Chapter 4

Rectilinear Motion

CHAPTER HIGHLIGHTS

- *Dynamics*
- *Rectilinear Motion: Displacement, Distance, velocity and Acceleration*
- *Motion at a Uniform Acceleration*
- *Vertical Motion Under Gravity*
- *Kinetics of a Particle*
- *Differential Equation of Rectilinear Motion*
- *Motion of a Particle Acted Upon by a Constant Force*
- *Freely Falling Body*
- *Dynamics of a Particle*
- *Work and Energy*
- *Law of Conservation of Energy*
- *Impact*
- *Elastic Impact*
- *Plastic or Inelastic Impact*
- *Coefficient of Restitution*

Dynamics

Dynamics is the branch of mechanics dealing with the motion of a particle or a system of particles under the action of a force. Dynamics is broadly divided into two categories:

1. Kinematics
2. Kinetics

Kinematics is the study of motion of a body without any reference to the forces or other factors which causes the motion. Kinematics relates displacement, velocity and acceleration of a particle of system of particles.

Kinetics studies the force which causes the motion. It relates the force and the mass of a body and hence the motion of the body. So in fact, the motion of a particle or body is largely covered and interpreted by Kinematics and Kinetics.

Types of Motion

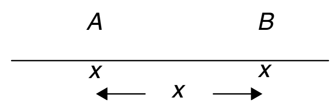
The rate of change of position is motion. The type of motion is explained by the type of path traced by it. If the path traced is a straight line, the motion is said to be rectilinear motion or translation.

If the path traced by the motion (or path traversed by the particle) is a curve, it is known as curvilinear motion. When the curve becomes a circle, then it is known as circular motion.

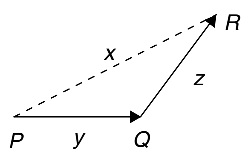
The two types of motion, i.e., rectilinear and curvilinear motions, explained above can be together termed as the general plane motion.

Rectilinear Motion: Displacement, Distance, Velocity and Acceleration

1. Displacement and distance:



Let the particle be at the position A at any point of time t. Let the position of the particle be at B at time $t + dt$ ($dt > 0$). Then the particle is said to move from A to B. The change in position is the displacement x. It is the shortest distance between A and B. Distance is the length of the path described by the particle from point A to point B.

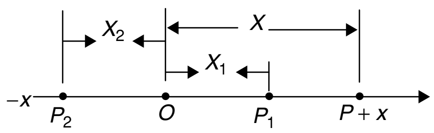


Let a body start from a point P and move towards a point Q and then turn and reach a point R. During this course of motion, the total displacement is denoted by x. The distance traversed is given by $y + z$.

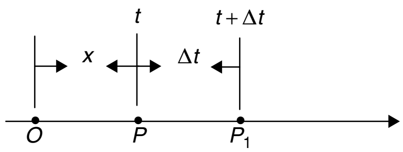
NOTE

When the motion of a particle is considered along a line segment, both distance and displacement are the same in magnitudes.

Motion can also be defined as the change in the position of a body with respect to a given object. The position of a point P at any time t is expressed in terms of the distance x from a fixed origin O on the reference x-axis, y-axis or z-axis and can be taken as positive or negative as per the usual sign convention.



2. **Average velocity:** The average velocity v_{av} of a point P, in the time interval between $t + \Delta t$ and t, i.e., in the time interval Δt, during which its position changes from x to $x + \Delta x$ is defined by $v_{av} = \dfrac{\Delta x}{\Delta t}$.



3. **Instantaneous velocity and speed:** The instantaneous velocity v of a point P at time t is the limiting value of the average velocity as the increment of time approaches zero as a limit. Mathematically it can be expressed as:

$$v = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The velocity v is positive if the displacement x is increasing and the particle is moving in a positive direction. The unit of velocity is metre per second (m/s).

If s is the distance covered by a moving particle at time t, then speed $= \dfrac{ds}{dt}$. The unit of speed is the same as that of the velocity.

4. **Average acceleration:** The average acceleration a_{av} of a point P, in the time interval between $t + \Delta t$ and t, i.e., in the time interval Δt, during which its velocity changes from v to $v + \Delta v$ is defined by $a_{av} = \dfrac{\Delta v}{\Delta t}$.

5. **Instantaneous acceleration:** The instantaneous acceleration of a point P is the limiting value of the average acceleration as the increment of time approaches zero. Mathematically it can be expressed as:

$$a = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Then,

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Now,

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{dv}{dx} \times v$$

Acceleration is positive when velocity is increasing. A positive acceleration means that the particle is either moving further in a positive direction or is slowing down in the negative direction.

Retardation or deceleration of a body in motion is the negative acceleration, i.e., retarding acceleration. Acceleration is the rate of increase in the velocity and deceleration is the rate of decrease in the velocity.

Uniform Motion

When a particle moves with a constant velocity so that its acceleration is zero, then the motion is termed as uniform motion.

Uniformly Accelerated Motion

When a particle moves with a constant acceleration, then the motion is termed as a uniformly accelerated motion.

Motion at a Uniform Acceleration

Let the uniform acceleration be 'a'. Then

$$v = u + at$$

And,

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

$$s_n = u + a\left(n - \frac{1}{2}\right)$$

Where,

v – Velocity at any time instant t (secs)
u – Initial velocity
s – Distance travelled during the time t (secs)
s_n – Distance travelled at the nth second

NOTE

For motion under constant retardation or deceleration, assign negative sign for acceleration (a).

Vertical Motion Under Gravity

A body in motion above the ground will be under influence of the gravitational force of attraction (g). If the body moves upwards then it is subjected to gravitational retardation, i.e., $a = -g$. Then, the equations for the upward motion of a body under gravity will be

$$v = u - gt$$

$$v^2 = u^2 - 2gs$$

$$s = ut - \frac{1}{2}gt^2$$

$$s_n = u - g\left(n - \frac{1}{2}\right)$$

If the body moves downwards then it is subjected to gravitational attraction and hence an acceleration, i.e., $a = g$. Then, the equations for the downward motion of a body under gravity will be

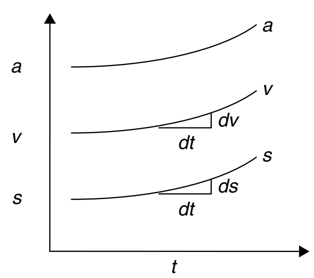
$$v = u + gt$$
$$v^2 = u^2 + 2gs$$
$$s = ut + \frac{1}{2} gt^2$$
$$s_n = u + g(n - \frac{1}{2})$$

NOTES

1. For a body that is just dropped, $a = g$ and $u = 0$.
2. The final vertical velocity of a body thrown upwards as it reaches the maximum height, will be zero, i.e., $v = 0$.

Motion Curves

These are the graphical representation of displacement, velocity and acceleration against time.



Considering the general case of acceleration not being a constant, the above graphical representation is made.

The slope of the displacement-time curve → Velocity
The slope of the velocity-time curve → Acceleration
The area under the velocity-time curve → Displacement
The area under the acceleration-time curve → Velocity

Solved Examples

Example 1: A particle has two velocities v_1 and v_2. Its resultant is v_1 in magnitude. When the velocity v_1 is doubled, the new resultant is

(A) Perpendicular to v_2 (B) Parallel to v_2
(C) Equal to v_2 (D) Equal to 2 v_2

Solution:
Applying the principle of vector, the magnitude of the resultant between $|\vec{v}_1 + \vec{v}_2|$
Given that $|\vec{v}_1 + \vec{v}_2| = \vec{v}_1$
Squring both side,

$$|\vec{v}_1 + \vec{v}_2|^2 = \vec{v}_1^{\,2}$$
$$\therefore (\vec{v}_1 + \vec{v}_2)\cdot(\vec{v}_1 + \vec{v}_2) = \vec{v}_1.\vec{v}_1$$
$$\vec{v}_1 \cdot \vec{v}_1 + 2\vec{v}_1 \cdot \vec{v}_2 + \vec{v}_2 \cdot \vec{v}_2 = \vec{v}_1 \cdot \vec{v}_1$$
$$2\vec{v}_1 \cdot \vec{v}_2 + \vec{v}_2 \cdot \vec{v}_2 = 0$$
$$(2\vec{v}_1 + \vec{v}_2)\cdot \vec{v}_2 = 0.$$

Dot product zero means the new resultant between $2v_1$ and v_2 is at right angles to v_2.

Example 2: If the two ends of a train, moving with a constant acceleration, pass a certain point with velocities u and v respectively, the velocity with which the middle point of the train passes through the same point is

(A) $\frac{u+v}{2}$ (B) $\sqrt{\frac{u^2+v^2}{u+v}}$

(C) $u - v$ (D) $\sqrt{\frac{u^2+v^2}{2}}$

Solution:
We have the relation $v^2 = u^2 + 2as$ (1)

If v is the velocity with which the mid point of the train crosses the point, we have

$$v^2 = u^2 + 2\,a\frac{s}{2} \qquad (2)$$

Eliminating s from (1) and (2),
We have,

$$v^2 - u^2 = as$$

And,

$$v^2 - u^2 = 2as$$

Now,

$$\frac{v^2 - u^2}{v^2 - u^2} = \frac{1}{2}$$

$$\therefore 2v^2 - 2u^2 = v^2 - u^2$$

Or, $$2v^2 = v^2 + u^2$$

Now, $$v^2 = \frac{v^2 + u^2}{2}$$

$$\therefore \quad v = \sqrt{\frac{v^2 + u^2}{2}}.$$

Direction for examples 3 and 4: The motion of a particle is defined as $s = 2t^3 - 6t^2 + 15$, where s is in metres and t is in seconds.

Example 3: The acceleration when the velocity is zero is

(A) 2 m/s^2 (B) 8 m/s^2

(C) 6 m/s^2 (D) 4 m/s^2

Solution:

$$s = 2t^3 - 6t^2 + 15$$
$$\frac{ds}{dt} = 6t^2 - 12t$$
$$a = \frac{ds^2}{dt^2} = 12t - 12$$

When velocity is zero, $6t^2 - 12t = 0$,

$\therefore\ t = 2$ sec

Then acceleration is, $a = 12 \times 2 - 12 = 12$ m/s^2

Example 4: The minimum velocity is

(A) −2 m/s (B) 6 m/s
(C) −6 m/s (D) 2 m/s

Solution:

Velocity is minimum when $\frac{dv}{dt} = 0$, i.e., when $12t - 12 = 0$,

$\therefore\ t = 1$ sec

$(\text{Velocity})_{\text{min}} = 6t^2 - 12t = 6 - 12 = -6$ m/s

Example 5: The velocity of a particle along the x-axis is given by $v = 5x^{3/2}$ where x is in metres and v is in m/s.

The acceleration when $x = 2$m is

(A) 300 m/s^2 (B) 200 m/s^2
(C) 180 m/s^2 (D) 150 m/s^2

Solution:

Given $v = 5x^{3/2}$, differentiating with respect to t, we have

$$\frac{dv}{dt} = 5 \times \frac{3}{2} x^{3/2-1}\left(\frac{dx}{dt}\right)$$

$$= \frac{15}{2} x^{1/2} \frac{dx}{dt},\ but\ \frac{dx}{dt} = v$$

$$\therefore\quad a = \frac{15}{2} x^{1/2} \times 5x^{3/2} = \frac{75}{2} x^2$$

When $x = 2$, $a = \frac{75}{2} \times 4 = 150$ m/s^2.

Example 6: A particle is moving in a straight line starting from rest. Its acceleration is given by the expression $a = 50 - 36t^2$, where t is in seconds. The velocity of the particle when it has travelled 52 m can be

(A) 2.3 m/s (B) 4 m/s
(C) 6.7 m/s (D) 8 m/s

Solution:

Given,

$$a = 50 - 36t^2$$

So, $\frac{dv}{dt} = 50 - 36t^2$

Or, $dv = 50dt - 36t^2\, dt$

Integrating the above equation, we have

$$v = 50t - 36\frac{t^3}{3} + C = 50t - 12t^3 + C$$

When $t = 0$, $v = 0$

$\therefore\ C = 0$

$\therefore\ v = 50t - 12t^3$

$\frac{ds}{dt} = 50t - 12t^3$

Integrating, $s = 50\frac{t^2}{2} - 12\frac{t^4}{4} + C_1$

$= 25t^2 - 3t^4 + C_1$

When $t = 0$, $s = 0$

$\therefore\ C_1 = 0$

$s = 25t^2 - 3t^4$

Here we can find the time when s = 52 m.

$\therefore\ 25t^2 - 3t^4 = 52$

Let $t^2 = u$, then $25u - 3u^2 = 52$

$$3u^2 - 25u + 52 = 0$$

$$u = \frac{25 \pm \sqrt{625 - 624}}{6}$$

$$u = \frac{25 \pm 1}{6} = \frac{26}{6} \text{ or } \frac{24}{6}$$

Case 1: when $t^2 = \frac{24}{6} = 4$

$\therefore\ t = 2$ sec

$v = 50t - 12t^3$

$= 50 \times 2 - 12 \times 8$

$= 100 - 96 = 4$ m/s

Case 2: when $t^2 = \frac{26}{6} = 4.333$

$\therefore\ t = 2.08$ sec

The value of the velocity calculated with this t value is not available in the options provided.

Example 7: A body dropped from a certain height covers $\frac{5}{9}$th of the total height in the last second, the height from which the body is dropped is

(A) 36.8 m (B) 40.3 m
(C) 44.1 m (D) 50.6 m

Solution:

Let 'h' be the height and let 'n' be the time taken for the fall. Then,

$$s = u + a\left(n - \frac{1}{2}\right)$$

$$\frac{5}{9}h = 0 + g\left(n - \frac{1}{2}\right)$$

$$\frac{5}{9}h = g\left(n - \frac{1}{2}\right) \quad (1)$$

Also, $h = un + \frac{1}{2}\ an^2$

$$h = 0 + \frac{1}{2}\ gn^2 \quad (2)$$

Putting (2) in (1),

$$\frac{5}{9}\times\frac{1}{2}gn^2 = g\left(n-\frac{1}{2}\right)$$

$\therefore\ 5n^2 - 18n + 9 = 0$

$5n^2 - 15n - 3n + 9 = 0$

$5n(n-3) - 3(n-3) = 0$

$\therefore\ (5n-3)(n-3) = 0$

$\therefore\ n = \frac{3}{5}$ or $n = 3$, but $n > 1$

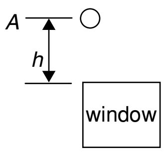
$\therefore\ n = 3$

$\therefore\ h = \frac{1}{2}\ gn^2 = \frac{1}{2}\ \times 9.81 \times 9 = 44.1$ m.

Example 8: A stone falls past a window 2 m high in a time of 0.2 seconds. The height above the window from where the stone has been dropped is

(A) 4.15 m (B) 5.23 m
(C) 5.87 m (D) 6.32 m

Solution:



The stone is dropped from A. Let the body reach the top of the window with a velocity of u m/s. Then,

$$u^2 = 0^2 + 2gh$$

$$u^2 = 2gh \qquad (1)$$

Falling with an initial velocity u, it covers the window 2 m high in 0.5 seconds.

$$s = ut + \frac{1}{2}\ at^2$$

$$2 = u \times 0.2 + \frac{1}{2}\ \times 9.81 \times 0.2^2$$

$$2 = 0.2u + \frac{1}{2}\ \times 9.81 \times 0.04$$

$$2 = 0.2u + 9.81 \times 0.02$$

$$u = 9.019 \text{ m/s}$$

From (1), $u^2 = 2gh$,

$$\therefore\quad h\frac{9.019^2}{2\times 9.81} = 4.145 \text{ m.}$$

Example 9: A ball is projected vertically upwards with a velocity of 49 m/s. If another ball is projected in the same manner after 2 seconds, and if both meet t seconds after the second ball is projected, then t is equal to

(A) 3 s (B) 10 s
(C) 5 s (D) 6 s

Solution:

Let both the balls meet T seconds after the first ball is projected. Therefore when the balls meet, for the first ball,

$$h = 49 \times T - \frac{1}{2}\ gT^2$$

For the second ball,

$$h = 49 \times (T-2) - \frac{1}{2}\ g(T-2)^2$$

Equating, $49T - \frac{1}{2}\ gT^2 = 49\ (T-2) - \frac{1}{2}\ g(T-2)^2$

$\therefore\ T = 11.99$ sec

$\therefore\ t = T - 2 = 9.99$ sec ≈ 10 sec

Example 10: Two bodies are moving uniformly towards each other. The distance between them decreases at a rate of 6 m/s. If both the bodies move in the same direction with the same speeds, then the distance between them increases at a rate of 4 m/s. The respective speeds of the bodies are

(A) 3 m/s and 1 m/s (B) 5 m/s and 1 m/s
(C) 4 m/s and 2 m/s (D) 3 m/s and 5 m/s

Solution:

Let u and v be the velocities of the bodies. From the statement of the problem,

$$u + v = 6$$

$$u - v = 4$$

$\therefore\ u = 5$ m/s and $v = 1$ m/s.

Example 11: Two cars are moving in the same direction each with a speed of 45 km/h. The distance separating them is 10 km. Another vehicle coming from the opposite direction meets these two cars in an interval of 6 minutes. The speed of the vehicle is

(A) 45 km/h (B) 50 km/h
(C) 55 km/h (D) 60 km/h

Solution:

The distance between the cars moves with a velocity of 45 km/h. If the speed of the vehicle is u, then its velocity relative to the moving distance is $45 + u$ m/s.

It takes 6 minutes to cover the distance of 10 km.

$$\therefore\quad (45+u)\times\frac{6}{60} = 10$$

$\therefore\ 45 + u = 100$

$u = 55$ km/h.

Motion under Variable Acceleration

In practical conditions a body may very often move with variable acceleration. The rate of change of velocity will not remain constant. We know that acceleration,

$$a = \frac{dv}{dt} = \frac{dv}{ds}\cdot\frac{ds}{dt}$$

Or

$$a = v\cdot\frac{dv}{ds}$$

Also when displacement can be expressed as a third degree or higher degree equation in time, the acceleration becomes a variable with respect to time.

For example, if $s = 4t^3 + 3t^2 + 5t + 1$

$$\frac{ds}{dt} = 12t^2 + 6t + 5$$

$$\frac{d^2s}{dt^2} = 24t + 6$$

The velocity and displacement are evaluated by integration.

Example 12: A body is starting from rest and moving along a straight line whose acceleration is given by $f = 10 - 0.006x^2$ where x is the displacement in m and f is the acceleration in m/s^2. The distance travelled by it when it comes to rest is

(A) 70.7 m (B) 68.3 m
(C) 62.6 m (D) 58.5 m

Solution:
Given that $f = 10 - 0.006x^2$

$$\frac{dv}{dt} = 10 - 0.006x^2$$

$$\frac{dv}{dx}\cdot\frac{dx}{dt} = 10 - 0.006x^2$$

$$v\cdot\frac{dv}{dx} = 10 - 0.066x^2$$

$vdv = (10 - 0.006x^2)dx$ integrating

$$\frac{v^2}{v} = 10x - 0.006\frac{x^3}{3} + C$$

when $x = 0$, $v = 0$
$\therefore\ C = 0$

$$\frac{v^2}{2} = 10x - 0.006\frac{x^3}{3}$$

$$v^2 = 20x - 0.004x^3$$

when $v = 0$; $20x - 0.004x^3 = 0$
$\therefore$ $0.004x^2 = 20$ (note that the solution of $x = 0$ is also possible for the above equation, but the value of $x > 0$ is sought for) $\therefore$ $x = 70.7$ m.

Direction for examples 13 and 14: An electric train starting from rest has an acceleration f in m/s^2. which vary with time as shown in the table below:

t(secs)	0	6	12	18
$f\left(\frac{m}{s^2}\right)$	12	10	9.5	8

Example 13: The velocity at the end of the first 6 seconds is
(A) 18 m/s (B) 27 m/s
(C) 43 m/s (D) 66 m/s

Solution:
During the first 6 seconds, the average acceleration $= \frac{12+10}{2} = 11$ m/s^2

$\therefore$ Increase in velocity during this interval of 6 seconds = average acceleration $\times$ 6 = 66 m/s

$\therefore$ Velocity at the end of 6 second = 66 m/s.

Example 14: The distance travelled during these six seconds is
(A) 242 m (B) 218 m
(C) 198 m (D) 124 m

Solution:
Average velocity during this interval

$$= \frac{0+66}{2} = 33 \text{ m/s}$$

$\therefore$ Distance travelled during this interval = 33 $\times$ 6 = 198 m.

Example 15: At any instant, the acceleration of a train starting from rest is given by $f = \frac{10}{u+1}$ where u is the velocity of the train in m/s. The distance at which the train will attain a velocity of 54 km/h is
(A) 123.7 m (B) 185.4 m
(C) 214.4 m (D) 228.2 m

Solution:
It is given $f = \frac{10}{u+1}$

$$u\cdot\frac{du}{dx} = \frac{10}{u+1}$$

$$u(u+1)du = 10dx$$

Integrating we have, $\frac{u^3}{3} + \frac{u^2}{2} = 10x + c$

when $x = 0$, $u = 0$.
$\therefore$ $c = 0$

$$\frac{u^3}{3} + \frac{u^2}{2} = 10x$$

when u = 54 km/h = 54 $\times$ 5/18 = 15 m/s

$$\frac{15^3}{3} + \frac{15^2}{2} = 10x$$

$$1125 + 112.5 = 10x$$

$$\therefore\ x = 123.7 \text{ m.}$$

Example 16: The motion of a particle is given by the equation $a = t^3 - 3t^2 + 5$, where 'a' is acceleration in m/s^2 and t is time in seconds. It is seen that the velocity and displacement of the particle at 't' = 1 sec are 6.25 m/s and 8.3 m respectively. Then the displacement at time t = 2 sec is
(A) 17.3 m (B) 15.6 m
(C) 14.8 m (D) 12.6 m

Solution:

Given $a = t^3 - 3t^2 + 5$

$$\frac{dv}{dt} = t^3 - 3t^2 + 5$$

Integrating, $v = \frac{t^4}{4} - 3\frac{t^3}{3} + 5t + c$ at $t = 1$ sec, $v = 6.25$ m/s i.e.,

$$6.25 = \frac{1}{4} - 1 + 5 + c$$
$$= 4.25 + c$$

$\therefore\ c = 2$

$$\therefore v = \frac{t^4}{4} - t^3 + 5t + 2$$

$$\frac{ds}{dt} = \frac{t^4}{4} - t^3 + 5t + 2$$

Integrating, $s = \frac{t^5}{20} - \frac{t^4}{4} + 5\cdot\frac{t^2}{2} + 2t + c$, at $t = 1$, $s = 8.3$ m

$$8.3 = \frac{1}{20} - \frac{1}{4} + \frac{5}{2} + 2 + c,$$

$$8.3 = \frac{1}{20} + 4.25 + c,$$

$c = 8.3 - 4.25 - 0.05 = 4.05 - 0.05 = 4$

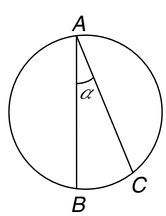
$$s = \frac{t^5}{20} - \frac{t^4}{4} + 5\cdot\frac{t^2}{2} + 2t + 4$$

s at $t = 2$ sec is

$$s = \frac{32}{20} - \frac{16}{4} + 10 + 4 + 4$$

$$= \frac{32}{20} + 14 = 15.6 \text{ m.}$$

Example 17: In the figure shown, AB is the diameter 'd' of the circle and AC is the chord of the same circle?



Making an angle α with AB. Two particles are dropped from rest one along AB and the other along AC. If t_1 is the time taken by the particle to slide along AB and t_2 is the time taken to slide along AC, then t_1: t_2 is

(A) 1:cosα (B) 1:secα
(C) 1:1 (D) 1:15

Solution:

Let $AB = l$, AC = l cosα

Consider sliding along AC, acceleration is g cosα

We have,

$$s = ut + \frac{1}{2}at^2$$

Now,

$$l\cos\alpha = 0 + \frac{1}{2}g\cos\alpha t_2^2$$

$$\therefore t_2^2 = \frac{2l}{g} \quad \text{or} \quad t_2 = \sqrt{\frac{2l}{g}}$$

Consider sliding along AB,

$$l = 0 + \frac{1}{2}gt_1^2$$

$$t_1 = \sqrt{\frac{2l}{g}}$$

$$\therefore\ t_1 : t_2 = 1:1.$$

Relative Velocity

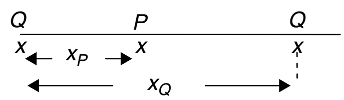
The motion of one body with respect to another moving body is known as relative motion.

Take the case of two bodies P and Q moving along the same straight line. The position of the bodies is specified with reference to an origin O.

x_P and x_Q are measured from the origin O. The difference $x_Q - x_P$ defines the relative position of Q with respect to P. It is denoted as

$$x_{Q/P} = x_Q - x_P$$
$$\therefore x_Q = x_P + x_{Q/P}$$

Consider the rate of change of displacement, then

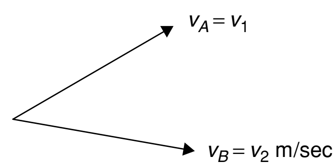


$$v_{Q/P} = v_Q - v_P$$
$$\therefore v_Q = v_P + v_{Q/P}$$

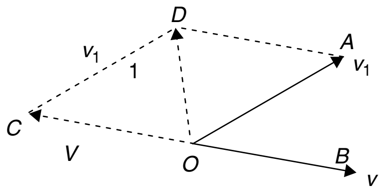
Similar relations hold good for acceleration also, i.e.,

$$\therefore a_Q = a_P + a_{Q/P}$$

Working rule: Let two particles A and B move with velocities v_1 m/s and v_2 m/s respectively in directions as shown in the following figure.



If we want to find out the velocity of A relative to B, the velocity of B is to be made zero. For that we provide velocity v_2 in the reverse direction of OB and find the vector sum with $v_1 = OA$.



The vector $\overrightarrow{OD}$ gives both the magnitude and direction of the velocity of A relative to B.

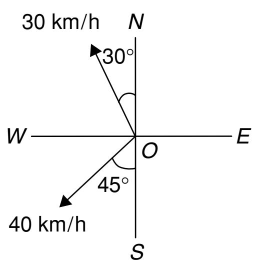
Another method is to resolve their velocities into their components with sign. Then evaluate the relative velocity in the x-direction and in the y-direction. Find their resultant vector. This vector will be the relative velocity, both in magnitude and in direction.

Example 18: Two boats start from a point at the same time. The boat A has a velocity of 30 km/h and move in the direction N 30° W. The boat moves in the south west direction with a velocity of 40 km/h. The distance between the boats after half an hour is

(A) 27.9 km (B) 32.3 km
(C) 36.7 km (D) 42.3 km

Solution:

Method 1:



Resolving along the x-axis,

$(v_A)_x = -(30\sin 30°)\vec{i}$ for A and

$(v_B)_x = -(40\cos 45°)\vec{i}$ for B, where $\vec{i}$ is a unit vector along the x-axis.

$$(v_{A/B})_x = (v_A)_x - (v_B)_x$$
$$= -(30\sin 30°)\vec{i} - (-40\cos 45°)\vec{i}$$
$$= \left(\frac{40}{\sqrt{2}} - 15\right)\vec{i}$$
$$= 13.28\vec{i} \text{ km/h}$$

Similarly, $(v_{A/B})_y = (v_A)_y - (v_B)_y$

$$= (30\cos 30°)\vec{j} - (-40\sin 45°)\vec{j},$$

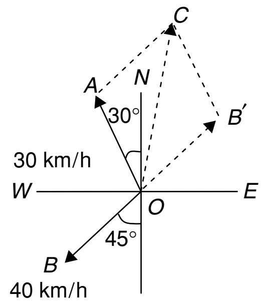
Where $\vec{j}$ is a unit vector along the y-axis.

$$= \left(\frac{40}{\sqrt{2}} + 30\frac{\sqrt{3}}{2}\right)\vec{j} = 54.26\vec{j} \text{ km/h}$$

$$v_{A/B} = \sqrt{54.26^2 + 13.28^2} = 55.86 \text{ km/h}$$

The relative distance after half an hour $= 55.86 \times \frac{1}{2} = 27.9$ km.

Method II:



The vector $\overrightarrow{OC}$ is the resultant velocity vector. Velocity of B is reversed and considered. Therefore the resultant is the velocity of A relative to B.

$$\overrightarrow{OC} = \sqrt{40^2 + 30^2 + 2 \times 40 \times 30 \times \cos 75°}$$
$$= \sqrt{1600 + 900 + 40 \times 60 \times 0.258} = 55.86 \text{ m/s}$$

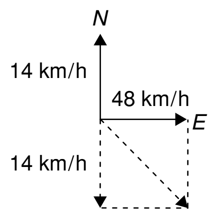
Relative distance after half and hour

$$= 55.86 \times \frac{1}{2} = 27.9 \text{ km.}$$

Example 19: A vessel which can steam in still water with a velocity of 48 km/h is steaming with its bow pointing due east. It is carried by a current which flows northward with a speed of 14 km/h. The distance it would travel in 12 minutes is

(A) 14 km (B) 12 km
(C) 10 km (D) 8 km

Solution:



To find the velocity of the steamer relative to the flow, the flow velocity is reversed and vector sum is found.

$$\text{Relative velocity } = \sqrt{48^2 + 14^2} = 50 \text{ km/h}$$

$$\text{Distance after 12 minutes } = 50 \times \frac{12}{60} = 10 \text{ km.}$$

Example 20: A man keeps his boat at right angles to the current and rows across a stream 0.25 km broad. He reaches the opposite bank 0.125 km below the point opposite to the starting point. If the speed of the boat in rowing alone is 6 km/ph, the speed of the current is

(A) 5 km/h (B) 4 km/h
(C) 3 km/h (D) 2 km/h

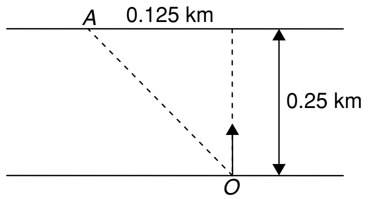
Solution:

The speed responsible for reaching the opposite side is the rowing velocity 6 km/h. Due to the velocity of the current by the time the boat can cross the stream with its absolute velocity, the boat flows down 0.125 km due to the speed of the current.

Time for crossing $= \frac{0.25}{6} = 0.04166$ hr

Let the stream velocity be v m/s.

$\therefore$ Resultant speed $= \sqrt{v^2 + 6^2}$



The distance covered by the boat within this time is

$$OA = \sqrt{0.25^2 + 0.125^2}$$

$$\therefore 0.04166 \times \sqrt{v^2 + 6^2} = \sqrt{0.25^2 + 0.125^2}$$

$$\therefore v = 3 \text{ km/h.}$$

Example 21: A boat of weight 45 kg is initially at rest. A boy of 32 kg is standing on it. If he jumps horizontally with a speed of 2 m/s relative to the boat, the speed of the boat is

(A) 2 m/s (B) 3.42 m/s
(C) 4.92 m/s (D) 5.36 m/s

Solution:

Given $v_{A/B} = 2$ m/s

It is relative velocity of the boy with respect to the boat.

$$v_{A/B} = v_A - v_B$$
$$2 = v_A - v_B$$
$$\therefore v_A = 2 + v_B$$

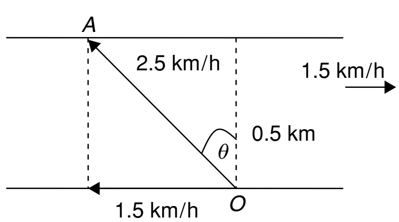
By conservation of momentum,

$$0 = 32\,(2 + v_B) - 45\,v_B = 64 - 13\,v_B$$
$$\therefore v_B = 4.92 \text{ m/s.}$$

Example 22: A stream of water flows with velocity of 1.5 km/h. A swimmer swims in still water with a velocity of 2.5 km/h. If the breadth of the stream is 0.5 km, the direction in which the swimmer should swim so that he can cross the stream perpendicularly is

(A) 26° with the vertical
(B) 29.4° with the vertical
(C) 32.5° with the vertical
(D) 36.8° with the vertical

Solution:



The swimmer must swim in the direction OA with velocity 2.5 m/s so that he can cross the stream at right angles.

From geometry 2.5 $\sin\theta = 1.5$

$$\therefore \sin\theta = \frac{1.5}{2.5} = 0.6$$

$$\theta = 36.8°.$$

Example 23: An aeroplane is flying in a horizontal direction with a velocity of 1800 km/h. At a height of 1960 metres, when it is above a point A on the ground, a body is dropped from it. If the body strikes the ground at point B, then the distance AB is

(A) 18 km (B) 15 km
(C) 10 km (D) 8 km

Solution:

The time taken by the body to fall down the distance 1960 m is

$$h = \frac{1}{2} g t^2$$

$$1960 = \frac{1}{2} 9.8\, t^2$$

$$\frac{2 \times 1960}{9.8} = t^2$$

$$400 = t^2;\ t = 20 \text{ sec}$$

$$AB = v \times t = \frac{1800}{60 \times 60} \times 20 = 10 \text{ km}$$

Example 24: Two ships leave the port at the same time. The first ship A steams north-west at 32 km/h and the second ship B 40° south of west at 24 km/h. The time after which they will be 160 km apart is

(A) 2.15 hrs (B) 2.86 hrs
(C) 3.46 hrs (D) 4.19 hrs

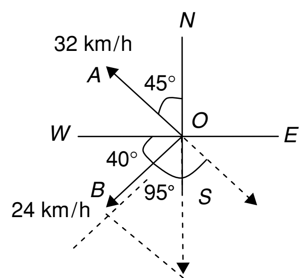
Solution:

Let us find the velocity of the second ship relative to the first. For that consider the velocity of the first ship in the reverse direction and evaluate the vector sum of the velocities.

Resultant or velocity of B relative to A is

$$= \sqrt{24^2 + 32^2 + 2 \times 32 \times 24 \cos 95°}$$

$$= \sqrt{1466} = 38.3 \text{ km/h}$$



Time for two ships to be 160 km apart $= \frac{160}{38.3} = 4.19$ hrs.

Example 25: A particle is accelerated from (1, 2, 3), where it is at rest, according to the equation $a = 6t\vec{i} - 24t^2\vec{j} + 10\vec{k}$ m/s^2, where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors along the x, y and z axes. The magnitude of the displacement after the lapse of 1 second is

(A) 5 m (B) $\sqrt{30}$ m
(C) 6 m (D) $\sqrt{47}$ m

Solution:
It is given that $a = 6t\vec{i} - 24t^2\vec{j} + 10\vec{k}$

$\therefore v = 3t^2\vec{i} - 8t^3\vec{j} + 10t\vec{k} + c$

when $t = 0$, $v = 0$

$\therefore c = 0$

$$\therefore v = 3t^2\vec{i} - 8t^3\vec{j} + 10t\vec{k}$$
$$\frac{dx}{dt} = 3t^2\vec{i} - 8t^3\vec{j} + 10t\vec{k}$$
$$x = 3\frac{t^3}{3}\vec{i} - 8\frac{t^4}{4}\vec{j} + 10\frac{t^2}{2}\vec{k} + C$$
$$x = t^3\vec{i} - 2t^4\vec{j} + 5t^2\vec{k} + C$$

when $t = 0$, position of the particle is at (1, 2, 3) i.e., at $t = 0$, $x = 1\vec{i} + 2\vec{j} + 3\vec{k}$

$$\therefore C = 1\vec{i} + 2\vec{j} + 3\vec{k}$$
$$\therefore x = t^3\vec{i} - 2t^4\vec{j} + 5t^2\vec{k} + 1\vec{i} + 2\vec{j} + 3\vec{k}$$
$$= (t^3 + 1)\vec{i} - (2 - 2t^4)\vec{j} + (3 + 5t^2)\vec{k}$$

When $t = 1$,

$$x = 2\vec{i} + 8\vec{k}$$

$\therefore$ Displacement vector

$$= 2\vec{i} + 8\vec{k} - (1\vec{i} + 2\vec{j} + 3\vec{k}) = 1\vec{i} - 2\vec{j} + 5\vec{k}$$

Magnitude of the displacement vector

$$= \sqrt{1 + 4 + 25} = \sqrt{30} \text{ m.}$$

Example 26: If a particle, moving with uniform acceleration, travels the distances of 8 and 9 cms in the 5th sec and 9th sec respectively, then its acceleration will be

(A) 1 cm/s^2 (B) 5 cm/s^2
(C) 25 cm/s^2 (D) 0.5 cm/s^2

Solution:

$$s \text{ in the } n\text{th sec } = u + \frac{a}{2}(2n - 1)$$
$$8 = u + \frac{a}{2}(2 \times 5 - 1) = u + 4.5a \quad (1)$$
$$9 = u + \frac{a}{2}(2 \times 9 - 1) = u + 8.5a \quad (2)$$

Subtracting Eq. (1) from Eq. (2),

$$1 = 4a \quad \text{or} \quad a = 0.25 \text{ cm/s}^2.$$

Example 27: The acceleration due to gravity on a planet is 200 cm/s^2. If it is safe to jump from a height of 2 m on earth, then the corresponding safe height on the planet is

(A) 2 m (B) 9.8 m
(C) 10 m (D) 8 m

Solution:
Let h_{se} and h_{sp} denote the safe heights on the earth and the planet.

On the earth, $v^2 = 2gh_{se} = 2 \times 9.8 \times 2$
$= 39.2$ m^2/s^2

On the planet, $v^2 = 2 \times 2 \times h_{sp}$

For a safe jump the final velocity (v) should be same on earth and the planet, hence, $2 \times 2 \times h_{sp} = 39.2$

$\therefore h_{sp} = 9.8$ m.

Example 28: A ball weighing 500 gm is thrown vertically upwards with a velocity of 980 cm/s. The time that the ball will take to return back to earth would be

(A) 1 s (B) 2 s
(C) 3 s (D) 4 s

Solution:
For the upward journey, $u = u_0 - gt$

$$0 = 980 \times 10^{-2} - 9.8\,t$$
$$\Rightarrow \quad t = 1\,s$$
$$v^2 - u^2 = 2gs \quad \Rightarrow \quad 0 - 9.8^2 = -2 \times 9.8\,s$$
$$s = 4.9 \text{ m}$$

For the downward journey,

$$s = ut + \frac{1}{2}gt^2$$
$$4.9 = 0 + \frac{1}{2} \times 9.8t^2$$
$$t = 1\,s$$

Total time taken to return back to earth = 1 + 1 = 2 s.

Kinetics of a Particle

Kinetics can be used to predict a particle's motion, given a set of forces (acting upon the particle) or to determine the forces necessary to produce a particular motion of the particle. Kinetics of the rectilinear motion of a particle are governed mainly by Newton's three laws of motion.

Newton's first law: Every body continues in its state of rest or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed upon it. This law is sometimes called as the law of inertia.

From Newton's first law, it follows that any change in the velocity of a particle is the result of a force. The question, of the relationship between this change in the velocity of the particle and the force that produces it, is answered by the second law of motion which is as follows.

Newton's second law: The acceleration of a given particle is proportional to the force applied to it and takes place in the direction of the straight line in which the force acts.

Newton's third law: Every action there is always an equal and opposite reaction, or the forces of two bodies on each other are always equal and directed in opposite directions.

General Equation of Motion for a Particle

From Newton's second law, the relationship between the acceleration 'a' produced in a body of mass 'm' (mass is always assumed to be invariant with time) by a resultant, 'F', of all the forces acting on the body can be derived as follows: $F = ma$, which is the general equation of motion for a particle.

For a stationary body lying on a surface (body with no motion), there is a force (F) exerted by the body on the surface which is equal to the weight of the body (W), i.e., $F = W = mg$, where 'm' is the mass of the body and 'g' is the acceleration due to gravity. There is an equal and opposite force exerted by the surface on the body (consequence of Newton's third law). Note that the weight of a body is obtained by multiplying the mass of the body by the acceleration due to gravity.

Differential Equation of Rectilinear Motion

The general equation of motion for a particle can be applied directly to the case of the rectilinear translation of a rigid body, since all the particles of the rigid body have the same velocity and acceleration (same motion) where the particles move in parallel straight lines. Here, the rigid body is considered as a particle concentrated at the center of gravity of the rigid body.

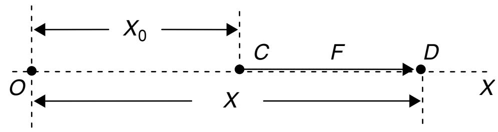
Whenever such a body or particle moves under the action of a force applied at its center of gravity and having a fixed line of action, acceleration of the body is produced in the same direction, and if any initial velocity of the body is also directed along this line, then the motion corresponding to this case is known as rectilinear translation.

If the line of motion of a particle is taken to be along the x-axis (i.e., displacement at a time t is denoted by x), $\ddot{x} = \dfrac{d^2x}{dt^2}$ represents the acceleration and F represents the resultant force acting, then the differential equation of the rectilinear motion of the particle is given by $F = m\ddot{x}$.

Two types of problems that can be solved by the above equation are: (a) Determination of the force necessary to produce a given motion of the particle where the displacement x is given as a function of time t, (b) Determination of the motion of a particle given a force F acting on the particle, i.e., to determine a function relating x and t such that the above equation is satisfied.

Motion of a Particle Acted upon by a Constant Force

A particle, acted upon by a force of constant magnitude and direction, will move rectilinearly in the direction of the force subjected to a constant acceleration. Let us consider a particle moving along the x-axis (see figure) where the initial (at $t = 0$) displacement and velocity of the particle is x_0 and $\dot{x}_0$ respectively.



If F is the magnitude of the constant force acting on the particle, then from the differential equation of rectilinear motion, $\ddot{x} = \dfrac{F}{m} = a$, where a is the constant acceleration produced in the particle due to the constant force. The equation $\ddot{x} = a$ can be written as $\dfrac{d(\dot{x})}{dt} = a$.

Integration of the above equation with the initial value condition, at $t = 0$ $\dot{x} = \dot{x}_0$, gives:

$$\dot{x} = \dot{x}_0 + at$$

Which is the general velocity-time equation for the rectilinear motion of a particle under the action of a constant force F producing the constant acceleration a in the particle. With $\dot{x} = \dfrac{dx}{dt}$, equation (1) can be rewritten as follows: $\dfrac{dx}{dt} = \dot{x}_0 + at$.

Integration of the above equation with the initial value condition, at $t = 0$, $x = x_0$, gives: $x = x_0 + \dot{x}_0 t + \dfrac{1}{2}at^2$, which is the general displacement-time equation for the rectilinear motion of a particle under the action of a constant force F producing the constant acceleration a in the particle.

Freely falling body

The force acting on a freely falling body is the weight of the body (assuming no friction in the motion) and therefore the acceleration produced in the body is the acceleration due to gravity, i.e., $F = W = mg$ and $\therefore\ a = g$. Hence, the velocity-time and displacement-time equations for a freely falling body are respectively as follows:

$$\boxed{\dot{x} = \dot{x}_0 + gt}$$

$$\boxed{x = x_0 + \dot{x}_0 t + \frac{1}{2}gt^2}$$

If the freely falling body starts to fall from a resting position, i.e., it has an initial velocity of zero $(\dot{x}(0) = 0)$ and if the origin of displacement of the body is taken to coincide with the initial position of the body, i.e., it has an initial displacement of zero ($x_0 = 0$), then the above equations reduce to:

$$\boxed{\dot{x} = gt}$$

$$\boxed{x = \frac{1}{2}gt^2}$$

Force as a Function of Time

If the force acting on the particle is a function of time t, i.e., the acting force = $F(t)$, then the acceleration $a(t)$, velocity $\dot{x}(t)$ and displacement $x(t)$ of the particle at time t (with initial time, $t = 0$) is given by the following respective equations.

$$a(t) = \frac{F(t)}{m}$$

$$\dot{x}(t) = \int_0^t a(t)\,dt$$

$$x(t) = \int_0^t \dot{x}(t)\,dt$$

Dynamics of a Particle

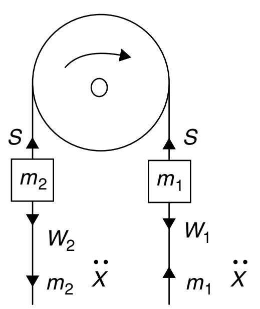
D'Alembert's Principle

Let ΣF_i, where F_i denotes the ith force, be the resultant of a set of forces acting on a particle in the x-axis direction. From the 0 differential equation of the rectilinear motion of a particle, we have

$$\sum F_i - m\ddot{x} = 0 \text{ or}$$
$$\sum F_i + (-m\ddot{x}) = 0$$

From the above equation, it can be seen that if a fictitious force $(-m\ddot{x})$ is added to the system of forces acting on the particle, then an equation resembling equilibrium is obtained. The force $(-m\ddot{x})$ which has the same magnitude as $m\ddot{x}$ but opposite in direction is called as the inertia force. Hence, it can be seen that if an inertia force is added to the system of forces acting on a particle, then the particle is brought into an equilibrium state called as dynamic equilibrium. This is called as D'Alembert's principle. The above equation thus represents the equation of dynamic equilibrium for the rectilinear translation of a rigid body.

Let us consider, now any system of particles connected between them and so constrained that each particle can have only a rectilinear motion. To exemplify such a system, the case of two weights W_1 and W_2 attached to the ends of a flexible but inextensible string overhanging a pulley, as shown in the figure below, is considered.



The inertia of the pulley and the friction on its axle are assumed to be negligible. If the motion of the system is assumed to be in the direction as shown by the arrow on the pulley, an upward acceleration $\bar{x}$ of the weight W_2 and a downward acceleration $\bar{x}$ of the weight W_1 is obtained. The inertia forces acting on the corresponding weights are shown in the above figure.

By adding the inertia forces to the real forces such as (W_1 and W_2 and the string reactions S), a system of forces in equilibrium is obtained for each particle. Hence, the entire system of forces can be concluded to be in equilibrium.

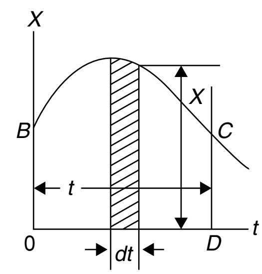
An equation of equilibrium can be written for the entire system (instead of separate equilibrium equations for the individual weights) by equating to zero the algebraic sum of moments of all the forces (including the inertia forces) with respect to the axis of the pulley or by using the principle of virtual work. In either case, the internal forces 'S' of the system need not be considered and the following equation of equilibrium can be obtained for the entire system.

$$\boxed{W_2 + m_2\ddot{x} = W_1 - m_1\ddot{x} \quad or \quad \ddot{x} = \left(\frac{W_1 - W_2}{W_1 + W_2}\right)g}$$

Momentum and Impulse

The differential equation of the rectilinear motion of a particle may be written as

$$\boxed{m\frac{d\dot{x}}{dt} = F \quad \text{or} \quad d(m\dot{x}) = Fdt} \qquad (1)$$

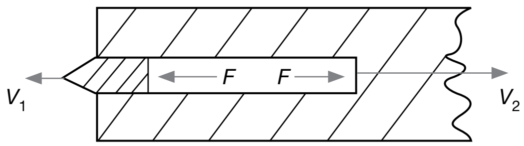


It is assumed that the force 'F' is known as a function of time and is given by the force-time diagram as shown in the above figure. The right hand side of equation (1) is then represented by the area of the shaded elemental strip of height F and width dt in the force-time diagram. This quantity is called as the impulse of the force F in the time interval dt. The expression $m\dot{x}$ on the left hand side of the equation is called as the momentum of the particle. The equation states that the differential change of the momentum of the particle during the time interval dt is equal to the impulse of the acting force during the same time interval. Impulse and momentum have the same dimensions of the product of mass and velocity.

Integrating equation (1), we get $\boxed{m\dot{x} - m\dot{x}_0 = \int_0^t Fdt}$,

where $\dot{x}_0$ is the velocity of the particle at time $t = 0$.

Thus the total change in the momentum of a particle during a finite time interval is equal to the impulse of the acting force during the same time interval. This impulse is represented by the area *OBCD* of the force-time diagram. The equation of momentum-impulse is particularly useful when dealing with a system of particles, since in such cases the calculation of the impulse can often be eliminated. As a specific example, consider the case of a gun and shell as shown in the figure below, which may be considered



as a system of two particles. During the extremely short interval of explosion, the forces 'F' acting on the shell and gun and representing the gas pressure in the barrel are varying in an unknown manner and a calculation of the impulses of these forces would be extremely difficult.

However, the relation between the velocity of the shell and velocity of recoil of the gun can be obtained without calculation of the impulse. Since the forces 'F' are in the nature of action and reaction between the shell and gun, they must at all times be equal and opposite, and hence their impulses for the interval of explosion are equal and opposite since the forces act exactly the same time 't'.

Let m_1 and m_2 be the masses of the shell and gun respectively. If the initial velocities of the shell and gun are assumed to be zero and if the external forces are neglected, then

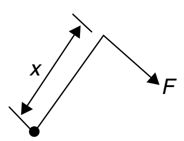
$$\boxed{m_1 v_1 = m_2 v_2, \text{ i.e., } \frac{v_2}{v_1} = \frac{m_1}{m_2}}$$

The velocities of the shell and gun after discharge are in opposite directions and inversely proportional to the corresponding masses. Internal forces in a system of particles always appear as pairs of equal and opposite forces and need not be considered when applying the equation of momentum and impulse. Thus it may be stated that, for a system of particles on which no external forces are applied, the momentum of the system remains unchanged, since the total impulse is zero. This is sometimes called as the principle of conservation of momentum.

Moment and Couple

Moment or moment of a force is the turning effect caused by the force. It is the force acting at a perpendicular distance d

Moment of a force = Force × Perpendicular distance.

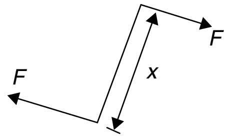


Moment $= F \times x$.

Couple

Tow equal and opposite forces with separate lines of action present in a system of forces constitute a couple. Both forces create their own moment of force. The net moment of the couple is independent of the location of the point considered.

Moment of couple = Force × Perpendicular distance between the forces.



Moment of couple $= F \cdot x$

- Moment is the measure of the turning effect produced by a force about a point. Couple consists of two forces, equal and opposite, acting in two different but parallel lines of action.
- Moment of a couple is independent of the location of the pivot or point considered.

Work and Energy

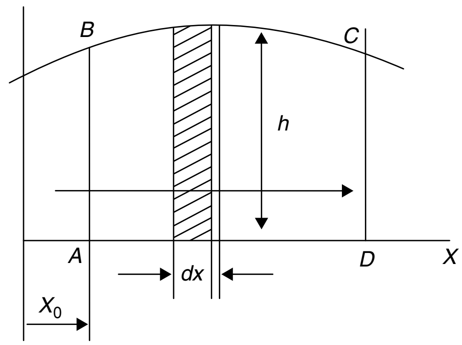
The differential equation of the rectilinear motion of a particle can be written in the following form:

$$\boxed{m\frac{d\dot{x}}{dt} = F}$$

Multiplying both sides of the above equation by $\dot{x}$ and with suitable modifications, the above equation can be written as follows:

$$\boxed{d\left(\frac{m\dot{x}^2}{2}\right) = F dx} \qquad (2)$$

It is assumed that the force F is known as a function of the displacement x of the particle and is represented by the below force-displacement diagram.



The right hand side of equation (2) is represented by the area of the elemental strip of the height h and width dx in the above figure. This quantity represents the work done by the force F on the infinitesimal displacement dx, and the expression in the parenthesis on the left hand side of equation (2) is called the as kinetic energy of the particle. Equation (2) thus states that the differential change in the kinetic energy of a moving particle is equal to the work done by the acting force on the corresponding infinitesimal displacement dx. Work and kinetic energy have the same dimensions of the product of force and length. They are usually expressed in the unit of Joules (J).

Integrating equation (2), with the assumption that the velocity of the particle is $\dot{x}_o$ when the displacement is x_0, we have

$$\boxed{\frac{m\dot{x}^2}{2} - \frac{m\dot{x}_0^{\,2}}{2} = \int_{x_0}^{x} F dx} \qquad (3)$$

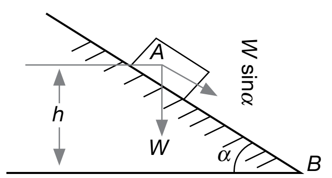
The definite integral on the right hand side of the above equation is represented by the area *ABCD* of the force-displacement diagram. This is the total work of the force F on the finite displacement of the particle from x_0 to x. The work of a force is considered positive if the force acts in the direction of the displacement and negative if it acts in the opposite direction. The total change in the kinetic energy of a particle during a displacement from x_0 to x is equal to the work of the acting force on the displacement.

The equation of work and energy is especially useful in cases where the acting force is a function of displacement and where the velocity of the particle as a function of displacement is of interest. For example, the velocity with which a weight W falling from a height h strikes the ground is to be determined. In this case, the acting force $F = W$ and the total work is Wh. Thus if the body starts from rest, the initial velocity $\dot{x}_0 = 0$ and hence equation (3) becomes

$$\frac{m\dot{x}^2}{2} = Wh \qquad (4)$$

Which yields $\boxed{\dot{x} = v = \sqrt{2gh}.}$

Let the same body slide, without friction, along an inclined plane *AB* starting from an elevation h above point *B* as shown in the figure below.



The equation of work and energy can be used to determine the velocity of the body when it reaches point *B*. Here only the component W sin a of the gravity force does work on the displacement and the component perpendicular to the inclined plane is at all times balanced by the reaction of the plane. In short, the resultant of all the forces acting on the body is $F = W\sin\alpha$ in the direction of motion, and this force acts through the distance $\left(\dfrac{h}{\sin\alpha}\right)$. The work of the force acting on the body is $= W\sin\alpha \times \dfrac{h}{\sin\alpha} = Wh$ and hence velocity at the point *B* (derived from equation (4)), $v = \sqrt{2gh}$. Hence, the velocity is the same as that gained in a free fall through the height h.

If (is the coefficient of friction between the block and the inclined plane, then the work of friction has to be considered in equation (3).

In such a case, the resultant acting force, in the direction of motion $F = W\sin\alpha - \mu W\cos\alpha$.

Then through the displacement $\left(\dfrac{h}{\sin\alpha}\right)$ between the points *A* and *B*, the work done is $= Wh - \mu Wh\cot\alpha$ Equation (3) would then yield

$$\boxed{v = \sqrt{2gh(1-\mu\cot(\alpha))}}$$

When $\alpha = \dfrac{\pi}{2}$, the above equation agrees with the velocity equation derived for a freely falling body and when $\mu = 0$, the above equation agrees with the velocity equation derived for the inclined plane motion of the body with no friction. Also from the above equation, it can be noted that to obtain a real value for the velocity, $\mu < \tan\alpha$, otherwise the block would not slide down.

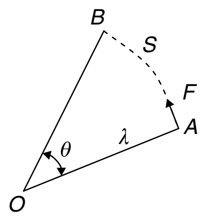
Work done by Torque

Consider a light rod of length l pin joined at one end and is turned by an angle θ by the force F from the position *A* to *B*. Work done by the constant torque is the product of the torque and the angle turned by the rod.

$\therefore$ Work done $= F \cdot s.$

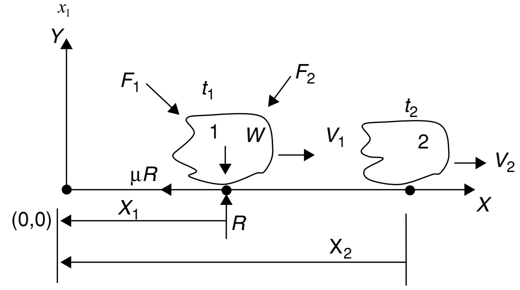
$= F \cdot r \cdot \theta$

$= T \cdot \theta$



Work Energy Formulations

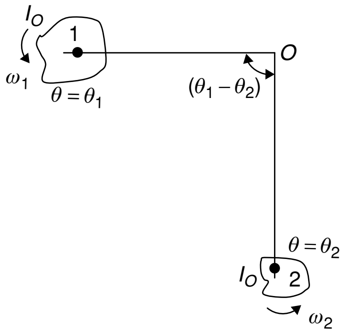
- Kinetic energy of a body/particle in translation $= \dfrac{1}{2}mv^2$.
- Kinetic energy of a body/particle in rotation and rotating about a point $= \dfrac{1}{2}IW^2$.
- Work Energy principle for a body/particle in translation. Work done on body/particle between points 1 and 2 is $W_{1-2} = \displaystyle\int_{x_1}^{x_2} \sum F_x dx.$



Change in kinetic energy from the positions 1 to 2 is $(\Delta K \cdot E)_{1-2} = \dfrac{1}{2}m(v_2^2 - v_1^2)$

$$\therefore W_{1-2} = \int_{x_1}^{x_2} \sum F_x dx = \frac{1}{2} m\left(v_2^2 - v_1^2\right).$$

Work energy principle for a body/particle in rotation.



Work done from 1 to 2 is given by.

$$W_{1-2} = \int_{\theta_1}^{\theta_2} \sum M_O \, d\theta.$$

Change in kinetic energy from 1 to 2 is

$$KE_{1-2} = \frac{1}{2} I_O\left(\omega_1^2 - \omega_2^2\right)$$

$\therefore$ Work done $W_{1-2} = \int_{\theta_1}^{\theta_2} \sum M_O \, d\theta = \frac{1}{2} I_O\left(\omega_1^2 - \omega_2^2\right)$

NOTES

1. Work done by a force is zero if either displacement is zero or the force acts normal to the displacement for example, gravity force does no work when a body moves horizontally.
2. Work done by a force is positive if the direction of force and the direction of displacement are same. For example, work done by force of gravity is positive when a body moves from a higher elevation to a lower elevation. A position work can be said as the work done by a force and negative work as the work done against a force.
3. Work is a scalar quantity and has magnitude but no direction.
4. Work done by a force depends on the path over which the force moves except in the case of conservative forces. Forces due to gravity, spring force are conservative forces, where as friction force is a non-conservative force.

Example 29: If a bucket of water weighing 15 N is pulled up from a well of 25 m depth by a rope weighing 1.5 N/m, then the work done is

(A) 843.75 Nm (B) 500 Nm
(C) 575 Nm (D) 600 Nm

Solution:

The work done to pull the rope

$= \int_0^{25} 1.5 \times (25 - h)\, dh$ (h is the tip of the rope from the bottom of the well)

$$= 1.5 \times \frac{25^2}{2} = 468.75 \text{ Nm}$$

Total work done = Work done to pull the bucket + work done to pull the rope

$= 15 \times 25 + 468.75 = 843.75$ Nm.

Example 30: A uniform chain of length 10 m and mass 100 kg is lying on a smooth table such that one third of its length is hanging vertically down over the edge of the table. If g is the acceleration due to gravity, then the work required to pull the hanging part of the chain is

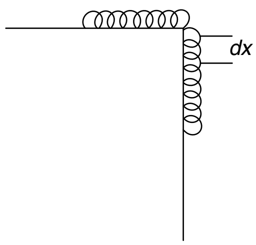
(A) 50g (B) 55.55g (C) 100g (D) 150g

Solution:

Work done = potential energy change in the raising of the centre of mass over the distance $\frac{L}{6}$.

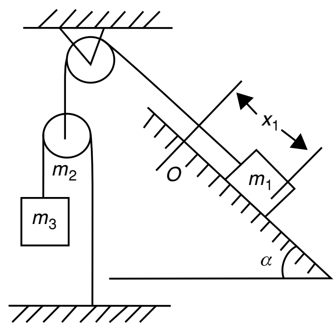
$$= \frac{m}{3} g \frac{L}{6} = \frac{100 \times g \times 10}{18} = \frac{1000g}{18} = 55.55g.$$

Alternate method:



$$W = \int_0^{\frac{L}{3}} \frac{mg}{L} x \, dx = \frac{mg}{L}\left(\frac{x^2}{2}\right)_0^{\frac{L}{3}} = \frac{\text{mg}}{L} \times \frac{L^2}{18} = \text{mg} \times \frac{L}{18}.$$

Ideal Systems: Conservation of Energy



The method of work and energy for a single particle can be extended to apply to a system of connected particles as shown in the above figure. In doing so, it is to be noted that the attention is limited to ideal systems with one degree of freedom. That is, it is assumed that the system has frictionless constraints and inextensible connections and that its configuration can be completely specified by one coordinate such as x_1 in the below figure. In the case shown in the above figure, for example, the assumptions involve a smooth inclined plane, frictionless bearings, inextensible strings and neglecting entirely the rotational inertia of the pulleys. Then the system may be regarded simply as three particles, m_1, m_2 and m_3, each of which performs a rectilinear motion. From kinematics, the displacements and velocities of all the three masses can be expressed in terms of one variable, say the coordinate x_1 of the particle m_1.

During motion of the system, an infinitesimal interval of time dt is considered during which the system changes its configuration slightly and each particle is displaced by a length of dx_i, along its line of motion. If F_i is the resultant force acting on any particle m_i, then the total increment of work of all the forces during such a displacement,

$$\boxed{dU = \sum F_i dx_i} \tag{5}$$

For the system of particles, it can be shown that

$$\boxed{dT = dU} \tag{6}$$

Where $T = \frac{1}{2}\sum(m_i \dot{x}_i^2)$, T is the total kinetic energy of the system of particles with the mass and velocity of the ith particle being m_i and $\dot{x}_i$ respectively. Equation (6) states that the differential change in the total kinetic energy of the system when it changes its configuration slightly is equal to the corresponding increment of work of all forces.

Consider any two configurations of the system denoted by the subscripts A and B, then from equation (6) we have

$$\boxed{\int_{T_A}^{T_B} dT = \int_{x_A}^{x_B} dU \ or}$$

$$\boxed{T_B - T_A = \int_{x_A}^{x_B} dU} \tag{7}$$

This is the equation of work and energy for a system of particles. It states that the total change in the kinetic energy of the system when it moves from configuration A to configuration B is equal to the corresponding work of all the forces acting upon it. In the case of an ideal system, the reactive forces will produce no work and work of all the internal forces which occur in equal and opposite pairs will cancel out each other. Thus for such systems, only the work of active external forces are to be considered on the right hand side of equation (7).

The potential energy of a system in any configuration (A or B) is defined as the work which will be done by the acting forces if the system moves from that configuration (A or B) back to a certain base or reference configuration (O). If V_A and V_B are the potential energies of the system in configurations A and B respectively, then

$$\boxed{V_A = \int_A^0 dU \quad \text{and} \quad V_B = \int_B^0 dU}$$

NOTE

If a particle of weight w is at an elevation x above a chosen datum plane, then the potential energy of the particle, $V = mx$. Similarly for a system of particles at an elevation, the potential energy of the system, $V = \sum w_i x_i = W x_c$,

Where w_i and x_i are the weight and elevation above a chosen datum plane for the ith particle, W is the total weight of the system and x_c is the elevation of the center of gravity of the system above the chosen datum plane.

For the system of particles moving from the configuration A to the configuration B, it can be shown that $T_B + V_B = T_A + V_A$.

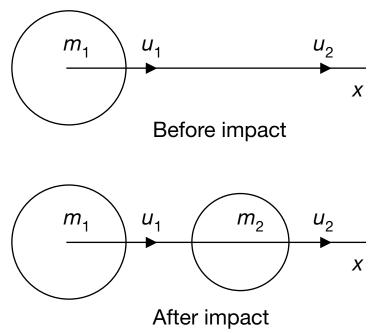
Law of Conservation of Energy

That is, as the system moves from one configuration to another, the total energy (kinetic + potential) remains constant. Kinetic energy may be transformed into potential energy and vice versa but the system as a whole can neither gain nor lose energy. This is the law of conservation of energy as it applies to a system of particles with ideal constraints. Such systems are sometimes called conservative systems.

Impact

The impact between two moving bodies refers to the collision of the two bodies that occurs in a very small time interval and during which the bodies exert a very large force (active and reactive force) on each other. The magnitudes of the forces and the duration of impact depend on the shapes of the bodies, their velocities, and their elastic properties.

Consider the impact of two spheres of masses m_1 and m_2 as shown in the below figure. Let the spheres have the respective velocities of u_1 and u_2, where $u_1 > u_2$, before impact and the respective velocities of v_1 and v_2 after impact.



It is assumed that these velocities are directed along the line joining the centers of the two spheres and are considered to be positive if they are in the positive direction of the x-axis. This is called the case of direct central impact. Two equal and opposite forces, i.e., action and reaction, are produced at the point of contact during impact. In accordance with the law of conservation of momentum, such forces cannot change the momentum of the system of two balls and hence,

$$\boxed{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2} \tag{8}$$

Elastic Impact

In an elastic impact, the momentum and kinetic energy is conserved. If the kinetic energy is conserved during impact, then

$$\boxed{\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2} \qquad (9)$$

Since momentum is conserved, equation (8) is also applicable in this type of impact. From equations (8) and (9), it can be shown that

$$\boxed{v_1-v_2=-(u_1-u_2)} \qquad (10)$$

This equation represents a combination of the law of conservation of momentum and conservation of energy. It states that for an elastic impact the relative velocity after impact has the same magnitude as that before impact but with reversed sign.

For two bodies of equal masses undergoing an elastic impact, from equations (8) and (10) it can be shown that they will exchange their velocities, i.e., $v_1 = u_2$ and $v_2 = u_1$. If the second body was at rest before the impact, i.e., $u_2 = 0$, then it would seem that the striking body stops, i.e., $v_1 = 0$, after having imparted its velocity to the other ball. This phenomenon can be observed in the case of a moving billiard ball which squarely strikes one that was at rest. Again, if the two balls were moving toward each other with equal speeds before impact, an exchange of velocities will simply mean that they rebound from one another with the same speed with which they collided. As another special case, we assume that $m_2 = \infty$ while m_1 remains finite and further $u_2 = 0$. This will represent the case of an elastic impact of a ball against a flat immovable obstruction, such as the dropping of a ball on a cement floor. In this case, it is obtained that $v_1 = -u_1$, i.e., the striking ball rebounds with the same speed with which it hits the obstruction.

Plastic or Inelastic Impact

In a plastic or inelastic impact, the momentum is conserved but the kinetic energy is not (part of the kinetic energy is converted to a different form of energy). In a perfectly plastic impact, the colliding bodies will stick to each other after collision and will move with a common velocity. If v is the common velocity of two colliding bodies after a perfectly plastic impact, then from equation (8), we have

$$\boxed{v=\frac{m_1u_1+m_2u_2}{m_1+m_2}}$$

Newton's experimental law of colliding bodies: Newton proposed an experimental law that describes how the impact of moving bodies was related to their velocities and found that:

$$\frac{\text{Speed of separation}}{\text{Speed of approach}}=\text{e}$$

e = coefficient of restitution

e satisfies the condition $0 \le e \le 1$.

If $e = 1$ $\Rightarrow$ the collision is perfectly elastic.

If $e = 0$ $\Rightarrow$ the collision is inelastic

If $0 < e < 1$ $\Rightarrow$ the collision is said to be elastic.

Energy loss due to impact: The energy lost in impact when $e \neq 1$, i.e., when the collision is not perfectly elastic is given by

$$\text{Loss in kinetic energy} = \frac{1}{2}\frac{m_1m_2}{m_1+m_2}(u_1-u_2)^2(1-e^2).$$

$\therefore$ When $e = 1$ the loss is zero.

Coefficient of restitution: It is defined as the ratio of the relative velocity of the impacting bodies after impact to their relative velocity before impact. The coefficient of restitution 'e' is given by the following equation.

$$\boxed{e=\frac{(v_2-v_1)}{(u_1-u_2)}}$$

Example 31: A bullet travelling with a velocity of 800 m/s and weighing 0.25 N strikes a wooden block of weight 50 Nresting on a horizontal floor. The coefficient of friction between floor and the block is 0.5. Determine the distance through which the block is displaced from its initial position.

Solution: Velocity of the bullet before impact, $v_a = 800$ m/s

Velocity of the block before impact, $v_b = 0$ m/s

$$\text{Mass of the bullet, } m_a = \frac{0.25}{g}\text{ kg}$$

$$\text{Mass of the block, } m_b = \frac{50}{g}\text{ kg}$$

The bullet after striking the block remains buried in the block and both move with a common velocity v.

Applying the principle of conservation of momentum,

$$m_av_a+m_bv_b=(m_a+m_b)v$$

$$\frac{0.25}{g}\times 800+\frac{50}{g}\times 0=\left(\frac{0.25}{g}+\frac{50}{g}\right)v$$

$$v = 3.98\text{ m/s}$$

To find the distance travelled by the block, apply the priniciple of work and energy. Kinetic energy lost by the block with the bullet buried = Work done to overcome the frictional force

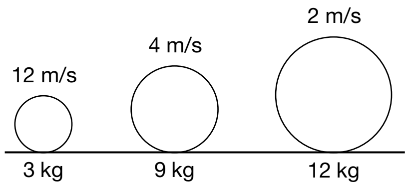
If s is the distance travelled by the block, then

$$\frac{1}{2}(m_a+m_b)v^2=\mu R\, s$$

$$=\mu g(m_a+m_b)\, s \;(\therefore\; R=g(m_a+m_b))$$

$$\therefore s=\frac{3.98^2}{2\times 9.81\times 0.5}=1.61\text{ m}$$

Example 32: Three spherical balls of masses 3 kg, 9 kg and 12 kg are moving in the same direction with velocities 12 m/s, 4 m/s and 2 m/s respectively. If the ball of mass 3 kg impinges with the ball of mass 9 kg which in turn impinges with the ball of mass 12 kg. Prove that the balls of masses 3 kg and 9 kg will be brought to rest by the impacts. Assume the balls to be perfectly elastic.



Solution: For perfectly elastic balls, $e = 1$

$m_a = 3$ kg, $m_b = 9$ kg, $m_c = 12$ kg

Impact of balls A and B: Conservation of momentum gives,

$$m_a v_a + m_b v_b = m_a v'_a + m_b v'_b$$

$$3 \times 12 + 9 \times 4 = 3v'_a + 9v'_b \qquad (1)$$

$$e = -\frac{v'_b - v'_a}{v_b - v_a}$$

$$v'_b - v'_a = e(v_a - v_b) = 1 \times (12 - 4) = 8 \qquad (2)$$

Solving Eqs. (1) and (2), we get $v'_b = 8$ m/s and $v'_a = 0$ m/s, i.e., the ball of mass 3 kg is brought to rest.

Impact of balls B and C: Consider now the impact of the ball B, of mass 9 kg and moving with the initial velocity of 8 m/s, with the ball C, of mass 12 kg and moving with the velocity of 2 m/s.

Conservation of momentum gives

$$m_b v_b + m_c v_c = m_b v'_b + m_c v'_c$$

$$9 \times 8 + 12 \times 2 = 9v'_b + 12v'_c \qquad (3)$$

$$e = -\frac{v'_c - v'_b}{v_c - v_b}$$

$$v'_c - v'_b = e(v_b - v_c)$$

$$= 1 \times (8 - 2) = 6 \qquad (4)$$

Solving Eqs. (3) and (4), we get $v'_c = 6$ m/s and $v'_b = 0$ m/s, i.e., the ball of mass 9 kg is brought to rest.

Direction for questions 33 and 34: The blocks 1 and 2, having a weight of 1 kg each and the respective velocities of 10 m/s and 4 m/s, undergo a perfect inelastic collision.

Example 33: The final velocity of the blocks is

(A) 7 m/sec (B) 6 m/sec
(C) 3 m/sec (D) 4 m/sec

Solution:

$$V = \frac{M_1V_1 + M_2V_2}{m_1 + m_2} = \frac{1\times10 + 4\times1}{1+1} = 7 \text{ m/s.}$$

Example 34: The energy converted into heat as a result of the collision is

(A) 40 J (B) 9 J
(C) 50 J (D) 54 J

Solution:

The original kinetic energy was,

$$K_1 = \frac{1}{2}\times1\times100 + \frac{1}{2}\times1\times16 = 58 \text{ J}$$

The final kinetic energy is,

$$K_2 = \frac{1}{2}\times2\times49 = 49 \text{ J}$$

Loss of Kinetic energy = 58 − 49 = 9 J (converted to heat energy).

Exercises

Practice Problems I

Direction for questions 1 to 10: Select the correct alternative from the given choices.

1. A car starts with an acceleration of 2 m/s^2. Another car starts from the same point after 5 seconds and chases the first car with a uniform velocity of 20 m/s. The time at which the second car, after it starts, will overtake the first car is
(A) 5 sec (B) 7 sec
(C) 9 sec (D) 11 sec

2. A body is moving with uniform acceleration. In the 4th second of its travel it covers 20 m and 30 m in the 8th second. The distance travelled at the 10th second is
(A) 24 m (B) 35 m
(C) 43 m (D) 52 m

3. A block is made to slide down an inclined plane which is smooth. It starts sliding from rest and takes a time of t to reach the bottom of the plane. An identical body is freely dropped from the same point. The time the body takes to reach the bottom of the plane is
(A) t (B) $\frac{t}{2}$
(C) $\frac{t}{3}$ (D) $\frac{t}{4}$

4. A stone is dropped into a well. The sound of the splash is heard 3.63 seconds later. Assume the velocity of the sound to be 331 m/s. The depth of the surface of water from the ground is
(A) 46.38 m (B) 51.36 m
(C) 58.39 m (D) 64.62 m

5. A motor cycle starts from rest from point A, 2 seconds after a car, speeding at a constant velocity of 120 km/h, passes that point. The motor cycle accelerates at a rate of 6 m/s^2 until the motor cycle attains a maximum speed of 150 km/h. The distance from the starting point to the point at which the motor cycle overtakes the car is
(A) 912 m (B) 1024 m
(C) 1286 m (D) 1356 m

6. A rail road coal car of tare weight m_o is moving at a constant speed v while being loaded with coal at a constant rate of w per second. The force necessary to sustain the constant speed, neglecting friction, is
(A) w^2v (B) wv
(C) $\frac{w^2v}{2}$ (D) w^2v^2

7. A 10 kg shell is fired with a velocity of 800 m/s at an angle of 30° from an old 2000 kg gun. Assuming that barrel and frame can recoil freely, the reaction of the gun, if the shell leaves the barrel 10 milliseconds after firing, is
(A) 400 kN (B) 450 kN
(C) 600 kN (D) 550 kN

8. A baggage truck pulls two carts A and B. If the mass of the truck is 400 kg and the carts A and B carry 800 and 400 kg respectively, and the truck develops a tractive force of 2 kN. The horizontal forces between the truck and the cart A and between the two carts, respectively, are
(A) 1200 N and 400 N (B) 1000 N and 450 N
(C) 1500 N and 500 N (D) 500 N and 500 N

9. A body of weight 200 N is placed on a rough horizontal plane. The coefficient of friction, if a horizontal force of 80 N just causes the body to slide over the horizontal plane, is
(A) 0.6 (B) 0.1
(C) 0.2 (D) 0.4

10. A body of weight 400 N is pulled up along an inclined plane having an inclination of 30° to the horizontal at a steady speed. The pulling force applied on the body is parallel to the inclined plane. The coefficient of friction between the body and the plane is 0.25. If the distance travelled by the body is 10 m along the plane, then the work done on the body is
(A) 3412 J (B) 2866 J
(C) 1002 J (D) 4956 J

Practice Problems 2

Direction for questions 1 to 10: Select the correct alternative from the given choices.

1. A boat goes 30 km down the stream in 75 minutes and the same distance up the stream in 90 minutes. The speed of the stream is
(A) 0.8 km/h (B) 1.2 km/h
(C) 1.6 km/h (D) 2 km/h

2. The motion of a body is explained by the equation: $s = t^3 - 3t^2 - 9t + 12$, where s is the displacement in metres at any time t in seconds. The acceleration of the particle when its velocity is zero is
(A) 4.5 m/s^2 (B) 6.2 m/s^2
(C) 8 m/s^2 (D) 12 m/s^2

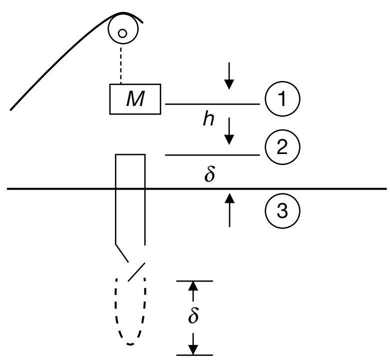
Direction for questions 3 and 4: There are three marks along a straight road at a distance of 100 m. A vehicle starting from rest and accelerating uniformly passes the first mark (P) and takes 10 seconds to reach the second mark (Q). Further it takes 8 seconds to reach the third mark (R).

3. The velocity of the car at Q is
(A) 11.38 m/s (B) 13.5 m/s
(C) 14.8 m/s (D) 15.5 m/s

4. The distance of mark P from the starting point is
(A) 218 m (B) 183 m
(C) 156 m (D) 134 m

5. An aircraft is flying at an elevation of 1500 m above the ground horizontally. The velocity is 100 km/h, horizontal and uniform. The aircraft releases a bomb at this elevation. If the target on the ground was just below the plane at the time of releasing the bomb, the distance away from the target, the bomb will hit the ground is
(A) 2.35 km (B) 3.42 km
(C) 4.86 km (D) 5.32 km

Direction for questions 6 and 7: A pile of mass 400 kg is driven by a distance of δ into the ground by the blow of a hammer of mass 800 kg through a height of h onto the top of the pile. Assume the impact between the hammer and pile to be plastic.



Given $M = 800$ kg, $m = 400$ kg, $h = 1.2$ m, $\delta = 10$ cm.

6. The work done is

(A) 5.28 kJ (B) 6.278 kJ
(C) 7.126 kJ (D) 6.8 kJ

7. The kinetic energy of the whole system in the position 3 is

(A) 0 J (B) 10 J
(C) 100 J (D) 20 J

Direction for questions 8 and 9: A gun of mass 2000 kg fires horizontally a shell of mass 50 kg with a velocity of 300 m/s.

8. The velocity with which the gun will recoil is

(A) –7.5 m/s (B) –8.4 m/s
(C) 9.2 m/s (D) 10 m/s

9. The uniform force required to stop the gun in 0.6 m is

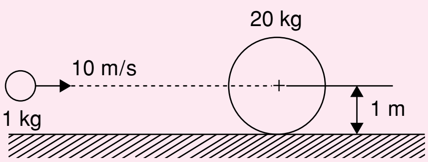
(A) 55310 N (B) 46875 N
(C) 55475 N (D) 82750 N

10. A tennis ball is having a velocity of 40 m/s at an angle of 30° with the horizontal just after being struck by the player. The radius of curvature of its trajectory is

(A) 188.2 m (B) 198.6 m
(C) 200 m (D) 168.2 m

Previous Years' Questions

1. A 1 kg mass of clay, moving with a velocity of 10 m/s, strikes a stationary wheel and sticks to it. The solid wheel has a mass of 20 kg and a radius of 1m. Assuming that the wheel and the ground are both rigid and that the wheel is set into pure rolling motion, the angular velocity of the wheel immediately after the impact is approximately **[2005]**



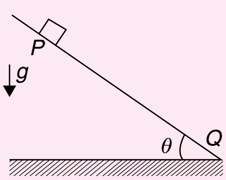
(A) Zero (B) $\frac{1}{3}$ rad/s

(C) $\sqrt{\frac{10}{3}}$ rad/s (D) $\frac{10}{3}$ rad/s

2. During inelastic collision of two particles, which one of the following is conserved? **[2007]**

(A) Total linear momentum only
(B) Total kinetic energy only
(C) Both linear momentum and kinetic energy
(D) Neither linear momentum nor kinetic energy

3. A block of mass M is released from point P on rough inclined plane with inclination angle θ, shown in the figure below. The coefficient of friction is μ. If $\mu < \tan\theta$, then the time taken by the block to reach another point Q on the inclined plane, where $PQ = s$, is **[2007]**



(A) $\sqrt{\frac{2s}{g\cos\theta(\tan\theta-\mu)}}$ (B) $\sqrt{\frac{2s}{g\cos\theta(\tan\theta+\mu)}}$

(C) $\sqrt{\frac{2s}{g\sin\theta(\tan\theta-\mu)}}$ (D) $\sqrt{\frac{2s}{g\sin\theta(\tan\theta+\mu)}}$

4. Match the approaches given below to perform stated kinematics/dynamics analysis of machine. **[2009]**

Analysis		**Approach**	
(P)	Continuous relative rotation	(1)	D' Alembert's principle
(Q)	Velocity and acceleration	(2)	Grubler's criterion
(R)	Mobility	(3)	Grashoff's law
(S)	Dynamicstatic analysis	(4)	Kennedy's theorem

(A) P–1, Q–2, R–3, S–4 (B) P–3, Q–4, R–2, S–1
(C) P–2, Q–3, R–4, S–1 (D) P–4, Q–2, R–1, S–3

5. The coefficient of restitution of a perfectly plastic impact is **[2011]**

(A) 0 (B) 1
(C) 2 (D) ∞

6. A truck accelerates up a 10° incline with a crate of 100 kg. Value of static coefficient of friction between the crate and the truck surface is 0.3. The maximum value of acceleration (in m/s^2) of the truck such that the crate does not slide down is ______ **[2014]**

7. A mass m_1 of 100 kg travelling with a uniform velocity of 5 m/s along a line collides with a stationary mass m_2 of 1000 kg. After the collision, both the masses travel together with the same velocity. The coefficient of restitution is **[2014]**

(A) 0.6 (B) 0.1
(C) 0.01 (D) 0

8. A swimmer can swim 10 km in 2 hours when swimming along the flow of a river. While swimming against the flow, she takes 5 hours for the same distance. Her speed in still water (in km/h) is _____. **[2015]**

9. A ball of mass 0.1 kg, initially at rest, is dropped from height of 1 m. Ball hits the ground and bounces off the ground. Upon impact with the ground, the velocity reduces by 20%. The height (in m) to which the ball will rise is _____. **[2015]**

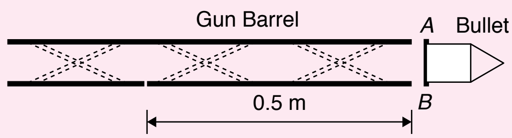
10. A small ball of mass 1 kg moving with a velocity of 12 m/s undergoes a direct central impact with a stationary ball of mass 2 kg. The impact is perfectly elastic. The speed (in m/s) of 2 kg mass ball after the impact will be ______. **[2015]**

11. The initial velocity of an object is 40 m/s. The acceleration a of the object is given by the following expression:

$a = -0.1v$,

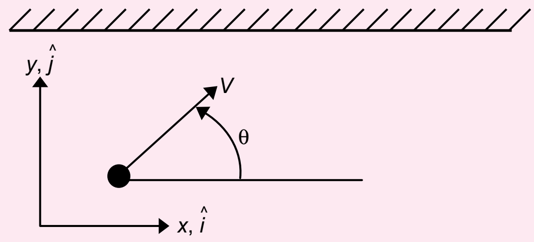
where v is the instantaneous velocity of the object. The velocity of the object after 3 seconds will be _______. **[2015]**

12. A bullet spins as the shot is fired from a gun. For this purpose, two helical slots as shown in the figure are cut in the barrel. Projections A and B on the bullet engage in each of the slots.



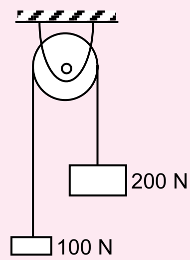
Helical slots are such that one turn of helix is completed over a distance of 0.5 m. If velocity of bullet when it exists the barrel is 20 m/s, it spinning speed in rad/s is _____. **[2015]**

13. A point mass having mass M is moving with a velocity V at an angle θ to the wall as shown in the figure. The mass undergoes a perfectly elastic collision with the smooth wall and rebounds. The total change (final minus initial) in the momentum of the mass is: **[2016]**



(A) $-2MV\cos\theta\hat{j}$
(B) $2MV\sin\theta\hat{j}$
(C) $2MV\cos\theta\hat{j}$
(D) $-2MV\sin\theta\hat{j}$

14. An inextensible mass less string goes over a frictionless pulley. Two weights of 100 N and 200 N are attached to the two ends of the string. The weights are released from rest, and start moving due to the gravity. The tension in the string (in N) is _______. **[2016]**



Answer Keys

Exercises

Practice Problems 1

1. A **2.** B **3.** B **4.** C **5.** A **6.** B **7.** A **8.** C **9.** D **10.** B

Practice Problems 2

1. D **2.** D **3.** A **4.** D **5.** C **6.** B **7.** A **8.** A **9.** B **10.** A

Previous Years' Questions

1. B **2.** A 3. A **4.** B **5.** A **6.** 1 to 1.3 **7.** D **8.** 3.5 **9.** 0.64
10. 7.8 to 8.2 **11.** 29.5 to 29.7 **12.** 251 to 252 **13.** D **14.** 133–134