15. Probability

(i) Probability = $\frac{\text{Favorable Cases}}{\text{Total Cases}}$.

(ii) $0 \leq$ Probability ≤ 1

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(iv) If events A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$

(v) Probability of an event A not happening = $P(A^c) = 1 - P(A)$

(vi) If A and B are Independent Events, then $P(A \cap B) = P(A) \times P(B)$

(vii) In case of experiments with only two outcomes possible (tossing a coin, passing or failing, hitting or not hitting a target etc.), the probability of getting r successes in n trials ($n \geq r$) is ${}^nC_r \times p^r \times q^{n-r}$, where p is the probability of success and q is the probability of failure.

(viii) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.

(ix) $P(A \cap B) = P(A) \times P(B)$, if A and B are independent events.

(x) $P(A \cap B) = 0$, if A and B are mutually exclusive events.

Mutually Exclusive Events: Let S be the sample space associated with a random experiment and let A_1 and A_2 be two events.

Then A_1 and A_2 are mutually exclusive if $A_1 \cap A_2 = \phi$.

Note–1: If A and B are two events associated with a random experiment, then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Note–2: If A and B are mutually exclusive events, then $P(A \cap B) = 0$, therefore $P(A \cup B) = P(A) + P(B)$.

Note–3: If A, B, C are three events associated with a random experiment, then

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.

Note–4: If A & B are two events associated with a random experiment, then

(i) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

(ii) (ii) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

Let A and B be two events associated with a random experiment. Then the probability of occurrence of A under the condition that B has already occurred and P(B) $\neq$ 0 is called conditional probability and it is denoted by P($\frac{A}{B}$).

Thus P($\frac{A}{B}$) = Probability of occurrence of A under the condition that B has already occurred.

P($\frac{B}{A}$) = Probability of occurrence of B under the condition that A has already occurred.

Note–1: If A & B are two events associated with a random experiment, then P(A $\cap$ B) = P(A) . P($\frac{B}{A}$) if P(A) $\neq$ 0 or P(A $\cap$ B) = P(B) . P($\frac{A}{B}$) if P(B) $\neq$ 0

Note–2: If $A_1, A_2, A_3, \_\_\_\_\_ A_n$ are n events related to a random experiment, then $P(A_1 \cap A_2 \cap A_3 \_\_\_ A_n) = P(A_1) \,.\, P(\frac{A_2}{A_1}) \,.\, P(\frac{A_3}{A_1 \cap A_2}) \_\_\_\_ P\left(\frac{A_n}{A_1 \cap A_2 \_\_ A_{n-1}}\right)$

Total Probability: Let S be the sample space and let $E_1, E_2, \_\_\_\_\_ E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or $\_\_\_\_\_\_\_\_ E_n$ then $P(A) = P(E_1) \,.\, P(\frac{A}{E_1}) + P(E_2) \,.\, P(\frac{A}{E_2}) + \_\_\_\_\_ + P(E_n) \,.\, P(\frac{A}{E_n})$.

Baye's Rule:

Let S be the sample space and let $E_1, E_2, \_\_\_\_\_ E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2, _ _ _ _ _ or E_n, then $P(\frac{E_2}{A})$ =

$$\frac{P(E_i).P\left(A/E_i\right)}{\sum_{i=1}^{n} P(E_i).P(A/E_i)}$$

1. P (A) + P (A^C) = 1.
2. P ($A^C \cap B^C$) = P [$(A \cup B)^C$] = 1 - P($A \cup B$).
3. P ($A^C \cup B^C$) = P [$(A \cap B)^C$] = 1 - P($A \cap B$).
4. If probability of an event in defined as the odds against or odds in favour then the logic to solve this is as follows. If the odd in favour of happening of an event are given to be in the ratio of p : q. Then the probability then the event will happen is $\frac{p}{p+q}$.

5. If the odds against happening of an event is given to be in the ratio of r :s, then the probability that the event will happen is $\frac{s}{r+s}$. Hence it is to be understood clearly what is being given in the question.