CBSE Class 11 Mathematics

Important Questions

Chapter 11

Conic Sections

1 Marks Questions

1. Find the equation of a circle with centre (P,Q) & touching the y axis

$(A)\, x^2 + y^2 + 2Qy + Q^2 = 0$

$(B)\, x^2 + y^2 - 2px + 2Qy + Q^2 = 0$

$(C)\, x^2 + y^2 - 2px + 2Qy + Q^2 = 0$

(D) none of these

Ans. $x^2 + y^2 - 2px + 2Qy + Q^2 = 0$

2.Find the equations of the directrix & the axis of the parabola $\Rightarrow 3x^2 = 8y$

$(A)\, 3y - 4 = 0, x = 0$

$(B)\, 3x - 4 = 0, y = 0$

$(C)\, 3y - 4x = 0$

(D) none of these

Ans. $3y - 4 = 0, x = 0$

3.Find the coordinates of the foci of the ellipse $\Rightarrow x^2 + 4y^2 = 100$

$(A)\, F\left(\pm 5\sqrt{3}, 0\right)$

$(B)\, F\left(\pm 3\sqrt{5}, 0\right)$

$(C)\, F\left(\pm 4\sqrt{5}, 0\right)$

(D) none of these

Ans. $F\left(\pm 5\sqrt{3}, 0\right)$

4.Find the eccentricity of the hyperbola: $3x^2 - 2y^2 = 6$

$(A)\, e = \sqrt{\frac{5}{2}}$ $(B)\, e = \frac{\sqrt{5}}{2}$ $(C)\, e = \frac{\sqrt{2}}{5}$ (D) **none of these**

Ans. $e = \sqrt{\frac{5}{2}}$

5.Find the equation of a circle with centre (b, a) **& touching** $x-$ **axis?**

$(A)\, x^2 + y^2 - 2bx + 2ay + b^2 = 0$

$(B)\, x^2 + y^2 + 2bx - 2ay + b^2 = 0$

$(C)\, x^2 + y^2 - 2bx - 2ay + b^2 = 0$

(D) none of these

Ans. $x^2 + y^2 - 2bx - 2ay + b^2 = 0$

6.Find the lengths of axes of $3x^2 - 2y^2 = 6$?

$(A)\, 2\sqrt{2}$ & $2\sqrt{5}$ units

$(B)\, 2\sqrt{2}$ & $2\sqrt{3}$ units

$(C) 2\sqrt{5}$ & $2\sqrt{2}$ units

(D) none of these

Ans. $2\sqrt{2}$ Units & $2\sqrt{3}$ units

7.Find the length of the latus rectum of $3x^2 + 2y^2 = 18$?

(A) 2 units (B) 3 units (C) 4 units (D) none of these

Ans.4 units

8.Find the length of the latus rectum of the parabola $3y^2 = 8x$

$(A) \frac{4}{3}$ units $(B) \frac{8}{3}$ units $(C) \frac{2}{3}$ units (D) none of these

Ans. $\frac{8}{3}$ units

9.The equation $x^2 + y^2 - 12x + 8y - 72 = 0$ **represent a circle find its centre**

$(A)(-6,-4)$ $(B)(6,-4)$ $(C)(6,4)$ $(D)(-6,4)$

Ans. $(6,-4)$

10.Find the equation of the parabola with focus $F(4,0)$ **& directrix** $x = -4$

$(A) y^2 = 32x$ $(B) y^2 = -16x$ $(C) y^2 = 8x$ $(D) y^2 = 16x$

Ans. $y^2 = 16x$

11.Find the coordinates of the foci of $\frac{x^2}{8} + \frac{y^2}{4} = 1$

$(A) F_1(2, 0) \& F_2(-2, 0)$

$(B) F_1(-2, 0) \& F_2(2, 0)$

$(C) F_1(-2, 0) \& F_2(-2, 0)$

(D) none of these

Ans. $F_1(-2, 0) \& F_2(2, 0)$

12.Find the coordinates of the vertices of $x^2 - y^2 = 1$

$(A) A(-1, 0), B(-1, 0)$

$(B) A(-1, 0), B(1, 0)$

$(C) A(1, 0), B(-1, 0)$

(D) none of these

Ans. $A(-1, 0), B(1, 0)$

13.Find the coordinates of the vertices of $x^2 - y^2 = 1$

$(A)\ A(-1, 0)\ \&\ B(5, 0)$

$(B) A(-5, 0)\ \&\ B(-1, 0)$

$(C) A(-1, 0)\ \&\ B(-5, 0)$

(D) none of these

Ans. $A(-1, 0) \& B(5, 0)$

14.Find the eccentricity of ellipse $4x^2+9y^2=1$

$(A)\ e=\frac{\sqrt{5}}{3}$ $(B)\ e=\frac{-\sqrt{5}}{3}$ $(C)\ e=\frac{\sqrt{3}}{5}$ $(D)\ e=\frac{3}{\sqrt{5}}$

Ans. $e=\frac{\sqrt{5}}{3}$

15.Find the length of the latus rectum of $9x^2+y^2=36$

$(A)\frac{1}{3}$ units $(B)\frac{1}{5}$ units $(C)1\frac{1}{3}$ units $(D)\frac{1}{6}$ units

Ans. $1\frac{1}{3}\,units$

16.Find the length of minor axis of $x^2+4y^2=100$

(A)10 units (B)12 units (C)14 units (D)8 units

Ans. $10units$

17.Find the centre of the circles $x^2+(y-1)^2=2$

$(A)\ (1,0)$ $(B)\ (0,1)$ $(C)\ (1,2)$ (D) **None of these**

Ans. $(0,1)$

18.Find the radius of circles $x^2+(y-1)^2=2$

$(A)\sqrt{2}$ $(B)\,2$ $(C)\,2\sqrt{2}$ (D) **None of these**

Ans. $\sqrt{2}$

19.Find the length of latcus rectum of $x^2 = -22y$

$(A) 11$ $(B) -22$ $(C) 22$ (D) **None of these**

Ans.22

20.Find the length of latcus rectum of $25x^2 + 4y^2 = 100$

$(A) \frac{3}{5}$ units $(B) \frac{1}{5}$ units $(C) \frac{8}{5}$ units (D) **None of these**

Ans. $\frac{8}{5}$ Units

CBSE Class 12 Mathematics

Important Questions

Chapter 11

Conic Sections

4 Marks Questions

1.Show that the equation $x^2+y^2-6x+4y-36=0$ represent a circle, also find its centre & radius?

Ans. This is of the form $x^2+y^2+2gx+2Fy+c=0,$

where $2g=-6, 2f=4 \& c=-36$

$\therefore q=-3, f=2 \& c=-36$

So, centre of the circle $=(-g,-f)=(3,-2)$

&

Radius of the circle $=\sqrt{q^2+f^2-c}=\sqrt{9+4+36}$

$=7$ units

2.Find the equation of an ellipse whose foci are $(\pm 8,0)$ & the eccentricity is $\frac{1}{4}$?

Ans. Let the required equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$ where $a^2>b^2$

let the foci be $(\pm c,0), c=8$

&

$$e=\frac{c}{a}\Leftrightarrow a=\frac{c}{e}=\frac{8}{\frac{1}{4}}=32$$

Now $c^2=a^2-b^2\Leftrightarrow b^2=a^2-c^2=1024-64=960$

$\therefore a^2=1024 \quad \& \quad b^2=960$

Hence equation is $\frac{x^2}{1024}+\frac{y^2}{960}=1$

3.Find the equation of an ellipse whose vertices are $(0,\pm 10)$ & $e=\frac{4}{5}$

Ans. Let equation be $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

& its vertices are $(0,\pm a) \quad \& \quad a=10$

Let $c^2=a^2-b^2$

Then $e=\frac{c}{a} \quad \Rightarrow \quad c=ae=10\times\frac{4}{5}=8$

Now $c^2=a^2-b^2\Leftrightarrow b^2=\left(a^2-c^2\right)=100-64=36$

$\therefore a^2=(10)^2=100 \quad \& \quad b^2=36$

Hence the equation is $\frac{x^2}{36}+\frac{y^2}{100}=1$

4.Find the equation of hyperbola whose length of latus rectum is 36 & foci are $(0,\pm 12)$

Ans. Clearly C = 12

Length of cat us rectum $= 36 \Leftrightarrow \frac{2b^2}{a} = 36$

$\Rightarrow b^2 = 18a$

Now $c^2 = a^2 + b^2 \Leftrightarrow a^2 = c^2 - b^2 = 144 - 18a$

$a^2 + 18a - 144 = 0$

$(a+24)(a-6) = 0 \Leftrightarrow a = 6 \quad [\because a \text{ is non negative}]$

This $a^2 = 6^2 = 36 \;\;\&\;\; b^2 = 108$

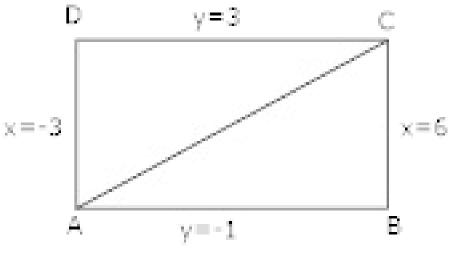
Hence, $\frac{x^2}{36} + \frac{y^2}{108} = 1$

5.Find the equation of a circle drawn on the diagonal of the rectangle as its diameter, whose sides are $x = 6,\ x = -3,\ y = 3 \;\;\&\;\; y = -1$

Ans. Let ABCD be the given rectangle &

$AD = x = -3,\ BC = x = 6,\ AB = y = -1 \;\&\; CD = y = -3$

Then $A(-3,-1) \;\;\&\;\; c(6,3)$



So the equation of the circle with AC as diameter is given as

$(x+3)(x-6) + (y+1)(y-3) = 0$

$\Rightarrow x^2 + y^2 - 3x - 2y - 21 = 0$

6.Find the coordinates of the focus & vertex, the equations of the diretrix & the axis &

length of latus rectum of the parabola $x = -8y$

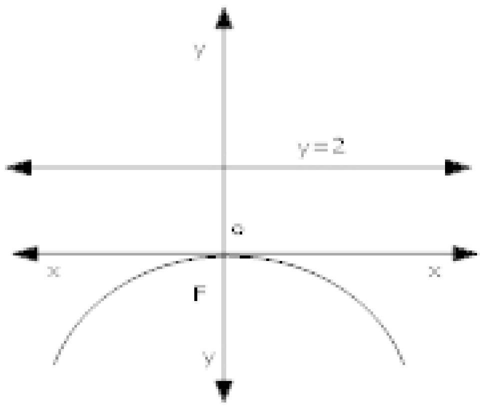
Ans. $x^2 = -8y$

& $x^2 = -4ay$

So, $4a = 8 \Leftrightarrow a = 2$

So it is case of downward parabola

o, foci is $F(0, -a)$ *ie* $F(0, -2)$



Its vertex is $0(0, 0)$

So, $y = a = 2$

Its axis is y – axis, whose equation is $x = 0$ length of lotus centum

$= 4a = 4 \times 2 = 8$ units.

7.Show that the equation $6x^2 + 6y^2 + 24x - 36y - 18 = 0$ **represents a circle. Also find its centre & radius.**

Ans. $6x^2 + 6y^2 + 24x - 36y + 18 = 0$

So $x^2 + y^2 + 4x - 6y + 3 = 0$

Where, $2g = 4, 2f = -6 \& C = 3$

$\therefore g = 2, f = -3 \& C = 3$

Hence, centre of circle $= (-g, -f) = (-2, 3)$

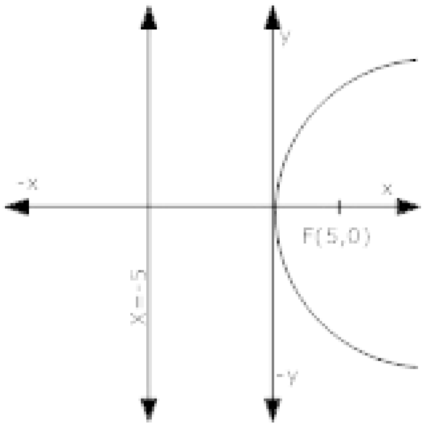
&

Radius of circle $= \sqrt{4+9+9} = \sqrt{20}$

$= 2\sqrt{5}$ units

8.Find the equation of the parabola with focus at $F(5,0)$ & directrix is $x = -5$

Ans. Focus $F(5,0)$ lies to the right hand side of the origin



So, it is right hand parabola.

Let the required equation be

$y^2 = 4ax \quad \& \quad a = 5$

So, $y^2 = 20x$

9.Find the equation of the hyperbola with centre at the origin, length of the transverse axis 18 & one focus at (0,4)

Ans. Let its equation be $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Clearly, C = 4.

length of the transverse axis $= 8 \Leftrightarrow 2a = 18$

$a = 9$

Also, $C^2 = \left(a^2 + b^2\right)$

$b^2 = c^2 - a^2 = 16 - 81 = -65$

So, $a^2 = 81 \;\; \& \;\; b^2 = -65$

So, equation is $\frac{y^2}{81} + \frac{x^2}{65} = 1$

10.Find the equation of an ellipse whose vertices are $(0, \pm 13)$ & the foci are $(0, \pm 5)$

Ans. Let the equation be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

& a = 13

Let its foci be $(0, \pm c)$, then $c = 5$

$\therefore b^2 = a^2 - c^2 = 169 - 25 = 144$

So, $a^2 = 169 \;\; \& \;\; b^2 = 144$

So, equation be $\frac{x^2}{144} + \frac{y^2}{169} = 1$

11.Find the equation of the ellipse whose foci are $(0, \pm 3)$ & length of whose major axis is 10

Ans. Let the required equation be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Let $c^2 = a^2 - b^2$

Its foci are $(0, \pm c)$ & $c = 3$

Also, a = length of the semi- major axis = $\frac{1}{2} \times 10 = 5$

Now, $c^2 = a^2 - b^2 \Rightarrow b^2 = a^2 - c^2 = 25 - 3 = 16.$

Then, $a^2 = 25$ & $b^2 = 16$

Hence the required equation is $\frac{x^2}{16} + \frac{y^2}{25} = 1.$

12.Find the equation of the hyperbola with centre at the origin, length of the transverse axis 8 & one focus at (0,6)

Ans. Let its equation by $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Clearly, C = 6

& length of the transverse axis $= 8 \Rightarrow 2a = 8 \Rightarrow a = 4$

Also, $c^2 = a^2 + b^2 \Leftrightarrow b^2 = c^2 - a^2 \Rightarrow 36 - 16 = 20$

So, $a^2 = 16$ & $b^2 = 20$

Hence, the required equation is $\frac{y^2}{16} - \frac{x^2}{20} = 1$

13.Find the equation of the hyperbola whose foci are at $(0, \pm B)$ & the length of whose conjugate axis is $2\sqrt{11}$

Ans. Let it equation be $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Let it foci be $\left(0,\pm C\right)$

$\therefore C=8$

Length of conjugate axis $=2\sqrt{11}$

$\Rightarrow 2b=2\sqrt{11}\Rightarrow b=\sqrt{11}\Rightarrow b^2=11$

Also, $C^2=\left(a^2+b^2\right)=\left(c^2-b^2\right)=64-11=53$

$a^2=53$

Hence, required equation is $\frac{y^2}{53}-\frac{x^2}{11}=1$

14.Find the equation of the hyperbola whose vertices are $\left(0,\pm 3\right)$ & foci are $\left(0,\pm 8\right)$

Ans. The vertices are $\left(0\pm a\right)$

But it is given that the vertices are $\left(0\pm 3\right)$

$\therefore a=3$

Let its foci be $\left(0,\pm c\right)$

But it is given that the foci are $\left(0,\pm 8\right)$

$\therefore c=8$

Now $b^2=\left(c^2-a^2\right)=8^2-3^2=64-9=55$

Then $a^2=3^2=9\ \&\ b^2=55$

Hence the required equation is $\frac{y^2}{9}-\frac{x^2}{55}=1$

15.Find the equation of the ellipse for which $e=\frac{4}{5}$ & whose vertices are $(0,\pm10)$.

Ans. Its vertices are $(0,\pm a)\&$ therefore a =10

Let $c^2=(a^2-b^2)$

Then, $e=\frac{c}{a}\Rightarrow c=ae=\left[10\times\frac{4}{5}\right]=8$

Now, $c^2=(a^2-b^2)\Rightarrow b^2=(a^2-c^2)=(100-64)=36$

$\therefore a^2=(10)^2=100\ \&\ b^2=36$

Hence the required equation is $\frac{x^2}{36}+\frac{y^2}{100}=1$

16.Find the equation of the ellipse, the ends of whose major axis are $(\pm7,0)$ & the ends of whose minor axis are $(0,\pm2)$

Ans. Its vertices are $(\pm a,0)$ & therefore, a = 5 ends of the minor axis are $c(0,-5)\ \&\ D(0,5)$

$\therefore CD=25$ i.e length of minor axis = 25 units

$\therefore 2b=25\Rightarrow\frac{25}{2}=12.5$

Now, $a=5\ \&\ b=12.5\ \Rightarrow\ a^2=25\ \&\ b^2=156.25$

Hence, the required equation $\frac{x2}{25}+\frac{y^2}{156.25}=1$

16.Find the equation of the parabola with vertex at the origin & y+5 = 0 as its directrix. Also, find its focus

Ans. Let the vertex of the parabola be $o(0,0)$

Now $y+5=0 \Rightarrow y=-5$

Then the directrix is a line parallel

To the x axis at a distance of 5 unite below the x axis so the focus is $F(0,5)$

Hence the equation of the parabola is

$x^2 = 4ay$ Where a = 5i.e, $x^2 = 20y$

17.Find the equation of a circle, the end points of one of whose diameters are $A(2,-3)$ & $B(-3,5)$.

Ans. Let the end points of one of whose diameters are (x_1, y_1) & (x_2, y_2) is given by

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

Hence $x_1 = 2,\ y_1 = -3$ & $x_2 = -3,\ y_2 = 5$

$\therefore$ The required equation of the circle is

$$(x-2)(x+3)+(y+3)(y-5)=0$$

$$\Rightarrow x^2+y^2+x-2y-21=0$$

18.Find the equation of ellipse whose vertices are $(0,\pm 13)$ & the foci are $(0,\pm 5)$

Ans. Let the required equation be $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ 5.

Its vertices are $(0 \pm a)$ & therefore a = 13

Let its foci be $(0 \pm C)$ then C = 5

$\therefore b^2 = a^2 - c^2 = 169 - 25 = 144$

This $b^2 = 144 \,\&\, a^2 = 169$

Hence, the required equation is $\frac{x^2}{144} + \frac{y^2}{169} = 1$

19.Find the equation of the hyperbola whose foci are $(\pm 5, 0)$ & the transverse axis is of length 8.

Ans. Let the required equation be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Length of its Trans verse axis =2a

$\therefore 2a = 8 \Leftrightarrow a = 4 \Leftrightarrow a^2 = 16$

Let its foci be $(\pm C, 0)$

Then C = 5

$\therefore b^2 = (c^2 - a^2) = 5^2 - 4^2 = 9$

This $a^2 = 16 \,\&\quad b^2 = 9$

Hence, the required equation is $\frac{x^2}{16} - \frac{y^2}{9} = 1$

20.Find the equation of a circle, the end points of one of whose diameters are

$A(-3, 2) \,\&\, B(5, -3)$.

Ans. Let the equation be $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Hence $x_1 = -3, y_1 = 2 \,\&\, x_2 = 5, y_2 = -3$

So $(x + 3)(x - 5) + (y - 2)(y + 3) = 0$

$$x^2 - 2x - 15 + y^2 + y - 6 = 0$$
$$x^2 + y^2 - 2x + y - 21 = 0$$

21.If eccentricity is $\frac{1}{5}$ & foci are $(\pm 7, 0)$ find the equation of an ellipse.

Ans. Let the required equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Let its foci be $(\pm C, 0)$, Then C =7

Also,

$$e = \frac{c}{a} \Leftrightarrow a = \frac{c}{e} = \frac{7}{\frac{1}{5}} = 35$$

Now $c^2 = (a^2 - b^2)$

$$b^2 = a^2 - c^2 = (35)^2 - 49 = 1225 - 49 = 1176$$
$$\therefore a^2 = 1225 \,\&\, b^2 = 1176$$

Hence the required equation is $\frac{x^2}{1225} + \frac{y^2}{1176} = 1$

22.Find the equation of the hyperbola where foci are $(\pm 5, 0)$ & the transverse axis is of length

Ans. Let the required equation be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Length of its transverse axis $= 2a$

$\therefore 2a = 8 \Leftrightarrow a = \frac{8}{2} = 4$

$a^2 = 16$

Let its foci be $(\pm C, 0)$

Then C = 5

$\therefore b^2 = c^2 - a^2 = 25 - 16 = 9$

Hence the required equation is $\frac{x^2}{16} - \frac{y^2}{9} = 1$

23.Find the length of axes & coordinates of the vertices of the hyperbola $\frac{x^2}{49} - \frac{y^2}{64} = 1$

Ans. The equation of the given hyperbola is $\frac{x^2}{49} - \frac{y^2}{64} = 1$

Comparing the given equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$ we get

$a^2 = 49 \;\&\; b^2 = 64$

$\therefore C^2 = \left(a^2 + b^2\right) = 49 + 64 = 113$

Length of transverse axis = $2a = 2 \times 7 = 14$ units

Length of conjugate axis = $2b = 2 \times 8 = 16$ units

The coordinators of the vertices are $A(-a, 0) \;\&\; B\,(a - 0)\; ie\, A(-7, 0) \;\&\; B\,(7, 0)$

24.Find the lengths of axes & length of lat us rectum of the hyperbola, $\frac{y^2}{9} - \frac{x^2}{16} = 1$

Ans. The given equation is $\frac{y^2}{9}-\frac{x^2}{16}=1$ means hyperbola

Comparing the given equation with $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1,$ we get

$a^2=9 \quad \& \quad b^2=16$

Length of transverse axis $=2a=2\times3=6$ units

Length of conjugate axis $=2b=2\times4=8$ units

The coordinates of the vertices are $A(0,-a) \quad \& \quad B(0,a)$ i.e $A(0,-3) \quad \& \quad B(0,3)$

25.Find the eccentricity of the hyperbola of $\frac{y^2}{9}-\frac{x^2}{16}=1$

Ans. As in above question

$a=3 \quad \& \quad b=4$

&

$c^2=a^2+b^2=9+16=25$

So, c = 5

Then $e=\frac{c}{a}=\frac{5}{3}$

26.Find the equation of the hyperbola with centre at the origin, length of the trans verse axis 6 & one focus at $(0,4)$

Ans. Let its equation be $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

Clearly c = 4

Length of transverse axis $= 6 \Leftrightarrow 2a = 6 \Leftrightarrow a = 3$.

Also, $c^2 = a^2 + b^2 \Leftrightarrow b^2 = c^2 - a^2 = 4^2 - 3^2 = 16 - 9 = 7$

Then $a^2 = 3^2 = 9 \quad \& \quad b^2 = 7$

Hence, the required equation is $\frac{y^2}{9} - \frac{x^2}{7} = 1$

27.Find the equation of the ellipse, the ends of whose major axis are $(\pm 3, 0)$ & at the ends of whose minor axis are $(0, \pm 4)$

Ans. Let the required equation be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Its vertices are $(\pm a, 0) \quad \& \quad a = 3$

Ends of minor axis are $C(0, -4) \quad \& \quad D(0, 4)$

$\therefore CD = 8$ i.e length of the minor axis = 8 units

Now, $2b = 8 \Leftrightarrow b = 4$

$\therefore a = 3 \quad \& \quad b = 4$

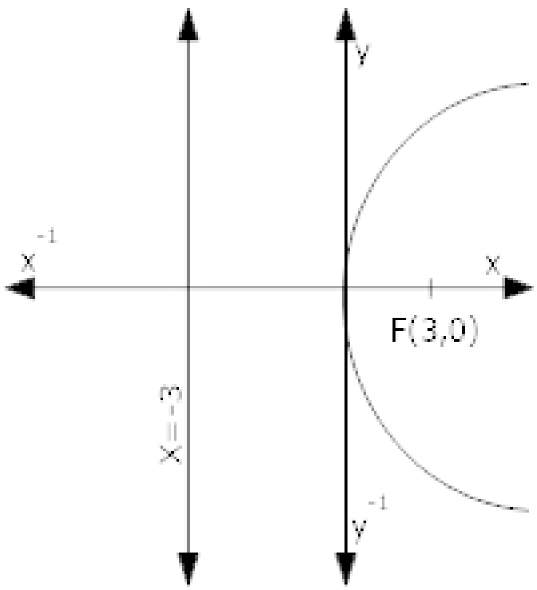
Hence the required equation is $\frac{x^2}{9} + \frac{y^2}{16} = 1$

28.Find the equation of the parabola with focus at $F(4, 0)$ & directrix $x = -3$

Ans. Focus $F(4, 0)$ lies on the axis hand side of the origin so, it is a right handed parabola. Let the required equation be $y^2 = 4ax$.

Than, a = 4

Hence, the required equation is $y^2 = 16x$



29.If $y = 2x$ is a chord of the circle $x^2 + y^2 - 10x = 0$, find the equation of the circle with this chord as a diameter

Ans. $y = 2x \quad \& \quad x^2 + y^2 - 10x = 0$

Putting $y = 2x$ in $x^2 + y^2 - 10x = 0$ we get

$5x^2 - 10x = 0 \Leftrightarrow 5x(x-2) = 0 \Leftrightarrow x = 0$ or $x = 2$

Now, $x = 0 \Rightarrow y = 0 \quad \& \quad x = 2 \Rightarrow y = 4$

$\therefore$ the points of intersection of the given chord & the given circle are

$A(0,0) \quad \& \quad B(2,4)$

$\therefore$ the required equation of the circle with AB as diameter is

$(x-0)(x-2)+(y-0)(y-4) = 0$

$\Rightarrow x^2 + y^2 - 2x - 4y = 0$

CBSE Class 12 Mathematics
Important Questions
Chapter 11
Conic Sections

6 Marks Questions

1. Find the length of major & minor axis- coordinate's of vertices & the foci, the eccentricity & length of latus rectum of the ellipse $16x^2 + y^2 = 16$

Ans. $16x^2 + y^2 = 16$

Dividing by 16,

$$x^2 + \frac{y^2}{16} = 1$$

So $b^2 = 1 \;\; \& \;\; a^2 = 16 \;\; \& \;\; b = 1 \;\; \& \;\; a = 4$

&

$$c = \sqrt{a^2 - b^2} = \sqrt{16 - 1}$$

$$= \sqrt{15}$$

Thus $a = 4, \;\; b = 1 \;\; \& \;\; c = \sqrt{15}$

(i) Length of major axis $= 2a = 2 \times 4 = 8$ units

Length of minor axis $= 2b = 2 \times 1 = 2$ units

(ii) Coordinates of the vertices are $A(-a, 0) \;\; \& \;\; B(a, 0) \;\; ie \;\; A(-4, 0) \;\; \& \;\; B(4, 0)$

(iii) Coordinates of foci are $F_1(-c, 0) \;\; \& \;\; F_2(c, 0) \;\; ie \;\; F_1\left(-\sqrt{15}, 0\right) \;\; \& \;\; F_2\left(\sqrt{15, 0}\right)$

(iv) Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{15}}{4}$

(v) Length of latus rectum $=\frac{2b^2}{a}=\frac{2}{4}=\frac{1}{2}$ units

2. Find the lengths of the axis , the coordinates of the vertices & the foci the eccentricity & length of the lat us rectum of the hyperbola $25x^2-9y^2=225$

Ans. $25x^2-9y^2=225 \Rightarrow \frac{x^2}{9}-\frac{y^2}{25}=1$

So, $a^2=9 \quad \& \quad b^2=25$

& $c=\sqrt{a^2+b^2}=\sqrt{9+25}=\sqrt{34}$

(i) Length of transverse axis $=2a=2\times 3=6$ units

Length of conjugate axis $=2b=2\times 5=10$ units

(ii) The coordinates of vertices are $A(-a,0) \quad \& \quad B(a,0) \quad ie \quad A(-3,0) \quad \& \quad B(3,0)$

(iii) The coordinates of foci are

$F_1,(-c,0) \quad \& \quad F_2(c,0) \quad ie \quad F_1\left(-\sqrt{34},0\right) \quad \& \quad F_2\left(\sqrt{34},0\right)$

(iv) Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{34}}{3}$

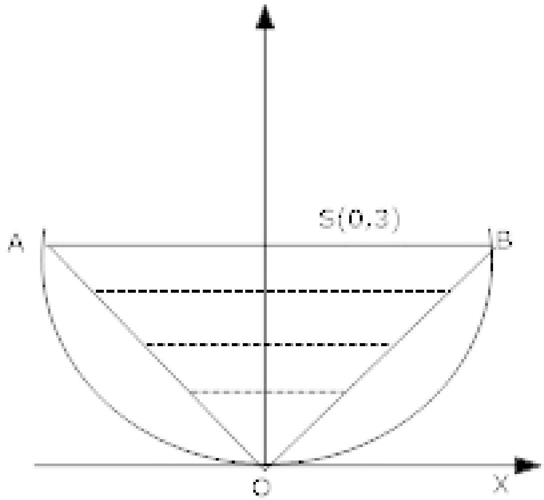
(v) Length of the lat us rectum $=\frac{2b^2}{a}=\frac{50}{3}$ units

3. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2=12y$ **to the ends of its latus rectum.**

Ans. The vertex of the parabola $x^2=12y \quad ie \quad o(0,0)$.

0	0	1
6	3	1
-6	3	1

Comparing $x^2 = 12y$ with $x^2 = 4ay$, we get $a = 3$ the coordinates of its focus S are $(0, 3)$.



Clearly, the ends of its latus rectum are : $A(-2a, a)$ & $B(2a, a)$

Ie $A(-6, 3)$ & $B(6, 3)$

$\therefore$ area of $\Delta OBA = \frac{1}{2}$

$= \frac{1}{2}\left[1 \times (18 + 18)\right]$

$= 18$ units.

4. A man running in a race course notes that the sum of the distances of the two flag posts from him is always 12 m & the distance between the flag posts is 10 m. find the equation of the path traced by the man.

Ans. We know that on ellipse is the locus of a point that moves in such a way that the sum of its distances from two fixed points (caked foci) is constant.

So, the path is ellipse.

Let the equation of the ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$$

where $b^2=a^2\left(1-c^2\right)$

Clearly, $2a=12 \quad \& \quad 2ae=10$

$$\Rightarrow a=b \quad \& \quad e=\frac{5}{6}$$

$$\Rightarrow b^2=a^2\left(1-e^2\right)=36\left(1-\frac{25}{36}\right)$$

$$\Rightarrow b^2=11$$

Hence, the required equation is $\frac{x^2}{36}+\frac{y^2}{11}=1$

5. An equilateral triangle is inscribed in the parabola $y^2=4ax$ so that one angular point of the triangle is at the vertex of the parabola. Find the length of each side of the triangle.

Ans. Let ΔPQR be an equilateral triangle inscribed in the parabola $y^2=4ax$

Let QP = QP = QR = PR = C

Let ABC at the $x-$ axis at M.

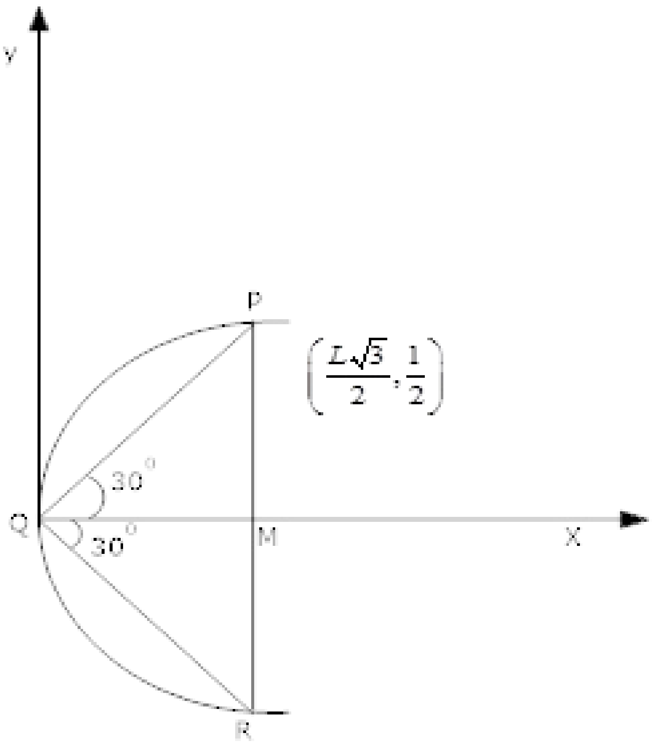
Then , $\angle \angle PQM=\angle RQWM=30^\circ$

$$\therefore \frac{QM}{QP}=\cos 30^\circ \Rightarrow QM=\angle \cos 30^\circ$$

$$\Rightarrow \frac{L\sqrt{3}}{2}$$

$\Rightarrow \frac{PM}{QP} = \sin 30^\circ \Rightarrow PM = \angle \sin 30^\circ$

$\Rightarrow \frac{L}{2}$



$\therefore$ the coordinates of are $\left[\frac{L\sqrt{3}}{2}, \frac{L}{2}\right]$

Since P lies on the parabola $y^2 = 4ax$, we have

$l^2 = 4a \times \frac{L\sqrt{3}}{2} \Rightarrow l = 8a\sqrt{3}$

Hence length of each side of the triangle is $8a\sqrt{3}$ units.

6. Find the equation of the hyperbola whose foci are at $\left(0, \pm\sqrt{10}\right)$ & which passes through the points $(2, 3)$

Ans. Let it equation be $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 ......(i)$

Let its foci be $(0, \pm C)$

But the foci are $(0, \pm\sqrt{10})$

$\therefore C = \sqrt{10} \Leftrightarrow C^2 = 10 \Leftrightarrow (a^2 + b^2) = 10 ......(ii)$

Since (i) passes through (2,3), we have $\frac{9}{a^2} - \frac{4}{b^2} = 1$

Now

$\frac{9}{a^2} - \frac{4}{b^2} = 1 \Leftrightarrow \frac{9}{a^2} - \frac{4}{(10 - a^2)} = 1 ......(iii)$

$\Rightarrow 9(10 - a^2) - 4a^2 = a^2(10 - a^2)$

$\Rightarrow a^2 - 23a^2 + 90 = 0$

$\Rightarrow (a^2 - 18)(a^2 - 5) = 0 \Leftrightarrow a^2 = 5$

[$\because a^2 = 18 \Rightarrow b^2 = -8$, which is not possible]

Then $a^2 = 5 \;\; \& \;\; b^2 = 5$

Hence, the required equation is $\frac{y^2}{5} - \frac{x^2}{5} = 1$,

i.e. $y^2 - x^2 = 5$

7. Find the equation of the curve formed by the set of all these points the sum of whose distance from the points $A(4, 0, 0) \;\; \& \;\; B(-4, 0, 0)$ is 10 units.

Ans. Let $P(x, y, z)$ be an arbitrary point on the given curve

Then $PA+PB=10$

$$\Rightarrow \sqrt{(x-4)^2+y^2+z^2}+\sqrt{(x+4)^2+y^2+z^2}=10$$

$$=\sqrt{(x+4)^2+y^2+z^2}=10-\sqrt{(x-4)^2+y^2+z^2}\ldots\ldots(i)$$

Squaring both sides

$$\Rightarrow (x+4)^2+y^2+z^2=100-(x-4)^2+y^2+z^2-20\sqrt{(x-4)^2+y^2+z^2}$$

$$\Rightarrow 16x=100-20\sqrt{(x-4)^2+y^2+z^2}$$

$$\Rightarrow 5\sqrt{(x-4)^2+y^2+z^2}=25-4x$$

$$\Rightarrow 25\left[(x-4)^2+y^2+z^2\right]=625+16x^2-200x$$

$$\Rightarrow 9x^2+25y^2+25z^2-225=0$$

Hence, the required equation of the curve is

$$9x^2+25x^2+25z^2-225=0$$

8. Find the equation of the hyperbola whose foci are at $\left(0,\pm\sqrt{10}\right)$ & which passes through the point $(2,3)$.

Ans. Let its equation be $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1\ldots\ldots(i)$

Let its foci be $(0,\pm c)$

But, the foci are $\left(0,\pm\sqrt{10}\right)$

$$\therefore C=\sqrt{10}\Leftrightarrow C^2=10$$

& $a^2 + b^2 = 10 ......(ii)$

Since (i) passes through $(2, 3)$, we have

$$\frac{9}{a^2} - \frac{4}{b^2} = 1$$

Now

$$\frac{9}{a^2} + \frac{4}{b^2} = 1 \Leftrightarrow \frac{9}{a^2} - \frac{4}{(10 - a^2)} = 1$$

$$\Rightarrow a^4 - 23a^2 + 90 = 0$$

$$\Rightarrow (a^2 - 18)(a^2 - 5) = 0$$

$$\Rightarrow a^2 = 5$$

Then $a^2 = 5 = b^2$

Hence, the required equation is $\frac{y^2}{5} - \frac{x^2}{5} = 1$

i.e. $y^2 - x^2 = 5$

9. Find the equation of the ellipse with centre at the origin, major axis on the y – axis & passing through the points $(3, 2)$ & $(1, 6)$

Ans. Let the required equation be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 ......(i)$

Since $(3, 2)$ lies on (i) we have $\frac{9}{b^2} + \frac{4}{a^2} = 1 ......(ii)$

Also, since $(1, 6)$ lies on (i), we have $\frac{1}{b^2} + \frac{36}{a^2} = 1 ......(iii)$

Putting $\frac{1}{b^2}=u$ & $\frac{1}{a^2}=v$ these equations become:

$9u+4v=1......(iv)$ & $u+36v=1......(v)$

On multiplying (v) by 9 & subtracting (iv) from it we get

$$320v=8 \Leftrightarrow v=\frac{8}{320}=\frac{1}{40} \Leftrightarrow \frac{1}{a^2}=\frac{1}{40} \Leftrightarrow a^2=40$$

Putting $v=\frac{1}{40}$ in (v) we get

$$u+\left[36\times\frac{1}{40}\right]=1 \Leftrightarrow u=\left[1-\frac{9}{10}\right]=\frac{1}{10} \Leftrightarrow \frac{1}{b^2}=\frac{1}{10} \Leftrightarrow b^2=10$$

Then, $b^2=10$ & $a^2=40$

Hence the required equation is $\frac{x^2}{10}+\frac{y^2}{40}=1$

10. Prove that the standard equation of an ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Where a & b are the lengths of the semi major axis & the semi- major axis respectively & a > b.

Ans. Let the equation of the given curve be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & *let*

$P(x,y)$ be an arbitrary point on this curve

Then,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow y^2=b^2\left[1-\frac{x^2}{a^2}\right]$$

$$\Rightarrow y^2 = \frac{b^2\left[a^2 - x^2\right]}{a^2} \text{......}(i)$$

Also, let $\left(a^2 - b^2\right) = c^2 \text{......}(ii)$

Let $F_1(-c, 0)$ & $F_2(c, 0)$ be two fixed points on the x- axis, than

$$PF_1 = \sqrt{(x+c)^2 + y^2}$$

$$= \sqrt{(x+c)^2 + \frac{b^2\left(a^2 - x^2\right)}{a^2}} \text{ using } (i)$$

$$= \sqrt{(x+c)^2 + \frac{\left(a^2 - c^2\right)\left(a^2 - x2\right)}{a^2}} \text{ using } (ii)$$

$$= \sqrt{a^2 + 2cx + \frac{c^2x^2}{a^2}}$$

$$= \sqrt{\left[a + \frac{cx}{a}\right]^2} = \left[a + \frac{cx}{a}\right]$$

Similarly, $PF_2 = \left[a - \frac{cx}{a}\right]$

$$\therefore PF_1 + PF_2 = \left[a + \frac{cx}{a} + a - \frac{cx}{a}\right]$$

$$\Rightarrow PF_1 + PF_2 = 2a$$

This shows that the given curve is an ellipse

Hence the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$