CBSE Sample Paper-03 SUMMATIVE ASSESSMENT-II MATHEMATICS Class - X

Time allowed: 3 hours

General Instructions:

- a) All questions are compulsory.
- b) The question paper consists of 31 questions divided into four sections A, B, C and D.
- c) Section A contains 4 questions of 1 mark each which are multiple choice questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
- d) Use of calculator is not permitted.

Section A

- 1. The probability that Apala and Meenu have same birthday (ignoring a leap year) is:
 - (a) $\frac{364}{365}$ (b) $\frac{1}{73}$ (c) $\frac{1}{365}$ (d) $\frac{3}{73}$
- 2. If A(5, y), B (1, 5), C (2, 1) and D (6, 2) are the vertices of a square, then the value of y is:
 - (a) 1 (b) 3 (c) 2 (d) 6
- 3. The list of numbers -12, -9, -6, -3, 0, 3, is:
 - (a) not an AP (b) an AP with d = 3
 - (c) an AP with d = -3 (d) an AP with d = 0
- 4. The height of a tree, if it casts a shadow 17 m long on the level of ground, when the angle of elevation of the Sun is 45°, is:
 - (a) 17 m (b) 8 m (c) 10 m (d) 14 m

Section B

5. Find the perimeter of the figure, where AED is a semi-circle and ABCD is a rectangle.



- 6. How many spherical lead shots of radius 2 cm can be made out of a solid cube of lead whose edge measures 44 cm?
- 7. The rain water from a roof 22 m x 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m of the vessel is just full, find the rainfall in cm.
- 8. Find the discriminant of the quadratic equation: $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

Maximum Marks: 90

- 9. If 3 times the 3rd term of an AP is equal to 7 times its 7th term, then find its 10th term.
- 10. In figure, PQL and PRM are tangents to the circle with centre O at the points P and R respectively and S is a point on the circle such that \angle SQL = 40° and \angle SRM = 70°. Then find \angle QSR.



Section C

- 11. If the point C(-1,2) divides the line segment AB in the ratio 3 : 4, where the coordinates of A are (2, 5), then find the coordinates of B.
- 12. Find the coordinates of the point equidistant from thee given points A (5, 1), B(-3,-7) and C (7,-1).
- 13. A paper is in the form of a rectangle ABCD with AB = 18 cm and BC = 14 cm. A semi-circular portion with BC as diameter is cut off. Find the area of the remaining paper.



14. PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in figure. Find the perimeter of the shaded region.



- 15. The radius of the base and the height of a solid right circular cylinder are in the ratio 2 : 3 and its volume is 1617 cm³. Find the total surface area of the cylinder.
- 16. Solve the quadratic equation: $4x^2 + 4\sqrt{3}x + 3 = 0$
- 17. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th term is 44. Find the first three terms of the AP.
- 18. A circle touching the side BC of $a\,\Delta\,ABC$ at P and touching AB and AC produced at Q and R

respectively. Prove that: $AQ = \frac{1}{2}$ (Perimeter of $\triangle ABC$)

- 19. The length of a shadow of a tower standing on level plane is found to be 20 m longer when the Sun's altitude is 30° , than when twas 60° . Find the height of the tower.
- 20. All the three face cards of spade are removed from a well shuffled pack of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting:
 - (i) a black face card.
 - (ii) a queen'
 - (iii) ablack card.

Section D

- 21. Construct a triangle ABC in which AB = 6.5 cm, $\angle B = 60^{\circ}$ and BC = 5.5 cm. Also construct a triangle AB'C' similar to \triangle ABC, whose each side is $\frac{3}{2}$ times the corresponding side of \triangle ABC.
- 22. A man standing on the deck of a ship, which is 10 m above the water level, observes the angle of elevation of the top of the hill as 60° and the angle of depression of the base of the hill as 30° . Calculate the distance of the hill from the ship and the height of the hill.
- 23. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is:
 - (i) acard of spades or an ace (ii) a red king
 - (iii) neither a king nor a queen (iv) either a king or a queen
- 24. Prove that the coordinates of the centroid of a \triangle ABC with vertices A(x_1, y_1), B(x_2, y_2) and C

$$(x_3, y_3)$$
 are given by $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

- 25. A right-angled triangle whose sides are 15 cm and 20 cm is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Use $\pi = 3.14$)
- 26. A bucket made up of a metal sheet in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the bucket, if the cost of the metal sheet used is Rs.15 per 100 cm². (Use $\pi = 3.14$)
- 27. A motorboat whose speed is 9 km/h in still water goes 12 km downstream and comes back in a total time of 3 h.
 - (i) Find the speed of the stream.
 - (ii) Explain the situation when speed of the stream is more than the speed of the boat in still water.
- 28. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.
- 29. How many terms of the series 54, 51, 48,be taken, so that their sum is 513? Explain the double answer.

30. X and Y are centres of circles of radius 9 cm and 2 cm and XY = 17 cm. Z is the centre of a circle of radius r cm, which touches the above circles externally. Given that \angle XZY = 90°, write an equation in r and solve it for r.



31. Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above result, prove the following:

If a circle touches all the four sides of a parallelogram, show that the parallelogram is a rhombus.

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(Solutions)

SECTION-A

- 1. (c)
- 2. (d)
- 3. (b)
- 4. (a)
- 5. Here, $a = 3\sqrt{3}, b = 10, c = \sqrt{3}$ Discriminant $= b^2 - 4ac$

$$= (10)^{2} - 4(3\sqrt{3}).(\sqrt{3})$$
$$= 100 - 36$$
$$= 64$$

6. $r = \frac{14}{2} = 7 \text{ cm}$

Length of arc =
$$\frac{\theta}{360^{\circ}} 2\pi r$$

 $\Rightarrow \qquad \widehat{\text{AED}} = \frac{180^{\circ}}{360^{\circ}} \times 2 \times \frac{22}{7} \times 7 = 22 \text{ cm}$

$$\therefore$$
 Perimeter of given figure = 20 + 14 + 20 + 22 = 76 cm

7. Let *n* spherical lead shots be made, then According to question,

 $n \times$ Volume of spherical lead shot = Volume of Solid Cube

$$\Rightarrow \qquad n \cdot \frac{4}{3} \pi r^3 = (a)^3$$
$$\Rightarrow \qquad n \cdot \frac{4}{3} \times \frac{22}{7} \times (2)^3 = 44 \times 44 \times 44$$
$$\Rightarrow \qquad n = \frac{44 \times 44 \times 44 \times 7 \times 3}{4 \times 22 \times 8} = 2541$$

8. Let the rainfall be *x* cm.

Volume of rain water = Volume of cylindrical vessel

$$\Rightarrow \qquad lbx = \pi r^2 h$$

$$\Rightarrow \qquad 22 \times 20 \times x = \frac{22}{7} \times 1 \times 1 \times 3.5$$

$$\Rightarrow x = \frac{22 \times 1 \times 3.5}{7 \times 22 \times 20} = 2.5 \text{ cm}$$
9. According to question, $3a_3 = 7a_7$

$$\Rightarrow 3(a+2d) = 7(a+6d)$$

$$\Rightarrow 3a+6d = 7a+42d$$

$$\Rightarrow 4a+36d = 0$$

$$\Rightarrow 4(a+9d) = 0$$

$$\Rightarrow a+9d = 0 \Rightarrow a_{10} = 0$$
10. $\angle OQS = \angle OQL - \angle SQL = 90^\circ - 40^\circ = 50^\circ$

$$\Rightarrow \angle OQS = 50^\circ \qquad \dots \dots \dots (i)$$

$$\angle ORS = \angle ORM - \angle SQM = 90^\circ - 70^\circ = 20^\circ \qquad \dots \dots \dots (ii)$$

$$\therefore \ \angle OSQ = \angle OQS = 50^\circ \qquad \dots \dots \dots (iii)$$

$$\therefore \ \angle OSQ = \angle OQS = 50^\circ \qquad \dots \dots \dots (iii)$$

$$\therefore \ \angle OSQ = \angle ORS = 20^\circ \qquad \dots \dots \dots (iii)$$

$$\therefore \ \angle OSR = \angle ORS = 20^\circ \qquad \dots \dots \dots (iv) \quad [From eq. (ii)]$$

$$\therefore \ \angle QSR = \angle ORS + \angle OSR$$

$$= 50^\circ + 20^\circ = 70^\circ$$

11. Let $B \rightarrow (x, y)$. Then

$$\frac{(3)(x)+(4)(2)}{3+4} = -1 \qquad \Rightarrow \qquad \frac{3x+8}{7} = -1$$
$$\Rightarrow \qquad 3x+8 = -7 \qquad \Rightarrow \qquad x = -5$$
Also,
$$\frac{(3)(y)+(4)(5)}{3+4} = 2 \qquad \Rightarrow \qquad \frac{3y+20}{7} = 2$$
$$\Rightarrow \qquad 3y+20 = 14 \qquad \Rightarrow \qquad y = -2$$

12. Let the required point be P(x, y).

Then PA = PB = PC

$$\Rightarrow PA^{2} = PB^{2} = PC^{2}$$

$$\Rightarrow (x-5)^{2} + (y-1)^{2} = (x+3)^{2} + (y+7)^{2} = (x-7)^{2} + (y+1)^{2}$$
Taking, $(x-5)^{2} + (y-1)^{2} = (x+3)^{2} + (y+7)^{2}$

$$\Rightarrow x+y=-2 \qquad(i)$$
Taking, $(x+3)^{2} + (y+7)^{2} = (x-7)^{2} + (y+1)^{2}$

$$\Rightarrow 5x+3y=-2 \qquad(ii)$$
Solving eq. (i) and eq. (ii), we get
 $x=2, y=-4$

13. Area of remaining paper =

Area of rectangle – Area of semi-circle

$$= l \times b - \frac{1}{2}\pi r^2$$

$$= 18 \times 14 - \frac{1}{2} \times \frac{22}{7} \times \frac{14}{2} \times \frac{14}{2}$$

$$= 252 - 77$$

$$= 175 \text{ cm}^{2}$$
14. PQ = QR = RS = $\frac{2\times6}{3} = 4 \text{ cm}$

$$\therefore \quad \text{Perimeter of the shaded region = $\widehat{PQ} + \widehat{PS} + \widehat{QS}$

$$= \pi \times 2 + \pi \times 6 + \pi \times 4$$

$$= 12\pi \text{ cm}$$
15. Let the radius and height be $2k$ cm and $3k$ cm respectively. Then,
Volume of cylinder $= \pi r^{2}h$

$$\Rightarrow \quad 1617 = \pi (2k)^{2} (3k)$$

$$\Rightarrow \quad k = \frac{7}{2} \text{ cm}$$

$$\therefore \quad \text{Total surface area of cylinder = $2\pi r (r+h)$

$$= 2 \times \frac{22}{7} \times 7 \left(7 + \frac{21}{2}\right)$$

$$= 770 \text{ cm}^{2}$$
16. $4x^{2} + 4\sqrt{3}x + 3 = 0$

$$\Rightarrow \quad (2x)^{2} + 2(2x) (\sqrt{3}) + (\sqrt{3}) = 0$$

$$\Rightarrow \quad (2x + \sqrt{3})^{2} = 0$$

$$\Rightarrow \quad (2x + \sqrt{3}) (2x + \sqrt{3}) = 0$$

$$\Rightarrow \quad x = \frac{-\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$
17. According to question, $a_{4} + a_{8} = 24$

$$\Rightarrow \quad (a + 3d) + (a + 7d) = 24$$

$$\Rightarrow \quad a + 5d = 12 \qquad \dots \dots \dots (i)$$
Again, $a_{6} + a_{10} = 44$

$$\Rightarrow \quad (a + 5d) + (a + 9d) = 44$$$$$$

$$\Rightarrow 2a+14d = 44$$

$$\Rightarrow a+7d = 22 \qquad \dots \dots \dots (ii)$$

 \Rightarrow

Solving eq. (i) and eq. (ii), we get,

 $a = -13, \quad d = 5$

- \therefore First three terms are -13, -8, -3.
- 18. :: Tangent segments from an external point to a circle are equal in length.

$$\therefore \qquad AQ = AR \qquad BP = BQ \qquad CP = CR$$

 $\therefore \qquad \text{Perimeter of } \Delta \text{ABC} = \text{AB} + \text{BC} + \text{AC}$

$$= AB + BP + CP + AC$$
$$= AB + BQ + CR + AC$$

$$= AQ + AR = AQ + AQ = 2AQ$$

$$\Rightarrow \qquad AQ = \frac{1}{2} \text{ (Perimeter of } \Delta \text{ ABC)}$$

19. In right triangle ABD,

$$\Rightarrow \sqrt{3} = \frac{AB}{BD}$$

In right triangle ABC,

$$\tan 30^{\circ} = \frac{AB}{BC} \qquad \Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{AB}{BD + 20}$$
$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{AB}{\frac{AB}{\sqrt{3}} + 20} \qquad \text{[From eq. (i)]}$$
$$\Rightarrow \qquad AB = 17.32 \text{ m}$$

20. There are 52 cards in a pack, out of which three cardsare removed.

- :. Total number of possible outcome = 52 3 = 49
- (i) Number of favourable outcomes = 3

Hence, Required probability = $\frac{3}{49}$

- (ii) Number of favourable outcomes = 3 Hence required probability = $\frac{3}{49}$
- (iii) Number of favourable outcomes = 23

Therefore there are 9 cards marked with the numbers which are perfect squares.

Hence, required probability = $\frac{23}{49}$

21. Steps of construction:

- (a) Draw a right angled triangle ABC with given measurements.
- (b) Draw any ray BY making an acute angle with BC on the side opposite to the vertex A.
- (c) Locate 3 points B_1 , B_2 and B_3 on BY so that $BB_1 = B_1B_2 = B_2B_3$.
- (d) Join B_2 to C and draw a line through B_3 parallel to B_2C , intersecting the extended line segment BC at C'.

(e) Draw a line through C' parallel to CA intersecting the extended line segment BA at A'. The A'BC' is the required triangle.



22. In right triangle AED,



- 23. Total number of cards in the deck = 52
 - \therefore Number of all possible outcomes = 52
 - Number of cards of spades or ace = 13 + 4 1 = 16(i)

$$\therefore$$
 Required probability = $\frac{16}{52} = \frac{4}{13}$

(ii) Number of a red king = 2

$$\therefore \text{ Required probability} = \frac{2}{52} = \frac{1}{26}$$

- (iii) Number of neither a king nor a queen = 52 (4 + 4) = 44
 - \therefore Required probability = $\frac{44}{52} = \frac{11}{13}$
- (iv) Number of either a king or a queen = (4 + 4) = 8



: Required probability =
$$\frac{8}{52} = \frac{2}{13}$$

24. \therefore D is the mid-point of BC.

$$\therefore \qquad \mathbf{D} \rightarrow \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

G divides AD internally in the ratio 2 : 1.

$$\therefore \qquad \overline{x} = \frac{2\left(\frac{x_2 + x_3}{2}\right) + 1(x_1)}{2 + 1} = \frac{x_1 + x_2 + x_3}{3}$$
And
$$= \frac{2\left(\frac{y_2 + y_3}{2}\right) + 1(y_1)}{y_1 + y_2 + y_3}$$

Ana

25. BC =
$$\sqrt{15^2 + 20^2} = \sqrt{225 + 400} = \sqrt{625} = 25$$
 cm



25

20

[AA similarity criterian]

A0 = 12 cm \Rightarrow $\frac{BO}{15} = \frac{15}{25}$ And BO = 9 cm \Rightarrow CO = 25 - 9 = 16 cm \Rightarrow Volume of the double cone = $\frac{1}{3}\pi r^2 h + \frac{1}{3}\pi r_1^2 h_1$ Now, $=\frac{1}{3}\times 3.14\times (12)^2 \times 9 + \frac{1}{3}\times 3.14\times (12)^2 \times 16$ $=\frac{1}{2}\times 3.14\times 144(9+16)$ $= 3.14 \times 48 \times 25 = 3768 \text{ cm}^3$ Surface area of the double cone = $\pi rl + \pi r_l l_1$ = 3.14 x 12 x 15 + 3.14 x 12 x 20 $= 3.14 \times 12 (15 + 20)$ = 3.14 x 12 x 35 = 1318.8 cm² 26. h = 16 cm, $r_1 = 20$ cm, $r_2 = 8$ cm $l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{(16)^2 + (20 - 8)^2} = \sqrt{256 + 144} = 20 \text{ cm}$ Total surface area of bucket = $\pi (r_1 + r_2) l + \pi r_2^2$ $= 3.14(20+8)20+3.14(8)^{2}$ $= 3.14 (28 \times 20 + 64)$ = 3.14(560 + 64)= 3.14 x 624 $= 1959.36 \text{ cm}^2$ $=\frac{15}{100}\times 1959.36$ Cost of bucket *.*.. = Rs. 293.90 (approx.) 27. (i) Let speed of the stream be x km/hSpeed of motorboat = 9 km/hSpeed of the motorboat to cover 12 km in downstream = (9+x) km/h Speed of the motorboat to cover 12 km in upstream = (9-x) km/h Time taken to cover 12 km in downstream = $\frac{12}{(9+x)}$ h Time taken to cover 12 km in upstream = $\frac{12}{(9-x)}$ h According to the question, $\frac{12}{9+x} + \frac{12}{9-x} = 3$

$$\Rightarrow 12\left[\frac{9-x+9+x}{81-x^2}\right] = 3 \qquad \Rightarrow 12\left[\frac{18}{81-x^2}\right] = 3$$
$$\Rightarrow 81-x^2 = \frac{12\times18}{3} \qquad \Rightarrow -x^2 = 72-81$$
$$\Rightarrow x^2 = 9 \qquad \Rightarrow x = \pm 3$$

Since the speed of stream cannot be negative.

 \therefore the speed of stream is 3 km/h.

- (ii) Logically if speed of stream is more than the speed of boat in still water, then the boat will not sail.
- 28. Let the speed of the train be x km/h.

Time taken by the train with this speed for a journey of 360 km = $\frac{360}{x}$ h

Increased speed of the train = (x+5) km/h

Time taken by train with increased speed for a journey of 360 km = $\frac{360}{x+5}$ h

According to the question,

$$\frac{360}{x} - \frac{360}{x+5} = 1$$

$$\Rightarrow \qquad 360 \left(\frac{x+5-x}{x^2+5x}\right) = 1 \qquad \Rightarrow \qquad x^2 + 5x = 1800$$

$$\Rightarrow \qquad x^2 + 5x - 1800 = 0 \qquad \Rightarrow \qquad x^2 + 45x - 40x - 1800 = 0$$

$$\Rightarrow \qquad x(x+45) - 40(x+45) = 0 \qquad \Rightarrow \qquad (x+45)(x-40) = 0 \qquad \Rightarrow \qquad x = -45, 40$$

x = -45 is inadmissible as x is the speed which cannot be negative.

Hence the speed of the train is 40 km/h.

29.
$$a = 54, d = -3, S_n = 513$$

 $S_n = \frac{n}{2} [2a + (n-1)d]$
 $\Rightarrow 513 = \frac{n}{2} [108 + (n-1)(-3)]$
 $\Rightarrow n^2 - 37n + 342 = 0$
 $\Rightarrow (n-18)(n-19) = 0$
 $\Rightarrow n = 18 \text{ or } 19$
Explanation
 $a_{19} = a + 18d = 54 + 8(-3) = 0$
30. $\because \angle XZY = 90^\circ$
 $\therefore (9+r)^2 + (2+r)^2 = (17)^2$ [By Pythagor.

$$\therefore \qquad (9+r)^{2} + (2+r)^{2} = (17)$$

$$\Rightarrow \qquad r^{2} + 11r - 102 = 0$$

$$\Rightarrow \qquad r^{2} + 17r - 6r - 102 = 0$$

[By Pythagoras theorem]

 $\Rightarrow \qquad r(r+17)-6(r+17)=0$

$$\Rightarrow (r+17)(r-6) = 0$$

 \Rightarrow r = -17, 6

Radius cannot be negative.

Hence r = 6 cm

31. **First part**: <u>Given</u> : A circle with centre O and a point P outside the circle. PT and PT' are tangents from P to the circle.



<u>To Prove</u>: We need to prove that PT = PT'<u>Construction</u>: Joined OP, OT and OT' <u>Proof</u> ::: OT is a radius and PT is a tangent.

> $\therefore \qquad \angle OTP = 90^{\circ}$ Similarly, $\angle OT' P = 90^{\circ}$ Now in right triangles OTP and OT'P, OT = OT' and OP = OP $\therefore \qquad \Delta OTP \cong \Delta OT'P$

...... (Radii of the same circle)(Common)(RHS congruency)

Hence, PT = PT'

Second part: Using the above, we get, AP = AS, BP = BQ, CR = CQ, DR = DS

On adding, we get,

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ) $\Rightarrow AB + CD + AD + BC$

 \Rightarrow 2AB = 2BC [:: ABCD is a parallelogram]

 $\Rightarrow AB = BC$

$$\Rightarrow$$
 AB = BC = CD = DA

[∵ ABCD is a parallelogram]

 \Rightarrow ABCD is a rhombus.