CHAPTER 9

Compound Interest

Compound Interest

If the principal does not remain the same for the entire loan period, due to addition of interest to the principal after a certain interest of time to form the new principal, then the interest obtained is called compound interest.

The borrower and the lender agree to fix up a certain limit of time say yearly, half-yearly (semi-annually) or quarterly (after every three month) to settle the previous account.

Compound interest can be calculated easily by using the formulae given below

Important Rules & Formulae

Consider P = Principal (amount borrowed)

A = Amount (principal + interest)

T = Time

R = Rate of interest

n = Number of time interest calculated

Rule 1 When interest is compounded annually.

Here, $n = T$

(i) $\text{Amount} = P\left(1+\frac{R}{100}\right)^n$

(ii) $\text{CI} = A - P = P\left[\left(1+\frac{R}{100}\right)^T - 1\right]$

Example 1. The amount and the compound interest on ₹ 24000 compounded annually for 2 yr at the rate of 10% per annum is

(a) ₹ 39040 and ₹ 4040 (b) ₹ 29040 and ₹ 5040

(c) ₹ 19040 and ₹ 3040 (d) None of the above

Sol. (b) Here, P = ₹ 24000, R = 10% per annum

T = 2 yr, n = 2 yr

$\therefore \quad A = P\left(1+\frac{R}{100}\right)^n = 24000\left(1+\frac{10}{100}\right)^2 = 24000\left(\frac{110}{100}\right)^2$

$= 24000 \times \frac{110}{100} \times \frac{110}{100} =$ ₹ 29040

$\therefore$ Compound interest = ₹ (29040 − 24000) = ₹ 5040

Here, amount = ₹ 29040 and compound interest = ₹ 5040.

Rule 2 When interest is compounded half-yearly.

Here, as interest would be calculated after every six months.

So, $n = T \times 2$ and rate $= \frac{R}{2}$

So, $\text{Amount} = P\left(1+\frac{R/2}{100}\right)^n$

Example 2. The compound interest on ₹ 24000 compounded semi-annually for $1\frac{1}{2}$ yr at the rate of 10% per annum is

(a) ₹ 3783 (b) ₹ 2783 (c) ₹ 2763 (d) ₹ 3763

Sol. (a) P = ₹ 24000

R = 10% per annum = 5% semi-annually

Here, $T = 1\frac{1}{2}$ yr = 3 half-year

$\therefore \quad n = 3$

$\therefore \quad A = 24000\left(1+\frac{5}{100}\right)^3 = 24000\left(1+\frac{1}{20}\right)^3 = 24000\left(\frac{21}{20}\right)^3$

$= 24000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} =$ ₹ 27783

And compound interest = $A - P$

= ₹ 27783 − ₹ 24000 = ₹ 3783

Example 3. What sum invested for $1\frac{1}{2}$ yr compounded half-yearly at the rate of 4% per annum will amount to ₹ 132651?

(a) ₹ 125000 (b) ₹ 120000

(c) ₹ 135000 (d) None of these

Sol. (a) Principal = ?

Time, $T = 1\frac{1}{2}$ yr = 3 half-year

$n = 3$

R = 4% per annum

= 2% semi-annually

A = ₹ 132651

$$\therefore \quad 132651 = P\left(1+\frac{2}{100}\right)^3 = P\times\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}$$

$$\therefore \quad P = \frac{132651\times 50\times 50\times 50}{51\times 51\times 51}$$

$$P = ₹\ 125000$$

Rule 3 When interest is compounded quarterly.

Here, as interest would be calculated after every three months, so

$$n = T\times 4 \text{ and Rate} = \frac{R}{4}$$

So, $$\text{Amount} = P\left(1+\frac{R/4}{100}\right)^n$$

Example 4. The amount and the compound interest on ₹ 100000 compounded quarterly for 9 months at the rate of 4% per annum is

(a) ₹ 106060 and ₹ 6060.10
(b) ₹ 103030 and ₹ 3030.10
(c) ₹ 103030.10 and ₹ 3030.10
(d) ₹ 106060.10 and ₹ 6060.10

Sol. (c) $P = ₹\ 100000$

Time, $T = 9$ months $= 3$ quarter year

$R = 4\%$ per annum $= 1\%$ quarterly

$$\therefore \quad A = 100000\left(1+\frac{1}{100}\right)^3 = 100000\left(\frac{101}{100}\right)^3$$

$$= 100000\times\frac{101}{100}\times\frac{101}{100}\times\frac{101}{100} = ₹\ 103030.10$$

Also, Compound interest $= A - P$

$= ₹\ 103030.10 - ₹\ 100000$

Compound interest $= ₹\ 3030.10$

Example 5. A sum of ₹ 3200 invested at 10% per annum compounded quarterly amounts to ₹ 3362. then the time period is

(a) 2 months (b) 4 months
(c) 6 months (d) 8 months

Sol. (c) Here, $P = ₹\ 3200$

Let time $= t$ years $\Rightarrow n = 4t$

$R = 10\%$ per annum $= \frac{10}{4}\%$ quarterly

$T = ?$

$$A = ₹\ 3362 \Rightarrow 3362 = 3200\left(1+\frac{10}{400}\right)^n$$

$$\Rightarrow \quad \frac{3362}{3200} = \left(\frac{410}{400}\right)^n \Rightarrow \frac{1681}{1600} = \left(\frac{41}{40}\right)^n$$

$$\left(\frac{41}{40}\right)^2 = \left(\frac{41}{40}\right)^n \Rightarrow n = 2$$

$$\Rightarrow \quad 4t = 2 \text{ quarter year}$$

$$t = \frac{1}{2} \text{ quarter year}$$

$$t = \frac{1}{2}\times 12 = 6 \text{ months}$$

Rule 4 When time is given in fraction of a year *i.e.*, $\frac{11}{3}$ yr, $\frac{7}{3}$ yr etc. Let the time be a fraction of a year say $4\frac{2}{3}$ yr and is compounded annually.

Then, amount $= P\left(1+\frac{R}{100}\right)^4\left(1+\frac{\frac{2}{3}R}{100}\right)$

Example 6. The amount and compound interest on ₹ 5000 compounded annually for 2 yr 6 months at the rate of 10% per annum is

(a) ₹ 6252.5 and ₹ 1342.5 (b) ₹ 6000 and ₹ 1300
(c) ₹ 6250 and ₹ 1340 (d) ₹ 6352.5 and ₹ 1352.5

Sol. (d) $P = ₹\ 5000$

$T = 2$ yr

$R = 10\%$ per annum

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$A = 5000\left(1+\frac{10}{100}\right)^2 = 5000\left(\frac{110}{100}\right)^2$$

$$= 5000\times\frac{110}{100}\times\frac{110}{100} = ₹\ 6050$$

For 6 months, $A = 6050 + \frac{6050\times 10\times 6}{100\times 12}$

$= 6050 + 302.50 = ₹\ 6352.50$

$\therefore$ Compound interest = Amount − Principal

$= 6352.50 - 5000 = ₹\ 1352.50$

Shortcut method

Here, $T = 2$ yr and 6 months

$$T = 2\frac{1}{2} \text{ yr}$$

$$\therefore \quad A = 5000\left(1+\frac{10}{100}\right)^2\left(1+\frac{\frac{1}{2}\cdot 10}{100}\right)$$

$$= 5000\left(\frac{110}{100}\right)^2\left(\frac{105}{100}\right)$$

$$= 5000\times\frac{11}{10}\times\frac{11}{10}\times\frac{21}{20} = ₹\ 6352.50$$

Rule 5 When rates are different for different years say R_1, R_2, R_3 per cent for first, second and third year, respectively.

Then, amount $= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

Example 7. The compound interest on ₹ 5000 for 4 yr if the rate of interest is 10% per annum for the first two years and 15% for the next two years is

(a) ₹ 3000 (b) ₹ 3001
(c) ₹ 3002 (d) None of these

Sol. (*b*) Here, $R_1 = 10\%$

$T = T_1 + T_2 = 4$ yr

$T_1 = 2$ yr

$R_2 = 15\%$

$T_2 = 2$ yr

and principal = ₹ 5000

$\therefore$ $\text{Amount} = 5000\left(1+\frac{10}{100}\right)^2\left(1+\frac{15}{100}\right)^2$

$= 5000 \times \left(\frac{110}{100}\right)^2\left(\frac{115}{100}\right)^2$

$= 5000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{23}{20} \times \frac{23}{20}$

$=$ ₹ 8001 (nearly)

Compound interest $= 8001 - 5000 =$ ₹ 3001

Example 8. The compound interest on ₹ 6000 for 3 yr at 8% for first year, 10% for second year and 12% for third year.

(a) ₹ 1982.185 (b) ₹ 1982.936

(c) ₹ 1983.250 (d) ₹ 1983.366

Sol. (*d*) Here, $R_1 = 8\%$

$R_2 = 10\%$

$R_3 = 12\%$

$P =$ ₹ 6000

$T = T_1 + T_2 + T_3 = 3$ yr

$T_1 = T_2 = T_3 = 1$ yr

$\therefore$ $A = 6000\left(1+\frac{8}{100}\right)\left(1+\frac{10}{100}\right)\left(1+\frac{12}{100}\right)$

$= ₹\left[6000 \times \left(\frac{108}{100}\right) \times \left(\frac{110}{100}\right) \times \left(\frac{112}{100}\right)\right]$

$= ₹\left[6000 \times \frac{27}{25} \times \frac{11}{10} \times \frac{28}{25}\right] = ₹\ 7983.366$

$\therefore$ Compound interest $= 7983.366 - 6000 = 1983.366$

Points to be Remember

- When interest is compounded yearly, then on the same sum and at the same rate, then compound interest for the first year = simple interest for the first year.
- If A and B are the amounts of a certain sum for two consecutive year, then simple interest for 1 yr $= B - A$

Example 9. The difference between the compound interest and the simple interest on a certain sum at 12% per annum for 2 yr is ₹ 90. Then, sum is

(a) ₹ 6250 (b) ₹ 6350 (c) ₹ 6520 (d) ₹ 6950

Sol. (a) Let the sum be ₹ 100.

Case I. Then, simple interest on ₹ 100 for 2 yr

$= ₹\left(\frac{100 \times 12 \times 2}{100}\right) =$ ₹ 24

Case II. Amount when ₹ 100 is borrowed for 2 yr on compound interest

$= ₹\ 100 \times \left(1+\frac{12}{100}\right)^2 = ₹\ 100 \times \frac{28}{25} \times \frac{28}{25} = ₹\left(\frac{3136}{25}\right)$

$\therefore$ Compound interest for 2 yr $= ₹\left(\frac{3136}{25} - 100\right) = ₹\left(\frac{636}{25}\right)$

Difference between CI and SI

$= ₹\left(\frac{636}{25} - 24\right) = ₹\ \frac{36}{25}$

If the difference between CI and SI is ₹ $\frac{36}{25}$ the sum = ₹ 100

If difference between CI and SI is ₹ 1 the sum $= ₹\left(\frac{100 \times 25}{36}\right)$

If the difference between CI and SI is ₹ 90

$= ₹\left(100 \times \frac{25}{36} \times 90\right) =$ ₹ 6250

Here, the sum is ₹ 6250.

Growth and Depreciation

Growth Certain things like the height of true, population, the weight and height of child etc., increase over a period of time. This increment in the things is called as growth while the growth per unit of time is called as the rate of growth.

Formulae to be Remember

- Let the present population be P.

I. When rate of growth = R% per annum

(a) Population after n years $= P \times \left(1+\frac{R}{100}\right)^n$

(b) Population n years ago $= \frac{P}{\left(1+\frac{R}{100}\right)^n}$

II. When rate of growth is R_1% during first year and R_2% during the next year.

Population after 2 yr $= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)$

- If population decreases regularly, then use negative sign in above formulae. Also, $n = T$ in most of the cases.

Example 10. The population of a city increases each year by 4% of what it had been at the beginning of each year. If the population in 1997 had been 6760000, then the population of the city in (i) 1999 and (ii) 1995.

(a) 7311616 and 6230016 (b) 7300600 and 6120012

(c) 7222617 and 6022313 (d) None of these

Sol. (*a*) (i) Population of town in 1997 = 6760000

Rate of growth = 4%

Let population in 1999 $= P_1$

and $n = 2$ yr

$\therefore$ $P_1 = P\left(1+\frac{R}{100}\right)^n = 6760000\left(1+\frac{4}{100}\right)^2$

$= 6760000 \times \left(\frac{104}{100}\right)^2$

$P_1 = 7311616$

$\therefore$ Population in 1999 = 7311616

(ii) Population in 1995 $= P_2$

$T = 2$ yr

$\because \quad P_2 = P\left(1-\frac{R}{100}\right)^n$

$\therefore \quad P_2 = 6760000\left(1-\frac{4}{100}\right)^2$

$= 6760000\left(\frac{96}{100}\right)^2 = \frac{6760000 \times 96 \times 96}{10000} = 6230016$

$\therefore$ Population in 1995 = 6230016

Depreciation The decrease in the value of an article is called its depreciation. Depreciation per unit time is called the rate of depreciation.

Formulae to be Remember

Let the present value of a article be P. Let it depreciate at rate of $R\%$ per annum.

Then,

(i) Value of article after n years $= P \times \left(1-\frac{R}{100}\right)^n$

(ii) Value of article n years ago $= \frac{P}{\left(1-\frac{R}{100}\right)^n}$

(iii) The unit's place digit is 4 and 9 respectively. If after spending $p_1\%$ first, then $p_2\%$ from the remaining and so on, B is the balance amount, then the total (original) amount is given by

$$\text{Total amount} = \frac{B \times 100 \times 100 \ldots}{(100-P_1)(100-P_2)\ldots}$$

Example 11. Given that carbon $-14(C_{14})$ decays at a constant rate in such a way that it reduces to 50% in 5568 yr. Then the age of an old wooden piece in which the carbon is only 12.5% of the original is

(a) 16000 yr (b) 16244 yr
(c) 16702 yr (d) None of these

Sol. (d) Let the rate of decay be $R\%$ and let the age of the wooden piece be n years.

Then, $\left(\frac{50}{100}\right) = \left(1-\frac{R}{100}\right)^{5568}$

$\left(1-\frac{R}{100}\right) = (0.5)^{1/5568}$ ...(i)

$\frac{125}{100} = \left(1-\frac{R}{100}\right)^n$

$(0.125) = \left(1-\frac{R}{100}\right)^n$

$(0.125)^{1/n} = \left(1-\frac{R}{100}\right)$

$(0.5)^{3/n} = \left(1-\frac{R}{100}\right)$ ...(ii)

From Eqs. (i) and (ii)

$\frac{3}{n} = \frac{1}{5568} \Rightarrow n = 3 \times 5568$

$n = 16704$ yr

Example 12. Yash started the business with an initial investment of ₹ 500000. In the first year he earned a loss of 4% however, during the second year, he earned a profit of 5% which in the third year a profit of 10%. Then the net profit for the entire period of 3 yr is

(a) ₹ 53300 (b) ₹ 53500
(c) ₹ 54400 (d) None of these

Sol. (c) Initial investment = ₹ 500000

In Ist year, loss of $4\% = R_1$
In IInd year, profit of $5\% = R_2$
In IIIrd year, profit of $10\% = R_3$
$\therefore$ Net amount obtained after 3 yr.

So, $A = 500000\left(1-\frac{4}{100}\right)\left(1+\frac{5}{100}\right)\left(1+\frac{10}{100}\right)$

$= 500000\left(\frac{96}{100} \times \frac{105}{100} \times \frac{110}{100}\right) =$ ₹ 554400

$\therefore$ Profit for entire period of 3 yr

$= A - P = 554400 - 500000$
$=$ ₹ 54400

Exercise

1. Kiran purchased a scooter for ₹ 24000. The value of scooter is depreciating at the rate of 5% per annum. Then, its value after three years is
(a) ₹ 20577 (b) ₹ 20977 (c) ₹ 20677 (d) ₹ 20877

2. If P be the principal amount and the rate of interest be $r\%$ per annum and the compound interest is calculated k times in a year, then what is the amount at the end of n years? **(CDS 2007 II)**

(a) $P\left(1+\frac{r}{100k}\right)^{nk}$ (b) $P\left(1+\frac{kr}{100}\right)^{nk}$

(c) $P\left(1+\frac{kr}{100}\right)^{n/k}$ (d) $P\left(1+\frac{kr}{100k}\right)^{n/k}$

3. The difference between the simple interest and the compound interest (compounded annually) on ₹ 1250 for 2 yr at 8% per annum will be
(a) ₹ 18 (b) ₹ 2 (c) ₹ 8 (d) ₹ 4

4. The amount of a certain sum at compound interest for 2 yr at 5% is ₹ 4410. The sum is
(a) ₹ 4000 (b) ₹ 4200 (c) ₹ 3900 (d) ₹ 3800

5. The difference between the simple interest and compound interest for 2 yr at 4% per annum is ₹ 20. The principal (in rupees) will be
(a) ₹ 12000 (b) ₹ 13000
(c) ₹ 12500 (d) ₹ 3080

6. At compound interest, if a certain sum of money doubles in 'n' years, then the amount will be four fold in
(a) $2n^2$ years (b) n^2 years (c) $2n$ years (d) $4n$ years

7. A saving bank gives interest which compounds annually. Raju deposited ₹ 100 and received ₹ 121 at the end of second year. Rate of compound interest per annum is
(a) 10% (b) 15% (c) 11.5% (d) 20.5%

8. A sum of ₹ 550 was taken as a loan. This is to be paid back in two equal instalments. If the rate of interest be 20% compounded annually, then the value of each instalment is
(a) ₹ 360 (b) ₹ 330 (c) ₹ 335 (d) ₹ 315

9. The compound interest on ₹ 5000 for 3 yr at 8% for first year, 10% for second year and 12% for third year will be
(a) ₹ 1560.40 (b) ₹ 1500 (c) ₹ 1565.60 (d) ₹ 1652.80

10. An amount of ₹ x at compound interest at 20% per annum for 3 yr becomes y. What is y : x? **(CDS 2008 I)**
(a) 3 : 1 (b) 36 : 25 (c) 216 : 125 (d) 125 : 216

11. The compound interest on ₹ 2000 for 1 yr at the rate of 8% per annum, when the interest is compounded semi-annually
(a) ₹ 163.20 (b) ₹ 2163 20 (c) ₹ 3153.20 (d) ₹ 1163

12. The amount Ram will pay on ₹ 8000 at the rate of 10% per annum compounded half-yearly for $1\frac{1}{2}$ yr
(a) ₹ 9263 (b) ₹ 9261 (c) ₹ 9267 (d) ₹ 9268

13. A certain sum invested at 4% per annum compounded semi-annually amounts to ₹ 7803 at the end of one year. Then the sum is
(a) ₹ 7508 (b) ₹ 7550
(c) ₹ 7500 (d) None of these

14. ₹ 16000 invested at 10% per annum compounded semi-annually amounts to ₹ 18522. Then, the period of investment is
(a) $1\frac{1}{2}$ yr (b) 3 yr (c) 2 yr (d) $\frac{5}{2}$ yr

15. Nagma invested ₹ 6000 in a company at compound interest compounded semi-annually. She receives ₹ 7986 after 18 months from the company, the rate of interest per annum is
(a) 10% (b) 20% (c) 30% (d) 15%

16. A sum compounded annually becomes $\frac{25}{16}$ times of itself in 2 yr. Then, the rate of interest per annum is
(a) 25% (b) 20% (c) 15% (d) $7\frac{1}{2}$%

17. A sum of ₹ 25000 invested at 8% per annum compounded semi-annually amounts to ₹ 28121.60. Then, the time period is
(a) $\frac{1}{2}$ yr (b) 3 yr (c) $1\frac{1}{2}$ yr (d) 2 yr

18. A sum of ₹ 3200 invested at 10% per annum compounded quarterly amounts to ₹ 3362, then the time period is
(a) $1\frac{1}{2}$ yr (b) $\frac{1}{2}$ yr (c) 2 yr (d) 1 yr

19. A sum amount to ₹ 9680 in 2 yr and to ₹ 10648 in 3 yr compounded annually. Then, the sum and rate of interest, respectively are
(a) ₹ 8000, 10% (b) ₹ 8500, 10%
(c) ₹ 8500, 9% (d) ₹ 8000, 9%

20. If the value of a machine depreciates by 10% of its value at the beginning of the year and its present value is estimated as ₹ 10935, what was its value three years back?
(a) ₹ 15000 (b) ₹ 7000
(c) ₹ 8050 (d) None of these

21. A sum of money doubles itself at compound interest in 15 yr. It will becomes 8 times in
(a) 30 yr (b) 40 yr (c) 60 yr (d) 45 yr

22. A sum of ₹ 24000 is borrowed for $1\frac{1}{2}$ yr at the rate of interest 10% per annum compounded semi-annually. What is the compound interest (x)? **(CDS 2007 I)**
(a) $x <$ ₹ 3000 (b) ₹ 3000 $< x <$ ₹ 4000
(c) ₹ 4000 $< x <$ ₹ 5000 (d) $x >$ ₹ 5000

23. A man borrows ₹ 4000 at 8% per annum on compound interest. At the end of every year he pays ₹ 1500 as part payment of loan and interest. How much does he still once to the bank after three such annual payments?
(a) ₹ 1799 (b) ₹ 2000
(c) ₹ 169.25 (d) None of these

24. A sum of ₹ 10000 deposited at compound interest becomes double after 5 yr. After 20 yr the amount will be
(a) ₹ 160000 (b) ₹ 40000 (c) ₹ 50000 (d) ₹ 60000

25. The compound interest on ₹ 5000 for 3 yr at 5% for the first year, 10% for the second year and 12% for the third year will be
(a) ₹ 5580 (b) ₹ 5850 (c) ₹ 1648 (d) ₹ 1468

26. The difference between simple and compound interest for two years at 5% per annum is ₹ 25. The sum is
(a) ₹ 10100 (b) ₹ 2500 (c) ₹ 10000 (d) ₹ 12000

27. The simple interest on a certain sum of money for 3 yr at 8% per annum is half the compound interest on ₹ 4000 for 2 yr at 10% per annum. What is the sum placed on simple interest? **(CDS 2010 I)**
(a) ₹ 1550 (b) ₹ 1650 (c) ₹ 1750 (d) ₹ 2000

28. An amount at compound interest doubles itself in 4 yr. In how many years will the amount become 8 times itself? **(CDS 2007 I)**
(a) 8 yr (b) 12 yr (c) 16 yr (d) 24 yr

29. The compound interest on a sum for 2 yr is ₹ 832 and the simple interest on the same sum at the same rate for the same period is ₹ 800. What is the rate of interest? **(CDS 2009 I)**
(a) 6% (b) 8% (c) 10% (d) 12%

30. A sum of money on compound interest amount to ₹ 9680 in 2 yr and to ₹ 10648 in 3 yr. What is the rate of interest per annum? **(CDS 2011 I)**
(a) 5% (b) 10% (c) 15% (d) 20%

Answers

1. (a)	2. (a)	3. (c)	4. (a)	5. (c)	6. (c)	7. (a)	8. (a)	9. (d)	10. (c)
11. (a)	12. (b)	13. (c)	14. (a)	15. (b)	16. (a)	17. (c)	18. (b)	19. (a)	20. (a)
21. (d)	22. (b)	23. (c)	24. (a)	25. (d)	26. (c)	27. (c)	28. (b)	29. (b)	30. (b)

Hints and Solutions

1. $P = ₹\ 24000$, $r = 5\%$ per annum $t = 3$ yr

$$A = 24000\left(1-\frac{r}{100}\right)^t = 24000\left(1-\frac{5}{100}\right)^3 = ₹\ 20577$$

2. Given, principal amount $= ₹\ P$

Rate of interest, $R = \frac{r}{k}\%$ and Time, $T = nk$

$$\therefore \quad A = P\left(1+\frac{r}{100k}\right)^{nk}$$

3. Simple interest $= \frac{1250\times 8\times 2}{100} = ₹\ 200$

$$\text{Compound interest} = 1250\left(1+\frac{8}{100}\right)^2 - 1250$$

$$= 1250\times\left(\frac{108}{100}\right)^2 - 1250$$

$$= 1458 - 1250 \quad = ₹\ 208$$

$\therefore$ Difference in SI and CI $= 208 - 200 = ₹\ 8$

4. Let principal $= ₹\ x$

Time $= 2$ yr, Rate $= 5\%$, Amount $= ₹\ 4410$

$$4410 = x\left(1+\frac{5}{100}\right)^2$$

$$4410 = x\left(\frac{21}{20}\right)^2 \Rightarrow x = \frac{4410\times 400}{441} = ₹\ 4000$$

5. Let principal $= ₹\ x$

Then, simple interest $= \frac{x\times 4\times 2}{100} = \frac{2x}{25}$

$$\text{Compound interest} = \left[x\times\left(1+\frac{4}{100}\right)^2 - x\right] = \frac{51x}{625}$$

$$\therefore \quad \frac{51x}{625} - \frac{2x}{25} = 20$$

$$\frac{51x - 50x}{625} = 20 \Leftrightarrow x = 20\times 625 = ₹\ 12500$$

6. Let principal $= ₹\ x$

$\therefore$ Compound interest $= ₹\ R$

$\therefore$ Amount $= ₹\ 2x$

Time $= n$ years

$$\therefore \quad x\left(1+\frac{R}{100}\right)^n = 2x$$

$$\Rightarrow \quad \left(1+\frac{R}{100}\right)^n = 2 \qquad \ldots(i)$$

Let it become four fold in N years.

Then, $$x\left(1+\frac{R}{100}\right)^N = 4x$$

$$\left(1+\frac{R}{100}\right)^N = 4 \Rightarrow 2^2 = \left(1+\frac{R}{100}\right)^N$$

$$\Rightarrow \quad \left(1+\frac{R}{100}\right)^{2n} = \left(1+\frac{R}{100}\right)^N \qquad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad N = 2yr$$

7. Principal $= ₹\ 100$

Amount received after 2 yr $= ₹\ 121$

Let rate of interest $= R\%$ per annum

$$\therefore \quad 121 = 100\left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \quad \frac{121}{100} = \left(\frac{100+R}{100}\right)^2 \Rightarrow \left(\frac{11}{10}\right)^2 = \left(\frac{100+R}{100}\right)^2$$

$$\Rightarrow \quad \frac{11}{10} = \frac{100+R}{100} \Rightarrow 100+R = \frac{11\times 100}{10}$$

$$\Rightarrow \quad 100+R = 110 \quad R = 110-100 = 10$$

$\therefore$ Rate of interest $= 10\%$

8. Let the value of each instalment be $₹\ x$.

Then, $$\frac{x}{\left(1+\frac{20}{100}\right)} + \frac{x}{\left(1+\frac{20}{100}\right)^2} = 550$$

$$\Rightarrow \quad \frac{5x}{6} + \frac{25x}{36} = 550 \Rightarrow x = ₹\ 360$$

9. Principal $= ₹\ 5000$

Time $= 3$ yr

Rate for first year $= 8\%$

Rate of second year $= 10\%$

and Rate for third year $= 12\%$

$$\therefore \quad \text{Amount} = ₹\left[5000\times\left(1+\frac{8}{100}\right)\left(1+\frac{10}{100}\right)\left(1+\frac{12}{100}\right)\right]$$

$$= ₹\left[5000\times\frac{27}{25}\times\frac{11}{10}\times\frac{28}{25}\right] = ₹\ 6652.80$$

$\therefore$ Compound interest $= ₹\ 6652.80 - 5000 = ₹\ 1652.80$

10. Let $P = ₹\ x$

$R = 20\%$

$T = 3$ yr

$A = ₹\ y$

$\because \quad A = P\left(1+\frac{R}{100}\right)^T$

$\therefore \quad y = x\left(1+\frac{20}{100}\right)^3 \Rightarrow y = x\left(\frac{6}{5}\right)^3$

$\Rightarrow \quad \frac{y}{x} = \frac{216}{125} \Rightarrow y : x = 216 : 125$

11. Principal = ₹ 2000

Time = 1 yr = 2 half-year $n = 2$

Rate = 8% per annum = 4% half-yearly

$CI = 2000\left(1+\frac{4}{100}\right)^2 - 2000$

$= 2163.20 - 2000 = ₹\ 163.20$

12. Principal = ₹ 8000

Rate = 10% per annum = 5% half-yearly

Time = $1\frac{1}{2}$ yr = 3 half-year

So, $n = 3$

$\therefore \quad \text{Amount} = 8000\left(1+\frac{5}{100}\right)^3$

$= 8000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = ₹\ 9261$

13. Let the sum be ₹ x.

Amount received after 1 yr = ₹ 7803

Rate = 4% per annum = 2% half-yearly

Time = 1 yr = 2 half-year

$\therefore \quad 7803 = x\left(1+\frac{2}{100}\right)^2$

$7803 = x \times \frac{102}{100} \times \frac{102}{100}$

$7803 = x \times \frac{51}{50} \times \frac{51}{50}$

$x = \frac{7803 \times 50 \times 50}{51 \times 51}$

$= ₹\ 7500$

14. Amount invested = ₹ 16000

Amount received at the end of period = ₹ 18522

Let time = t years

So, $n = 2t$

Rate = 10% per annum = 5% half-yearly

$\because \quad 18522 = 16000\left(1+\frac{5}{100}\right)^n$

$\Rightarrow \quad \left(\frac{18522}{16000}\right) = \left(\frac{21}{20}\right)^n \Rightarrow \left(\frac{9261}{8000}\right) = \left(\frac{21}{20}\right)^n$

$\Rightarrow \quad \left(\frac{21}{20}\right)^3 = \left(\frac{21}{20}\right)^n$

$\Rightarrow \quad n = 3$

$\Rightarrow \quad 2t = 3 \text{ as } n = 2t$

$\Rightarrow \quad t = \frac{3}{2}\text{ yr} = 1\frac{1}{2}\text{ yr}$

15. Amount invested = ₹ 6000

Let rate be R% per annum = $\frac{R}{2}$ half-yearly

T = 18 months = $\frac{3}{2}$ yr, $n = 3$

$\therefore \quad 7986 = 6000\left(1+\frac{R}{2\times 100}\right)^3$

$\Rightarrow \quad \frac{7986}{6000} = \left(1+\frac{R}{200}\right)^3 \Rightarrow \left(\frac{11}{10}\right)^3 = \left(1+\frac{R}{200}\right)^3$

$\Rightarrow \quad 1+\frac{R}{200} = \frac{11}{10} \Rightarrow \frac{R}{200} = \frac{11}{10} - 1 \Rightarrow \frac{R}{200} = \frac{1}{10} \Rightarrow R = 20\%$

16. Let sum = ₹ x Amount = $\frac{25}{16}x$

$t = 2$ yr

Let Rate = R% per annum

$\therefore \quad \frac{25}{16}x = x\left(1+\frac{R}{100}\right)^2$

$\Rightarrow \quad \frac{25}{16} = \left(1+\frac{R}{100}\right)^2 = \left(\frac{5}{4}\right)^2 \Rightarrow \frac{5}{4} = \left(1+\frac{R}{100}\right) \Rightarrow \frac{R}{100} = \frac{1}{4}$

$\Rightarrow \quad R = 25\%$

17. Let P = ₹ 25000

Let time = t years $\Rightarrow n = 2t$

r = 8% per annum = 4% semi-annually

A = ₹ 28121.60

$\therefore \quad 28121.60 = 25000\left(1+\frac{4}{100}\right)^n$

$\frac{28121.60}{25000} = \left(\frac{104}{100}\right)^n$

$\frac{17576}{15625} = \left(\frac{26}{25}\right)^n$

$\left(\frac{26}{25}\right)^3 = \left(\frac{26}{25}\right)^n$

$\Rightarrow \quad n = 3 \text{ yr} \Rightarrow 2t = 3 \text{ yr}$

$t = 18 \text{ months} = 1\frac{1}{2}\text{ yr}$

18. Here, P = ₹ 3200

R = 10% per annum = $\frac{10}{4}$% quarterly

$t = ? \Rightarrow n = 4t$

Amount = ₹ 3362

$3362 = 3200\left(1+\frac{10}{4\times 100}\right)^n$

$\frac{3362}{3200} = \left(\frac{410}{400}\right)^n$

$\left(\frac{41}{40}\right)^2 = \left(\frac{41}{40}\right)^n \Rightarrow n = 2$

$\Rightarrow \quad 4t = 2 \text{ yr} \quad (\text{as } n = 4t)$

$t = \frac{1}{2}\text{ yr}$

19. Let principal be ₹ x.
Rate = R% per annum

Amount, A_1 = ₹ 9680
$t_1 = 2$ yr
Amount, A_2 = ₹ 10648
$t_2 = 3$ yr

$$A_1 = P\left(1+\frac{R}{100}\right)^{t_1}$$

$$9680 = x\left(1+\frac{R}{100}\right)^{2} \quad \ldots(i)$$

$$A_2 = x\left(1+\frac{R}{100}\right)^{t_2}$$

$$10648 = x\left(1+\frac{R}{100}\right)^{3} \quad \ldots(ii)$$

From Eqs. (i) and (ii)

$$9680\left(1+\frac{R}{100}\right)^{-2} = 10648\left(1+\frac{R}{100}\right)^{-3} \quad (\because A_1 = A_2)$$

$$\left(1+\frac{R}{100}\right) = \frac{10648}{9680}$$

$$\frac{R}{100} = \frac{10648}{9680} - 1 = \frac{968}{9680}$$

$$\frac{R}{100} = \frac{1}{10} \Rightarrow R = 10\%$$

So, from Eq. (i)

$$9680 = x\left(1+\frac{10}{100}\right)^{2} = x\left(\frac{11}{10}\right)^{2}$$

$$x = 9680 \times \frac{10}{11} \times \frac{10}{11} \Rightarrow x = 8000$$

$\therefore$ Principal = ₹ 8000

20. Let the value be ₹ x, thus

$$x\left(1-\frac{10}{100}\right)^{3} = 10935 \Rightarrow x\left(\frac{90}{100}\right)^{3} = 10935$$

$$x = \frac{10935 \times 10 \times 10 \times 10}{9 \times 9 \times 9} = ₹\ 15000$$

21. Let the sum be ₹ x.
Let rate be R% per annum.

Then, $$x\left(1+\frac{R}{100}\right)^{15} = 2x \quad \text{(by given condition)}$$

$$\Rightarrow \left(1+\frac{R}{100}\right)^{15} = 2 \quad \ldots(i)$$

Suppose, the sum becomes eight times in 'x' years, then

$$x\left(1+\frac{R}{100}\right)^{n} = 8x \quad \text{(by given condition)}$$

$$\Rightarrow \left(1+\frac{R}{100}\right)^{n} = 8 = 2^3 \quad \ldots(ii)$$

$$[\text{But } 2^3 = \left[\left(1+\frac{R}{100}\right)^{15}\right]^3 \text{ from Eq. (i)}]$$

$$\therefore \left(1+\frac{R}{100}\right)^{n} = 2^3 = \left(1+\frac{R}{100}\right)^{45} \quad \text{(on comparing)}$$

$$\Rightarrow n = 45 \text{ yr}$$

22. Given that, P = ₹ 24000

R% = 10% per annum = 5% per half-yearly

$$T = 1\frac{1}{2} \text{ yr} = \frac{3}{2} \times 2 \text{ yr} = 3 \text{ half-year}$$

$\because$ $$CI = P\left[\left(1+\frac{R}{100}\right)^{T} - 1\right]$$

$\therefore$ $$CI = 24000\left[\left(1+\frac{5}{100}\right)^{3} - 1\right]$$

$$= 24000\left[\left(\frac{21}{20}\right)^{3} - 1\right] = 24000\left[(1.05)^3 - 1\right]$$

$$= 24000 \times 0.1576 = 3782.4$$

But $CI = x$

Therefore, ₹ 3000 < x < ₹ 4000

23. Amount due at the end of 1 yr

$$= 4000\left(1+\frac{8}{100}\right) = ₹\ 4320$$

$\therefore$ Amount due after 1 yr = ₹ (4320 − 1500) = ₹ 2820
Amount due at the end of second year

$$= 2820\left(1+\frac{8}{100}\right) = ₹\ 3045.60$$

$\therefore$ Amount received after second year
= ₹ (3045.60 − 1500) = ₹ 1545.60
Amount due at the end of third year

$$= 1545.60\left(1+\frac{8}{100}\right) = ₹\ 1669.25$$

Amount received after the payment of third instalment
= ₹ (1669.25 − 1500) = ₹ 169.25

24. Let the sum be ₹ x, then $x\left(1+\frac{R}{100}\right)^{5} = 2x$

$$\Rightarrow \left(1+\frac{R}{100}\right)^{5} = 2 \quad \ldots(i)$$

The amount after 20 yr

$$\Rightarrow x\left(1+\frac{R}{100}\right)^{20} = x\left[\left(1+\frac{R}{100}\right)^{5}\right]^4$$

$$= 2^4 x = 16x$$

$$= 16 \times 10000 = ₹\ 160000$$

25. Sum = ₹ 5000
Time = 3 yr
Rates are $R_1 = 5\%$, $R_2 = 10\%$, $R_3 = 12\%$ for each year

$$\text{Amount} = 5000\left(1+\frac{5}{100}\right)\left(1+\frac{10}{100}\right)\left(1+\frac{12}{100}\right)$$

$$= 5000 \times \frac{21}{20} \times \frac{11}{10} \times \frac{28}{25} = ₹\ 6468$$

$\therefore$ Compounded interest = ₹ (6468 − 5000)
= ₹ 1468

26. Let the principal be ₹ x.

∴ Rate = 5% per annum

Time = 2 yr

By condition,

$$\therefore \quad \left[x\left(1+\frac{5}{100}\right)^2 - x\right] - \frac{x \times 2 \times 5}{100} = 25$$

$$x\left(\frac{21}{20}\right)^2 - x - \frac{x}{10} = 25$$

$$\frac{441x}{400} - \frac{11x}{10} = 25$$

$$\frac{441x - 440x}{400} = 25$$

$$x = 25 \times 400 \Rightarrow 10000$$

27. Let the principal amount be ₹ P.

By given condition, $SI = \frac{1}{2} CI$

$$\Rightarrow \quad \frac{P \times 8 \times 3}{100} = \frac{1}{2}\left[4000\left(1+\frac{10}{100}\right)^2 - 4000\right]$$

$$\Rightarrow \quad \frac{24P}{100} = \frac{1}{2}\left[4000 \times \frac{121}{100} - 4000\right]$$

$$= \frac{1}{2}[4840 - 4000] \Rightarrow \frac{24P}{100} = 420$$

$$\Rightarrow \quad P = \frac{420 \times 100}{24}$$

$$\Rightarrow \quad P = ₹\ 1750$$

28. Let ₹ P be the given sum of money and $T_1 = 4$ yr

$$2P = P\left(1+\frac{R}{100}\right)^4 \quad \text{(by given condition)}$$

$$\Rightarrow \quad 2 = \left(1+\frac{R}{100}\right)^4$$

$$\Rightarrow \quad 2^{1/4} = \left(1+\frac{R}{100}\right) \quad \ldots(i)$$

Let the sum become 8 times in T years.

Then, $8P = P\left(1+\frac{R}{100}\right)^T$ (by given condition)

$$\Rightarrow \quad 8 = \left(1+\frac{R}{100}\right)^T$$

$$\Rightarrow \quad 8 = (2^{1/4})^T \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad 8 = 2^{T/4}$$

$$\Rightarrow \quad 2^3 = 2^{T/4}$$

On comparing, we get

$$3 = \frac{T}{4} \Rightarrow T = 12 \text{ yr}$$

29. Given, CI = ₹ 832, SI = ₹ 800, $T = 2$ yr

Since, $CI = P\left\{\left(1+\frac{R}{100}\right)^T - 1\right\}$

$$\therefore \quad 832 = P\left\{\left(1+\frac{R}{100}\right)^2 - 1\right\} \quad \ldots(i)$$

Also, $SI = \frac{P \times R \times T}{100}$

$$\Rightarrow \quad 800 = \frac{P \times R \times 2}{100} \Rightarrow P = \frac{40000}{R}$$

∴ From Eq. (i),

$$832 = \frac{40000}{R}\left(\frac{R^2}{10000} + \frac{2R}{100}\right)$$

$$\Rightarrow \quad 832 = 4R + 800 \Rightarrow R = \frac{32}{4} = 8\%$$

30. Let the rate of interest per annum be R% and principal amount be ₹ P, then

amount in 2 yr, $P\left(1+\frac{R}{100}\right)^2 = 9680 \quad \ldots(i)$

and amount in 3 yr, $P\left(1+\frac{R}{100}\right)^3 = 10648 \quad \ldots(ii)$

Dividing Eq. (ii) by Eq. (i),

$$1 + \frac{R}{100} = \frac{10648}{9680}$$

$$\Rightarrow \quad R = \left(\frac{10648}{9680} - 1\right) \times 100$$

$$\therefore \quad R = \frac{968}{9680} \times 100 = 10\%$$