Chapter-01
Introduction to Control sys.

Control System (1)

* Consider liquid level control system whose control objective is to keep the water level into tank at a height 'h'.

* Controller is a automatic device with error signal E(s) as i/p & controller o/p P(s) affecting the dynamics of plant to acheive the control objective.

Therefore controller o/p $P = f(e)$

where e = steady state error.

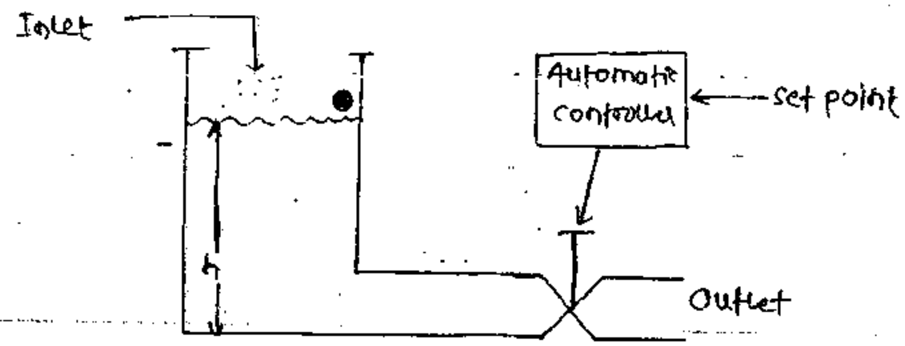
* The different modes of controller operation are proportional, proportional + integral & proportional + integral + derivative.

* There are 2 basic control loop configurations:-

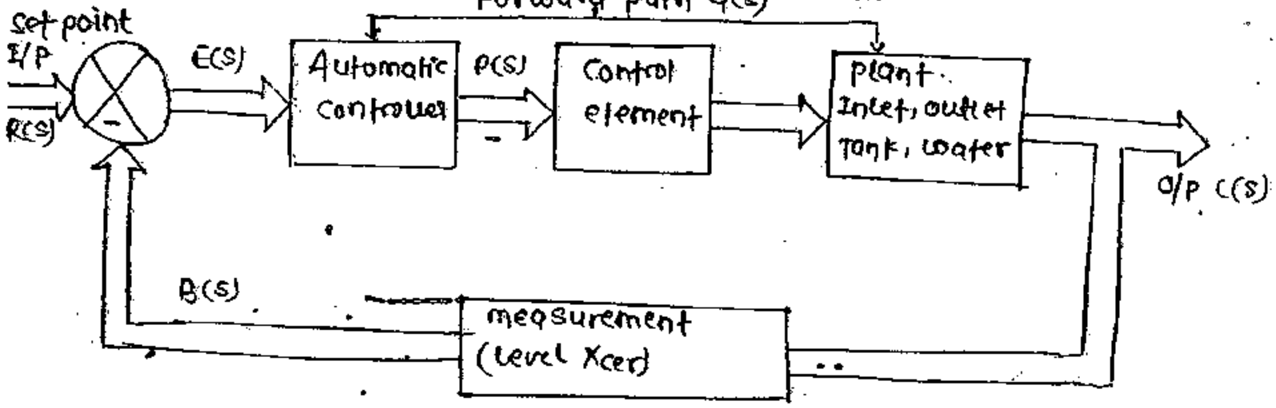
(1.) Closed loop (or) Feedback control system

⇒ In this configuration the changes in the o/p are measured through F/b & compared with the i/p (or) set point to acheive control objective.

⇒ Feeback employs measurement (sensor or Xcer)



(control element)



Backward path H(s)

$$E(s) = R(s) - B(s)$$

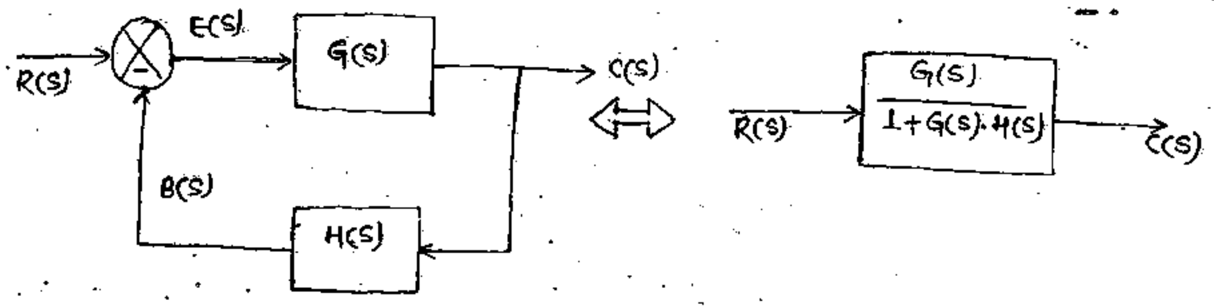
$$\frac{C(s)}{G(s)} = R(s) - C(s)\cdot H(s)$$

$$C(s) = G(s)R(s) - G(s)\cdot H(s)\cdot C(s)$$

$$C(s)\left[1 + G(s)\cdot H(s)\right] = G(s)\, R(s)$$

$$\boxed{C(s) = \left[\frac{G(s)}{1+G(s)\cdot H(s)}\right] R(s)}$$

Control canonical form $\rightarrow$ Eq. mathematical model $\rightarrow$



* Sensitivity Analysis $\rightarrow$

Let α = A variable that changes its value

β = A parameter that changes the value of 'α'.

$$S_{\beta}^{\alpha} = \frac{\%\text{ change in }\alpha}{\%\text{ change in }\beta}$$

$$\boxed{S_{\beta}^{\alpha} = \frac{\frac{\partial\alpha}{\alpha}}{\frac{\partial\beta}{\beta}} = \frac{\beta}{\alpha}\cdot\frac{\partial\alpha}{\partial\beta}}$$

Sensitivity of a closed loop control system (CLCS) →

Let $\alpha = m(s) =$ CLCS

$\beta =$ disturbances in forward path elements

$$S^{m(s)}_{G(s)} = \frac{G(s)}{m(s)} \cdot \frac{\partial m(s)}{\partial G(s)}$$

Since $m(s) = \dfrac{G(s)}{1+G(s)\cdot H(s)}$

$$\frac{G(s)}{m(s)} = 1+G(s)\cdot H(s)$$

$$\frac{\partial m(s)}{\partial G(s)} = \frac{\partial}{\partial G(s)}\left[\frac{G(s)}{1+G(s)\cdot H(s)}\right]$$

$$= \frac{1+G(H)\cdot H(s) - G(s)\cdot H(s)}{[1+G(s)\cdot H(s)]^2}$$

$$= \frac{1}{[1+G(s)\cdot H(s)]^2}$$

$$S^{m(s)}_{G(s)} = [1+G(s)\cdot H(s)] \times \frac{1}{[1+G(s)\cdot H(s)]^{2}}$$

$$\boxed{S^{m(s)}_{G(s)} = \frac{1}{1+G(s)\cdot H(s)}}$$

$1+G(s)\cdot H(s) =$ Noise reduction Factor (OR) Return diff.

* Sensitivity of ~~clcs~~ CLCS with respect to disturbances in H(s) i.e. F/b elements →

$\alpha = m(s) =$ CLCS

$\beta =$ disturbances in F/b elements i.e. $H(s)$

$$S^{m(s)}_{H(s)} = \frac{H(s)}{m(s)} \cdot \frac{\partial m(s)}{\partial H(s)}$$

Since $m(s) = \dfrac{G(s)}{1+G(s)\cdot H(s)}$

$$\frac{m(s)}{H(s)} = \frac{G(s)}{H(s)\left[1+G(s)\cdot H(s)\right]}$$

$$\frac{H(s)}{m(s)} = \frac{H(s)\left[1+G(s)\cdot H(s)\right]}{G(s)}$$

$$\frac{\partial m(s)}{\partial H(s)} = \frac{\partial}{\partial H(s)}\cdot\left[\frac{G(s)}{1+G(s)\cdot H(s)}\right]$$

$$= \frac{\left[1+G(s)\cdot H(s)\right]\times 0 - G(s)\cdot G(s)}{\left[1+G(s)\cdot H(s)\right]^2}$$

$$= \frac{-\left[G(s)\right]^2}{\left[1+G(s)\cdot H(s)\right]^2}$$

$$S^{m(s)}_{H(s)} = \frac{H(s)\left[1+\cancel{G(s)\cdot H(s)}\right]}{G(s)} \times \frac{-\left[G(s)\right]^2}{\left[1+G(s)\cdot H(s)\right]^{\cancel{2}}}$$

$$\boxed{S^{m(s)}_{H(s)} = \frac{-G(s)\cdot H(s)}{1+G(s)\cdot H(s)}}$$

$$\boxed{\left|S^{m(s)}_{H(s)}\right| = \frac{G(s)\cdot H(s)}{1+G(s)\cdot H(s)}}$$

★ **Note →** The CLCS is more sensitive to disturbances in F/b elements i.e. $H(s)$ then forward path element i.e. $G(s)$.

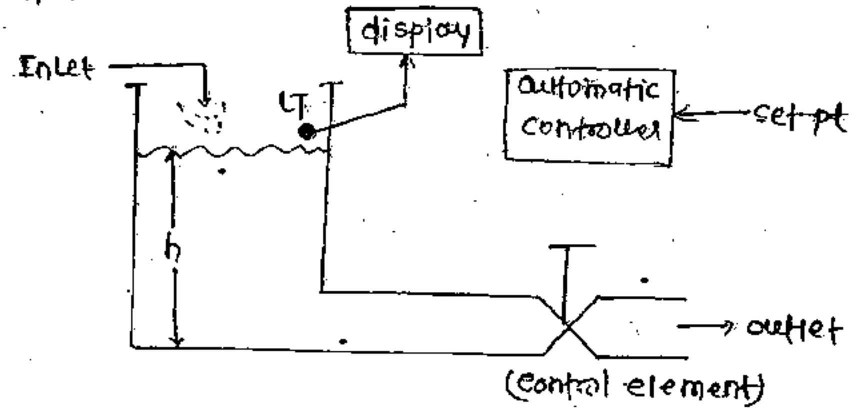
Open loop Control system (OLCS) →

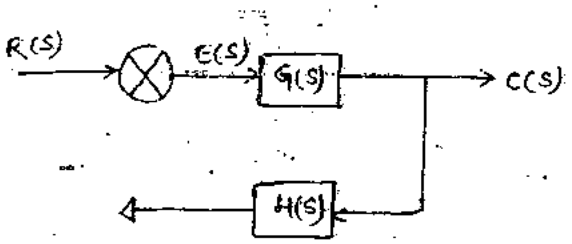
(1) They are conditional control sys. formulated under the basic condn that the sys is not subjected to any type of disturbances.

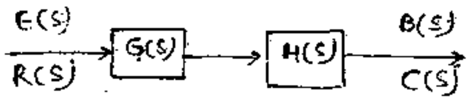
(2) In this configuration the F/b (or) measurement is not connected to ~~forward~~ forward path (or) controller (open loop).

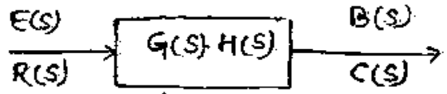
(3) F/b in open loop sys. except for displaying the information about o/p do not have any major significance. This insignificantness of F/b is known as elimination of F/b

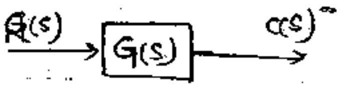
4.) Performance analysis is not applicable to these systems because they are not subjected to any type of disturbances & give out a desired o/p for the desired i/p.











Sensitivity of OLCS wrt disturbances in G(S)

Let $\alpha = m(s) = $ OLCS

$\beta = G(s)$

$$S^{m(s)}_{G(s)} = \frac{G(s)}{m(s)} \cdot \frac{\partial m(s)}{\partial G(s)}$$

$$m(s) = G(s) \cdot H(s)$$

$$\frac{G(s)}{H(s)} = \frac{1}{G(s)}$$

$$\frac{\partial m(s)}{\partial G(s)} = \frac{\partial}{\partial G(s)}\left[G(s)\cdot H(s)\right] = H(s)$$

$$S^{m(s)}_{G(s)} = \frac{1}{\cancel{H(s)}} \times \cancel{H(s)}$$

$$\boxed{S^{m(s)}_{G(s)} = 1}$$

Que.→ The OL & CL sys. are shown in Figs

For 10% change in the forward path, the responces of openloop sys. & CL sys. will be affected by:-

(a) 10%, 1%

(b) 10%, 0.1%

(c) 1%, 10%

(d) 1%, 0.1%

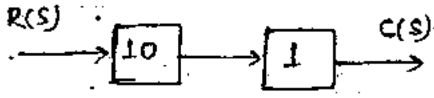


Fig.01

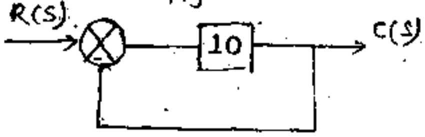


Fig:-02

Sol^n→ OPCS → 10%

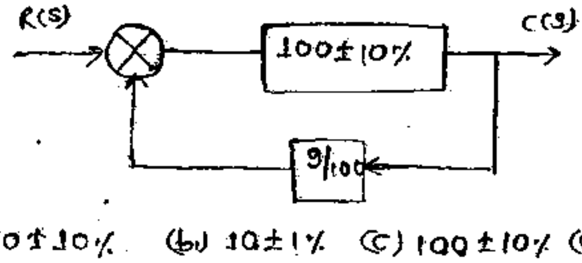
CLCS → $\frac{1}{1+G(s)\cdot H(s)} \times 10\%$

$= \frac{1}{1+10\times 1} \times 10\%$

$= 0.1\%$

Ans. (b)

Que.(2) The closed loop gain when f/b of gain 9/100 is connected in the sys. as shown below will be:-



(a) 10±10% (b) 10±1% (c) 100±10% (d) 100±1%

Sol^n → C.L. gain $= \frac{G}{1+GH}$

$$= \frac{100}{1+100\times\frac{9}{100}}$$

CL gain $= 10$

$$= \frac{1}{1+GH}\times 10\%$$

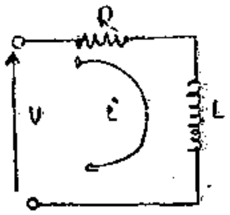
$$= \frac{1}{1+100\times\frac{9}{100}}\times 10\%$$

$$= \frac{1}{10}\times 10\% = 1\%$$

* Concept of TF →

A TF is a mathematical model representing the CS relating i/p & o/p in the form of a ratio i.e. o/p devided by i/p.

Ex:-



$$V = i\cdot R + L\frac{di}{dt}$$

Applying LT

$$V(S) = I(S)\cdot R + LS\,I(S)$$

$$V(S) = I(S)[R+LS]$$

$$\frac{I(S)}{V(S)} = \frac{1}{R+LS}$$

$$\frac{I(S)}{V(S)} = \frac{1/L}{S+R/L}$$

$$TF = F(S) = \frac{C(S)}{R(S)} = \frac{N(S)}{D(S)} = \frac{K(S+z_1)(S+z_2)\ldots}{(S+P_1)(S+P_2)\ldots} \quad \text{—— (i)}$$

where $k =$ sys. gain

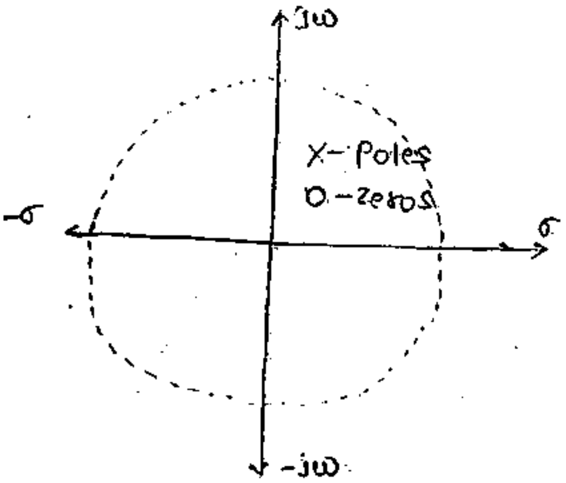
Time constant form $(1+TS)$

$$TF = F(S) = \frac{C(S)}{R(S)} = \frac{N(S)}{D(S)} = \frac{K'(1+T_aS)(1+T_bS)}{(1+T_1S)(1+T_2S)} \quad \text{—— (ii)}$$

where; $K' = \frac{k\cdot z_1 z_2}{P_1\cdot P_2}$ = sys. dc gain

$T_a = \frac{1}{z_1}$, $T_b = \frac{1}{z_2}$

$T_1 = \frac{1}{P_1}$, $T_2 = \frac{1}{P_2}$



S-plane
Space plane

$$S = \lim_{R \to \infty} Re^{j\theta}$$
$$(\theta = 0 - 360^\circ)$$

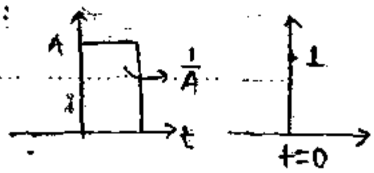
From eq^n (i) the TF of LTI sys. may be defined as the ratio of Laplace Xform of o/p to Laplace Xform of i/p under the assumption that the sys. initials cond^n are zero.

Poles & zeros are those critical freq. which make the TF ∞ (OR) zero.

* Impulse Responce & TF →

* Impulse signal

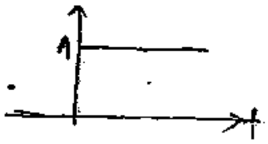
$r(t) = 1 \quad , t = 0$
$\quad\;\; = 0 \quad , t \neq 0$



LT → $R(S) = \frac{1}{S}$

* Step signal

$r(t) = A\,u(t)$
$u(t) = 1 \quad t > 0$
$\quad\;\; = 0 \quad t < 0$

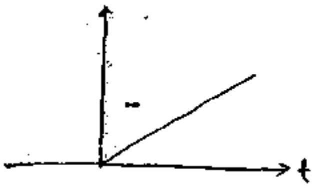


LT → $R(S) = \frac{A}{S}$

* Ramp signal

$r(t) = At, \quad t>0$

$\quad = 0, \quad t<0$



L.T $\rightarrow R(s) = \frac{A}{s^2}$

* Parabolic signal

$r(t) = A\frac{t^2}{2}, \quad t>0$

$\quad = 0 \quad t<0$

L.T $\rightarrow R(s) = \frac{A}{s^3}$

$$T.F = F(s) = \frac{C(s)}{R(s)}$$

$$C(s) = F(s).R(s)$$

Let $R(s)$ = Impulse Responce = 1

$C(s)$ = Impulse Responce = $F(s) \times 1$

$\quad$ = T.F ——— (3)

$L\left(\begin{matrix}\text{Impulse}\\ \text{Responce}\end{matrix}\right)$ = T.F "Weighting Function"

$\frac{d}{dt}$(Parabolic Responce) = Ramp Responce

$\frac{d}{dt}$(Ramp Responce) = Step Responce

$\frac{d}{dt}$(Step Responce) = Impulse Responce

$$\boxed{L\left(\begin{matrix}\text{Impulse}\\ \text{Responce}\end{matrix}\right) = T.F}$$

Chap.(1.)

Que.-1.) The IR of a system is

$$c(t) = -te^{-t} + 2e^{-t} \quad (t>0)$$

Its open loop TF will be :-

(a.) $\frac{2s+1}{(s+1)^2}$ (b.) $\frac{2s+1}{s^2}$ (c.) $\frac{2s+1}{s}$ (d.) $\frac{2s+1}{(s+1)}$

Solⁿ → $L(IR) = T.F.$

$$TF = \frac{-1}{(s+1)^2} + \frac{2}{(s+1)}$$

$$TF = \frac{-1+2(s+1)}{(s+1)^2}$$

$$\frac{G(s)}{1+G(s).H(s)} = \frac{2s+1}{(s+1)^2}$$

$$G(s).H(s) = 1$$

put $H(s) = 1$

$$\frac{G(s)}{1+G(s)} = \frac{2s+1}{(s+1)^2}$$

$$G(s)\left[(s+1)^2-(2s+1)\right] = 2s+1$$

$$\boxed{G(s) = \frac{2s+1}{s^2}}$$

Short cut method

(for unity f/b sys.)

$$\text{open loop TF} = \frac{\text{Num.}}{\text{Den}-\text{Num}}$$

$$TF = \frac{2s+1}{(s+1)^2-(2s+1)}$$

$$\boxed{G(s) = \frac{2s+1}{s^2}}$$

Chap.(2.)

Que(5.) What is the open loop dc gain of unity -ve f/b sys. having closed loop TF $\frac{s+4}{s^2+7s+13}$

(a.) $\frac{4}{13}$ (b.) $\frac{4}{9}$ (c.) 4 (d.) 13

Solⁿ →

$$\text{Open loop TF } G(s) = \frac{s+4}{s^2+7s+13-s-4}$$

$$= \frac{s+4}{s^2+6s+9}$$

$$= \frac{4\left(1+\frac{s}{4}\right)}{9\left(1+\frac{6s}{9}+\frac{s^2}{9}\right)}$$

$$\therefore \boxed{k = \frac{4}{9}}$$

Que.→ Find the dc. gain of TF.

$$\frac{C(s)}{R(s)} = \frac{10(s+5)}{s(s+1)(s+2)}.$$

Sol^n→

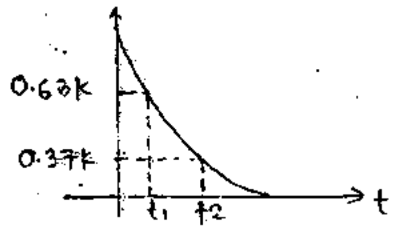
$$\frac{C(s)}{R(s)} = \frac{10\times5\,(1+0.2s)}{(s)\,1\,(1+s)\,2\,(1+0.5s)}$$

$$= \frac{25(1+0.2s)}{s(1+s)(1+0.5s)}$$

$$\boxed{K=25}$$

Que.→ The IR of the sys. is

$$\frac{C(s)}{R(s)} = \frac{K}{s+\alpha}$$ is shown in fig.



The value of α will be

(a.) t_1 (b.) $1/t_1$ (c) t_2 (d) $1/t_2$

Sol^n→ Impulse Responce $= \mathcal{L}^{-1}(T.F.)$

$$= \mathcal{L}^{-1}\left(\frac{K}{s+\alpha}\right)$$

$$= Ke^{-\alpha t}$$

At $t = t_2$

$$Ke^{-\alpha t_2} = K(0.37)$$

Since $e^{-1} = 0.37$

$$Ke^{-\alpha t_2} = Ke^{-1}$$

$$\alpha t_2 = 1$$

$$\boxed{\alpha = \frac{1}{t_2}}$$

Chap(01)
Que.(2)

$$R(s) \longrightarrow \boxed{\frac{1}{s+1}} \longrightarrow \boxed{\frac{1}{s}} \longrightarrow C(s)$$

Impulse Responce = ?

Ans:→ Impulse Responce = $\mathcal{L}^{-1}(T.F.)$

$$= \mathcal{L}^{-1}\frac{1}{s(s+1)} = \mathcal{L}^{-1}\left[\frac{1}{s}-\frac{1}{s+1}\right]$$

$$= e^{t}-e^{-t}$$

Q.→ A certain cs has i/p $r(t)$ & o/p $c(t)$. If the i/p is 1st passed through a block whose TF is e^{-s} & then applied to the sys. the modified o/p will be.

(a) $c(t)\,u(t-1)$ (c) $c(t-1)\,u(t-1)$

(b) $c(t-1)\,u(t)$ (d) None.

Soln→

$$R(s) \longrightarrow \boxed{F(s)} \longrightarrow C(s)$$

$$C(s) = R(s)\,F(s)$$

$$R(s) \longrightarrow \boxed{e^{-s}} \longrightarrow \boxed{F(s)} \longrightarrow C_m(s)$$

$$C_m(s) = R(s)\,e^{-s}F(s)$$

$$C_m(s) = C(s)\,e^{-s}$$

$$\boxed{\mathcal{L}^{-1}F(s)\,e^{-as} = f(t-a)\,u(t-a)}$$

$$\boxed{C_m(t) = C(t-1)\,u(t-1)}$$

Cha(2)(a) (59) Ans.(b)

(2) CL gain = $\frac{G}{1+GH}$

$$100 = \frac{10^3}{1+10^3\times B}$$

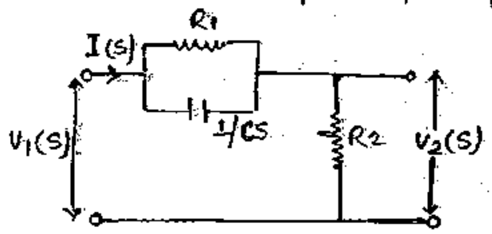
$$\boxed{B = 9\times10^{-3}}.$$

* TF For Compensators →

* Compensators in CS are used for improving the performings specifications i.e. transient & steady state responce c/s.

* There are 3 type:-

(i) Lead compensator → It is used for improving the transient state or Speed of responce of sys.



$$V_i(s) = I(s)\left[\frac{R_1}{R_1CS+1}\right] + I(s)\cdot R_2$$

$$= I(s)\left[\frac{R_1+R_2+R_1R_2CS}{R_1CS+1}\right]$$

$$V_0(s) = I(s)\,R_2$$

$$\boxed{\frac{V_0(s)}{V_i(s)} = \frac{R_2(R_1CS+1)}{R_1+R_2+R_1R_2CS}}$$

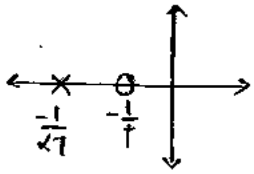
$$T = R_1C\ ; \quad \alpha = \frac{R_2}{R_1+R_2} \quad (\alpha<1)$$

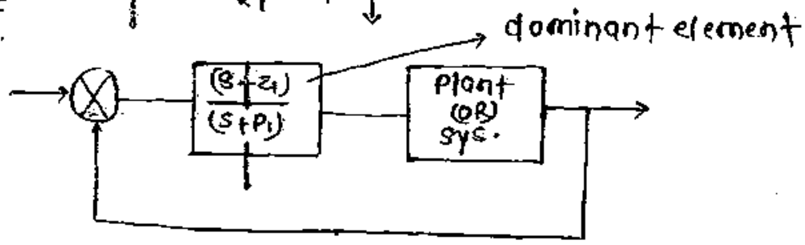
$$\frac{R_2(R_1CS+1)}{R_1+R_2\left[1+\frac{R_1R_2CS}{R_1+R_2}\right]}$$

$$\boxed{\frac{V_0(s)}{V_i(s)} = \frac{\alpha(1+Ts)}{(1+\alpha Ts)}}$$

Zero at $s = \frac{-1}{T}$

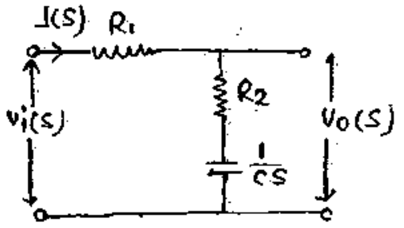
Poles at $s = \frac{-1}{\alpha T}$





Note → Adding a zero to a sys. TF in terms of compensators represents a lead-compensator.

(2) Lag compensator → It is used for improving steady state response c/s of the sys. i.e. elemination of steady state error b/w i/p & o/p.



$$V_i(S) = I(S)\left[R_1 + R_2 + \frac{1}{CS}\right]$$

$$= I(S)\left[\frac{R_1CS + R_2CS + 1}{CS}\right]$$

$$V_o(S) = I(S)\left[R_2 + \frac{1}{CS}\right]$$

$$V_o(S) = I(S)\left[\frac{R_2CS + 1}{CS}\right]$$

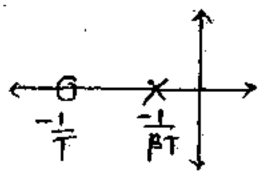
$$\boxed{\frac{V_o(S)}{V_i(S)} = \frac{R_2CS + 1}{R_1CS + R_2CS + 1}}$$

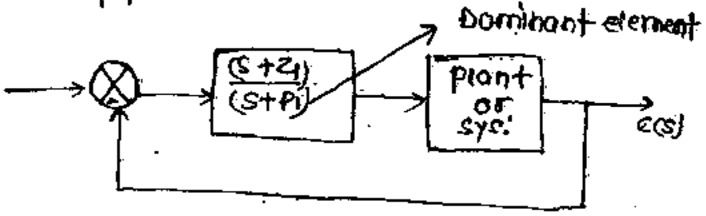
$T = R_2C$; $\beta = \frac{R_1 + R_2}{R_2} = \frac{1}{\alpha}$ $(\beta > 1)$

$$\frac{R_2CS + 1}{R_2CS\left(\frac{R_1 + R_2}{R_2}\right) + 1}$$

$$\boxed{\frac{V_o(S)}{V_i(S)} = \frac{1 + TS}{1 + \beta TS}}$$

zero at $s = \frac{-1}{T}$; pole at $s = \frac{-1}{\beta T}$

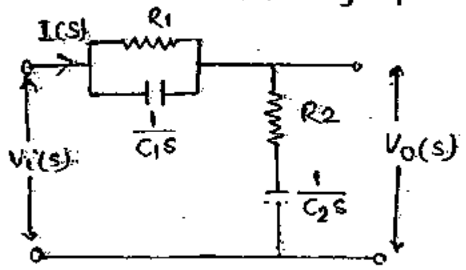




Note → Adding the pole to a sys. TF in terms of compensators represent lag compensator.

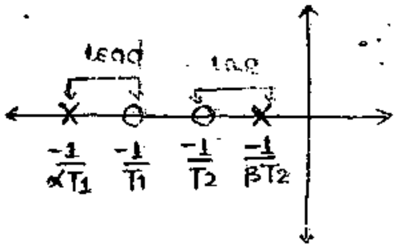
(3.) LAG- LEAD compensator → It improves both transient & steady state responce c/s.

* It exibits both lead & lag c/s in its freq. responce.



Lead	Lag
$\alpha = \frac{R_2}{R_1+R_2}$	$\beta = \frac{R_1+R_2}{R_2}$
$T_1 = R_1C_1$	$T_2 = R_2C_2$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{\alpha(1+T_1s)(1+T_2s)}{(1+\alpha T_1 s)(1+\beta T_2 s)}}$$



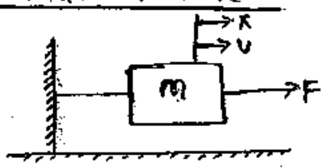
* mechanical systems → All mech. sys. are classified into 2 types:-

(1.) mechanical translational sys. →

I/P = Force (F) ; O/P = linear disp (x) (or) linear velocity (v)

* The 3 ideal elements are:-

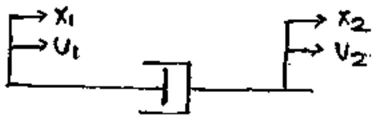
(1.) mass element →



(a) $F = m\frac{dv}{dt}$

(b.) $F = m\frac{d^2x}{dt^2}$

(2.) Damper element →

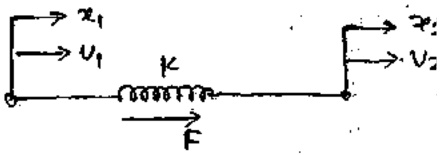


f (or) B

(a) $F = f(v_1 - v_2) = fv$ $[\because v = v_1 - v_2]$

(b) $F = f\frac{d}{dt}(x_1 - x_2) = f\frac{dx}{dt}$ $(x = x_1 - x_2)$

(3.) Spring element →



(a) $F = K\int(v_1 - v_2)\,dt = K\int v\,dt \quad (v = v_1 - v_2)$

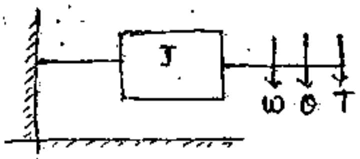
(b) $F = k(x_1 - x_2) = kx \quad (x = x_1 - x_2)$

(2.) Mech. Rotational sys. →

I/p = torque (T) ; o/p = Angular disp (θ) or Angular velocity (ω)

3 ideal elements are →

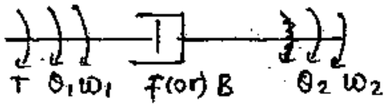
(1.) Inertia element



(a) $T = J\dfrac{d\omega}{dt}$

(b) $T = J\dfrac{d^2\theta}{dt^2}$

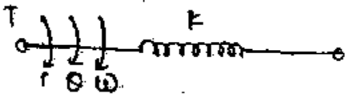
(2.) Torsional Damper element (Friction)



(a) $T = f(\omega_1 - \omega_2) = f\omega \quad (\omega_1 - \omega_2 = \omega)$

(b) $T = f\dfrac{d}{dt}(\theta_1 - \theta_2) = f\dfrac{d\theta}{dt} \quad (\theta = \theta_1 - \theta_2)$

(3.) Torsional Spring element (stiffness)



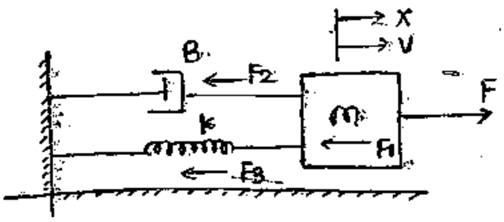
(a) $T = k\int \omega\,dt$

(b) $T = k\theta$

* Analogeous system →

The electrical eq. of mech. elements are known as analogeous sys.

(1.) Mech. translational sys. →

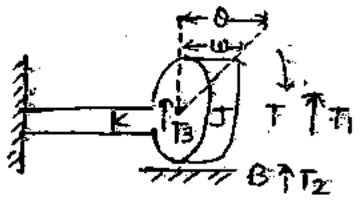


$F = F_1 + F_2 + F_3$

$F = m\frac{dv}{dt} + BV + k\int v\,dt$

$F = m\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kX$ ——(i)

(2.) Mech. Rotational sys. →

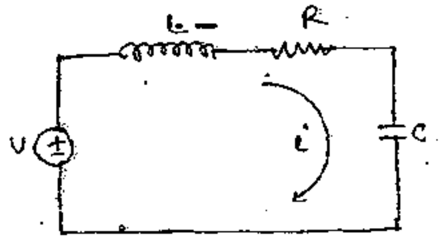


$T = T_1 + T_2 + T_3$

$T = J\frac{d\omega}{dt} + B\omega + k\int \omega\,dt$

$T = J\frac{d^2\theta}{dt^2} + \frac{Bd\theta}{dt} + k\theta$ ——(ii)

(3.) Electrical system →



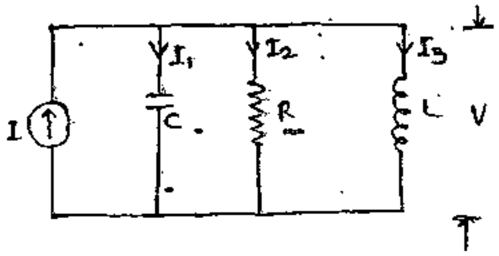
$V = V_1 + V_2 + V_3$

$V = L\frac{di}{dt} + iR + \frac{1}{c}\int i\,dt$

$\left[i = \frac{dq}{dt}\ ,\ q = \text{charge}\right]$

$V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c}$ ——(iii)

(4.) Electrical system →



$I = I_1 + I_2 + I_3$

$I = C\frac{dv}{dt} + \frac{V}{R} + \frac{1}{L}\int v\,dt$

$\left[V = \frac{d\phi}{dt}\ ;\ \phi = \text{flux}\right]$

$I = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L}$ ——(iv)

Comparing eqn (1) – (4.)

(1.) F - T - V Analogy (2.) F - T - I Analogy

F - T - V - I

m - J - L - C

B - B - R - $\frac{1}{R}$

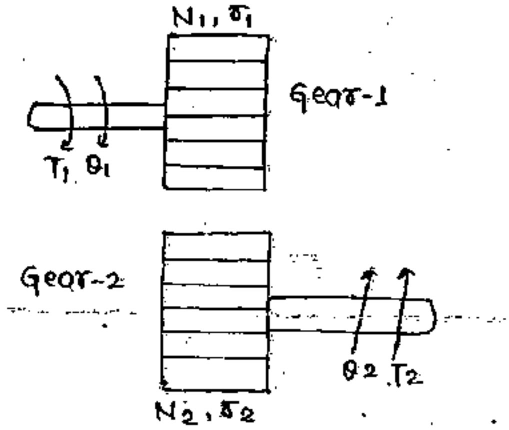
k - k - $\frac{1}{c}$ - $\frac{1}{L}$

V - ω - i - V

X - θ - q - φ

GEARS →

* These are mech. devices which are used as intermediate elements b/n electrical motor & load.
* They are used for stepping up (or) stepping down either torqe (or) speed.
* They are analogeous to electrical TF.



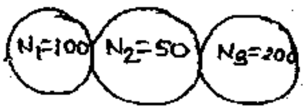
N = No. of teeths
r = Radius of gear wheel (m)
T = Torque on gear wheel (N-m)
θ = Angular disp (radians)

Dynamics of gear train

$$\frac{N_1}{N_2} = \frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\omega_2}{\omega_1} = \frac{\alpha_2}{\alpha_1}$$

$$\boxed{\frac{N_x}{N_y} = \frac{T_x}{T_y} = \frac{r_x}{r_y} = \frac{\theta_y}{\theta_x} = \frac{\omega_y}{\omega_x} = \frac{\alpha_y}{\alpha_x}}$$

En:—



(i) $T_1 = 10$ N-m, Find T_2 & T_3?

(ii) $\omega_1 = 20$ r/s (cw); Find ω_2 & ω_3?

(i) $\frac{N_1}{N_2} = \frac{T_1}{T_2} = \frac{100}{50} = \frac{10}{T_2} = T_2 = 5$ N-m

$\frac{N_1}{N_3} = \frac{T_1}{T_3} = \frac{100}{200} = \frac{10}{T_3} = T_3 = 20$ N-m

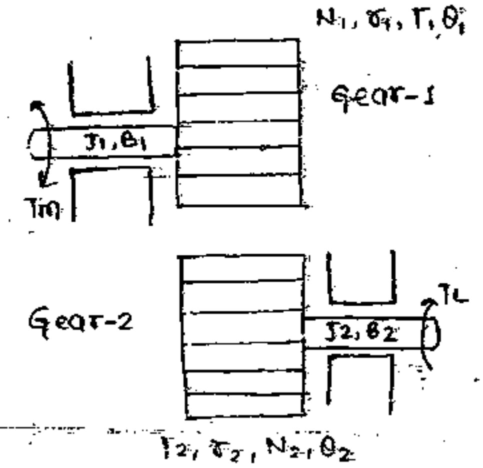
(ii) $\frac{N_1}{N_2} = \frac{\omega_2}{\omega_1} = \frac{100}{50} = \frac{\omega_2}{20} = \omega_2 = 40$ r/s (ccw)

$\frac{N_1}{N_3} = \frac{\omega_3}{\omega_1} = \frac{100}{200} = \frac{\omega_3}{20} = \omega_3 = 10$ r/s (cw)

Observations →

(1) $N_1 > N_2 \Rightarrow T\downarrow,\ \omega\uparrow$

(2) $N_1 = N_2 \Rightarrow$ There will be no change on T & ω (speed & torqe)



T_m = motor torque (N-m)

T_1 = Torque on gear-1 due to T_m (N-m)

T_2 = Torque on gear-2 due to T_1 (N-m)

T_L = Torque on load due to T_2 (N-m)

J, B = Inertia & Friction of gears.

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + T_1$$

$$T_2 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$$\frac{N_1}{N_2} = \frac{T_1}{T_2} \;;\quad T_1 = \left(\frac{N_1}{N_2}\right) T_2$$

$$T_1 = \left(\frac{N_1}{N_2}\right) J_2 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) J_2 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

* (I) eq. Inertia & Friction of motor side gear (gear-1)

$$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{\dot\theta_2}{\dot\theta_1} = \frac{\ddot\theta_2}{\ddot\theta_1} \;;\quad \dot\theta_2 = \left(\frac{N_1}{N_2}\right)\dot\theta_1 \;;\quad \ddot\theta_2 = \left(\frac{N_1}{N_2}\right)\ddot\theta_1$$

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right)^2 J_2 \frac{d^2\theta_1}{dt^2} + \left(\frac{N_1}{N_2}\right)^2 B_2 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$T_m = \left[J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2\right] \frac{d^2\theta_1}{dt^2} + \left[B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2\right] \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$\boxed{J_{eq} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2} \quad \boxed{B_{eq} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2}$$

* (ii) Eq. inertia & friction of Load side gear (gear-2) →

$$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{\dot\theta_2}{\dot\theta_1} = \frac{\ddot\theta_2}{\ddot\theta_1} \quad ; \quad \dot\theta_1 = \left(\frac{N_2}{N_1}\right)\dot\theta_2 \ , \ \ddot\theta_1 = \left(\frac{N_2}{N_1}\right)\ddot\theta_2$$

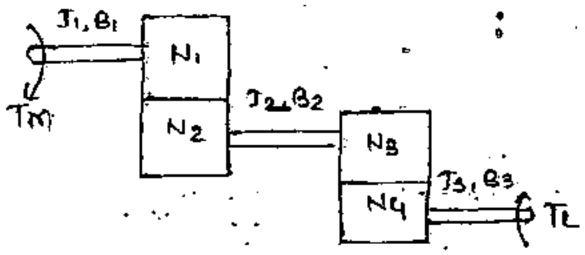
$$T_m = \left(\frac{N_2}{N_1}\right) J_1 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_2}{N_1}\right) B_1 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) \cdot \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$\left(\frac{N_2}{N_1}\right) T_m = \left(\frac{N_2}{N_1}\right)^2 J_1 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_2}{N_1}\right)^2 B_1 \frac{d\theta_2}{dt} + J_2\frac{d^2\theta_2}{dt^2} + B_2\frac{d\theta_2}{dt} + T_L$$

$$\left(\frac{N_2}{N_1}\right) T_m = \left[\left(\frac{N_2}{N_1}\right)^2 J_1 + J_2\right] \frac{d^2\theta_2}{dt^2} + \left[\left(\frac{N_2}{N_1}\right)^2 B_1 + B_2\right] \frac{d\theta_2}{dt} + T_L$$

$$\boxed{J_{eq(2)} = \left(\frac{N_2}{N_1}\right)^2 J_1 + J_2} \qquad \boxed{B_{eq(2)} = \left(\frac{N_2}{N_1}\right)^2 B_1 + B_2}$$

Q →



Soln → (i) motor side gear →

$$J_{eq} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2 + \left(\frac{N_1 N_3}{N_2 N_4}\right)^2 J_3$$

$$B_{eq} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2 + \left(\frac{N_1 N_3}{N_2 N_4}\right)^2 B_3$$

(ii) Load side gear →

$$J_{eq} = \left(\frac{N_2 N_4}{N_1 N_3}\right)^2 J_1 + \left(\frac{N_4}{N_3}\right)^2 J_2 + J_3$$

$$B_{eq} = \left(\frac{N_2 N_4}{N_1 N_3}\right)^2 B_1 + \left(\frac{N_4}{N_3}\right)^2 B_2 + B_3$$

* NODAL METHOD →

(i) No. of nodes = No. of displacements.

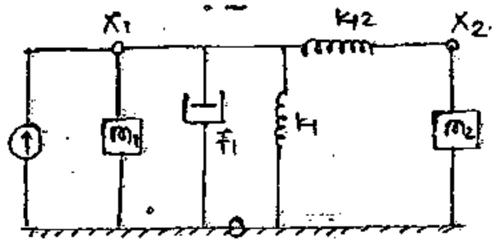
(ii) Take an additional Node which is a ref node.

(iii) Connect the mass or inertia elements b/n the principal node & ref node only.

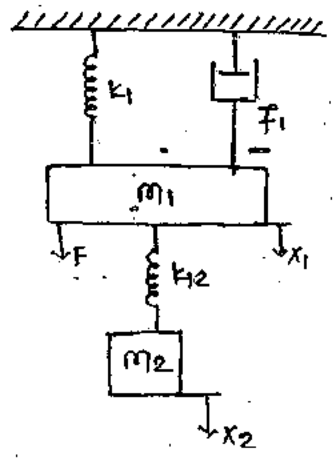
(iv.) Connect an spring & damping elements either b/n the principal node (or) b/n the principal node & ref. depending on there position.

(v.) Obtain the nodel dia. & write the describing diffn eqn at each node.

* Nodal diagram → (mech. eq. diagram)



* mech. system →



At Node X_1

$$F = m_1\frac{d^2x_1}{dt^2} + f_1\frac{dx_1}{dt} + k_1x_1 + k_{12}(x_1 - x_2)$$

at node X_2

$$0 = m_2\frac{d^2x_2}{dt^2} + k_{12}(x_2 - x_1)$$

* Transfer function →

$$F(s) = [m_1 s^2 + f_1 s + k_1 + k_{12}]\, X_1(s) - k_{12}X_2(s)$$

$$0 = [m_2 s^2 + k_{12}]\, X_2(s) - k_{12}X_1(s)$$

$$X_2(s) = \left[\frac{k_{12}}{m_2 s^2 + k_{12}}\right] X_1(s)$$

$$\boxed{\frac{X_1(s)}{X_2(s)} = \frac{m_2 s^2 + k_{12}}{(m_1 s^2 + f_1 s + k_1 + k_{12})(m_2 s^2 + k_{12}) - k_{12}^2}}$$

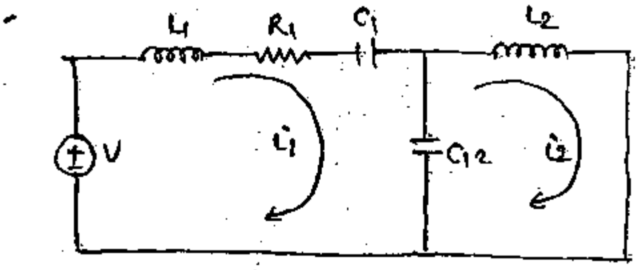
* Order →

(1.) mass element → Order (2)
(2.) mass element → order - 4
3 mass element → order - 6
n mass element → order - 2n

F-V Analogy →

$$V = L_1\frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1\,dt + \frac{1}{C_{12}}\int (i_1 - i_2)\,dt$$

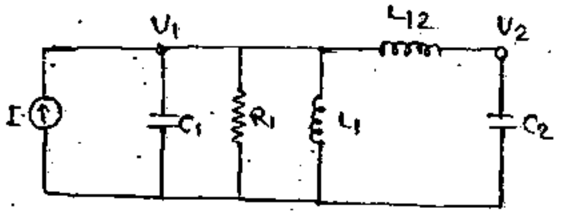
$$0 = L_2\frac{di_2}{dt} + \frac{1}{C_{12}}\int (i_2 - i_1)\,dt$$



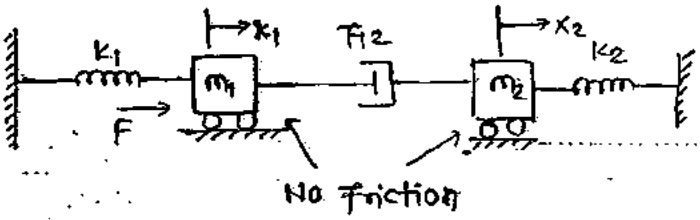
F-I Analogy →

$$I = C_1\frac{dv_1}{dt} + \frac{v_1}{R_1} + \frac{1}{L_1}\int v_1\,dt + \frac{1}{L_{12}}\int (v_1 - v_2)\,dt$$

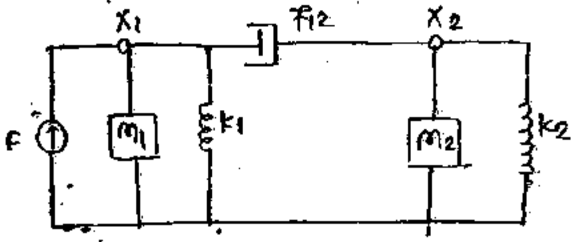
$$0 = C_2\frac{dv_2}{dt} + \frac{1}{L_{12}}\int (v_2 - v_1)\,dt$$



Conv(2)
Q.(2)



Sol^n → Nodal diagram → (mech. eq. dia.)



At node x_1 →

$$F = m_1\frac{d^2x_1}{dt^2} + f_{12}\frac{d}{dt}(x_1-x_2) + k_1x_1$$

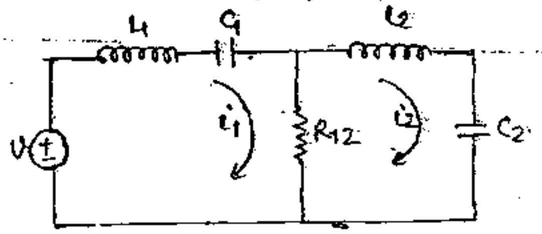
at node x_2 →

$$0 = m_2\frac{d^2x_2}{dt^2} + f_{12}\frac{d}{dt}(x_2-x_1) + k_2x_2$$

F-V analogy →

$$V = L_1\frac{di_1}{dt} + R_{12}(i_1-i_2) + \frac{1}{C_1}\int i_1 dt$$

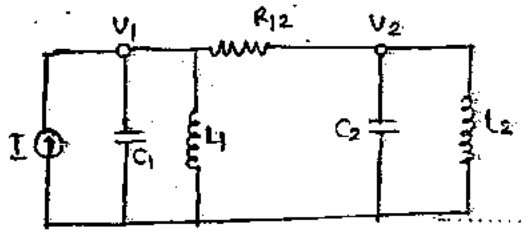
$$0 = L_2\frac{di_2}{dt} + R_{12}(i_2-i_1) + \frac{1}{C_2}\int i_2 dt$$



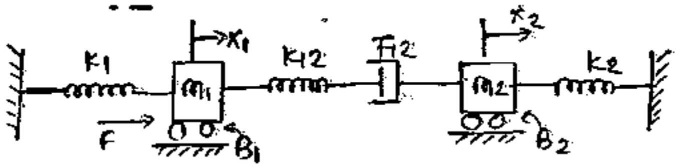
F-I analogy →

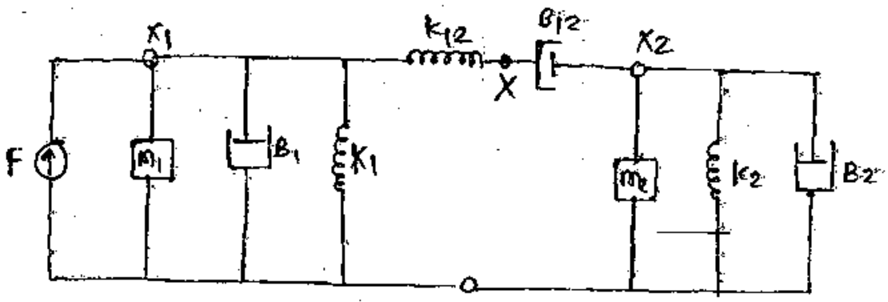
$$I = C_1\frac{dv_1}{dt} + \frac{v_1-v_2}{R_{12}} + \frac{1}{L_1}\int v_1 dt$$

$$0 = C_2\frac{dv_2}{dt} + \frac{v_2-v_1}{R_{12}} + \frac{1}{L_2}\int v_2 dt$$



Conv. Que.





at node $X_1 \rightarrow$

$$F = m_1\frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + k_1x_1 + k_{12}(x_1 - x_2)$$

at dummy node $x \rightarrow$

$$0 = k_{12}(x - x_1) + B_{12}\frac{d}{dt}(x - x_2)$$

at Node $x_2 \rightarrow$

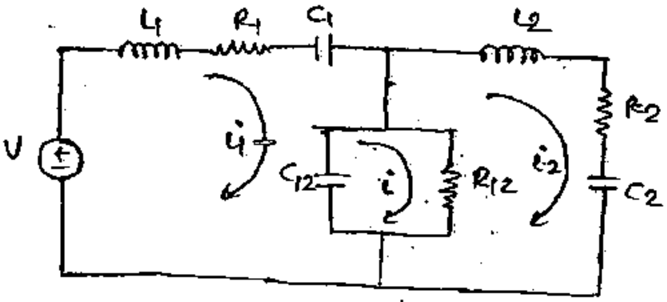
$$0 = m_2\frac{d^2x_2}{dt^2} + B_{12}\frac{dx_2}{dt} + k_2x_2 + B_{12}\frac{d}{dt}(x_2 - x)$$

F-V analogy $\rightarrow$

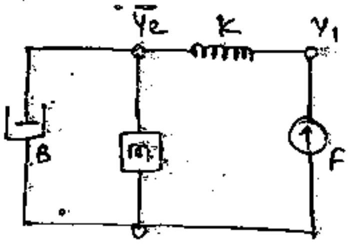
$$V = L_1\frac{di_1}{dt} + R_1i_1 + \frac{1}{C_1}\int i_1dt + \frac{1}{C_{12}}\int (i_1 - i)\,dt$$

$$0 = \frac{1}{C_{12}}\int (i - i_1)\,dt + R_{12}(i - i_2)$$

$$0 = L_2\frac{di_2}{dt} + R_2i_2 + \frac{1}{C_2}\int i_2dt + R_{12}(i_2 - i)$$



Q.5 / 55



at dummy node (y_1)

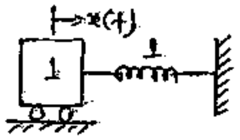
$$F = k(y_1 - y_2)$$

at node (y_2)

$$0 = m\frac{d^2y_2}{dt^2} + B\frac{dy_2}{dt} + k(y_2 - y_1)$$

$$\boxed{k(y_1 - y_2) = m\frac{d^2y_2}{dt^2} + B\frac{dy_2}{dt}}$$

4 / 55



$$F = m\frac{d^2x}{dt^2} + kx$$

Given $m = k = 1$

$$F = \frac{d^2x}{dt^2} + x$$

$$F(s) = (s^2+1)\,x(s)$$

$$X(s) = \frac{1}{s^2+1}\cdot F(s)$$

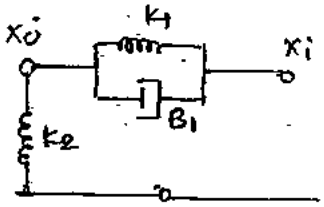
Given

$F(s)$ = unit impulse force = 1

$$X(s) = \frac{1}{s^2+1} \times (1)$$

$$\boxed{x(t) = \sin t}$$

3 / 55



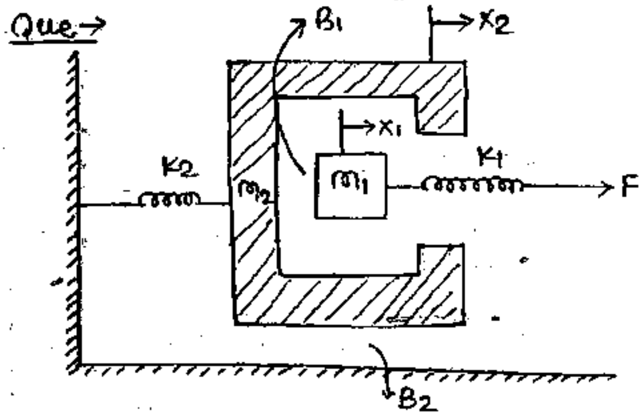
at node X_o →

$$0 = k_2 X_o + k_1(X_o - X_i) + B_1\frac{d}{dt}(X_o - X_i)$$

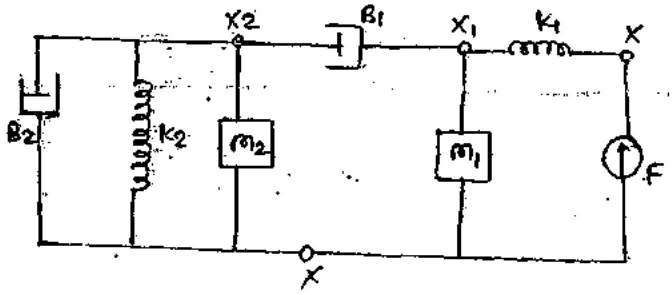
$$[k_2 + k_1 + B_1 s]\,X_o(s) = X_i(s)\,[B_1(s)] + k_1$$

$$\frac{X_o(s)}{X_i(s)} = \frac{k_1 + B_1(s)}{(k_1 + k_2 + B_1 s)}$$

Que →



Sol^n → Nodal diagram →



At dummy node 'x' →

$$F = K_1(x - x_1) \quad \text{—— (i)}$$

at node 'x_1' →

$$0 = m_1\frac{d^2x_1}{dt^2} + B_1\frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x)$$

$$K_1(x - x_1) = m_1\frac{d^2x_1}{dt^2} + B_1\frac{d}{dt}(x_1 - x_2)$$

$$F = m_1\frac{d^2x_1}{dt^2} + B_1\frac{d}{dt}(x_1 - x_2) \quad \text{—— (2)}$$

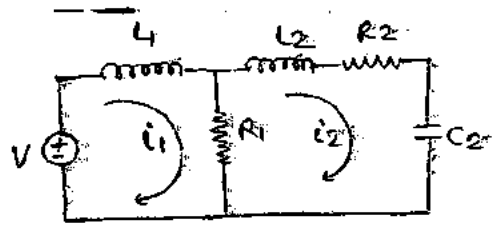
At node 'x_2' →

$$0 = m_2\frac{d^2x_2}{dt^2} + B_2\frac{dx_2}{dt} + K_2x_2 + B_1\frac{d}{dt}(x_2 - x_1) \quad \text{—— (iii)}$$

F-V analogy →

$$V = L_1\frac{di_1}{dt} + R_1(i_1 - i_2)$$

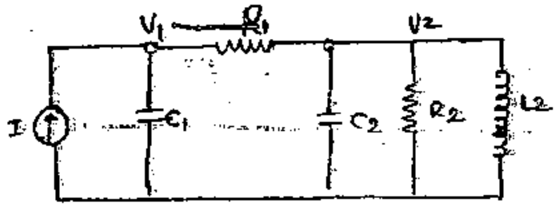
$$0 = L_2\frac{di_2}{dt} + R_2i_2 + \frac{1}{C_2}\int i_2\,dt + R_1(i_2 - i_1)$$



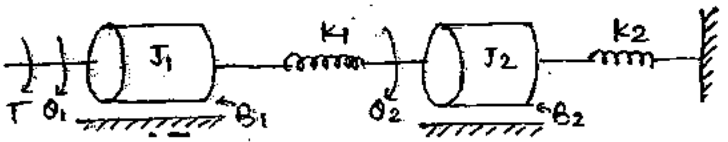
F-I Analogy →

$$I = C_1\frac{dv_1}{dt} + \frac{v_1 - v_2}{R_1}$$

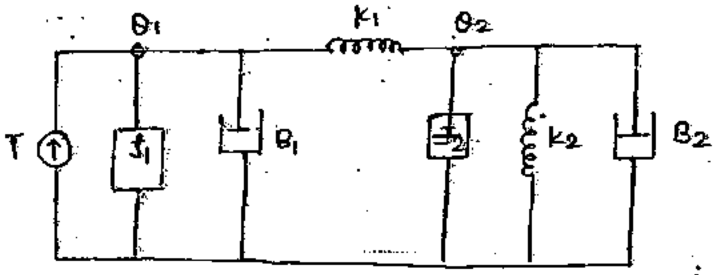
$$0 = C_2\frac{dv_2}{dt} + \frac{v_2}{R_2} + \frac{1}{L_2}\int v_2\,dt + \frac{v_2 - v_1}{R_1}$$



Que →



Soln →



At node θ1 →

$$T = J_1\frac{d^2\theta_1}{dt^2} + B_1\frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2)$$

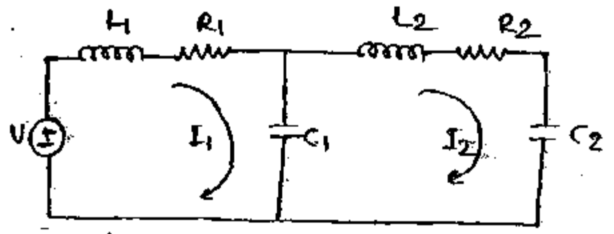
At node θ2 →

$$0 = J_2\frac{d^2\theta_2}{dt^2} + B_2\frac{d\theta_2}{dt} + K_2\theta_2 + K_1(\theta_2 - \theta_1)$$

T-V Analogy →

$$v = L_1\frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int (i_1 - i_2)\,dt$$

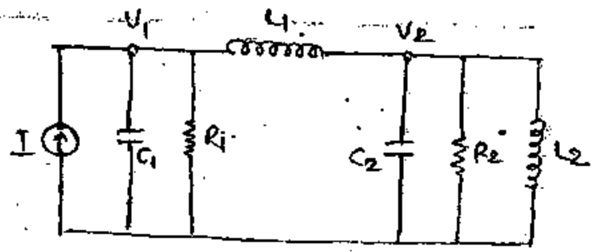
$$0 = L_2\frac{di_1}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + \frac{1}{C_1}\int (i_2 - i_1)\,dt$$



T-I analogy →

$$I = C_1\left(\frac{dV_1}{dt}\right) + \left(\frac{V_1}{R_1}\right) + \frac{1}{L_1}\int (V_1 - V_2)\,dt$$

$$0 = C_2\frac{dV_2}{dt} + \left(\frac{V_2}{R_2}\right) + \frac{1}{L_2}\int (V_2 - V_1)\,dt$$

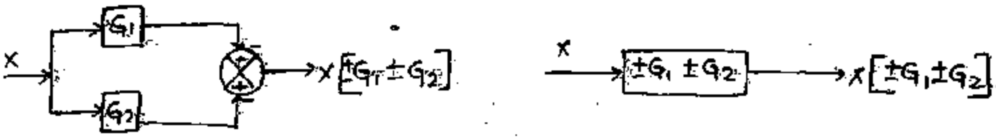


∴ Block diagram →

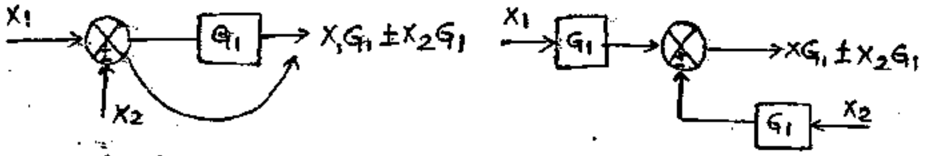
Rules | **Original diagram** | **Eq. diagram**

i) Combining bloks in series.

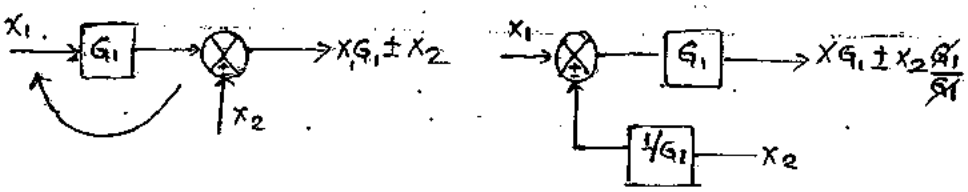
ii) Combining blocks in parallel.



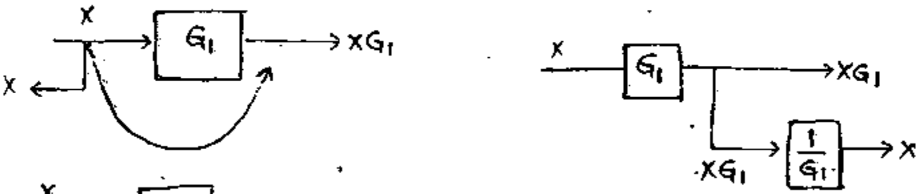
iii) Shifting the summing elements after the blocks.



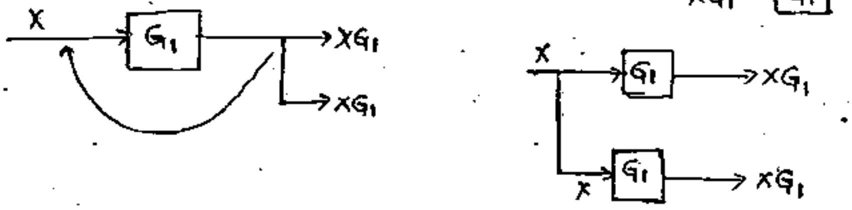
iv) Shifting the summing elements before blocks



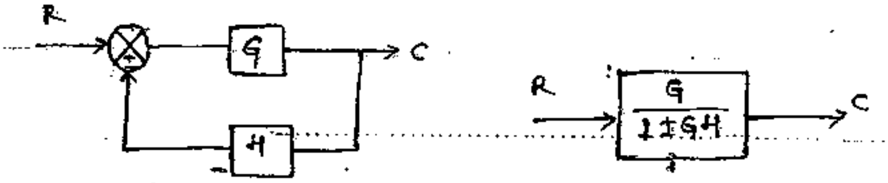
v) Shifting the take off point after the block.



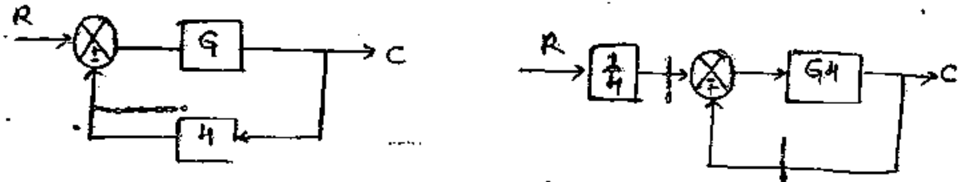
vi) —||— before the block.



vii) TF of CLCS.

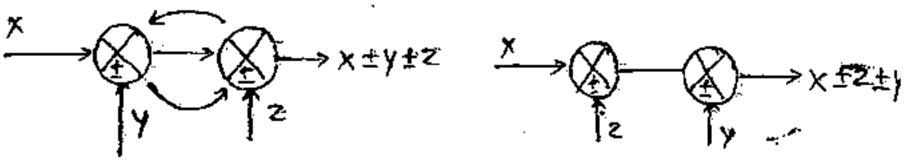


viii) Block dia transformation



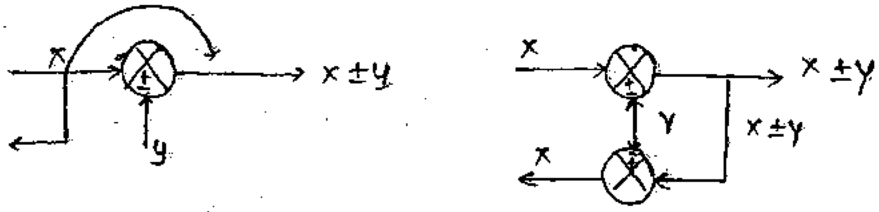
$$\boxed{\frac{C}{R} = \frac{1}{H} \times \frac{GH}{1 \pm GH}}$$

9.) Interchanging the summing elements.

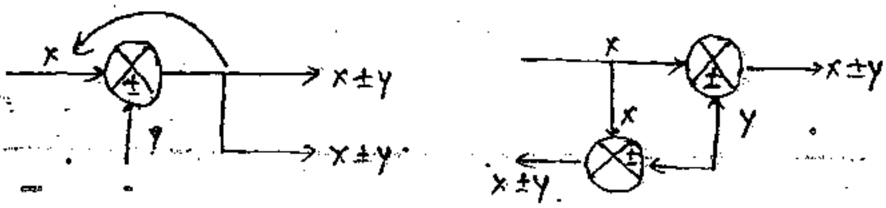


CRITICAL RULE →

10) Shifting the take-off point after the summing element



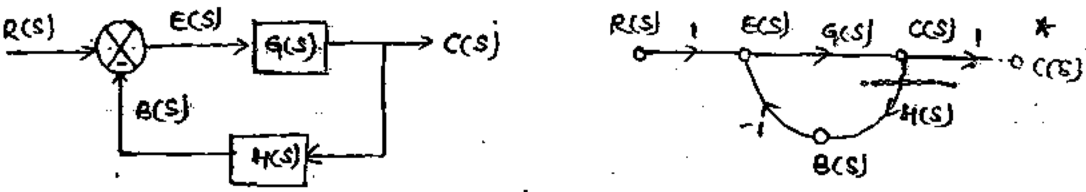
11) —11— Before



7/56 (d) 8/56 (b) 9/56 (c)

* SIGNAL FLOW GRAPH → It is the graphical representation of cs in which nodes representing each of the system variable & are connected by direct branches.

SFG for CLCS →



Terminology of SFG's →

(1) Node:- It represents sys variable & is equal to the sum of all incoming signal at it.

(2) I/p Node (OR) Source node → It is a node having only outgoing branches

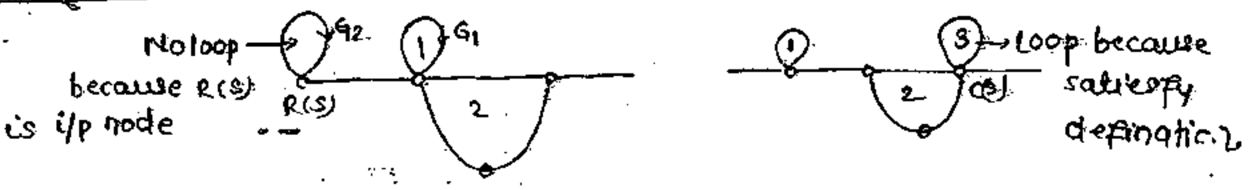
*(3) O/p Node (OR) Sink node → It is a node having only incoming branches.

(4) mix (OR) chain node → It is a node having both incoming & outgoing branches.

(5) Path → It is the ~~traversal~~ of the connected branches in the dirn of branch arrow such that no node is traversed more than one.

(6) forward path → It is a path from i/p node to o/p node.

(7) Loop → It is a path which starts & ends at the same node.



Note → Self loops on the defined i/p nodes are not valid loops.
Loops (or) self loops on the defined o/p nodes are valid loops.

(8) Non-touching loops → 2 (or) more loops are said to be non-touching if they do not have a common node.

* MASON'S GAIN Formula →

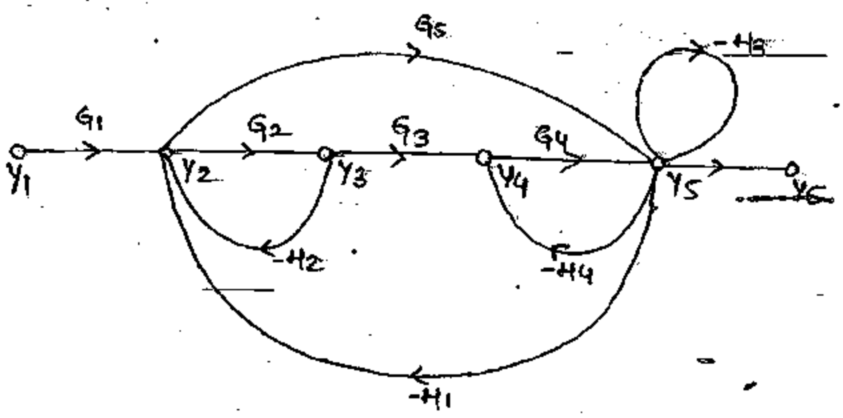
The overall Gain (OR) transfer Function $= \dfrac{\Sigma P_K \Delta_K}{\Delta} = \dfrac{P_1\Delta_1 + P_2\Delta_2 + \cdots + P_R\Delta_R}{\Delta}$

where; P_K = Path gain of K^{th} forward path.

$$\Delta = 1 - \begin{bmatrix}\text{Sum of loop gains} \\ \text{of all individual} \\ \text{loops}\end{bmatrix} + \begin{bmatrix}\text{Sum of gain products} \\ \text{of 2 Non-touching} \\ \text{loops}\end{bmatrix}$$

$$- \begin{bmatrix}\text{Sum of gain products} \\ \text{of 3 non-touching loops}\end{bmatrix} + \cdots$$

Δ_K = It is that value of Δ obtained by removing all the loops touching K^{th} forward path.



Case-(i.) $\frac{Y_5}{Y_1}$

(i) Forward paths

$P_1 = G_1G_2G_3G_4$

$P_2 = G_1G_5$

(ii) To find $\Delta \rightarrow$

(a) Individual loops

$I_1 = -G_2H_2$ $\quad I_3 = -H_3$ $\quad I_5 = -G_5H_1$

$I_2 = -G_4H_4$ $\quad I_4 = -G_2G_3G_4H_1$

(b) Two NTL's $\rightarrow$

$L_1 = I_1 I_2 = G_2H_2 = G_2H_2H_4G_4$

$L_2 = I_1 I_3 = G_2H_2H_3$

(iii) To find Δ_1 & Δ_2

$\Delta_1 = \Delta_2 = 1$

$$\frac{Y_5}{Y_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} \qquad \frac{Y_5}{Y_1} = \frac{G_1G_2G_3G_4 + G_1G_5}{\Delta} \text{ ——(i)}$$

$$\Delta = 1 - \begin{bmatrix} -G_2H_2 - G_4H_4 - H_3 \\ -G_2G_3G_4H_1 - G_5H_1 \end{bmatrix} + \begin{bmatrix} G_2H_2G_4H_4 \\ + G_2H_2H_3 \end{bmatrix}$$

Case(ii) $\rightarrow$ To find $Y_5/Y_3 = \dfrac{\left(\frac{Y_5}{Y_1}\right)}{\left(\frac{Y_3}{Y_1}\right)}$

To Find $\frac{Y_3}{Y_1}$.

(i) forward path

$P_1 = G_1G_2$

(ii) To Find Δ ——

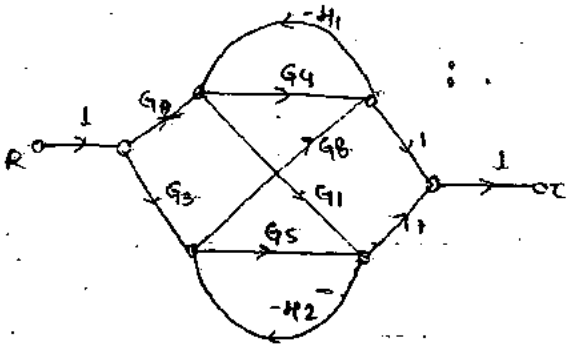
Δ is independent of forward path

(iii) To Find Δ_1

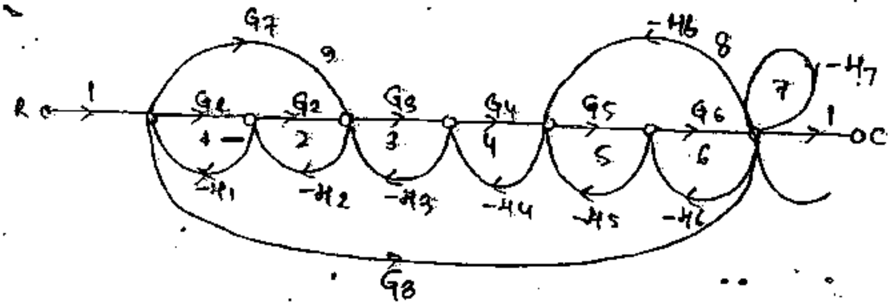
$\Delta_1 = 1 - (-G_4H_4 - H_3)$

$$\frac{Y_3}{Y_1} = \frac{P_1\Delta_1}{\Delta} = \frac{G_1G_2(1+G_4H_4+H_3)}{\Delta} \quad \text{—— (2)}$$

$$\frac{Y_5}{Y_3} = \frac{G_1G_2G_3G_4+G_1G_5}{G_1G_2(1+G_4H_4+H_3)} \quad \text{—— (3)}$$



$$\frac{C}{R} = \frac{G_2G_4(1+G_5H_2)\cancel{+G_3}G_5(1+G_4H_1)+G_2G_1(1)+G_3G_8(1)+G_2G_1(-H_2)G_8(1)+G_3G_8(-H_1)G_1(1)}{1-[-G_4H_1-G_5H_2+G_1G_8H_1H_2]+[G_4H_1G_5H_2]}$$



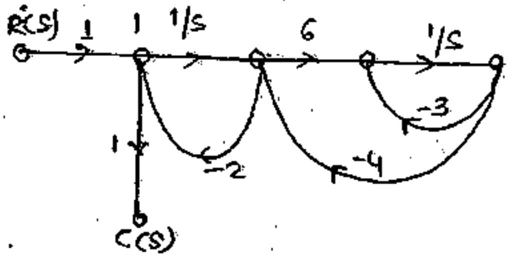
Sol^n → $L_{10} = G_8(-H_8)(-H_4)(-H_3)(-H_2)(-H_1)$

$L_{11} = G_8(-H_6)(-H_5)(-H_4)(-H_3)(-H_2)(-H_1)$

No. of forward paths = 3

No. of loops = 11

Q(11)/56



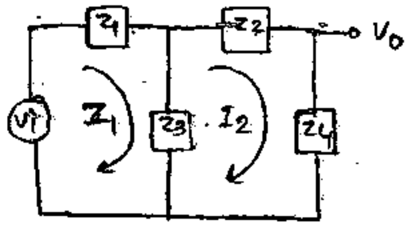
calc. $\frac{C(s)}{R(s)} = ?$

~~Sol^n~~

$$\frac{C(s)}{R(s)} = \frac{1\left\{1-\left(\frac{-3}{s}-\frac{24}{s}\right)\right\}}{1-\left[\frac{-2}{s}-\frac{3}{s}-\frac{24}{s}\right]+\frac{6}{s^2}}$$

$$= \frac{\frac{s+27}{s}}{\frac{s^2+29s+6}{s^2}} = \frac{s(s+27)}{s^2+29s+6}$$

12/56



cal. G_2 & $H = ?$

Sol^n → Loop-(1) →

$V_i = I_1 Z_1 + (I_1 - I_2) Z_3$

$V_i = I_1(Z_1 + Z_3) - I_2 Z_3$

$I_1 = \frac{V_i}{Z_1+Z_3} + \frac{I_2 Z_3}{Z_1+Z_3}$

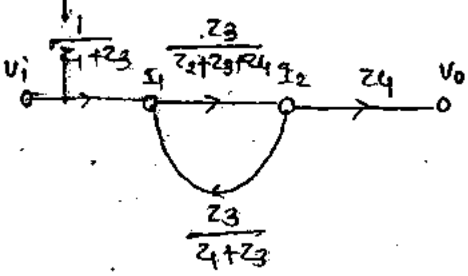
loop(2) →

$0 = I_2 Z_2 + I_2 Z_4 + (I_2 - I_1) Z_3$

$I_2(Z_2 + Z_3 + Z_4) = I_1 Z_3$

$I_2 = I_1 \cdot \frac{Z_3}{Z_2+Z_3+Z_4}$

Consider the variables as node in the given que. eq.

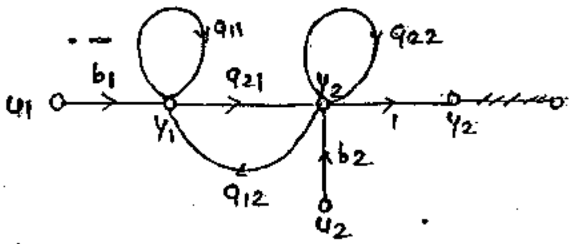


$$\boxed{G_2 = \frac{Z_3}{Z_2+Z_3+Z_4} \qquad H = \frac{Z_3}{Z_1+Z_3}}$$

Q.→ Find $\frac{y_2}{u_1}$ & $\frac{y_2}{u_2}$ using SFG

$$y_1 = a_{11}y_1 + a_{12}y_2 + b_1u_1$$
$$y_2 = a_{21}y_1 + a_{22}y_2 + b_2u_2$$

Sol^n→



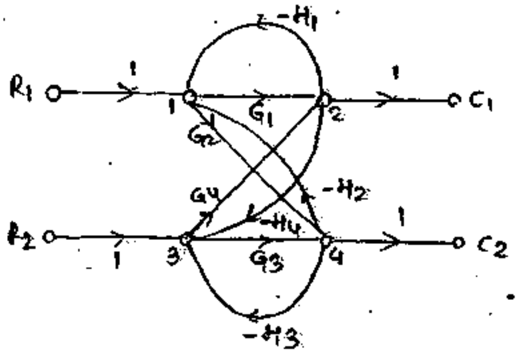
case(1)→ $\left.\frac{y_2}{u_1}\right|_{u_2=0}$

$$\frac{y_2}{u_1} = \frac{b_1 a_{21}(1)}{1-(a_{11}+a_{22}+a_{12}a_{21})+(a_{11}a_{22})}$$

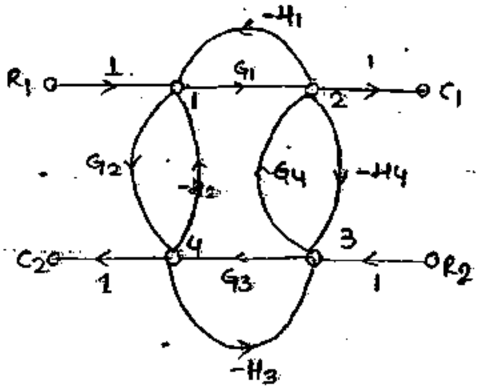
case(2) $\left.\frac{y_2}{u_2}\right|_{u_1=0}$

$$\frac{y_2}{u_2} = \frac{b_2(1-a_{11})}{1-(a_{11}+a_{22}+a_{12}a_{21})+(a_{11}a_{22})}$$

Q.→



Sol^n→



case(1) $\left.\frac{C_1}{R_1}\right|_{R_2=C_2=0}$

$$\frac{C_1}{R_1} = \frac{G_1(1+G_3H_3)+G_2(-H_3)G_4(1)}{1+\left[-G_1H_1-G_4H_4-G_2H_2-G_3H_3+G_2H_3G_4H_1+G_1H_2G_3H_4\right]+(G_1H_1G_3H_3+G_2H_2G_4H_4)}$$

<u>case(2)</u> $\left.\frac{C_2}{R_2}\right|_{C_1=R_1=0}$; $\frac{C_2}{R_2} = \frac{G_3(1+G_1H_1) + G_4(-H_1)G_2(1)}{\Delta}$

<u>case(3)</u> $\left.\frac{C_2}{R_1}\right|_{C_1=R_2=0}$; $\frac{C_2}{R_1} = \frac{G_3(1+G_4H_4) + G_1(-H_4)G_3(1)}{\Delta}$

<u>case(4)</u> $\left.\frac{C_1}{R_2}\right|_{C_2=R_1=0}$; $\frac{C_1}{R_2} = \frac{G_4(1+G_2H_2) + G_3(-H_2)G_1(1)}{\Delta}$

conq(3) 58

$$F = m\frac{d^2x}{dt^2} + f\frac{dx}{dt} + kx$$

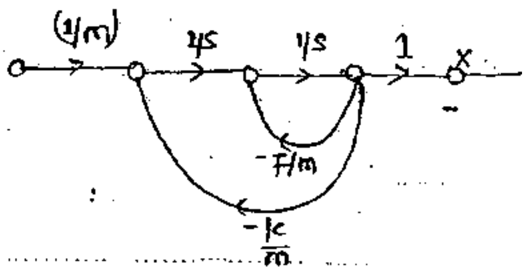
$$F(s) = (ms^2 + fs + k)\,X(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + fs + k}$$

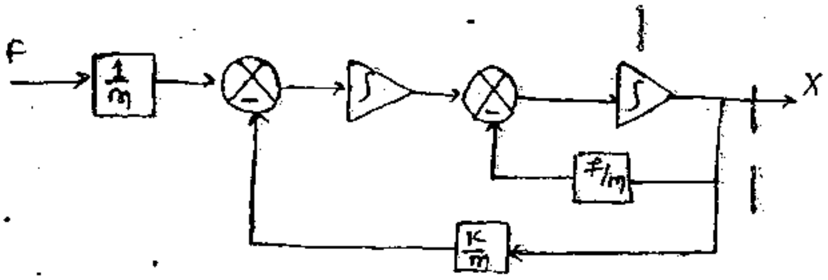
$$= \frac{(1/m)}{s^2 + \left(\frac{f}{m}\right)s + \left(\frac{K}{m}\right)}$$

<u>State diagram →</u>

$$\frac{\frac{(1/m)}{s^2}}{1 + \frac{(f/m)}{s} + \frac{(K/m)}{s^2}}$$



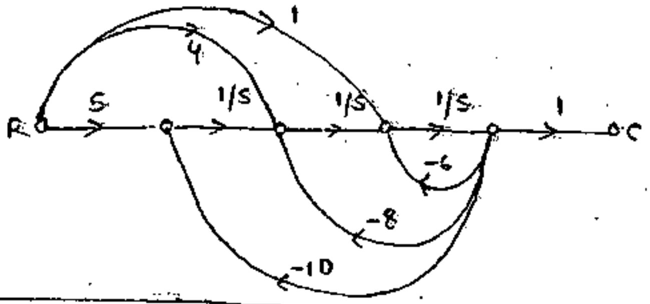
$$\frac{\frac{(1/m)}{s^2}}{1 - \left[\frac{-f/m}{s} - \frac{K/m}{s^2}\right]}$$

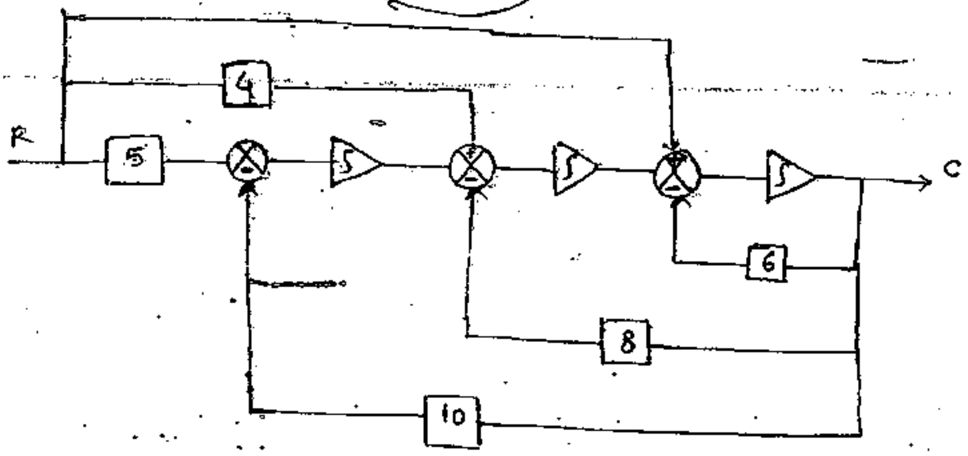


Q→ $\dfrac{C(s)}{R(s)} = \dfrac{s^2+4s+5}{s^3+6s^2+8s+10}$

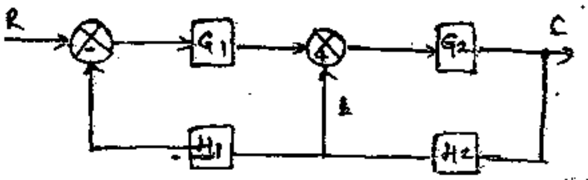
Sol^n→

$$\frac{C(s)}{R(s)} = \frac{\frac{s^2}{s^3}+\frac{4s}{s^3}+\frac{5}{s^3}}{1+\frac{6s^2}{s^3}+\frac{8s}{s^3}+\frac{10}{s^3}} = \frac{\frac{1}{s}+\frac{4}{s^2}+\frac{5}{s^3}}{1-\left(\frac{-6}{s}-\frac{8}{s^2}-\frac{10}{s^3}\right)}$$

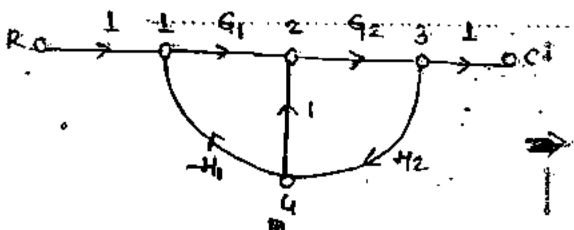


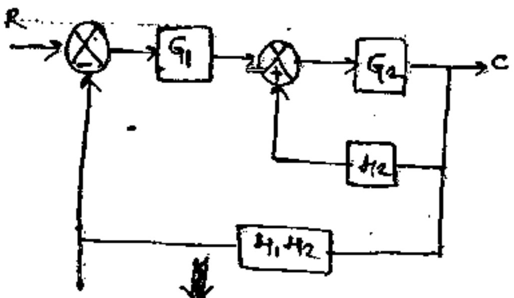


Q→

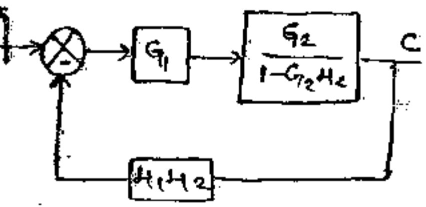


Sol^n→

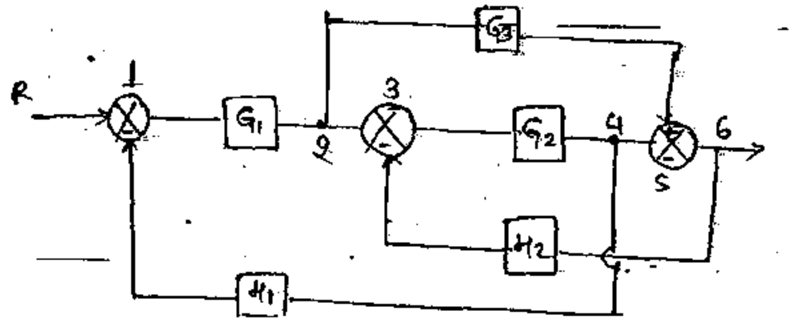




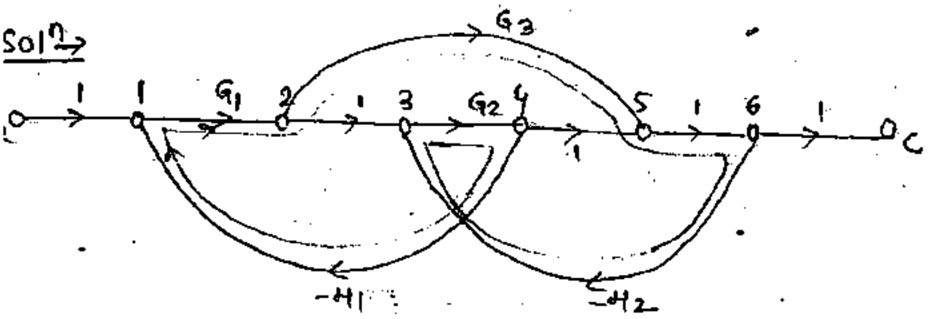
$$\frac{C}{R} = \frac{G_1G_2(1)}{1-(G_2H_2-G_1G_2H_1H_2)} = \frac{G_1G_2}{1-G_2H_2+G_1G_2H_1H_2}$$



1(Conv.)
56

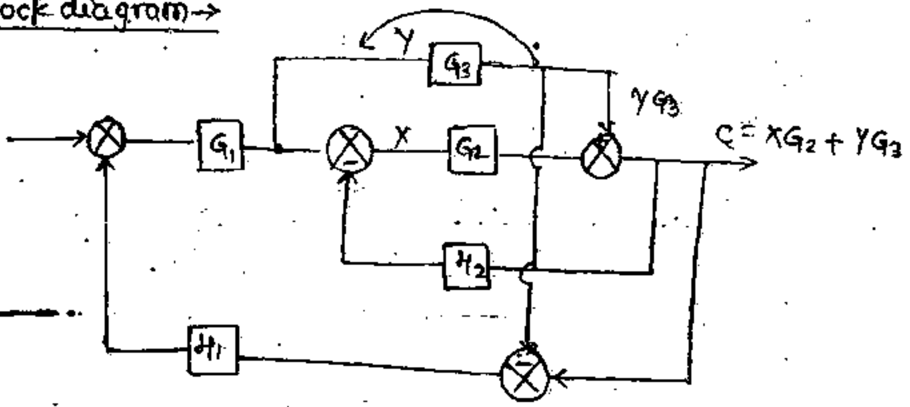


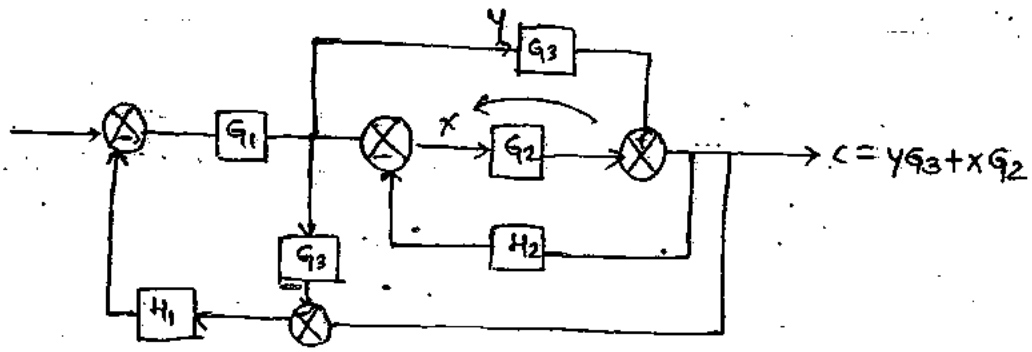
Sol^n→

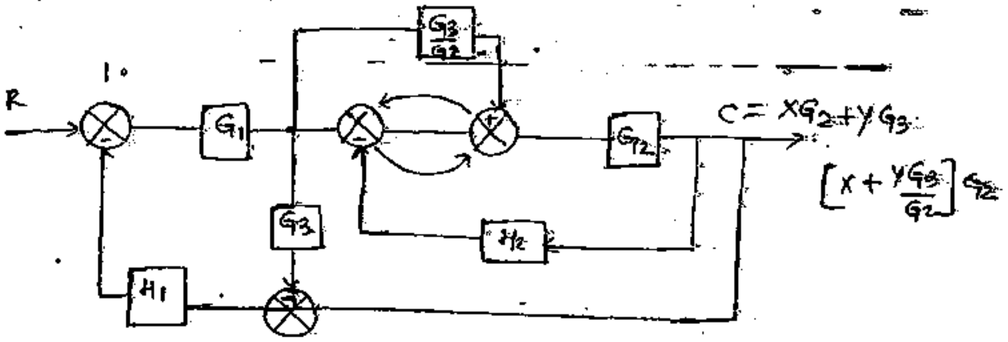


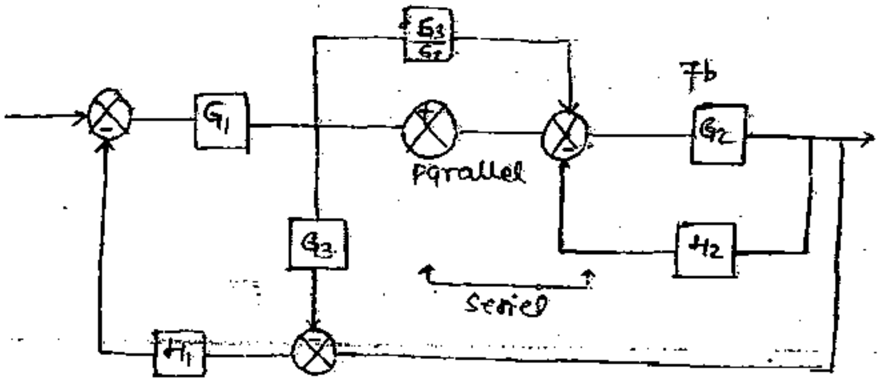
$$\frac{C}{R} = \frac{G_1G_2(1) + G_1G_3(1)}{1 - (-G_1G_2H_1 - G_2H_2 + G_1G_2G_3H_1H_2)}$$

Block diagram→







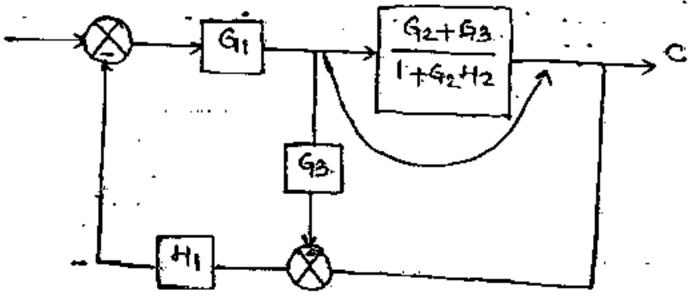


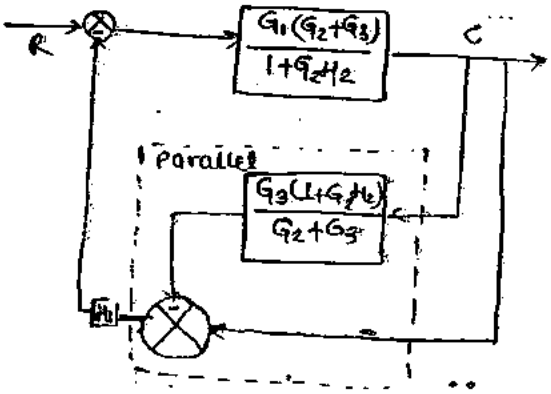
Parallel $= 1 + \frac{G_3}{G_2}$

$= \frac{G_2+G_3}{G_2}$

Feedback $= \frac{G_2}{1+G_2H_2}$

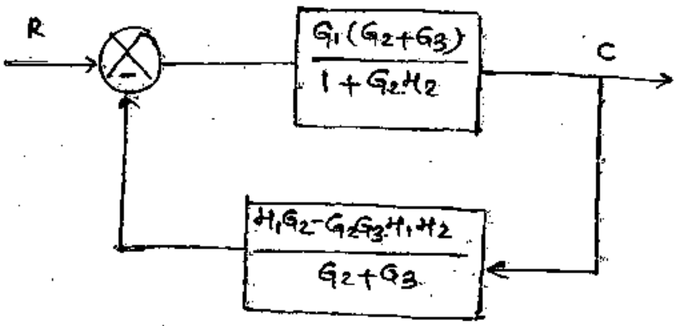
Series $= \frac{G_2+G_3}{1+G_2H_2}$





Parallel

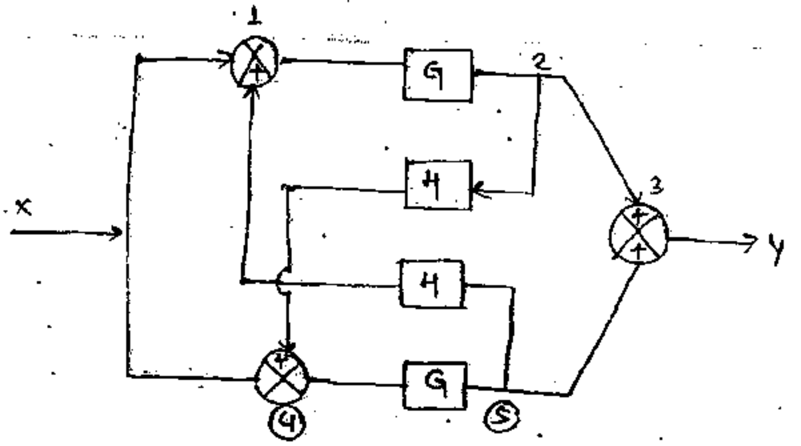
$$1 - \frac{G_3(1+G_2H_2)}{G_2+G_3} = \frac{G_2+\cancel{G_3}-\cancel{G_3}-G_2G_3H_2}{G_2+G_3}$$



$$\frac{C}{R}=\frac{G_1(G_2+G_3)}{1+G_2H_2+G_1G_2H_1-G_1G_2G_3H_1H_2}$$

Q.→ $\frac{Y}{X}$ equals

(a.) $\frac{2G}{1-2GH}$ (b.) $\frac{2G}{1-GH}$ (c.) $\frac{G}{1-2GH}$ (d.) $\frac{G}{1-GH}$

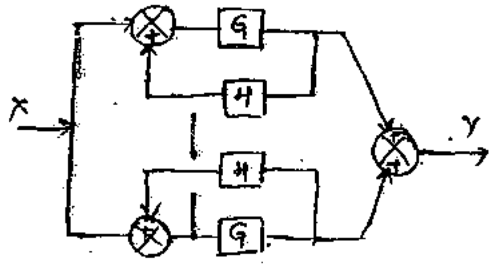


Soln→

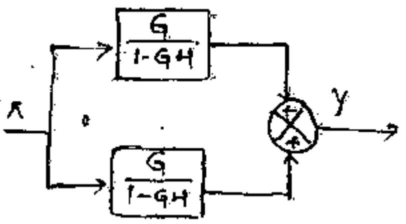
$$\frac{Y}{X}=\frac{G(1)+G(1)+G^2H(1)+G^2H(1)}{1-G^2H^2}=\frac{2G+2G^2H}{1-G^2H^2}=\frac{2G\cancel{(1+GH)}}{\cancel{(1+GH)}(1-GH)}$$

$$\boxed{\frac{Y}{X}=\frac{2G}{1-GH}}$$

By Block dia. reduction→

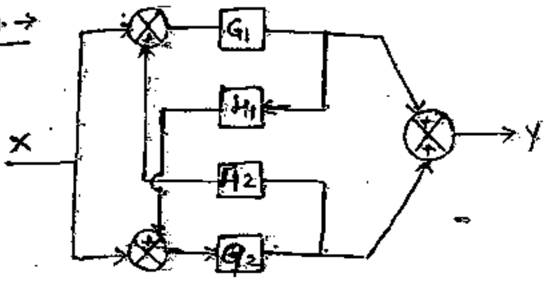


⇒



$$\boxed{\frac{Y}{X}=\frac{2G}{1-GH}}$$

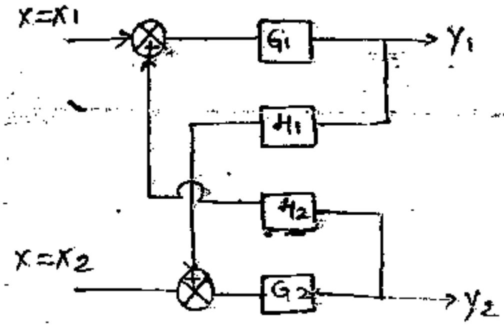
Q →



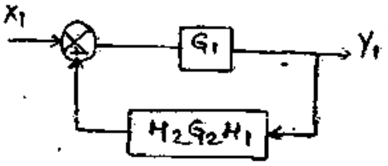
Sol^n →

$$\frac{Y}{X} = \frac{G_1 + G_2 + G_1 H_1 G_2 + G_2 H_2 G_1}{1 - G_1 H_1 G_2 H_2}$$

By Block diagram →

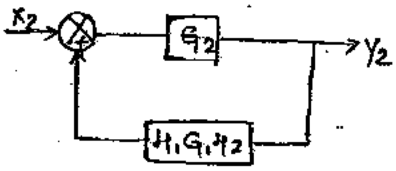


case(i) → $\left.\frac{Y_1}{X_1}\right|_{Y_2 = X_2 = 0}$



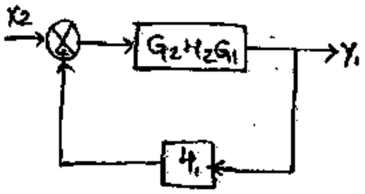
$$\frac{Y_1}{X_1} = \frac{G_1}{1 - G_1 H_1 G_2 H_2}$$

case(2) → $\left.\frac{Y_2}{X_2}\right|_{Y_1 = X_1 = 0}$



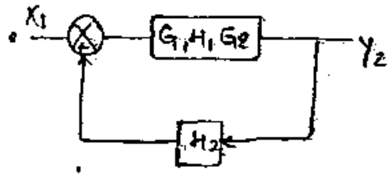
$$\frac{Y_2}{X_2} = \frac{G_2}{1 - G_1 H_1 G_2 H_2}$$

Case(3) → $\left.\frac{Y_1}{X_2}\right|_{Y_2 = X_1 = 0}$



$$\frac{Y_1}{X_2} = \frac{G_1 H_1 G_2}{1 - G_1 H_1 G_2 H_2}$$

Case(4) $\left.\frac{Y_2}{X_1}\right|_{X_2 = Y_1 = 0}$



$$\frac{Y_2}{X_1} = \frac{G_2 H_2 G_1}{1 - G_1 H_1 G_2 H_2}$$

TF for physical systems →

* A general physical sys. is said to be constituted of 5 elements :-

1.) Resistance type element.

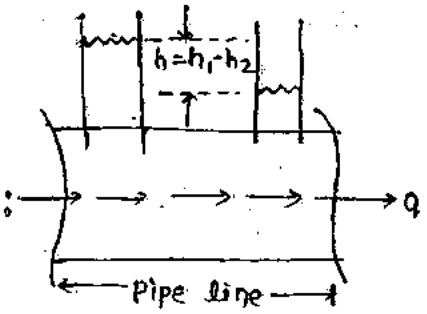
2.) Capacitance type elemet.

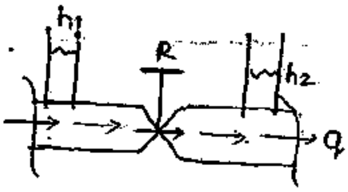
3.) Time-constant element.

4.) Oscillatory element.

5.) Dead time element.

(1.) Resistance type element →





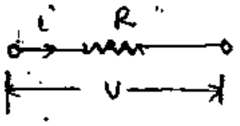
$h \propto q$

$h = Rq$ R = Hydraulic Resistance

$H(s) = RQ(s)$

$Q(s) \longrightarrow \boxed{R} \longrightarrow H(s)$

It is analouges to electrical resistance

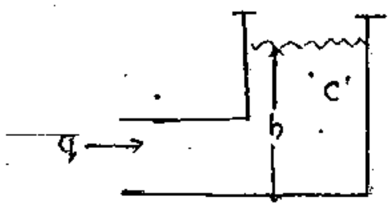


$V = Ri$

$V(s) = RI(s)$

$I(s) \longrightarrow \boxed{R} \longrightarrow V(s)$

(2) Capacitance type element →



C = Hydraulic capacitance
= (Area/volume)

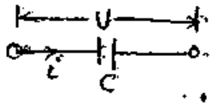
$$q \propto \frac{dh}{dt}$$

$$q = C\frac{dh}{dt}$$

$$Q(s) = CS\,H(s)$$

$$Q(s) \longrightarrow \boxed{1/CS} \longrightarrow H(s)$$

It is analouges to "electrical capacitance"

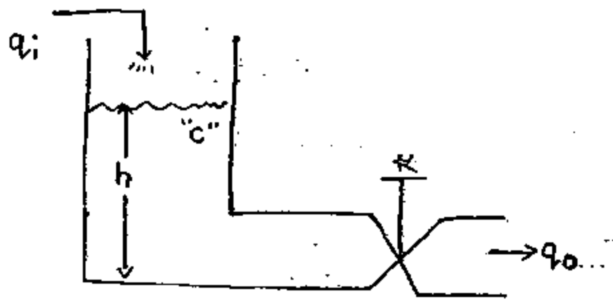


$$i = C\frac{dv}{dt}$$

$$I(s) = CS\,V(s)$$

$$I(s) \longrightarrow \boxed{\frac{1}{CS}} \longrightarrow V(s)$$

3) Time-constant element →



$$q_i - q_o = C\frac{dh}{dt}$$

$$q_i = C\frac{dh}{dt} + q_o$$

since $h = Rq_o$

$$q_o = \frac{h}{R}$$

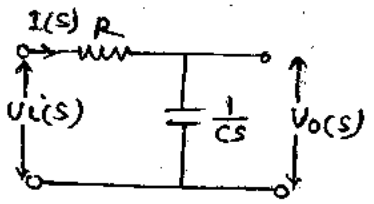
$$q_i = C\frac{dh}{dt} + \frac{h}{R}$$

$$Rq_i = RC\frac{dh}{dt} + h$$

$$RQ_i(s) = (RCS+1)\,H(s)$$

$$\boxed{\frac{H(s)}{Q_i(s)} = \frac{R}{RCS+1}}$$

It is analogous to RC n/w.



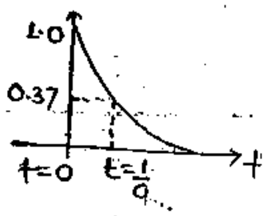
$$V_i(s) = I(s)\left[R + \frac{1}{CS}\right]$$

$$= I(s)\left[\frac{RCS+1}{CS}\right]$$

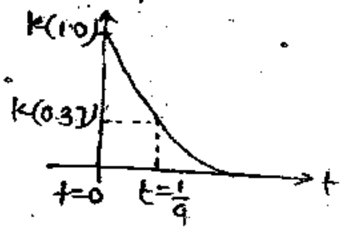
$$V_o(s) = I(s)\frac{1}{CS}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{RCS+1}}$$

Eg:- $\frac{1}{(S+a)} \Rightarrow e^{-at} \Rightarrow$

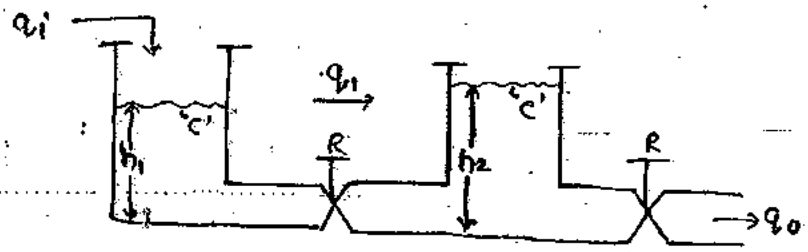


$\frac{K}{S+a} \Rightarrow Ke^{-at} \Rightarrow$



~~(4) Oscillatory element →~~

*** Interacting & non-interacting system →**



$$q_i - q_1 = C\frac{dh_1}{dt}$$

$$Q_i(s) - Q_1(s) = CSH_1(s) \text{ ——— (i)}$$

$$h_1 - h_2 = Rq_1$$

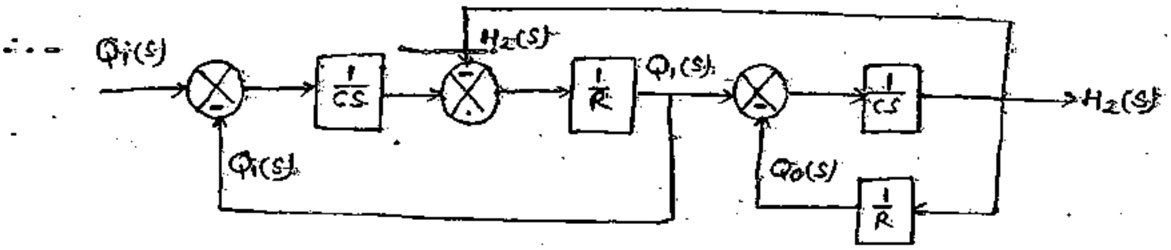
$$H_1(s) - H_2(s) = RQ_1(s) \text{ ——— (ii)}$$

$$q_1 - q_0 = C\frac{dh_2}{dt}$$

$$Q_1(s) - Q_0(s) = CSH_2(s) \text{ ——— (iii)}$$

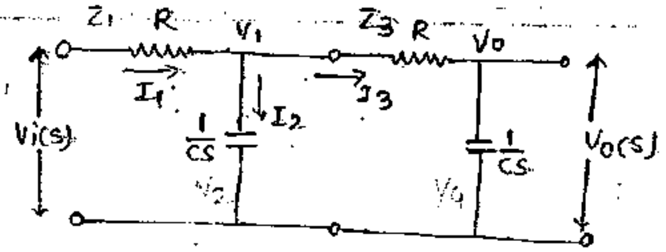
$$h_2 = Rq_0$$

$$H_2(s) = RQ_0(s) \text{ ——— (iv)}$$



$$\frac{H_2(s)}{Q_i(s)} = \frac{\frac{1}{R^2C^2S^2}(1)}{1-\left[\frac{-1}{RCS}-\frac{1}{RCS}-\frac{1}{RCS}\right]+\frac{1}{R^2C^2S^2}} = \frac{\frac{1}{R^2C^2S^2}}{\frac{R^2C^2S^2+3RCS+1}{R^2S^2C^2}}$$

$$\boxed{\frac{H_2(s)}{Q_i(s)} = \frac{R}{R^2C^2S^2+3RCS+1}}$$



(1.) Current through $Y_4\ (I_3) = V_o \cdot Y_4 = V_o CS$.

(2.) To Find V_1

$$I_3 = \frac{V_1 - V_o}{Z_3} \Rightarrow \quad V_1 = I_3 Z_3 + V_o = V_1 = V_o CS \cdot R + V_o$$

$$V_1 = V_o(1+RCS)$$

(3.) To find I_2

$$I_2 = V_1 \cdot Y_2 = V_o(1+RCS)\cdot CS \Rightarrow V_o(CS+RC^2S^2)$$

(4.) To find I_1

$$I_1 = I_2 + I_3 = V_o(CS+RC^2S^2) + V_o CS = V_o(2CS+RC^2S^2)$$

(5.) To Find V_i

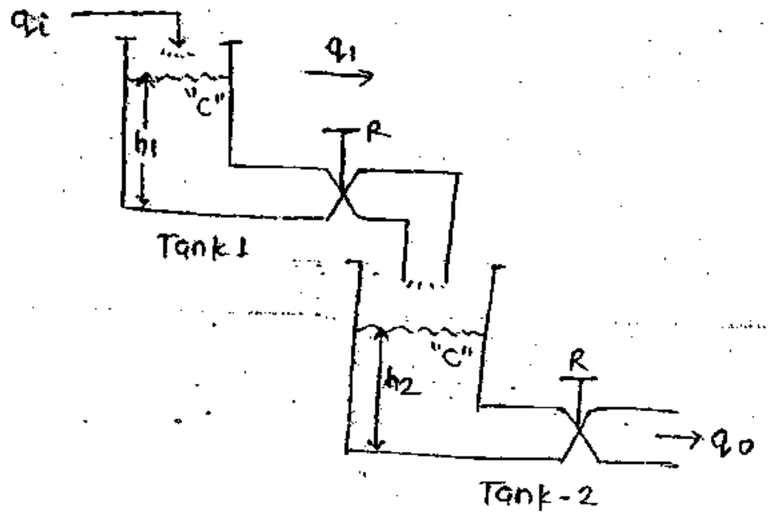
$$I_1 = \frac{V_i - V_1}{Z_1} \Rightarrow V_i = I_1 Z_1 + V_1 \Rightarrow V_o(2CS+RC^2S^2)R + V_o(1+RCS)$$

$$V_i = V_o(2RCS + R^2C^2S^2 + 1 + RCS)$$

$$\boxed{\frac{V_o(S)}{V_i(S)} = \frac{1}{R^2C^2S^2 + 3RCS + 1}}$$

Note→ When 2 time constant elements are cascoded interactively the overall TF of such an arrangement is not the product of 2 individual TF.

*** Non-interacting system →**



Tank-(01)

$$q_i - q_1 = c\frac{dh_1}{dt}$$

$$q_i = c\frac{dh_1}{dt} + q_1$$

Since $h_1 = Rq_1$

$$q_1 = h_1$$

$$q_i = c\frac{dh_1}{dt} + \frac{h_1}{R}$$

$$Rq_i = Rc\frac{dh_1}{dt} + h_1$$

$$RQi(s) = (Rcs+1)\,H_1(s) \text{ ---- (i)}$$

Tank (02)

$$q_1 - q_o = c\frac{dh_2}{dt}$$

$$q_1 = c\frac{dh_2}{dt} + q_o$$

$$\frac{h_1}{R} = c\frac{dh_2}{dt} + \frac{h_2}{R}$$

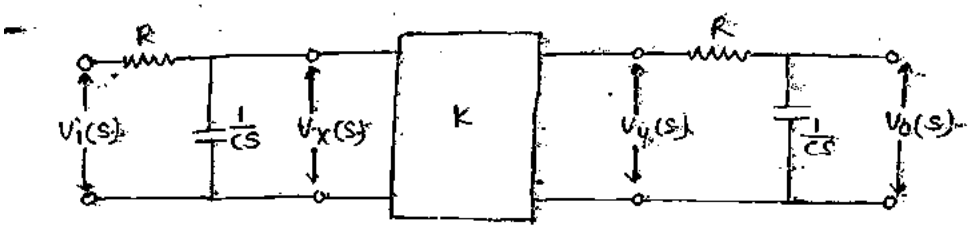
$(\because h_2 = Rq_o)$

$$h_1 = Rc\frac{dh_2}{dt} + h_2$$

$$H_1(s) = (Rcs+1)\,H_2(s) \text{ ---- (ii)}$$

$$Q_i(s) \longrightarrow \boxed{\frac{R}{Rcs+1}} \xrightarrow{H_1(s)} \boxed{\frac{1}{Rcs+1}} \xrightarrow{H_2(s)}$$

$$\boxed{\frac{H_2(s)}{H_1(s)} = \frac{R}{(Rcs+1)^2}}$$



$$\frac{V_x(s)}{V_i(s)} = \frac{1}{RCs+1} \qquad k = \frac{V_y(s)}{V_x(s)} \qquad \frac{V_o(s)}{V_y(s)} = \frac{1}{RCs+1}$$

$$V_i(s) \longrightarrow \boxed{\frac{1}{RCs+1}} \xrightarrow{V_x(s)} \boxed{K} \xrightarrow{V_y(s)} \boxed{\frac{1}{RCs+1}} \longrightarrow V_o(s)$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{k}{(RCs+1)^2}}$$

<u>Note→</u> when 2 time constant elements are cascaded non-interactively the overall TF of such an arrangement is the product of 2 individual TF.

<u>* Control sys components→</u> CS there are 2 types:-

<u>(1) Tranducers →</u> A ~~Xcer~~ is a device which when actuated with one form of energy is capable of converted it into any other related form; the converrion is usualy from non-electrical to electrical.

<u>(2) Servomechanisms →</u> *They are electromech. sys. whose i/p is ele. voltage & o/p is mech. position (OR) speed.

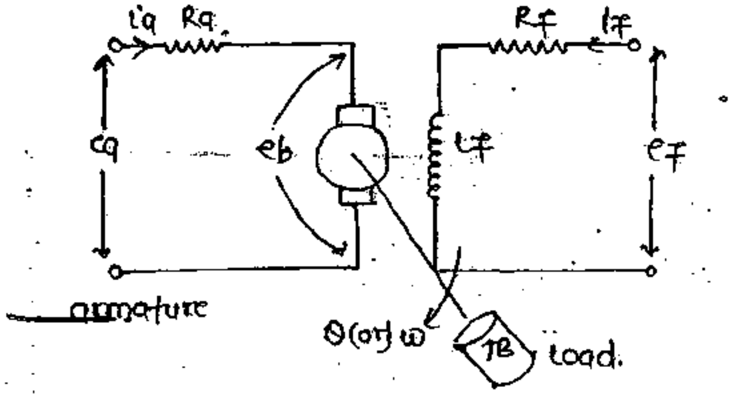
* They are also known as inverse Xcers.

servo - low power low ~~freq.~~ application

* The term servo indicates low power & low freq. appli^n. It also indicates that the i/p & o/p c/s must be approximately linear.

Servomotors
- Ac Servomotors
- dc Servomotor
  - Armature Controlled
  - Field Controlled

* DC Servomotor →



- armature Controlled dc servomotors →

Input = e_a ; output = θ (OR) ω

Principle of operation →

(i) Air gap flux (ϕ) $\propto$ Field current (i_f)

$$\phi = K_f \cdot i_f$$

(ii) $T_m \propto \phi \cdot i_a$

$T_m \propto K_f \cdot i_f \cdot i_a$

$$T_m = \underbrace{K_1 K_f \cdot i_f}_{K_T = \text{motor torque const}} \cdot i_a$$

$T_m = K_T i_a$

$T_m(s) = K_T I_a(s)$ —— (i)

(3.) Back emf (e_b) $\propto$ speed (ω)

$e_b = K_b \omega$

$E_b(s) = K_b \omega(s)$ —— (2)

(4.) Analysis of arm. current

$e_a = i_a R_a + e_b$

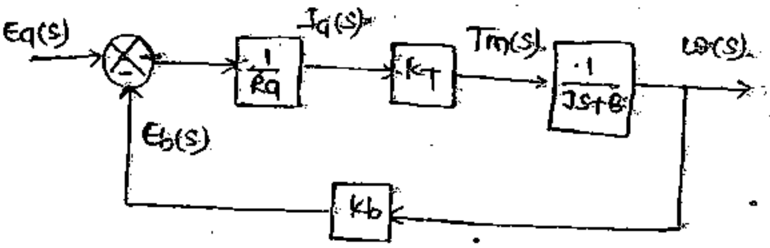
$e_a - e_b = i_a R_a$

$E_a(s) - E_b(s) = I_a(s) R_a$ —— (3.)

(5.) At load

$T_m = J\frac{d\omega}{dt} + B\omega$

$T_m(s) = (Js + B)\, \omega(s)$ —— (4.)



$$\frac{\omega(S)}{E_a(S)} = \frac{\dfrac{k_T}{R_a(JS+B)}}{1+\dfrac{k_T k_b}{R_a(JS+B)}} = \frac{k_T}{R_a(JS+B)+k_T k_b}$$

$$= \frac{k_T/R_a}{JS+\underbrace{B+\frac{k_T k_b}{R_a}}_{f}} = \frac{k_T/R_a}{JS+f}$$

$$\boxed{\frac{\omega(S)}{E_a(S)} = \frac{k_T/R_a}{JS+f}}$$

Note:- The TF of arm. control dc servomotor has a single time constant element & it represents the f/b control mechanism as seen from the block dia.

* Field control servomotor →

$I/p = e_f$; $o/p = \theta$ (or) ω

Principle of operation →

(1.) Air gap flux ∝ field current
(ϕ) (i_f)

$\phi = k_f i_f$

(2.) $T_m \propto \phi \cdot i_a$

$T_m \propto k_f i_f i_a$

$T_m = \underbrace{k_1 k_f i_a}_{k_T} \cdot i_f$

k_T = motor torque const.

$T_m = k_T i_f$

$T_m(S) = k_T I_f(S)$ ——(1)

(3.) Back emf (e_b) ∝ speed (ω)

$e_b = k_b \omega$

$E_b(S) = k_b \omega(S)$ ——(2)

(4) Analysis of field circuit

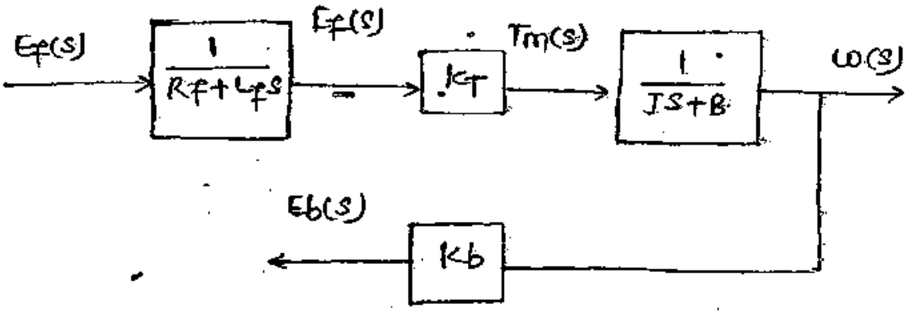
$e_f = i_f R_f + L_f \dfrac{di_f}{dt}$

$E_f(S) = I_f(S)\left[R_f + L_f \cdot S\right]$ ——(3.)

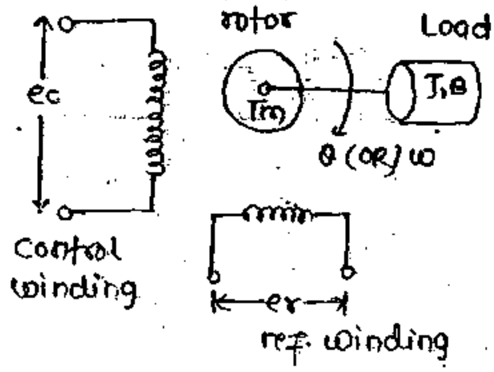
(5.) At load.

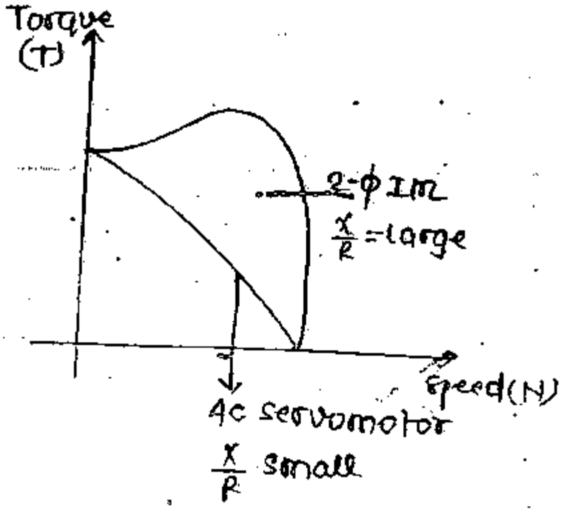
$T_m = J\dfrac{d\omega}{dt} + B\omega$

$T_m(S) = (JS+B)\,\omega(S)$ ——(4)



Ac servomotor →





* It is constrctionaly similar to a 2-φ indⁿ motor.
* Out of the 2 wdgs placed in quadrature. One of the wdg is excited by ~~for~~ a constant vol. & is known as ref wdg.
* The torque developed by the rotor is proportional to control winding vol.
* The rotor of Ac servomotor is built with high resistance so that; its $\frac{X}{R}$ ratio is small & torque-speed c/s are approximately linearised.
* In the TF modelling since the torque developed by the rotor is proportional to control winding vol.; the resistance & inductance of the control wdg is assumed to be negligible.

$$T_m \propto e_c$$

$$T_m = k_m e_c$$

$$\boxed{k_m = \frac{T_a}{e_c} \quad \text{where; } T_a = \text{stall torque}}$$

$$T_m(s) = k_m E_c(s) \text{ ------ (i)}$$

at load →

$$T_m = J\frac{d\omega}{dt} + B\omega$$

$$T_m(s) = (Js+B)\cdot\omega(s) \text{ ----- (2)}$$

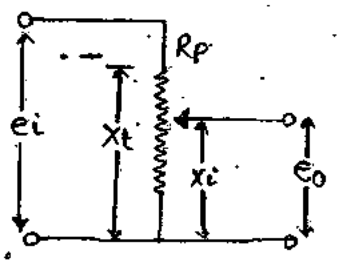
From eq^n (i) & (ii)

$$k_m E_c(s) = (Js+B)\,\omega(s)$$

$$\boxed{\frac{\omega(s)}{E_c(s)} = \frac{k_m}{Js+(B-m)}}$$

where; m = correction factor slope of N-T charts.

* Potentiometer →



* It is a variable resistive displacement Xeer used as error detector in CS app^n

* A pair of potentiometer act as error detector.

Input = wiper disp. X_i , o/p = e_0

Total resistance of POT = R_p

Resistance/for unit length = $\frac{R_p}{X_t}$

Resistance for wiper disp of X_i units = $\frac{R_p}{X_t}.X_i$

Applying Voltage divider;

$$e_0 = \frac{\frac{R_P}{X_t} \cdot X_i}{R_P} \times e_i$$

$$e_0 = \frac{X_i}{X_t} \cdot e_i$$

Let k_p = pot gain $= \frac{e_i}{X_t} \left(\frac{V}{mm}\right)$

$$e_0 = k_p X_i$$

$$E_0(s) = k_p X_i(s)$$

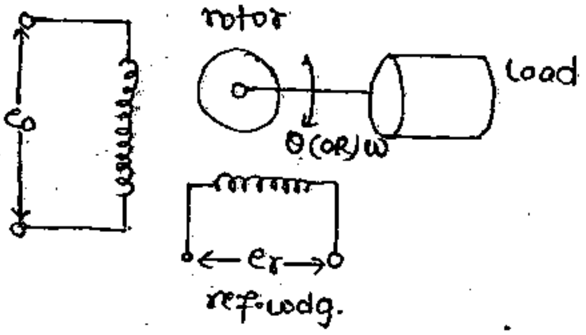
$$X_i(s) \longrightarrow \boxed{k_p} \longrightarrow E_0(s)$$

* Tachometer → * They are speed Xcer used as F/b elements in CS appⁿ.

* There of 2 types :-

(i) DC tachometer → These is a small dc genʳ whose i/p is mech. speed & o/p is ele. voltage proportional to the speed.

~~(ii) AC~~ tachometer →



* Out of the 2 wdgs placed in quadrature only one of the wdg is excited by a constant vol. & is known as ref. wdg.

* When the rotor is stationary the peripheral flux links the ref. wdg only. As the rotor rotates the rate of change of flux induces an emf

which is directly proportional to speed of rotor.
The ac tachometer is also known as drag cup gen^r because the rotor ~~Transfer f^n~~ is of drag cup shape.

Transfer $f^n \rightarrow$

$e_0 \propto$ speed

(a.) Input $= \theta$, O/p $= e_0$

$$e_0 \propto \frac{d\theta}{dt}$$

$$e_0 = k_T \cdot \frac{d\theta}{dt}$$

$$E_0(s) = k_T \cdot s \cdot \theta(s)$$

$$\theta(s) \longrightarrow \boxed{k_T s} \longrightarrow E_0(s)$$

(b.) I/p $= \omega$

O/p $= e_0$

$$e_0 \propto \omega$$

$$E_0(s) = k_T\, \omega(s)$$

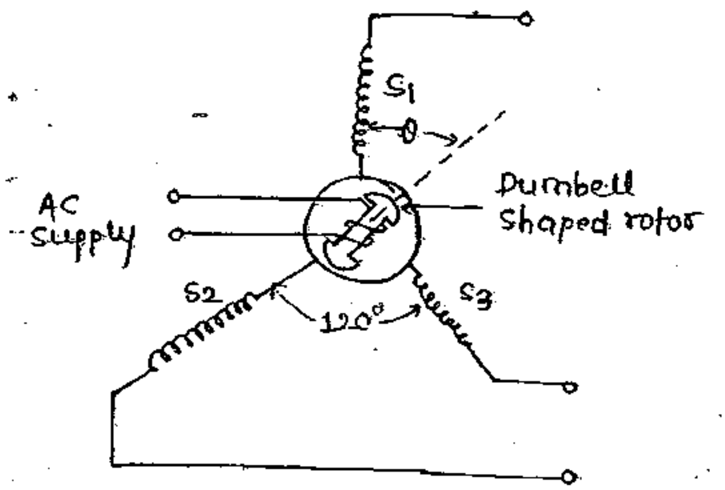
$$\omega(s) \longrightarrow \boxed{k_T} \longrightarrow E_0(s)$$

where; k_T = Tachometer gain

Ex $\rightarrow$ The T.F. of a tachometer $\frac{E(s)}{\theta(s)}$ is

(a) ks (b) ks^2 (c) $\frac{k}{s}$ (d) k

* SYNCHRO $\rightarrow$

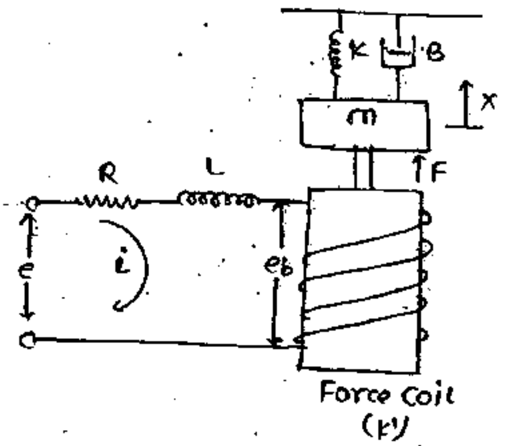


It is commercially known as SELSYN (OR) autosyn.

It is an electromagnetic Xcer which converts angular portion of the rotor into proportional voltages.

* It is constructionaly similar to a 3-ϕ alternator but operationaly the principle is based on Xmer action.
* when the rotor is Inclined along any one of the stator access completely then max^m vol. will be induced in that wdg. & negligible vol. is induced in the other 2 wdgs.
* Such a position of rotor is known as ele. zero position.
* A pair of synchro known as synchro Xmitter & synchro control Xmer (receiver) act as error detector.
* The Xmitter rotor is dumbell shape & th

Que.→



Solⁿ→ I/p = e ; O/p = x

$$e = iR + L\frac{di}{dt} + e_b$$

$$e - e_b = iR + L\frac{di}{dt}$$

$$E(s) - E_b(s) = I(s)\left[R + Ls\right] \text{------ (i)}$$

e_b (tranduced vol.)

$$e_b \propto K_T\frac{dx}{dt}$$

$$E_b(s) = K_T\, s X(s) \text{------ (ii)}$$

where; K_T = Tachometer gain.

At Force coil

$$F \propto i$$

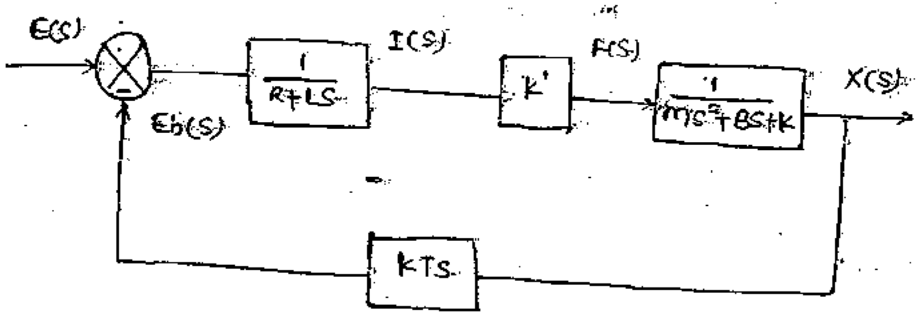
$$F = K'i$$

$$F(s) = K' I(s) \text{------ (iii)}$$

At mech. sys.

$$F = m\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

$$F(s) = (ms^2 + Bs + K)X(s) \text{------ (iv)}$$



E(S)
Eb(S)
1
R+LS
I(S)
K'
F(S)
1
ms²+BS+K
X(S)
KTS