Class-X Session 2022-23

Subject - Mathematics (Standard)

Sample Question Paper - 38

With Solution

BLUE PRINT

Ch. No.	Chapter Name	Per Unit Marks	Section-A (1 Mark)		Section-B (2 Marks)	Section-C (3 Marks)	Section-D (5 Marks)	Section-E (4 Marks)	Total Marks
			MCQ	A/R	VSA	SA	LA	Case-Study	
1	Real Number	6	1(Q1)	1(Q19)				1(Q36)	6
2	Polynomials	20	1(Q3)		1(Q21)	1(Q26)			6
3	Pair of Linear Equations in Two Variables		1(Q2)		1(Q22)				3
4	Quadratic Equations		1(Q4)				1(Q32)		6
5	Arithmetic Progression		2(Q5, 8)			1(Q27)			5
6	Triangles	15	2(Q6, 9)		1(Q23)			1(Q37)	8
7	Circles		2(Q10, 18)		1(Q25)	1(Q30)			7
8	Coordinate Geometry	6	3(Q7, 12, 14)	1(Q20)	1(Q24)				6
9	Introduction to Trigonometry	12	1(Q11)			1(Q28)	1(Q33)		9
10	Some Applications of Trigonometry					1(Q29)			3
11	Areas Related to Circles	10					1(Q34)		5
12	Surface Areas and Volumes		1(Q13)					1(Q38)	5
13	Statistics	11					1(Q35)		5
14	Probability		3(Q15, 16, 17)			1(Q31)			6
Total Marks (Total Questions)		80	18(18)	2(2)	10(5)	18(6)	20(4)	12(3)	80(38)

Note : The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

Time : 3 Hours **Max. Marks : 80**

General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 case based integrated units of assessment (4 marks each) with sub parts of values of 1, 1 and 2 marks each respectively.*

SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. A class of 20 boys and 15 girls is divided into n groups so that each group has x boys and y girls. Values of x, y and n respectively are
(a) 3, 4 and 8 (b) 4, 3 and 6 (c) 4, 3 and 7 (d) 7, 4 and 3

2. Which of the following is the other name for a pair of linear equations in two variables?
(a) Consistent equations (b) Simultaneous equations
(c) Inconsistent equations (d) Dependent equations

3. Let $f(x) = x^2 - 27x + 196$. If $f(a) = a$, then what is the value of a.
(a) 7 (b) 14 (c) 21 (d) 6

4. If the roots of $5x^2 - kx + 1 = 0$ are real and distinct, then
(a) $-2\sqrt{5} < k < 2\sqrt{5}$ (b) $k > 2\sqrt{5}$ only
(c) $k < -2\sqrt{5}$ only (d) either $k > 2\sqrt{5}$ or $k < -2\sqrt{5}$

5. The first term of an A.P. is 5 and its 100^{th} term is –292. The 50^{th} term of this A.P. will be
(a) 142 (b) –142 (c) 130 (d) –140

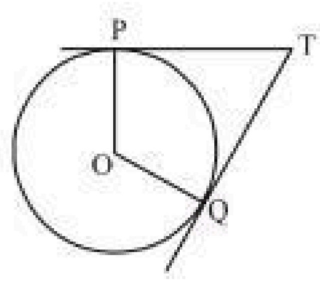
6. If in ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when
(a) $\angle B = \angle E$ (b) $\angle A = \angle D$ (c) $\angle B = \angle D$ (d) $\angle A = \angle F$

7. If the point $P(p, q)$ is equidistant from the points $A(a + b, b - a)$ and $B(a - b, a + b)$, then
(a) $ap = by$ (b) $bp = ay$ (c) $ap + bq = 0$ (d) $bp + aq = 0$

8. The n^{th} term of the A.P. a, $3a$, $5a$, ..., is
(a) na (b) $(2n - 1)a$ (c) $(2n + 1)a$ (d) $2na$

9. If ΔABC is an equilateral triangle such that $AD \perp BC$, then $AD^2 =$
A. $\frac{3a^2}{4}$ B. $\frac{3a^2}{2}$ C. $\frac{3}{4}BC^2$ D. $\frac{\sqrt{3}}{2}a$
(a) A and C (b) A (c) D (d) B and C

10. In the adjoining figure, TP and TQ are the two tangents to a circle with centre O. If $\angle POQ = 110°$, then $\angle PTQ$ is



(a) 60° (b) 70° (c) 80° (d) 90°

11. If cosec x – cot $x = \frac{1}{3}$, where $x \neq 0$, then the value of $\cos^2 x - \sin^2 x$ is

(a) $\frac{16}{25}$ (b) $\frac{9}{25}$ (c) $\frac{8}{25}$ (d) $\frac{7}{25}$

12. The point which divides the line joining the points $A(1, 2)$ and $B(-1, 1)$ internally in the ratio 1 : 2 is

(a) $\left(\frac{-1}{3}, \frac{5}{3}\right)$ (b) $\left(\frac{1}{3}, \frac{5}{3}\right)$ (c) (–1, 5) (d) (1, 5)

13. The radii of the top and bottom of a bucket of slant height 45 cm are 28 cm and 7 cm, respectively. The curved surface area of the bucket is

(a) 4950 cm^2 (b) 4951 cm^2 (c) 4952 cm^2 (d) 4953 cm^2

14. If the points (a, 0), (0, b) and (1, 1) are collinear then which of the following is true :

(a) $\frac{1}{a}+\frac{1}{b}=2$ (b) $\frac{1}{a}-\frac{1}{b}=1$ (c) $\frac{1}{a}-\frac{1}{b}=2$ (d) $\frac{1}{a}+\frac{1}{b}=1$

15. The probability of getting a number greater than 2 in throwing a die is

(a) 2/3 (b) 1/3 (c) 4/3 (d) 1/4

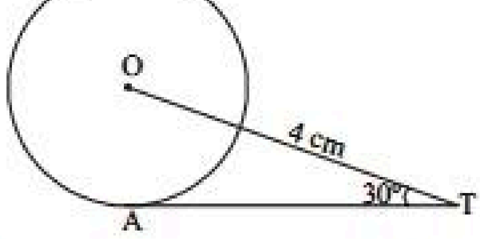
16. A bag contains card numbers 3, 4, 5, 6, 7....27. One card is drawn, then probability of prime number card is

(a) $\frac{9}{25}$ (b) $\frac{8}{27}$ (c) $\frac{8}{25}$ (d) $\frac{1}{5}$

17. A coin is tossed. Then the probability of getting either head or tail is

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

18. In figure, AT is a tangent to the circle with centre O such that OT = 4 cm and ∠OTA = 30°. Then, AT is equal to



(a) 4 cm (b) 2 cm (c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false but R is true.***

19. **Assertion :** The H.C.F. of two numbers is 16 and their product is 3072. Then, their L.C.M = 162.
Reason : If a, b are two positive integers, then H.C.F × L.C.M. = $a \times b$.

20. **Assertion :** If $A(2a, 4a)$ and $B(2a, 6a)$ are two vertices of an equilateral triangle ABC then, the vertex C is given by $(2a + a\sqrt{3}, 5a)$.
Reason : In equilateral triangle, all the coordinates of three vertices can be rational.

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. If α and β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$ then evaluate $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$

22. Find the solution of the pair equations $\frac{x}{10} + \frac{y}{5} - 1 = 0$ and $\frac{x}{8} + \frac{y}{6} = 15$. Hence, find λ, if $y = \lambda x + 5$.

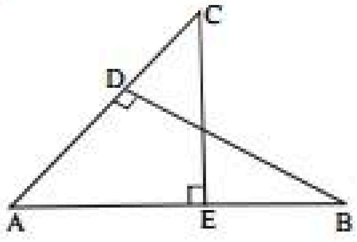
OR

A part of monthly hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay ₹ 3,000 as hostel charges whereas Mansi who takes food for 25 days ₹ 3,500 as hostel charges. Find the fixed charges and the cost of food per day.

23. In the given fig, $BD \perp AC$ and $CE \perp AB$. Prove that

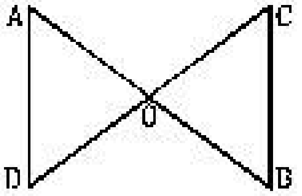
(i) $\Delta AEC \sim \Delta ADB$

(ii) $\frac{CA}{AB} = \frac{CE}{DB}$



OR

In the given fig, $\frac{OA}{OC} = \frac{OD}{OB}$, prove that $\angle A = \angle C$ and $\angle B = \angle D$



24. In what ratio, the line segment joining the points (3, 5) & (–4, 2) is divided by y-axis?

25. Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. If α, β are the roots of the polynomial $f(x) = 2x^2 + 5x + k$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k for this to be possible.

27. If $a\left(\frac{1}{b} + \frac{1}{c}\right), b\left(\frac{1}{a} + \frac{1}{c}\right), c\left(\frac{1}{a} + \frac{1}{b}\right)$, are in A.P., prove that a, b, c are in A.P.

28. Prove that : $\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \frac{1 + \cos A}{\sin A} = \operatorname{cosec} A + \cot A$

29. The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground is 45°. If the angle of elevation of the top of a flagstaff fixed at the top of the tower, at A is 60°, then find the height of the flagstaff. [Use $\sqrt{3} = 1.73$]

30. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

OR

Prove that the length of the tangents drawn from an external point to a circle are equal.

31. A piggy bank contains hundred 50p coins, fifty ₹ 1 coins, twenty ₹ 2 coins and ten ₹ 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 p coin ? (ii) will not be a ₹ 5 coin?

OR

A book containing 100 pages is opened at random. Find the probability that a doublet page is found.

SECTION-D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Solve the equation by using quadratic formula : $(x + 4)(x + 5) = 3(x + 1)(x + 2) + 2x$

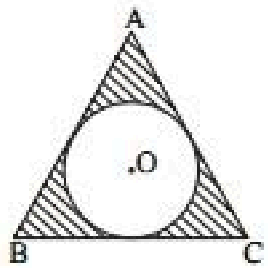
OR

If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are equal, prove that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

33. If $\cos\theta + \sqrt{3}\sin\theta = 2\sin\theta$. Show that $\sin\theta - \sqrt{3}\cos\theta = 2\cos\theta$.

34. In fig., a circle is inscribed in an equilateral triangle ABC of side 12 cm. Find the radius of inscribed circle and the area of the shaded region.

[Use $\pi = 3.14$ and $\sqrt{3} = 1.73$]



35. Following frequency distribution shows the daily expenditure on milk of 30 households in a locality:

Daily expenditure on milk (in ₹)	0 – 30	30 – 60	60 – 90	90 – 120	120 – 150
Number of households	5	6	9	6	4

Find the mode for the above data.

OR

If the mean of the following data is 14.7, find the value of p and q.

Class	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30	30 – 36	36 – 42	Total
Frequency	10	p	4	7	q	4	1	40

SECTION-E

This section comprises of 3 case study/passage - based questions of 4 marks each with three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively.

36. Case - Study 1: Read the following passage and answer the questions given below.

A seminar is being conducted by an Educational Organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.

(i) What is the LCM of 60, 84 and 108?

(ii) How 108 can be expressed as a product of its primes?

(iii) In each room the same number of participants are to be seated and all of them being in the same subject. What are the maximum number of participants that can accommodated in each room?

OR

What is the minimum number of rooms required during the event?

37. Case - Study 2: Read the following passage and answer the questions given below.

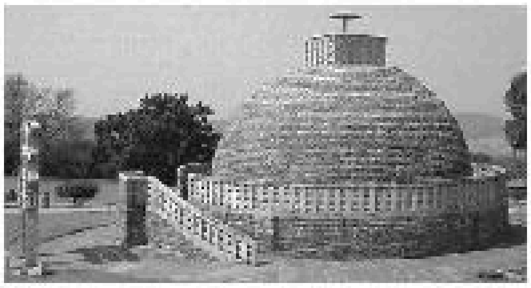
Rohan wants to measure the distance of a pond during the visit to his native. He marks points A and B on the opposite edges of a pond as shown in the figure below. To find the distance between the points, he makes a right-angled triangle using rope connecting B with another point C are a distance of 12m, connecting C to point D at a distance of 40m from point C and the connecting D to the point A which is are a distance of 30m from D such the $\angle ADC = 90°$.

(i) What is the distance AC?
(ii) Find the length of the rope used.
(iii) Which is the following does not form a Pythagoras triplet?

OR

Find the length AB?

38. **Case - Study 3:** Read the following passage and answer the questions given below.



The Great Stupa at Sanchi is one of the oldest stone structures in India, and an important monument of Indian Architecture. It was originally commissioned by the emperor Ashoka in the 3rd century BCE. Its nucleus was a simple hemispherical brick structure built over the relics of the Buddha. .It is a perfect example of combination of solid figures. A big hemispherical dome with a cuboidal structure mounted on it. $\left(\text{Take } \pi = \frac{22}{7}\right)$

(i) Calculate the volume of the hemispherical dome if the height of the dome is 21 m?
(ii) What is the lateral surface area of cuboidal shaped top with dimensions 10m × 8m × 6m?
(iii) How much the cloth require to cover the hemispherical dome if the radius of its base is 14m?

OR

What is the total surface area of the combined figure i.e. hemispherical dome with radius 14m and cuboidal shaped top with dimensions 8m × 6m × 4m?

Solution

SAMPLE PAPER-2

1. **(c)** H.C.F. of 20 and 15 = 5
So, 5 students are in each group.

$\therefore \quad n = \frac{20+15}{5} = \frac{35}{5} = 7$

Hence, x = 4, y = 3 and n = 7

2. **(b)** The pair of linear equations in two variables is also known as simultaneous equations.

3. **(b)** Equate value of polynomial at $x = a$ with a
$a^2 - 27a + 196 = a \Rightarrow a^2 - 28a + 196 = 0 \Rightarrow a = 14$

4. **(d)** The roots of $5x^2 - kx + 1 = 0$ are real and distinct.
$\therefore \quad (k^2 - 4 \times 5 \times 1) > 0 \Rightarrow k^2 > 20$

$\Rightarrow \quad k > \sqrt{20}$ or $k < -\sqrt{20} \Rightarrow k > 2\sqrt{5}$ or $k < -2\sqrt{5}$.

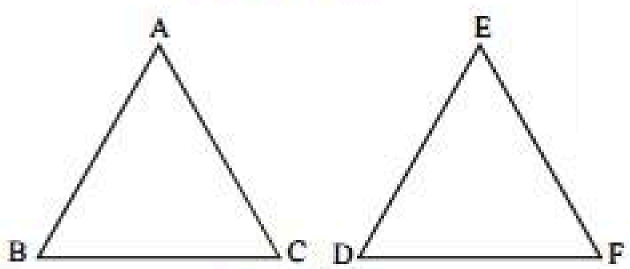
5. **(b)** $a = 5, t_{100} = -292$
$t_{100} = 5 + (100 - 1)d \qquad$ [using $t_n = a + (n-1)d$]
$\Rightarrow -292 = 5 + 99d$
$\Rightarrow -292 - 5 = 99d$

$\Rightarrow d = \frac{-297}{99} \Rightarrow d = -3$

$\therefore t_{50} = 5 + (50 - 1)(-3) = 5 + (-147)$
$= 5 - 147 \Rightarrow t_{50} = -142$

6. **(c)** In ΔABC and ΔEDF,

$$\frac{AB}{DE} = \frac{BC}{FD} \qquad \text{[Given]}$$

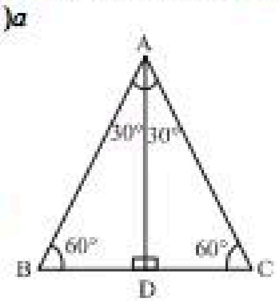


$\Delta ABC \sim \Delta EDF$
By converse of basic proportionality theorem
Then, $\quad \angle B = \angle D, \angle A = \angle E$
$\angle C = \angle F$

7. **(b)** **Hint:** Using distance formula

8. **(b)** $a_n = a + (n-1)d = a + (n-1)2a \quad [\because d = 3a - a = 2a]$
$= a + 2an - 2a = 2an - a = (2n - 1)a$

9. **(a)** We know that height of an equilateral triangle $\frac{\sqrt{3}}{2}a$, where a is the side of equilateral triangle



$\therefore AD^2 = \frac{3}{4}a^2 = \frac{3}{4}BC^2$

10. **(b)** [**Hint.** $OP \perp PT, OQ \perp QT$.
In quad. OPTQ, $\angle POQ + \angle OPT + \angle PTQ + \angle OQT = 360°$
$\Rightarrow \quad 110° + 90° + \angle PTQ + 90° = 360°$
$\Rightarrow \quad \angle PTQ = 70°$]

11. **(d)** Let $\operatorname{cosec} x - \cot x = \frac{1}{3}$

$\Rightarrow \quad \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \frac{1}{3}$

$\Rightarrow \quad \frac{1 - \cos x}{\sin x} = \frac{1}{3} \Rightarrow \frac{2\sin^2 \frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \frac{1}{3}$

$\Rightarrow \quad \tan\frac{x}{2} = \frac{1}{3}$

Consider

$\tan x = \frac{2\tan\frac{x}{2}}{1 - \tan^2\frac{x}{2}} = \frac{\frac{2}{3}}{1 - \frac{1}{9}} = \frac{3}{4}$

Thus $\sin x = \frac{3}{5}, \cos x = \frac{4}{5}$

$\therefore \quad \cos^2 x - \sin^2 x = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

12. **(b)** **Hint:** Using section formula

13. **(a)** The curved surface area of new solid
$= 2\pi r^2 + 2\pi r^2 = 4\pi r^2$

14. **(d)** As $(a, 0), (0, b)$ and $(1, 1)$ are collinear
By using distance formula,

we get, $1 = \frac{1}{a} + \frac{1}{b}$

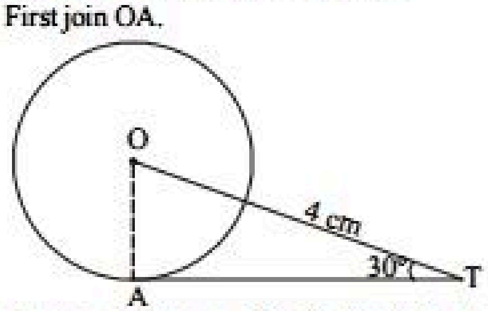
15. **(a)** Required probability $= \frac{4}{6} = \frac{2}{3}$.

16. **(c)** Total number of cards = 25
Prime number are 3, 5, 7, 11, 13, 17, 19, 23,

$\therefore \quad$ Probability of prime number card $= \frac{8}{25}$

17. **(a)** **Hint:** Probability of head and tail.

18. **(c)** First join OA.



Then the tangent at any point of a circle is $\perp$ to the radius through the point of contact.

$\therefore \quad \angle OAT = 90^\circ$

In ΔOAT, $\cos 30^\circ = \frac{AT}{OT}$

$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AT}{4} \Rightarrow AT = 2\sqrt{3}$ cm

19. **(d)** Here, reason is true [standard result]

Assertion is false. $\because \frac{3072}{16} = 192 \neq 162$

20. **(c)** Let $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ are all rational coordinates of a triangle ABC.

According to question, $\frac{\sqrt{3}}{4}a^2 = \frac{1}{2} \times b \times h$.

Suppose AB as base which will provide us the rational number then, height would be irrational.

Hence, (x_1, y_1) (x_2, y_2) & (x_3, y_3) cannot be all rational. Then, third coordinate would be irrational.

21. For the given quadratic polynomial $f(x) = ax^2 + bx + c$

Sum of roots = $(\alpha + \beta) = -\frac{b}{a}$

and product of roots = $\alpha\beta = \frac{c}{a}$

$$\therefore \quad \frac{\alpha^4 + \beta^4}{\alpha^2\beta^2} = \frac{(\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$$

$$= \frac{\left[(\alpha + \beta)^2 - 2\alpha\beta\right]^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$$ [1 Mark]

$$= \frac{\left[\left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)\right] - 2\left(\frac{c}{a}\right)^2}{\left(\frac{c}{a}\right)^2} = \frac{\left[\frac{b^2}{a^2} - 2\frac{c}{a}\right]^2 - 2\frac{c^2}{a^2}}{\frac{c^2}{a^2}}$$

$$= \frac{\frac{b^4}{a^4} + 4\frac{c^2}{a^2} - 4\frac{cb^2}{a^3} - 2\frac{c^2}{a^2}}{\frac{c^2}{a^2}}$$ [½ Mark]

$$= \frac{b^4 + 4a^2c^2 - 4acb^2 - 2a^2c^2}{a^4 \times \frac{c^2}{a^2}}$$

$$= \frac{b^4 + 2a^2c^2 - 4acb^2}{a^2c^2}$$ [½ Mark]

22. $\frac{x}{10} + \frac{y}{5} - 1 = 0$ and $\frac{x}{8} + \frac{y}{6} = 15$

i.e. $x + 2y = 10$ ...(1)

and $3x + 4y = 360$ ... (2)

Multiplying equation (1) by 2 and equation (2) by 1. We have

$2x + 4y = 20$ ... (3)

$3x + 4y = 360$ ... (4) [½ Mark]

Subtracting (3) from (4), we get

$x = 340$

and $y = -165$ (from (1)) [½ Mark]

Now, $y = \lambda x + 5$

$\Rightarrow \quad \lambda = \frac{y-5}{x} = \frac{-165-5}{340} = \frac{-170}{340} = \frac{-1}{2}$ [1 Mark]

$x = 340, y = -165$ and $\lambda = -\frac{1}{2}$

OR

Let fixed charge be ₹ x and cost of food per day be ₹ y

$x + 20y = 3000$ ...(i)

$x + 25y = 3500$ ...(ii)

By using Elimination Method

$x + 25y = 3500$

$x + 20y = 3000$

$- \quad - \quad -$

$5y = 500$ [1 Mark]

$\Rightarrow \quad y = 100$

$x + 20(100) = 3000$ From (i)

$\Rightarrow \quad x = 1000$

$\therefore \quad x = 1000$ and $y = 100$ [1 Mark]

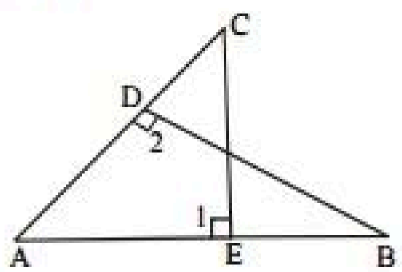
Fixed charge and cost of food per day are ₹ 1000 and ₹ 100.

23. **Given:** In the fig., $BD \perp AC$ and $CE \perp AB$

To prove:

(i) $\Delta AEC \sim \Delta ADB$

(ii) $\frac{CA}{AB} = \frac{CE}{DB}$



Proof:

(i) In ΔAEC and ΔADB

$\angle 1 = \angle 2$ (each 90°)

$\angle A = \angle A$ (common)

$\therefore \Delta AEC \sim \Delta ADB$ (by AA rule) [1 Mark]

(ii) $\Delta AEC \sim \Delta ADB$

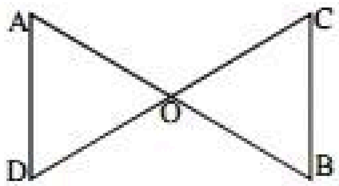
$\frac{CA}{AB} = \frac{CE}{DB}$ ($\because$ Angles are similar) [1 Mark]

$\therefore$ corresponding sides are proportional

Hence proved

OR

Given : $\frac{OA}{OC} = \frac{OD}{OB}$



To prove: $\angle A = \angle C$ and $\angle B = \angle D$

Proof: In ΔAOD and ΔBOC

$\frac{OA}{OC} = \frac{OD}{OB}$ (Given)

and $\angle AOD = \angle BOC$ (Vertically opposite angles)

$\therefore \Delta AOD \sim \Delta BOC$ (by SAS) [1 Mark]

$\therefore \angle A = \angle C$ and $\angle B = \angle D$ (C.P.S.T.) [1 Mark]

24. Let the required ratio be K : 1

$\therefore$ The co-ordinates of the required point on the y-axis is

$x = \frac{K(-4)+1(3)}{K+1}$; $y = \frac{K(2)+1(5)}{K+1}$ [½ Mark]

Since, it lies on y-axis

$\therefore$ Its x-coordinates = 0

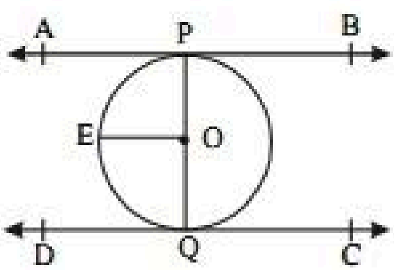
$\therefore \frac{-4K+3}{K+1} = 0 \Rightarrow -4K+3 = 0 \Rightarrow K = \frac{3}{4}$ [1 Mark]

$\Rightarrow$ Required ratio $= \frac{3}{4} : 1 = 3 : 4$ [½ Mark]

25. Given : Tangents AB and DC are parallel

Prove : PQ passing through centre O

Const : Draw EO ∥ AB



Proof : AB ∥ DC

$\therefore$ Sum of the angles on the same side of transversal is 180°

$\angle APO + \angle EOP = 180°$ [1 Mark]

$\angle EOP = 180° - 90° = 90°$

Similarly, $\angle EOB = 90°$ [1 Mark]

$\therefore$ $\angle EOP + \angle EOB = 90° + 90° = 180°$

$\therefore$ PQ is a straight line

Hence PQ passing through the centre O.

26. $\alpha + \beta = -\frac{5}{2}$ and $\alpha\beta = \frac{k}{2}$ [$\because$ Sum of roots $= -\frac{b}{a}$ and product of roots $= \frac{c}{a}$] [1 Mark]

$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$

Now $\alpha^2 + \beta^2 + \alpha\beta = (\alpha+\beta)^2 - \alpha\beta$

$\Rightarrow (\alpha+\beta)^2 - \alpha\beta = \frac{21}{4} \Rightarrow \frac{25}{4} - \frac{k}{2} = \frac{21}{4}$

$\Rightarrow \frac{25-2k}{4} = \frac{21}{4} \Rightarrow 2k = 4 \Rightarrow k = 2$ [2 Marks]

27. Since, $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{a}+\frac{1}{c}\right)$ and $c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

So, $2b\left(\frac{1}{a}+\frac{1}{c}\right) = a\left(\frac{1}{b}+\frac{1}{c}\right) + c\left(\frac{1}{a}+\frac{1}{b}\right)$ [1 Mark]

$\Rightarrow \frac{2b}{a} + \frac{2b}{c} = \frac{a}{b} + \frac{a}{c} + \frac{c}{a} + \frac{c}{b} \Rightarrow \frac{2b}{a} - \frac{c}{a} + \frac{2b}{c} - \frac{a}{c} = \frac{a}{b} + \frac{c}{b}$

$\Rightarrow \frac{2b-c}{a} + \frac{2b-a}{c} = \frac{a+c}{b}$

$\Rightarrow [2bc - c^2 + 2ab - a^2]\, b = [a+b]\, ac$

$\Rightarrow 2b^2c - b^2c + 2ab^2 - a^2b = a^2c + ac^2$

$\Rightarrow 2b^2(a+c) = a^2(b+c) + c^2(a+b)$ [½ Mark]

$\Rightarrow 2b^2(a+c) = a^2c + c^2a + a^2b + c^2b$

$\Rightarrow 2b^2(a+c) = ac(a+c) + b(a^2 + c^2 + 2ac - 2ac)$ [½ Mark]

$\Rightarrow 2b^2(a+c) + 2abc = ac(a+c) + b(a+c)^2$ [½ Mark]

$\Rightarrow 2b(ab+bc+ca) = (a+c)(ab+bc+ca)$ [½ Mark]

$\Rightarrow 2b = a + c$ ($\because ab+bc+ca \neq 0$)

So, a, b, c are in A.P. (Hence proved.)

28. LHS $= \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$

$= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1}$

$= \frac{\cot A + \operatorname{cosec} A - [(\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)]}{\cot A - \operatorname{cosec} A + 1}$

[2 Marks]

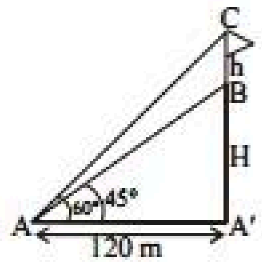
$= \frac{(\cot A + \operatorname{cosec} A)[1 - \operatorname{cosec} A + \cot A]}{\cot A - \operatorname{cosec} A + 1} = \cot A + \operatorname{cosec}$

$$A = \frac{\cos A}{\sin A} + \frac{1}{\sin A} = \frac{1 + \cos A}{\sin A}$$ [1 Mark]

Also $\frac{1}{\sin A} + \frac{\cos A}{\sin A} = \operatorname{cosec} A + \cot A$

$$\therefore \quad \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \frac{1 + \cos A}{\sin A} = \operatorname{cosec} A + \cot A$$

Hence LHS = RHS

29. 

[1 Mark]

Let A′B is the tower of height 'H' and BC is the height of flag 'h'. A is the point on ground

Now, in ΔBA'A

$$\tan 45° = \frac{H}{120} \Rightarrow H = 120 \text{ m}$$ [1 Mark]

Now, in ΔCA'A

$$\tan 60° = \frac{h + H}{120} \Rightarrow \sqrt{3} = \frac{h + 120}{120}$$

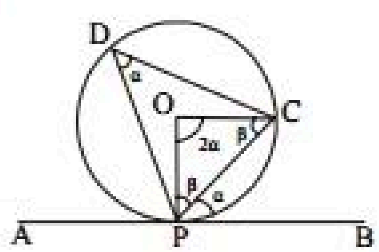
$\Rightarrow h = 120\sqrt{3} - 120$

$\Rightarrow h = 120\left(\sqrt{3} - 1\right) \Rightarrow h = 120\,(1.73 - 1)$

$\Rightarrow h = 120 \times 0.73 \Rightarrow h = 87.6 \text{ m}$ [1 Mark]

Height of flag = 87.6 m

30. Suppose there is a circle with centre O and APB is tangent at the point of contact P.



Let $\angle CPB = \alpha$ [1 Mark]

Then, $\angle PDC = \alpha = \angle CPB$

[Angle made in remaining portion of circle]

Consider $\angle OPC = \beta$

$\Rightarrow \angle OCP = \beta$ $(\because OP = OC = \text{radii of circle})$

Now, $\angle POC = 2\angle PDC$ [Angle subtends on centre]

$= 2\alpha$ [1 Mark]

In ΔOPC,

$\angle OPC + \angle PCO + \angle POC = 180°$ [By angle sum property]

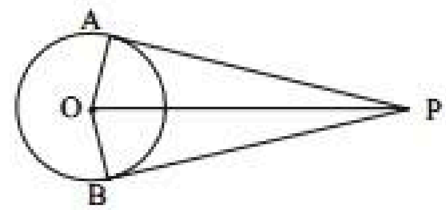
$\Rightarrow \beta + \beta + 2\alpha = 180°$

$\Rightarrow 2\beta + 2\alpha = 180°$

$\therefore \alpha + \beta = 90° = \angle CPB + \angle OPC = \angle OPB$ [1 Mark]

Hence, tangent at any point of a circle is perpendicular to the radius through the point of contact.

OR



Given : PA and PB are two tangents from an external point P.

Prove : PA = PB

Const : Join OA, OB and OP.

Proof : We know that tangent is perpendicular to the radius of circle at point of contact.

$\therefore \quad \angle OAP = \angle OBP = 90°$ [1 Mark]

In ΔAOP and ΔBOP

OP = OP (Common)

$\angle OAP = \angle OBP$ (each 90°) [1 Mark]

OA = OB (Radii)

$\Delta AOP \cong \Delta BOP$ (RHS)

$\therefore$ PA = PB (C.P.C.T) [1 Mark]

Hence Proved.

31. Number of 50 p coins = 100

Number of ₹ 1 coins = 50

Number of ₹ 2 coins = 20

Number of ₹ 5 coins = 10

Total number of coins = 180

$\Rightarrow$ Total no. of outcomes = 180 [½ Mark]

Then (i) $P\,(50 \text{ p coin}) = \frac{100}{180} = \frac{5}{9}$ [1 Mark]

(ii) P (not a ₹ 5 coin) = 1 – P (a ₹ 5 coin) [½ Mark]

$$= 1 - \frac{10}{180} = 1 - \frac{1}{18} = \frac{17}{18}$$ [1 Mark]

OR

Here sample space is given as: S = {1, 2, 3, ..., 100 }

$\therefore$ n(S) = 100 [1 Mark]

Let E be an event when doublet page is found.

E = {11, 22, 33, 44, 55, 66, 77, 88, 99} [1 Mark]

n(E) = 9 [½ Mark]

$\therefore$ required probability = $P(E) = \frac{9}{100}$ [½ Mark]

32. $(x + 4)(x + 5) = 3(x + 1)(x + 2) + 2x$

$\Rightarrow \quad x^2 + 9x + 20 = 3(x^2 + 3x + 2) + 2x$ [½ Mark]

$\Rightarrow \quad x^2 + 9x + 20 = 3x^2 + 9x + 6 + 2x$ [½ Mark]

$\Rightarrow\ x^2 - 3x^2 + 9x - 9x - 2x + 20 - 6 = 0$ [½ Mark]

$\Rightarrow\ -2x^2 - 2x + 14 = 0 \Rightarrow x^2 + x - 7 = 0$ [½ Mark]

We have

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-1 \pm \sqrt{1 - 4 \times 1(-7)}}{2 \times 1}$ [1 Mark]

$\Rightarrow x = \frac{-1 \pm \sqrt{1+28}}{2} \Rightarrow x = \frac{-1 \pm \sqrt{29}}{2}$ [2 Marks]

OR

The equation

$(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$

As, the roots are equal, D = 0 [½ Mark]

$\Rightarrow\ [-2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$ [½ Mark]

$\Rightarrow\ 4(a^4 + b^2c^2 - 2a^2bc - c^2b^2 + ac^3 + ab^3 - a^2bc) = 0$

[½ Mark]

$\Rightarrow\ a^4 + b^2c^2 - 2a^2bc - c^2b^2 + ac^3 + ab^3 - a^2bc = 0$

[½ Mark]

$\Rightarrow\ a^4 - 3a^2bc + ac^3 + ab^3 = 0$ [½ Mark]

$\Rightarrow\ a[a^3 - 3abc + c^3 + b^3] = 0$ [½ Mark]

Either $a = 0$ or $a^3 - 3abc + c^3 + b^3 = 0$ [1 Mark]

$\Rightarrow\ a^3 + b^3 + c^3 = 3abc.$ [1 Mark]

33. Let $\cos\theta + \sqrt{3}\sin\theta = 2\sin\theta$

$\Rightarrow \cos\theta = 2\sin\theta - \sqrt{3}\sin\theta = (2 - \sqrt{3})\sin\theta$ [1 Mark]

Multiplying both sides by $2 + \sqrt{3}$, we get

$(2+\sqrt{3})\cos\theta = (2+\sqrt{3})(2-\sqrt{3})\sin\theta$ [1 Mark]

$\Rightarrow (2+\sqrt{3})\cos\theta = \{(2)^2 - (\sqrt{3})^2\}\sin\theta$ [1 Mark]

$\Rightarrow 2\cos\theta + \sqrt{3}\cos\theta = (4-3)\sin\theta$ [1 Mark]

$\Rightarrow 2\cos\theta + \sqrt{3}\cos\theta = \sin\theta \Rightarrow \sin\theta - \sqrt{3}\cos\theta = 2\cos\theta.$

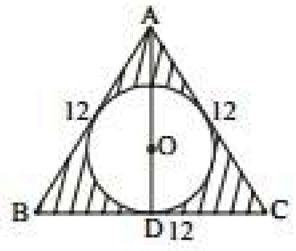
[1Mark]

34. Median and altitude of an equilateral triangle are same and passing through the centre of incircle and centre divides the median in ratio 2 : 1.

$\therefore$ In ΔABD, $\angle D = 90°$

By Pythagoras theorem

$AD^2 = AB^2 - BD^2 \Rightarrow AD^2 = 12^2 - 6^2$



$\Rightarrow AD^2 = 144 - 36 \Rightarrow AD = \sqrt{108} = 6\sqrt{3}$cm

AO : OD = 2 : 1 [1 Mark]

$\therefore\ OD = \frac{1}{3}AD = \frac{1}{3} \times 6\sqrt{3} = 2\sqrt{3}$ [½ Mark]

Now, radius of circle $= 2\sqrt{3} = 2 \times 1.73 = 3.46$cm

[½ Mark]

Area of shaded region = Area of equilateral ΔABC – Area of circle [1 Mark]

$= \frac{\sqrt{3}}{4}(12)^2 - (2\sqrt{3})^2 \times \pi = 1.73 \times 36 - 12 \times 3.14$

$= 62.28 - 37.68 = 24.6\ cm^2.$ [2 Marks]

35. Here, maximum frequency = 9, so modal class is 60 – 90

Mode $= L + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$ [2 Marks]

Here, $L = 60$, $f_1 = 9$, $f_0 = 6$, $f_2 = 6$ and $h = 30$. [1 Mark]

Mode $= 60 + 30\left(\frac{9-6}{2 \times 9 - 6 - 6}\right) = 60 + \frac{30 \times 3}{6} = 75$

[2 Marks]

OR

x_i	f_i	$x_i f_i$
3	10	30
9	p	$9p$
15	4	60
21	7	147
27	q	$27q$
33	4	132
39	1	39
Total	$\Sigma f_i = 26 + p + q$	$\Sigma x_i f_i = 408 + 9p + 27q$

[2 Marks]

Given $\Sigma fi = 40 \Rightarrow 26 + p + q = 40 \Rightarrow p + q = 14$ ...(i)

[½ Mark]

$\therefore$ Mean, $\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} \Rightarrow 14.7 = \frac{408 + 9p + 27q}{40}$

$\Rightarrow\ 588 = 408 + 9p + 27q \Rightarrow p + 3q = 20$...(ii) [1 Mark]

Subtracting eq.(i) from eq. (ii),

$2q = 6 \Rightarrow q = 3$ [½ Mark]

Putting this value of q in eq. (i),

$p = 14 - q = 14 - 3 = 11$

$\therefore\ p = 11, q = 3$ [1 Mark]

36. **(i)** LCM of 60, 84, 108 is

$12 \times 5 \times 7 \times 9 = 3780$ [1 Mark]

(ii) $108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$ [1 Mark]

(iii) For maximum number of participants, taking HCF of 60, 84 and 108

$$\begin{array}{c|l} 12 & 60,84,108 \\ \hline & 5,7,9 \end{array}$$ [2 Marks]

= 12

OR

Minimum number of rooms required are

$5 + 7 + 9 = 21$ [2 Marks]

37. **(i)** $AC^2 = 30^2 + 40^2 = 2500 \Rightarrow AC = 50m$ [1 Mark]

(ii) 82m [1 Mark]

(iii) $(21, 20, 28) \because 28^2 \neq (21)^2 + (20)^2$ [2 Marks]

OR

$AB = 50 - 12 = 38m$ [2 Marks]

38. **(i)** Volume of hemispherical dome

$$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 = 19404 \text{ cu. m}$$

[1 Mark]

(ii) $216 m^2$. [1 Mark]

(iii) Cloth required to cover hemispherical done = curved surface area of hemisphere

$= 2\pi r^2 = 1232$ sq. m [2 Marks]

OR

Surface area of combined figure

$= 2\pi r^2 + 2(l + b)h$

$= 1232 + 2(6 + 4)8 = 1392$ sq. m