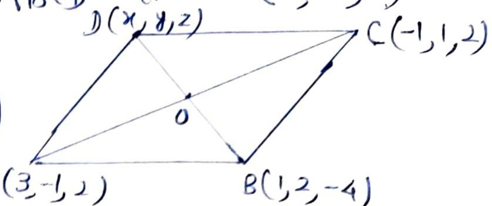
**Q No 1.** Three vertices of a parallelogram ABCD are $A(3,-1,2)$, $B(1,2,-4)$ and $C(-1,1,2)$. Find the coordinates of fourth vertex.

**Sol** Three vertices of parallelogram ABCD are $A(3,-1,2)$, $B(1,2,-4)$ and $C(-1,1,2)$




Let fourth vertex D be $(x,y,z)$

Since ABCD is a llgm.

$\therefore$ Mid point of both the diagonals will be same

ie Mid-point of AC = Mid-point BD.

$$\therefore \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) = \left(\frac{1+x}{2}, \frac{2+y}{2}, \frac{-4+z}{2}\right)$$

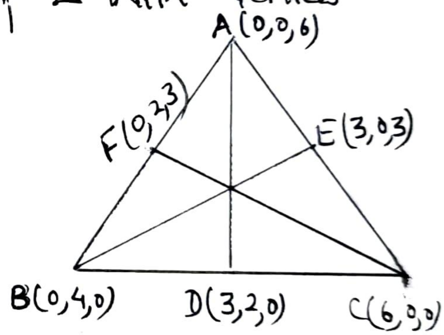
$$\Rightarrow (1,0,2) = \left(\frac{1+x}{2}, \frac{2+y}{2}, \frac{-4+z}{2}\right)$$

$$\Rightarrow 1+x = 2 \quad ; \quad 2+y = 0 \quad ; \quad -4+z = 4.$$

$$\Rightarrow x = 1 \quad ; \quad y = -2 \quad ; \quad z = 8$$

$\therefore$ D is $(1,-2,8)$

**Q No. 2 :** Find the lengths of the medians of $\Delta$ with vertices $A(0,0,6)$; $B(0,4,0)$; $C(6,0,0)$




**Sol:** Vertices of $\Delta ABC$ are $A(0,0,6)$; $B(0,4,0)$; $C(6,0,0)$

Let D, E, F be mid points of BC, CD and AB respectively.

$\therefore$ D is $\left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right)$ ie $(3,2,0)$

E is $\left(\frac{6+0}{2}, \frac{0+0}{2}, \frac{0+6}{2}\right)$ ie $(3,0,3)$

F is $\left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right)$ ie $(0,2,3)$

$\therefore$ Length of Median $AD = \sqrt{(3-0)^2+(2-0)^2+(0-6)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$

Median $BE = \sqrt{(3-0)^2+(0-4)^2+(3-0)^2} = \sqrt{9+16+9} = \sqrt{34}$

Median $CF = \sqrt{(0-6)^2+(2-0)^2+(3-0)^2} = \sqrt{36+4+9} = \sqrt{49} = 7.$

$\therefore$ Lengths of medians are $7, \sqrt{34}, 7$.

**Q No 3 :** If the origin is the centroid of the $\Delta PQR$ with vertices $P(2a, 2, 6)$; $Q(-4, 3b, -10)$ and $R(8, 14, 2c)$, then find values of $a, b, c$.

**Sol.** Since origin $O(0,0,0)$ is the centroid of $\Delta PQR$ with vertices $P(2a,2,6)$; $Q(-4,3b,-10)$ and $R(8,14,2c)$

$\therefore \dfrac{2a-4+8}{3} = 0$ ; $\dfrac{2+3b+14}{3} = 0$ ; $\dfrac{6-10+2c}{3} = 0$ [Using Centroid formula]

ie $2a+4=0$ ; $3b+16=0$ ; $2c-4=0$

$\therefore 2a=-4$ ; $3b=-16$ ; $2c=4$

$\therefore a=-2$ ; $b=\dfrac{-16}{3}$ ; $c=2.$

**Q No 4** Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point $P(3,-2,5)$

**Sol :** Let $R(0,y,0)$ be point on y-axis

$\therefore$ Distance of $R(0,y,0)$ from $P(3,-2,5)$ is $5\sqrt{2}$

ie $RP = 5\sqrt{2}$

$\Rightarrow RP^2 = 50.$

$\Rightarrow (3-0)^2+(-2-y)^2+(5-0)^2 = 50$

$\therefore 9+y^2+4y+4+25 = 50$

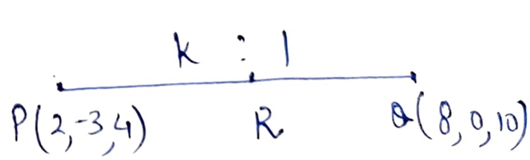
$\therefore y^2+4y-12=0 \Rightarrow (y-2)(y+6)=0$

$\Rightarrow y=2,-6$

$\therefore$ Required point is $(0,2,0)$ or $(0,-6,0)$

**Q No 5 :** A point R with x-coordinate 4 lies on the line segment joining the points $P(2,-3,4)$; $Q(8,0,10)$ Find the coordinates of R.

**Sol :** Let R divides the line segment PQ in the ratio $k:1$




$\therefore$ R is $\left(\dfrac{8k+2}{k+1}, \dfrac{-3}{k+1}, \dfrac{10k+4}{k+1}\right)$

From the given condition $\frac{8K+2}{K+1} = 4$. $\therefore 8K+2 = 4K+4$

$\Rightarrow 4K = 2 \Rightarrow K = \frac{1}{2}$

$\therefore$ R is $\left(4, \frac{-3}{\frac{1}{2}+1}, \frac{10\times\frac{1}{2}+4}{\frac{1}{2}+1}\right)$ i.e. $(4, -2, 6)$.

**Q No 6.** If A and B be the points $(3,4,5)$ and $(-1,3,-7)$ resp. find equation of the set of points P such that $PA^2 + PB^2 = K^2$; where K is any constant.

Sol: Given points are $A(3,4,5)$ and $B(-1,3,-7)$ and let $P(x,y,z)$

From given condition

$$PA^2 + PB^2 = K^2$$

$\therefore (x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (z+7)^2 = K^2$

$\therefore x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z + x^2 + 1 + 2x + y^2 + 9 - 6y + z^2 + 49 + 14z = K^2$

$\therefore 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = K^2 - 100$

$\therefore x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{K^2 - 100}{2}$.

Which is required equation

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