Chapter 4 Matrices and Determinants

Ex 4.3

Answer 1e.

The zeros of a function are the values for which the function becomes zero. This means that the value of y in $y = f(x)$ will be zero for those values.

When the graph of this function is drawn, it crosses or touches the x-axis at these points. Thus, the zeros of a function are also the x-intercepts of the function.

Answer 1gp.

The given expression is of the form $x^2 + bx + c$. You have to find two factors $x + m$ and $x + n$ such that mn is -18 and $m + n$ is -3.
$x^2 - 3x - 18 = (x + m)(x + n)$

List all the factors of -18 and find the sum of factors.

Factors of -18: m, n	$-1, 18$	$1, -18$	$-2, 9$	$2, -9$	$-3, 6$	$3, -6$
Sum of factors: $m + n$	17	-17	7	-7	3	-3

You can notice that the factors $m = 3$ and $n = -6$ gives mn as -18 and $m + n$ as -3.

Therefore, $x^2 - 3x - 18 = (x + 3)(x - 6)$.

Answer 2e.

A monomial is an expression containing only one term.
For example, $3x$ is a monomial.
A binomial is an expression containing two terms.
For example, $3x+2$ is a binomial.
A trinomial is an expression containing three terms.
For example, x^2+2x+3 is a trinomial.

Answer 2gp.

Consider the expression

$n^2 - 3n + 9$

It is need to factor the expression.

To factor $n^2 - 3n + 9$, we need to find integers a and b such that

$$n^2 - 3n + 9 = (n+a)(n+b)$$
$$= n^2 + (a+b)n + ab$$

That is,

$$n^2 - 3n + 9 = n^2 + (a+b)n + ab$$

Comparing both sides the coefficients of n^2, n and constant terms, we get

$a + b = -3$ and $ab = 9$

Factors of 9 : a, b	1,9	−1,−9	3,3	−3,−3
Sum of factors : $a+b$	10	−10	6	−6

There are no factor a and b such that $a + b = -3$.

So $n^2 - 3n + 9$ cannot be factored.

Answer 3e.

The given expression is of the form $x^2 + bx + c$. You have to find two factors $x + m$ and $x + n$ such that mn is 6 and $m + n$ is 5.

$$x^2 + 6x + 5 = (x + m)(x + n)$$

List all the factors of 5 and find the sum of factors.

Factors of 5: m, n	$-1, -5$	$1, 5$
Sum of factors: $m + n$	-6	6

You can notice that the factors $m = 1$ and $n = 5$ gives mn as 5 and $m + n$ as 6.

Therefore, $x^2 + 6x + 5 = (x + 1)(x + 5)$.

Answer 3gp.

The given expression is of the form $x^2 + bx + c$. You have to find two factors $x + m$ and $x + n$ such that mn is -63 and $m + n$ is 2.

$r^2 + 2r - 63 = (r + m)(r + n)$

List all the factors of -63 and find the sum of factors.

Factors of -63: m, n	$-1, 63$	$1, -63$	$-3, 21$	$3, -21$	$-7, 9$	$7, -9$
Sum of factors: $m + n$	62	-62	18	-18	2	-2

You can notice that the factors $m = -7$ and $n = 9$ gives mn as -63 and $m + n$ as 2.

Therefore, $r^2 + 2r - 63 = (r - 7)(r + 9)$.

Answer 4e.

To factor $x^2 - 7x + 10$, we need to find integers m and n such that

$$x^2 - 7x + 10 = (x + m)(x + n)$$
$$= x^2 + (m + n)x + mn$$

That is,

$$x^2 - 7x + 10 = x^2 + (m + n)x + mn$$

Comparing both sides the coefficients of x^2, x and constant terms, we get

$m + n = -7$ and $mn = 10$

Factors of 10 : m, n	$1, 10$	$-1, -10$	$2, 5$	$-2, -5$
Sum of factors : $m + n$	11	-11	7	-7

Notice that $m = -2$ and $n = -5$

So $x^2 - 7x + 10$ can be factored as $(x - 2)(x - 5)$

Answer 4gp.

Consider the quadratic expression

$x^2 - 9$

To factorize the above quadratic expression

$x^2 - 9 = x^2 - 3^2$ Difference of two squares

$= (x + 3)(x - 3)$ Using $a^2 - b^2 = (a - b)(a + b)$

Answer 5e.

The given trinomial is of the form $x^2 + bx + c$, which when factored will be $(x + m)(x + n)$, where the product of m and n gives c, and their sum gives b.

Compare the given equation with $x^2 + bx + c = 0$. The value of b is -13 and of c is 22. We need to find m and n such that their product gives 22 and sum gives -13.

List the factors of 22 and find their sums.

Factors of 22: m, n	1, 22	2, 11	$-1, -22$	$-2, -11$
Sum of factors: $m + n$	23	13	-23	-13

From the table, it is clear that $m = -2$ and $n = -11$ gives the product 22 and sum -13.

Therefore, the given trinomial can be factored as $(a - 2)(a - 11)$.

Answer 5gp.

Rewrite the given expression as difference of two squares.
$q^2 - 100 = (q)^2 - (10)^2$

Use the formula $a^2 - b^2 = (a + b)(a - b)$.
$(q)^2 - (10)^2 = (q + 10)(q - 10)$

Therefore, the given expression can be factored as $(q + 10)(q - 10)$.

Answer 6e.

To factor $r^2 + 15r + 56$, we need to find integers m and n such that

$$r^2 + 15r + 56 = (r + m)(r + n)$$
$$= r^2 + (m + n)r + mn$$

That is,

$$r^2 + 15r + 56 = r^2 + (m + n)r + mn$$

Comparing both sides the coefficients of r^2, r and constant terms, we get
$m + n = 15$ and $mn = 56$

Factors of 56: m, n	1, 56	$-1, -56$	2, 28	$-2, -28$	4, 14	$-4, -14$	7, 8	$-7, -8$
Sum of factors: $m + n$	57	-57	30	-30	18	-18	15	-15

Notice that $m = 7$ and $n = 8$
So $r^2 + 15r + 56$ can be factored as $(r + 7)(r + 8)$

Answer 6gp.

Consider the quadratic expression

$y^2+16y+64$

To factorize the quadratic expression

$$y^2+16y+64=y^2+2(y)(8)+8^2 \quad \text{Perfect square trinomial}$$

$$=(y+8)^2 \quad \text{Using } a^2+2ab+b^2=(a+b)^2$$

Answer 7e.

The given expression is of the form x^2+bx+c. You have to find two factors $p+m$ and $p+n$ such that mn is 4 and $m+n$ is 2.

$p^2+2p+4=(p+m)(p+n)$

List all the factors of 4 and find the sum of factors.

Factors of 4: m, n	$-1, -4$	1, 4	$-2, -2$	2, 2
Sum of factors: $m+n$	-5	5	-4	4

You can notice that no factors gives mn as 4 and $m+n$ as 2.

Therefore, the given expression cannot be factored.

Answer 7gp.

Check whether the given expression is a perfect square trinomial.

The first condition for an expression to be a perfect square trinomial is that the first and last terms of the trinomial must be perfect squares.

The first term of the given expression is w^2, which is the perfect square of w. Similarly, the last term 81 is the perfect square of 9. Thus, the first condition is satisfied.

The second condition is that the middle term must be twice the product of the square roots of the first and last terms of the trinomial.

The square root of w^2 is w and of 81 is 9. Twice the product of w and 9 will be $2(w)(9)$ or $18w$. Since the middle term of the given expression is also $18w$ (except for the negative sign), the second condition has also been satisfied.

Factor using the special factoring pattern $a^2-2ab+b^2=(a-b)^2$.

$$w^2-18w+81 = w^2-2(w)(9)+9^2$$

$$= (w-9)^2$$

Therefore, the given expression can be factored as $(w-9)^2$.

Answer 8e.

To factor $q^2-11q+28$, we need to find integers m and n such that

$$q^2-11q+28=(q+m)(q+n)$$
$$=q^2+(m+n)q+mn$$

That is,

$$q^2-11q+28=q^2+(m+n)q+mn$$

Comparing both sides the coefficients of q^2, q and constant terms, we get
$m+n=-11$ and $mn=28$

Factors of 28 : m, n	$1, 28$	$-1, -28$	$2, 14$	$-2, -14$	$4, 7$	$-4, -7$
Sum of factors : $m+n$	29	-29	16	-16	11	-11

Notice that $m=-4$ and $n=-7$
So $q^2-11q+28$ can be factored as $(q-4)(q-7)$

Answer 8gp.

Consider the equation
$x^2-x-42=0$
It is need to solve the equation.
If the left side of $x^2-x-42=0$ can be factored, then the equation can be solved using the zero product property.
To factor x^2-x-42, we need to find integers m and n such that

$$x^2-x-42=(x+m)(x+n)$$
$$=x^2+(m+n)x+mn$$

That is,

$$x^2-x-42=x^2+(m+n)x+mn$$

Comparing both sides the coefficients of x^2, x and constant terms, we get
$m+n=-1$ and $mn=-42$

Factors of -42 : m, n	$2, -21$	$3, -14$	$6, -7$	$7, -6$
Sum of factors : $m+n$	-19	-11	-1	1

Notice that $m=6$ and $n=-7$
So $x^2-x-42=(x+6)(x-7)$

Answer 9e.

The given trinomial is of the form $x^2 + bx + c$, which when factored will be $(x + m)(x + n)$, where the product of m and n gives c, and their sum gives b.

Compare the given equation with $x^2 + bx + c = 0$. Thus, we need to find m and n such that their product gives -40 and sum gives 3.

List the factors of -40 and find their sums.

Factors of -40: m, n	$-1, 40$	$-2, 20$	$-4, 10$	$-5, 8$	$1, -40$	$2, -20$	$4, -10$	$5, -8$
Sum of factors: $m + n$	39	18	6	3	-39	-18	-6	-3

From the table, it is clear that $m = -5$ and $n = 8$ gives the product -40 and sum 3.

Therefore, the given trinomial can be factored as $(b - 5)(b + 8)$.

Answer 9gp.

The new length of the rectangular field will be $1000 + x$ and the new width will be $300 + x$. The new area thus obtained will be $2(1000)(300)$.

$$\begin{array}{ccccc} \text{New area} & = & \text{New length} & \cdot & \text{New width} \\ \text{(square meters)} & & \text{(meters)} & & \text{(meters)} \\ \Downarrow & & \Downarrow & & \Downarrow \\ 2(1000)(300) & = & (1000 + x) & \cdot & (300 + x) \end{array}$$

Multiply the right side using the FOIL method.

$$600{,}000 = 300{,}000 + 1000x + 300x + x^2$$

$$600{,}000 = 300{,}000 + 1300x + x^2$$

Subtract 600,000 from both the sides to rewrite the equation in the standard form.

$$600{,}000 - 600{,}000 = 300{,}000 + 1300x + x^2 - 600{,}000$$

$$0 = x^2 + 1300x - 300{,}000$$

Factor the right side of the equation. For this, find two numbers with product $-300{,}000$ and sum 1300. Two such numbers are 1500 and -200.
Thus, the equation becomes
$0 = (x + 1500)(x - 200)$.

Use the zero product property. According to this property, if the product of two expressions is zero, then one or both of the expressions equal zero.
Thus, either
$x + 1500 = 0$ or
$x - 200 = 0$.

Solve both the equations.
Subtract 1500 from the first equation.

$$x + 1500 - 1500 = 0 - 1500$$
$$x = -1500$$

Add 200 to both the sides of the second equation.

$$x - 200 + 200 = 0 + 200$$
$$x = 200$$

Since the increase in length cannot be negative, reject $x = -1500$.
The new length will be $1000 + 200 = 1200$, and the new width will be $300 + 200$.

Therefore, the new dimensions of the field will be 1200 meters by 500 meters.

Answer 10e.

Consider the quadratic expression $x^2 - 4x - 12$
Let $x^2 - 4x - 12 = (x+m)(x+n)$ Where $m \cdot n = -12$
Construct the table,

Factors of mn	$-1,12$	$1,-12$	$6,-2$	$-6,2$
Sum of factors, $m+n$	11	-11	4	-4

Using this table, $x^2 - 4x - 12$ can be written as

$$x^2 - 4x - 12 = x^2 + 2x - 6x - 12$$
$$= x(x+2) - 6(x+2)$$
$$= \boxed{(x+2)(x-6)}.$$

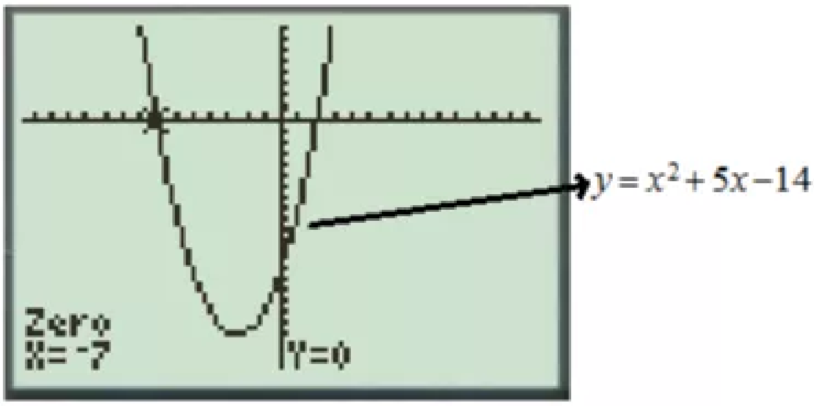
Answer 10gp.

Consider the function
$y = x^2 + 5x - 14$
It is need to find the zeros of the function by rewriting the function in intercept form.

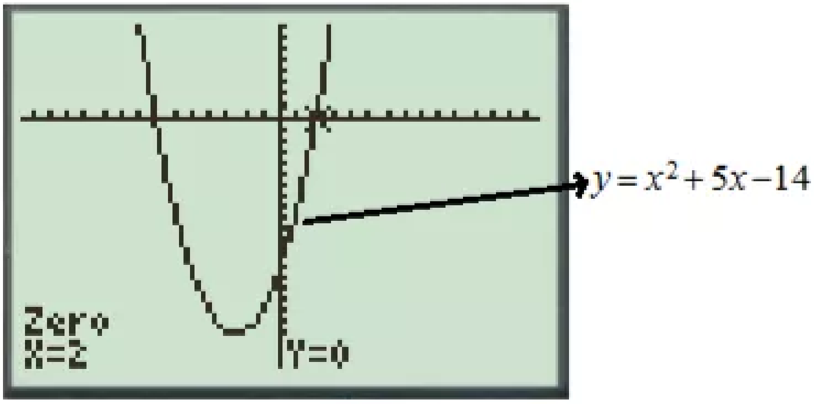
$y = x^2 + 5x - 14$ Write original function
$= x^2 + 7x - 2x - 14$
$= x(x+7) - 2(x+7)$
$= (x+7)(x-2)$ Factor

Therefore, -7 and 2 are the zeros of the given quadratic function.

Check:
Graph $y = x^2 + 5x - 14$
One of the zero $(-7, 0)$ is shown.



The second zero $(2, 0)$ is shown



Answer 11e.

The given expression is of the form $x^2 + bx + c$. You have to find two factors $x + m$ and $x + n$ such that mn is -18 and $m + n$ is -7.
$x^2 - 7x - 18 = (x + m)(x + n)$

List all the factors of -18 and find the sum of factors.

Factors of -18: m, n	$-1, 18$	$1, -18$	$-2, 9$	$2, -9$	$-3, 6$	$3, -6$
Sum of factors: $m + n$	17	-17	7	-7	3	-3

You can notice that the factors $m = 2$ and $n = -9$ gives mn as -18 and $m + n$ as -7.

Therefore, $x^2 - 7x - 18 = (x + 2)(x - 9)$.

Answer 11gp.

Factor the right side of the given function. For this, you have to find two numbers with product -30 and sum -7. Two such numbers are -10 and 3.

Rewrite the given function.
$y = (x - 10)(x + 3)$

The value of the function will be zero when x takes the value 10 and -3.

Therefore, the zeros of the given function are 10 and -3.

Answer 12e.

Consider the quadratic expression $c^2-9c-18$
Let $c^2-9c-18=(c+m)(c+n)$ where, $mn=-18$
We need to factorize the above quadratic expression

Factors of $-18: m,n$	$-2,9$	$2,-9$	$3,-6$	$-3,6$	$-1,18$	$1,-18$
Sum of factors: $m+n$	7	-7	-3	3	17	-17

There are no factors m and n such that $m+n=9$
Therefore the given expression is not easily factorizable.

Answer 12gp.

Consider the function
$f(x)=x^2-10x+25$
It is need to find the zeros of the function by rewriting the function in intercept form.

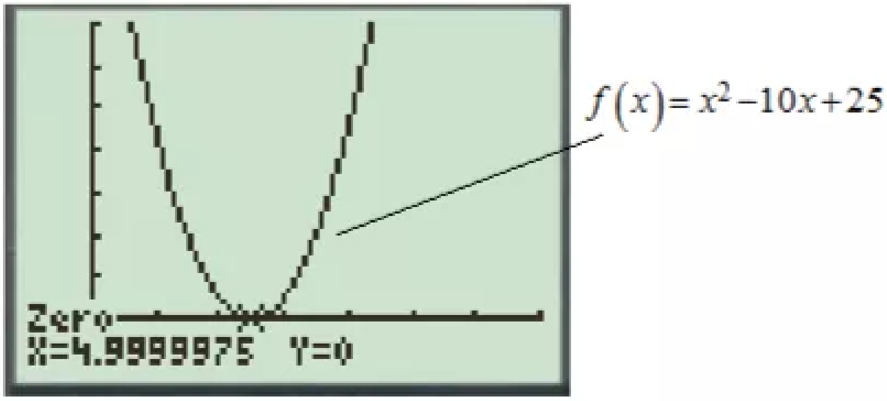
$f(x)=x^2-10x+25$ Write original function
$=x^2-2(x)(5)+5^2$ Perfect square trinomial
$=(x-5)^2$ Using $a^2-2ab+b^2=(a-b)^2$
$=(x-5)(x-5)$ Factor

Therefore, 5 and 5 are the zeros of the given quadratic function

Check:

Graph $f(x)=x^2-10x+25$

The graph passes through $(5,0)$ as shown.



Answer 13e.

The given trinomial is of the form x^2+bx+c, which when factored will be $(x+m)(x+n)$, where the product of m and n gives c, and their sum gives b.

Compare the given equation with $x^2+bx+c=0$. Thus, we need to find m and n such that their product gives -36 and sum gives 9.

List the factors of -36 and find their sums.

Factors of $-36: m, n$	$-1, 36$	$-2, 18$	$-3, 12$	$-4, 9$	$6, -6$	$1, -36$	$2, -18$	$3, -12$	$4, -9$
Sum of factors: $m+n$	35	16	9	5	0	-35	-16	-9	-5

From the table, it is clear that $m=-3$ and $n=12$ gives the product -36 and sum 9.

Therefore, the given trinomial can be factored as $(x-3)(x+12)$.

Answer 14e.

Consider the quadratic expression $m^2+8m-65$

Factorize the above quadratic expression

$$m^2+8m-65=m^2+13m-5m-65$$

$$=m(m+13)-5(m+13) \qquad (\text{Since}, 8m=13m-5m)$$

$$=\boxed{(m+13)(m-5)}$$

Answer 15e.

Rewrite the given expression as difference of two squares.
$x^2 - 36 = (x)^2 - (6)^2$

Use the formula $a^2 - b^2 = (a + b)(a - b)$.
$(x)^2 - (6)^2 = (x + 6)(x - 6)$

Therefore, the given expression can be factored as $(x + 6)(x - 6)$.

Answer 16e.

Consider the quadratic expression $b^2 - 81$
We need to factorize the above quadratic expression

$b^2 - 81 = b^2 - 9^2$ It is in the form $a^2 - b^2$

$= \boxed{(b+9)(b-9)}$ $a^2 - b^2 = (a+b)(a-b)$

Answer 17e.

Check whether the given expression is a perfect square trinomial.

The first condition for an expression to be a perfect square trinomial is that the first and last terms of the trinomial must be perfect squares.

The first term of the given expression is x^2, which is the perfect square of x. Similarly, the last term 144 is the perfect square of 12. Thus, the first condition is satisfied.

The second condition is that the middle term must be twice the product of the square roots of the first and last terms of the trinomial.

The square root of x^2 is x and of 144 is 12. Twice the product of x and 12 will be $2(x)(12)$ or $24x$. Since the middle term of the given expression is also $24x$ (except for the negative sign), the second condition has also been satisfied.

Factor using the special factoring pattern $a^2 - 2ab + b^2 = (a - b)^2$.

$$x^2 - 24x + 144 = x^2 - 2(x)(12) + 12^2$$
$$= (x - 12)^2$$

Therefore, the given expression can be factored as $(x - 12)^2$.

Answer 18e.

Consider the quadratic expression,

$t^2 - 16t + 64$

Factorize the above quadratic expression

$$t^2 - 16t + 64 = t^2 - 2(t)(8) + 8^2 \qquad \text{(Since, } (a-b)^2 = a^2 - 2ab + b^2\text{)}$$

$$= \boxed{(t-8)^2}$$

Answer 19e.

Check whether the given expression is a perfect square trinomial.

The first condition for an expression to be a perfect square trinomial is that the first and last terms of the trinomial must be perfect squares.

The first term of the given expression is x^2, which is the perfect square of x. Similarly, the last term 16 is the perfect square of 4. Thus, the first condition is satisfied.

The second condition is that the middle term must be twice the product of the square roots of the first and last terms of the trinomial.

The square root of x^2 is x and of 16 is 4. Twice the product of x and 4 will be $2(x)(4)$ or $8x$. Since the middle term of the given expression is $8x$, the second condition has also been satisfied.

Factor using the special factoring pattern $a^2 + 2ab + b^2 = (a+b)^2$.

$$x^2 + 8x + 16 = x^2 + 2(x)(4) + 4^2$$

$$= (x+4)^2$$

Therefore, the given expression can be factored as $(x+4)^2$.

Answer 20e.

618-4.3-20E

AID: 484 | 16/10/2012
RID: 1372 | 07/11/2012

Consider the quadratic expression,

$c^2 + 28c + 196$

Factorise the above quadratic expression

$$c^2 + 28c + 196 = c^2 + 2(c)(14) + 14^2 \qquad \text{(Since, } (a+b)^2 = a^2 + 2ab + b^2\text{)}$$

$$= (c+14)^2$$

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Answer 21e.

Check whether the given expression is a perfect square trinomial.

The first condition for an expression to be a perfect square trinomial is that the first and last terms of the trinomial must be perfect squares.

The first term of the given expression is n^2, which is the perfect square of n. Similarly, the last term 49 is the perfect square of 7. Thus, the first condition is satisfied.

The second condition is that the middle term must be twice the product of the square roots of the first and last terms of the trinomial.

The square root of n^2 is n and of 49 is 7. Twice the product of n and 7 will be $2(n)(7)$ or $14n$. Since the middle term of the given expression is also $14n$, the second condition has also been satisfied.

Factor using the special factoring pattern $a^2 + 2ab + b^2 = (a + b)^2$.

$$\begin{aligned} n^2 + 14n + 49 &= n^2 + 2(n)(7) + 7^2 \\ &= (n + 7)^2 \end{aligned}$$

Therefore, the given expression can be factored as $(n + 7)^2$.

Answer 22e.

618-4.3-22E

AID: 484 | 16/10/2012
RID: 1372 | 07/11/2012

Consider the quadratic expression,

$s^2 - 26s + 169$

Factorise the above quadratic expression

$$\begin{aligned} s^2 - 26s + 169 &= s^2 - 2(s)(13) + 13^2 \\ &= \boxed{(s-13)^2} \end{aligned}$$

(Since, $(a-b)^2 = a^2 - 2ab + b^2$)

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Answer 23e.

Rewrite the given expression as difference of two squares.

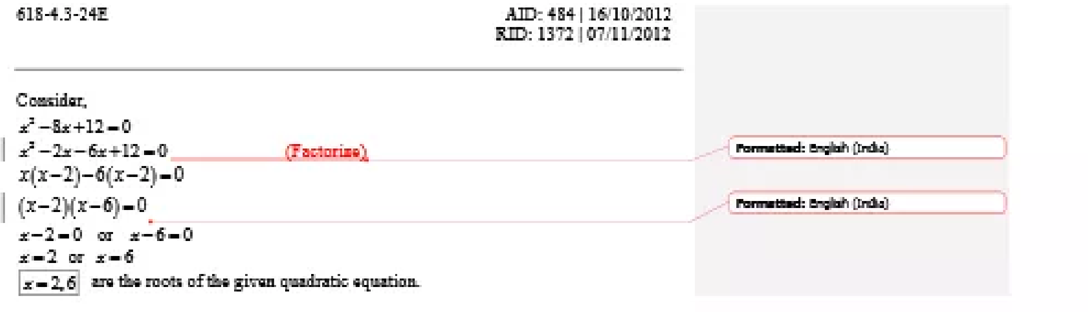
$z^2 - 121 = (z)^2 - (11)^2$

Use the formula $a^2 - b^2 = (a + b)(a - b)$.

$(z)^2 - (11)^2 = (z + 11)(z - 11)$

Therefore, the given expression can be factored as $(z + 11)(z - 11)$.

Answer 24e.



618-4.3-24E

AID: 484 | 16/10/2012
RID: 1372 | 07/11/2012

Consider,

$x^2 - 8x + 12 = 0$

$x^2 - 2x - 6x + 12 = 0$ (Factorise)

$x(x-2) - 6(x-2) = 0$

$(x-2)(x-6) = 0$

$x - 2 = 0$ or $x - 6 = 0$

$x = 2$ or $x = 6$

$\boxed{x = 2, 6}$ are the roots of the given quadratic equation.

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Answer 25e.

The given equation is of the form $ax^2 + bx + c = 0$. This equation can be solved using the Zero product property only if the left side can be factored.

The trinomial on the left side of the given equation is of the form $x^2 + bx + c$, which when factored will be of the form $(x + m)(x + n)$, where the product of m and n gives c, and their sum gives b.

Compare the trinomial with $x^2 + bx + c = 0$. The value of b is -11 and of c is 30. We need to find m and n such that their product gives 30 and sum gives -11.

List the factors of 30 and find their sums.

Factors of 30: m, n	1, 30	2, 15	3, 10	5, 6	$-1, -30$	$-2, -15$	$-3, -10$	$-5, -6$
Sum of factors: $m + n$	31	17	13	11	-31	-17	-13	-11

From the table, it is clear that $m = -5$ and $n = -6$ gives the product 30 and sum -11.

Thus, the equation becomes $(x - 5)(x - 6) = 0$.

Apply the Zero product property. This property states that if the product of two expressions is zero, then one or both of the expressions equal zero.

Thus, $x - 5 = 0$ or $x - 6 = 0$.

Solve the equations.

Add 5 to both sides of $x - 5 = 0$, and 6 to both sides of $x - 6 = 0$.

$$x - 5 + 5 = 0 + 5 \quad \text{or} \quad x - 6 + 6 = 0 + 6$$
$$x = 5 \quad \text{or} \quad x = 6$$

Thus, the solutions to the given equation are 5 and 6.

Answer 26e.

Consider,

$x^2+2x-35=0$

$x^2+7x-5x-35=0$

$x(x+7)-5(x+7)=0$ (Factorize)

$(x+7)(x-5)=0$

$x+7=0$ or $x-5=0$

$x=-7$ or $x=5$

$\boxed{x=-7,5}$ are the roots of the given quadratic equation.

Answer 27e.

Rewrite the given equation as difference of two squares.

$(a)^2-(7)^2=0$

Factor the right side of the equation using. $a^2-b^2=(a+b)(a-b)$.

$(a+7)(a-7)=0$

Use the zero product property. According to this property, if the product of two expressions is zero, then one or both of the expressions equal zero.

Thus, either

$a+7=0$ or $a-7=0$.

Subtract 7 from the both sides of the first equation.

$$a+7-7=0-7$$
$$a=-7$$

Add 7 to both sides of the second equation.

$$a-7+7=0+7$$
$$a=7$$

Therefore, the solutions are 7 and -7.

Answer 28e.

618-4.3-28E

AID: 484 | 16/10/2012
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Consider,

$b^2-6b+9=0$

$b^2-2(b)(3)+3^2=0$ Since, $(a-b)^2=a^2-2ab+b^2$

$(b-3)^2=0$

$b-3=0$

$\boxed{b=3}$ is the root of the given quadratic equation.

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Answer 29e.

The given equation is of the form $x^2 + bx + c = 0$. This equation can be solved using the Zero product property only if the left side can be factored.

The trinomial on the left side of the given equation is of the form $x^2 + bx + c$, which when factored will be of the form $(x + m)(x + n)$, where the product of m and n gives c, and their sum gives b.

Compare the trinomial with $x^2 + bx + c = 0$. The product is 4 and the sum is 5. We need to find m and n such that their product gives 4 and sum gives 5.

List the factors of 4 and find their sums.

Factors of 4 : m, n	1, 4	2, 2	−1, −4	−2, −2
Sum of factors : $m + n$	5	4	−5	−4

From the table, it is clear that $m = 1$ and $n = 4$ gives the product 4 and sum 5.

Thus, the equation becomes $(c + 1)(c + 4) = 0$.

Apply the Zero product property. This property states that if the product of two expressions is zero, then one or both of the expressions equal zero.

Thus, $c + 1 = 0$ or $c + 4 = 0$.

Solve the equations.

Subtract 1 from both sides of $c + 1 = 0$, and 4 from both sides of $c + 4 = 0$.

$c + 1 - 1 = 0 - 1$ or $c + 4 - 4 = 0 - 4$

$c = -1$ or $c = -4$

Thus, the solutions to the given equation are −1 and −4.

Answer 30e.

Consider the quadratic equation $n^2 - 6n = 0$

Solve the quadratic equation $n^2 - 6n = 0$ by factorization.

$n^2 - 6n = 0$ Write the original equation

$\Rightarrow n(n-6) = 0$ Factor

$\Rightarrow n = 0$ or $n - 6 = 0$ Zero product property

$\Rightarrow n = 0$ or $n = 6$ Solve for n

Therefore, $\boxed{n = 0,\ 6}$ are the roots of the quadratic equation $n^2 - 6n = 0$.

Answer 31e.

Factor the rightside of the equation using $a^2 + 2ab + b^2 = (a + b)^2$.
$(t + 5)^2 = 0$

Use the zero product property. According to this property, if the product of two expressions is zero, then one or both of the expressions equal zero.
Thus, $t + 5 = 0$.

Subtract 5 from both the sides.

$$t + 5 - 5 = 0 - 5$$
$$t = -5$$

Therefore, the solution is -5.

Answer 32e.

Consider the quadratic equation
$w^2 - 16w + 48 = 0$
Solve the quadratic equation $w^2 - 16w + 48 = 0$ by factorization.

	$w^2 - 16w + 48 = 0$	Write the original equation
$\Rightarrow$	$w^2 - 4w - 12w + 48 = 0$	Since $-16w = -4w - 12w$
$\Rightarrow$	$w(w-4) - 12(w-4) = 0$	Factor
$\Rightarrow$	$(w-4)(w-12) = 0$	Factor
$\Rightarrow$	$w - 4 = 0$ or $w - 12 = 0$	Zero product property
$\Rightarrow$	$w = 4$ or $w = 12$	Solve for w

Therefore, $\boxed{w = 4, 12}$ are the roots of the quadratic equation $w^2 - 16w + 48 = 0$.

Answer 33e.

Rewrite the given equation in the form $x^2 + bx + c = 0$.
$z^2 - 3z - 54 = 0$

This equation can be solved using the Zero product property only if the left side can be factored.

The trinomial on the left side of the given equation is of the form $x^2 + bx + c$, which when factored will be of the form $(x + m)(x + n)$, where the product of m and n gives c, and their sum gives b.

Compare the trinomial with $x^2 + bx + c = 0$. The product is -54 and the sum is -3. We need to find m and n such that their product gives -54 and sum gives -3.

List the factors of -54 and find their sums.

Factors of $-54: m, n$	$1, -54$	$2, -27$	$3, -18$	$6, -9$	$-1, 54$	$-2, 27$	$-3, 18$	$-6, 9$
Sum of factors: $m + n$	-53	-25	-15	-3	53	25	15	3

From the table, it is clear that $m = 6$ and $n = -9$ gives the product -54 and sum -3.

Thus, the equation becomes $(z + 6)(z - 9) = 0$.

Apply the Zero product property. This property states that if the product of two expressions is zero, then one or both of the expressions equal zero.

Thus, $z + 6 = 0$ or $z - 9 = 0$.

Solve the equations.

Subtract 6 from both sides of $z + 6 = 0$, and add 9 to both sides of $z - 9 = 0$.

$$z + 6 - 6 = 0 - 6 \quad \text{or} \quad z - 9 + 9 = 0 + 9$$

$$z = -6 \quad \text{or} \quad z = 9$$

Thus, the solutions to the given equation are -6 and 9.

Answer 34e.

Consider the quadratic equation

$r^2 + 2r = 80$

Solve the quadratic equation $r^2 + 2r = 80$ by factorization.

	$r^2 + 2r = 80$	Write the original equation
$\Rightarrow$	$r^2 + 2r - 80 = 0$	Subtract 80 from the sides
$\Rightarrow$	$r^2 + 10r - 8r - 80 = 0$	Since $2r = 10r - 8r$
$\Rightarrow$	$r(r + 10) - 8(r + 10) = 0$	Factor
$\Rightarrow$	$(r + 10)(r - 8) = 0$	Factor
$\Rightarrow$	$r + 10 = 0$ or $r - 8 = 0$	Zero product property
$\Rightarrow$	$r = -10$ or $r = 8$	Solve for r

Therefore, $\boxed{r = -10, 8}$ are the roots of the quadratic equation $r^2 + 2r = 80$.

Answer 35e.

Add $9u$ to both the sides.

$$u^2 + 9u = -9u + 9u$$

$$u^2 + 9u = 0$$

Factor the right side of the equation.

$u(u+9) = 0$

Use the zero product property. According to this property, if the product of two expressions is zero, then one or both of the expressions equal zero.
Thus, either
$u = 0$ or $u + 9 = 0$.

Subtract 9 from both the sides of $u + 9 = 0$.

$$u + 9 - 9 = 0 - 9$$

$$u = -9$$

Therefore, the solutions are 0 and -9.

Answer 36e.

Consider the quadratic equation

$m^2 = 7m$

Solve the quadratic equation $m^2 = 7m$ by factorization.

	$m^2 = 7m$	Write the original equation
$\Rightarrow$	$m^2 - 7m = 0$	Subtract $7m$ from both the sides
$\Rightarrow$	$m(m-7) = 0$	Factor
$\Rightarrow$	$m = 0$ or $m - 7 = 0$	Zero product property
$\Rightarrow$	$m = 0$ or $m = 7$	Solve for m

Therefore, $\boxed{m = 0, 7}$ are the roots of the quadratic equation $m^2 = 7m$.

Answer 37e.

Rewrite the given equation in the form $x^2 + bx + c = 0$.

$x^2 - 14x + 49 = 0$

This equation can be solved using the Zero product property only if the left side can be factored.

The first term, x^2 or $(x)^2$, and the last term, 49 or 7^2, of the equation are perfect squares. This means that the equation may be a perfect square trinomial.

Check whether the middle term is twice the product of the square roots of the first and last terms. Since $2(x)(7)$ or $14x$ is the required middle term (except for the negative sign), the trinomial is a perfect square trinomial.

When factored using the special factoring pattern $a^2 - 2ab + b^2 = (a - b)^2$, we obtain the equation $(x - 7)^2 = 0$ or $(x - 7)(x - 7) = 0$.

Apply the Zero product property. This property states that if the product of two expressions is zero, then one or both of the expressions equal zero.

Thus, $x - 7 = 0$.

Solve the equation. Add 7 to both sides of $x - 7 = 0$.

$$x - 7 + 7 = 0 + 7$$
$$x = 7$$

Thus, the solution to the given equation is 7.

Answer 38e.

Consider the quadratic equation

$-3y + 28 = y^2$

Solve the quadratic equation $-3y + 28 = y^2$ by factorization.

	$-3y+28=y^2$	Write the original equation
$\Rightarrow$	$y^2+3y-28=0$	Subtract 28 and add $3y$ from both the sides
$\Rightarrow$	$y^2+7y-4y-28=0$	Since $3y=7y-4y$
$\Rightarrow$	$y(y+7)-4(y+7)=0$	Factor
$\Rightarrow$	$(y+7)(y-4)=0$	Factor
$\Rightarrow$	$y+7=0$ or $y-4=0$	Zero product property
$\Rightarrow$	$y=-7$ or $y=4$	Solve for y

Therefore, $\boxed{y=-7, 4}$ are the roots of the equation $-3y + 28 = y^2$.

Answer 39e.

Let us factor the equation to find the error.

Factor the left side of the equation first. We need to find two numbers such that their product is -6 and sum is -1. These two numbers are 2 and -3.

Thus, the equation can be rewritten in factored form as $(x + 2)(x - 3) = 0$.

The error has occurred while factoring the left side of the equation. The correct factorization is $(x + 2)(x - 3) = 0$.

Apply the Zero product property. This property states that if the product of two expressions is zero, then one or both of the expressions equal zero.

Thus, $x + 2 = 0$ or $x - 3 = 0$.

Solve the equations. Subtract 2 from both sides of $x + 2 = 0$, and add 3 to both sides of $x - 3 = 0$.

$$x + 2 - 2 = 0 - 2 \quad \text{or} \quad z - 3 + 3 = 0 + 3$$
$$x = -2 \quad \text{or} \quad x = 3$$

Thus, the solutions to the given equation are −2 and 3.

Answer 40e.

Consider

$x^2 + 7x + 6 = 14$

Error: $(x+6)(x+1) = 14$

$x+6 = 14$ or $x+1 = 14$

Because, if product of two numbers is 14.
It need not necessarily imply that one of them must be 14.

Consider

	$x^2 + 7x + 6 = 14$	Write the original equation
$\Rightarrow$	$x^2 + 7x + 6 - 14 = 0$	Subtract 14 from both the sides
$\Rightarrow$	$x^2 + 8x - x - 8 = 0$	Since $7x = 8x - x$
$\Rightarrow$	$x(x+8) - (x+8) = 0$	Factor
$\Rightarrow$	$(x+8)(x-1) = 0$	Factor
$\Rightarrow$	$x+8 = 0$ or $x-1 = 0$	Zero product property
$\Rightarrow$	$x = -8$ or $x = 1$	Solve for x

Therefore, $\boxed{x = -8, 1}$ are the roots of the quadratic equation $x^2 + 7x + 6 = 14$.

Answer 41e.

Factor the right side of the equation.

$(x + 9)(x - 7) = 0$

Use the zero product property. According to this property, if the product of two expressions is zero, then one or both of the expressions equal zero.

Thus, either

$x + 9 = 0$ or $x - 7 = 0$

Subtract 9 from both sides of the first equation.

$$x + 9 - 9 = 0 - 9$$
$$x = -9$$

Add 7 to both sides of the second equation.

$$x - 7 + 7 = 0 + 7$$
$$x = 7$$

Thus, the solutions are −9 and 7.

Therefore, the roots of the equation match with choice **A**.

Answer 42e.

A rectangular site measures 24 feet by 10 feet.
The area of a rectangular site is Length $\times$ Width

Therefore, the area of a rectangular site is $(24)(10)$.

Double the area by adding the same distance x feet to the length and the width
That is,

$$(24+x)(10+x)=2(24)(10)$$

$(24)(10)+24x+10x+x^2=480$ — Simplify

$x^2+34x+240-480=0$ — Combine like terms and subtract 480 from both the sides

$x^2+34x-240=0$

$x^2+40x-6x-240=0$ — Since $34x=40x-6x$

$x(x+40)-6(x+40)=0$

$(x+40)(x-6)=0$

$x+40=0$ or $x-6=0$ — Zero product property

$x=-40$ or $x=6$ — Solve for x

$x=6$ — Since x cannot be negative

Therefore, $\boxed{x=6}$

Answer 43e.

The new length of the rectangular field will be $10+x$ and the new width will be $12+x$. The new area thus obtained will be $3(10)(12)$.

New area (square meters)	=	New length (meters)	$\cdot$	New width (meters)
$\Downarrow$		$\Downarrow$		$\Downarrow$
$3(10)(12)$	=	$(10+x)$	$\cdot$	$(12+x)$

Multiply the right side using the FOIL method.

$360 = 120 + 10x + 12x + x^2$

$360 = 120 + 12x + x^2$

Subtract 360 from both the sides to rewrite the equation in the standard form.

$360 - 360 = 120 + 22x + x^2 - 360$

$0 = x^2 + 22x - 240$

Factor the right side of the equation. For this, find two numbers with product −240 and sum 22. Two such numbers are 30 and −8.
Thus, the equation becomes
$0 = (x + 30)(x - 8)$.

Use the zero product property. According to this property, if the product of two expressions is zero, then one or both of the expressions equal zero.
Thus, either
$x + 30 = 0$ or
$x - 8 = 0$.

Solve both the equations.
Subtract 30 from each side of the first equation.

$$x + 30 - 30 = 0 - 30$$
$$x = -30$$

Add 8 to both the sides of the second equation.

$$x - 8 + 8 = 0 + 8$$
$$x = 8$$

Since the increase in length cannot be negative, reject $x = -30$.
The new length will be $10 + 8 = 18$, and the new width will be $12 + 8 = 20$.

Therefore, the new dimensions of the field will be 18 feet by 20 feet.

Answer 44e.

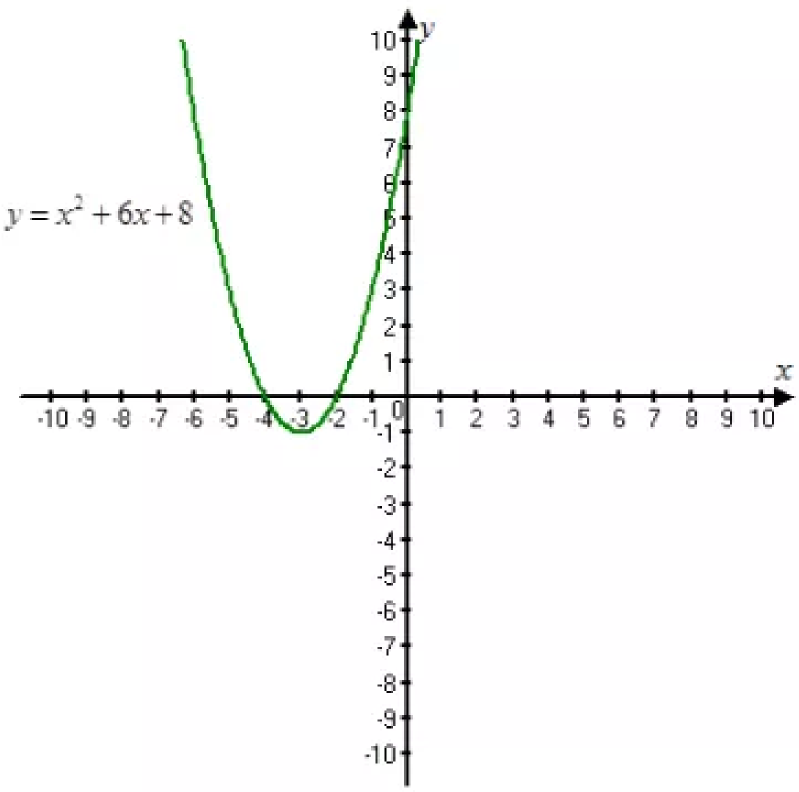
Consider the function
$y = x^2 + 6x + 8$
Find the zeros of the quadratic function $y = x^2 + 6x + 8$ by rewriting the function in intercept form.

$y = x^2 + 6x + 8$	Write original function
$= x^2 + 2x + 4x + 8$	Since $2x + 4x = 6x$
$= x(x+2) + 4(x+2)$	Factor
$= (x+2)(x+4)$	Factor

Therefore, −2 and −4 are the zeros of the quadratic function $y = x^2 + 6x + 8$.

CHECK:

Graph of the function $y = x^2 + 6x + 8$ is shown below:



Therefore, the above graph passes through the points $(-4,0)$ and $(-2,0)$.

Thus, the zeros of the function $y = x^2 + 6x + 8$ are $\boxed{-2 \text{ and } -4}$.

Answer 45e.

Factor the right side of the given function. We can see that the first and last terms are perfect squares, x^2 and 4^2. Since $2(x)(4)$ or $8x$ is the required middle term, the expression is a perfect square trinomial.

Thus, we can rewrite the given equation as $y = (x - 4)^2$ or $y = (x - 4)(x - 4)$.

We know that the x-intercepts of the graph of $y = a(x - p)(x - q)$ are the zeros of the function. This means that the function's value is zero when $x = p$ and $x = q$. Thus, p and q are zeros of the function.

In $y = (x - 4)(x - 4)$, the value of the function will be zero when x takes the value 4.

Therefore, the zero of the given function is 4.

Answer 46e.

Consider a quadratic function

$y = x^2 - 4x - 32$

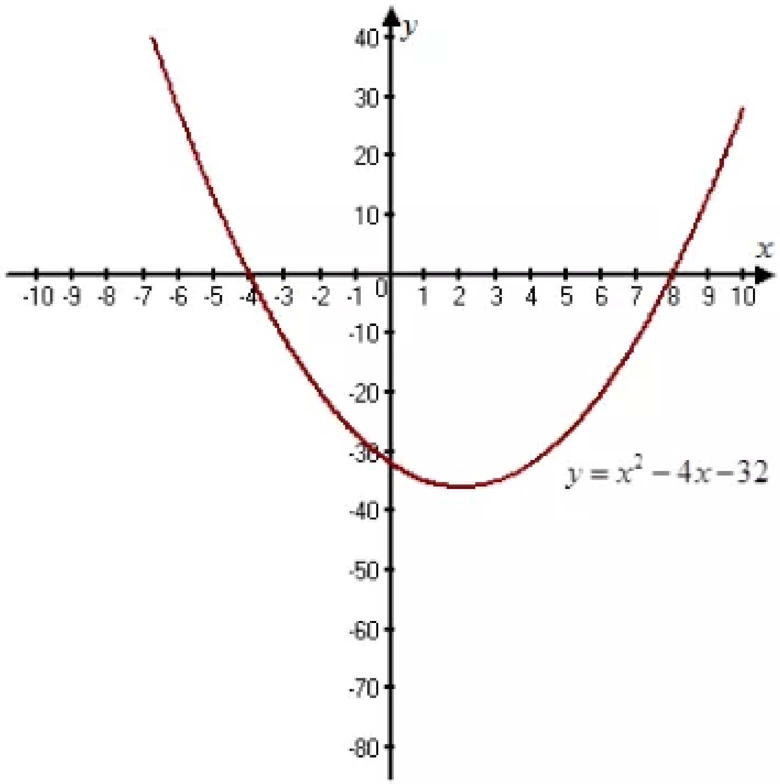
Find the zeros of the quadratic function $y = x^2 - 4x - 32$ by rewriting the function in intercept form.

$y = x^2 - 4x - 32$	Write original function
$= x^2 + 4x - 8x - 32$	Since $4x - 8x = -4x$
$= x(x+4) - 8(x+4)$	Factor
$= (x+4)(x-8)$	Factor

Therefore, -4 and 8 are the zeros of the quadratic function $y = x^2 - 4x - 32$.

CHECK:

Graph of the function $y = x^2 - 4x - 32$ is shown below:



Therefore, the above graph passes through the points $(-4, 0)$ and $(8, 0)$.

Thus, the zeros of the function $y = x^2 - 4x - 32$ are $\boxed{8 \text{ and } -4}$.

Answer 47e.

Factor the right side of the given function. For this, you have to find two numbers with product −30 and sum 7. Two such numbers are 10 and −3.

Rewrite the given function.

$y = (x + 10)(x - 3)$

The value of the function will be zero when x takes the value −10 and 3.

Therefore, the zeros of the given function are −10 and 3.

Answer 48e.

Consider the function

$f(x) = x^2 + 11x$

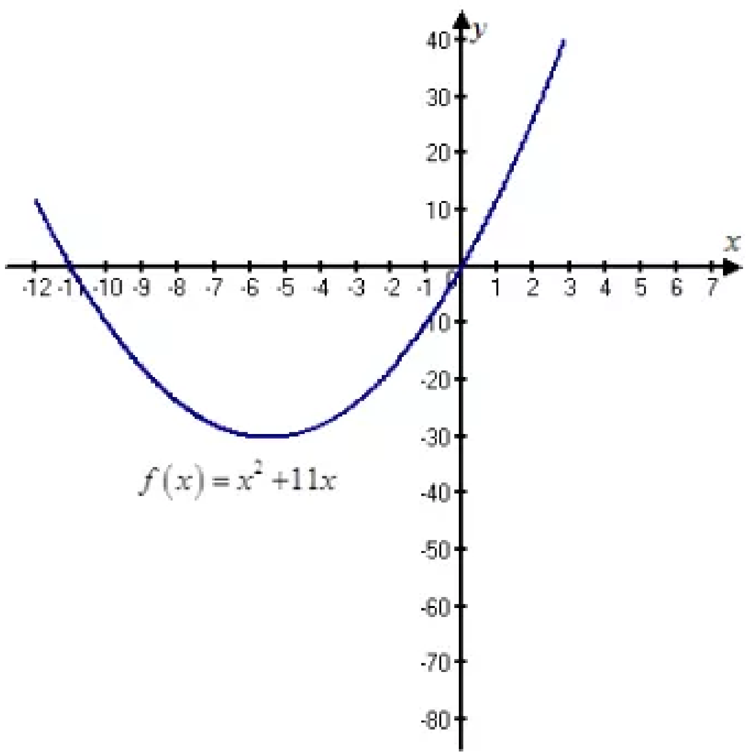
Find the zeros of the quadratic function $f(x) = x^2 + 11x$ by rewriting the function in intercept form.

$f(x) = x^2 + 11x$	Write original function
$= x(x+11)$	Factor

Therefore, 0 and −11 are the zeros of the quadratic function $f(x) = x^2 + 11x$.

CHECK:

Graph of the function $f(x) = x^2 + 11x$ is shown below:



Therefore, the above graph passes through the points $(-11, 0)$ and $(0, 0)$.

Thus, the zeros of the function $f(x) = x^2 + 11x$ are $\boxed{-11 \text{ and } 0}$.

Answer 49e.

Factor the right side of the given function. There is a common factor, x, that can be factored out.

$g(x) = x(x - 8)$

We know that the x-intercepts of the graph of $y = a(x - p)(x - q)$ are the zeros of the function. This means that the function's value is zero when $x = p$ and $x = q$. Thus, p and q are zeros of the function.

In $g(x) = x(x - 8)$, the value of the function will be zero when x takes the value 0 or 8.

Therefore, the zeros of the given function are 0 and 8.

Answer 50e.

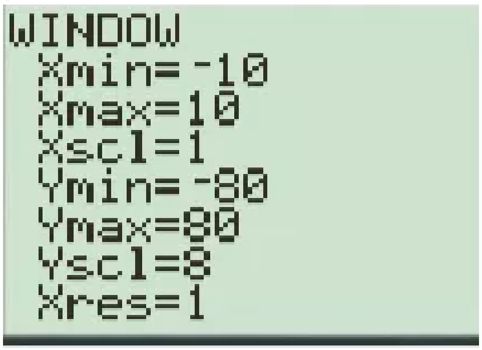
Consider the function $y = x^2 - 64$

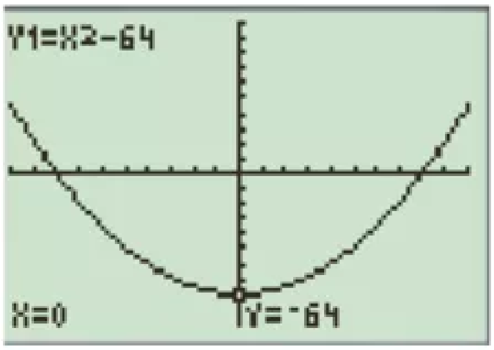
We need to find the zeros of the following quadratic function by rewriting the function in intercept form.

$$\begin{aligned} y &= x^2 - 64 \\ &= x^2 - 8^2 \qquad \text{Since } a^2 - b^2 = (a+b)(a-b) \\ &= (x+8)(x-8) \end{aligned}$$

Therefore, -8 and 8 are the zeros of the given quadratic function.

Check: the graph of the function $y = x^2 - 64$





From the above graph clearly $x = 8$ and $x = -8$ are zeroes of the functions.

Answer 51e.

Factor the right side of the given function. For this, use the difference of two squares formula.

$y = (x + 5)(x - 5)$

The value of the function will be zero when x takes the value -5 and 5.

Therefore, the zeros of the given function are -5 and 5.

Answer 52e.

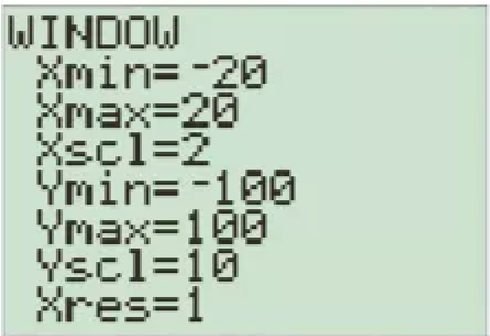
Consider the function $f(x) = x^2 - 12x - 45$

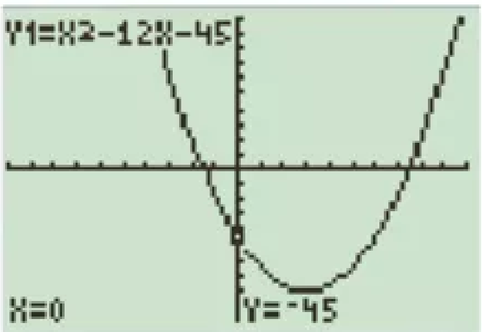
We need to find the zeros of the following quadratic function by rewriting the function in intercept form.

$$\begin{aligned} f(x) &= x^2 - 12x - 45 \\ &= x^2 + 3x - 15x - 45 \\ &= x(x+3) - 15(x+3) \\ &= (x+3)(x-15) \end{aligned}$$

Therefore, -3 and 15 are the zeros of the given quadratic function.

Check: The graph of the function $f(x) = x^2 - 12x - 45$





Clearly the zeroes of the function $f(x) = x^2 - 12x - 45$ are $x = -3$ and $x = 15$

Answer 53e.

Factor the right side of the given function. We need to find two factors such that their product is 84 and sum is 19. The two factors are 12 and 7.

After factorization, the equation can be written as $g(x) = (x + 12)(x + 7)$.

We know that the x-intercepts of the graph of $y = a(x - p)(x - q)$ are the zeros of the function. This means that the function's value is zero when $x = p$ and $x = q$. Thus, p and q are zeros of the function.

In $g(x) = (x + 12)(x + 7)$, the value of the function will be zero when x takes the value -12 or -7.

Therefore, the zeros of the given function are -7 and -12.

Answer 54e.

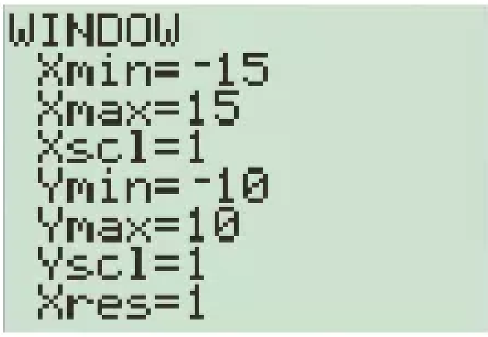
Consider the function $y = x^2 + 22x + 121$

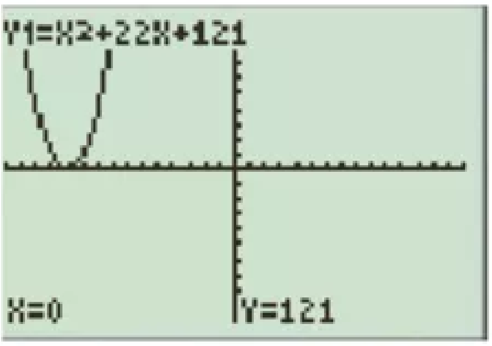
We need to find the zeros of the following quadratic function by rewriting the function in intercept form.

$$y = x^2 + 22x + 121$$
$$= x^2 + 2(x)(11) + 11^2$$
$$= (x + 11)^2$$
$$= (x + 11)(x + 11)$$

Therefore, -11 and -11 are the zeros of the given quadratic function.

Sketch: The graph of the function $y = x^2 + 22x + 121$





Clearly the zeroes of the function $y = x^2 + 22x + 121$ are $x = -11$

Answer 55e.

Factor the right side of the given function. We can see that the given trinomial is a perfect square trinomial, since the first and last terms are perfect squares and the middle term is twice the product of the square roots of the first and last terms.

Rewrite the given function.
$y = (x + 1)(x + 1)$

The value of the function will be zero when x takes the value -1.

Therefore, the zero of the given function is -1.

Answer 56e.

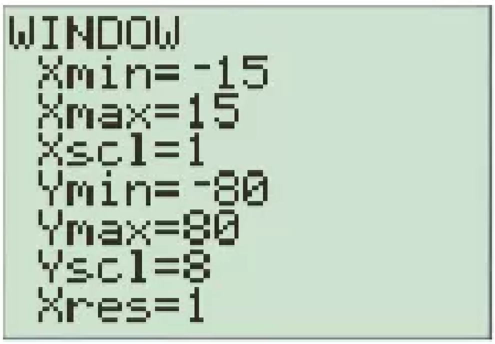
Consider the function $f(x) = x^2 + 6x - 55$

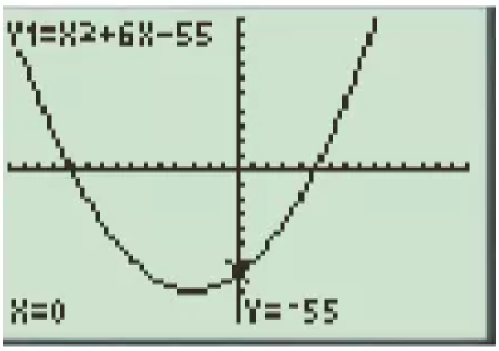
We need to find the zeros of the following quadratic function by rewriting the function in intercept form.

$$f(x) = x^2 + 6x - 55$$
$$= x^2 + 11x - 5x - 55$$
$$= x(x+11) - 5(x+11)$$
$$= (x+11)(x-5)$$

Therefore, -11 and 5 are the zeros of the given quadratic function.

Check: The graph of the function $f(x) = x^2 + 6x - 55$ is





The zeroes of the function $f(x) = x^2 + 6x - 55$ are $x = -11$ and $x = 5$

Answer 57e.

The roots of an equation $x^2 + bx + c = 0$ are the x-values of that equation that make the equation zero.

Since the roots of the equation are 8 and 11, we have $x = 8$ and $x = 11$. We know that the zeros of $y = a(x - p)(x - q)$ are p and q as the function's value is zero when $x = p$ and $x = q$.

Thus, we get $(x - 8)(x - 11) = 0$. This can be expanded as $x^2 - 19x + 88 = 0$.

A possible equation that has the given roots is $x^2 - 19x + 88 = 0$.

Answer 58e.

Consider the expression $x^2 + bx + 7$

We need to find for what integers b, the expression $x^2 + bx + 7$ can be factorized.

Factors of 7 : m, n	$1, 7$	$-1, -7$
Sum of factors : $m + n$	8	-8

From the above table, we get $b = \pm 8$.

Therefore, the value of b is $\boxed{\pm 8}$

Answer 59e.

We know that the area of a rectangle is the product of its length and width.

$x(x + 5) = 36$

Use the distributive property to open the parentheses.

$$x \cdot x + x \cdot 5 = 36$$
$$x^2 + 5x = 36$$

Subtract 36 from both the sides to rewrite in the standard form.

$$x^2 + 5x - 36 = 36 - 36$$
$$x^2 + 5x - 36 = 0$$

Factor the right side of the equation. For this, we have to find two numbers with product -36 and sum 5. Two such numbers are -9 and 4.

$(x + 9)(x - 4) = 0$

Use the zero product property. According to this property, if the product of two expressions is zero, then one or both of the expressions equal zero.
Thus, either
$x+9=0$ or $x-4=0$.

Subtract 9 from both the sides of first equation.

$$x+9-9=0-9$$
$$x=-9$$

Add 4 to both sides of the second equation.

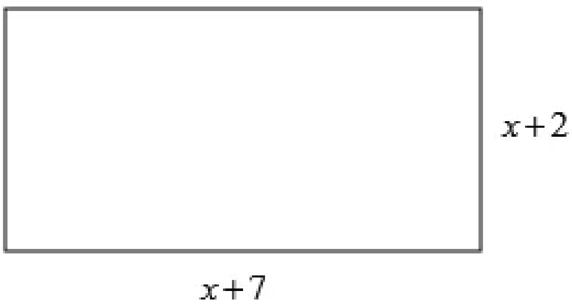
$$x-4+4=0+4$$
$$x=4$$

We get the value of x as 4 and -9. Since length cannot be negative discard the value -9.

Therefore, the value of x is 4.

Answer 60e.

Let us consider the following figure



Given a rectangle with side lengths $x+7$ and $x+2$, and the area is 84.
Then, $(x+7)(x+2)=84$

$\Rightarrow \quad x^2+7x+2x+14=84$

$\Rightarrow \quad x^2+9x-70=0$

$\Rightarrow \quad x^2+14x-5x-70=0$

$\Rightarrow \quad x(x+14)-5(x+14)=0$

$\Rightarrow \quad (x-5)(x+14)=0$

$\Rightarrow \quad x-5=0 \qquad$ (Since $x+14>0$)

$\Rightarrow \quad x=5$

Answer 61e.

We know that the area of a triangle is half the product of its base and altitude.

$$\frac{1}{2}(2x+8)(x+3) = 42$$

Simplify.

$$(2x+8)(x+3) = 84$$
$$2x^2 + 6x + 8x + 24 = 84$$
$$2x^2 + 14x + 24 = 84$$

Subtract 84 from both the sides to rewrite in the standard form.

$$2x^2 + 14x + 24 - 84 = 84 - 84$$
$$2x^2 + 14x - 60 = 0$$
$$x^2 + 7x - 30 = 0$$

Factor the right side of the equation. For this, we have to find two numbers with product −30 and sum 7. Two such numbers are −3 and 10.

$$(x-3)(x+10) = 0$$

Use the zero product property. According to this property, if the product of two expressions is zero, then one or both of the expressions equal zero.

Thus, either

$x - 3 = 0$ or $x + 10 = 0$.

Add 3 to both the sides of first equation.

$$x - 3 + 3 = 0 + 3$$
$$x = 3$$

Subtract 10 from both sides of the second equation.

$$x + 10 - 10 = 0 - 10$$
$$x = -10$$

We get the value of x as −10 and 3. Since length cannot be negative discard the value −10.

Therefore, the value of x is 3.

Answer 62e.

Consider the following figure

Given a trapezoid with parallel sides of length $x+2$ and $x+6$, height is x and the area is 32.

Then $\frac{1}{2}\{(x+2)+(x+6)\}x=32$

$\Rightarrow \quad \frac{1}{2}(2x+8)x=32$

$\Rightarrow \quad (x+4)x=32$

$\Rightarrow \quad x^2+4x-32=0$

$\Rightarrow \quad x^2+8x-4x-32=0$

$\Rightarrow \quad x(x+8)-4(x+8)=0$

$\Rightarrow \quad (x-4)(x+8)=0$

$\Rightarrow \quad x-4=0 \qquad$ (Since $x+8>0$)

$\Rightarrow \quad x=4$

Answer 63e.

Let the two numbers be 6 and 14. Thus, we get $x = 6$ and $x = 14$.

The roots of an equation $x^2 + bx + c = 0$ are the x-values of that equation that make the equation zero. We know that the zeros of $y = a(x - p)(x - q)$ are p and q as the function's value is zero when $x = p$ and $x = q$. Thus, we get $(x - 6)(x - 14) = 0$. This can be expanded as $x^2 - 20x + 84 = 0$.

A possible equation is $x^2 - 20x + 84 = 0$.

Answer 64e.

(a) Consider the sum of two squares x^2+16.
If the above sum is factorizable then there are integers m and n such that
$x^2+16=(x+m)(x+n)$.

$\Rightarrow \quad x^2+16=x^2+mx+nx+mn$

$\Rightarrow \quad x^2+16=x^2+(m+n)x+mn$

By comparing the coefficients, we get
$m+n=0$ and $mn=16$

(b) The above two conditions $m+n=0$ and $mn=16$ cannot be satisfied by any pair of integers m,n.
Because,

Factors of $16: m,n$	$1,16$	$-1,-16$	$2,8$	$-2,-8$	$4,4$	$-4,-4$
Sum of factors $: m+n$	17	-17	10	-10	8	-8

If $mn=16$
Then $m+n$ can never be zero.
Therefore, the sum of the squares x^2+16 is not factorable.

Answer 65e.

We know that the area of a rectangle is the product of its length and width. In order to find the area of the skate park, multiply its length, 100 ft, by its width, 50 ft.

$$\text{Area} = (100)(50)$$
$$= 5000 \text{ sq. ft.}$$

Therefore, the area of the skate park is 5000 square feet.

It is given that the new area will be thrice the old area, which is 3(5000) or 15,000 square feet. The length and the width are increased by x feet each.

$$\underset{\text{(square meters)}}{\text{New area}} = \underset{\text{(meters)}}{\text{New length}} \cdot \underset{\text{(meters)}}{\text{New width}}$$
$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$
$$15,000 = (100+x) \cdot (50+x)$$

Use the FOIL method to simplify the right side of the equation.

$$15,000 = 5000 + 100x + 50x + x^2$$

$$15,000 = 5000 + 150x + x^2$$

Subtract 15,000 from both the sides.

$$15,000 - 15,000 = 5000 + 150x + x^2 - 15,000$$

$$0 = x^2 + 150x - 10,000$$

Factor the right side of the equation. For this, find two numbers with product as $-10,000$ and sum as 150. Two such numbers are 200 and -50.

$$0 = (x + 200)(x - 50)$$

Use the Zero product property. According to this property, if the product of two expressions is zero, then one or both of the expressions equal zero.

Thus, either $x + 200 = 0$ or $x - 50 = 0$.

Solve both the equations. Subtract 200 from the first equation.

$$x + 200 - 200 = 0 - 200$$

$$x = -200$$

Add 8 to both the sides of the second equation.

$$x - 50 + 50 = 0 + 50$$

$$x = 50$$

Since the increase in length cannot be negative, reject $x = -200$. Therefore, the length and the width must be expanded by 50 feet.

Therefore, the new length of the skate park is 150 feet and the new width is 100 feet.

Answer 66e.

Given that a rectangular enclosure at a zoo is of length 35 feet and width 18 feet.
If the zoo wants to double the area of the enclosure by adding the same distance x to the length and width.
We need to find x.

Here, $(x+35)(x+18)=2(35)(18)$

$\Rightarrow \quad x^2+53x-630=0$

$\Rightarrow \quad x^2+63x-10x-630=0$

$\Rightarrow \quad x(x+63)-10(x+63)=0$

$\Rightarrow \quad (x-10)(x+63)=0$

$\Rightarrow \quad x-10=0 \qquad$ (Since $x+63>0$)

$\Rightarrow \quad x=10$

And the dimensions of the new enclosure are 45 feet long and 28 feet wide.

Answer 67e.

a. We know that the area of a rectangle is the product of its length and width. In order to find the area of the existing patio, multiply its length, 30 ft, by its width, 20 ft.

$$\text{Area} = (20)(30)$$
$$= 600 \text{ sq. ft.}$$

Therefore, the area of the existing patio is 600 square feet.

b. It is given that the new area will be 464 square feet more, that is $600 + 464$ or 1064 square feet. The length and the width are increased by x feet each.

$$\underset{\text{(square meters)}}{\text{New area}} = \underset{\text{(meters)}}{\text{New length}} \cdot \underset{\text{(meters)}}{\text{New width}}$$
$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$
$$1064 = (30 + x) \cdot (20 + x)$$

c. Use the FOIL method to simplify the right side of the equation.

$$1064 = 600 + 30x + 20x + x^2$$
$$1064 = 600 + 50x + x^2$$

Subtract 1064 from both the sides.

$$1064 - 1064 = 600 + 50x + x^2 - 1064$$
$$0 = x^2 + 50x - 464$$

Factor the right side of the equation. For this, find two numbers with product as -464 and sum as 50. Two such numbers are 58 and -8.

$0 = (x + 58)(x - 8)$

Use the Zero product property. According to this property, if the product of two expressions is zero, then one or both of the expressions equal zero.

Thus, either $x + 58 = 0$ or $x - 8 = 0$.

Solve both the equations. Subtract 58 from the first equation.

$$x + 58 - 58 = 0 - 58$$
$$x = -58$$

Add 8 to both the sides of the second equation.

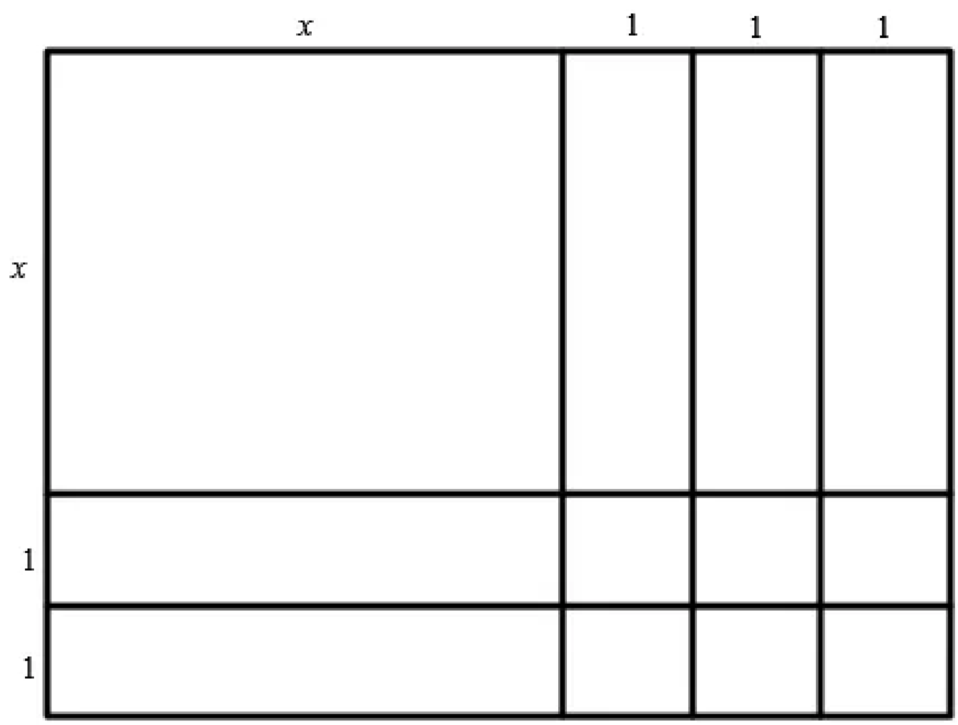
$$x - 8 + 8 = 0 + 8$$
$$x = 8$$

Since the increase in length cannot be negative, reject $x = -58$.

Therefore, the length and the width must be expanded by 8 feet.

Answer 68e.

We consider the diagram as shown below.



a.

Hence, a quadratic trinomial that represents the area of the diagram as shown above is $\boxed{x^2+5x+6}$

b.

We want to factor the expression x^2+5x+6 from part (a).

We have to find integers m and n to factor x^2+bx+c, such that

$$\begin{aligned} x^2+bx+c &= (x+m)(x+n) \\ &= x^2+(m+n)x+mn \end{aligned}$$

So, the sum of m and n must equal b and the product of m and n must equal c.

We want $x^2+5x+6=(x+m)(x+n)$ where $mn=6$ and $m+n=5$.

Factor of 6 : m, n	$1, 6$	$-1, -6$	$2, 3$	$-2, -3$
Sum of factors : $m+n$	7	-7	5	-5

From the table, we see that the fourth column is suitable to factorization of x^2+5x+6.

We notice that $m=2$ and $n=3$.

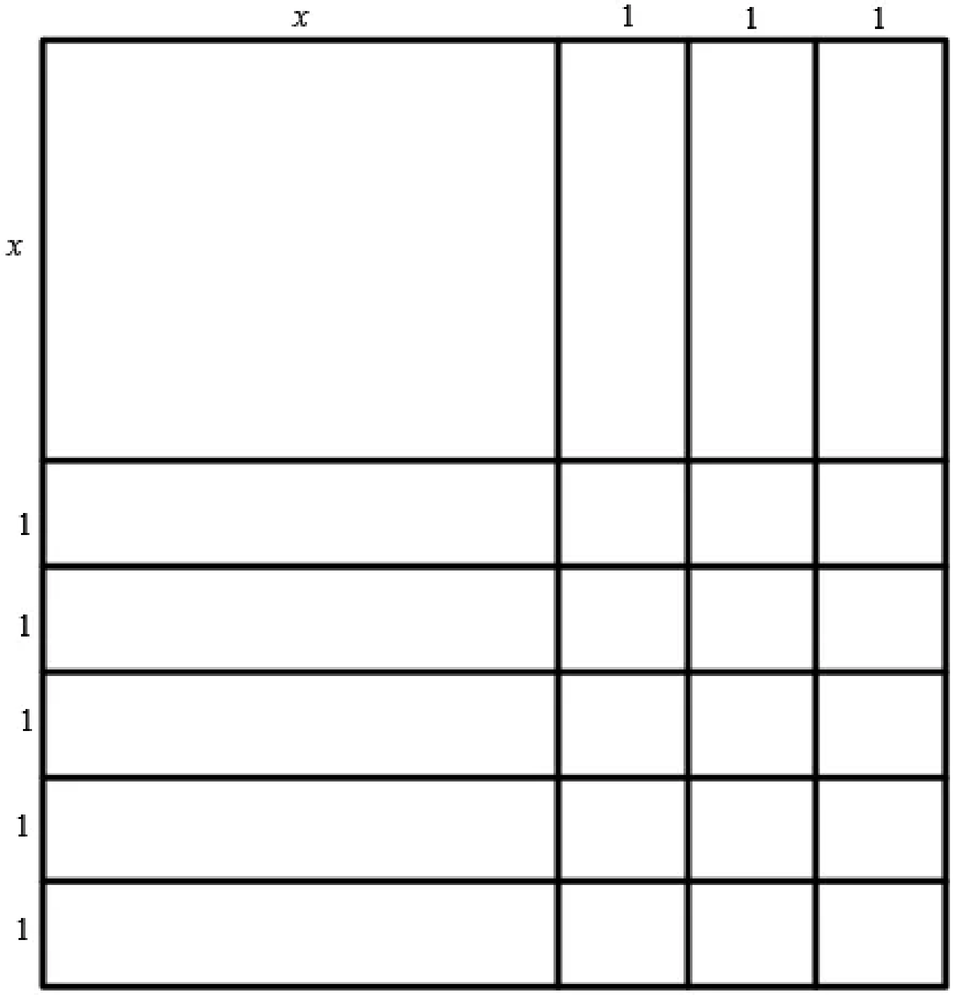
Hence, $x^2+5x+6=\boxed{(x+2)(x+3)}$.

The diagram models the factorization $x^2+5x+6=(x+2)(x+3)$ is that the area that initially measures x units by x units wants to be adding 2 units to the length and 3 units to the width.

c.
We want to draw a diagram that models the factorization

$$x^2+8x+15=(x+5)(x+3)$$

So, we can draw a diagram that models the factorization $x^2+8x+15=(x+5)(x+3)$ as given below.



Answer 69e.

We know that the area of a rectangle is the product of its length and width. In order to find the area of the skate park, multiply its length, 18 ft, by its width, 15 ft.

$$\text{Area} = (18)(15)$$
$$= 270 \text{ sq. ft.}$$

Therefore, the area of the skate park is 270 square feet.

It is given that the new area will be thrice the old area, which is 3(270) or 810 square feet. The length and the width are increased by x feet each.

$$\begin{array}{ccccc} \text{New area} & = & \text{New length} & \cdot & \text{New width} \\ \text{(square meters)} & & \text{(meters)} & & \text{(meters)} \\ \Downarrow & & \Downarrow & & \Downarrow \\ 810 & = & (18+x) & \cdot & (15+x) \end{array}$$

Use the FOIL method to simplify the right side of the equation.

$$810 = 270 + 18x + 15x + x^2$$

$$810 = 270 + 33x + x^2$$

Subtract 810 from both the sides.

$$810 - 810 = 270 + 33x + x^2 - 810$$

$$0 = x^2 + 33x - 540$$

Factor the right side of the equation. For this, find two numbers with product as -540 and sum as 33 Two such numbers are 45 and -12.

$0 = (x+45)(x-12)$

Use the Zero product property. According to this property, if the product of two expressions is zero, then one or both of the expressions equal zero.

Thus, either $x + 45 = 0$ or $x - 12 = 0$.

Solve both the equations. Subtract 45 from the first equation.

$$x + 45 - 45 = 0 - 45$$

$$x = -45$$

Add 12 to both the sides of the second equation.

$$x - 12 + 12 = 0 + 12$$

$$x = 12$$

Since the increase in length cannot be negative, reject $x = -45$. Therefore, the length and the width must be expanded by 12 feet.

Therefore, the new length of the skate park is 30 feet and the new width is 27 feet.

Answer 70e.

A rectangular deck is 21 feet long by 20 feet wide.
If its area is to be halved by subtracting the same distance x from the length and the width.
We need to find x

Here $(21-x)(20-x)=\frac{1}{2}(21)(20)$

$\Rightarrow \quad 420-41x+x^2=210$

$\Rightarrow \quad x^2-41x+210=0$

$\Rightarrow \quad x^2-35x-6x+210=0$

$\Rightarrow \quad x(x-35)-6(x-35)=0$

$\Rightarrow \quad (x-6)(x-35)=0$

$\Rightarrow \quad x-6=0 \qquad$ (Since $x<20$)

$\Rightarrow \quad x=6$

And the dimensions of the new rectangular deck is 15 feet long by 14 feet wide.

Answer 71e.

The new length of the square field will be $10+x$. The new area thus obtained will be $2(10)^2$.

New area (square meters)	=	$(\text{New length})^2$ (meters)
⇓		⇓
$2(10)^2$	=	$(10+x)^2$

Multiply the right side using $(a+b)^2=a^2+2ab+b^2$.
$200=100+20x+x^2$

Subtract 200 from both the sides to rewrite the equation in the standard form.

$200-200 = 100+20x+x^2-200$

$0 = x^2+20x-100$

In order to solve the equation, we have to factor the right side of the equation. For this, we have to find two numbers with product as -100 and sum as 10.

List the factors of -100 and find the sums.

Factors of -100: m, n	$-1, 100$	$1, -100$	$-2, 50$	$2, -50$	$-4, 25$	$4, -25$	$5, -10$	$-5, 10$
Sum of factors: $m+n$	17	-17	7	-7	3	-3	-5	5

There are no factors with product -100 and sum 20.

Therefore, the equation cannot be factored.

Answer 72e.

A grocery store wants to double the area of its parking lot by expanding the Existing lot By a distance x feet in both sides

Area of the existing parking lot is

$$=(165+75)(300+75)-(165)(300)$$
$$=40500$$

Area of the expanded parking lot is

$$=(240+x)(375+x)-(165)(300)$$
$$=40500+615x+x^2$$

And

$$x^2+615x+40500=2(40500)$$
$$\Rightarrow\ x^2+615x-40500=0$$
$$\Rightarrow\ x^2+675x-60x-40500=0$$
$$\Rightarrow\ x(x+675)-60(x+675)=0$$
$$\Rightarrow\ (x-60)(x+675)=0$$
$$\Rightarrow\ x-60=0 \qquad \text{Since } x+675>0$$
$$\Rightarrow\ x=60$$

Answer 73e.

Add 1 to both sides of the equation to bring the constants on one side.

$$2x-1+1 = 0+1$$
$$2x = 1$$

Divide both the sides by 2.

$$\frac{2x}{2} = \frac{1}{2}$$
$$x = \frac{1}{2}$$

CHECK

Substitute $\frac{1}{2}$ for x in the original equation.

$$2\cdot x-1 = 2\cdot\frac{1}{2}-1$$
$$= 1-1$$
$$= 0 \quad \checkmark$$

Therefore, $\frac{1}{2}$ is the solution of the equation.

Answer 74e.

Consider the equation

$$3x+4=0$$

$$\Rightarrow 3x+4+(-4)=0+(-4) \quad \text{Adding both sides } -4$$

$$\Rightarrow 3x+(4-4)=-4$$

$$\Rightarrow 3x=-4 \quad \text{Dividing with 3}$$

$$\Rightarrow 3x\left(\frac{1}{3}\right)=-4\left(\frac{1}{3}\right)$$

$x=-\frac{4}{3}$ is the only solution of the given equation.

Answer 75e.

Subtract 7 from both sides of the equation to bring the constants on one side.

$$-8x+7-7 = 0-7$$

$$-8x = -7$$

Divide both the sides by -8.

$$\frac{-8x}{-8} = \frac{-7}{-8}$$

$$x = \frac{7}{8}$$

CHECK

Substitute $\frac{7}{8}$ for x in the original equation.

$$-8x+7 = -8\left(\frac{7}{8}\right)+7$$

$$= -7+7$$

$$= 0 \quad \checkmark$$

Therefore, $\frac{7}{8}$ is the solution of the equation.

Answer 76e.

Consider the equation

$$6x+5=0$$

$$\Rightarrow 6x+5+(-5)=0+(-5) \quad \text{Adding both sides } -5$$

$$\Rightarrow 6x+(5-5)=-5$$

$$\Rightarrow 6x=-5 \quad \text{Divide with 6}$$

$\Rightarrow x=-\frac{5}{6}$ is the only solution of the given equation.

Answer 77e.

Add 5 to both sides of the equation to bring the constants on one side.

$$4x - 5 + 5 = 0 + 5$$
$$4x = 5$$

Divide both the sides by 4.

$$\frac{4x}{4} = \frac{5}{4}$$
$$x = \frac{5}{4}$$

CHECK

Substitute $\frac{5}{4}$ for x in the original equation.

$$\begin{aligned} 4x - 5 &= 4 \cdot \frac{5}{4} - 5 \\ &= 5 - 5 \\ &= 0 \quad \checkmark \end{aligned}$$

Therefore, $\frac{5}{4}$ is the solution of the equation.

Answer 78e.

Consider the equation

$3x + 1 = 0$

$\Rightarrow 3x + 1 + (-1) = 0 + (-1)$ Adding -1 both sides

$\Rightarrow 3x + (1 - 1) = -1$

$\Rightarrow 3x = -1$ Divide by 3

$\Rightarrow x = -\frac{1}{3}$ is the only solution of the given equation.

Answer 79e.

We know that, for an equation of the form $|x - b| = k$

$x - b = k$ or $x - b = -k$.

Here, b is 6, and k is 7.

Thus,

$x - 6 = 7$ or $x - 6 = -7$.

Add 6 to both sides of both the equation to solve for x.

$x - 6 + 6 = 7 + 6$ or $x - 6 + 6 = -7 + 6$

$x = 13$ or $x = -1$

Therefore, the solutions for the equation are -1, and 13.

Answer 80e.

Consider the equation

$|2x-5|=10$

$$|2x-5|=10 \quad \text{Original equation}$$
$$\Rightarrow 2x-5=\pm 10 \quad \text{Definition of modulus}$$
$$\Rightarrow 2x=5\pm 10$$
$$\Rightarrow 2x=-5 \text{ or } 2x=15 \quad \text{Divide with 2}$$
$$\Rightarrow x=-\frac{5}{2} \text{ or } x=\frac{15}{2}$$
$$\Rightarrow x=-\frac{5}{2},\frac{15}{2} \text{ are the solutions of the given equation}$$

Answer 81e.

We know that, for an equation of the form $|ax-b|=k$

$ax-b=k$ or $ax-b=-k$.

Thus,

$4-3x=8$ or $4-3x=-8$.

Subtract 4 from both sides of both the equations.

$4-4-3x=8-4$ or $4-4-3x=-8-4$

$-3x=4$ or $-3x=-12$

Divide both sides of both the equations by -3.

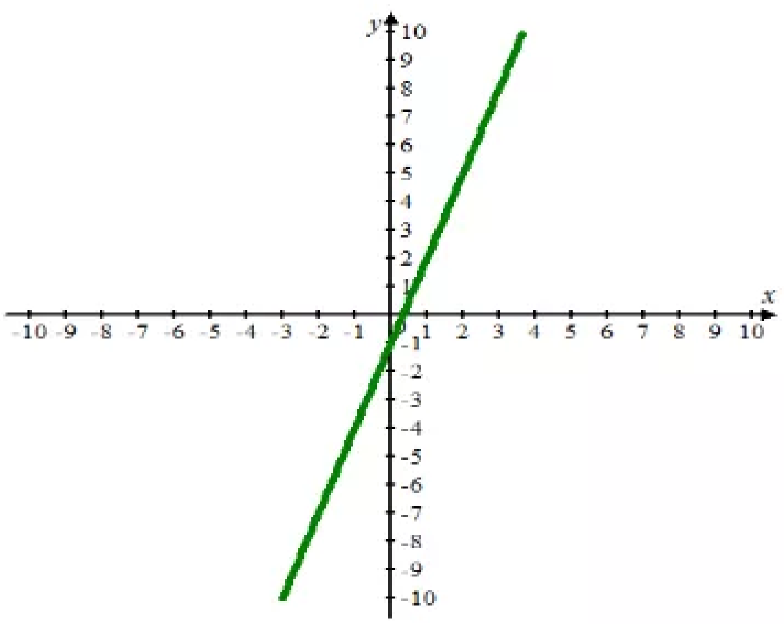
$$\frac{-3x}{-3}=\frac{4}{-3} \quad \text{or} \quad \frac{-3x}{-3}=\frac{-12}{-3}$$

$$x=-\frac{4}{3} \quad \text{or} \quad x=4$$

Therefore, the solutions for the equation are $\frac{-4}{3}$, and 4.

Answer 82e.

The following diagram contains the graph of the function $y=3x-1$

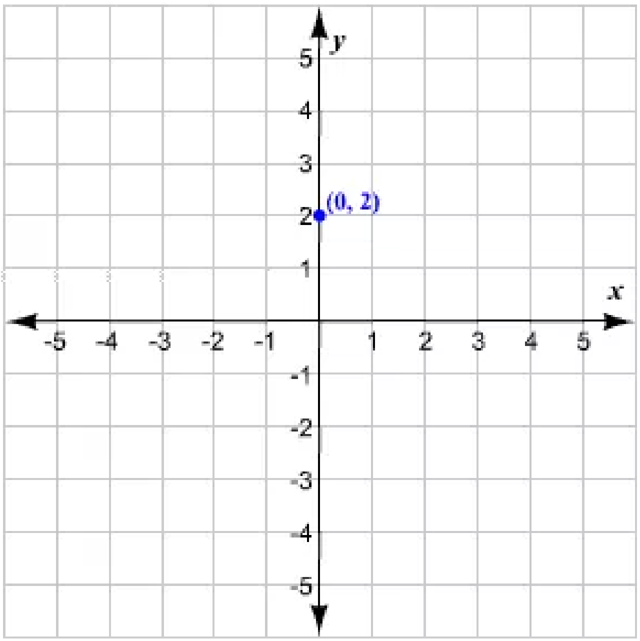


Answer 83e.

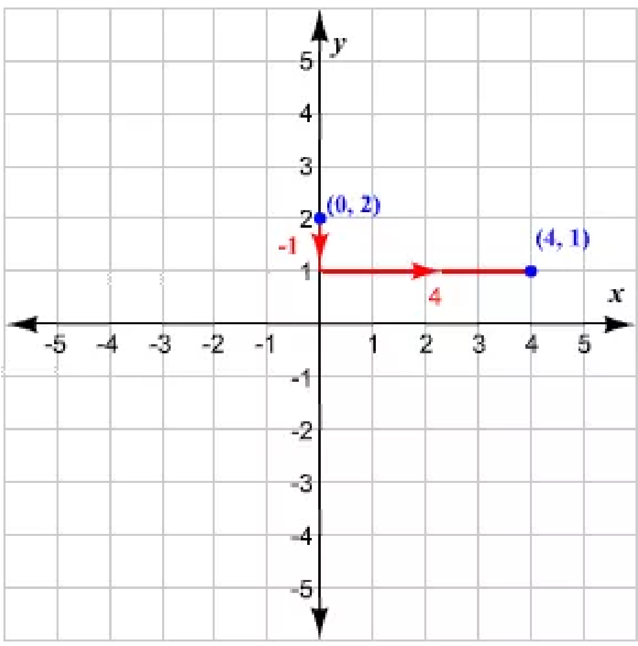
STEP 1 The slope-intercept form of a line is $y = mx + c$, where m is the slope and c is the y-intercept. Here, m is $-\frac{1}{4}$, and c is 2.

The equation is already in slope-intercept form.

STEP 2 First, identify the y-intercept. The y-intercept is 2, so we can plot the point (0, 2) on the y-axis where the line crosses.

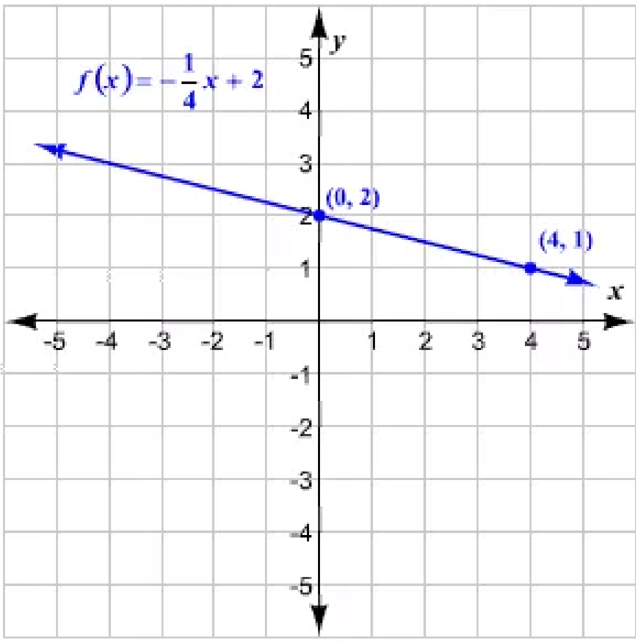


STEP 3 Now, we can identify the slope. The slope is $-\frac{1}{4}$. Start from point (0, 2) move 1 unit down, and then 4 units to the right.



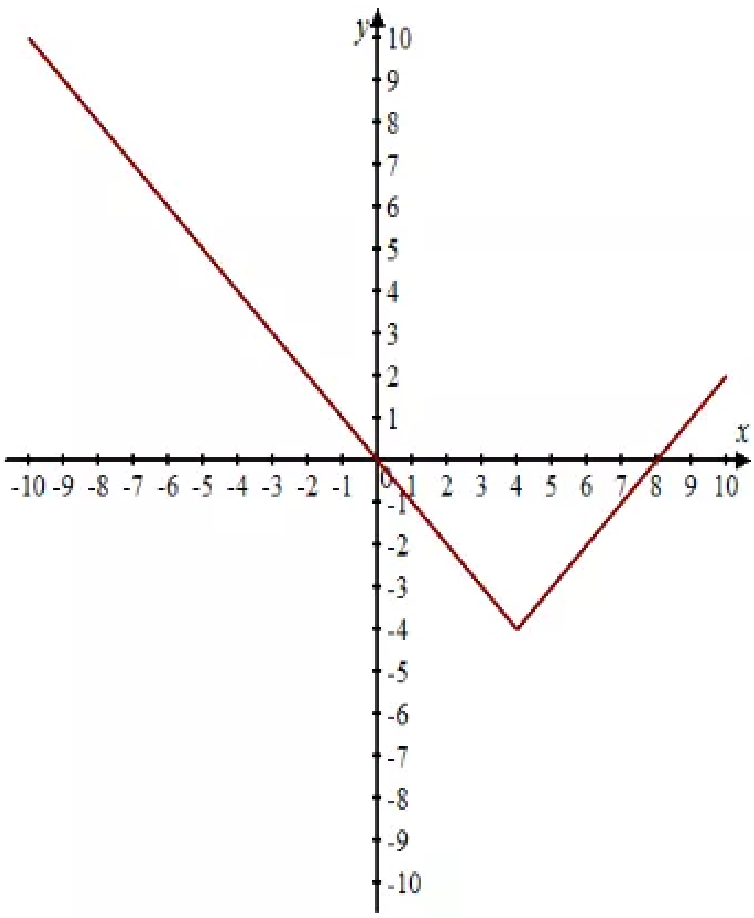
The second point is (4, 1).

STEP 4 Draw a straight line joining the two points.



Answer 84e.

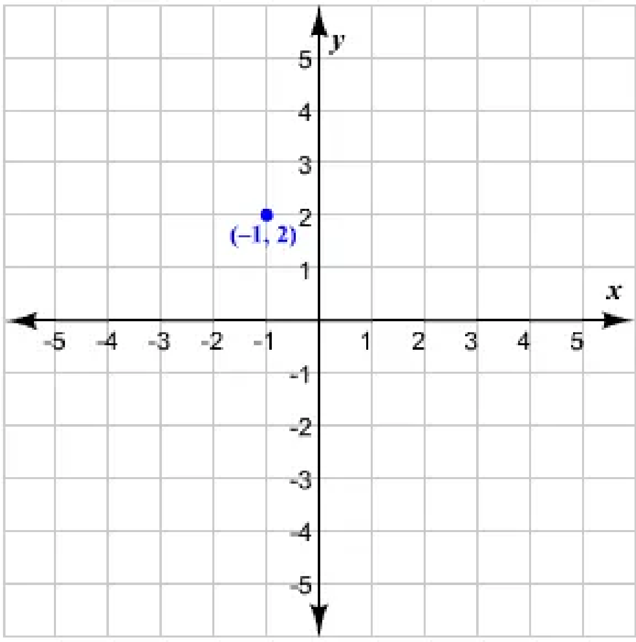
The following diagram contains the graph of the function $y = |x-4| - 4$



Answer 85e.

STEP 1 We know that a graph of the form $y = a\,|\,x - h\,| + k$ has vertex at (h, k). On comparing the given function with $y = a\,|\,x - h\,| + k$, we get h as -1 and k is 2. So the vertex of the graph is $(-1, 2)$.

Plot the vertex on the graph.



STEP 2 Now, find the symmetric points on the graph. Substitute 3 for y in the original equation.

$$3 = \frac{1}{2}\,|\,x + 1\,| + 2$$

Multiply both sides by 2.

$$3 \cdot 2 = \frac{1}{2} \cdot 2\,|\,x + 1\,| + 2 \cdot 2$$
$$6 = |\,x + 1\,| + 4$$

Subtract 4 from both the sides.

$$6 - 4 = |\,x + 1\,| + 4 - 4$$
$$2 = |\,x + 1\,|$$

We know that for an equation of the form $|\,x + b\,| = k$,
$x + b = k$ or $x + b = -k$.

On comparing the function with $|\,x + b\,| = k$, we get b as 1 and k as 2. Thus,
$x + 1 = 2$ or $x + 1 = -2$.

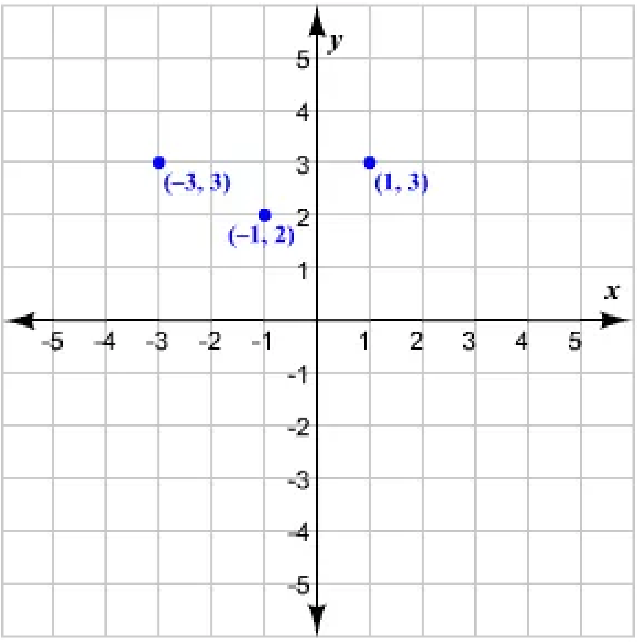
Subtract 1 from both sides of both the equations.

$x + 1 - 1 = 2 - 1$ or $x + 1 - 1 = -2 - 1$

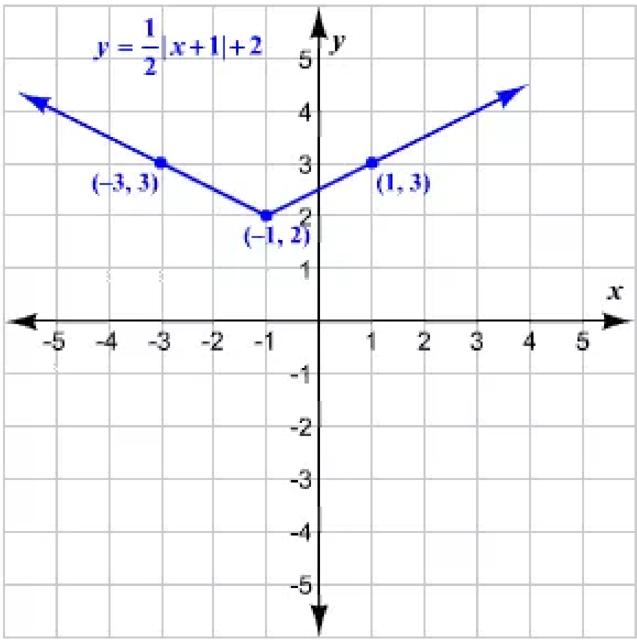
$x = 1$ or $x = -3$

Thus, the symmetric points are (1, 3) and (–3, 3).

Now, plot the symmetric points on the graph.



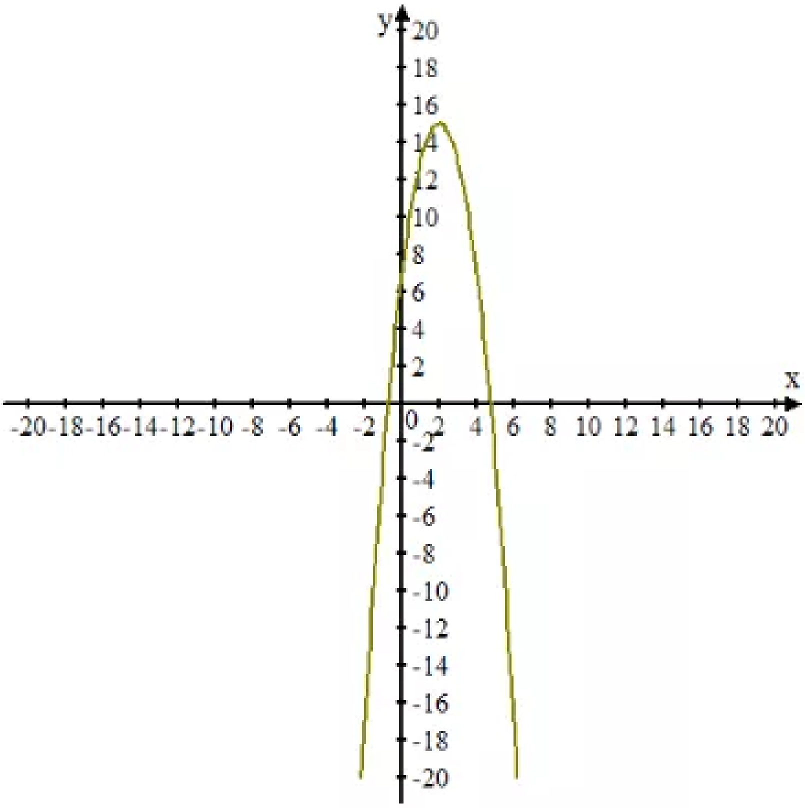
STEP 3 Join the points using straight lines.



The resulting graph is a V-shaped graph.

Answer 86e.

The following diagram contains the graph of the function $y = -2x^2 + 8x + 7$

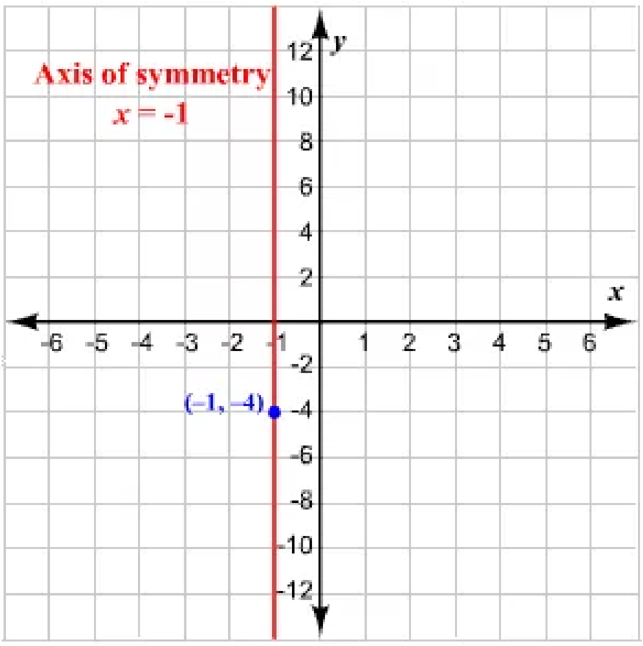


Answer 87e.

STEP 1 The vertex form of a quadratic function is $y = a(x - h)^2 + k$, where the vertex is (h, k) and axis of symmetry is $x = h$.

On comparing the given function with $y = a(x - h)^2 + k$, we get a as -2, h as -1, and k as -4. Since $a < 0$, the parabola opens down. So the axis of symmetry is $x = -1$ and the vertex is $(-1, -4)$.

STEP 2 Draw the axis of symmetry and plot the vertex.



STEP 3 Now, find the values for y by substituting values for x. Substitute 0 for x in the equation.

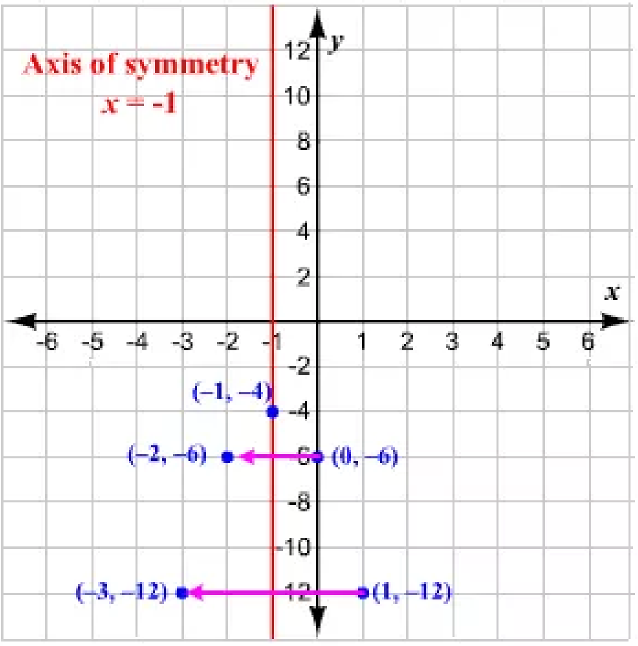
$$y = -2(0+1)^2 - 4 = -2-4 = -6$$

Similarly, substitute 1 for x in the equation.

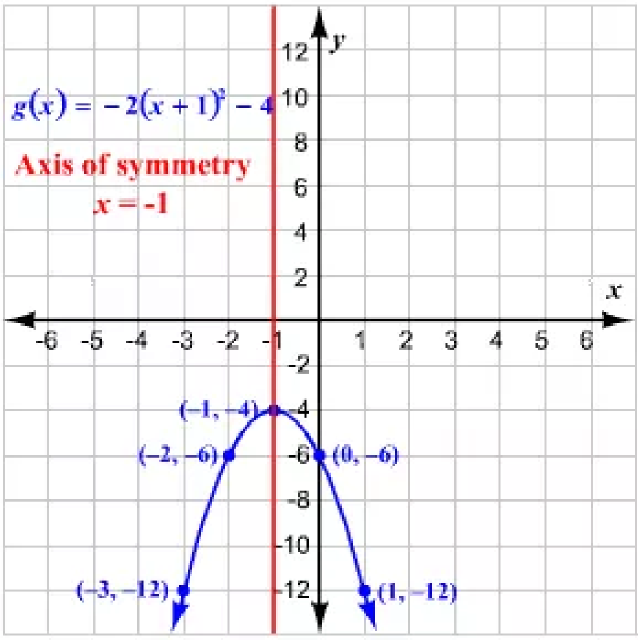
$$y = -2\,(1+1)^2 - 4 = -2 \cdot 4 - 4 = -8 - 4 = -12$$

Thus, the points obtained are (0,−6) and (1, −12).

Plot the obtained points on the graph. The symmetric points are (−2, −6) and (−3, −12).

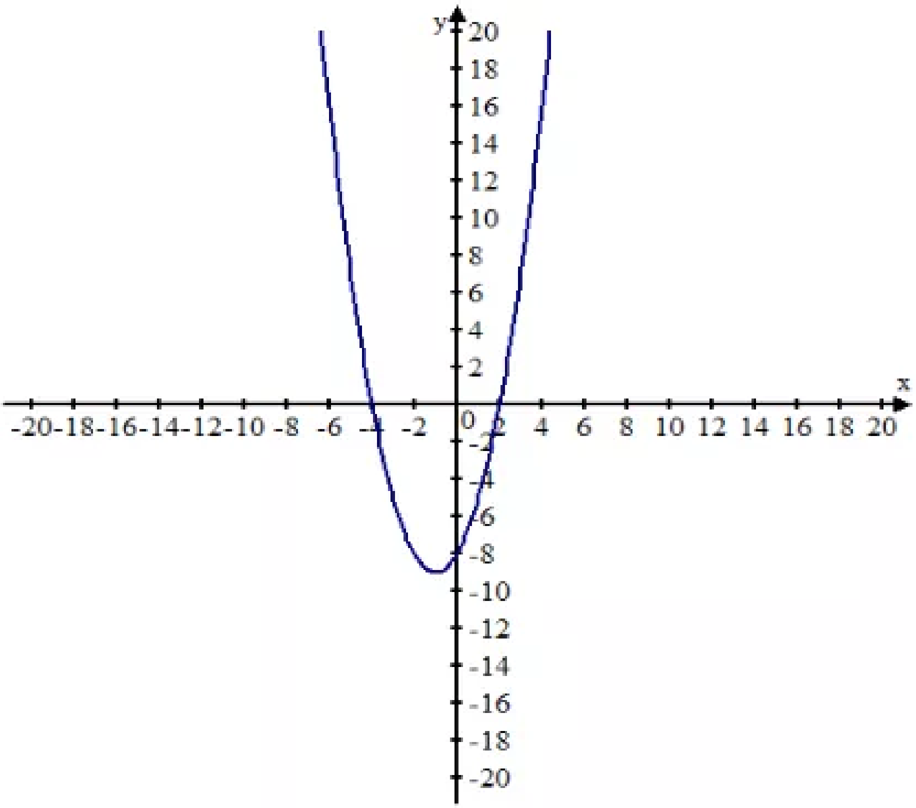


STEP 4 The parabola can be drawn through the plotted points.



Answer 88e.

The following diagram contains the graph of the function $f(x)=(x+4)(x-2)$

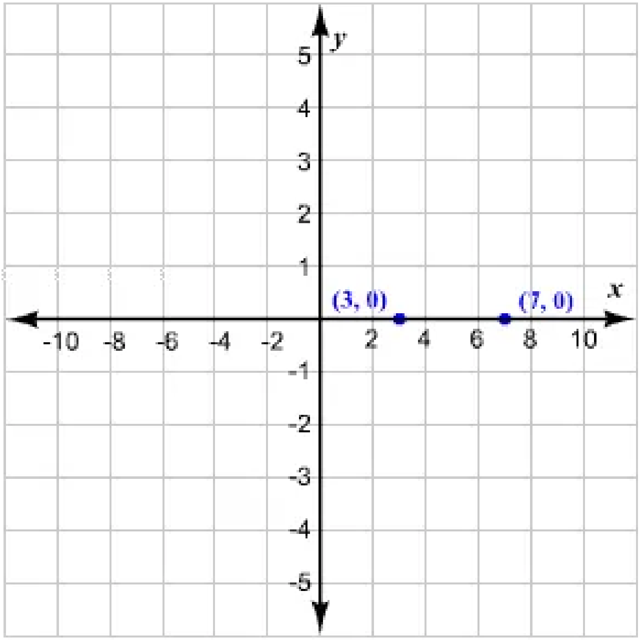


Answer 89e.

STEP 1 The intercept form of a quadratic equation is $y=a(x-p)(x-q)$. The x-intercept of the functions are $(p,\ 0)$ and $(q,\ 0)$. The axis of symmetry can be obtained by the equation $x\ =\ \dfrac{p+q}{2}$, and is halfway between p and q.

On comparing the given function with $y=a(x-p)(x-q)$, we get a as -1, p as 3 and q as 7. So, the x-intercepts are (3, 0) and (7,0). Since $a < 0$, the graph opens down.

Plot the x-intercepts.



STEP 2 Substitute 3 as p and 7 as q in the equation for axis of symmetry.

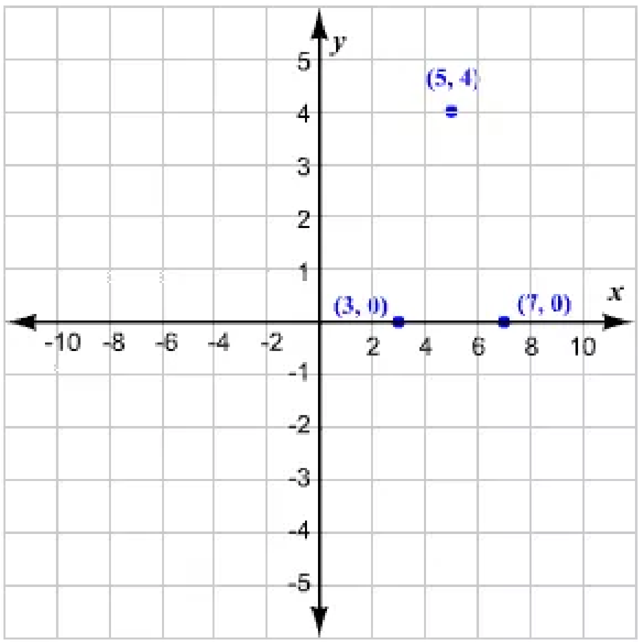
$$x = \frac{3+7}{2}$$
$$= 5$$

Similarly, substitute 5 as x in the given equation.

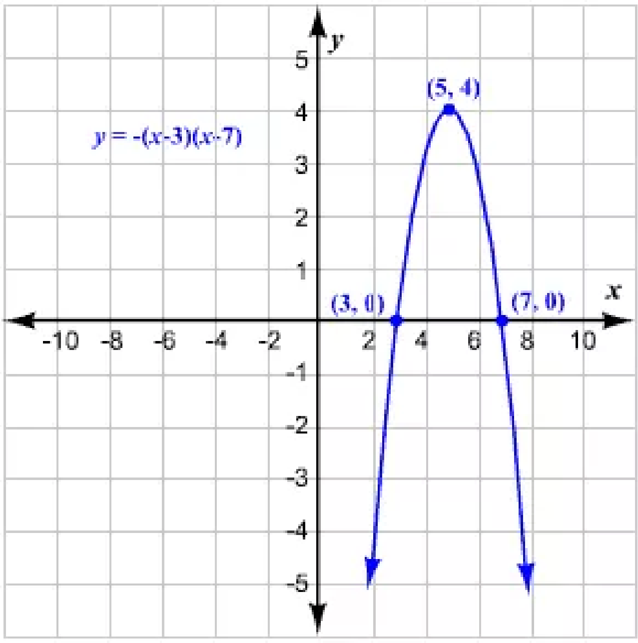
$$y = -1(5-3)(5-7)$$
$$= -1(2)(-2)$$
$$= 4$$

The vertex is (5, 4).

Plot the vertex on the graph.



STEP 3 The parabola can be drawn through the plotted points.



Answer 90e.

A triangular playground has the vertices $(0,0)$, $(14,3)$ and $(6,25)$ where the coordinates are given in feet

The area of the triangle is

$$\text{Area } = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 14 & 3 & 1 \\ 6 & 25 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left|1\left((14)(25)-(3)(6)\right)\right|$$

$$= \frac{1}{2}(350-18)$$

$$= \frac{1}{2}(332)$$

$$= 166 \text{ square feet}$$