.5 s (B) 0.6 s

(C) 0.8 s (D) 0.95 s

(iii) The maximum velocity of the particle is

(A) 19.58 m/s (B) 20.53 m/s

(C) 18.65 m/s (D) 13.5 m/s

Solution:

(i) Given, $s = 18t + 3t^2 - 2t^3$

$$v = \frac{ds}{dt} = 18 + 6t - 6t^2$$

From equation, $a = \frac{d^2s}{dt^2} = 6 - 12t$

At the starting point when $t = 0$,
Acceleration

$$a = 6 - 0 = 6 \text{ m/s}^2.$$

(ii) For determining the condition for maximum velocity, we have

$$\frac{d^2s}{dt^2} = 6 - 12t = 0 = 0.5 \text{ secs}$$

(iii) When $t = 0.5$ s,

$$v_{\max} = 18 + 3 - 1.5 = 19.5 \text{m/s}$$

Example 2: A particle moving along curved path has the law of motion $v_x = 2t - 4$, $v_y = 3t^2 - 8t + 8$ where v_x and v_y are the rectangular components of the total velocity in the x and y co-ordinates. The co-ordinates of a point on the path at an instant when $t = 0$, are (4, –8). The equation of the path is

(A) $x^2 + 3x - 2$ (B) $x^3 + 4x + 2$

(C) $x^{\frac{1}{2}} + 3x + 2$ (D) $x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + 2$

Solution:

$$v_x = 2t - 4$$
$$v_y = 3t^2 - 8t + 8$$

Integrating both sides, we have $\int v_x dt = \int (2t - 4)\, dt$

$$x = 2 \times \frac{t^2}{2} - 4t + C_1 = t^2 - 4t + C_1$$

$$\int v_y dt = \int (3t^2 - 8t + 8)\, dt$$

$$y = 3 \times \frac{t^3}{3} - 8 \times \frac{t^2}{2} + 8t + C_2 = t^3 - 4t^2 + 8t + C_2$$

Where C_1 and C_2 are constants

Given $x = 4$, $y = -8$ when $t = 0$

Substituting for x, y and t in equation $4 = 0 - 0 + C_1$

$$\therefore\ C_1 = 4$$
$$-8 = 0 - 0 + 0 + C_2$$
$$\therefore\ C_2 = -8$$

Now the equations of displacement are $x = t^2 - 4t + 4$ and $y = t^3 - 4t^2 + 8t - 8$

$$x = (t - 2)^2$$

$$x^{\frac{1}{2}} = t - 2$$

$$t = x^{\frac{1}{2}} + 2 \qquad (1)$$

$$y = t^3 - 4t^2 + 8t - 8 \qquad (2)$$

Substituting the value of t from (1) in (2), we get

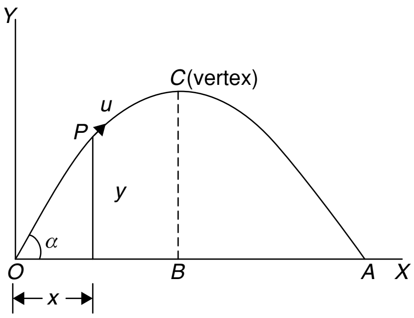
$$y = x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + 2x$$

Projectile Motion

Definitions

1. **Projectiles:** A particle projected at a certain angle is called projectile.
2. **Angle of Projection:** Angle between the direction of projection and the horizontal plane through the point of projection is called as the angle of projection. It is denoted by α.
3. **Trajectory:** The path traced out by the projectile is called the trajectory of the projectile.
4. **Velocity of projection (u):** The initial velocity of projectile is the velocity of projection.
5. **Time of flight (T):** The total time taken by the projectile is termed as the time of flight.
6. **Horizontal range (R):** It is the distance between the point of projection and the point where the trajectory meets the horizontal plane.

Equations of the Path of Projectile



P is the position occupied by the projectile after t sec and x and y are the two co-ordinates of P along the x-axis and y-axis respectively.

Along the x-axis, $u_x = u \cos \alpha$.

Along the y-axis $u_y = u \sin \alpha$

The component u_x remains constant all throughout u_y retards due to the action of gravitational force.

We know $S = vt$, for horizontal motion

$x = u \cos \alpha\, xt$

$$t = \frac{x}{u \cos \alpha}$$

$$s = ut + \frac{1}{2}at^2, \text{ for vertical motion}$$

Therefore $y = u \sin \alpha t - \frac{1}{2}gt^2$

Substituting value of t we can write

$$y = u \sin \alpha \frac{x}{u \cos \alpha} - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \alpha}$$

$$y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$$

This is the equation of the path of a projectile which represents a parabola.

Horizontal range,

$$\boxed{R = \frac{2u^2}{g}\sin\alpha\cos\alpha = \frac{u^2\sin 2\alpha}{g}}$$

Time of Flight, $\boxed{T = \left[\frac{2u\sin\alpha}{g}\right]}$

Maximum height when the vertical component of the velocity is zero.

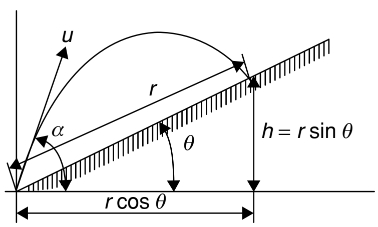
$$v_y = 0 \,.\, y_{\max} = \frac{u_y{}^2}{2g}$$

$$\boxed{y_{\max} = \frac{u^2\sin^2\alpha}{2g}}, \quad (\text{since } u_y = u\sin\alpha)$$

Co-ordinates of vertex C $\boxed{\frac{u^2\sin^2\alpha}{2g}, \frac{u^2\sin^2\alpha}{2g}}$

Motion of a Projectile on an Inclined Plane

Consider the motion of projectile with an initial velocity u and making an angle α with the horizontal on an inclined plane of inclination θ, taking the coordinate axes x, y the expressions for the distance r and height h can be derived.



$$x = u(\cos\alpha)t = r\cos\theta$$

$$y = u(\sin\alpha)t - \frac{1}{2}gt^2 = h = r\sin\theta$$

By eliminating t, we get

$$r\sin\theta = r\cos\theta\tan\alpha - \frac{gr^2\cos^2\theta}{2u^2\cos^2\alpha}$$

$$\Rightarrow \quad r = \frac{2u^2\cos^2\alpha}{g\cos\theta}(\tan\alpha - \tan\theta). \qquad (1)$$

$\therefore$ The distance r is given by equation (1) and thus the height h and the distance on the horizontal plane can be found.

i.e., $h = r\sin\theta$ and $x = r\cos\theta$.

The maximum range possible on the inclined place is found out by differentiation equation 1 with respect to α and equating it to zero.

$\therefore$ $\tan 2\alpha = -\cot\theta$.

$\therefore$ for maximum range the angle made by the velocity vector α should be equal to $(45° + \theta/2)$ with the horizontal.

Example 3: Find the least initial velocity which a projectile may have so that it may clear a wall of 3.6 m high and 6 m distant and strike the horizontal plane through the foot of the wall at a distance of 3.6 m beyond the wall. The point of projection being at the same level as the foot of the wall. Take $g = 9.81$ m/sec^2

(A) 10.2 m/s (B) 11 m/s

(C) 12 m/s (D) 13.5 m/s

Solution:

Let u be the least initial velocity of the projectile and α be the angle of projection with the horizontal plane.

Horizontal range of projectile, $R = 6 + 3.6 = 9.6$ m

$$R = \frac{2u^2\sin\alpha\cos\alpha}{g}$$

$$\therefore 9.6 = \frac{2u^2\sin\alpha\cos\alpha}{g}$$

Now,

$$u^2 = \frac{9.6\,g}{2\sin\alpha\cos\alpha}$$

Putting value,

$$u^2 = \frac{4.8\,g\times\sec^2\alpha}{\tan\alpha} \qquad (1)$$

Equation for the path of projectile

$$y_{\max} = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$3.6 = 6\tan\alpha - \frac{6^2 g}{2u^2\cos^2\alpha}$$

Substituting for u^2, we have,

$$3.6 = 6\tan\alpha - \frac{6^2\tan\alpha}{9.6}$$

$$3.6 = \tan\alpha\left[6 - \frac{6^2}{9.6}\right]$$

$$3.6 = 2.25\tan\alpha$$

$$\tan\alpha = \frac{3.6}{2.25} = 1.6$$

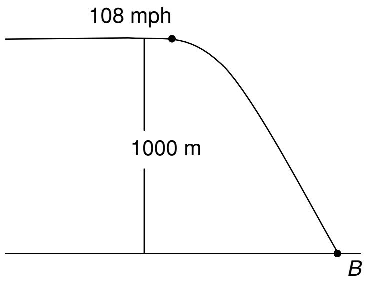
$$\alpha = 57.9°$$

From equation (1)

$$u^2 = \frac{4.8\,g\times\sec^2 57.9}{\tan 57.9} = \frac{4.8\,g\times 3.54}{1.594} = 104.57$$

$$u = 10.2\text{m/s}$$

Example 4: An aeroplane is moving horizontally at 108 km/h at an altitude of 1000 m towards a target on the ground which is intended to be bombed.



(a) The distance from the target where the bomb must be released in order to hit the target is

(A) 428.35 m (B) 450.54 m
(C) 580.2 m (D) 800 m

(b) The velocity, with which the bomb hits the target is

(A) 143 m/s (B) 148 m/s
(C) 150 m/s (D) 161.2 m/s

Solution:

(a) Let B be the point of target and A be the position of the aeroplane and the bomb is released from A to hit at B. The horizontal component of the bomb velocity, which is uniform, is

$$v = 108 \text{ km/h} = \frac{108 \times 1000}{60 \times 60} = 30 \text{ m/sec.}$$

Considering the vertical component of bomb velocity,

At $A, u = 0,\ g = 9.81 \text{m/sec}^2$

$$S = \frac{1}{2} g t^2$$

Let t be the time required to hit B, then

$$1000 = \frac{1}{2} \times 9.81 \times t^2$$

$$t^2 = \frac{2000}{9.81} = 203.87$$

$$t = 14.278 \text{ sec}$$

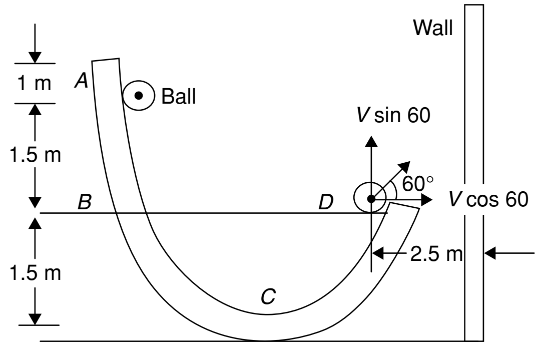
Horizontal distance covered by the bomb $S = Vt = 30 \times 14.278 = 428.35$ m i.e., the bomb is released from plane when the horizontal distance is 428.35 from B

(b) Vertical component velocity at $B = u + gt = 0 + 9.81 \times 14.278 = 140.06$ m/sec

Resultant velocity at $B = \sqrt{30^2 + 140.06^2}$

$$= \sqrt{20518.8} = 143 \text{ m/sec}$$

Example 5: A ball weighing 10 N starts from the position A as shown in figure and slides down a frictionless chute under its own weight. After leaving the chute 1 at point D, the ball hits the wall as depicted in the figure.



(a) The time interval of the ball's travel from the point D to the point of hit is

(A) 0.88 s (B) 0.92 s
(C) 0.733 s (D) 0.898 s

(b) The distance on the wall above the point D to the point of hit is

(A) 0.21 m (B) 0.158 m
(C) 0.32 m (D) 0.168 m

Solution:

(a) The ball starts from point A. The vertical distance from A to C is equal to 3 m. Considering the motion of ball from A to C,

$V^2 = 2as$

Since initial velocity is zero, $a = g$

$= 9.81 \text{ m/sec}^2$

or $v_C{}^2 = 2 \times 9.81 \times 3$

$v_C = 7.67$ m/s,

This is the velocity of the ball at C.

The motion of the ball from C to D

$v_D{}^2 = v_C{}^2 - 2as \quad 7.67^2 = 2 \times 9.81 \times 1.5$

$= 58.82 - 29.43 = 29.39$

$v_D = 5.42$ m/s

On reaching the point D, the horizontal component of the velocity of the ball

$$= v \cos 60° = 5.42 \times \frac{1}{2} = 2.71 \text{ m/s}$$

Let t be the time taken by the ball to hit the wall from point D. Then,

$$t = \frac{2.5}{2.71} = 0.922 \text{ sec.}$$

(b) Finally considering the vertical motion of the ball beyond the point D

$$s = ut - \frac{1}{2}gt^2$$

Here $u = v_D = 5.42\frac{m}{s}$

$$= 5.42 \times 0.922 - \frac{1}{2} \times 9.81(0.922)^2$$

$$= 4.327 - 4.169 = 0.158 \text{ m.}$$

Hence the ball will hit the wall 0.158 m above the point D after 0.922 sec

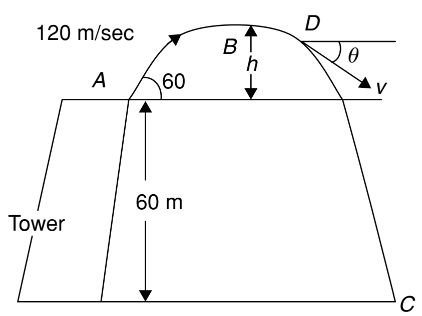
Example 6: From the top of a tower 60 m high, a bullet is fired at an angle of 60° with the horizontal, with an initial velocity of 120 m/s as shown in figure. Neglect air resistance.

(a) The maximum height from the ground that would be attained by the bullet, is

(A) 528 m (B) 611 m
(C) 680 m (D) 720 m

(b) The velocity of bullet, 12 seconds after it is fired, is

(A) 55 m/s (B) 58 m/s
(C) 61 m/s (D) 80 m/s



Solution:

(a) Height

$$h = \frac{u^2 \sin^2 \alpha}{2g}$$

$$= \frac{120 \times 120 \times (\sin 60)^2}{2 \times 9.81} = \frac{120 \times 120 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}}{2 \times 9.81}$$

$$= \frac{10800}{2 \times 9.81} = 551 \text{ m}$$

Maximum height above ground = 551 + 60 = 611 m.

(b) Time of travel upto highest point B is given by,

$$t = \frac{u \sin \alpha}{g} = \frac{120 \times \sin 60}{9.81} = 10.6 \text{ sec.}$$

Let D be the point reached by the bullet, 12 seconds after it is fired. Time taken by the bullet to reach point B from A (point at which it is fired from) = 10.6 sec.

∴ Time taken by the bullet to travel from point B to point D = 12 – 10.6 = 1.4 sec.

Horizontal velocity at B, $v_H = 120 \cos 60°$

$$= 120 \times 0.5 = 60 \text{ m/s}$$

The vertical velocity after 1.4 sec of travel from point B,

$$v_v = 0 + \frac{1}{2} \times 9.81 \times 1.4^2 = 9.62 \text{ m/s}$$

Velocity at point D

$$v = \sqrt{v_H{}^2 + v_v{}^2} = \sqrt{60^2 + 9.62^2} = 60.8 \text{ m/s}$$

Kinematics of Rotation

When a moving body follows a circular path it is known as circular motion. In circular motion the centre of rotation is stationary.

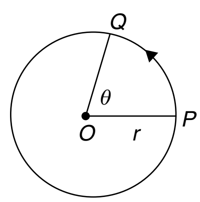
Angular Displacement and Angular Velocity

Angular displacement is defined as the change in angular position (usually referred to as the angle θ), with respect to time.

Angular velocity is defined as the rate of change of angular displacement with respect to time. Let a body, moving along a circular path, be initially at P and after time t seconds be at Q.

Let $\angle POQ = \theta$

Then angular displacement = $\angle POQ = \theta$



Time taken = t

$$\text{Angular velocity} = \frac{\text{Angular displacement}}{\text{Time}} = \frac{\theta}{t}$$

Mathematically, it is expressed as $\frac{d\theta}{dt}$.

It is denoted by the symbol ω

$$\boxed{\omega = \frac{d\theta}{dt}}$$

It is measured in radian/sec or rad/sec

Relation between Linear Velocity and Angular Velocity

Let v = linear velocity $= \dfrac{\text{Linear displacement}}{\text{Time}}$

But linear displacement = Arc $PQ = OP \times \theta = r\theta$

$$v = \frac{r \times \theta}{t} = r \times \text{Angular velocity}$$

$$\left(\because \frac{\theta}{t} = \text{angular velocity}\right)$$

$$\boxed{v = r \times \omega}$$

Where ω = angular velocity

Angular Acceleration

It is defined as the rate of change of angular velocity. It is measured in radians per sec^2 and written as rad/sec^2 and is denoted by the symbol α.

α = Rate of change of angular velocity

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right)\left(\because \omega = \frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}.$$

Also $\dfrac{d\omega}{dt} = \dfrac{d\omega}{d\theta} \times \dfrac{d\theta}{dt} = \dfrac{d\omega}{d\theta} \times \omega = \omega\dfrac{d\omega}{d\theta}$

It has two components:

Normal component $= \dfrac{V^2}{r} = \omega^2 r$ and tangential component $= \dfrac{dV}{dt} = r\dfrac{d\omega}{dt} = r\alpha$

If a is the linear acceleration, then

$$\boxed{a = r\alpha}$$

Equations of Motion Along a Circular Path

$$\boxed{\alpha = \frac{\omega - \omega_0}{t}}$$

$$\boxed{\begin{array}{l} \theta = \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega^2 - \omega_0^2 = 2\alpha\theta \end{array}}$$

If N is the r.p.m.

$$\boxed{\begin{array}{l} \omega = \dfrac{2\pi N}{60} \text{ radians/sec} \\ v = r\omega = \dfrac{2\pi N}{60} \times r = \dfrac{\pi DN}{60} \text{ m/s} \end{array}}$$

Where,

ω_0 = initial angular velocity in cycles/sec,
ω = final angular velocity in cycles/sec,
t = time (in seconds) during which angular velocity changes from ω_0 to ω,
v = linear speed in m/s,

The rotational speed is N revolutions per minute or N r.p.m.

Example 7: A wheel rotates for 5 seconds with a constant acceleration and describes during the time 100 radians. It then rotates with a constant angular velocity and during the next 5 seconds, it describes 70 radians. The initial angular velocity and angular acceleration respectively are,

(A) 15 rad/s, 2.5 rad/s^2 (B) 13 rad/s, 2 rad/s^2
(C) 15 rad/s, –2 rad/s^2 (D) 26 rad/s, –2.4 rad/s^2

Solution:

Angular velocity

$$\omega = \frac{\theta}{t} = \frac{70}{5} = 14 \text{ rad/s}$$

α is constant angular acceleration and ω_0 be initial angular velocity.

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$100 = (\omega_0 5) + \frac{1}{2}\alpha \times 5^2$$

$$5\omega_0 + 12.5\alpha = 100 \quad (1)$$

$$\omega = \omega_0 + \alpha t$$

$$14 = \omega_0 + 5\alpha \quad (2)$$

Solving equations (1) and (2) $\omega_0 = 26$ rad/sec
$\alpha = -2.4$ rad/sec^2 (Retardation)

Example 8: A wheel rotating about a fixed axis at 20 r.p.m. is uniformly accelerated for 80 seconds during which time it makes 60 revolutions.

(a) The angular velocity at the end of the time interval is
(A) 7.294 rad/s (B) 8.384 rad/s
(C) 6.812 rad/s (D) 7.829 rad/s

(b) The time required for the speed to reach 100 r.p.m.
(A) 3.65 min (B) 2.14 min
(C) 1.85 min (D) 2.58 min

Solution:

(a)

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

ω_0 = initial angular velocity

$$\omega_0 = \frac{2\pi \times 20}{60} = 2.094$$

$$2\pi \times 60 = (2.094 \times 80) + \frac{1}{2}\alpha(80)^2$$

$$2\pi \times 60 = 167.52 + 3200\alpha$$

$$\alpha = \frac{2\pi 60 - 167.52}{3200} = 0.065 \text{ rad/sec}^2$$

Let ω be the angular velocity at the end of 80 seconds in rad/sec. Then $\omega = \omega_0 + \alpha t \quad \omega = 2.094 + (0.065 \times 80) = 7.294$ rad/sec.

(b)

$$7.294 = \frac{2\pi \times N}{60}$$

$$N = 69.65 \text{ r.p.m}$$

$$\omega_1 = \omega_0 + \alpha t_1$$

Where $\omega_1 = \dfrac{2\pi \times 100}{60}$ rad/sec

$$= 10.466 \text{ rad/sec}$$

$$10.466 = 2.094 + 0.065 \times t_1$$

$$t_1 = \frac{8.372}{0.065} = 128.8 \text{ sec} = 2.14 \text{ min}.$$

Curvilinear and Rotary Motion

Kinetics of Curvilinear and Rotary Motion

For a particle or a body moving in a curved path with particular emphasis to the circular path comes under this section.

In order to maintain the circular motion, an inward radial force called 'centripetal force' is acted upon the body, which is equal and opposite to the centrifugal force that is directed away from the centre of curvature. If r is the radius of the circular path, v is the linear velocity, ω is the angular velocity and t is the time, then

Angular acceleration $= \dfrac{d\omega}{dt}$

Tangential acceleration $= r\dfrac{d\omega}{dt}$,

Normal acceleration $= \dfrac{v^2}{r} = \omega^2 r$,

Centripetal or centrifugal force $= \dfrac{W}{g} \times \dfrac{v^2}{r} = \dfrac{W}{g}\omega^2 r$.

Laws for Rotary Motion

First Law

It states that a body continues in its state of rest or of rotation about an axis with constant or uniform angular velocity unless it is compelled by an external torque to change that state.

Second Law

It states that the rate of change of angular momentum of a rotating body is proportional to the external torque applied on the body and takes place in the direction of the torque.

$$I = Mk^2$$

where M = mass of the body and k = radius of gyration
= moment of inertia × initial angular velocity
Initial angular momentum $= I\omega_0$
Final angular momentum $= I\omega$
Change of angular momentum $= I(\omega - \omega_0)$
Rate of change of angular momentum

$$= \frac{\text{change of angular momentum}}{\text{Time}}$$

$$= I\frac{(\omega - \omega_0)}{t} = I\alpha\left[\because \alpha = \frac{\omega - \omega_0}{t} = \text{angular acceleration}\right]$$

From second law of motion of rotation,
Torque α rate of change of angular momentum

$$T = I\alpha$$

$$T = KI\alpha,$$

where K is a constant of proportionality. SI unit of torque is Nm.

Angular momentum or moment of momentum: Moment of momentum of the body about $O = I\omega$,

Where the rigid body undergoes rotation about O.

Angular momentum is the moment of linear momentum

Rotational kinetic energy: Rotational kinetic energy $= \dfrac{1}{2}I\omega^2$

Angular impulse or impulsive torque: Angular impulse or impulsive torque $= I\, d\omega$

Work done in rotation: Work done in rotation $= T \times \theta$

Kinetic energy in combined motion: Kinetic energy due to translatory motion $= \dfrac{1}{2}mv^2$

Kinetic energy due to rotation $= \dfrac{1}{2}I\omega^2$

Kinetic energy due to combined motion $= \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$.

Conservation of Angular Momentum

The law of conservation of angular momentum states that the angular momentum of a body or a system will remain unaltered if the external torque acting on it is zero.

D'alemberts' Principle for Rotary Motion

D'Alemberts' principle for rotary motion states that the sum of the external torques (also termed as active torques) acting on a system, due to external forces and the reversed active torques including the inertia torques (taken in the opposite direction of the angular momentum) is zero.

Suppose a disc of moment of inertia I rotates at an angular acceleration α under the influence of a torque T, acting in the clockwise direction. Inertia torque $= I\alpha$ (acting in the anti-clockwise direction)

From D'Alemberts' principle, $T - I\alpha = 0$, the dynamic equation of equilibrium for a rotating system.

Rotation caused by a weight W attached to one end of a string passing over a pulley of weight W_0

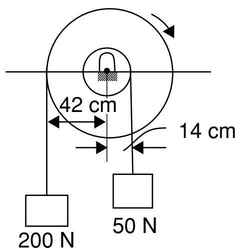
From D'Alemberts' principle, it can be shown that,

$$\boxed{a = \frac{gW}{\left(W + \frac{W_0}{2}\right)}}$$, when the pulley is considered as a disc.

Rotation caused due to two weights W_1 and W_2 attached to the two ends of a string which passes over a rough pulley of weight W_0

$$\boxed{a = \frac{g(W_1 - W_2)}{\left(W_1 + W_2 + \frac{W_0}{2}\right)}}$$

Example 9: In a pulley system shown in figure the pulley weighs 20 N and its radius of gyration is 40 cm. A 200 N weight is attached to the end of a string and a 50 N is attached to the end of the other string as shown in the figure.



(a) The torque to be applied to the shaft to raise the 200 N weight at an acceleration of 1.5 m/s^2 is

(A) 6812 Ncm (B) 9136 Ncm
(C) 700 Ncm (D) 7832 Ncm.

(b) The tensions in the strings are respectively

(A) 170.4 N, 35.6 N (B) 180 N, 40 N
(C) 190.2 N 35 N (D) 180.6 N, 42.34 N

Solution:

(a) Moment of inertia of the pulley $I = \frac{W}{g}k^2$

$$I = \frac{20}{981} \times (40)^2 \text{ Ncm}^2 = 32.62 \text{ Ncm}^2$$

T_1 = Torque produced by 200 N
= 200 × 42 = 8400 Ncm

T_2 = Torque developed by 50 N = 50 × 14 = 700 Ncm

Inertia torque due to angular rotation of the pulley with angular acceleration $\alpha = I\alpha = 32.62\alpha$ Ncm.

Torque due to inertia force on

$$200 \text{ N} = (ma)r = \frac{200}{981} r\alpha r = \frac{200}{981} \times \alpha \times (42)^2$$
$$= 359.63\alpha \text{ Ncm}$$

Torque due to inertia force on

$$50 \text{ N} = \frac{50}{981} \times \alpha \times 14^2 = 9.99\ \alpha\text{Ncm}$$

Let T be the torque applied to the shaft for dynamic equilibrium $\Sigma T = 0$

$$T + 700 = 8400 + 32.62\alpha + 359.63\alpha + 9.99\alpha$$
$$T = 8400 + 312.33 = 9136 \text{ Ncm},$$

Since $\alpha = \frac{150}{42} = 3.57$ rad/s^2.

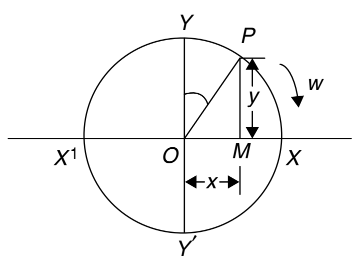
(b) Let F_1 and F_2 be the tensions in the strings. Applying D'Almberts' principle for linear motion, we get

$$F_1 - 200 - \frac{200}{9.8} \times 1.5 = 0$$
$$F_2 + 50 - F_2 = \frac{50}{9.8} \times 1.5$$
$$F_1 = 200 + \frac{200}{9.8} \times 1.5 = 200 + 22.96$$
$$= 180.6\ N$$
$$F_2 = \frac{50 \times 9.8 - 50 \times 1.5}{9.8} = 42.34\text{N}$$

Simple Harmonic Motion and Free Vibrations

Simple harmonic motion: It is defined as the type of motion in which the acceleration of the body in its path of motion, varies directly as its displacement from the equilibrium position and is directed towards the equilibrium point.

Oscillation, Amplitude, Frequency and Period



In the above figure, when a particle P is describing a circular path, M being the projection of P, it describes a simple harmonic motion.

The motion of M from X to X' and back to X is called an oscillation or simple harmonic motion.

$OX = OX'$ is the amplitude.

This amplitude is the distance between the centre of simple harmonic motion and the point where the velocity is zero.

The period of one complete oscillation is the period of simple harmonic motion.

Thus the period of simple harmonic motion is the time in which M describes 2π radians at ω radians/sec.

$T = \frac{2\pi}{\omega}$, where T is the time period in seconds.

Velocity and Acceleration

The simple harmonic displacement

$$X = r \sin \omega t$$

$$v = \omega\sqrt{r^2 - x^2}$$

$$\text{Acceleration } = \frac{d^2x}{dt^2} = -\omega^2 r \sin \omega t$$

$$a = -\omega^2 x$$

$$\text{Frequency } = \frac{1}{2\pi}\sqrt{\frac{a}{x}}$$

Frequency of Vibration of a Spring Mass System

Consider a helical spring subjected to a load W. The static equilibrium position is 0-0. Let S be the stiffness of the spring which is defined as force required to cause one unit extension. If the weight is displaced and stretched to position 1-1′ by an amount 'y', as shown in the below figure, then the acceleration with which the load springs back,

$$\frac{w}{g}a = -sy$$

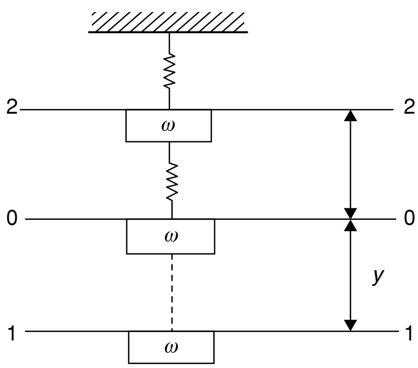
$$\therefore\ a = \frac{s \times g}{-W}\cdot y$$

This is of the form $a = -\omega_n^2 y$

Where $\omega_n^2 = \frac{sg}{w} = \frac{g}{\delta}$,

δ being $\frac{\omega}{s}$

$$\text{Frequency } f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}.$$



Oscillations of a Simple Pendulum

Period of oscillation $T = \frac{1}{f} = 2\pi\sqrt{\frac{l}{g}}$ (for 2 beats)

l = length of pendulum. Half of an oscillation is called a beat or swing. A pendulum executing one half oscillation per second is called seconds pendulum. Time of one beat or swing $= \pi\sqrt{\frac{l}{g}} = \frac{T}{2}$. For n number of beats, time $= n\pi\sqrt{\frac{l}{g}}$.

For a compound pendulum $T = 2\pi\sqrt{\frac{K_G^2 + h^2}{gh}}$

Where h is the distance between the point of suspension and centre of gravity. Where k_G = radius of gyration about O, the centre of suspension. A compound pendulum is a rigid body free to oscillate about a smooth horizontal axis passing through it.

A simple pendulum whose period of oscillation is the same as that of a compound pendulum is called as a simple equivalent pendulum $\boxed{1 = \frac{k_G^2}{h} + h}$.

Example 10: A body performing simple harmonic motion has a velocity 12 m/s when the displacement is 50 mm and 3 m/s when the displacement is 200 mm, the displacement being measured from the mean position.

(a) Calculate the frequency of the motion.

(A) 35 cycles/sec (B) 40.5 cycles/sec
(C) 31.8 cycles/sec (D) 35.5 cycles/sec

(b) What is the acceleration when the displacement is 75 mm.

(A) 15 m/s^2 (B) 16.5 m/s^2
(C) 13.8 m/s^2 (D) 15.6 m/s^2

Solution:

(a) In simple harmonic motion

$$V^2 = \omega^2(r^2 - x^2)$$

V = velocity, r = amplitude

x = distance from mid positions

x_1 = 50 mm, x_2 200 mm

V_1 = 12 m/s V_2 = 3 m/s

$$12^2 = \omega^2\left[r^2 - \left(\frac{50}{1000}\right)^2\right]$$

$$3^2 = \omega^2\left[r^2 - \left(\frac{200}{1000}\right)^2\right] \quad (1)$$

Dividing we get $\frac{144}{9} = \frac{r^2 - \frac{1}{400}}{r^2 - \frac{4}{100}}$

$$16 = \frac{r^2 - \frac{1}{400}}{r^2 - \frac{4}{100}}$$

$$16r^2 - \frac{16\times 4}{100} = r^2 - \frac{1}{400}$$

$$15r^2 = \frac{16\times 2}{50} - \frac{1}{400}$$

$$15r^2 = \frac{2\times 64\times 4}{400} - \frac{1}{400} = \frac{511}{400}$$

$$r^2 = \frac{511}{400\times 15} = 0.085$$

$$r = 0.29,\ m = 290 \text{ mm.}$$

Putting the value of r^2 in equation (1), we get

$$9 = \omega^2[0.085 - 0.04]$$

Or, $$\omega^2 = \frac{9}{0.045}$$

We get, $\omega = 200$ rad/s

So, $$f = \frac{\omega}{2\pi} = \frac{200}{2\pi} = 31.83 \text{ cycles/sec.}$$

(b) If a be the acceleration when displacement $x = 75$ mm

$$a = \omega^2 x = \left(\frac{9}{0.045}\times\frac{75}{1000}\right) = 15 \text{ m/s}^2.$$

Example 11: The amount of seconds a clock would loose per day, if the length were increased in the ratio 800 : 801 is

(A) 48 *s* (B) 54 *s*
(C) 50 *s* (D) 60 *s*

Solution:

Given $I = 800$ units
$I + dI = 801$ units
$dI = 1$ unit

We get, $$\frac{dl}{I} = \frac{I}{800}$$

$$\frac{dn}{n} = \frac{-dI}{2I} = \frac{I}{1600}$$

$$dn = -\frac{n}{1600} = -\frac{86400}{1600} = -54$$

Where $n = 86400$, as a seconds pendulum will beat 86400 times/day. The clock will loose 54 seconds a day.)

Exercises

Practice Problems I

Direction for questions 1 to 10: Select the correct alternative from the given choices.

Direction for questions 1 to 3: A force of $2t$ Newton, where t in seconds, acts on a mass of 100 kg initially at rest, for a period of 20 seconds.

1. The impulse on the mass is
(A) 400 Ns (B) 300 Ns
(C) 350 Ns (D) 500 Ns

2. The velocity of the mass is
(A) 1 m/s (B) 2 m/s
(C) 3 m/s (D) 2.5 m/s

3. The average force, which would have resulted in the same velocity, is
(A) 15 N (B) 30 N
(C) 20 N (D) 10 N

4. A car of mass 1500 kg descends a hill of 1 in 5 incline. The average braking force required to bring the car to rest from a speed of 80 km per hour in a distance of 50 m is (take the frictional resistance as 300 N)
(A) 10 N (B) 15 N
(C) 8 N (D) 12 N

Direction for questions 5 and 6: A thin circular ring of mass 200 kg and radius 2 m resting flat on a smooth surface is subjected to a sudden application of a force of 300 N at a point of its periphery.

5. The angular acceleration is
(A) 0.75 rad/s^2 (B) 1.5 rad/s^2
(C) 2 rad/s^2 (D) 2.5 rad/s^2

6. The acceleration of mass centre is
(A) 1 m/s^2 (B) 1.5 m/s^2
(C) 2 m/s^2 (D) 3 m/s^2

7. A particle traveling in a curved path of radius of curvature 500 m with a speed of 108 km/h and a tangential acceleration of 4 m/s^2. The total acceleration of the particle is
(A) 4.38 m/s^2 (B) 5 m/s^2
(C) 3.5 m/s^2 (D) 8 m/s^2

Direction for questions 8 and 9: A solid cylinder 80 cm in diameter is released from the top of an inclined plane 2.0 m high surface and rolls down the inclined surface without any loss of energy due to friction.

8. The energy equation for the system is

(A) $mgh = \frac{1}{2}mv^2$ (B) $mgh = \frac{1}{3}mv^2$

(C) $mgh = \frac{3}{4}mv^2$ (D) $mgh = \frac{2}{3}mv^2$

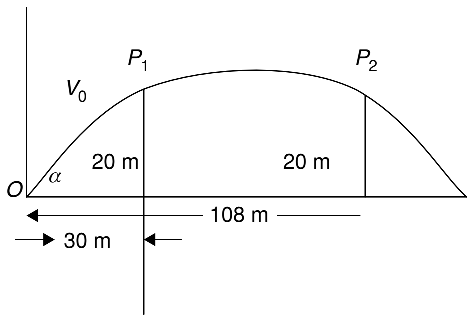
9. The linear and angular speeds, at the bottom respectively are
(A) 6.1 m/s and 12.75 rad/s
(B) 5.5 m/s and 34 rad/s
(C) 5.1 m/s and 12.75 rad/s
(D) 6.1 m/s and 34 rad/se

10. A disc shaped frictionless pulley $I = \frac{1}{2}MR^2$ has a mass of 80 kg and radius of 2 m. A rope is wound round the pulley and supports a 4 kg mass. The angular acceleration of the pulley $(g = 10\ \text{m/s}^2)$ is
(A) $\frac{1}{4}$ rad/ s^2 (B) $\frac{1}{2}$ rad/ s^2
(C) 1 rad/ s^2 (D) $\frac{3}{4}$ rad/ s^2

Practice Problems 2

Direction for questions 1 to 10: Select the correct alternative from the given choices.

1. A bullet is projected so as to graze the top of two walls each of height 20 m located at distances of 30 m and 180 m in the same line from the point of projection as shown in figure. The angle and speed of projection of the bullet, respectively, are



(A) 34.1° and 44 m/s
(B) 38.2° and 48 m/s
(C) 35.29° and 49.5 m/s
(D) 37.87° and 46.1 m/s

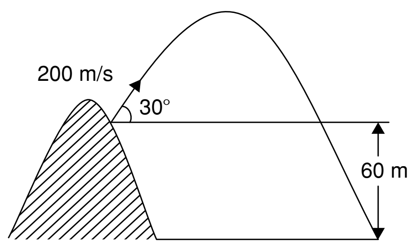
2. For a given value of initial velocity for a projectile, the maximum range, on an inclined plane inclined to the horizontal at an angle of β (in degrees), can be obtained if the angle of projection is
(A) 45° (B) $90° - 0.5\beta$
(C) $45° + 0.5\beta$ (D) $45° - 0.5\beta$

3. A shell bursts on contact with the ground and the pieces of it fly off in all directions with speeds up to 40 m/s. A person, standing 40 m away from the point of burst, can be hit by a piece in a time duration of
(A) 1.5 sec (B) 1 sec
(C) 2 sec (D) 3 sec

4. The coefficient of restitution is defined as the
(A) Negative of the ratio of the velocity of separation to the velocity of approach
(B) Ratio of the velocity components in the line of impact
(C) Ratio of the velocity vectors before and after collision
(D) Negative of the ratio of the energies of the bodies before and after the impact

5. A cylinder of radius of r and mass m rest on a rough horizontal rug. If the rug is pulled from under it with an acceleration, a perpendicular to the axis of the cylinder, the angular acceleration of the centre of mass of the cylinder, assuming that it does not slip, is
(A) $\frac{2}{3}\frac{A}{r}$ (B) $\frac{1}{3}\frac{A}{r}$
(C) $\frac{3}{4}\frac{A}{r}$ (D) $\frac{2}{3}A$

Direction for questions 6 to 8: A soldier positioned on a hill fires a bullet at an angle of 30° upwards from the horizontal as shown in the figure. The target lies 60 m below him and the bullet is fired with a velocity of 200 m/s.



6. The maximum height, to which the bullet will rise above the position of the soldier, is
(A) 615 m (B) 490 m
(C) 509.7 m (D) 710.6 m

7. The velocity with which the bullet will hit the target is
(A) 202.9 m/s (B) 245.3 m/s
(C) 312.7 m/s (D) 343.6 m/s

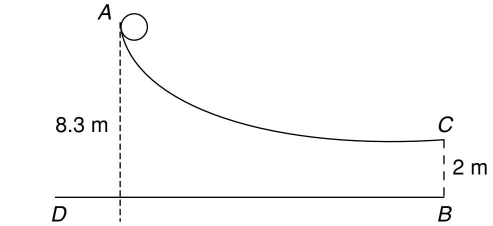
8. The time required to hit the target is
(A) 21.7 sec (B) 20.97 sec
(C) 15.6 sec (D) 23 sec

9. A carpet of mass m made of an inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough horizontal floor. When a small push, of negligible force, is given to the carpet, it starts unrolling without sliding on the floor. The horizontal velocity of the axis of the cylindrical part of the carpet is $\sqrt{\frac{63}{3}gR}$ when the radius of the carpet reduces to

(A) $\frac{3R}{4}$ (B) $\frac{R}{4}$

(C) $\frac{R}{2}$ (D) $\frac{R}{5}$

10. A small sphere rolls down without slipping from the top most point of a track, with an elevated section and a horizontal part, as shown in the following figure, in a vertical plane. The horizontal part is 2 m above the ground level and the top of the track is 8.3 m above the ground. The distance on the ground, with respect to the point B (which is vertically below the end of the track), where the sphere would land is

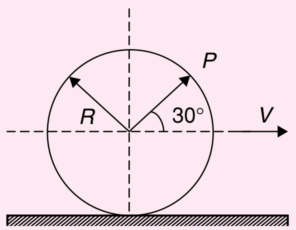


(A) 6 m (B) 10 m

(C) 3 m (D) 2 m

Previous Years' Questions

1. A circular disk of radius R rolls without slipping at a velocity v. The magnitude of the velocity at point P (see figure) is **[2008]**



(A) $\sqrt{3}v$ (B) $\sqrt{3}\frac{v}{2}$

(C) $\frac{v}{2}$ (D) $\frac{2v}{\sqrt{3}}$

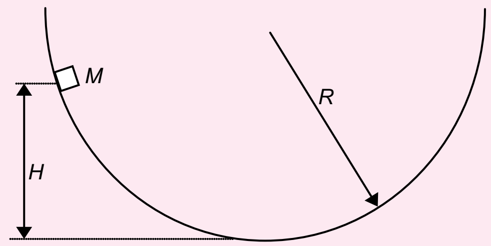
2. An annular disc has a mass m, inner radius R and outer radius $2R$. The disc rolls on a flat surface without slipping. If the velocity of the centre of mass is v, the kinetic energy of the disc is **[2014]**

(A) $\frac{9}{16}mv^2$ (B) $\frac{11}{16}mv^2$

(C) $\frac{13}{16}mv^2$ (D) $\frac{15}{16}mv^2$

3. Consider a steel (Young's modulus E = 200 GPa) column hinged on both sides. Its height is 1.0 m and cross-section is 10 mm × 20 mm. The lowest Euler critical buckling load (in N) is ______. **[2015]**

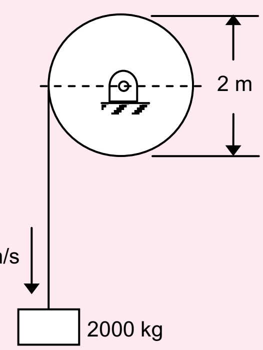
4. A point mass M is released from rest and slides down a spherical bowl (of radius R) from a height H as shown in the figure below. The surface of the bowl is smooth (no friction). The velocity of the mass at the bottom of the bowl is: **[2016]**



(A) $\sqrt{gH}$ (B) $\sqrt{2gR}$

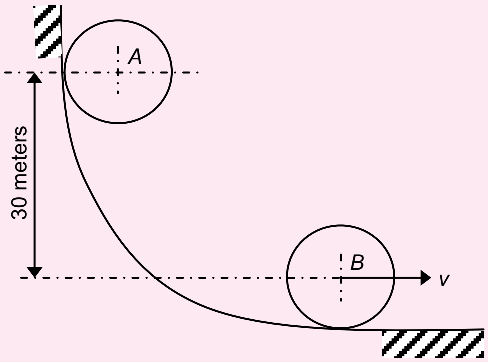
(C) $\sqrt{2gH}$ (D) 0

5. A mass of 2000 kg is currently being lowered at a velocity of 2 m/s from the drum as shown in the figure. The mass moment of inertia of the drum is 150 kg-m^2. On applying the brake, the mass is brought to rest in a distance of 0.5 m. The energy absorbed by the brake (in kJ) is _______. **[2016]**

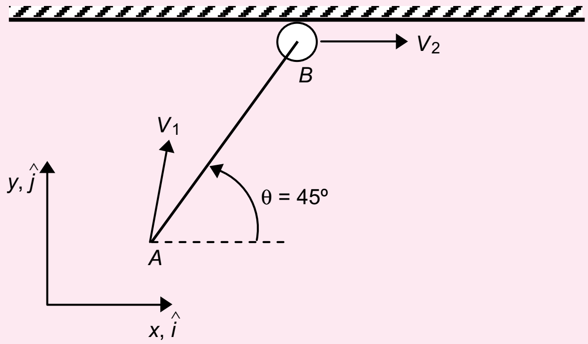


6. A circular disc of radius 100 mm and mass 1 kg, initially at rest at position A, rolls without slipping down a curved path as shown in figure. The speed v of the disc when it reaches position B is _______ m/s. **[2016]**

Acceleration due to gravity g = 10 m/s^2.



7. A rigid rod (AB) of length $L = \sqrt{2}$ m is undergoing translational as well as rotational motion in the x-y plane (see the figure). The point A has the velocity $V_1 = \hat{i} + 2\hat{j}$ m/s. The end B is constrained to move only along the x direction.



The magnitude of the velocity V_2 (in m/s) at the end B is ______. **[2016]**

Answer Keys

Exercises

Practice Problems 1

1. A **2.** B **3.** C **4.** A **5.** A **6.** B **7.** A **8.** C **9.** C **10.** A

Practice Problems 2

1. D **2.** C **3.** B **4.** A **5.** A **6.** C **7.** A **8.** B **9.** B **10.** A

Previous Years' Questions

1. A **2.** C **3.** 3285 to 3295 **4.** C **5.** 14.1 to 14.3 **6.** 20 **7.** 3

TEST

ENGINEERING MECHANICS

Time: 60 Minutes

Direction for questions 1 to 30: Select the correct alternative from the given choices.

1. Two equal and opposite co-planar couples
(A) Balance each other.
(B) Produce a couple and unbalanced force.
(C) Cannot balance each other.
(D) Give rise to a couple of double the magnitude.

2. In a perfect frame, the number of members are
(A) $2j-3$ (B) $2j+3$
(C) $2j-2$ (D) $2j-1$
Where j = number of joints.

3. The state of equilibrium of a body implies that the body must (with respect to some inertial frame) be:
(A) At rest or with uniform acceleration.
(B) Uniform velocity or uniform acceleration.
(C) At rest or with uniform velocity.
(D) At rest or with uniform velocity or uniform acceleration.

4. The distance of the centroid of a semicircle of radius 'r' from its base is
(A) $\frac{4r}{3\pi}$ (B) $\frac{3\pi}{4r}$
(C) $\frac{4\pi}{3r}$ (D) $\frac{2\pi}{3r}$

5. A machine requires an effort of 10 Kg to lift a load of 200 Kg an effort of 12 Kg for a load of 300 Kg. The effort required to lift a load of 500 Kg will be
(A) 16 Kg (B) 15 Kg
(C) 14 Kg (D) 17 Kg

6. The moment of a force
(A) Ocures about a point
(B) Measures the capacity to do useful work.
(C) Occurs only when bodies are in motion
(D) Measures the abilities to turning or twisting about axes.

7. The required condition of equilibrium of a body is that
(A) The algebraic sum of horizontal components of all the forces must be zero.
(B) The algebraic sum of the vertical components of all the forces must be zero.
(C) The algebraic sum of moments about a point must be zero.
(D) All the above.

8. The unit of the moment of Inertia of an area is
(A) Kg-m (B) Kg-m^2
(C) Kg-m^4 (D) m^4

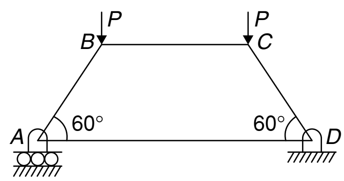
9. Moment of Inertia of a square of side 'a' about an axis passing through its C. G is equal to
(A) $\frac{a^3}{12}$ (B) $\frac{a^4}{12}$
(C) $\frac{a^3}{36}$ (D) $\frac{a^4}{36}$

10. According to the law of the machine, the relation between effort 'P' and load W is given by
(A) $W = mP + C$ (B) $W = mP - C$
(C) $P = mW + C$ (D) $P = mW - C$

11. Weight of 150 kN is being supported by a tripod whose leg is of the length of 13 m. If the vertical height of the point of attachment of the load is 12, the force on the tripod leg would be
(A) 48.24 N (B) 54.16 N
(C) 50.8 N (D) 45.3 N

12. The resultant of two forces $4P$ and $3P$ is R. If the first force is doubled the resultant is also doubled. The angle between the two forces is
(A) 48.25° (B) 95.73°
(C) 32.5° (D) 45.53°

13. In the truss shown the force in the member BC is



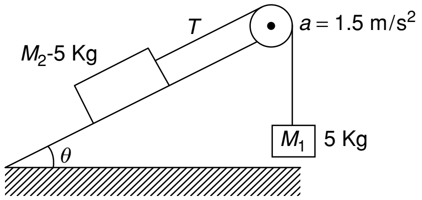
(A) 0 (B) 0.577 $P(T)$
(C) 0.577 P (comp) (D) 0.866 P (comp)

Direction for questions 14 and 15: A body is weighing 500 N is just moved along a horizontal plane by a pull of $100\sqrt{2}$ N making 45° with horizontal.

14. Find the value of normal reaction R
(A) 300 N (B) 400 N
(C) 200 N (D) 500 N

15. Find the coefficient of friction
(A) 0.32 (B) 0.33
(C) 0.25 (D) 0.28

Direction for question 16, 17, 18: For the mass-pulley system shown, the mass m_2 = 5 Kg is placed on a smooth inclined plane of inclination θ where as mass m_1 = 5 Kg is a hanging force. If acceleration of the system is 1.5 m/s^2.



16. The inclination of the plane will be
(A) 41.52° (B) 35.50°
(C) 52.15° (D) 43.96°

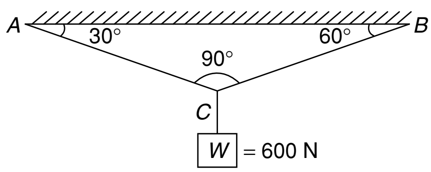
17. The tension in the string will be
(A) 41.55 N (B) 35.15 N
(C) 21.5 N (D) 25.28 N

18. How the acceleration of the system would be affected of each mass is doubted
(A) 3 m/s^2 (B) 2 m/s^2
(C) 1.5 m^2 (D) 2.5 m/s^2

19. A block is sliding down an incline of 30° with an acceleration $\frac{g}{4}$. Then the kinetic coefficient of friction is

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{\sqrt{3}}$
(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2\sqrt{3}}$

Direction for question for 20 and 21: A 600 N weight is suspended by flexible cables as shown in figure



20. The tension in the wire BC will be
(A) 519.6 (B) 613.4
(C) 318 (D) 435.5

21. The tension in the wire AC will be
(A) 256 (B) 300
(C) 311 (D) 288

22. The smallest angle θ for equilibrium of the homogenous ladder of length l is, when coefficient of friction for all surfaces is assumed as μ:

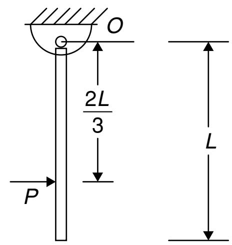
(A) $\tan^{-1}\left(\frac{1-\mu^2}{2\mu}\right)$

(B) $\tan^{-1}\frac{\mu^2}{2}$

(C) $\tan^{-1}\left(\frac{2\mu}{1-\mu}\right)$

(D) $\tan^{-1}\left(\frac{\mu^2-1}{2}\right)$

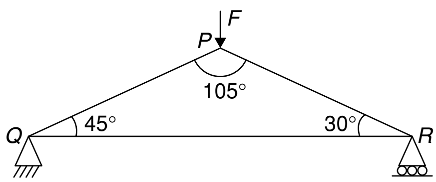
23. The reaction at the hinge when a rigid rod of mass 'm' and length 'L' is subjected to a force 'P' as shown



(A) $-P$ (B) 0

(C) $\frac{P}{3}$ (D) $\frac{2P}{3}$

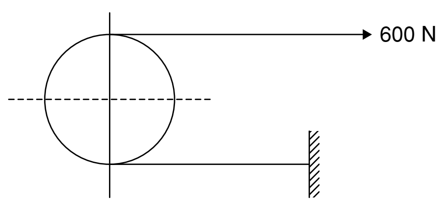
24. In the figure shown tension in the member QR is



(A) 0.732 F (B) 0.63 F
(C) 0.433 F (D) 0.75 F

25. Force in member QR
(A) 0.633 F (B) 0.75 F
(C) 0.732 F (D) 0.433 F

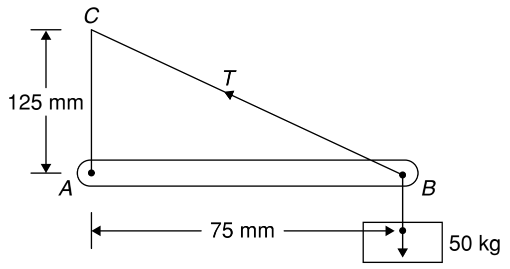
26. A force of 600 N is applied to the brake drum of 0.6 m diameter in a band brake. System as shown in below figure, where the wrapping angle is 180°c. If the coefficient of friction between the drum and band is 0.25, the breaking lorque applied, in Nm is



(A) 97.8 N (B) 16 N
(C) 22.1 N (D) 15.7 N

27. A circular roller of weight 200 N and radius of 0.8 m hangs by a tie rod of length 2 m and rests on a smooth vertical wall as shown in figure. The tension 'T' in the tie rod will be
(A) 219.78
(B) 239.2
(C) 310.30
(D) 250.5

28. A mass of 50 kg is suspended from a weight less bar '*AB*' which is supported by a cable *BC* and pinned at '*A*' as shown in figure. The Pin reactions at '*A*' on the bar *AB* are



(A) $R_x = 343.4$ N, $R_y = 755.4$ N
(B) $R_x = 343.4$ N, $R_y = 0$
(C) $R_x = 1080$ N; $R_y = 0$
(D) $R_x = 755.$ N, $R_y = 0$

Direction for questions 28 and 29: All the forces acting on a particle are situated at the origin of the two dimensional reference frame. One force has a magnitude of 10 N acting in the positive '*X*' direction, whereas the other has a magnitude of 5 N acting at an angle of 120° directed away from the origin

29. The value of the resultant force will be.
(A) 5.88 N (B) 7.2 N
(C) 7.98 N (D) 8.66 N

30. The value of α made by resultant with the horizontal force will be
(A) 43° (B) 30°
(C) 78° (D) 80°

Answer Keys

1. D	**2.** A	**3.** C	**4.** A	**5.** A	**6.** A	**7.** D	**8.** D	**9.** B	**10.** C
11. B	**12.** B	**13.** C	**14.** B	**15.** C	**16.** D	**17.** A	**18.** C	**19.** D	**20.** A
21. B	**22.** A	**23.** C	**24.** A	**25.** A	**26.** A	**27.** A	**28.** C	**29.** D	**30.** B