

Class XI Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 8

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there is some internal choice in some questions.
2. Section A has 18 MCQ's and 02 Assertion Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer(VSA) questions of 2 marks each.
4. Section C has 6 Short Answer(SA) questions of 3 marks each.
5. Section D has 4 Long Answer(LA) questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (04 marks each) with sub parts.
7. Internal Choice is provided in 2 questions in Section-B, 2 questions in Section-C, 2 Questions in Section-D. You have to attempt only one alternatives in all such questions.

Section A

1. If $P(A \cap B) = \frac{1}{2}$ and $P(A' \cap B') = \frac{1}{3}$, $P(A) = p$ and $P(B) = 2p$, then the value of p is: [1]
a) $\frac{4}{9}$ b) $\frac{1}{3}$
c) $\frac{7}{18}$ d) $\frac{1}{9}$
2. How many types of measures of dispersion are there? [1]
a) 3 b) 5
c) 2 d) 4
3. The amount at the compound interest which is calculated yearly on a certain sum of money is ₹ 1250 is one year [1]
and ₹ 1375 in two years. The rate of interest per annum is:
a) 9% b) 10%
c) 8% d) 11%
4. If $\frac{\log 243}{\log 27} = x$, then x is [1]
a) $\frac{5}{3}$ b) $\frac{3}{5}$
c) 3 d) 5
5. A relation ϕ from C to R is defined by $x \phi y \Leftrightarrow |x| = y$. Which one is correct? [1]
a) $(1 + i) \phi 2$ b) $3 \phi (-3)$
c) $i \phi 1$ d) $(2 + 3i) \phi 13$

6. Standard form of 192.8 is [1]
- a) 19.28×10^1 b) 1.928×10^2
c) 1.928 d) 0.1928×10^{-1}
7. 6 boys and 6 girls sit in a row at random. The probability that all the girls sit together is [1]
- a) $\frac{12}{431}$ b) $\frac{1}{432}$
c) $\frac{1}{112}$ d) $\frac{1}{132}$
8. Equation of the circle with centre lies on the y-axis and passing through the origin and the point (2, 3) is: [1]
- a) $3x^2 + 3y^2 - 13y = 0$ b) $x^2 + y^2 + 13y = 0$
c) $6x^2 + 6y^2 - 13x = 0$ d) $x^2 + y^2 + 13x + 3 = 0$
9. How many odd days are there in a century? [1]
- a) 4 b) 2
c) 3 d) 5
10. The mean deviation of the data 3, 10, 10, 4, 7, 10, 5 from the mean is [1]
- a) 3.75 b) 2.57
c) 3 d) 2
11. The product $\sqrt[3]{2} \cdot \sqrt[4]{2} \cdot \sqrt[12]{32}$ equals [1]
- a) $\sqrt[12]{2}$ b) $\sqrt[12]{32}$
c) 2 d) $\sqrt{2}$
12. Find the simple interest on ₹ 5200 for 2 years at 6% per annum: [1]
- a) ₹ 600 b) ₹ 624
c) ₹ 524 d) ₹ 450
13. The number of ways in which the letters of the word **CONSTANT** can be arranged without changing the relative positions of the vowels and consonants is [1]
- a) 444 b) 372
c) 360 d) 256
14. There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin? [1]
- a) $\frac{4}{9}$ b) $\frac{5}{9}$
c) $\frac{1}{9}$ d) $\frac{2}{9}$
15. If A and B are independent events, then $P(A \cap B)$ is equal to: [1]
- a) $\frac{P(A)}{P(B)}$ b) $P(A) \cdot P(B)$
c) $P(A) - P(B)$ d) $P(A) + P(B)$
16. The present value of an annuity of ₹ 3993 payable at the end of each year for 3 years, if money is worth 10% effective. [1]

a) ₹ 9039

b) ₹ 9903

c) ₹ 9930

d) ₹ 9390

17. 20 students can compete for a race. The number of ways in which they can win the first three places is (given that no two students finish in the same place). [1]

a) 1140

b) 8000

c) 6840

d) 6000

18. Let $A = \{1, 2, 3\}$, then the domain of the relation $R = \{(1, 1), (2, 3), (2, 1)\}$ defined on A is [1]

a) $\{1, 3\}$

b) $\{1, 2\}$

c) None of these

d) $\{2, 3\}$

19. **Assertion (A):** Mean deviation about median of the following data: 1, 3, 5, 7, 9, 11 is 3. [1]

Reason (R): Median of the observations 1, 3, 5, 7, 9, 11 is 5.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If the product of positive real numbers a, b, c is 27, then minimum value of $a + b + c$ is 9. [1]

Reason (R): For positive real numbers, $A.M. \leq G.M.$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. In an examination the average of 150 students is 25. The average of passed students is 40 and average of failed students is 15. Find the number of students who passed the examination. [2]

22. In a survey of 425 students in a school, it was found that 115 drink apple juice, 160 drink orange juice and 80 drink both apple as well as orange juice. How many drink neither apple juice nor orange juice? [2]

OR

If $A = \{x : x = 2n, n \in \mathbb{Z}\}$ and $B = \{x : x = 3n, n \in \mathbb{Z}\}$, then find $A \cap B$.

23. A can do a piece of work in 80 days. He works at it for 10 days and then B alone finishes the remaining work in 42 days. In how much time will A and B, working together, finish the work? [2]

24. Find the derivative of the given function: $\frac{ax+b}{cx+d}$ [2]

OR

Differentiate the function with respect to x : $\log \{x + 2 + \sqrt{x^2 + 4x + 1}\}$

25. Convert the decimal number to the binary number: 250 [2]

Section C

26. If AM and GM are between two positive numbers x and y are 13 and 12 respectively, find the numbers. [3]

OR

Find three numbers in G.P. whose product is 216 and the sum of their products in pairs is 156.

27. Find the equations of the lines for which $\tan \theta = \frac{1}{2}$, where θ is the inclination of the line and [3]

i. y-intercept is $-\frac{3}{2}$

ii. x-intercept is 4

28. Find the domain and the range of the given function: $f(x) = \frac{1}{\sqrt{5-x}}$ [3]
29. a. In what time will ₹ 85000 amount to ₹ 157675 at 4.5% p.a? [3]
 b. A sum of ₹ 46875 was lent out at simple interest and at the end of 1 year 8 months the total amount was ₹ 50,000. Find the rate of interest percent per annum.
30. Using logarithmic table, find the value of $\sqrt{\frac{41.32 \times 20.18}{12.69}}$. [3]
31. For a group of 60 boys students, the mean and S.D. of statistics marks are 45 and 2 respectively. The same figures for a group of 40 girls students are 55 and 3 respectively. What is the mean and S.D. of marks if the two groups are pooled together? [3]

Section D

32. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that one of them is black and other is red. [5]

OR

In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII, what is the probability that a student chosen randomly studies in class XII, given that the chosen student is a girl?

33. Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$ [5]
34. Find the mean deviation about the mean for the following data: [5]

Marks obtained	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Number of students	8	6	12	5	2	7

OR

Find the mean deviation from the median of the following frequency distribution:

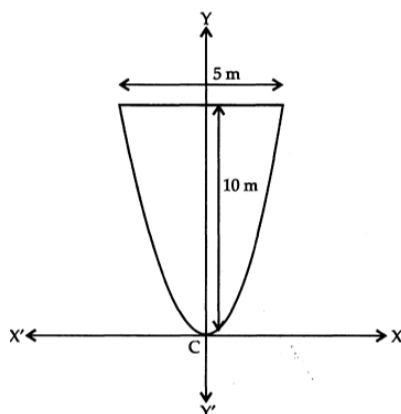
x_i	10	11	12	13	14	15	16
f_i	3	8	14	19	7	6	3

35. Find the equations of the lines through the point (3, 2) which make an angle of 45° with the line $x - 2y = 3$. [5]

Section E

36. Read the text carefully and answer the questions: [4]

Sachin is playing with long string, he hang the ends of the string at two points on the wall. Now, it is in the form of parabola with its vertical axis and is 10m high and 5 m wide at its base as shown in the following figure:



- (a) What is the standard equation of parabola in this case?
 (b) Parabola passes through the which point?
 (c) Find the value of a in the standard equation?

- (d) What is particular equation of parabol?

37. **Read the text carefully and answer the questions:**

[4]

In a library 25 students read physics, chemistry and mathematics books. It was found that 15 students read mathematics, 12 students read physics while 11 students read chemistry. 5 students read both mathematics and chemistry, 9 students read physics and mathematics. 4 students read physics and chemistry and 3 students read all three subject books.



- (a) Find the number of students who read none of the subject.
- (b) Find the number of students who read atleast one of the subject.
- (c) Find the number of students who read only one of the subjects.
- (d) Find the number of students who read only mathematics.

38. **Read the text carefully and answer the questions:**

[4]

Five friends Mohit, Sachin, Rohit, Mohan and kapil were playing in a ground, where they sit in a row in a straight line.



- (a) In how many ways these five students can sit in a row?
- (b) Total number of sitting arrangements if Mohit and Sachin sit together:
- (c) What are the possible arrangements if Rohit and Mohan sits at the extrement positions?
- (d) What are the possible orders if Kapil is sitting in the middle?

Solution

Section A

1.

(c) $\frac{7}{18}$

Explanation: Given $P(A' \cap B') = \frac{1}{2}$, $P(A' \cap B) = \frac{1}{3}$, $P(A) = p$, $P(B) = 2p$

Now, $P(A' \cap B') = (A \cup B)' = 1 - (A \cup B) = \frac{1}{3}$

$$\therefore P(A \cup B) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = \frac{2}{3}$$

$$\Rightarrow p + 2p - \frac{1}{2} = \frac{2}{3}$$

$$\Rightarrow 3p = \frac{2}{3} + \frac{1}{2} = \frac{4+3}{6} = \frac{7}{6} \Rightarrow p = \frac{7}{18}$$

2.

(b) 5

Explanation: There are five most commonly used measures of dispersion. These are range, variance, standard deviation, mean deviation and quartile deviation.

3.

(b) 10%

Explanation: ₹ 1250 is the interest of first year and ₹ 1375 is the interest in second year. Here, the difference is of ₹ 125 which is the interest obtained ₹ 1250.

Let rate be $r\%$

$$\therefore \frac{1250 \times r \times 1}{100} = 125$$

$$\Rightarrow r = \frac{125 \times 100}{1250} = 10\%$$

4.

(a) $\frac{5}{3}$

Explanation: $\frac{\log 243}{\log 27} = x$

$$\frac{\log 3^5}{\log 3^3} = x$$

$$x \log 3^3 = \log 3^5$$

$$\log (3^3)^x = \log 3^5$$

$$\log 3^{3x} = \log 3^5$$

$$\therefore 3x = 5$$

$$x = \frac{5}{3}$$

5.

(c) i ϕ 1

Explanation: We have $x \phi y$ given by, $|x| = y$

i ϕ 1

$x = i$;

$$|x| = \sqrt{1^2}$$

$$= 1$$

$$1 = 1$$

$$|x| = y.$$

6.

(b) 1.928×10^2

Explanation: Standard form of

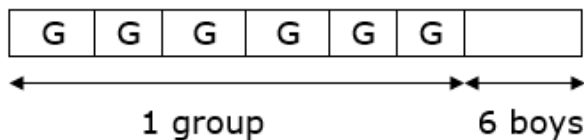
192.8

$$= 1.928 \times 10^2$$

7.

(d) $\frac{1}{132}$

Explanation: If all the girls sit together, then consider it as 1 group.



Total number of persons = 6 + 1 = 7 persons

\therefore Total number of arrangements in a row of 7 persons = 7!

and the girls interchange their seats in 6! ways.

Required Probability = $\frac{\text{Number of favourable outcome}}{\text{Total number of outcomes}}$

\therefore Required probability = $\frac{6!7!}{12!}$

= $\frac{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 7!}{12 \times 11 \times 10 \times 9 \times 8 \times 7!}$

= $\frac{6 \times 5 \times 4 \times 3 \times 2}{12 \times 11 \times 10 \times 9 \times 8}$

= $\frac{1}{2 \times 11 \times 2 \times 3 \times 2}$

= $\frac{1}{132}$

Hence, the correct option

8. (a) $3x^2 + 3y^2 - 13y = 0$

Explanation: Given that the circle with centre lies on the y-axis and passing through the origin.

General equation of the circle is

$(x - 0)^2 + (y - k)^2 = k^2$

It passes through (2, 3)

i.e. $2^2 + (3 - k)^2 = k^2$

$\Rightarrow 4 + 9 + k^2 - 6k = k^2$

$\Rightarrow k = \frac{13}{6}$

Therefore, the required equation of the circle is

$x^2 + \left(y - \frac{13}{6}\right)^2 = \left(\frac{13}{6}\right)^2$

$\Rightarrow 3x^2 + 3y^2 - 13y = 0$

$3x^2 + 3y^2 - 13y = 0$

9.

(d) 5

Explanation: In a century, we have 24 leap years and 76 non-leap years, so number of odd days

= $2 \times 24 + 1 \times 76 = 48 + 76 = 124$

= $7 \times 17 + 5 = 5$ odd days

10.

(b) 2.57

Explanation: Given data is 3, 10, 10, 4, 7, 10, 5. They are total 7.

Here mean, $\bar{x} = \frac{3+10+10+4+7+10+5}{7} = \frac{49}{7} = 7$

This can be written in table form as,

Data (x_i)	$d_i = x_i - \bar{x} $
3	= $ 3 - 7 = 4$
10	= $ 10 - 7 = 3$
10	= $ 10 - 7 = 3$
4	= $ 4 - 7 = 3$
7	= $ 7 - 7 = 0$
10	= $ 10 - 7 = 3$

5	$= 5 - 7 = 2$
Total	$\Sigma d_i = 18$

Hence Mean Deviation becomes,

$$\text{M.D} = \frac{\Sigma d_i}{7} = \frac{18}{7} = 2.57$$

Therefore, the mean deviation about the mean of the distribution is 2.57

11.

(c) 2

$$\begin{aligned} \text{Explanation: } \sqrt[3]{2} \cdot \sqrt[4]{2} \cdot \sqrt[12]{32} &= 2^{\frac{1}{3}} \cdot 2^{\frac{1}{4}} \cdot (2^5)^{\frac{1}{12}} = 2^{\frac{1}{3}} \cdot 2^{\frac{1}{4}} \cdot 2^{\frac{5}{12}} = 2^{\frac{1}{3} + \frac{1}{4} + \frac{5}{12}} \\ &= 2^{\frac{12}{12}} = 2^1 \end{aligned}$$

12.

(b) ₹ 624

$$\text{Explanation: Given, } P = 5200, n = 2, i = 6\% = \frac{6}{100} = 0.06$$

$$I = Pit$$

$$\therefore I = 5200 \times 0.06 \times 2$$

$$\text{or, } I = 624.$$

13.

(c) 360

Explanation: The word CONSTANT consists of two vowels that are placed at the 2nd and 6th position, and six consonants.

The two vowels can be arranged at their respective places, i.e. 2nd and 6th place, in 2! ways.

The remaining 6 consonants can be arranged at their respective places in $\frac{6!}{2!2!}$ ways.

$$\therefore \text{Total number of arrangements} = 360$$

14. (a) $\frac{4}{9}$

Explanation: Let

E_1, E_2 and E_3 are events of selection of a two headed coin, biased coin and unbiased coin respectively.

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}.$$

Let A = event of getting head.

$$P(A/E_1) = 1, P(A/E_2) = \frac{3}{4}, P(A/E_3) = \frac{1}{2}.$$

$$\begin{aligned} P(E_1/A) &= \frac{P(A/E_1) \cdot P(E_1)}{P(A/E_1) \cdot P(E_1) + P(A/E_2) \cdot P(E_2) + P(A/E_3) \cdot P(E_3)} \\ &= \frac{\frac{1}{3} \cdot 1}{\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{1}{2}} = \frac{4}{9} \end{aligned}$$

15.

(b) $P(A) \cdot P(B)$

Explanation: Given A and B are independent events, then

$$P(A \cap B) = P(A) \cdot P(B)$$

16.

(c) ₹ 9930

$$\text{Explanation: } R = ₹ 3993, n = 3, i = 0.1$$

$$\therefore \text{Present value} = ₹ 3993 \left[\frac{1 - (1.1)^{-3}}{0.1} \right]$$

$$= ₹ 39930[1 - 0.7513] = ₹ 9930.$$

17.

(c) 6840

Explanation: For first place we have 20 students, for second we have 19 and for the third we have 18

$${}^{20}P_3 = 20 \times 19 \times 18$$

18.

(b) {1, 2}

Explanation: Since the domain is represented by the x- coordinate of the ordered pair (x, y). Therefore, the domain of the given relation is { 1, 2 }.

19.

(c) A is true but R is false.

Explanation: The data values in ascending order are 1, 3, 5, 7, 9, 11.

Here, total number of observations (n) = 6, which is even.

$$\begin{aligned}\text{So, median} &= \frac{\left(\frac{6}{2}\right)^{\text{th observation}} + \left(\frac{6}{2} + 1\right)^{\text{th observation}}}{2} \\ &= \frac{3^{\text{rd observation}} + 4^{\text{th observation}}}{2} \\ &= \frac{5+7}{2} = 6\end{aligned}$$

∴ R is false.

Now, mean deviation about median

$$\begin{aligned}\text{M.D.} &= \frac{|1-6| + |3-6| + |5-6| + |7-6| + |9-6| + |11-6|}{6} \\ &= \frac{5+3+1+1+3+5}{6} = 3\end{aligned}$$

∴ A is true.

20.

(c) A is true but R is false.

Explanation: We know that for positive numbers A.M. \geq G.M.

$$\begin{aligned}\Rightarrow \frac{a+b+c}{3} &\geq \sqrt[3]{a \cdot b \cdot c} \\ \Rightarrow \frac{a+b+c}{3} &\geq \sqrt[3]{27} \Rightarrow a + b + c \geq 3 \times 3 \\ \Rightarrow a + b + c &\geq 9 \\ \therefore \text{Minimum value of } a + b + c &\text{ is } 9. \\ \therefore \text{A is true. R is false.}\end{aligned}$$

Section B

21. Average of 150 students = 25

$$\therefore \text{Total of 150 students} = 150 \times 25 = 3750 \dots(i)$$

Let number of passed students be x

$$\therefore \text{Average of x students} = 40$$

$$\therefore \text{Total of x students} = 40x \dots(ii)$$

Average of (150 - x) students = 15

$$\therefore \text{Total of (150 - x) students} = (150 - x) \times 15 \dots(iii)$$

From (i), (ii) and (iii), we get

$$40x + (150 - x)15 = 3750$$

$$\Rightarrow 40x + 2250 - 15x = 3750 \Rightarrow 25x = 1500 \Rightarrow x = 60$$

$$\therefore \text{Passed students} = 60$$

22. Let U = set of all students surveyed;

A = set of all students who drink apple juice and B = set of all students who drink orange juice.

Then, we know that n(U) = 425, n(A) = 115, n(B) = 160 and n(A ∩ B) = 80.

$$\therefore \text{We have, } n(A \cup B) = n(A) + n(B) - n(A \cap B) = (115 + 160 - 80) = 195.$$

Set of students who drink neither apple juice nor orange juice = (A' ∩ B') = (A ∪ B)'

$$\Rightarrow n\{(A \cup B)'\} = n(U) - n(A \cup B) = (425 - 195) = 230.$$

Therefore, 230 students drink neither apple juice nor orange juice.

OR

$$A \cap B = \{x : x = 2n, n \in \mathbb{Z}\} \cap \{x : x = 3n, n \in \mathbb{Z}\}$$

$$= \{\dots -6, -4, -2, 0, 2, 4, 6, \dots\} \cap \{\dots, -9, -6, -3, 0, 3, 6, 9, \dots\}$$

$$= \{\dots, -6, 0, 6, 12, \dots\}$$

$$= \{x : x = 6n, n \in \mathbb{Z}\}$$

23. Work done by A in 10 days = $\frac{1}{80} \times 10 = \frac{1}{8}$

$$\text{Remaining work} = \left(1 - \frac{1}{8}\right) = \frac{7}{8}$$

Now, $\frac{7}{8}$ work is done by B in 42 days

$$\text{Whole work will be done by B in } \left(42 \times \frac{8}{7}\right) = 48 \text{ days}$$

$$\therefore \text{A's 1 day's work} = \frac{1}{80} \text{ and B's 1 days' work} = \frac{1}{48}$$

$$\therefore (\text{A} + \text{B})\text{'s 1 days' work} = \frac{1}{80} + \frac{1}{48} = \frac{8}{240} = \frac{1}{30}$$

Hence, both will finish the work in 30 days.

24. Let us assume, $f(x) = \frac{ax+b}{cx+d}$

$$\text{Let } u = ax + b \text{ and } v = cx + d$$

$$\therefore f(x) = \frac{u}{v}$$

$$\text{So, } f'(x) = \left(\frac{u}{v} \right)'$$

$$f'(x) = \frac{u'v - v'u}{v^2}$$

Finding u and v

$$u = ax + b$$

$$u' = a + 0$$

$$= a$$

$$v = cx + d$$

$$v' = c + 0$$

$$= c$$

$$f'(x) = \left(\frac{u}{v} \right)'$$

$$= \frac{u'v - v'u}{v^2}$$

$$= \frac{a(cx+d) - c(ax+b)}{(cx+d)^2}$$

$$= \frac{acx + ad - acx - cb}{(cx+d)^2}$$

$$= \frac{acx + ad - acx - cb}{(cx+d)^2}$$

$$= \frac{ad - cb}{(cx+d)^2}$$

$$\text{Hence, } f'(x) = \frac{ad - cb}{(cx+d)^2}$$

OR

$$\text{Let } y = \log \{x + 2 + \sqrt{x^2 + 4x + 1}\}$$

Differentiate both side with respect to x we get,

$$\frac{dy}{dx} = \frac{d}{dx} \log [x + 2 + \sqrt{x^2 + 4x + 1}]$$

$$= \frac{1}{[x+2+\sqrt{x^2+4x+1}]} \cdot \frac{d}{dx} [x + 2 + (x^2 + 4x + 1)^{\frac{1}{2}}] \text{ [using chain rule]}$$

$$= \frac{1}{[x+2+\sqrt{x^2+4x+1}]} \times [1 + 0 + \frac{1}{2} (x^2 + 4x + 1)^{-\frac{1}{2}} \frac{d}{dx} (x^2 + 4x + 1)]$$

$$= \frac{1 + \frac{1}{2(\sqrt{x^2+4x+1})}}{[x+2+\sqrt{x^2+4x+1}]}$$

$$= \frac{\sqrt{x^2+4x+1} + x + 2}{[x+2+\sqrt{x^2+4x+1}] \times \sqrt{x^2+4x+1}}$$

$$= \frac{1}{\sqrt{x^2+4x+1}}$$

$$\text{So, } \frac{d}{dx} \log [x + 2 + \sqrt{x^2 + 4x + 1}] = \frac{1}{\sqrt{x^2+4x+1}}$$

25. Given decimal number is 250

2	250	
2	125	0
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
2	0	1

Put all the remainders together in reverse order.

The required binary number is 11111010

Section C

26. Given: $\frac{x+y}{2} = 13 \Rightarrow x + y = 26 \dots(i)$

and $\sqrt{xy} = 12 \Rightarrow xy = 144 \dots(ii)$

From (i) and (ii)

$$x(26 - x) = 144 \Rightarrow 26x - x^2 = 144$$

$$\Rightarrow x^2 - 26x + 144 = 0$$

$$\Rightarrow (x - 18)(x - 8) = 0$$

$$\Rightarrow x - 18 = 0 \text{ or } x - 8 = 0$$

$$\Rightarrow x = 18 \text{ or } 8$$

\therefore Numbers are 18, 8 or 8, 18.

OR

Let three numbers in G.P. be $\frac{a}{r}$, a , ar

\therefore Their product = $\frac{a}{r} \cdot a \cdot ar = 216$ (given)

$$\Rightarrow a^3 = 216 = (6)^3 \Rightarrow a = 6.$$

Also sum of their products in pairs = 156 (given)

$$\Rightarrow \frac{a}{r} \cdot a + a \cdot ar + ar \cdot \frac{a}{r} = 156$$

$$\Rightarrow a^2 \left(\frac{1}{r} + r + 1 \right) = 156$$

$$\Rightarrow 6^2 \cdot \frac{1+r^2+r}{r} = 156$$

$$\Rightarrow 3 \cdot \frac{r^2+r+1}{r} = 13$$

$$\Rightarrow 3r^2 + 3r + 3 = 13r$$

$$\Rightarrow 3r^2 - 10r + 3 = 0$$

$$\Rightarrow (r - 3) \left(r - \frac{1}{3} \right) = 0 \Rightarrow r = 3, \frac{1}{3}$$

When $r = 3$, numbers are 2, 6, 18 and when $r = \frac{1}{3}$, numbers are 18, 6, 2

27. Here slope of the line $m = \tan \theta = \frac{1}{2}$

i. y-intercept is $-\frac{3}{2}$ i.e. $c = -\frac{3}{2}$

Using slope-intercept form case I, the equation of the line is

$$y = \frac{1}{2}x + \left(-\frac{3}{2} \right) [\because y = mx + c]$$

$$\Rightarrow 2y = x - 3$$

$$\Rightarrow 2y - x + 3 = 0$$

ii. x-intercept is 4 i.e. $d = 4$

Using slope-intercept form case II, the equation of the line is

$$y = \frac{1}{2}(x - 4) [\because y = m(x - d)]$$

$$\Rightarrow 2y = x - 4$$

$$\Rightarrow 2y - x + 4 = 0$$

28. Given $f(x) = \frac{1}{\sqrt{5-x}}$

For D_f , $f(x)$ must be a real number

$$\Rightarrow \frac{1}{\sqrt{5-x}} \text{ must be a real number}$$

$$\Rightarrow 5 - x > 0 \Rightarrow 5 > x \Rightarrow x < 5$$

$$\Rightarrow D_f = (-\infty, 5)$$

For R_f , let $y = \frac{1}{\sqrt{5-x}}$

As $x < 5$, $0 < 5 - x$

$$\Rightarrow 5 - x > 0 \Rightarrow \sqrt{5-x} > 0$$

$$\Rightarrow \frac{1}{\sqrt{5-x}} > 0 \left(\because \frac{1}{a} > 0 \text{ if and only if } a > 0 \right)$$

$$\Rightarrow y > 0$$

$$\Rightarrow R_f = (0, \infty)$$

29. a. We know,

$$A = P(1 + it)$$

$$\Rightarrow 157675 = 85000 \left(1 + \frac{4.5}{100} \times t \right)$$

$$\Rightarrow \frac{157675}{85000} = \frac{100 + 4.5t}{100}$$

$$\Rightarrow 4.5t = \left[\frac{157675}{85000} \times 100 \right] - 100$$

$$\Rightarrow 4.5t = \frac{85.5}{4.5} = 19$$

\therefore In 19 years ₹ 85000 will amount to ₹ 157675 at 4.5% p.a. simple interest rate.

b. We know,

$$A = P(1 + it)$$

$$\Rightarrow 50000 = 46875 \left(1 + i \cdot 1 \frac{8}{12} \right)$$

$$\Rightarrow \frac{50000}{46875} = 1 + \frac{5}{3}i$$

$$\Rightarrow (1.067 - 1) \times \frac{3}{5} = i$$

$$\Rightarrow i = 0.04$$

$$\Rightarrow i = 4\%$$

30. Let $x = \sqrt{\frac{41.32 \times 20.18}{12.69}}$. Then,

$$\log x = \log \left\{ \frac{41.32 \times 20.18}{12.69} \right\}^{\frac{1}{2}} = \frac{1}{2} \log \left\{ \frac{41.32 \times 20.18}{12.69} \right\}$$

$$\Rightarrow \log x = \frac{1}{2} [\log (41.32 \times 20.18) - \log 12.69] = \frac{1}{2} [\log 41.32 + \log 20.18 - \log 12.69]$$

$$\Rightarrow \log x = \frac{1}{2} [1.6162 + 1.3049 - 1.1035] = \frac{1}{2} (2.9211 - 1.1035) = \frac{1}{2} (1.8176) = 0.9088$$

$$\therefore x = \text{antilog } (0.9088) = 8.106$$

31. As given $n_1 = 60$, $\bar{x}_1 = 45$, $\sigma_1 = 2$, $n_2 = 40$, $\bar{x}_2 = 53$, $\sigma_2 = 3$

Thus, combined mean is given by

$$\begin{aligned} \bar{x} &= \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} \\ &= \frac{60 \times 45 + 40 \times 55}{60 + 40} \end{aligned}$$

$$= 49$$

$$\text{Thus, } d_1 = \bar{x}_1 - \bar{x} = 45 - 49 = -4$$

$$d_2 = \bar{x}_2 - \bar{x} = 55 - 49 = 6$$

$$\text{Combined S.D.} = \sqrt{\frac{n_1 \sigma_1^2 + n_2 \sigma_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

$$= \sqrt{\frac{60 \times (2)^2 + 40 \times (3)^2 + 60 \times (-4)^2 + 40 \times (6)^2}{60 + 40}}$$

$$= \sqrt{30} = 5.48.$$

Section D

32. Given: A box containing 10 black and 8 red balls.

Total number of balls in box = 18

$$\text{Probability of getting a black ball in first draw} = \frac{10}{18} = \frac{5}{9}$$

As the ball is replaced after first throw,

Hence, Probability of getting a red ball in second draw = $\frac{8}{18} = \frac{4}{9}$

Now, Probability of getting first ball is black and second is red = $\frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$

Probability of getting a red ball in first draw = $\frac{8}{18} = \frac{4}{9}$

As the ball is replaced after first throw,

Hence, Probability of getting a black ball in second draw = $\frac{10}{18} = \frac{5}{9}$

Now, Probability of getting first ball is red and second is black = $\frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$

Therefore, Probability of getting one of them is black and other is red :

= Probability of getting first ball is black and second is red + Probability of getting first ball is red and second is black

$$= \frac{20}{81} + \frac{20}{81} = \frac{40}{81}$$

OR

Let 'A' be the event that the chosen student studies in class XII and B be the event that the chosen student is a girl.

There are 430 girls out of 1000 students

So, $P(B) = P(\text{Chosen student is girl}) = \frac{430}{1000} = \frac{43}{100}$

Since, 10% of the girls studies in class XII

So, total number of girls studies in class XII

$$= \frac{10}{100} \times 430 = 43$$

Then, $P(A \cap B) = P(\text{Chosen student is a girl of class XII})$

$$= \frac{43}{1000}$$

\therefore Required probability = $P(A/B)$

$$= \frac{P(A \cap B)}{P(B)} \quad \left[\because P(A/B) = \frac{P(A \cap B)}{P(B)} \right]$$

$$= \frac{43/1000}{43/100} = \frac{1}{10}$$

$$33. \lim_{x \rightarrow 0} \frac{1+x-1+x}{x[\sqrt{1+x}+\sqrt{1-x}]} \quad [\text{By rationalising}]$$

$$= \lim_{x \rightarrow 0} \frac{2}{\sqrt{1+x}+\sqrt{1-x}} = \frac{2}{1+1} = 1$$

$$34. \text{ Here length of class interval, } h = 10. \text{ Let the assumed mean be } A = 35.$$

Then, we prepare the following table by using assumed mean as given.

Marks obtained	Number of students (f_i)	Midpoint x_i	$d_i = \frac{x_i - 35}{10}$	$f_i d_i$	$ x_i - \bar{x} $	$f_i \times x_i - \bar{x} $
10 - 20	8	15	-2	-16	22	176
20 - 30	6	25	-1	-6	12	72
30 - 40	12	35 = A	0	0	2	21
40 - 50	5	45	1	5	8	40
50 - 60	2	55	2	4	18	36
60 - 70	7	65	3	21	28	196
	$N = \sum f_i$ = 40			$\sum f_i d_i = 8$		$\sum f_i \times x_i - \bar{x} $ = 544

$$N = \sum f_i = 40, \bar{x} = A + \left\{ \frac{\sum f_i d_i}{N} \times h \right\} = 35 + \left\{ \frac{8}{40} \times 10 \right\} = 37$$

$$\therefore MD(\bar{x}) = \frac{\sum f_i \times |x_i - \bar{x}|}{N} = \frac{544}{40} = \frac{136}{10} = 13.6.$$

OR

Here, $N = \sum f_i = 60$, which is even

$$\therefore \text{Median (M)} = \frac{\left(\frac{N}{2}\right)^{\text{th term}} + \left(\frac{N}{2} + 1\right)^{\text{th term}}}{2} = \frac{\left(\frac{60}{2}\right)^{\text{th term}} + \left(\frac{60}{2} + 1\right)^{\text{th term}}}{2} = \frac{30^{\text{th term}} + 31^{\text{th term}}}{2}$$

$$= \frac{13+13}{2} = 13$$

We make the table from the given data:

Age (in yr), x_i	f_i	cf	$ x_i - M $	$f_i x_i - M $
10	3	3	3	9
11	8	11	2	16

12	14	25	1	14
13	19	44	0	0
14	7	51	1	7
15	6	57	2	12
16	3	60	3	9
Total	N=60			$\sum f_i x_i - M = 67$

Mean deviation from median, $MD = \frac{\sum f_i |x_i - M|}{N} = \frac{67}{60} = 1.12 \text{ yr}$

35. The given line is $x - 2y - 3 = 0$

$$\Rightarrow y = \frac{x}{2} - \frac{3}{2}$$

$$\therefore \text{Slope, } m_1 = \frac{1}{2}$$

Let ' m_2 ' be the slope of a line AB which passes through (3, 2).

Since the angle between the two line is 60°

$$\therefore \tan 45^\circ = \pm \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$\Rightarrow 1 = \pm \frac{m_2 - \frac{1}{2}}{1 + \frac{1}{2} m_2}$$

$$\Rightarrow 1 = \pm \frac{2m_2 - 1}{m_2 + 2}$$

$$\therefore \frac{2m_2 - 1}{m_2 + 2} = 1 \text{ or } \frac{2m_2 - 1}{m_2 + 2} = -1$$

$$\Rightarrow m_2 = 3 \text{ or } m_2 = -\frac{1}{3}$$

\therefore Equation of AB is

$$y - 2 = 3(x - 3)$$

$$\Rightarrow 3x - y = 7 \text{ (} m_2 = 3 \text{)}$$

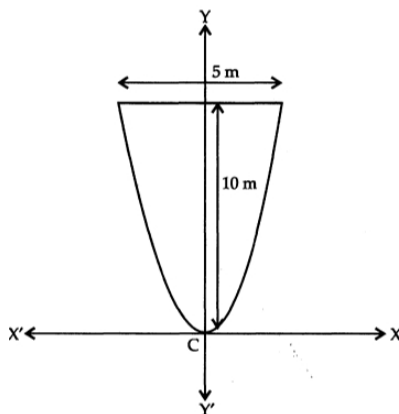
$$\text{or, } y - 2 = -\frac{1}{3}(x - 3) \text{ (} m_2 = -\frac{1}{3} \text{)}$$

$$\Rightarrow x + 3y = 9$$

Section E

36. Read the text carefully and answer the questions:

Sachin is playing with long string, he hang the ends of the string at two points on the wall. Now, it is in the form of parabola with its vertical axis and is 10m high and 5 m wide at its base as shown in the following figure:



(i) the Equation of the parabola is of the form $x^2 = 4ay$ (as it opening Upwards).

(ii) It can be clearly seen from the given figure that parabola passes through point $\left(\frac{5}{2}, 10\right)$.

(iii) It can be clearly seen that the parabola passes through point $\left(\frac{5}{2}, 10\right)$.

$$\left(\frac{5}{2}\right)^2 = 4a(10)$$

$$\Rightarrow a = \frac{25}{4 \times 4 \times 10}$$

$$= \frac{5}{32}$$

(iv) The equation of parabola is

$$x^2 = 4ay$$

$$x^2 = 4 \left(\frac{5}{32} \right) y = \left(\frac{5}{8} \right) y$$

37. Read the text carefully and answer the questions:

In a library 25 students read physics, chemistry and mathematics books. It was found that 15 students read mathematics, 12 students read physics while 11 students read chemistry. 5 students read both mathematics and chemistry, 9 students read physics and mathematics. 4 students read physics and chemistry and 3 students read all three subject books.



$$\begin{aligned} \text{(i) Atleast one} &= 11 + 9 + 5 + 4 - 2(3) \\ &= 29 - 6 = 23 \\ \Rightarrow \text{None} &= 25 - 23 = 2 \end{aligned}$$

(ii) The number of students who reading atleast one of the subject is 23.

$$\text{(iii) Only maths} = 15 - 9 - 5 + 3 = 4$$

$$\text{Only physics} = 12 - 9 - 4 + 3 = 2$$

$$\text{Only chemistry} = 5 \Rightarrow \text{Total} = 11$$

(iv) The number of students who reading only mathematics is 4.

38. Read the text carefully and answer the questions:

Five friends Mohit, Sachin, Rohit, Mohan and kapil were playing in a ground, where they sit in a row in a straight line.



$$\text{(i) Total number of ways} = 5! = 120$$

(ii) Two position are fixed for Mohit and Sachin therefore considering it as one unit, total students

$$\text{left} = 3 + 1 = 4$$

$$\text{Total possible arrangement} = 4! \times 2! = 48$$

$$\text{(iii) Total possible arrangements} = 3! \times 2! = 12$$

$$\text{(iv) Total possible arrangements} = 4! = 24$$