35. Magnetic Field due to a Current

Short Answer

Answer.1

An electric current flows in a wire from north to south. So the direction of the magnetic field associated with it can be predicted by Maxwell's Right hand Cork-Screw rule.

So we can conclude that,

(a) The magnetic field to the east of the wire will be directed vertically upwards.

(b) The magnetic field to the west of the wire will be directed vertically downwards.

(c) The magnetic field vertically above the wire will be directed towards the west.

(d) The magnetic field vertically below the wire will be directed towards the east.

Answer.2

Magnetic field at a point d distance apart from a long straight current carrying wire is given by $B = \frac{\mu_0 i}{2\pi d}$ from Ampere's circuital law. Where μ_0 is the magnetic permeability of free space, i denotes the current through the wire and d denotes the distance from the wire, of the point where magnetic field is to be calculated.

Now we know that, speed of light, $c = \sqrt{\mu_0 \epsilon_0}$ (ϵ_0 denotes the electric permittivity of free space)

$c^2 = \mu_0 \epsilon_0$

$\mu_0 = \frac{c^2}{\epsilon_0}$

Thus we get magnetic field to be $B = \frac{c^2 i}{2\pi\epsilon_0 d}$

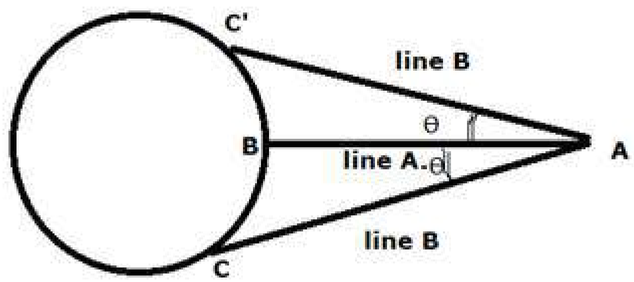
Answer.3

We can predict the direction of the magnetic field by Maxwell's Right hand Cork-Screw rule. So we put the thumb of our right hands pointing towards the direction of current and now the direction the other fingers curl toward will be direction of the magnetic field.

So, we get the field at the center is moving away from me if the current is going in the clockwise direction.

Answer.4

The contribution to the magnetic field B by currents outside the Amperian loop turns out to be zero because if we draw a straight line on any fixed point on the loop from the current vector(the direction vector of current), let line A and draw another line from the current vector to a movable point on the loop, let line B, and move that point in one direction along the loop until the full loop is traversed then we can see the net positive angle travelled by line B with respect to line A is equal to the net negative value travelled by the same. So the total angle travelled is zero. The line integral has a finite value for a current that is within the loop as it traverses an angle of 2ϖ. So, we can say that Ampere's law takes into account every current be it inside the Amperian loop/curve or outside but as for outside currents, the integral is zero, so only the enclosed current is taken into account for calculation but actually takes into consideration the contribution of all the currents.



we consider the circle to be the Amperian loop. Point B is the fixed point C is the moving point. At some point of time C coincides with C' where ∠BAC = -∠BAC'. We will get a point like this for every arbitrary point on the circle, so total angle for all the circle will be zero as stated before.

Answer.5

Magnetic field on the axis of a circular current carrying loop is given by,

$$B = \frac{\mu_0 i a^2}{2(a^2 + x^2)^{\frac{3}{2}}}$$

where a denotes the radius of the loop and x is the distance of the point on the axis at which the magnetic field is to be calculated. i is the current flowing in the loop. μ_0 is the magnetic permeability of free space.

And B = $\mu_0$ni (where μ_0 is the magnetic permeability of free space, n is the number of turns of the wire per unit length of the solenoid, and i is current in the wire) holds true for a very long solenoid where at the terminal points x >> a. We can say that if we keep adding more loops at the ends, the total number of loops increase but with that length also increases so the turns per unit length n is unvaried. We can also think of it this way,

The value of B for loops added to the ends of the solenoid will have dropped to almost zero as for the condition x >> a,

This can be shown mathematically by taking the limit of B for a circular loop. So, let us take x to be tending to infinity and a to have a finite value.

$$\lim_{x \to \infty} B$$

$$= \lim_{x \to \infty} \frac{\mu_0 i a^2}{2(a^2 + x^2)^{\frac{3}{2}}}$$

$$= \lim_{x \to \infty} \frac{\mu_0 i \frac{a^2}{x^3}}{2(\frac{a^2}{x^2} + 1)^{\frac{3}{2}}}$$ (Dividing both sides by x^3)

$$= 0$$

So if we add more loops at the ends of a very long solenoid, the magnetic field due to each one of them will be zero at the centre of the solenoid thus won't have considerable effect.

Answer.6

Ampere's law holds true for any closed loop of magnetic field containing constant electric current. The left hand side of Ampere's circuital law $\oint \vec{B} . \vec{dl} = \mu_0 i$

Where

μ_0 is the magnetic permeability of free space,

i is the current enclosed by the loop and B is the magnetic field

The above equation deals with a closed line integral along the path of the magnetic field. The line integral will not have a finite value if the loop is not closed. So, Ampere's law will not hold for a loop that does not enclose the wire but it will hold for any shape of the loop even if it's not circular.

Answer.7

No, there won't be any magnetic force acting on the wire in this case because, if the ring starts rotating, it will create a magnetic field which is parallel or antiparallel to the direction of current or the length vector of the wire at the center of the ring. i.e. angle between the is zero or ϖ respectively. According to F = ILBsinΘ

for $\Theta = 0$ or ϖ,

F= 0.

Here I is the current, L is the length of the wire and B is the magnitude of the magnetic field.

So, there will be no force on the wire if the ring starts rotating about the wire.

Answer.8

Using the relation for force acting on a conductor

$$\vec{F} = I(\vec{L} \times \vec{B})$$

Where

I is the current in the wire,

L is the length vector of the wire

B is the external electric field.

Thus, if the wires are kept perpendicular, the angle between them will be 90° and thus the angle between the movable wire and the magnetic field due to the fixed wire is 0°(from Maxwell's Cork-Screw rule). Thus the force is the minimum and it is in an unstable equilibrium. Thus the will rotate and make itself parallel to the fixed wire and attain the maximum attractive force and thus a stable equilibrium.

Answer.9

The magnetic field is not as strong as electric field and the magnitude of magnetic field due to a moving charge only equals that of its electric field only when the velocity is equal to the speed of light. So for two protons moving in the same direction, Force due to electric field repulsion will exceed that due to magnetic field attraction. So the proton beams moving in the same direction repel each other.

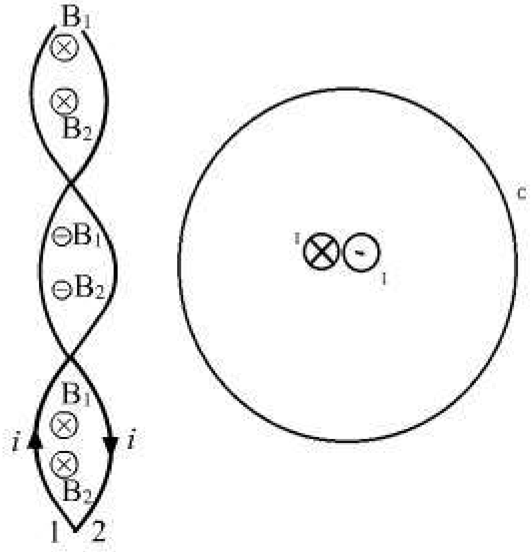
But two parallel current carrying wire attract due to magnetic attraction as the electric field is inside the conductor and is shielded by it and can't be felt outside. The magnetic attraction can be further reinforced by Fleming's left rule by considering the magnetic field produced by one wire and electric field in the other wire in the direction of current.

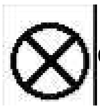
Answer.10

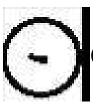
We can always find the magnetic field due to a long, straight current carrying wire along any loop which only contains that wire by using Ampere's circuital law. The law only takes into account the current bound in the loop irrespective of the source. If we take the whole circuit along with the battery within that Amperian loop, then the total current contained in that loop will be zero so the resultant magnetic field will be zero too even if the wire itself produces a non zero magnetic field.

Answer.11

If we choose an Amperian loop containing both the wires, then the magnetic field along that loop will turn out to be zero as the wires carry currents in the opposite direction and the total current turns out to be lesser than the maximum of that of the two wires. And if the magnitudes of currents are the same then the total current in that loop turns out to be zero. If they are kept very close together, this will hold for even very small Amperian loops. Thus twisting two wires, which carry currents in the opposite directions, minimizes the magnetic field produced and can even cancel it out.



denotes current going perpendicularly inside the screen.

denotes current coming perpendicularly out of the screen.

If this both currents are equal in magnitude then the total current enclosed by the Amperian Loop C will be zero.

Answer.12

The displacement of the wire due to magnetic forces happens in the direction of the distance vector between the two wires. The magnetic field due to the wires is perpendicular to the wire and the distance vector between the wires so work done is by the field is zero as usual. As the wires move towards each other there is a change in magnetic field and according to Lenz law there will be an induced current in the wires which will be directed opposite to the current initially carried by them. As these electrons cannot come out of the wire, they hit the edges of the wire starts moving and gains kinetic energy.

Objective I

Answer.1

With the right hand cork screw we can get the direction of the magnetic field due to the vertical current carrying wire. And we find it to be directed downwards. Now applying Fleming's Left hand rule, we can find the direction of force as we put the direction of the electron beam as the opposite direction of current. We can find that force is pointed upwards. So the electron beam will be deflected upwards.

Answer.2

There won't be any magnetic force acting on the wire in this case because, according the Cork-Screw rule, the magnetic field due to the straight wire will be parallel or anti parallel to the direction of current in the loop, thus creating an angle of 0 or ϖ with the direction of current.

According to $F = ILB\sin\Theta$

Where

I is the current,

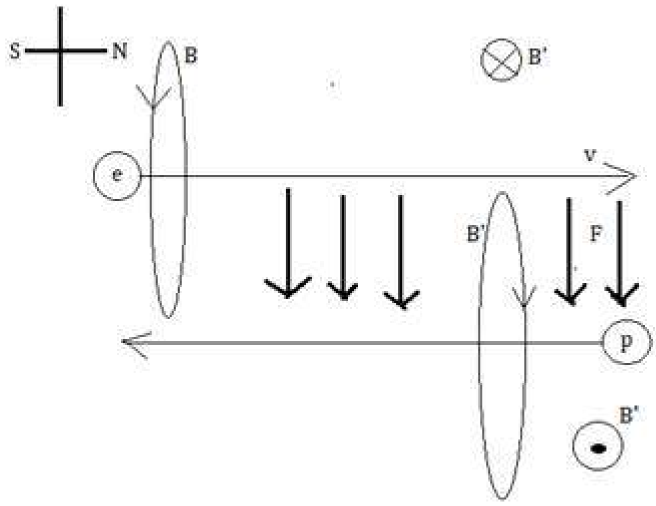
L is the length of the wire

B is the magnitude of the magnetic field.

for $\Theta = 0$ or ϖ, $F = 0$.

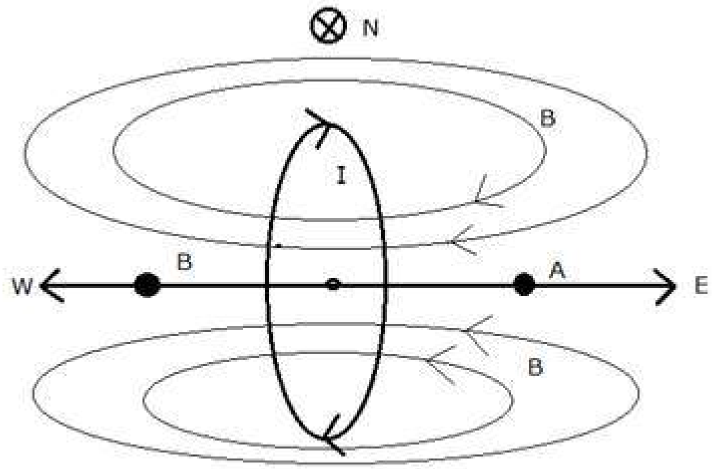
Answer.3

We can consider the beams to be current carrying wires. The proton beam is going from north to south so the current for it will be directed from north to south. And the electron beam is moving from South to north so the current will be directed opposite to the direction of the beam, i.e. from north to south. So we get the case of two current carrying wires carrying current in the same direction and they attract each other. So the electron beam will be deflected towards the proton beam.



Answer.4

If the current is directed towards north at the topmost point of that circular loop that means, from point A's perspective, current is in clockwise direction and will cause a magnetic field which is moving inwards i.e. to the west at point A and subsequently to the west at point B.



Answer.5

The closest side of the loop to the straight wire carries a current in the same direction as the straight wire. So there will be an attractive force. And for the opposite side of the loop the current is moving in the opposite direction to that of the straight wire, so there will be a repulsive force. As the closest side shows attractive force and the farthest side shows repulsive force, the attractive force will be dominant. To explain it mathematically, we introduce the expression for force per unit length on each of the wires.

$$F = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Where

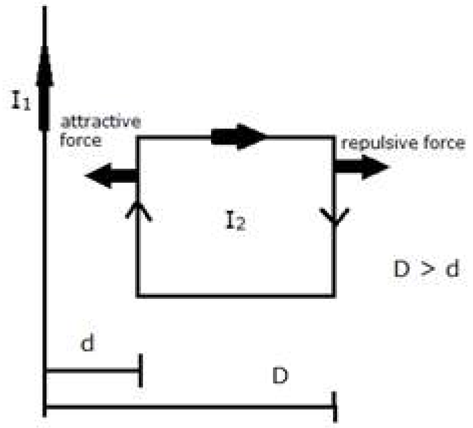
F denote the force per unit length,

I_1, I_2 denote the currents in the two wires,

d is the distance between them

μ_0 is the magnetic permeability of free space.

The current in the loop is of same magnitude everywhere. The parallel sides of the loop to the long straight wire will encounter all the force. As the side closer to the straight wire carries a current in the same direction, I_1I_2 will be positive thus it will encounter attractive force and the opposite side will encounter repulsive force as the, I_1I_2 is negative as the currents are in opposite directions. We can see that the force is inversely proportional to the distance. As the wire carrying current in the same direction is closer than the wire carrying current in the opposite direction, the attractive force will be greater than the repulsive force. The other two sides will experience forces of equal amplitude but opposite directions and thus will cancel each other out. Thus the loop will move towards the wire.



Answer.6

If a charged particle is moving along a magnetic field line, that means it is moving either parallel or anti parallel to the direction of magnetic field. i.e. the direction of velocity of the particle is either making an angle of 0 or ϖ. According to F = qvBsinΘ (where q is the charge of the particle, v is the velocity of it and B is the magnitude of the external Magnetic field and Θ is the angle between the direction of magnetic field and the velocity) for Θ = 0 or ϖ, F= 0.

Answer.7

Under any circumstances, a charge particle will produce an electric field. And if we imagine that the moving charge particle is analogous to an electric current then it is clear to us that it, most certainly, will produce a magnetic field as well.

Answer.8

If a charged particle is projected with velocity v on a plane perpendicular to a uniform magnetic field B, then the particle will travel in a circle. Whose radius is $R = \frac{mv}{qB}$ where m and q are the mass and the charge of the particle respectively.

So the area enclosed by the path will be A = $\varpi R^2 = \pi \frac{m^2 v^2}{q^2 B^2} = \frac{2\pi m}{q^2 B^2} \times \frac{1}{2} m v^2 = \frac{2\pi m}{q^2 B^2} \times$ kinetic energy

Thus, the area will be proportional to the kinetic energy of the particle.

Answer.9

let the charge of both of them be q and mass be m_x and m_y respectively. If they are driven through the same potential V then, $\frac{1}{2}mv^2 = qV$

$$v = \sqrt{\frac{2qV}{m}}$$

Thus, v_x and v_y (their respective velocities after leaving the potential) will be $v_x = \sqrt{\frac{2qV}{m_x}}$ and $v_y = \sqrt{\frac{2qV}{m_y}}$

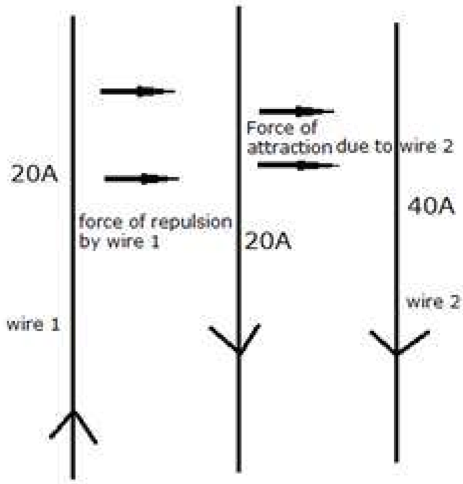
So from $R = \frac{mv}{qB}$ we get, $R \propto \sqrt{m}$ (all the other terms are constant for both the particles)

So, according to the question $\frac{R_1}{R_2} = \sqrt{\frac{m_x}{m_y}}$

Thus, $\frac{m_x}{m_y} = (\frac{R_1}{R_2})^2$

Answer.10

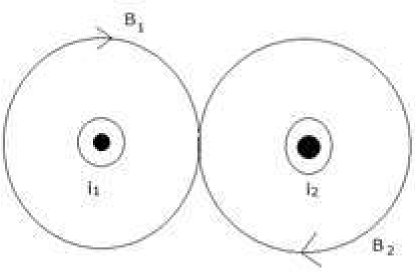
The third wire is carrying a current 20A antiparallel to the previously set 20A current wire. So these wires will repel each other. As the 40A wire is carrying a current in the opposite direction to the previously set 20A wire, the newly set 20A wire will be having the same current direction to it. So they will attract. So the newly set wire in the midway will move towards the 40A wire.



Answer.11

The magnetic field caused by a long wire is given by $B = \frac{\mu_0 i}{2\pi d}$ where i is the current in the wire, d is the distance at which B is to be calculated, μ_0 is the magnetic permeability of free space. Let the distance between the two wires be 2d.

Then, when the currents are parallel, one's magnetic field will negate the other's because at the point midway between the two wires, the magnetic fields generated by them will be directed in the opposite directions and it can be verified by the right hand Cork-Screw rule.



So the total B will be

$B = \frac{\mu_0 i_1}{2\pi d} - \frac{\mu_0 i_2}{2\pi d}$ which is given to be 10μT and when the direction of i_2 is reversed then, $B = \frac{\mu_0 i_1}{2\pi d} + \frac{\mu_0 i_2}{2\pi d}$ is given to be 30μT. So, solving these, we get, $\frac{\frac{\mu_0 i_1}{2\pi d}+\frac{\mu_0 i_2}{2\pi d}}{\frac{\mu_0 i_1}{2\pi d}-\frac{\mu_0 i_2}{2\pi d}} = \frac{30}{10}$

$\frac{\frac{\mu_0 i_1}{\pi d}}{\frac{\mu_0 i_2}{\pi d}} = \frac{40}{20}$ (Componendo- Dividendo rule)

$\frac{i_1}{i_2} = 2$

Answer.12

We know, drift velocity of electron is

vd = i/(nAe)

Where

i is the current in the wire,

n is the electron number density of the wire a,

A is the area of cross section of the wire,

e is the charge of the electron.

We can see that the trolley is moving with drift velocity but current travels in a conductor with a speed comparable to that of light in free space. So the velocity of the trolley is very low compared to the velocity at which the current travels. So there will be no significant change in the value of B and it will be $\frac{\mu_0 i}{2\pi r}$.

Objective II

Answer.1

According to Biot-Savart law:The magnetic field due to a current element is given as$\overrightarrow{dB} = \frac{\mu_0 i}{4\pi} \frac{\overrightarrow{dl} \times \vec{r}}{r^3}$ Where,

i is the current,

is the length element vector,

is the vector joining the required point and the length element,

is the permeability of vacuum and $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}.Also, by vector properties: A× B = -(B× A)Thus, $\overrightarrow{dB} = -\frac{\mu_0 i}{4\pi} \frac{\vec{r} \times \overrightarrow{dl}}{r^3}$ Thus, options (A) and (B) are the correct options

Answer.2

(A)Dimension for electric field E is: F/q = $[MLT^{-3}A^{-1}]$Force due to magnetic field is given as: F=qvB.Where q is the charge, v is the velocity of charge and B is the magnetic field.Hence $B = \frac{F}{qv}$

The dimensions of charge, q is [AT]

The dimension of the velocity, V is [LT-1]

Putting the value in the above formula, we get

Dimensions of B = $\left(\frac{[MLT^{-2}]}{[AT][LT^{-1}]}\right)$

∴ Dimensions of B = $[MT^{-2}A^{-1}]$Thus, $\boldsymbol{x} = \frac{\boldsymbol{E}}{\boldsymbol{B}} = \frac{[\mathbf{MLT^{-3}A^{-1}}]}{[\mathbf{MT^{-2}A^{-1}}]} = [\mathbf{LT^{-1}}]$

(B) $y = \sqrt{\frac{1}{\mu_0\epsilon_0}}$We know that $\frac{1}{\mu_0\epsilon_0} = c^2$∴ $\sqrt{\frac{1}{\mu_0\epsilon_0}} = c$Where

μ_0 is permeability of free space,

ϵ_0 is the permittivity of free space and

c is the speed of light.Thus, y = c, c is the speed and unit of speed is m/s**Dimensions of y = $[LT^{-1}]$**(C)

Reactance, $z = \frac{l}{CR}$Where

C is the capacitance

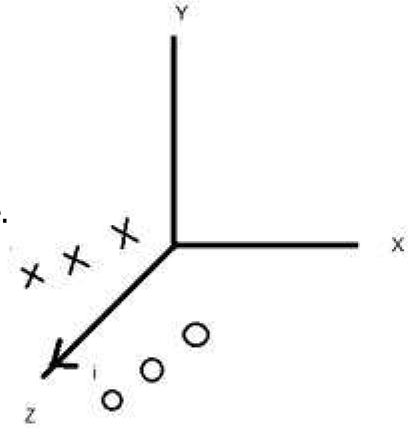
R is the resistance

l is the length of the wire.

In a CR circuit the product CR is the time constant and unit is seconds.Thus, **Dimensions of z = l/CR = m/s= $[LT^{-1}]$**Hence x, y and z have same dimensions.Thus, options (A),(B) and (C) are the correct options.

Answer.3

When a current carrying wire is along z-axis, the magnetic field can be found using right hand thumb rule.



Here X represents the field is going in and O represents field is coming out.First case, if two points in the XY plain are above the wire and at same distance from z-axis, then both will lie in the X region(field going in), thus have same direction of magnetic field and magnitudes of magnetic field would be same.Second case , if the two points in XY plane are at same distance from z axis but one point is above the wire and one is below it, then the direction of the electric field would be opposite for both the points. One point having X direction (going in) and other having O direction (coming out).First two cases satisfy options (B),(C) and (D).Thus, options (B),(C) and (D) are the correct options.

Answer.4

According to Ampere's Law:$\oint B.dl = \mu_0 i$Where,

dl is the current element,

B is the magnetic field,

μ_0 is the permeability of free space and

i is the current flowing.

Thus, at the cross-section the formula becomes,$B.\,2\pi R = \mu_0 i$2πR is the circumference of the wire and R is the radius. We get,$B = \frac{\mu_0 i}{2\pi R}$Now, at the axis of the wire. R=0, and so no area to integrate and hence zero current is enclosed. Thus magnitude of magnetic field is minimum at axis of the wire.At the surface of the wire, R= some minimum value.As R increases , magnitude of magnetic field decreases.Hence, B will be maximum at the surface of the wire.Thus, options (B) and (C) are correct options.

Answer.5

By Ampere's Law ,$\oint B.\,dl = \mu_0 i$Here, dl is the current element, B is the magnetic field, μ_0 is the permeability of free space and i is the current flowing.We know that , magnetic field due to a circular wire is given$B = \frac{\mu_0 i}{2\pi R}$R is the radius of the wire.Consider a Hollow tube as the series of circular wires.Inside the tube at a point, r<R $\oint B_{inside}.\,dl = \mu_0 i_{inside}$Current inside the hollow tube is zero: i_{inside} = 0.∴ $\oint B.\,dl = 0$∴ $B_{inside} = Constant$Hence , the magnetic field is constant inside the hollow tube.Now, at the axis, R=0.

Therefore, zero current is enclosed.Hence, B=0.Thus, options (B) and (C) are the correct options.

Answer.6

According to Ampere's Law:$\oint B.\,dl = \mu_0 i$Where,

B is the magnetic field,

dl is the current element,

μ_0 is the permeability of free space

i is the current flowing in the conductor

Magnetic field outside the cable: B = B

$_I + B_O B_I$ and B_O are the magnetic field of the inner conductor and outer cable respectively.Due to wire,$B = \frac{\mu_0 i}{2\pi R}$R is the radius of the wire.$B = \frac{\mu_0 i}{2\pi R} + \frac{\mu_0 (-i)}{2\pi R} \therefore B = \frac{\mu_0 i}{2\pi R} - \frac{\mu_0 i}{2\pi R} \therefore B = 0$Thus, option (A) is the correct option.

Magnetic field inside the inner conductor:

$\oint B.\, dl = \mu_o I$ (I=0(inside the wire))

Magnetic field between the two conductors and inside the outer conductor.

$\oint B.\, dl = \mu_o I$

On integrating the length element over the complete loop, we get dl= 2π r

$B.2\pi r = \mu_o I$

On solving the above equation for B, we get

$$B = \frac{\mu_o I}{2\pi r}$$

Thus, option A and B is the correct option.

Answer.7

Using Ampere's law, the electric field due to a current carrying conductor of cross section areas is:magnetic field, $B = \frac{\mu_0 i}{2\pi R}$Where, dl is the current element,

μ_0 is the permeability of free space,

i is the current flowing

R is the radius of the conductor.At the axis,

R=0 and so no area to enclose current.

Hence, B=0 at the axis of the conductor.

Electric field in the vicinity of the conductor is zero as a minimum distance between the point and the conductor is required to have electric field at that point.Thus, options (B) and (C) are correct options.

Exercises

Answer.1

Given:$\vec{F} = q\vec{v} \times \vec{B}$

$B = \frac{\mu_0 i}{2\pi r}$**Formula used:**Lorentz force: $\vec{F} = q\vec{v} \times \vec{B}$Where, $\vec{F}$ is the magnetic force vector, q is the charge, $\vec{v}$ is the velocity vector and $\vec{B}$ is the magnetic field vector.

Magnetic field can be written as:$B = \frac{F}{qv}$Unit of force is Newton: NCharge: q = ItUnit of current : I = Amperes= AUnit of time : t = seconds = sUnit of charge : q = AsUnit of velocity : v = m/sThus, unit of Magnetic field: $B = \frac{N}{As \times \frac{m}{s}} \therefore B = \frac{N}{Am}$
Therefore, unit of Magnetic field : B = N $m^{-1}A^{-1}$.Now,Ampere's Law:$B = \frac{\mu_0 i}{2\pi r}$
Where,

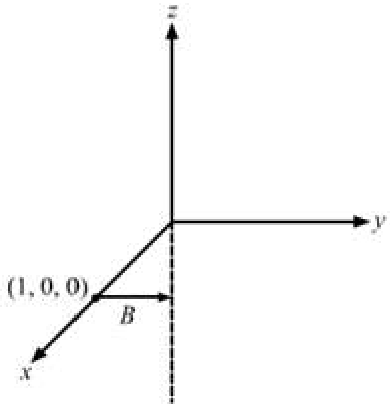
μ_0 is the permeability of free space,

I is the current flowing

r is the radius of the current carrying wire.SI Unit of B = N mA^{-1}SI Unit of I = ASI Unit of r = m $\therefore \mu_0 = \frac{B 2\pi r}{i} \therefore \mu_0 = \frac{N\, m^{-1} A^{-1}\, m}{A} \therefore \mu_0 = N\, A^{-2}$Thus, unit of permeability constant μ_0 is N A^{-2}.Hence proved that unit of Magnetic field is N $m^{-1}A^{-1}$ and unit of permeability constant μ_0 is N A^{-2}.

Answer.2

Given: Current : I = 10 A



Point is located along x-axis at 1 m.**Formula used:**Using Ampere's Law for a current carrying straight wire: $B = \frac{\mu_0 i}{2\pi d}$ Here, B is the magnitude of magnetic field, μ_0 is the permeability of free space and μ_0= 4π × 10^{-7} T mA^{-1} and d is the distance between the current carrying wire and the required point.Substituting the values,$B = \frac{4\pi \times 10^{-7} \times 10}{2 \times \pi \times 1}$ **∴ B = 2× 10^{-6} T**Hence, magnetic field at a point (1m, 0, 0) is 2× 10^{-6} T.

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Answer.3

Given: Diameter of the copper wire: d = 1.6mm = 1.6× 10^{-3} mCurrent through the wire: I = 20 A**Formula used:**By Ampere's law for current carrying wire of cross section area,$B = \frac{\mu_0 i}{2\pi r}$Where,

B is the magnitude of magnetic field,

μ_0 is the permeability of free space and μ_0= 4π × 10^{-7} T mA^{-1} r is the radius of the wire.

Radius of the wire : r = d/2 = 8× 10^{-3} mSubstituting we get, $B = \frac{4\pi \times 10^{-7} \times 20}{2 \times \pi \times 8 \times 10^{-3}}$ ∴ B = 5× 10^{-3} THence, maximum magnitude of magnetic field due to a current of 20 A is 5× 10^{-3} T.

Answer.4

Given: Current through the transmission wire: I = 100 ADistance between wire and the road : d = 8 m**Formula used:**By Ampere's Law for a current carrying wire is $B = \frac{\mu_0 i}{2\pi d}$ Where,

B is the magnitude of magnetic field,

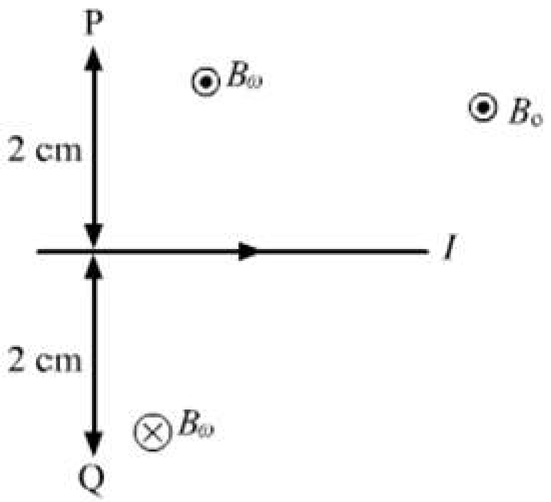
μ_0 is the permeability of free space

$\mu_0 = 4\pi \times 10^{-7}$ T mA^{-1}

d is the distance between the current carrying wire and the required point.Substituting we get,$B = \frac{4\pi \times 10^{-7} \times 100}{2\pi \times 8}$ B = 2.5×10^{-6} THence, magnitude of magnetic field at a point on the road due to current carrying transmission wire is 2.5×10^{-6} T.

Answer.5

Given : Current in the wire : i = 1.0 AMagnitude of Horizontal magnetic field: $B_o = 1.0 \times 10^{-5}$ TDistance between points P,Q and the wire :d = 2.0 cm = 0.02 m



Right hand rule is used to determine the direction of the magnetic field due to the current carrying wire. Cross determines the field going into the plane and dot determines the field coming out of the field.B_w is the magnetic field due to wire.**Formula used:**By Ampere's Law for a current carrying wire is$B = \frac{\mu_0 i}{2\pi d}$Where,

B is the magnitude of magnetic field,

μ_0 is the permeability of free space, μ_0= 4π × 10^{-7} T mA^{-1}

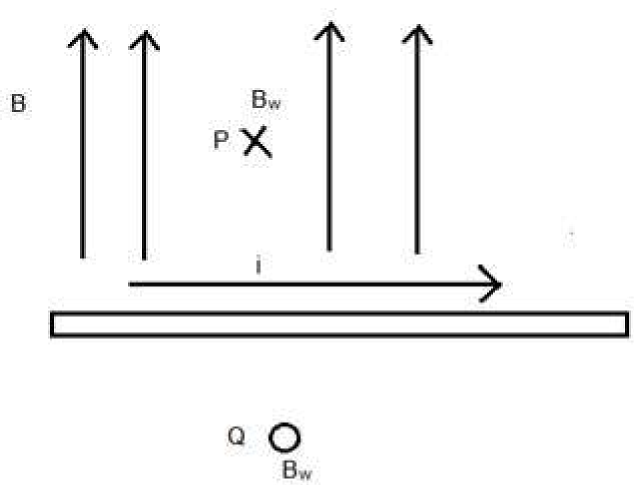
d is the distance between the current carrying wire and the required point.

Magnetic field at point P due to wire,$B_w = \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0.02}$$\therefore B_w = 1 \times 10^{-5}\, T$

Resultant magnetic field at P$\mathbf{B_P = B_o + B_w}$ ∴ B_P = 1× 10^{-5} + 1× 10^{-5} ∴ B_P = 2 × 10^{-5} TMagnetic field at point Q due to wire,$B_w = -\frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0.02}$ ∴ B_W = - 1 × 10^{-5} TThe negative sign is because the field at Q is opposite to the direction of horizontal field and at P.Resultant magnetic field at P$\mathbf{B_Q = B_o + B_w}$ ∴ B_Q = 1× 10^{-5} - 1× 10^{-5} ∴ B_Q = 0Hence, resultant magnetic field at P is 2 × 10^{-5} T and at Q is 0.

Answer.6

Given: Radius of the wire : rCurrent through the wire : iUniform magnetic field pointing upward : B



Formula used: Magnetic field due to a current carrying wire is

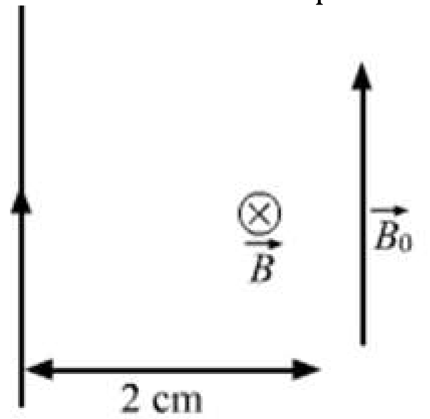
Magnitude of magnetic field due to wire, $B_w = \frac{\mu_0 i}{2\pi r}$

Where, μ_0 is the permeability of free space and μ_0= 4π × 10^{-7} T mA^{-1}.

r is the radius of the wire.(a)Maximum net magnetic field would be above the wire at P.Considering direction of the current as shown in the diagram, right hand rule will give us direction of the magnetic field due to wire.thus,B_{net} = B+B_W ∴ B_{net} = B + $\frac{\mu_0 i}{2\pi r}$(b)Similarly, the field at Q would be in opposite direction to the P.Minimum magnitude of the magnetic field will be a QB_{net} = B-B_W ∴ B_{net} = B - $\frac{\mu_0 i}{2\pi r}$Hence, magnitude of magnetic field will be maximum at points above at wire and minimum at points below the wire only if the direction of the current is as shown in the figure.

Answer.7

Given: Current in the wire : I = 30 AExternal Uniform magnetic field: B_O = 4.0 ×10^{-4} TDistance between the point and the wire : d = 2 cm = 0.02 m

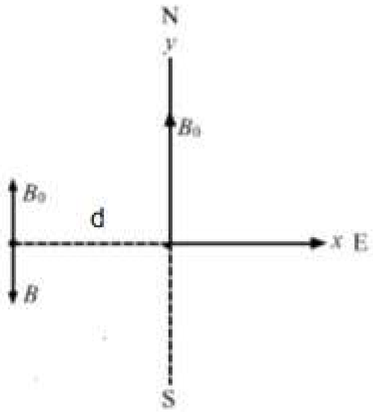


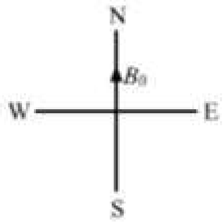
Here, B is the magnetic field due to wire. B_O is parallel to the current in the wire. Using Right hand rule we can see that direction of magnetic field at the desired point is going into the plane (cross). This direction due to wire is perpendicular to the External magnetic field.$\vec{B} \perp \vec{B_O}$**Formula used:**By Ampere's Law for a current carrying wire is$B = \frac{\mu_0 i}{2\pi d}$Here, B is the magnitude of magnetic field, μ_0 is the permeability of free space and μ_0= 4π × 10^{-7} T mA^{-1} and d is the distance between the current carrying wire and the required point.Substituting we get,$B = \frac{4\pi \times 10^{-7} \times 30}{2\pi \times 0.02}$ ∴ B = 3× 10^{-4} TAs Magnetic field due to wire is perpendicular to the external magnetic field, the resultant net magnetic field would be:$B_{net} = \sqrt{B^2 + B_O^2}$

∴ $B_{net} = \sqrt{(3\times\ 10^{-4})^2 + (4.0\ \times 10^{-4})^2}$ ∴ B_{net} = 5 × 10^{-4} T Hence, the magnitude of the resultant magnetic field at a point 2.0 cm away from the wire is 5 × 10^{-4} T.

Answer.8

Given: Current in the wire : I = 10 AMagnitude of Horizontal magnetic field : B_O = 2.0×10^{-3} T





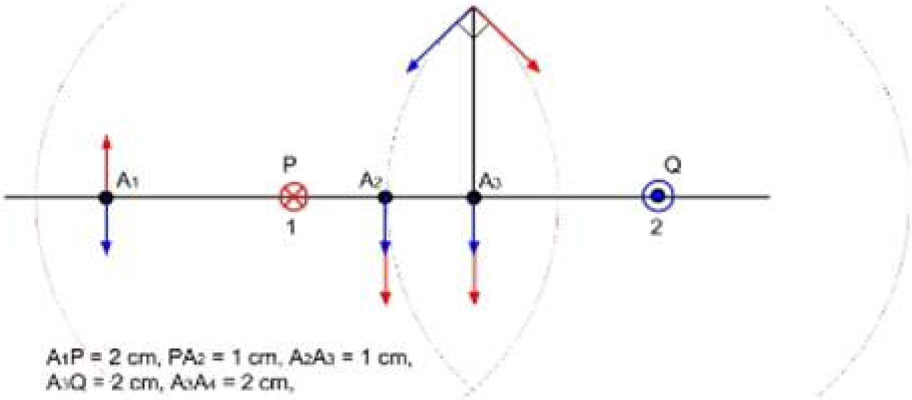
For the resultant magnetic field to be zero, we need to have a field in opposite direction to that of the existing field.Hence, magnetic field B due to wire in the direction from north to south as shown in the diagram..**Formula used:**By Ampere's Law for a current carrying wire is$B = \dfrac{\mu_0 i}{2\pi d}$Where,

B is the magnitude of magnetic field,

μ_0 is the permeability of free space and μ_0= 4π × 10^{-7} T mA^{-1} d is the distance between the current carrying wire and the required point.To get zero resultant,B=B_O∴ $\dfrac{\mu_0 i}{2\pi d} = 2.0 \times 10^{-3}$∴ $d = \dfrac{4\pi \times 10^{-7} \times 10}{2\pi \times 2.0 \times 10^{-3}}$ ∴ d = 0.001 m=1 mmHence, the point should be place at a distance of 0.001 m from the wire in west direction

Answer.9

Given: Magnitude of current in both the wires: i = 10 ADistance between 2 wires : d = 4cm = 0.04 m



P and Q are the two wires having opposite direction of currents. From the diagram, P has current into the plane and Q has current coming out of the plane.By right hand rule, P will have field coming up at A_1 and going down at A_2 and A_3 and tangent to A_4 as shown by the red arrow.Similarly, Q will have field going down as A_1,A_2 and A_3 aand tangent to A_4 as shown by blue arrow.Distance between, A_1,A_2 and A_3 is 2 cm each.**Formula used:**By Ampere's Law for a current carrying wire is $B = \frac{\mu_0 i}{2\pi d}$ Here, B is the magnitude of magnetic field, μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7}$ T mA^{-1} and d is the distance between the current carrying wire and the required point.At A_1 :Net magnetic field due to P and Q would be subtraction of individual fields at A_1.PA_1 :d_1 = 0.02 mQA_1 : d_2 = 0.06m$B_{net} = B_P - B_Q$Where, B_P is the magnetic field due to wire P and B_Q is the magnetic field due to wire Q.

$\therefore B_{net} = \frac{\mu_0 i}{2\pi d_1} - \frac{\mu_0 i}{2\pi d_2}$ $\therefore B_{net} = \frac{\mu_0 i}{2\pi}\left(\frac{1}{0.02} - \frac{1}{0.06}\right)$

$\therefore B_{net} = \frac{4\pi \times 10^{-7} \times 10}{2\pi} \times 33.33$ $\therefore B_{net} = 6.66 \times 10^{-5}$ THence, magnetic field at A_1 is 6.66×10^{-5} T.At A_2Effect of magnetic field at A_2 will be sum of magnetic field due to P and Q as both are in same direction as shown.PA_2: d_1 = 0.01 mQA_3: d_2 = 0.03 m$B_{net} = B_P + B_Q$Where, B_P is the magnetic field due to wire P and B_Q is the magnetic field due to wire Q.$\therefore B_{net} = \frac{\mu_0 i}{2\pi d_1} + \frac{\mu_0 i}{2\pi d_2}$

$\therefore B_{net} = \frac{\mu_0 i}{2\pi}\left(\frac{1}{0.01} + \frac{1}{0.03}\right)$ $\therefore B_{net} = \frac{4\pi \times 10^{-7} \times 10}{2\pi} \times 133.33$ $\therefore B_{net} = 2.66 \times 10^{-4}$ THence , magnetic field at A_2 is 2.66×10^{-4} TAt A_3Effect of magnetic field at A_3 will be sum of magnetic field due to P and Q as both are in same direction as shown.PA_3: d_1 = 0.02 mQA_3 : d_2 = 0.02 m$B_{net} = B_P + B_Q$Where, B_P is the magnetic field due to wire P and B_Q is the magnetic field due to wire Q.

$\therefore B_{net} = \frac{\mu_0 i}{2\pi d_1} + \frac{\mu_0 i}{2\pi d_2}$ $\therefore B_{net} = \frac{\mu_0 i}{2\pi}\left(\frac{1}{0.02} + \frac{1}{0.02}\right)$

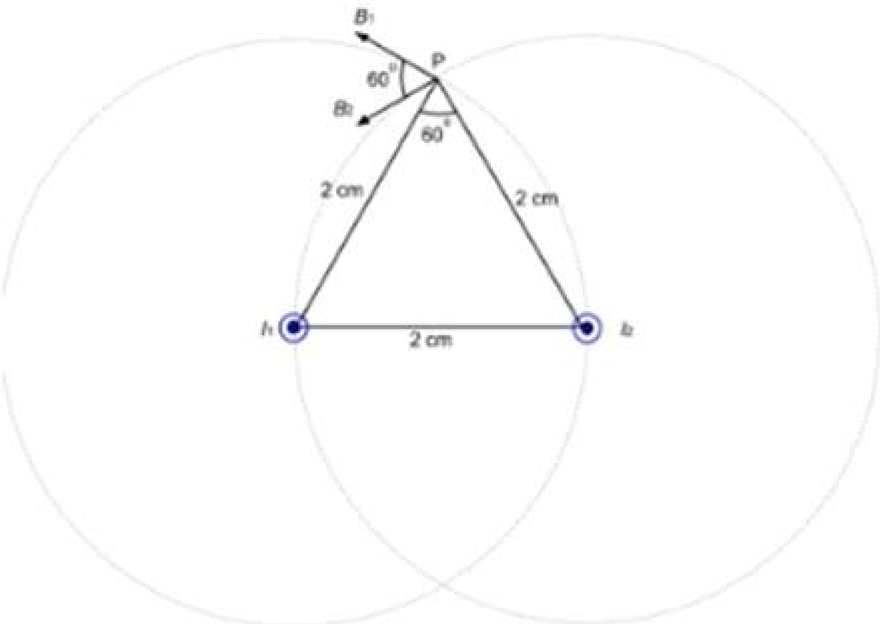
$\therefore B_{net} = \frac{4\pi \times 10^{-7} \times 10}{2\pi} \times 100$ $\therefore B_{net} = 2\times 10^{-4}$ THence, magnetic field at A_3 is 2×10^{-4} TAt A_4:The resultant magnetic field at A_4 would be $B_{net} = \sqrt{B_P^2 + B_Q^2}$From Pythagoras theorem,$(PA_4)^2 = (PA_3)^2 + (A_3A_4)^2$$\therefore (PA_4)^2 = 0.02^2 + 0.02^2$$\therefore PA_4$ = 0.028 m = d_1Now,$B_P = \frac{\mu_0 i}{2\pi d_1}$$\therefore B_P = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.028}$ $\therefore B_P = 7.14 \times 10^{-5}$

TSimilarly,QA_4 = 0.028 m = d_2Thus,$B_Q = \frac{\mu_0 i}{2\pi d_2}$∴ $B_Q = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.028}$ ∴ B_Q = 7.14 × 10^{-5} T Substituting to get net magnetic field at A_4,

$B_{net} = \sqrt{(7.14 \times 10^{-5})^2 + (7.14 \times 10^{-5})^2}$ ∴ B_{net} = 1× 10^{-4} THence magnetic field at A_4 is 1× 10^{-4} T.

Answer.10

Given: Current in the equal wires: i= 10 ADistance between the two wires: a = 2 cm = 0.02 mDistance between the wire and the point : d = 2 cm = 0.02 m



From the diagram l_1 and l_2 are the wires coming out of the plane.These two wires and point P form an equilateral triangle of side 0.02 m.By right hand rule, Magnetic field at P due to l_1 is shown by arrow B_1 tangent to the field's path and Magnetic field at P due to l_2 is shown by B_2 tangent to the field's path.By geometry we can calculate the angle between B_1 and B_2 which turns out to be 60° .**Formula used:**By Ampere's Law for a current carrying wire is$B = \frac{\mu_0 i}{2\pi d}$Where,

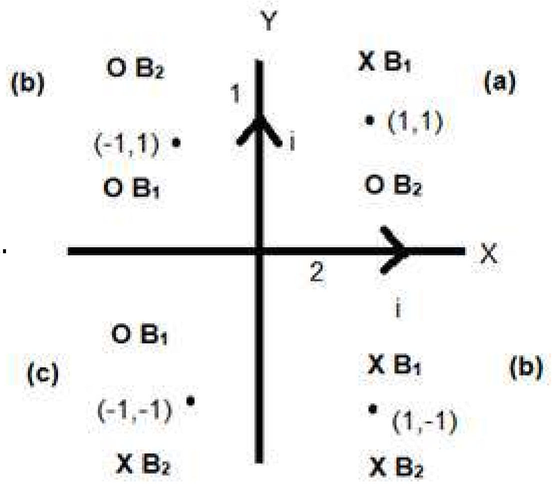
B is the magnitude of magnetic field,

μ_0 is the permeability of free space, μ_0= 4π × 10^{-7} T mA^{-1}

d is the distance between the current carrying wire and the required point.Magnetic field due to wire l_1:$B_1 = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.02}$∴ B_1 = 1 × 10^{-4} TMagnetic Field due to l_2 :$B_2 = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.02}$ ∴ B_2 = 1 × 10^{-4} TB_1=B_2 as same magnitude of current is flowing through both the wires and point P is located at same distance from each wires.Now, angle between $\overrightarrow{B_1}$ and $\overrightarrow{B_2}$: θ = 60°Resultant of two vectors is given as $B_R = \sqrt{B_1^2 + B_2^2 + 2B_1B_2cos\theta}$ B_R is the resultant magnetic field at P.∴ $B_R = \sqrt{(1 \times 10^{-4})^2 + (1 \times 10^{-4})^2 + 2(1 \times 10^{-4})^2 \cos(60)}$ ∴ B_R = 1.73 × 10^{-4} T Hence, resultant magnetic field at a point 2 cm away from due to two current carrying wires is 1.73 × 10^{-4} T.

Answer.11

Given: Current in both the wires : I = 5 A.



Wires 1 and 2 generate magnetic fields B_1 and B_2 respectively.By right hand rule, direction of magnetic field in each quadrant due to wires 1 and is shown by X and O.X : Field is going into the planeO : Field is coming out of the plane.**Formula used:**By Ampere's Law for a current carrying wire is $B = \frac{\mu_0 i}{2\pi d}$ Where,

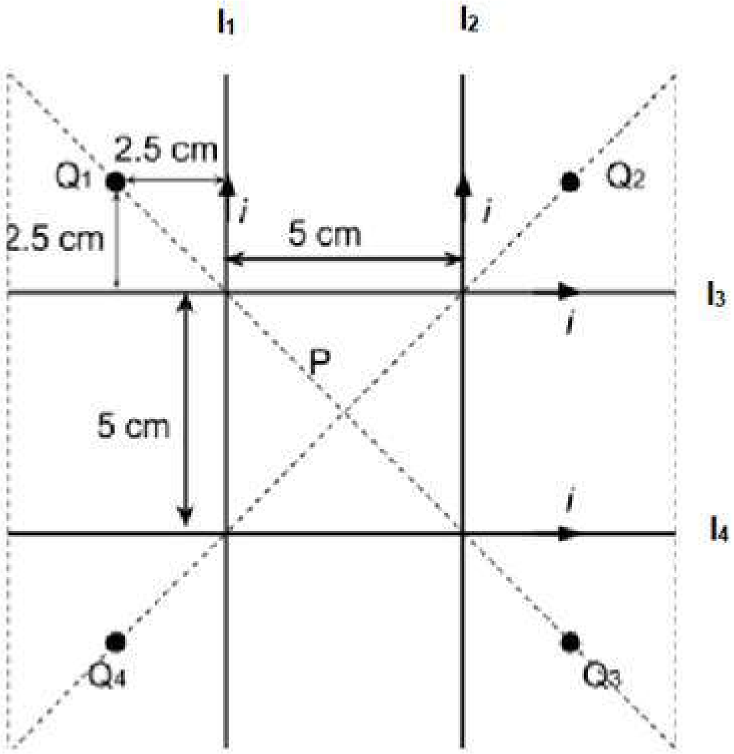
B is the magnitude of magnetic field,

μ_0 is the permeability of free space and μ_0= 4π × 10^{-7} T mA^{-1} d is the distance between the current carrying wire and the required point.(a) At (1m,1m)As we can see from the diagram at (1,1) the magnetic fields due to wire 1 and 2 are in opposite direction. (X and O)Hence, net magnetic field would be zero.(b) At (-1m , 1m)Here, magnetic field due to wire 1 and 2 would add up as direction of magnetic field is same. (O and O)Thus B = B_X + $B_Y$$B_X$ is the magnetic field due to wire on x axisB_Y is the magnetic field due to wire on y axis

$B = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 1} + \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 1}$ ∴ B = 2× 10^{-6} TThis B would be along z-Axis.(c) At (-1m,-1m)As we can see from the diagram at (-1,-1) the magnetic fields due to wire 1 and 2 are in opposite direction. (O and X)Hence, net magnetic field would be zero.(d) At (1m,-1m)Here, magnetic field due to wire 1 and 2 would add up as direction of magnetic field is same. (X and X)Thus B = B_X + $B_Y$$B_X$ is the magnetic field due to wire on x axisB_Y is the magnetic field due to wire on y axis $B = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 1} + \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 1}$ ∴ B = 2× 10^{-6} TThis B would be along negative z-axis.

Answer.12

Given: Current in the 4 wires: i = 5.0 ASide of the square : d = 5 cm = 0.05 m



In the diagram above, l_1 and l_2 represents vertical wires. l_3 and l_4 represent horizontal wires.Arrows represent the direction of current in the wire. We can determine the direction of magnetic field due to any of the wire using Right hand rule.**Formula used:**By Ampere's Law for a current carrying wire is$B = \frac{\mu_0 i}{2\pi d}$Here, B is the magnitude of magnetic field, μ_0 is the permeability of free space and μ_0= 4π × 10^{-7} T mA^{-1} and d is the distance between the current carrying wire and the required point.(a)At point P, if we use

Right hand rule for every wire.Field due to l_1 would be into the plane and field due to l_2 would come out of the plane. Thus canceling out each other.Similarly, field due to l_3 would go into the plane and field due to l_4 would come out, canceling each other.Thus, net field at P is zero.(b)At Q_1, By geometry:Distance between wire l_1 and Q_1 : d_1 = 0.025 mDistance between wire l_2 and Q_1 : d_2 = 0.075 mDistance between wire l_3 and Q_1 : d_3 = 0.025 mDistance between wire l_4 and Q_1 : d_3 = 0.075 mNet magnetic field at Q_1 due to wires $l_{1,}l_{2,}l_{3,}l_4$ is$B_{Q1} = B_{l1}+B_{l2}+B_{l3}+B_{l4}$

$$\therefore B_{Q_1} = \frac{\mu_0 i}{2\pi d_1} + \frac{\mu_0 i}{2\pi d_2} + \frac{\mu_0 i}{2\pi d_3} + \frac{\mu_0 i}{2\pi d_4}$$

$$\therefore B_{Q_1} = \frac{4\pi \times 10^{-7} \times 5}{2\pi}\left(\frac{1}{0.025} + \frac{1}{0.075} + \frac{1}{0.025} + \frac{1}{0.075}\right)$$ $\therefore B_{Q1} = 1.06 \times 10^{-4}$ THence, magnetic field at Q_1 is 1.06×10^{-4} T and in upward direction.At Q_2, Similarly by geometryDistance between wire l_1 and Q_2 : d_1 = 0.075 mDistance between wire l_2 and Q_2 : d_2 = 0.025 mDistance between wire l_3 and Q_2 : d_3 = 0.025 mDistance between wire l_4 and Q_2 : d_3 = 0.075 mNow, Net magnetic field at Q_2 due to wires $l_{1,}l_{2,}l_{3,}l_4$ is$B_{Q2} = B_{l1}+B_{l2}+B_{l3}+B_{l4}$ $\therefore B_{Q_2} = \frac{\mu_0 i}{2\pi d_1} + \frac{\mu_0 i}{2\pi d_2} - \frac{\mu_0 i}{2\pi d_3} - \frac{\mu_0 i}{2\pi d_4}$

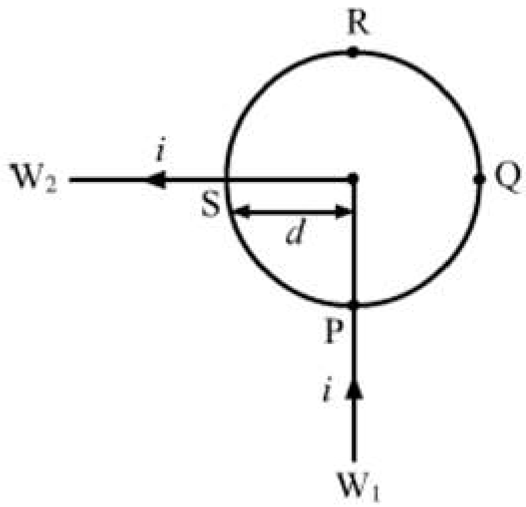
$$\therefore B_{Q_2} = \frac{4\pi \times 10^{-7} \times 5}{2\pi}\left(\frac{1}{0.075} + \frac{1}{0.025} - \frac{1}{0.025} - \frac{1}{0.075}\right)$$ $\therefore B_{Q2} = 0$Hence, magnetic field at Q_2 is zero.Negative sign for B_{l3} and B_{l4} as direction of magnetic field due to l_1(going in) is opposite to l_4(coming out) and l_2(going in) is opposite to $l_3$9coming out).At Q_3, By geometryDistance between wire l_1 and Q_3 : d_1 = 0.075 mDistance between wire l_2 and Q_3 : d_2 = 0.025 mDistance between wire l_3 and Q_3 : d_3 = 0.075 mDistance between wire l_4 and Q_3 : d_3 = 0.025 mNet magnetic field at Q_3 due to wires $l_{1,}l_{2,}l_{3,}l_4$ is$B_{Q3} = B_{l1}+B_{l2}+B_{l3}+B_{l4}$

$$\therefore B_{Q_3} = \frac{\mu_0 i}{2\pi d_1} + \frac{\mu_0 i}{2\pi d_2} + \frac{\mu_0 i}{2\pi d_3} + \frac{\mu_0 i}{2\pi d_4}$$

$$\therefore B_{Q_3} = \frac{4\pi \times 10^{-7} \times 5}{2\pi}\left(\frac{1}{0.075} + \frac{1}{0.025} + \frac{1}{0.075} + \frac{1}{0.025}\right)$$ $\therefore B_{Q3} = 1.06 \times 10^{-4}$ THence, magnetic field at Q_3 is 1.06×10^{-4} T and in downward direction.At Q_4,Magnetic field at Q_4 would be same as that of Q_2 because at Q_4, l_1 and l_4 have opposite fields, l_2 and l_3 have opposite fields. Thus canceling out each other and B_{Q4} is zero.Hence, Magnetic field at Q_1 and Q_3 have same magnitude of1.06×10^{-4} T but in opposite direction and magnetic field at Q_2 and Q_4 is zero.

Answer.13

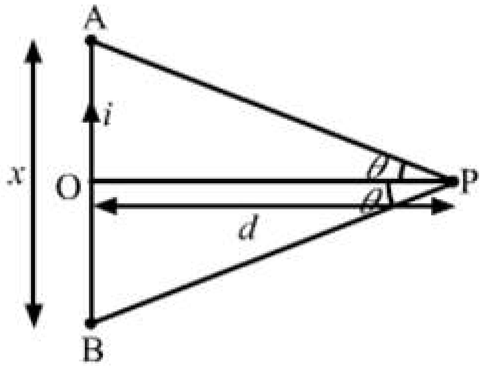
Given: Current through the wire : iradius of the circle formed: d



Let W_1 and W_2 be the terms for wire 1 and wire 2.Arrow shows the direction of the current in the wire.By right hand rule we can determine the direction of the magnetic field due to any of the two wire at any point.**Formula used:**By Ampere's Law for a current carrying wire is$B = \frac{\mu_0 i}{2\pi d}$ Here, B is the magnitude of magnetic field, μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7}$ T mA^{-1} and d is the distance between the current carrying wire and the required point. In this case, d is the radius of the circle.(1) At point P,Magnetic field at P due to W_1 is zero as P is on the axis of W_1.Magnetic field at P due to W_2 is$B_{Pw_2} = \frac{\mu_0 i}{2\pi d}$ Hence, net magnetic field at P due to both the wires is$B_P = \frac{\mu_0 i}{2\pi d}$ Direction of B_P is perpendicular to the plane and in outward direction.(2) At point Q,Magnetic field at Q due to W_1,$B_{QW_1} = \frac{\mu_0 i}{2\pi d}$ Magnetic field at Q due to W_2 is zero as Q is not in the effective zone of W_2.Hence, net magnetic field at Q due to both the wires is$B_Q = \frac{\mu_0 i}{2\pi d}$Direction of B_Q is perpendicular to the plane and inward direction(3) At point R,Magnetic field at R due to W_1 is zero as R is not in the effective zone pf W_1.Magnetic field at R due to W_2 is$B_{Rw_2} = \frac{\mu_0 i}{2\pi d}$ Hence, net magnetic field at R due to both the wires is$B_R = \frac{\mu_0 i}{2\pi d}$Direction of B_R is perpendicular to the plane and in inward direction.(4) At point S,Magnetic field at S due to W_2 is zero as S is on the axis of W_2.Magnetic field at S due to W_1 is $B_{Sw_1} = \frac{\mu_0 i}{2\pi d}$ Hence, net magnetic field at S due to both the wires is$B_S = \frac{\mu_0 i}{2\pi d}$ Direction of B_P is perpendicular to the plane and in outward direction.

Answer.14

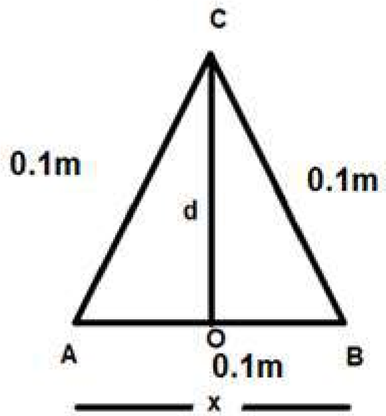
Given: Length of the piece: xCurrent in the piece : iDistance between midpoint of piece and P : d



P is on the perpendicular bisector of AB meeting at O.**Formula used:** By Biot-Savart Law: $dB = \frac{\mu_0 i}{4\pi}\frac{dl \times \bar{r}}{r^3}$ Here, dB is the magnitude of magnetic field element, μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7}$ T mA^{-1},dl is the length element, r is the distance between the current carrying wire and the required point.We know that, magnetic field at a point on the perpendicular bisector is $B = \frac{\mu_0 i}{2\pi d}\left(\frac{a}{\sqrt{a^2+4d^2}}\right)$ Here, a is the length of the wire. In this case a=xd is the distance between point P and the midpoint of AB.For d>>x x^2 can be neglected. $B = \frac{\mu_0 i}{2\pi d}\left(\frac{x}{\sqrt{4d^2}}\right) \therefore B = \frac{\mu_0 i}{2\pi d}\frac{x}{2d} \therefore B = \frac{\mu_0 i x}{4\pi d^2}$ ∴ B α 1/d^2Hence for d>>x magnetic field is inversely proportional to the square of the distance d.For d<<xd^2 can be neglected. $B = \frac{\mu_0 i}{2\pi d}\left(\frac{x}{\sqrt{x^2}}\right) \therefore B = \frac{\mu_0 i}{2\pi d}$ ∴ B α 1/dHence, when d<<x the magnetic field is inversely proportional to the distance d.

Answer.15

Given: Length of the wire piece: x = 10 cm = 0.1 mCurrent in the wire piece: i= 10 A



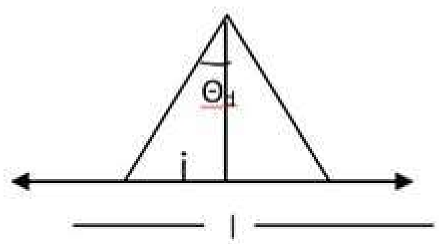
Formula used: By Biot-Savart Law: $dB = \frac{\mu_0 i}{4\pi}\frac{dl \times \bar{r}}{r^3}$

Here, dB is the magnitude of magnetic field element, μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7}$ T mA^{-1},dl is the length element, r is the distance between the current carrying wire and the required point.We know that, magnetic field at a point on the perpendicular bisector is $B = \frac{\mu_0 i}{2\pi d}\left(\frac{x}{\sqrt{x^2+4d^2}}\right)$ Here, x is the length of the wire.d is the distance between point C and the midpoint O.From Pythagoras theorem,$(BC)^2 = (OB)^2+(OC)^2$ ∴ $(OC)^2 = (BC)^2 - (OB)^2$ ∴ $(OC)^2 = (0.1)^2 – (0.05)^2$ ∴ $OC = (7.5\times 10^{-3})^{1/2}$ ∴ OC = 0.086 m = dSubstituting for B we get, $B = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.086} \times \frac{0.01}{\sqrt{0.01^2 + 4 \times (0.086)^2}}$ ∴ $B = 2.32\times 10^{-5} \times 0.502$ ∴ $B = 1.16 \times 10^{-5}$ THence the magnitude of the magnetic field due to the piece at a point which makes an equilateral triangle with the ends of the piece is 1.16×10^{-5} T.

Answer.16

Magnitude of current = i

Distance of point P from wire = d



The magnetic field induced in the wire is defined as

$$B_1 = \frac{\mu_0 i}{2\pi d}$$

Also, the magnetic field due to a section of length l on the perpendicular bisector is given by

$$B_2 = \frac{\mu_0 i}{4\pi d} \times \frac{2l}{\sqrt{l^2 + 4d^2}}$$

$$\frac{\mu_0 i}{4\pi d} \times \frac{2l}{d\sqrt{\frac{l^2}{d^2} + 4}}$$

Neglecting $\frac{l^2}{d^2}$ (as it is very small), we get

$$B_2 = \frac{\mu_0 il}{4\pi d^2} \times \frac{2}{\sqrt{2}}$$

$$B_2 = \frac{\sqrt{2}\mu_0 il}{4\pi d^2}$$

now, $B_1 > B_2$

We are given that, $\frac{B_1 - B_2}{B_1} = \frac{1}{100}$

$\Rightarrow B_2 = 0.99 B_1$

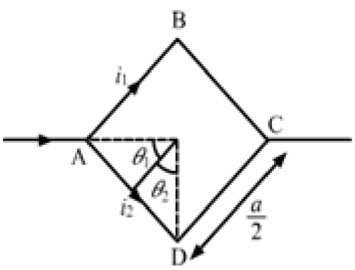
substituted the value of B_1 and B_2 in the above equation we get,

$$\Rightarrow \frac{\sqrt{2}\mu_0 il}{4\pi d^2} = 0.99 \times \frac{\mu_0 i}{2\pi d}$$

$$\Rightarrow \frac{d}{l} = \frac{1.414}{1.98}$$

$$\Rightarrow \frac{d}{l} = 0.71$$

Answer.17



Resistance of wire ADC is given twice that of wire ABC. hence current flow through ADC is half of ABC

i.e.,

$\frac{i_2}{i_1} = \frac{1}{2}$ --------------------(1)

also $i_1 + i_2 = i$ --------------(2)

Equating equations (1) and (2), we get

$i_1 = \frac{2i}{3}$ and $i_2 = \frac{i}{3}$

Hence, magnetic field at centre O due to wire AB and BC

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \times \frac{2i_1 \, sin \, sin \, 45}{\frac{a}{2}}$$

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \times \frac{2\sqrt{2}i_1}{a}$$

and magnetic field at centre o due to wire AD and DC

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \times \frac{2i_2 \, sin \, sin \, 45}{\frac{a}{2}}$$

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \times \frac{2\sqrt{2}i_2}{a}$$

Also $i_1 = 2i_2$, so $(B_1 = B_2) > (B_3 = B_4)$

Hence net magnetic field at centre O

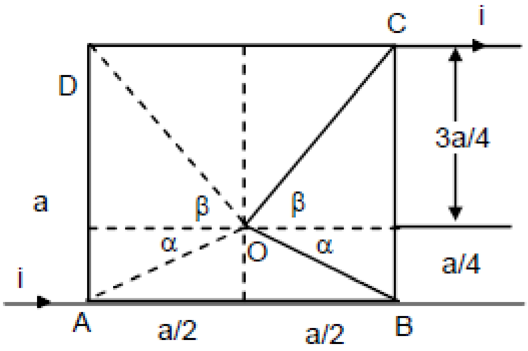
$$B_{net} = (B_1 + B_2) - (B_3 + B_4)$$

$$B_{net} = \left(2 \times \frac{\mu_0}{4\pi} \times \frac{2\sqrt{3} \times \frac{2}{3}(i)}{a}\right) - \left(\frac{\mu_0}{4\pi} \times \frac{2\sqrt{3} \times \frac{1}{3}(2)}{a}\right)$$

$$B_{net} = \frac{\mu_0}{4\pi} \times \frac{4\sqrt{2}i}{3a}(2 - 1)$$

$$B_{net} = \frac{\sqrt{2}\mu_0 i}{3\pi a}$$

Answer.18



$$A_0 = \sqrt{\frac{a^2}{16} + \frac{a^2}{4}} = \sqrt{\frac{5a^2}{16}} = \frac{a\sqrt{5}}{4}$$

$$D_0 = \sqrt{\left(\frac{3a}{4}\right)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{\frac{9a^2}{16} + \frac{a^2}{4}} = \sqrt{\frac{13a^2}{16}} = \frac{a\sqrt{13}}{4}$$

magnetic field due to AB

$$B_{AB} = \frac{\mu_0}{4\pi} \times \frac{i}{2\left(\frac{a}{4}\right)} [sin\, sin\, (90° - i)\ + sin\, sin\, (90° - \alpha)\ ]$$

$$B_{AB} = \frac{\mu_0 \times 2i}{4\pi a} \times 2cos\alpha$$

$$B_{AB} = \frac{\mu_0 \times 2i}{4\pi a} \times 2\frac{\frac{a}{2}}{a\left(\frac{\sqrt{5}}{4}\right)}$$

$$B_{AB} = \frac{2i\mu_0}{\pi\sqrt{5}}$$

magnetic field due to DC

$$B_{DC} = \frac{\mu_0}{4\pi} \times \frac{i}{2\left(\frac{3a}{4}\right)} (90° - \alpha)$$

$$B_{DC} = \frac{\mu_0 i \times 4 \times 2}{4\pi \times 3a} \times cos\beta$$

$$B_{DC} = \frac{\mu_0 \times i}{3\pi a} \times \frac{\frac{a}{2}}{\left(\frac{\sqrt{13}a}{4}\right)}$$

$$B_{DC} = \frac{2i\mu_0}{a^3\pi\sqrt{13}}$$

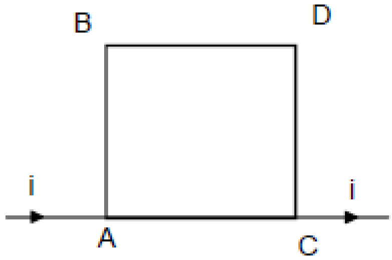
The magnetic field due to AD and BC are equal and opposite hence nullify each other

Hence, net magnetic field

$$B_{net} = \frac{2i\mu_0}{\pi\sqrt{5}} - \frac{2i\mu_0}{a^3\pi\sqrt{13}}$$

$$B_{net} = \frac{2i\mu_0}{\pi a} \times \left[\frac{1}{\sqrt{5}} - \frac{1}{3\sqrt{13}}\right]$$

Answer.19



Magnetic vector B due to BC and due to AD at PT equal and opposite.

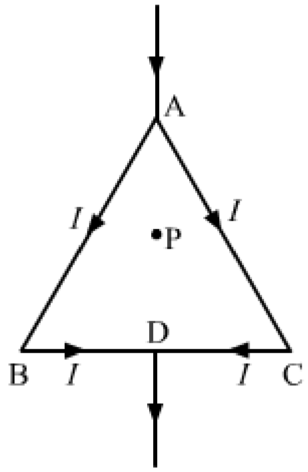
Hence net vector B = 0

Similarly, due to AB and CD at P = 0

∴ **The net vector B at the centre of the square loop = 0**

Answer.20

Let current 2*I* flow through the circuit.Since the wire is uniform, the current will enter the loop at point A and divide equally between the two arms AB and AC (as shown in the figure)



Now,Magnetic field induced at point P due to wire AB and wire AC = B (say)(Perpendicular to the plane in outward direction)

$$B = \frac{\mu_0 i}{4\pi r}(sin\, sin\, 60° \ + sin\, sin\, 60°)$$

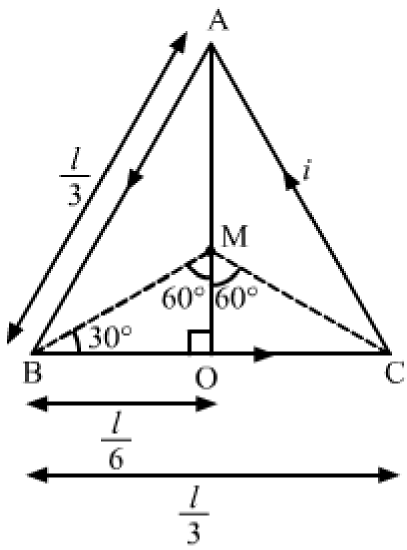
Magnetic field at P due to wire BD and DC = B' (say)(Perpendicular to the plane in outward direction)

$$B' = \frac{\mu_0 i}{4\pi r}(sin\, sin\, 60°)$$

∴ Net magnetic field at $P \ = \ B \ + \ B' \ - \ B \ - \ B'$ **= 0**

Answer.21

(a)



Let M be the midpoint of equilateral ΔABC

$$\therefore AB = BC = CA = \frac{l}{2}$$

current is given to be i

Hence, in ΔAOB

$$AO = \sqrt{\left(\frac{l}{3}\right)^2 - \left(\frac{l}{6}\right)^2}$$

$$AO = l\sqrt{\frac{1}{9} - \frac{1}{36}}$$

$$AO = l\sqrt{\frac{9-1}{36}}$$

$$AO = l\sqrt{\frac{1}{12}}$$

$$MO = \frac{1}{3} \times l\sqrt{\frac{1}{12}}$$

$$MO = \frac{l}{6\sqrt{3}}$$

The angles made by points B and C with centre M are $\theta_1 = \theta_2 = 60°$respectively,

Hence separation of the point from the wire is given by, $d = MO = \frac{l}{6\sqrt{3}}$

Thus, the magnetic field induced due to current in wire BC is-

$$B = \frac{\mu_0 i}{4\pi d}(\sin \sin \theta_1 \ + \sin \sin \theta_2 \ )$$

$$B = \frac{\mu_0 i}{4\pi \frac{l}{6\sqrt{3}}}(\sin \sin 60° \ + \sin \sin 60° \ )$$

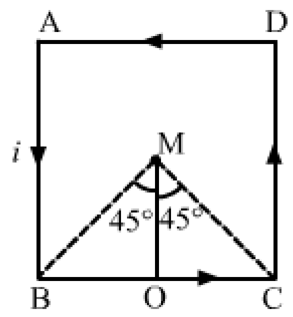
$$B = \frac{\mu_0 i}{4\pi l} \times 6\sqrt{3} \times \sqrt{3}$$

Now,Net magnetic field at point M is given as Magnetic field due to wire BC + Magnetic field due to wire CA + Magnetic field due to wire ABSince all wires are equal,

$$B_{net} = 3B = \frac{27\mu_0 i}{\pi l}$$

It is perpendicular to the plane in outward direction if current is anticlockwise and in an inward direction if the current is clockwise.

(b)



The angles made by B and C with centre M are $\theta_1 = 45°, \theta_2 = 60°$

Distance between point from the wire, $d = \frac{l}{8}$

Thus, the induced magnetic field due to electric current in wire BC is given by

$$B = \frac{\mu_0 i}{4\pi d}(sin\ sin\ \theta_1\ + sin\ sin\ \theta_2\ )$$

$$B = \frac{\mu_0 i}{4\pi \frac{l}{8}}(sin\ sin\ 45°\ + sin\ sin\ 45°\ )$$

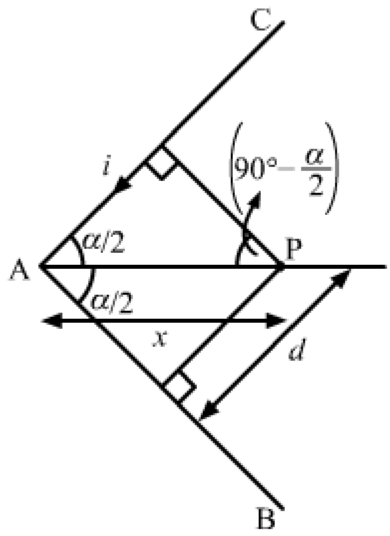
$$B = \frac{2\sqrt{2}\mu_0 i}{\pi l}$$

Since all wires are equal,Net magnetic field at point M = 4 × Magnetic field due to wire BC

$$B_{net} = 3B = \frac{8\sqrt{2}\mu_0 i}{\pi l}$$

Answer.22

Let CAB be the wire making an angle α, P be a point on the bisector of this angle situated at a separation *x* from the vertex A and *d* be the perpendicular distance of AC and AB from P.



From the diagram, $sin\left(\frac{\alpha}{2}\right) = \frac{r}{x}$

$\Rightarrow r = x\ sin\left(\frac{\alpha}{2}\right)$

The induced magnetic field B due to wire AB

$$\frac{\mu_0 i}{4\pi r}\left[sin\ sin\ \left(180 - \left(90 - \left(\frac{\alpha}{2}\right)\right)\right) + 1\right]$$

$$\Rightarrow \frac{\mu_0 i\left[sinsin\left(180-\left(90-\left(\frac{\alpha}{2}\right)\right)\right)+1\right]}{4\pi\times sin\left(\frac{\alpha}{2}\right)}$$

$$\Rightarrow \frac{\mu_0 i\left[coscos\left(\frac{\alpha}{2}\right)+1\right]}{4\pi\times sin\left(\frac{\alpha}{2}\right)}$$

$$= \frac{\mu_0 i2cos^4\left(\frac{\alpha}{4}\right)}{4\pi\times sin\left(\frac{\alpha}{4}\right)coscos\left(\frac{\alpha}{4}\right)}$$

$$= \frac{\mu_0 i}{4\pi x}cot\left(\frac{\alpha}{4}\right)$$

The magnetic field due to current in both the wires

$$= \frac{2\mu_0 i}{4\pi x}cot\left(\frac{\alpha}{4}\right)$$

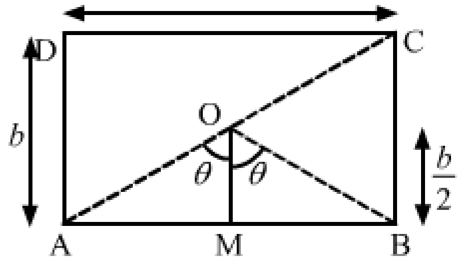
$$= \frac{\mu_0 i}{2\pi x}cot\left(\frac{\alpha}{4}\right)$$

Answer.23

Let us consider two angles made by points A and B with point O be θ_1 and θ_2 respectively.

$\theta_1 = \theta_2 = \theta$

Let distance of the point from the wire $d = \frac{b}{2}$



In triangle ΔAOM

$$AO = \sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{l}{2}\right)^2} = \frac{1}{2}\sqrt{b^2 + l^2}$$

$$sin\theta = \frac{\frac{l}{2}}{\frac{1}{2}\sqrt{b^2 + l^2}} = \frac{l}{\sqrt{b^2 + l^2}}$$

Thus induced magnetic field due to AB is given by,

$$B = \frac{\mu_0 i}{4\pi d}(sin\theta_1 + sin\theta_2)$$

$$\Rightarrow B = \frac{\mu_0 i}{4\pi d} \times 2sin\theta$$

$$\Rightarrow B = \frac{\mu_0 i}{4\pi \frac{b}{2}} \times \frac{2l}{\sqrt{l^2 + b^2}}$$

$$\Rightarrow B = \frac{\mu_0 i}{\pi b} \times \frac{l}{\sqrt{l^2 + b^2}}$$

Similarly the induced magnetic field due to current in BC is given as:

$$B' = \frac{\mu_0 i}{\pi l} \times \frac{b}{\sqrt{l^2 + b^2}}$$

Now, magnetic field due to CD = magnetic field due to current in wire AB =B

Also, magnetic field due to current in wire DA = magnetic field due to current in wire BC =B'

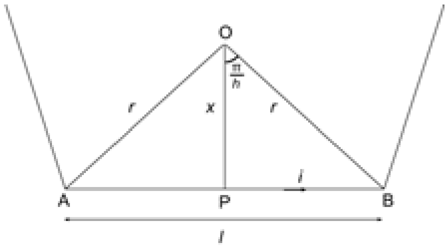
$$\therefore B_{nut} = l(B + B')$$

$$B_{nut} = 2\left[\frac{\mu_0 i}{\pi b} \times \frac{l}{\sqrt{l^2 + b^2}} + \frac{\mu_0 i}{\pi l} \times \frac{b}{\sqrt{l^2 + b^2}}\right]$$

$$B_{nut} = \frac{2\mu_0 i}{\pi\sqrt{l^2 + b^2}}\left[\frac{l}{b} + \frac{b}{l}\right]$$

$$B_{nut} = \frac{2\mu_0 i\sqrt{b^2 + l^2}}{\pi bl}$$

Answer.24



a) Using the diagram,

For a polygon of n equal sides, the angle at the centre is $\frac{2\pi}{n}$.

$$Tan\theta = \frac{l}{2\pi}$$

$$\Rightarrow x = \frac{l}{2tan\theta}$$

Considering angle to be very small

$$\frac{l}{2} = \frac{\pi r}{n}$$

Using Biot–Savart's law for one side of the n-sided polygon,

$$B = \frac{\mu_0}{4\pi}\frac{idlsin\theta}{x^2}$$

$$\Rightarrow B = \frac{\mu_0}{4\pi}\frac{i(sin\theta + sin\theta)}{x^2}$$

$B = \frac{\mu_0 i\, 2(tan\theta)(2sin\theta)}{4\pi l}$ (substituting the value of r)

$B = \frac{\mu_0 i\, 2n\left[tan\left(\frac{\pi}{n}\right)\right]\left[2sin\left(\frac{\pi}{n}\right)\right]}{4\pi(2\pi r)}$ (substituting the value of l)

For n-sided polygon

B' =nB

$\Rightarrow B = \frac{\mu_0 in^2 tan\left(\frac{\pi}{n}\right)sin\left(\frac{\pi}{n}\right)}{2\pi^2 r}$

b) When n → ∞, polygon tends to a circle with radius r and magnetic field will tend toward

$$B = \frac{\mu_0 i}{2r}$$

Answer.25

By applying Kirchhoff's Voltage Law, we can observe that the current in the circuit is zero

Now, since net current in the circuit = 0

Induced magnetic field is always proportional to the current flowing in the circuit. Hence,

Net magnetic field $\hat{A}$ at the point P=0

Induced magnetic field at point P is independent of the values of the various resistances in the electric circuit.

Answer.26

Current on both wires $i_1=i_2=20A$

Force acting on 10 cm of the second wire, F =20 x 10^{-5}

$\therefore$ force per unit length $= \frac{2\times10^{-5}}{0.1} = 20 \times 10^{-4}$

Now, let the distance between the two wires be d. Thus, the force per unit length is given as:

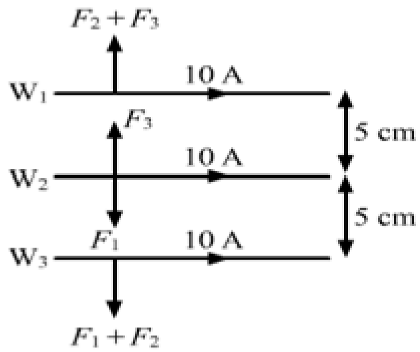
$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

$$\Rightarrow 2 \times 10^{-4} = \frac{2\times10^{-7}\times20\times20}{d}$$

$\Rightarrow$ distance d = 0.4m =40cm

Answer.27

-



Given: - Current flowing in each wire is $i_1=i_2=i_3=10A$

The magnetic force acting per unit length on wire due to the flow of the current is

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

So, for wire w_1,

$\frac{F}{l} = \frac{F}{l}$ by wire w_2 + $\frac{F}{l}$ by wire w_3

$\frac{F}{l} = \frac{\mu_0\times10\times10}{2\pi\times5\times10^{-2}} + \frac{\mu_0\times10\times10}{2\pi\times10\times10^{-2}}$

$\frac{F}{l} = 6.4 \times 10^{-4}$ N

And for wire w_2

$\frac{F}{l} = \frac{F}{l}$ by wire w_1 - $\frac{F}{l}$ by wire w_3

$\frac{F}{l} = \frac{\mu_0 \times 10 \times 10}{2\pi \times 5 \times 10^{-2}} - \frac{\mu_0 \times 10 \times 10}{2\pi \times 5 \times 10^{-2}}$

$\frac{F}{l} = 0$

Similarly for wire w_3

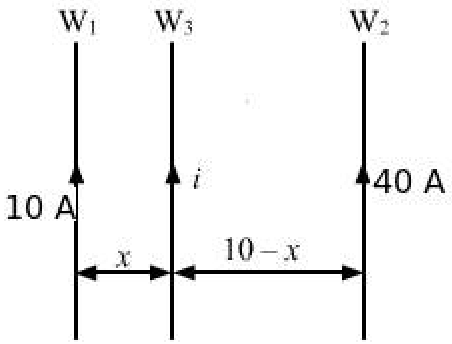
$\frac{F}{l} = \frac{F}{l}$ by wire w_1 + $\frac{F}{l}$ by wire w_2

$\frac{F}{l} = \frac{\mu_0 \times 10 \times 10}{2\pi \times 5 \times 10^{-2}} + \frac{\mu_0 \times 10 \times 10}{2\pi \times 5 \times 10^{-2}}$

$\frac{F}{l} = 6.4 \times 10^{-4}$ N

Answer.28

Let x be the distance of the wire W_3 from the wire carrying a current of 10A.



Magnetic force acting per unit length on a wire due to a parallel current-carrying wire is given by

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

In this case as the wire W_3 experiences no magnetic force.

$\therefore \frac{F}{l}$ by wire w_1 = $\frac{F}{l}$ by wire w_2

$\Rightarrow \frac{\mu_0 10 i}{2\pi x} = \frac{\mu_0 40 i}{2\pi(10-x)}$

$\Rightarrow 10 - x = 4x$

$\Rightarrow x = 2$ cm

Thus, wire W_3 is placed 2 cm from the 10 A current wire.

Answer.29

Current will equally divide into three parts since wires AB, CD and EF have identical resistance, 10 A each.

So, the Magnetic force on wire AB due to wire CD is,

$$\frac{F_1}{l} = \frac{\mu_0 i_1 i_2}{2\pi d} = \frac{\mu_0 \times 10 \times 10}{2\pi \times 1 \times 10^{-2}} = \frac{\mu_0}{2\pi} \times 10^4 \text{ N/m}$$

and Magnetic force on wire AB due to wire EF is,

$$\frac{F_2}{l} = \frac{\mu_0 i_1 i_2}{2\pi d'} = \frac{\mu_0 \times 10 \times 10}{2\pi \times 2 \times 10^{-2}} = \frac{\mu_0}{4\pi} \times 10^4 \text{ N/m}$$

Total force per unit length,

$$\frac{F}{l} = \frac{F_1}{l} + \frac{F_2}{l} = \frac{3\mu_0}{4\pi} \times 10^4 = 3 \times 10^{-3} \frac{N}{m}$$

Now, magnetic force on wire CD due to wire AB,

$$\frac{F_1}{l} = \frac{\mu_0 i_1 i_2}{2\pi d} = \frac{\mu_0 \times 10 \times 10}{2\pi \times 1 \times 10^{-2}} = \frac{\mu_0}{2\pi} \times 10^4 \text{ N/m}$$

and magnetic force on wire CD due to wire EF is

$$\frac{F_2}{l} = \frac{\mu_0 i_1 i_2}{2\pi d} = \frac{\mu_0 \times 10 \times 10}{2\pi \times 1 \times 10^{-2}} = \frac{\mu_0}{2\pi} \times 10^4 \text{ N/m}$$

$$\frac{F}{l} = \frac{F_1}{l} + \frac{F_2}{l} = \frac{4\mu_0}{4\pi} \times 10^4 = 4 \times 10^{-3} \frac{N}{m}$$

Answer.30

The current in the coil is 10A

There will be induction of magnetic force due to two current carrying wire placed in each other's vicinity.

The force is given by

$$F_m = \frac{\mu_0 i_1 i_2}{2\pi d}$$

Where l is the length of the wire and i_1 i_2 are the currents flowing through the wires.

To balance the weight, the gravitational force should be equal to the magnetic force that is exerted.

Hence

$$F_g = \lambda \times g \times l$$

Where λ is the linear mass density of the wire.

Hence, equating these forces, we get,

$$\frac{\mu_0 i_1 i_2}{2\pi d} = \lambda \times g \times l$$

$$i_2 = \frac{\lambda \times g \times l \times 2\pi d}{\mu_0 i_1}$$

Substituting the numerical values, we get

$$i_2 = \frac{2 \times 10^{-7} \times 50}{5 \times 10^{-8}}$$

$$i_2 = \frac{9.8 \times 10^{-7}}{20 \times 10^{-7}} = 0.49A$$

Answer.31

Here, current in the wire is $I_1 = 10A$

Current in the loop, $I_2 = 6A$

Force on an element dx in the arm PQ a distance x away is

$$dF_{PQ} = \frac{\mu_0 I_1 I_2}{2\pi x} dx$$

$$\Rightarrow F_{PQ} = \int_1^2 \frac{\mu_0 I_1 I_2}{2\pi x} dx = \frac{\mu_0 I_1 I_2}{2\pi} \ln x|_1^2 = 120 \times 10^{-7} [\log 3 - \log 1]$$

similarly, force on RS is,

$$\Rightarrow F_{RS} = \int_1^2 \frac{\mu_0 I_1 I_2}{2\pi x} dx = \frac{\mu_0 I_1 I_2}{2\pi} \ln x|_1^2 = 120 \times 10^{-7} [\log 3 - \log 1]$$

Hence, $F_{PQ} = F_{RS}$.

B distance of PS from wire, $x = 1cm$

distance of PS from RQ, $x' = 2cm$

Force on element PS is

$$F_{PS} = \frac{\mu_0 I_1 I_2}{2\pi x} - \frac{\mu_0 I_1 I_2}{2\pi x'}$$

$$F_{PS} = \frac{4\pi \times 10^{-7} \times 10 \times 6}{2\pi \times 10^{-2}} - \frac{4\pi \times 10^{-7} \times 6 \times 6}{2\pi \times 2 \times 10^{-2}} = 8.4 \times 10^{-6} N \text{ towards right}$$

Now distance of RQ from wire, $x = 3cm$

distance of RQ from PS, $x' = 2cm$

similarly, force on RQ is

$$F_{RQ} = \frac{\mu_0 I_1 I_2}{2\pi x} - \frac{\mu_0 I_1 I_2}{2\pi x'}$$

$F_{RQ} = \frac{4\pi\times10^{-7}\times6\times10}{2\pi\times3\times10^{-2}} - \frac{4\pi\times10^{-7}\times6\times6}{2\pi\times2\times10^{-2}} = 7.6\times10^{-6}N$ towards left

Net force, $F = F_{RQ} + F_{PS} = (8.4 + 7.6)\times10^{-6} = 16\times10^{-6}N$

Answer.32

Given:No. of turns, n = 1Magnitude of current, i = 5.00 Athe magnetic field at the center due to the current in the loop is given by

We know that

$$B = \frac{\mu_0 ni}{2r}$$

Now,

Where B=Magnetic Field= 0.2 mT

r= radius of loop

$$r = \frac{\mu_0 nI}{2B}$$

$$r = \frac{4\pi\times10^{-7}\times5}{2\times0.2\times10^{-3}}$$

$$r = 1.57\times10^{-2}\ m = 1.57\ cm$$

Answer.33

number of turns n =100 turns

Radius r = 5cm =0.05m

Magnetic field B =6.0×10^{-3} T

Now,

$$B = \frac{n\mu_0 i}{2r}$$

$$6.0\times10^{-3} = \frac{100\times4\pi\times10^{-7}\times i}{2\times0.05}$$

i = 4.7740 A

Answer.34

no of revolutions = 3×10^5

radius = 0.5 angstrom = 5×10^{-11} m

Thus, the magnetic field at the centre due to the current in the loop is given by

We know that,

$$B = \frac{\mu_0 nI}{2r}$$

Here n=1,

$I = \frac{q}{t} = 1.6 \times 10^{-19}$

and r= 5×10^{-11} m

Hence,

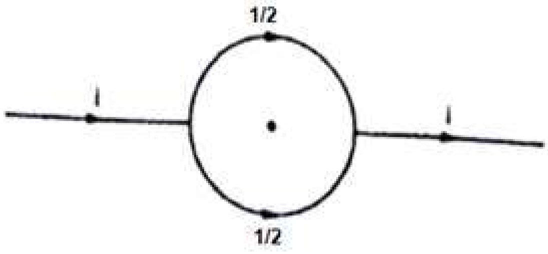
$$B = \frac{4\pi \times 10^{-7} \times 1.6 \times 10^{-19}}{2 \times 0.5 \times 10^{-10} \times 3 \times 10^5}$$

$$B = 6 \times 10^{-10}\, T$$

Answe.35r

The current flows into the circular coil and using the KCL, splits equally between the two arms because both the arms are equal in length and ae made up of the same material.

Hence the current in the upper section as well as the lower section will be i/2.



The current leaves the loop again and the resultant current leaving the coil will again be i.

The magnitude of the current is same and the direction is opposite at every point of time. Magnetic fields produced will hence also be zero as both the arms produce equivalent magnetic field which neutralize each other. Hence net magnetic field at the center will be 0

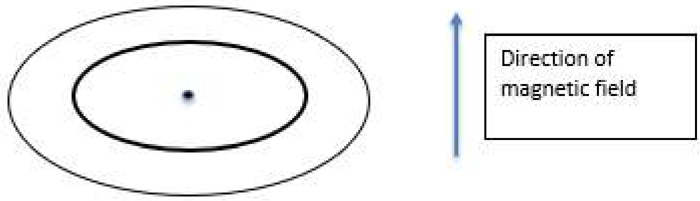
Answer.36

n_l = 100

n_s = 50

r_l = 10 cm

r_s = 5 cm



Both the coils have a common center and hence the effect due to the current flowing will depend on the direction of flow. We can calculate the individual results and then do the vector addition of both the forces to get the resultant magnetic fields

1) In the same sense

Field due to larger coil

$$B_l = \frac{\mu_0 n_l I}{2r_l}$$

Field due to smaller coil

$$B_l = \frac{\mu_0 n_s I}{2r_s}$$

As the direction is same and both the fields are acting in the same direction, that outside the plane of the paper, we can simply perform a scalar addition

$$B_t = \frac{\mu_0 n_l I}{2r_l} + \frac{\mu_0 n_s I}{2r_s}$$

$$B_t = \frac{\mu_0 I}{2}\left(\frac{n_s}{r_s} + \frac{n_l}{r_l}\right)$$

$$B_t = \frac{4 \times \pi \times 10^{-7}}{2}\left(\frac{100}{10 \times 10^{-2}} + \frac{50}{5 \times 10^{-2}}\right)$$

$$B \quad = 25.12\, mT$$

Similarly for

2) Opposite sense

As the direction is opposite, we can simply perform scalar subtraction to get the resultant

$$B_t = \frac{\mu_0 n_l I}{2r_l} - \frac{\mu_0 n_s I}{2r_s}$$

$$B_t = \frac{\mu_0 I}{2}\left(\frac{n_s}{r_s} - \frac{n_l}{r_l}\right)$$

$$B_t = \frac{4 \times \pi \times 10^{-7}}{2}\left(\frac{100}{10 \times 10^{-2}} - \frac{50}{5 \times 10^{-2}}\right)$$

$$\therefore B_t = 0$$

Answer.37

Given, for outer circle,

Number of turns $n = 100$,

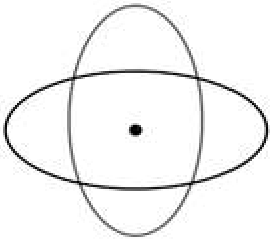
current, $I_o = 2A$

Radius, $r = 0.1m$

therefore, magnetic field is pointing west,

$$\vec{B}_o = \frac{\mu_0 nI}{2r} = \frac{4\pi \times 10^{-7} \times 2 \times 100}{2 \times 0.1} = 4\pi \times 10^{-4}T$$

for inner circle,



Number of turns $n = 100$,

current, $I_o = 2A$

Radius, $r = 0.1m$

therefore, magnetic field is pointing downwards,

$$\vec{B}_i = \frac{\mu_0 nI}{2r} = \frac{4\pi\times10^{-7}\times2\times50}{2\times0.05} = 4\pi \times 10^{-4}T.$$

Therefore, total magnetic field is

$$B = \sqrt{B_o^2 + B_i^2} = \sqrt{2 \times (4 \times 10^{-4})^2} \approx 1.8 \times 10^{-3}T \approx 1.8mT$$

Answer.38

Given, Radius,

$$r = 20cm = 0.2m$$

Velocity of electron, $v = 2 \times 10^6 m/s$

direction of motion, $\theta = 30^\circ$

Current in the loop, $I = 10A$

Therefore, force, $F = e(v \times B) = evBsin\theta = evsin\theta \times \frac{\mu_0 I}{2r}$

$$F = 1.6 \times 10^{-19} \times 2 \times 10^{6} \times \frac{4\pi \times 10^{-7}}{2 \times 0.2} \times \frac{1}{2} = 16\pi \times 10^{-19} N$$

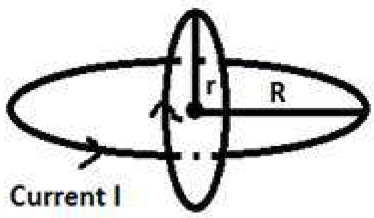
Answer.39

Given:

Current carried by circular loop of radius R = I

Current carried by circular loop of radius r(<<R) = i

Diagram:



Two loops of radius R and r carrying currents I and i respectively, passing through a common centre with their planes perpendicular to each other

Formula used:

Magnetic field at the center of a circular loop = $\mu_0 I/2r$,

where I = current through the loop, r = radius of the circle,

$\mu_0 = 4\pi \times 10^{-7}$ kgm s^{-2} A^{-2}

Magnetic torque acting on a loop of radius r = I (A X B), where I = current carried by the loop, A = area of loop, B = magnetic field

Therefore, magnetic field acting at the center of the loops due to the bigger loop of radius R(B) = $\mu_0 I/2R$.

Area of the smaller loop(A) = πr^2

Hence, torque acting on the smaller loop = I (A X B), since current carried by smaller loop is i. Here A = area of smaller loop = πr^2 and B = magnetic field due to bigger loop.

Now, we know that A X B = |A||B|sinθ, where θ is the angle between the area vector of the smaller loop and the magnetic field of the larger loop. |A| = modulus value of area of smaller loop, |B| = modulus value of magnetic field due to larger loop.

Since the planes of the two loops are perpendicular to each other, $\theta = 90^0$.

Hence, torque acting on the smaller loop = i (|A||B|sin 90^0) = i(|A||B|)

We know, $|A| = \pi r^2$, $|B| = \frac{\mu_0 I}{2R}$

Hence, torque on smaller loop = $i\left(\pi r^2 \times \frac{\mu_0 I}{2R}\right) = \frac{\mu_0 I i \pi r^2}{2R}$(Answer)

Answer.40

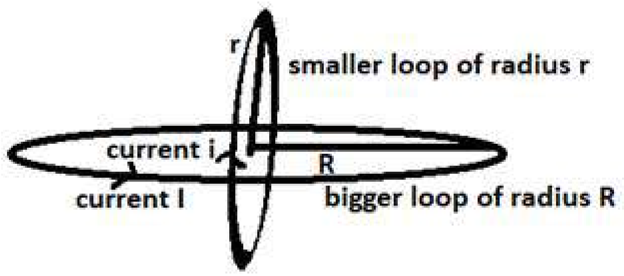
Given:

Current carried by circular loop of radius r = i

Current carried by circular loop of radius R(>>r) = I

Angle between the planes of the two loops = 30^0

Diagram:



Two loops of radius R and r, carrying currents I and i, with their planes making an angle of 30 degree with each other

Formula used:

Magnetic field at the center of a circular loop of radius r

M= $\frac{\mu_0 I}{2r}$,

where

I = current through the loop,

r = radius of the circle,

$\mu_0 = 4\pi \times 10^{-7}$ kgm s^{-2} A^{-2}

Magnetic torque acting on a loop of radius r

$$\boldsymbol{\tau} = \mathbf{I}(\mathbf{A} \times \mathbf{B}) = \mathbf{F} \times \mathbf{r},$$

where

I = current carried by the loop,

A = area of loop,

B = magnetic field,

F = force on periphery

Therefore, magnetic field at the center of the loops due to the bigger loop(B) = $\frac{\mu_0 I}{2R}$

Area of the smaller loop(A) = πr^2

Therefore, torque on the smaller loop = i (A X B), since the smaller loop carries current i.

A = area of smaller loop, B = magnetic field due to bigger loop.

Now, we know that A X B = |A||B|sinθ,

Where

θ is the angle between the area vector of the smaller loop and the magnetic field of the larger loop.

|A| = magnitude of area of smaller loop,

|B| = magnitude of magnetic field due to larger loop.

Given:

The angle between the area vector of the smaller loop and the magnetic field of the larger loop is $\theta = 30^0$.

Hence, torque acting on the smaller loop = $i(|A||B|\sin 30^0) = \frac{i(|A||B|)}{2}$

We know, $|A| = \pi r^2$, $|B| = \frac{\mu_0 I}{2R}$

Hence,

Torque on smaller loop due to magnetic field,

$$\frac{i\left(\pi r^2 \times \frac{\mu_0 I}{2R}\right)}{2} = \frac{\mu_0 I i \pi r^2}{4R}$$

Torque due to external force on its periphery,

$$\frac{\mu_0 Ii\pi r^2}{4R}$$

Therefore

$$\frac{\mu_0 Ii\pi r^2}{4R} = F \times r$$

Where

F = external force,

r = perpendicular distance between the line of action of force and the point at which the torque is acting.

Here, F and r are perpendicular to each other, since the force is being applied to the periphery.

Here r = radius of smaller loop.

Hence, $\frac{\mu_0 Ii\pi r^2}{4R} = F \times r$

which gives us

$F = \frac{\mu_0 Ii\pi r}{4R}$ (Answer)

Answer.41

Given:

Radius of semicircular wire(r) = 10 cm = 0.1 m

Current carried by it(i) = 5 A

Formula used: **Magnetic field at the center of a semicircular loop =**

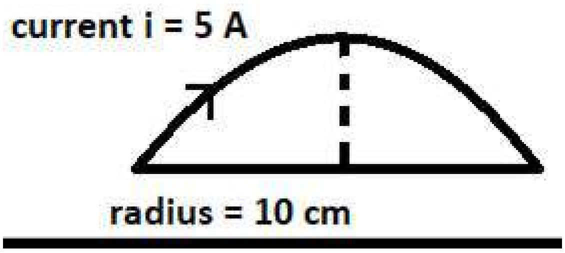
$$\frac{1}{2} \times \frac{\mu_0 i}{2r} = \frac{\mu_0 i}{4r}$$

Where

i **=** current carried by loop,

r = radius of loop,

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ kg m s^{-2} A^{-2}



Hence, required magnetic field(B) =$\frac{\mu_0 \times 5}{4 \times 0.1}$Tesla(T) ,

where $\boldsymbol{\mu}_0$ = 4π x 10^{-7} kg m s^{-2} A^{-2}

$$\mathbf{B} = \frac{4\pi \times 10 - 7 \times 5}{4 \times 0.1} \text{ T} = 1.57 \times 10^{-5} \text{ T}$$

Answer.42

Given:

Radius of circular arc(r) = 10 cm = 0.1 m

Current carried by it(i) = 6 A

Angle subtended at the center = 1200

Formula used: **Magnetic field at the center of a circular arc,**

$$\frac{\mu_0 i}{2r} \times \frac{\theta}{2\pi}$$

Where

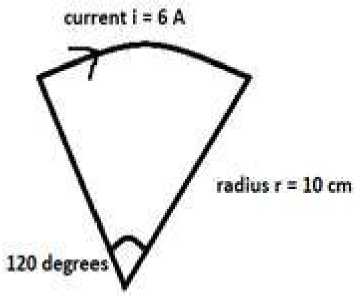
μ_0 = magnetic permeability of vacuum = 4π x 10^{-7} kg m s^{-2} A^{-2},

i = current carried by the wire,

r = radius of the wire,

θ = angle subtended at the center of the arc

Diagram:



Hence, required magnetic field

$$B = \frac{\mu_0 i}{4\pi r}(\theta)$$

where, i = 6 A, r = 0.1 m, θ = 120^0 = 2π/3 radian, μ_0 = 4π x 10^{-7} kg m s^{-2} A^{-2}

Hence, B = $\frac{4\pi \times 10{-}7 \times 6}{4\pi \times 0.1} \times \frac{2\pi}{3}$ = **1.256 x 10^{-5} T** = **12.56 μT** (Ans)

Answer.43

For the magnetic field at the center to be zero, the magnetic field due to the circular loop has to be exactly equal and opposite to the long straight wire at the center.

Given:

Current carried by circular loop of radius r = i

Current carried by long straight wire = 4i

Formula used:

Magnetic field at the center of a circular loop due to current in the loop(B_l) = $\frac{\mu_0 i}{2r}$, which points into the plane of the paper.

Here

μ_0 = magnetic permeability of vacuum,

i = current carried by the loop,

r = radius of the loop

Magnetic field due to an infinitely long wire at a distance x from it(B_w) = $\frac{\mu_0 i}{2\pi x}$, which points out of the plane of the paper.

Here

μ0 = magnetic permeability of vacuum,

i = current carried by the wire,

x = distance of the center of the circle from the wire

Hence, from given information,

$B_l = \frac{\mu_0 i}{2r}$, and $B_w = \frac{\mu_0 \times 4i}{2\pi x}$

where B_l = magnetic field due to loop, B_W = magnetic field due to wire, r = radius of loop, x = distance of the center of the circle from the wire

Hence, for the magnetic field at the center to be 0,

$$\mu_0 \frac{i}{2r} = \mu_0 \frac{4i}{2\pi x}$$

$$\Rightarrow \frac{1}{r} = \frac{4}{\pi x} \Rightarrow x = \frac{4r}{\pi}$$

Hence, for the magnetic field to be 0, the wire has to be placed at a distance of 4r/π from the center of the circular loop. (Answer)

Answer.44

Given:

Number of turns(n) = 200

Radius of circular coil(r) = 10 cm = 0.1 m

Current carried by the coil(i) = 2 A

Formula used: **(a) Magnetic field at the center of a circular coil of n turns(B) =**

M=$\frac{n\mu_0 i}{2r}$,

where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A^{-1},

i = current carried by coil,

r = radius of coil

Hence, from the given information,

Magnetic field at the center of the given coil

$= \frac{200\times 4\pi\times 10{-}7\times 2}{2\times 0.1}$ T = 2.51×10^{-3} T = 2.51 mT

(b) The magnetic field at a distance point in the coil is given by B_p,

$$\frac{n\mu_0 i r^2}{2(x^2+r^2)^{\frac{3}{2}}}$$

The magnetic field at the center of the coil is $\frac{n\mu_0 i}{2r}$

Given: the magnetic field at the center is 1/2 of the initial

The magnetic field at the center of the coil is $\frac{n\mu_0 i}{4r}$

On equation the magnetic field at the center and the magnetic field at the any distant point from the coil, we get

$$\frac{n\mu_0 i}{4r} = \frac{n\mu_0 i r^2}{2(x^2+r^2)^{\frac{3}{2}}}$$

On equating the above equation, we get

$(r^2 + x^2)^3 = 4r^6$

We get

$x^2 + r^2 = 4^{1/3} r^2$

$x^2 = 0.58\ r^2$

$x^2 = 0.5\times 100 = 58$

$x = \pm\ 7.66$ cm

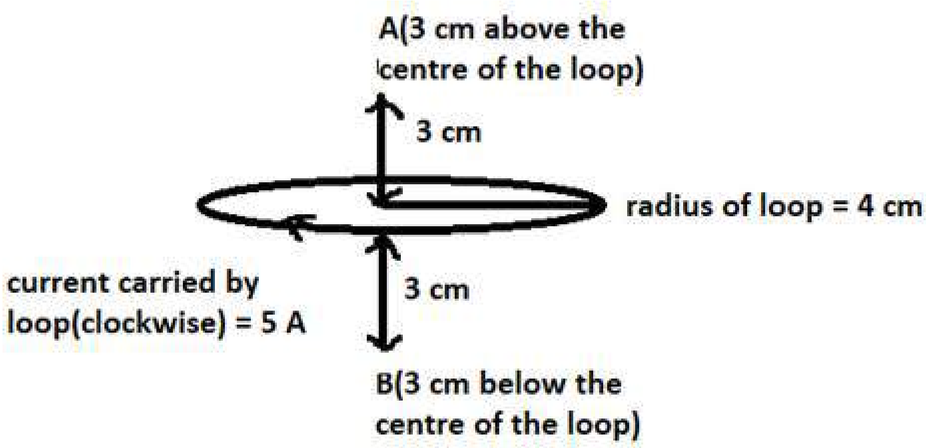
Magnetic field will drop to half of its value at the center if the distance of that point from the center of the coil along the axis of coil is equal to 7.66 cm.

Answer.45

Given:

Radius of circular loop(r) = 4 cm = 0.04 m

Current carried by it(i) = 5 A



Formula used:

Magnetic field at a distance x from the center of a current carrying circular loop(B),

$$\frac{\mu_0 i r^2}{2(x^2 + r^2)^{3/2}}$$

Where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A^{-1},

i = current carried by coil,

r = radius of coil

(a) To find the magnetic field 3 cm above the loop:

Magnetic field,

$$B = \frac{\mu_0 i r^2}{2(x^2 + r^2)^{3/2}}$$

Here, x = 3 cm = 0.03 m

Hence, substituting the given values:

$$B = \frac{(4\pi \times 10-7 \times 5 \times 0.042)}{\left(2 \times \frac{(0.032 + 0.042)3}{2}\right)} T$$

= **4.019 x 10^{-5} T** (in the downward direction, by right hand thumb rule) (Ans)

(b) To find the magnetic field 3 cm below the loop:

Here, all the values are same as the previous part. Hence,

B = **4.019 x 10^{-5} T** (in the upward direction, by right hand thumb rule)

Answer.46

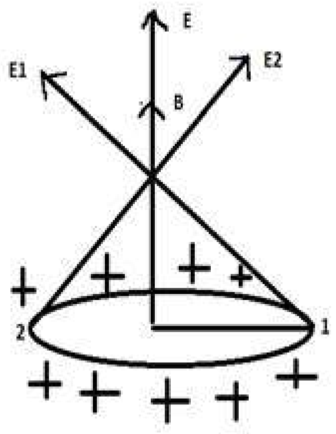
Given:

Charge of the ring(q) = 3.14 × 10^{-6} C

Radius of circular ring(r) = 20 cm = 0.2 m

Angular velocity with which the ring rotates(w) = 60 rad s^{-1}

Diagram:



$_1$ and E_2 denote the electric field due to two point elements 1 and 2 on the circumference of the circular loop. The resultant electric field due to them is E. Similarly, the resultant magnetic field due to the them is B.

Formula used:

Electric field due to a circular loop(E),

$$\frac{qx}{4\pi\epsilon_o(x^2 + r^2)^{\frac{3}{2}}}$$

where q = charge carried by the ring, x = distance from the center, r = radius of the ring, ϵ_0 = electric permittivity of vacuum = 8.85 X 10^{-12} N m^2 C^{-2}

Magnetic field due to a current carrying loop(B),

$$\frac{\mu_0 ir^2}{2(x^2 + r^2)^{\frac{3}{2}}}$$

where μ_0 = magnetic permeability of vacuum = 4π x 10^{-7} T m A^{-1}, i = current carried by the loop, r = radius of the loop, x = distance from the center

$$\frac{\mathbf{E}}{\mathbf{B}} = \frac{\dfrac{\mathbf{qx}}{\mathbf{4\pi\epsilon_o(x^2 + r^2)^{\frac{3}{2}}}}}{\dfrac{\mu_0 ir^2}{2(x^2 + r^2)^{\frac{3}{2}}}} = \frac{2qx}{4\pi\epsilon_o\mu_o ir^2}$$

Now, i = q/t, where i = current, q = charge, t = time taken.

Angular velocity(w) = 60 rad/s (given)

Hence, time period of one revolution(T) = 2π/w = 2π/60 s

Hence, current(i) = charge per unit time = q/T = 60q/2π = 30q/π A, where q = charge, T = time period of revolution

Therefore, E/B = (2 x 3.14 × 10^{-6} x 0.05 x 9 x 10^9 x 3.14)/ (4 x 3.14 x 10^{-7} x 30 x 3.14 × 10^{-6} x 0.2^2) = **1.88 x 10^{15} ms^{-1}** (Ans)

Answer.47

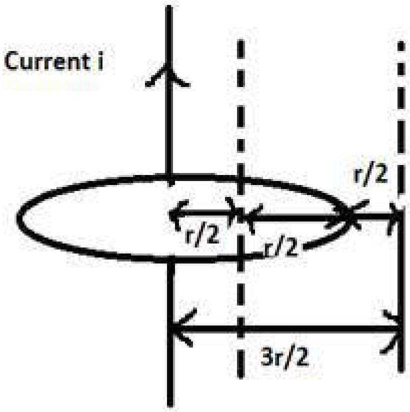
Formula used:

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$$\oint B.dl = \mu_o I,$$

where B = magnetic field, dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

Diagram:



(a) Now, since inside a conductor, the current enclosed is 0, in Ampere's circuital law, we put I = 0.

Hence, the magnetic field at a distance r/2 from the surface inside the tube is 0. (Ans).

(b) Now, for any point outside the tube, the current enclosed will be given by Ampere's circuital law:

$$\oint B.dl = \mu_o I$$

Where

B = magnetic field,

dl = line element,

μ_0 = magnetic permeability of vacuum,

I = current enclosed.

Now, since we have to find the magnetic field at a distance r/2 from the surface,

we consider an Amperian loop of radius $\left(r + \frac{r}{2}\right) = \frac{3r}{2}$ from the centre of the loop.

The total current enclosed will be i, since we are considering a loop outside the tube.

Hence,

$$\oint B.dl = B.\oint dl = B \times 2\pi \times \frac{3r}{2} = \mu_0 \times i,$$

since $\oint dl = 2\pi \times \frac{3r}{2}$ = circumference of the Amperian loop.

Hence, we get, B(at a distance r/2 from the surface outside the tube) $= \frac{\mu_0 i}{3\pi r}$ (Ans)

Answer.48

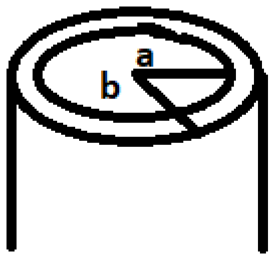
Given:

Inner radii of tube = a

Outer radii = b

Current distributed over its cross section = i

Diagram:



Formula used:

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$$\oint B.dl = \mu_o I,$$

Where

B = magnetic field, dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

(a) Inside a conducting tube, I (current enclosed) is 0.

Hence, by Ampere's circuital law, $\oint B.dl$ = 0 => B = 0.

Hence, the magnetic field just inside the tube is 0.

(b) Just outside the tube, the distance from the center is b.

Hence, we consider an Amperian loop of radius b from the center.

Hence, from Ampere's circuital law:

$$\oint \mathrm{B.dl} = \mu_o I,$$

Where B = magnetic field,

dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

In this case, $\oint dl = 2\pi b$ (circumference of the loop of radius b)

Therefore, B x $2\pi b = \mu_0 i$ (since the total current enclosed is i).

Hence, the magnetic field at a point just outside the tube = $\mathbf{\mu_0 i/2\pi b}$(Ans)

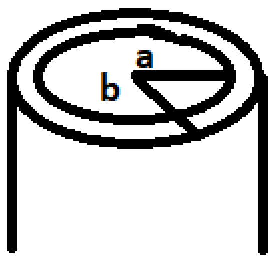
Answer.49

Given:

Radius of wire = b

Current distributed throughout the cross section = i

Diagram:



Formula used:

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$\oint B.dl = \mu_o I$, where B = magnetic field, dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

Since the current i is uniformly distributed throughout the cross section of the wire,

For an amperian loop of radius a(a<b) from the center of the wire,

I (current enclosed)

$$\left(\frac{i}{\pi b^2}\right)\times \pi a^2 = \frac{ia^2}{b^2}$$

Where

i= total current distributed throughout the wire,

b = radius of the wire,

a = distance from the axis at which the magnetic field is to be found.

Hence, from Ampere's circuital law,

$$\oint B.dl = B \times 2\pi a = \mu_0 I = \frac{\mu_0 i a^2}{b^2}$$

since $2\pi a$ = circumference of Amperian loop of radius a, and I = enclosed current = ia^2/b^2.

Therefore, magnetic field at a distance a from the axis inside the wire, $\frac{\mu_0 ia}{2\pi b^2}$

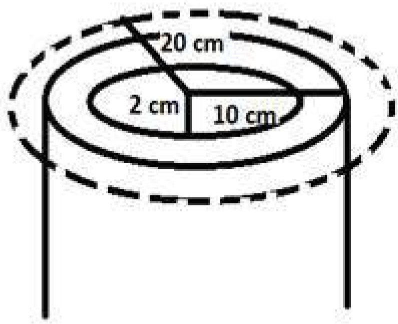
Answer.50

Given:

Radius of wire(r) = 10 cm = 0.1 m

Current carried by it(i) = 5 A

Diagram:



Formula used:

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$$\oint B.dl = \mu_o I,$$

Where B = magnetic field, dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

(a) Since the current is uniformly distributed over the cross section of the wire, at a distance of 2 cm (0.02 m) from the axis (inside the wire), I (current enclosed)

$$= \left(\frac{5}{\pi(0.1)^2}\right) \times (\pi(0.02)^2)$$

Where,

A = 0.2 A, where 0.1 m is the radius of the wire.

Hence, from Ampere's circuital law,

$$\oint B.dl = \mu_0 I$$

where μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A^{-1}, B = magnetic field, dl = line element, I = current enclosed

Substituting the values, we get

B x 2π x 0.02 = 4π x 10^{-7} x 0.2 (since 2π x 0.02 = circumference of the loop of radius 2 cm)

=> Magnetic field at a distance of 2 cm from the axis

= **2 x 10^{-6} T = 2μT** (Ans)

(b) At a distance of 10 cm from the axis, we are basically on the surface of the wire. Hence the whole current of 5 A is enclosed by it.

Hence, from Ampere's circuital ,$\oint B.dl = \mu_o I$,

where μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A^{-1}, B = magnetic field, dl = line element, I = current enclosed

Here, I = 5 A.

Substituting the values, we get

B X 2π x 0.1 = 4π x 10^{-7} x 5(since 2π x 0.1 is the circumference of the loop of radius 10 cm)

=> Magnetic field at a distance of 10 cm from the axis

= **10^{-5} T = 10μT** (Ans)

(c) At a distance of 20 cm from the axis, we are outside the wire. Hence the whole current of 5 A is enclosed by it.

Hence, from Ampere's circuital law, $\oint B.dl = \mu_o I$, where μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A^{-1}, B = magnetic field, dl = line element, I = current enclosed

Here, I = 5 A.

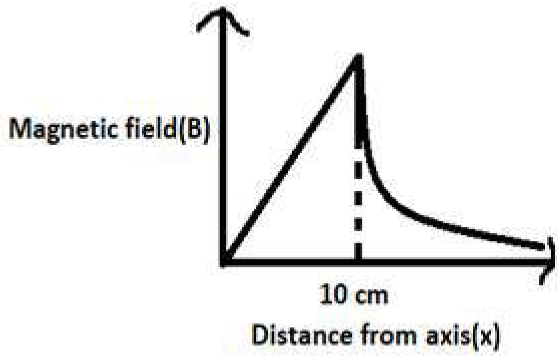
Substituting the values, we get

B X $2\pi \times 0.2 = 4\pi \times 10^{-7} \times 5$(since $2\pi \times 0.2$ is the circumference of the loop of radius 20 cm)

=> Magnetic field at a distance of 20 cm from the axis

= **5×10^{-6} T = 5μT** (Ans)

Graph for B vs x for 0 < x < 20 cm:



Answer.51

Half of the given loop is in a region with zero magnetic field, while half of it is in a region with magnetic field B.

Now, we take the Amperian loop PQRS.

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$$\oint B.dl = \mu_o I$$

where

B = magnetic field,

dl = line element,

μ_0 = magnetic permeability of vacuum,

I = current enclosed.

Hence, I (current enclosed) is not 0, but is finite (since half of it lies in a region of magnetic field B). Hence, this states the magnetic field inside the loop PQRS is not zero.

However, it is obvious that the magnetic field at any point in the region outside the magnetic field lines = 0.

These are two contradictory statements.

Hence, such a field is not possible. (proved)

Answer.52

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$\oint B.dl = \mu_o I$, where B = magnetic field, dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

(i) At point P, the current enclosed(I) = 0.

Hence, from Ampere's circuital law, $\oint$B.dl = 0 => B (magnetic field at P) = **0**. (Ans)

(ii) Point Q is in between the two strips carrying current.

Hence, from Ampere's circuital law, we consider only B.dl for the strip of width dl carrying surface current K. Hence, I (current enclosed by the strip) = Kdl.

Therefore, B.dl = μ_0Kdl => B (magnetic field at Q) = $\boldsymbol{\mu_0 K}$ (Ans)

(iii) At point Q, again, current enclosed(I) = 0.

Hence, from Ampere's circuital law, $\oint$B.dl = 0 => B (magnetic field at Q) = **0**. (Ans)

Answer.53

Given:

Charge of the particle = q

Mass of the particle = m

Radius of circle described by the particle between the two plates = r

Formula used:

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$$\oint B.dl = \mu_o I$$

where B = magnetic field, dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

Now, from the previous problem, we found out that the magnetic field(B) at point Q is $\mu_0 K$, where μ_0 = magnetic permeability of vacuum, K = surface current.

Also, we know that the velocity of a particle in a magnetic field is given as

$$v = \frac{qBr}{m}$$

Where

q = charge of particle, B = magnetic field, r = radius of circle described by particle, m = mass of particle.

Substituting the value of B, we get

Velocity of the particle is $q\mu_0 K\frac{r}{m}$

Answer.54

Given:

Current(i) = 5 A

Magnetic field(B) = 3.14×10^{-2} T

Formula used:

Magnetic field inside a long solenoid $B = \mu_0 ni$,

Where

$\mu 0$ = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A^{-1}, n = number of turns per unit length, i = current carried by the wire

Hence, from the given information:

$3.14 \times 10^{-2} = 4\pi \times 10^{-7} \times n \times 5$

=> n = 5000 turns/m

Therefore, number of turns per unit length of the solenoid = **5000**(Ans)

Answer.55

Given:

Radius of cylinder(r) = 0.5 mm = 5×10^{-4} m

Current(i) = 5 A

Formula used:

Magnetic field inside a long solenoid $B = \mu_0 ni$,

where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A^{-1}, n = number of turns per unit length, i = current carried by the wire

Now, width of each turn = diameter of the wire = (0.5 x 2) mm = 1 mm = 10^{-3} m.

Hence, number of turns per unit length(n) (per m) =10^3 = 1000

Therefore, substituting the values:

B (magnetic field at the center) = $4\pi \times 10^{-7} \times 1000 \times 5$ T

= **6.28×10^{-3} T** (Ans)

Answer.56

Given:

Resistance per unit length(R_0) = 0.01 ohm

Total number of turns(N) = 400

Radius of the wire(r) = 1 cm = 0.01 m

Length of the wire(l) = 20 cm = 0.2 m

Magnetic field near the center(B) = 1.0×10^{-2} T

Formula used:

Magnetic field at the center of a solenoid, B = μ0ni,

Where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A^{-1},

n = number of turns per unit length,

i = current carried by the wire

Emf(E) = iR,

where i = current, R = resistance

Now, n(number of turns per unit length) = Total number of turns(N)/Length of wire(l) = $\left(\frac{400}{0.2}\right)$ = 2000

Total resistance (R of all turns) = $R_0 \times 2\pi r \times 400 = (0.01 \times 2\pi \times 0.01 \times 400) = 0.25$ ohm

Now, substituting the given values:

$1.0\times10^{-2} = 4\pi \times 10^{-7} \times 2000 \times i \Rightarrow i = 3.98$ A

Therefore, Emf(E) = iR = (3.98 X 0.25) = 0.995 V which is almost equal to 1 V.

Emf of the battery = **1 V** (Ans)

Answer.57

Given

Radius of solenoid is a

Length of the solenoid is l

Number of turns per unit length is n

The current in the circular loop is indx

The magnetic field due to the circular ring at the any distance from the ring is

$$B = \frac{\mu_o I r^2}{4\pi(x^2 + r^2)^{\frac{3}{2}}}$$

Where

μ_o is the permeability of the free space,

I is the current in the ring

r is the radius of the loop

x is the distance from the loop at which the magnetic field to be found.

Integrating the above equation for x-0 and r

$$B = \int dB = \int_0^l \frac{\mu_o nI a^2 dx}{4\pi\left(\left(\frac{l}{2} - x\right)^2 + a^2\right)^{\frac{3}{2}}}$$

$$B = \int_0^l \frac{\mu_o nI a^2 dx}{4\pi\left((\frac{l}{2} - x)^2 + a^2\right)^{\frac{3}{2}}} = \frac{\mu_o nI}{4\pi}\int_0^l \frac{a^2 dx}{a^3\left(\left(\frac{l - 2x}{2a}\right)^2 + 1\right)^{\frac{3}{2}}}$$

On solving the above equation, we get

$$B = \frac{\mu_o\, nI}{4\pi a}\int_0^l \frac{dx}{\left(\left(\frac{l-2x}{2a}\right)^2+1\right)^{\frac{3}{2}}} = \frac{\mu_o\, nI}{4\pi a} \times \frac{4\pi a}{\left(1+\left(\frac{2a}{l}\right)^2\right)^{1/2}}$$

The magnetic field at the center of the solenoid due to this circular current

$$B = \frac{\mu_o\, nI}{\left(1+\left(\frac{2a}{l}\right)^2\right)^{1/2}}\ (i)$$

(b) When $l \gg a$

Putting in the equation (i), we get

$$\frac{2a}{l} \ll 1 \text{ or } \left(1+\left(\frac{2a}{l}\right)^2\right)^{\frac{1}{2}} = 1$$

Putting the values in the above formula, we get

$B=\mu_0\, n\, I$

When $a \gg l$

Putting in the equation (i), we get

$$\frac{2a}{l} \gg 1 \text{ or } \left(1+\left(\frac{2a}{l}\right)^2\right)^{\frac{1}{2}} = \frac{2a}{l}$$

Putting the above value in the equation (i), we get

$$B = \frac{\mu_o nIl}{2a}$$

Hence, proved if $l >> a$, the field tends to $B = \mu_0 ni$ and

if $a >> l$, the field tends to B, $\frac{\mu_0 nil}{2a}$.

Answer.58

Given:

Current(i) = 2 A

Frequency(f) = 1.00×10^8 rev s^{-1}.

Formula used:

Magnetic field inside a solenoid(B) = $\mu_0 ni$,

Where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A^{-1},

n = number of turns per unit length,

i = current carried by the wire

Now, charge of electron(q) = 1.6×10^{-19} C

Mass of electron(m) = 9.1×10^{-31} kg

Now, frequency of the particle in uniform circular motion in a magnetic field is given by

$f = \frac{qB}{2\pi m}$,

Where

q is the charge,

B is the magnetic field,

m is the mass of particle

Hence, $B = \frac{2\pi mf}{q}$

Substituting the given values, we obtain B as,

$$\frac{2 \times 3.14 \times 9.1 \times 10^{-31} \times 1.00 \times 108}{1.6} \times 10^{-19} = 0.0036\, T$$

Hence, B = $\mu_0 ni$,

Where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A^{-1},

n = number of turns per unit length,

i = current carried by the wire

=> Number of turns per meter(n) = $\frac{B}{\mu_0 i}$ = 0.0036/($4\pi \times 10^{-7}$ x 2) = **1420 turns/meter**. (Ans)

Answer.59

Given:

Number of turns per unit length = n

Current = i

Radius = r

Charge of particle = q

Mass = m

Formula used:

Magnetic field inside a solenoid(B) = $\mu_0 ni$,

where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A^{-1},

n = number of turns per unit length,

i = current carried by the wire

Now, when a particle is projected perpendicular to a magnetic field, it describes a circle. Now, for the particle to not strike to solenoid, the required radius is r/2.

Since it is moving in a circular path, centripetal acceleration = mv^2/r,

Where

m = mass of particle,

v = velocity,

r = radius

Force due to the magnetic field = qvB,

Where

q = charge of particle,

v = velocity,

B = magnetic field.

Here, r => r/2.

The centripetal and magnetic forces balance each other.

Hence, using the given data:

$\frac{mv^2}{\frac{r}{2}} = qv\mu_0 ni$ => v = $\mathbf{q\mu_0 nir/2m}$ (Ans)

Answer.60

Given:

Number of turns per unit length = n

Surface current of sheet = K

Formula used:

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$$\oint B.dl = \mu_o I,$$

Where

B = magnetic field,

dl = line element,

μ_0 = magnetic permeability of vacuum,

I = current in the circuit.

(a) Since the magnetic field near the center of the solenoid is 0, we infer that the magnetic field due to the solenoid = magnetic field due to the sheet

For the sheet, using Ampere's circuital law, we get:

B x 2l = μ_0 x Kl, where Kl = current enclosed by sheet and we take 2l because of the two surfaces of the sheet.

=> $B_{solenoid} = \mu_0 K/2$

Now, magnetic field due to a solenoid $B_{solenoid} = \mu_0 ni$, where μ_0 = magnetic permeability of vacuum, i = current, n = number of turns per unit length

Hence, $\mu_0 K/2 = \mu_0 ni$ => i (current in solenoid) = **K/2n** (Ans)

(b) Now, if the axes of the solenoid and the plate are perpendicular to each other, their magnetic fields are also perpendicular to each other.

Hence, net magnetic field at center,

$$\sqrt{\left(\frac{\mu_o k}{2}\right)^2 + \left(\frac{\mu_o k}{2}\right)^2} = \frac{\mu_0 K}{\sqrt{2}}$$

Answer.61

Given:

Capacitance(C) = 100 μF = 10^{-4} F

Initial Voltage of battery(V) = 20 V

Number of turns per meter of solenoid(n) = 4000

Final potential difference(V’) = 90% of 20 V = 18 V

Time taken(t) = 2 s

Formula used:

Initial charge stored in capacitor(Q) = CV,

where C = capacitance, V = potential difference

Therefore Q = (10^{-4} x 20) C = 2 x 10^{-3} C

New potential difference = V’ = 18 V

Therefore, New charge Q’ = CV’ = (10^{-4} x 18) = 1.8 x 10^{-3} C

Hence, current flowing in conductor(i) = (Q-Q’)/t, where Q = initial charge, Q’ = final charge, t = time taken

Therefore, i = $(2.0 - 1.8) \times 10 - \frac{3}{2}\, A$ = 10^{-4} A

Thus, average magnetic field at the center of solenoid = B = $\mu_0 ni$,

where μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A^{-1}, n = number of turns per unit length, i = current carried by the wire

Hence, from given data, B = $(4\pi \times 10^{-7} \times 4000 \times 10^{-4})$ T

= **0.5 x 10-6 T** (Ans)