

1. Sets and Relations.

Exercise: 1.1

1) Describe the following sets in roster form.

i) $\{x/x \text{ is a letter of the word 'MARRIAGE'}\}$

Ans: Let $A = \{x/x \text{ is a letter of the word 'MARRIAGE'}\}$
In that word 'MARRAGE' the letter A and R are repeated.
Hence, in roster form, $A = \{M, A, R, I, G, E\}$

ii) $\{x/x \text{ is a integer, } -\frac{1}{2} < x < \frac{9}{2}\}$

Ans:

Let $B = \{x/x \text{ is an integer, } -\frac{1}{2} < x < \frac{9}{2}\}$

Integers between $-\frac{1}{2}$ and $\frac{9}{2}$ are 0, 1, 2, 3, 4.

In roster form, $B = \{0, 1, 2, 3, 4\}$

iii) $C = \{x/x = 2n, n \in \mathbb{N}\}$

Ans: Let $C = \{x/x = 2n, n \in \mathbb{N}\}$

As $n \in \mathbb{N}$, if $n = 1$ Therefore, $x = 2(1) = 2$

If $n = 2$ Therefore, $x = 2(2) = 4$ and so on.

Therefore, $n = 1, 2, 3, 4, 5 \dots$

Consequently, $x = 2, 4, 6, 8, 10 \dots$

Hence, In roster form, $C = \{2, 4, 6, 8, 10\}$

2) Describe the following sets in the set builder form

i) $\{0\}$

Ans: Let $A = \{0\}$

In Set-builder form $A = \{x/x \in \mathbb{I}, -1 < x < 1\}$

ii) $\{0, \pm 1, \pm 2, \pm 3\}$

Ans: Let $B = \{0, \pm 1, \pm 2, \pm 3\}$

In Set Builder form

$$B = \{x/x \in \mathbb{I}, -3 \leq x \leq 3\}$$

iii) $\left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\right\}$

Ans:

$$\text{Let } C = \left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\right\}$$

In set builder form

$$C = \left\{x/x = \frac{n}{n^2 + 1}, n \in \mathbb{N}, n \leq 7\right\}$$

3. If $A = \{x/6x^2 + x - 15 = 0\}$

$B = \{x/2x^2 - 5x - 3 = 0\}$

$C = \{x/2x^2 - x - 3 = 0\}$ then

Find: i) $A \cup B \cup C$ ii) $A \cap B \cap C$

Solution:

We have

$$A = \{x/6x^2 + x - 15 = 0\}$$

$$\text{Now } 6x^2 + x - 15 = 0$$

$$6x^2 - 10x - 9x - 15 = 0$$

$$2x(3x+5) - 3(3x+5) = 0$$

$$(3x+5) 2x-3 = 0$$

$$3x+5=0 \quad \text{or} \quad 2x-3=0$$

$$X = \frac{-5}{3} \quad \text{or} \quad x = \frac{3}{2}$$

$$A = \left\{ \frac{-5}{3}, \frac{3}{2} \right\}$$

$$\text{Similarly, } B = \left\{ \frac{-1}{2}, 3 \right\}$$

$$\text{and } C = \left\{ -1, \frac{3}{2} \right\}$$

$$\text{i) } A \cup B \cup C = \left\{ \frac{-5}{3}, \frac{3}{2} \right\} \cup \left\{ \frac{-1}{2}, 3 \right\} \cup \left\{ -1, \frac{3}{2} \right\}$$

$$= \left\{ \frac{-5}{3}, \frac{3}{2}, \frac{-1}{2}, 3, -1 \right\}$$

$$\text{ii) } A \cap B \cap C$$

$$= \left\{ \frac{-5}{3}, \frac{3}{2} \right\} \cap \left\{ \frac{-1}{2}, 3 \right\} \cap \left\{ -1, \frac{3}{2} \right\}$$

$$= \{ \}$$

4) If A, B, C are the sets for the letters in the word 'College', 'Marriage' and 'Luggage' respectively, then verify that
 $[A - (B \cup C)] = [(A - B) \cap (A - C)]$

Solution:

$$\text{Given: } A = \{c, o, l, e, g\},$$

$$B = \{m, a, r, i, g, e\}$$

$$C = \{l, u, g, a, e\}$$

$$B \cup C = \{m, a, r, i, g, e, l, u\}$$

$$\text{L.H.S} = A - (B \cup C)$$

$$\text{L.H.S} = \{c, o, l, e, g\} - \{m, a, r, i, g, e, l, u\}$$

$$\text{L.H.S} = \{c, o\} \quad \dots (I)$$

$$\text{Now, } A - B = \{c, o, l, e, g\} - \{m, a, r, i, g, e\}$$

$$A - B = \{c, o\}$$

$$\text{R.H.S} = (A-B) \cap (A-C)$$

$$= (c, o, l) \cap \{c, o\}$$

$$= \{c, o\} \quad \dots(\text{II})$$

From (i) and (ii), we get,

$$\text{L.H.S} = \text{R.H.S.}$$

$$\text{i.e. } [A-(B \cup C)] = [(A-B) \cap (A-C)]$$

5) If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{4, 5, 6, 7, 8\}$ and Universal Set $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ then verify the following.

$$(i) A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Solution:

$$B \cap C = \{4, 5, 6\}$$

$$\text{L.H.S} = A \cup (B \cap C)$$

$$= \{1, 2, 3, 4\} \cup \{4, 5, 6\}$$

$$= \{1, 2, 3, 4, 5, 6\} \quad \dots (I)$$

$$\text{Now, } A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$\text{R.H.S} = (A \cup B) \cap (A \cup C)$$

$$= \{1, 2, 3, 4, 5, 6\} \quad \dots (II)$$

$$\text{L.H.S} = \text{R.H.S} \quad \dots [\text{From (I) And (II)}]$$

$$A (B \cap C) = (A \cup B) \cap (A \cup C) \quad \dots (\text{Verified})$$

$$(ii) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Solution:

$$B \cup C = \{3, 4, 5, 6, 7, 8\}$$

$$\text{L.H.S} = A \cap (B \cup C)$$

$$= (1, 2, 3, 4) \cap (3, 4, 5, 6, 7, 8)$$

$$= \{3, 4\} \quad \dots (i)$$

$$A \cap B = \{3, 4\}$$

$$A \cap C = \{4\}$$

$$\text{RHS} = (A \cap B) \cup (A \cap C)$$

$$\text{RHS} = \{3, 4\}$$

$$\text{LHS} = \text{RHS} \quad \dots [\text{From (i) and (ii)}]$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \quad \dots (\text{Verified})$$

$$(iii) (A \cup B)' = A' \cap B'$$

Solution:

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$\text{L.H.S} = (A \cup B)'$$

$$= \{7, 8, 9, 10\} \quad \dots (i)$$

$$A' = \{5, 6, 7, 8, 9, 10\}$$

$$B' = \{1, 2, 7, 8, 9, 10\}$$

$$\text{R.H.S} = A' \cap B'$$

$$= \{5, 6, 7, 8, 9, 10\} \cap \{1, 2, 7, 8, 9, 10\}$$

$$= \{7, 8, 9, 10\} \quad \dots (ii)$$

$$\text{L.H.S} = \text{R.H.S} \quad \dots [\text{from (i) and (ii)}]$$

$$(A \cup B)' = A' \cap B' \quad \dots (\text{Verified})$$

$$iv) (A \cap B)' = A' \cup B'$$

Solution:

$$A \cap B = \{3, 4\}$$

$$\text{L.H.S} = (A \cap B)'$$

$$= \{1, 2, 5, 6, 7, 8, 9, 10\} \quad \dots(i)$$

$$\text{R.H.S} = A' \cup B'$$

$$= \{5, 6, 7, 8, 9, 10\} \cup \{1, 2, 7, 8, 9, 10\}$$

$$= \{1, 2, 5, 6, 7, 8, 9, 10\} \quad \dots(ii)$$

$$\text{L.H.S} = \text{R.H.S} \quad [\text{from (i) and (ii)}]$$

$$(A \cap B)' = A' \cup B'$$

$$\text{v) } A = (A \cap B) \cup (A \cap B')$$

Solution:

$$\text{L.H.S} = A = \{1, 2, 3, 4\} \quad \dots (i)$$

$$A \cap B = \{3, 4\}$$

$$A \cap B' = \{1, 2, 3, 4\} \cap \{1, 2, 7, 8, 9, 10\}$$

$$= \{1, 2\}$$

$$\text{R.H.S} = \{A \cap B\} \cup \{A \cap B'\}$$

$$= \{3, 4\} \cup \{1, 2\}$$

$$= \{1, 2, 3, 4\} \quad \dots (ii)$$

$$\text{L.H.S} = \text{R.H.S} \quad \dots [\text{From (i) and (ii)}]$$

$$A = (A \cap B) \cup (A \cap B') \quad \dots (\text{Verified})$$

$$\text{Vi) } B = (A \cap B) \cup (A' \cap B)$$

Solution:

$$\text{L.H.S} = B = \{3, 4, 5, 6\} \quad \dots (i)$$

$$A \cap B = \{3, 4\}$$

$$A' \cap B = \{5, 6, 7, 8, 9, 10\} \cap \{3, 4, 5, 6\}$$

$$A' \cap B = \{5, 6\}$$

$$\text{R.H.S} = \{A \cap B\} \cup \{A' \cap B\}$$

$$= \{3, 4, 5, 6\} \quad \dots (ii)$$

$$\text{L.H.S} = \text{R.H.S} \quad \dots [\text{From (i) and (ii)}]$$

$$B = \{A \cap B\} \cup \{A' \cap B\} \quad \dots (\text{Verified})$$

$$\text{Vii) } n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Solution:

$$A \cup B = \{1, 2, 3, 4, 5, 6\} \text{ and } A \cap B = \{3, 4\}$$

$$n(A \cup B) = 6, n(A) = 4, n(B) = 4, n(A \cap B) = 2$$

$$\text{L.H.S} = n(A \cup B)$$

$$= 6 \quad \dots (i)$$

$$\text{R.H.S} = n(A) + n(B) - n(A \cap B)$$

$$= 4 + 4 - 2$$

$$= 6 \quad \dots (ii)$$

$$\text{L.H.S} = \text{R.H.S} \quad \dots [\text{From (i) and (ii)}]$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

6. If A and B are subsets of the universal set X and $n(x) = 50$, $n(A) = 35$, $n(B) = 20$, $n(A' \cap B) = 5$.

Find: i) $n(A \cup B)$ ii) $n(A \cap B)$ iii) $n(A' \cap B)$ iv) $n(A \cap B')$

Solution: A and B are subset of the universal set X.

Where, $n(x) = 50$, $n(A) = 35$

$$n(B) = 20, \quad n(A' \cap B') = 5,$$

$$\text{i) } n(A \cup B) = n(X) - n(A \cap B')$$

$$= 50 - 5$$

$$= 45$$

ii) We know that,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$= 35 + 20 - 45$$

$$= 10$$

$$\text{iii) } n(A' \cap B) = n(A \cup B) - n(A) = n(B) - n(A \cap B)$$

$$= 45 - 35 = 20 - 10$$

$$= 10 \quad = 10$$

$$\text{iv) } n(A \cap B') = n(A \cup B) - n(B) = n(A) - n(A \cap B)$$

$$= 45 - 20 = 35 - 10$$

$$= 25 \quad = 25$$

7) Out of 200 students, 35 students failed in MHT-CET, 40 in AIEEE and 40 in IIT, 20 failed in MHT-CET and AIEEE, 17 in AIEEE and IIT, 15 in MHT-CET and IIT and 5 failed in all three examinations, find how many students,

i) Did not fail in any examination.

ii) Failed in AIEEE or IIT entrance.

Solution:

Let X = Set of students in a class

A = Set of students failed in MHT-CET

B = Set of students failed in AIEEE

C = Set of students failed in IIT

$$n(x) = 200, n(A) = 35, n(B) = 40, n(C) = 40,$$

$$n(A \cap B) = 20, n(B \cap C) = 17,$$

$$n(A \cap C) = 16, n(A \cap B \cap C) = 5,$$

(i) We have to find the number of students who did not fail in any examination , i.e. $n(A' \cap B' \cap C')$

$$n(A' \cap B' \cap C') = n(A \cup B \cup C)'$$

$$n(A' \cap B' \cap C') = n(x) - n(A \cup B \cup C) \quad \dots\dots(i)$$

$$\text{But } n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$$

$$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$= 35 + 40 + 40 - 20 - 17 - 16 + 5$$

$$= 68$$

From (I), we get

$$n(A' \cap B' \cap C') = 200 - 68 = 132$$

Number of the students who did not fail in any examination are 132.

(ii) Now, we find the number of students who failed in AIEEE or IIT entrance i.e. $n(B \cup C)$.

$$n(B \cup C) = n(B) + n(C) - n(B \cap C)$$

$$= 40 + 40 - 17$$

$$= 63$$

Therefore. 63 students were failed in AIEEE or IIT entrance.

8. From amongst 2000 literates individual of a town, 70% read Marathi newspaper, 50% read English newspaper and 32.5% read both Marathi and English newspapers.

Find the number of individual who read, at least one of the newspaper

i) at least one of the newspaper.

ii) neither Marathi nor English newspaper.

iii) only one of the newspaper.

Solution:

Let, X = Set of literate individuals of a town.

M = Set of individual who read Marathi newspapers.

E = Set of individual who read English newspaper.

$$n(X) = 2000$$

$$n(M) = 70\% = \frac{70}{100} \times 2000 = 1400$$

$$n(E) = 50\% = 1000$$

$$n(M \cap E) = 32.5\% = 650$$

(i) We have to find the number of individual who read at least one of the newspaper, i.e. $n(M \cup E)$.

$$n(M \cup E) = n(M) + n(E) - n(M \cap E)$$

$$= 1400 + 1000 - 650$$

$$= 1750$$

1750 individual who read at least one of the newspaper.

(ii) we have to find the number of individual who neither read Marathi nor English newspaper i.e. $(M \cap E)'$

$$n(M \cap E)' = n(M \cup E)'$$

$$= n(X) - n(M \cup E)$$

$$= 2000 - 1750$$

$$= 250$$

250 individual who neither read Marathi nor English newspapers.

(iii) we have to find the number of individual who read only one of the newspapers (i.e. only Marathi or only English), i.e. $n(M \cap E) + n(M \cap E)'$

$$n(M' \cap E) + n(M \cap E')$$

$$= n(E) - n(M \cap E) + n(E) - n(M \cap E)$$

$$= n(E) + n(M) - 2n(M \cap E)$$

$$= 1000 + 1400 - 2(650)$$

$$= 1100$$

1100 individual who read only one of the newspaper.

9. In a hostel, 25 students take tea, 20 students take coffee, 15 students take milk, 10 students take both tea and coffee, 8 students take both milk and coffee. None of them take tea and milk both and everyone take at least one beverage, find the total number of students in the hostel.

Solution:

Let X = Set of student in a hostel.

T = Set of student who take tea.

C = Set of students who take coffee.

M = Set of students who take milk.

Since every students take at least one beverage

$$X = T \cup C \cup M$$

$$n(T) = 25, \quad n(C) = 20, \quad n(M) = 15,$$

$$n(T \cap C) = 10, \quad n(M \cap C) = 8$$

Since none of the students take tea and milk both

$$n(T \cap M) = 0$$

$$n(T \cap C \cap M) = 0$$

We have to find the number of students in hostel,

$$\text{I.e. } n(X) = n(T \cap C \cap M)$$

$$\begin{aligned}
&= n(T) + n(C) + n(M) - n(T \cap C) - n(C \cap M) - n(T \cap M) + n(T \cap C \cap M) \\
&= 25 + 20 + 15 - 10 - 8 - 0 + 0 \\
&= 42
\end{aligned}$$

There are 42 students in a hostel.

10. There are 260 persons with a skin disorder. If 150 had been exposed to a chemical A, 74 to the chemical B and 36 to both chemical A and B, find the number of person who exposed to,

- i) Chemical A but not chemical B.
- ii) Chemical B but not chemical A.
- iii) Chemical A or Chemical B.

Solution:

Let X = Set of persons with skin disorder.

A = Set of persons who had been exposed to chemical A

B = Set of persons who had been exposed to chemical B

$$n(X)=260, n(A)= 150, n(B)=74, \text{ and } n(A \cap B)= 36$$

(i) We have to find the number of persons exposed to chemical A but not chemical B
i.e. $n(A \cap B')$

$$n(A \cap B') = n(A) - n(A \cap B) = 150 - 36 = 114$$

(ii) We have to find the number of persons exposed to chemical B but not chemical A i.e. $n(A' \cap B)$

$$n(A' \cap B) = n(B) - n(A \cap B) = 74 - 36 = 38$$

(iii) We have to find the number of persons exposed to Chemical A or chemical B
i.e. $n(A \cup B)$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 150 + 74 - 36$$

$$= 188$$

11. If $A = \{1, 2, 3\}$ Write the set of all possible subsets of A

Solution:

$$\text{Given } A = \{1, 2, 3\}$$

Subset of A are

$$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$$

12. Write the following intervals on set builder form:

(i) $(-3,0)$ (ii) $[6,12]$ (iii) $(6,12)$ (iv) $(-23,5)$

Solution:

The given intervals in set – builder form:

$$(i) (-3,0) = \{x/x \in R, -3 < x < 0\}$$

$$(ii) [6, 12] = \{x/x \in R, 6 \leq x \leq 12\}$$

$$(iii) (6, 12) = \{x/x \in R, 6 < x < 12\}$$

$$(iv) (-23,5) = \{x/x \in R, -23 < x < 5\}$$

Exercise 1.2

1. If $(x-1, y+4) = (1, 2)$ Find the value of x and y.

Solution:

$$\text{Given : } (x-1, y+4) = (1, 2)$$

By equality of an ordinary pairs, we get

$$\begin{aligned} x-1 &= 1, & y+4 &= 2, \\ x &= 2, & y &= -2 \end{aligned}$$

$$2. \text{ If } \left(x + \frac{1}{3}, \frac{y}{3} - 1\right)$$

$$= \left(\frac{1}{3}, \frac{3}{2}\right),$$

Find x and y.

Solution.

$$\text{Given: } \left(x + \frac{1}{3}, \frac{y}{3} - 1\right)$$

$$= \left(\frac{1}{3}, \frac{3}{2}\right)$$

$$x + \frac{1}{3} = \frac{1}{3}$$

$$\text{and } \frac{y}{3} - 1 = \frac{3}{2}$$

$$x = \frac{1}{3} - \frac{1}{3}$$

$$= \frac{3}{2} + 1$$

$$x = 0 \quad \frac{y}{3} = \frac{5}{2}$$

$$x = 0 \quad y = \frac{15}{2}$$

3. If $A = \{a, b, c\}$, $B = \{x, y\}$, Find $A \times B$, $B \times A$, $A \times A$, $B \times B$.

Solution:

We have $A = \{a, b, c\}$ $B = \{x, y\}$

$$A \times B = \{(a,x), (a,y), (b,x), (b,y), (c,x), (c,y)\}$$

$$B \times A = \{(x,a), (y,a), (x,b), (y,b), (x,c), (y,c)\}$$

$$A \times A = \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c)$$

$$= (c,a), (c,b), (c,c)\}$$

$$B \times B = \{(x,x), (x,y), (y,x), (y,y)\}$$

4. If $P = \{1, 2, 3\}$ and $Q = \{6, 4\}$, find the sets $P \times Q$ and $Q \times P$.

Solution.

We have $P = \{1, 2, 3\}$ $Q = \{6, 4\}$

$$P \times Q = \{(1,6), (2,6), (3,6), (1,4), (2,4), (3,4)\}$$

$$\text{And } Q \times P = \{(6,1), (6,2), (6,3), (4,1), (4,2), (4,3)\}$$

5. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$, $C = \{5, 6\}$.

Find: i) $A \times (B \cap C)$

ii) $(A \times B) \cap (A \times C)$

iii) $A \times (B \cup C)$

iv) $(A \times B) \cup (A \times C)$

Solution.

We have,

$$A = \{1, 2, 3, 4\}, B = \{4, 5, 6\}, C = \{5, 6\}$$

$$\text{i) } B \cap C = \{5, 6\}$$

$$A \times (B \cap C) = \{1, 2, 3, 4\} \times \{5, 6\}$$

$$A \times (B \cap C) = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)\}$$

$$\text{ii) } A \times B = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$$

$$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)\}$$

$$(A \times B) \cap (A \times C) = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)\}$$

$$\text{iii) } B \cup C = \{4, 5, 6\}$$

$$A \times (B \cup C) = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$$

$$\text{iv) } A \times B = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$$

$$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)\}$$

$$(A \times B) \cup (A \times C) = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$$

6. Express $\{(x, y) / x^2 + y^2 = 100\}$,

where $x, y \in W$
as a set of ordered pairs.

Solution:

Let $A = \{(x, y) / x^2 + y^2 = 100$
, where $x, y \in W\}$

$$x^2 + y^2 = 100, x, y \in W$$

If $x = 0$

$$y^2 = 100 = y = \pm 10$$

But $y \in W$ $y = 10$

$$(x, y) = (0, 10)$$

If $x = 6$

$$y^2 = 64 = y = \pm 8$$

But $y \in W$ $y = 8$

$$(x, y) = (6, 8)$$

If $y = 0$

$$x^2 = 100 = y = \pm 10$$

But $x \in W$ $x^2 = 10$

$$(x, y) = (10, 0)$$

If $y = 6$

$$x^2 = 64 = y = \pm 8$$

But $x \in W$ $x = 8$

$$(x, y) = (8, 6)$$

Hence, the set of ordered pairs is

$$A = \{(0,10),(6,8),(10,0),(8,6)\}$$

7. Write the domain and range of the following relation:

i) $\{(a,b)/a \in \mathbb{N}, a < 6 \text{ and } b = 4\}$

Solution:

This relation is expressed as a set of ordered pairs is

$$\{(1,4),(2,4),(3,4),(4,4),(5,4)\}$$

$$\text{Domain} = \{1, 2, 3, 4, 5\} \text{ and range} = \{4\}$$

ii) $\{(a,b)/a,b \in \mathbb{N}, a + b = 12\}$

Solution:

Here, the relation is $a+b = 12$ and $a, b \in \mathbb{N}$.

If $a = 1$ then $b = 11$,

If $a = 2$ then $b = 10$,

And If $a = 11$ then $b = 1$.

(Zero and negative values are not allowed).

The relation expressed as set of ordered pairs is

$$\{(1,11),(2,10),(3,9),(4,8),(5,7),(6,6),(7,5),(8,4),(9,3),(10,2),(11,1)\}$$

$$\text{Domain} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$$

$$\text{Range} = \{11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1\}$$

iii) $\{(2,4),(2,5),(2,6),(2,7)\}$

Solution:

$$\text{Domain} = \{2\} \text{ and range} = \{4, 5, 6, 7\}$$

8. Let $A = \{6, 8\}$ and $B = \{1, 3, 5\}$

Show that: $R = \{(a,b)/a \in A, b \in B, a - b \text{ is an even number}\}$.

Show that R is an empty relation from A to B .

Solution:

Given: $A = \{6, 8\}$ and $B = \{1, 3, 5\}$

$R = \{(a, b) / a \in A, b \in B, a - b \text{ is an even number}\}$

We have,

$$6 - 1 = 5,$$

$$6 - 3 = 3, \quad 6 - 5 = 1$$

$$8 - 1 = 7,$$

$$8 - 3 = 5,$$

$$8 - 5 = 3$$

None of these is an even number

$$R = \{\}$$

Hence, R is an empty relation from A to B .

9. Write the relation in the roster form. State its domain and range.

i) $R_1 = \{(a, a^2)$

/ a is prime number less than 15}

Solution.

$$R_1 = \{(a, a^2)$$

/ a is prime number less than 15}

$$a = 2, 3, 5, 7, 11, 13$$

$$a^2 = 4, 9, 25, 49, 121, 169$$

Relation R_1 in roster form: $R_1 = \{(2, 4), (3, 9), (5, 25),$

$(7, 49), (11, 121), (13, 169)\}$

Domain of $R_1 = \{2, 3, 5, 7, 11, 13\}$

Range of R_1

$$= \{4, 9, 25, 49, 121, 169\}$$

$$\text{ii) } R_1 = \{(a, \frac{1}{a}) / 0 <$$

$$\rightarrow a \leq 5, a \in N\}$$

Solution:

$$\text{Given : } R_2 = \{(a, \frac{1}{a}) / 0$$

$$\rightarrow < a \leq 5, a \in N\}$$

Since, $a \in N$ and $0 < a \leq 5$

$$a = 1, 2, 3, 4, 5$$

$$\frac{1}{a} = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$$

Relation R_2 in roster form

$$R_2 = \{(1, 1), (2, \frac{1}{2}), (3, \frac{1}{3})$$

$$\rightarrow, (4, \frac{1}{4}), (5, \frac{1}{5})\}$$

$$\text{Domain of } R_2 = \{1, 2, 3, 4, 5\}$$

$$\text{Range of } R_2 = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\}$$

$$10. R = (a, b) / b = a + 1, a \in Z, 0 < a < 5\}$$

Find the range of R.

Solution:

Here, $0 < a < 5, a \in Z$

$$a = 1, 2, 3, 4 \text{ and } b = a + 1$$

$$R = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$$

$$\text{Range of } R = \{2, 3, 4, 5\}$$

11. Find the following relation as sets of ordered pairs:

i) $\{(x, y) / y = 3x,$

$$x \in \{1, 2, 3\},$$

$$y \in \{3, 6, 9, 12\}$$

Given:

$$R = \{(x, y) / y = 3x,$$

$$x \in \{1, 2, 3\},$$

$$y \in \{3, 6, 9, 12\}$$

$$\text{Since } y = 3x$$

$$\text{If } x = 1 \quad y = 3(1) = 3 \quad (x, y) = (1, 3)$$

$$\text{If } x = 2 \quad y = 3(2) = 6 \quad (x, y) = (2, 6)$$

$$\text{If } x = 3 \quad y = 3(3) = 9 \quad (x, y) = (3, 9)$$

Relation R in roster form

$$R = \{(1, 3), (2, 6), (3, 9)\}$$

ii) $\{(x, y) / y > x + 1, x \in \{1, 2\}$

$$\text{and } y \in \{2, 4, 6\}$$

Solution:

Given.

$$R = \{(x, y) / y > x + 1, x \in \{1, 2\}$$

$$\text{and } y \in \{2, 4, 6\}$$

$$\text{Since } y > x + 1$$

$$\text{If } x = 1 \quad y = 4 \text{ and } 6$$

$$(x,y) = (1,4) \text{ and } (1,6)$$

$$\text{If } x=1 \quad y=4 \text{ and } 6$$

$$(x,y) = (2, 4) \text{ and } (2,6)$$

Relation R in roster form

$$R = \{(1,4),(1,6),(2,4),(2,6)\}$$

$$\text{iii) } R = \{(x,y) / x + y = 3, x,y \in \{0, 1, 2, 3\}\}$$

Solution:

$$\text{Given. } R = \{(x,y) / x + y = 3, x,y \in \{0, 1, 2, 3\}\}$$

$$\text{Since } x + y = 3$$

$$\text{If } x = 0 \quad 0 + y = 3 \quad y = 3 \quad (x,y) = (0,3)$$

$$\text{If } x = 1 \quad 1 + y = 3 \quad y = 2 \quad (x,y) = (1,2)$$

$$\text{If } x = 2 \quad 2 + y = 3 \quad y = 1 \quad (x,y) = (2,1)$$

$$\text{If } x = 3 \quad 3 + y = 3 \quad y = 0 \quad (x,y) = (3,0)$$

Relation R in Roster form

$$R = \{(0,3),(1,2),(2,1),(3,0)\}$$